Lab 11

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Spring 2022 DA 350

April 15, 2022

Q1. Let's start simple - one way to prevent serving such mass quantities of similar products is to restrict how much of each product we are allowed to buy. How would you write such a constraint? **Show your new optimization model** (Baseline plus these constraints) and solve in your spreadsheet with Solver. You can do this by writing the maximum purchasable amount in one cell, and adding new constraints restricting each variable to be less than than this value (one constraint per variable). Picking a reasonable restriction is at your discretion. **Report the number of customers served, the food each customer is served per day, and the price per customer per day.**

: Our new optimization model is the following.

Food	Sign	Amount (g)	Food	Sign	Amount(g)
Peanut Butter	≤	50	Pork	≤	300
Chocolate	≤	60	Mushroom	≤	100
Apple	≤	1360	Turkey	≤	150
Cheese	≤	120	Greek Yogurt	≤	450
Black Bean	≤	200	Broccoli	≤	150
Potato	≤	600	Sweet Potato	≤	200
Egg	≤	450	Bread	≤	180
Brown Rice	≤	600	Tomato	≤	300
Carrot	≤	75	Corn	≤	100

Info	peanut butter	chocolate	apple	cheese	black bean(1cup-172g)	potato(size1-173g)	egg(size1-50g)	brown rice(0.5cup-98g)	carrot(1-46g)
Price(g)	0.0028	0.0135	0.002	0.0146	0.0023	0.0013	0.0022	0.0016	0.0022
cal(g)	5.875	5.2857	0.522	4.2857	1.3198	0.9306	1.44	1.1122	0.3478
protein(g)	0.2188	0.0714	0.0027	0.2381	0.0872	0.0249	0.126	0.0235	0.0065
carbs(g)	0.2438	0.6	0.044	0.0476	0.2384	0.2139	0.008	0.2347	0.0826
fibers(g)	0.0563	0.0286	0.0242	0	0.0872	0.022	0	0.0184	0.0304
fat(g)	0.5	0.3	0.0016	0.3333	0.0052	0.0012	0.096	0.0082	0.0022
Purchased	50	0	0	0	162.651394	0	450	327.9238287	0
Objective	2.498926404								

Info	corn(1-103g)	pork(1oz-28.3g)	mushroom(1-12g)	turkey(1oz-28.3g)	greek yogurt(1oz-	brocoli(1floret-10	sweet potato (1-1	bread(1slice-29g	tomato(1-123g)	Amount
Price(g)	0.0017	0.0055	0.0079	0.0088	0.0044	0.0042	0.0022	0.0026	0.0044	2.4989264
cal(g)	0.9612	2.3781	0.2833	1.8905	0.5873	0.35	0.9035	2.6552	0.1789	1999.9998
protein(g)	0.034	0.258	0.025	0.2827	0.1005	0.02	0.0202	0.0897	0.0089	105.77627
carbs(g)	0.2136	0	0.05	0	0.0353	0.07	0.2105	0.4828	0.039	218.43381
fibers(g)	0.0243	0	0.025	0	0	0.03	0.0333	0.0276	0.0122	28
fat(g)	0.0146	0.1413	0.0083	0.0742	0.0035	0	0.0018	0.0345	0.0024	78
Purchased	0	0.3909223	0	0	0	0	0	180	0	

Also, the number of customers we can serve per day is **114 customers** (285.7143/2.4989=114.336028). Per day, **each customer is served with 50g of peanut butter, 162.6514g of black bean, 450g of egg, 372.9238g of brown rice, 0.3909g of pork, and 180g of bread**. The price per customer per day is **$2.4989**.

