Department of Electrical, Electronic and Computer Engineering

(EECE)

Fault Analysis of Permanent Magnet Machines Using Finite Element

Literature review

Written by

AL-Nasir, Zuher Ali
ID# 069171376
Electrical Power (Msc)

Under Supervision of

Prof. Barrie Mecrow
Professor of Electrical Power

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ABSTRACT

The aim of this review is to examine some recent and relevant literature for analysis of Permanent Magnet (PM) machines under different types of faults. Some peripheral literature is included.

This goal will be achieved by starting with overview some of the previous research papers written and experimental works done on PM machines operating under different types of faults. Effects of faults on the PM machines characteristics will be mainly reviewed and discussed. Some techniques developed to detect such faults will also be considered. Moreover, some control strategies, their benefits and their effectiveness of controlling the PM machines under faults will be briefly described.

The main idea of these researches is to analyze very reliable and effective PM machines under different types of failures. To perform such analysis, a deep study of how PM machines operate with failures must be performed. There are some modeling techniques used to help in this issue. A Finite element modeling method has been suggested to be one of the best techniques used in modeling PM machines.

Further work will be performed in this project in order to study and demonstrate the properties of the finite element model and how helpful it is in modeling PM machines under faults.

Literature Review

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1. Permanent Magnet AC Machines

Electrical machines consist mainly of two parts: stationary part called a stator and rotating part called a rotor. Depending on type of power fed and produced by the machines, they are classified into direct current machines (DC machines) and alternating current machines (AC machines). AC machines can be categorized into two main types: synchronous machines and asynchronous machines (induction machines). The former ones rotate normally at synchronous speed while the later ones rotate at speed slightly different from the synchronous speed. A traditional AC machine has windings in the stator and windings in the rotor. However, the windings of the rotor can be replaced by permanent magnets introducing a new type of AC machines called permanent magnet (PM) AC machines. In general, PM AC machines can be analyzed as conventional AC machines with an assumption of a constant field current excitation. One type of PM machines widely used is the permanent magnet synchronous machine. It can be referred as (PMSM) for simplicity. PMSM is much similar to synchronous machine except that the rotor is excited by permanent magnets instead of the normal field windings. Due to its simple construction and control and nonexistence of reluctance torque, the most popular arrangement of magnets is to mount them on the rotor surface. However, there are other different methods of design. Fig (1) shows two typical PMSM rotor configurations.

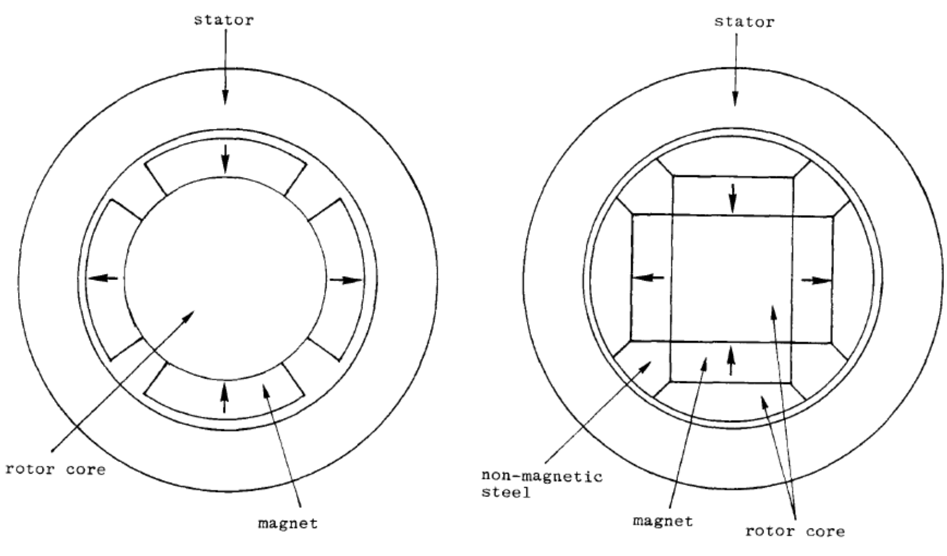


Fig (1): PM motors with: (a) surface –mounted magnet; (b) buried magnet. (Taken From [1])

Literature Review

Choosing magnet materials is a matter of design depending on their magnetic efficiency, cost and applications. Examples of some magnets materials commonly used are ferrite, neodymium iron boron (NdFeB) and samarium cobalt (SmCo) magnets. For PMSM, slip rings are no longer required and the electrical rotor dynamics complicating control is eliminated by using permanent magnets. It is true that such achievement will increase the capital cost but it will guarantee minimum long term maintenance cost due to higher efficiency of PMSM. In the other hand, one of the worst points in using permanent magnets is the interaction of the magnets and the stator teeth. This interaction leads to parasitic tendency of the rotor to align at discrete positions causing what is called cogging torque. This phenomenon, indeed, gets worse at low speed. Therefore, appropriate design of the machine or electronic circuit is needed to overcome this phenomenon.

Due to their high efficiency, low torque ripples and high power density, PMSMs become widely used in many low power applications such as machine tools, actuators and robotics. They are also used in aerospace applications where high power is needed. Therefore, it is very important to have a reliable PMSM that can offer good operation under normal and fault situations. This review will concentrate on the analysis of low power PM machines under fault conditions. Main types of faults, their effects on the machine performance, new techniques for detecting them and fault tolerant PM machines have been of interest for many papers and works published through the previous few years. This review will concentrate on what have been discussed and achieved yet in this area.

2. Main modes of Failure

According to previous works published by *Mecrow, B.C. et al.*, Faults may either occur inside the PM machine itself or within the power converter circuit (controller) [2].

A- Faults that can occur within the machine are
   - (i) Winding short circuit(phase to ground or within a phase)
   - (ii) Winding open circuit
   - (iii) Winding short circuit at the terminals

B- Faults that can occur within the power converter are
   - (i) Power device short circuit
   - (ii) Power device open circuit
   - (iii) DC supply failure

Furthermore, another paper published by *Neely, J.D. et al.,* Classified open and short circuit faults as primary faults introducing another class of faults called secondary faults that might occur as results of primary once. These secondary faults include permanent magnet and shaft bearing faults [3].

2.1. Short Circuit Faults (within winding or power electronic circuit)

The flux, voltage, current and torque are the main values which need to be considered and analyzed under fault conditions. A previous study done on a 100HP, 6-phase PM machine at 60 Hz, illustrated that although the damper cage can minimize the flux demagnetization stress on the magnets under single phase short circuit transients, demagnetization cannot be avoided [4]. This can be seen in fig (2). Obviously, Flux density curve with damper cage (B) is less demagnetized than the flux density curve without damper cage (A).

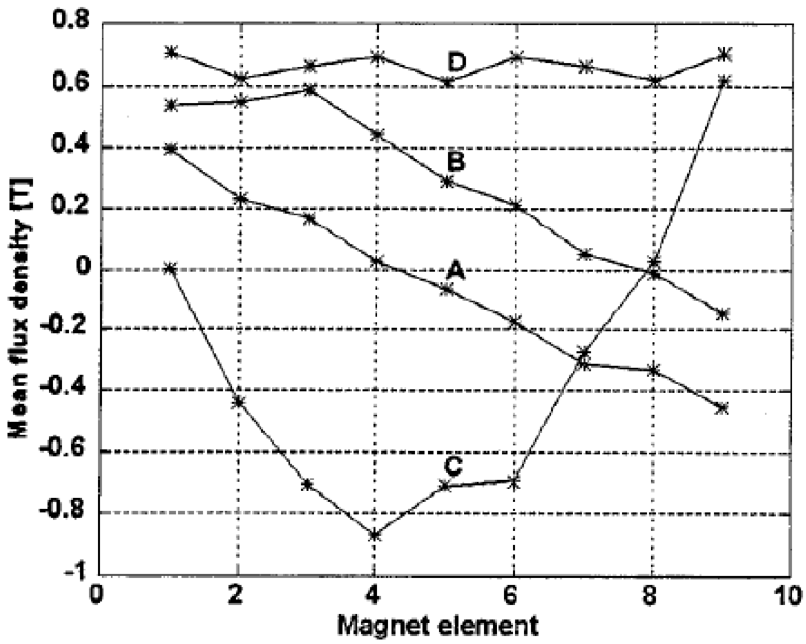


Fig (2): Mean flux density in the magnetic elements along the pole. (Taken from [4]).

(A) Single phase short circuit without damper cage
(B) Single phase short circuit with damper cage
(C) Six-phase short circuit with damper cage
(D) No loaded machine for reference

If a short circuit occurs, the voltage drops to nil while the current goes rapidly up due to the back EMF. Fig (3) shows a simulation of three phase current and voltage during short circuit fault. In this illustration, a single phase short fault occurred at t=16.67ms [3].