Q2. Let's improve the model from 1) and consider more constraints. We've constrained having too much of one food, but the diet may not still be nutritionally balanced. While the Baseline model only considered the minimum number of nutrients required, clearly only serving bread and potatoes every day would drastically exceed an acceptable carbs amount. Or, the pound of flour might meet macronutrient minimum requirements, but might give far too many calories for a healthy diet. So, let's consider more constraints on the macronutrients. **Create additional constraints to limit the maximum number of each macronutrient to 125% of the minimum required value.** For example, if an individual needs 56g of protein in a day, the maximum they can receive is 56*1.25 = 70g. **You will need to add one additional constraint per macronutrient. Report the number of customers served, the food each customer is served per day, and the price per customer per day. (**Note that if our solution says to serve many foods on a given day, it's perfectly acceptable to space these out over a week or so - practically speaking, it's fine if we get slightly less of a nutrient on a given day and more on the next, as long as the macronutrients are reaching the required levels on average.)

	min	max
calories: 2000	2000	2500
protein: 56(g)	56	70
carbs: 130(g)	130	162.5
fiber: 28(g)	28	35
fat: 78~89(g)(mediterian)	78	89

Info	peanut butter	chocolate	apple	cheese	black bean(1cup-172g)	potato(size1-173g)	egg(size1-50g)	brown rice(0.5cup-98g)	carrot(1-46g)
Price(g)	0.0028	0.0135	0.002	0.0146	0.0023	0.0013	0.0022	0.0016	0.0022
cal(g)	5.875	5.2857	0.522	4.2857	1.3198	0.9306	1.44	1.1122	0.3478
protein(g)	0.2188	0.0714	0.0027	0.2381	0.0872	0.0249	0.126	0.0235	0.0065
carbs(g)	0.2438	0.6	0.044	0.0476	0.2384	0.2139	0.008	0.2347	0.0826
fibers(g)	0.0563	0.0286	0.0242	0	0.0872	0.022	0	0.0184	0.0304
fat(g)	0.5	0.3	0.0016	0.3333	0.0052	0.0012	0.096	0.0082	0.0022
Purchased	50	60	1110.764176	3.589043212	0	0	311.585167	0	0
Objective	4.247463485								

Info	corn(1-103g)	pork(1oz-28.3g)	mushroom(1-12g)	turkey(1oz-28.3g)	greek yogurt(1oz	brocoli(1floret-1(	sweet potato (1-1	bread(1slice-29g	tomato(1-123g)	Amount
Price(g)	0.0017	0.0055	0.0079	0.0088	0.0044	0.0042	0.0022	0.0026	0.0044	4.2474635
cal(g)	0.9612	2.3781	0.2833	1.8905	0.5873	0.35	0.9035	2.6552	0.1789	1999.9999
protein(g)	0.034	0.258	0.025	0.2827	0.1005	0.02	0.0202	0.0897	0.0089	69.999992
carbs(g)	0.2136	0	0.05	0	0.0353	0.07	0.2105	0.4828	0.039	162.50001
fibers(g)	0.0243	0	0.025	0	0	0.03	0.0333	0.0276	0.0122	35
fat(g)	0.0146	0.1413	0.0083	0.0742	0.0035	0	0.0018	0.0345	0.0024	80.37126
Purchased	1.16209E-14	0	0	0	0	0	0	130.01836	0	
Objective										

: The number of customers we can serve per day is **67 customers** (285.7143/4.2475=67.2665). Per day, each customer is served with **50g of peanut butter , 60g of chocolate, 1110.7642g of apple, 3.5890g of cheese, 311.5852g of egg, and 130.0184g of bread**. Since corn has a value close to 0, we are not considering buying corn. The price per customer per day is **$4.2475**.

Q3. If we let $X_i \geq My_i$, **why does this accomplish the goal?** Add these two sets of constraints to your model from 2) and solve. **Report the number of customers served, the food each customer is served over the course of a week, and the price per customer per week.**

Info	chicken()	black bean(1cu	potato(size1-17	egg(size1-50g)	brown rice(0.5c	carrot(1-46g)	corn(1-103g)	pork(1oz-28.3g)	mushroom(1-12g)	turkey(1oz-28.3
Price(g)	0.0984	0.0023	0.0013	0.0022	0.0016	0.0022	0.0017	0.0055	0.0079	0.0088
cal(g)	2.2014	1.3198	0.9306	1.44	1.1122	0.3478	0.9612	2.3781	0.2833	1.8905
protein(g)	0.2367	0.0872	0.0249	0.126	0.0235	0.0065	0.034	0.258	0.025	0.2827
carbs(g)	0	0.2384	0.2139	0.008	0.2347	0.0826	0.2136	0	0.05	0
fibers(g)	0	0.0872	0.022	0	0.0184	0.0304	0.0243	0	0.025	0
fat(g)	0.1307	0.0052	0.0012	0.096	0.0082	0.0022	0.0146	0.1413	0.0083	0.0742
maxAmount(g)	100	200	600	450	600	75	100	300	100	150
Purchased	0	0	0	253.7972722	100	50	30	0	5.32907E-15	0
y	1	0	0	1	1	1	1	0	0	0
minimum	0	30	150	250	100	50	30	20	40	20
MY	0	0	0	250	100	50	30	0	0	0
Objective	4.840565658									