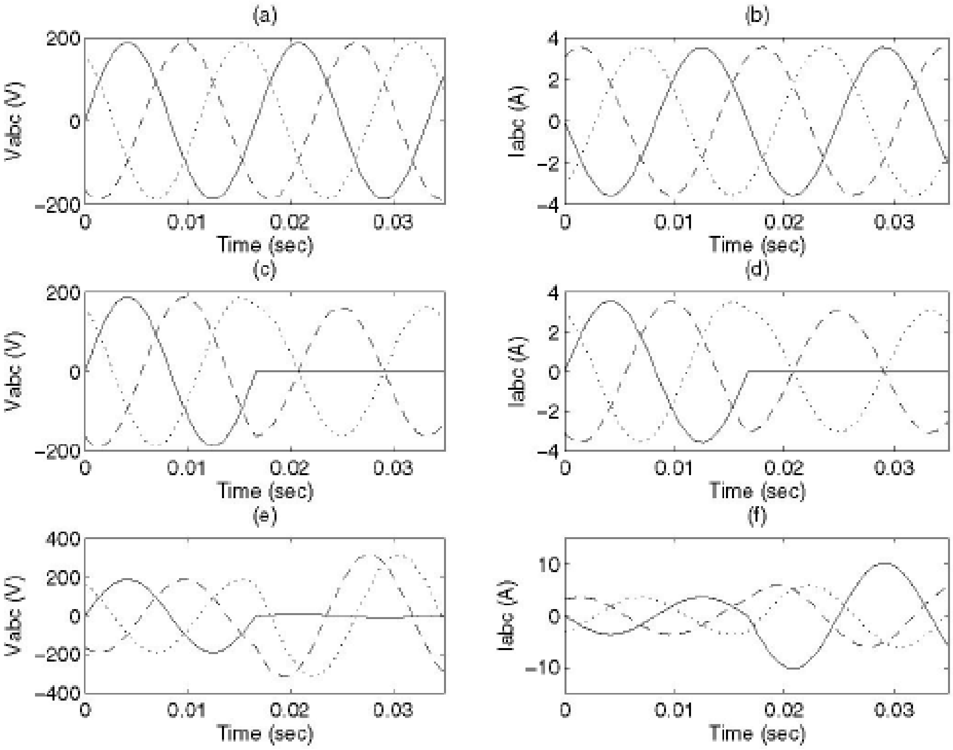


Fig (3): Three phase voltages and current:(a)normal voltage,(b)normal current
(c)open fault voltage,(d)open fault Current (e)short fault voltage,(f)short fault current. (Taken from [3]).

It becomes more serious if two phases are shorted. Phase to Phase short circuit can occur in case of winding insulation failure. Such failure can also lead to turn to turn and phase to frame faults. However, turn to turn fault appears to be the reason behind most of phase to frame faults. In particular, the single shorted turn fault is the worst case because of inverse proportion between the number of shorted turn and the short circuit current [5]. In other words, the magnitude of fault current can be N (no. of turns) times the rated current in a one per unit machine [6]. Similarly , if compared to a symmetrical three-phase short circuit, it has been proved that single-phase asymmetrical short circuit faults produce more sever fault responses with high pulsating torque and a significant threat of rotor demagnetization [7]. To verify this statement, a 2.2 KW interior (IPM) machine with commercially available 4 pole, 50 Hz induction motor stator was tested as in fig (4).

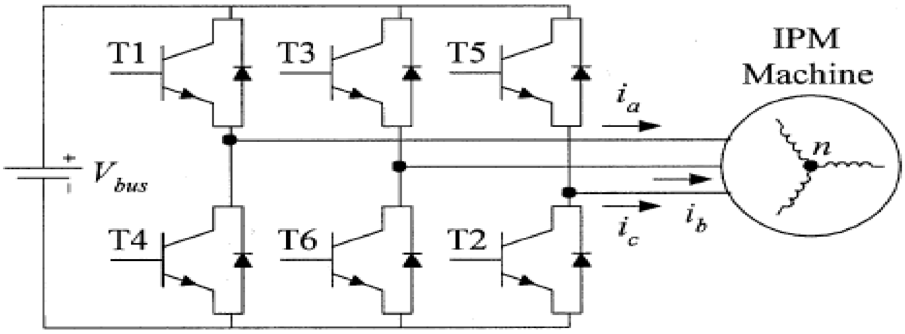


Fig (4): Basic IPM machine drive configuration. (Taken from [7]).