Info	greek yogurt(1c	brocoli(1floret-1	sweet potato (1	bread(1slice-29	tomato(1-123g)	Chedder Chees	Chocolate	apple	Peanut butter	Amount	Recommended	Maximum	
Price(g)	0.0044	0.0042	0.0022	0.0026	0.0044	0.0146	0.0135	0.002	0.0028	4.840565658			
cal(g)	0.5873	0.35	0.9035	2.6552	0.1789	4.2857	5.2857	0.522	5.875	2000.000001	2000	2500	cal(g)
protein(g)	0.1005	0.02	0.0202	0.0897	0.0089	0.2381	0.0714	0.0027	0.2188	70.0000002	56	70	protein(g)
carbs(g)	0.0353	0.07	0.2105	0.4828	0.039	0.0476	0.6	0.044	0.2438	162.5000003	130	162.5	carbs(g)
fibers(g)	0	0.03	0.0333	0.0276	0.0122	0	0.0286	0.0242	0.0563	35.00000009	28	35	fibers(g)
fat(g)	0.0035	0	0.0018	0.0345	0.0024	0.3333	0.3	0.0016	0.5	87.42713919	78	89	fat(g)
maxAmount(g)	450	150	200	180	300	120	60	1360	50				
Purchased	0	0	50	45.69929327	50	46.19756434	60	943.9545287	50				
y	0	0	1	0	1	1	1	1	1	11			
minimum	100	50	50	100	50	3	50	800	40				
MY	0	0	50	0	50	3	50	800	40				
Objective													

: Letting $X_i \geq My_i$ accomplishes the goal because the solution guarantees the purchase of a minimum amount of an ingredient if its "y" value is 1. For instance, in lab 10, we purchased only eggs, bread, and black beans. However, because an ingredient will at least be purchased by its minimum amount, it will thus diversify the meal (we also constrain the minimum sum of y, such that a minimum number of ingredients are served).

The number of customers we can serve per week is 413 customers (285.7143/4.8406*7). Per week, each customer is served 1776.5811g of egg (253.7973*7), 700g of brown rice (100*7), 350g of carrot (50*7), 210g of corn (30*7), 350g of sweet potato (50*7), 319.89503g of bread (45.69929*7), 350g of tomato (50*7), 323.3832g of cheese (46.1976*7), 420g of chocolate (60*7), 6607.6815g of apple (943.9545*7), and 350g of peanut butter (50*7). The price per customer per week is $33.8842 (4.8406*7). Here, we are assuming that a customer who comes every day (Monday to Sunday) is counted as an individual customer, so if customer A comes every day then we are counting as 7 customers in a week rather than one customer.

Q4. Consider your answers to 1), 2), and 3). By this point you should have some fairly realistic, practical solutions. Write a paragraph to the soup kitchen explaining your proposed solution and arguing why it meets their criteria.

:We are focusing on two primary goals. First, we wanted to find a solution to serve more customers within our budget, and second, to provide all nutrients needed for a healthy lifestyle. By using our first model which adds the maximum amount of each ingredient we can buy as a constraint, we found a solution that we can serve the most customers per day (114 customers) with the price per customer per day of $2.4989. Here, we have to buy six ingredients to make three meals per day: peanut butter (50g), black bean

(162.6514g), egg (450g), brown rice (372.9238), pork (0.3909g), and bread (180g). However, using these six ingredients faced the problem of having too much protein and carbs (105g and 218g which is over the recommended macronutrient). Thus, we used the second model by adding a constraint of the maximum of each macronutrient. With this model, the price per customer per day increased to $4.2475 meaning we serve fewer customers per day. If we decide to use this solution, to meet recommended macronutrients, we have to buy six ingredients to make three meals per day: peanut butter (50g), chocolate (60g), apple (1110.7642g), cheese (3.5890g), egg (311.5852g) and bread (130.0184g). Both the first and second models faced the same problem of having too limited ingredients to make a meal. Thus, our final model found a solution to have five more ingredients compared to previous solutions. Here, the price per customer per day increased to $4.8406 but we can use 11 ingredients to make three meals per day. Using this solution we have to buy egg (253.7973g), brown rice (100g), carrot (50g), corn (30g), sweet potato (50g), bread (45.69929g), tomato (50g), cheese (46.1976g), chocolate (60g), apple (943.9545g), and peanut butter (50g). Comparing three models, in order to meet two criteria of the soup kitchen, we need to use the third model with 11 ingredients.

Q5. Finally, a modeling question that does not need to be implemented or solved - assume the kitchen is still not perfectly happy, as they have more considerations. Meat cannot be served on Fridays because of religious traditions, potatoes must be served on Wednesdays, and we don't want to serve both fish and fruit on the same day. Show how you could write an optimization problem with linear and integer variables to accommodate these restrictions. Hint : you will have many more variables than in any of the models you solved above.

:On top of our constraints and variables from the optimization above, we will introduce additional integer programming formulation(binary) for each day of the week:

- $\mathbf{W_{mon}}$, $\mathbf{W_{tue}}$, $\mathbf{W_{wed}}$, $\mathbf{W_{thu}}$, $\mathbf{W_{fri}}$, $\mathbf{W_{sat}}$, $\mathbf{W_{sun}}$

The variable, W, will be 1 if purchased on that day of the week, and 0, otherwise.

We also introduce the variable, $\mathbf{P_{DayOfTheWeek}}$, which indicates how much of each ingredient is purchased on that day of the week:

- $\mathbf{P_{mon}}$, $\mathbf{P_{tue}}$, $\mathbf{P_{wed}}$, $\mathbf{P_{thu}}$, $\mathbf{P_{fri}}$, $\mathbf{P_{sat}}$, $\mathbf{P_{sun}}$

We then introduce another variable, $\mathbf{W_{DayOfTheWeek}P_{DayOfTheWeek}}$, which indicates how much of each ingredient is purchased on that day of the week if the $\mathbf{W_{DayOfTheWeek}}$ is **1**, and **0** otherwise:

- $\mathbf{W_{mon}P_{mon}}$ **,** $\mathbf{W_{tue}P_{tue}}$ **,** $\mathbf{W_{wed}P_{wed}}$ **,** $\mathbf{W_{thu}P_{thu}}$ **,** $\mathbf{W_{fri}P_{fri}}$ **,** $\mathbf{W_{sat}P_{sat}}$ **,** $\mathbf{W_{sun}P_{sun}}$

We modify the decision variable, "purchased", which is the amount purchased of each variable:

- **"Purchased" =** $\mathbf{W_{mon}P_{mon} + W_{tue}P_{tue} + W_{wed}P_{wed} + W_{thu}P_{thu} + W_{fri}P_{fri} + W_{sat}P_{sat} + W_{sun}P_{sun}}$

In other words, we are summing the purchased amount of each day of the week to derive the total purchased amount.

We also introduce additional constraints to satisfy the clients' requests:

- Potatoes must be served on Wednesdays:
  - For potatoes, its $\mathbf{W_{Wed}P_{Wed} > 0}$
- Can't serve fish & fruit on the same day:
  - For each W of fish and fruit, **(fish's W + fruit's W) < 2**

*Note) If the sum of W for fish and fruit is equal to 2, it means they are served together on that day, thus we set it less than 2

- Meat cannot be served on Fridays:
  - For each meat ingredient, the **WP = 0**

Other than this, we may have to introduce further constraints such as price and macronutrient amounts for each day of the week, etc.

As we can see above, we have introduced numerous additional linear programming formulations, constraints, and changes in our model just to implement 3 more requests from our clients, which can be very computationally expensive.