A symmetrical three phase short circuit can occur if (T1, T3 &T5) or (T4, T6 & T2) get closed at the same time. In addition, the inverter dc bus can be shorted so that the six free-wheeling diodes conduct the fault currents. It is also possible to have a symmetrical three phase short circuit if the connection cable between the machine and the inverter is physically damaged. The measured phase current and torque for a symmetrical three phase short circuit are shown in figs (5) and (6) respectively. Fig(6) indicates that three phase symmetrical short circuit causes low steady state torque at moderate to high speed, going to peak at low speed where the stator resistance plays a role [7].

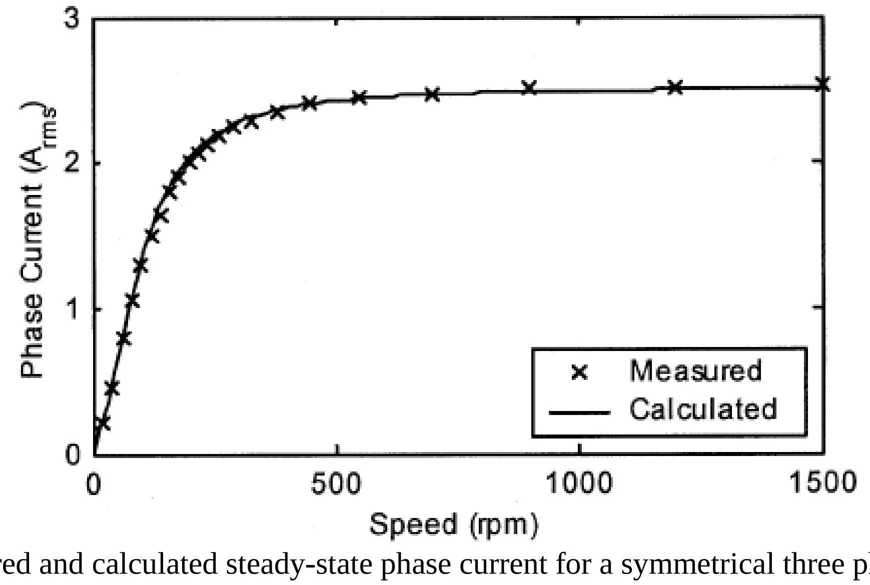


Fig (5): Measured and calculated steady-state phase current for a symmetrical three phase short circuit fault. (Taken from [7]).

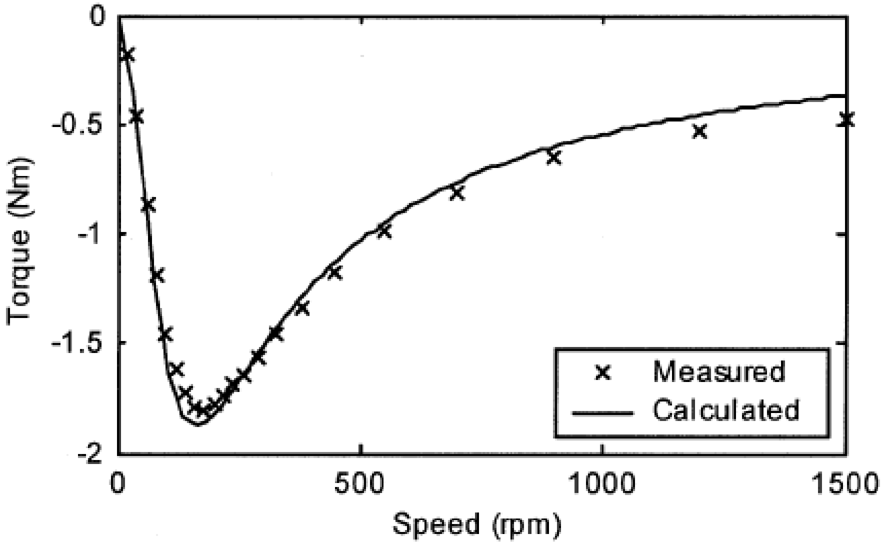


Fig (6): Measured and calculated steady-state motor torque for a symmetrical three phase short circuit fault. (Taken from [7]).

For the same machine, the asymmetrical single phase short circuit was performed with phase (a) short circuited to the positive DC supply keeping all other inverter switches off so that the current passed only through the anti-parallel diodes. A reduced bus voltage (10V) was used in this test in order to avoid the dangerous rotor demagnetization. The experimental results of currents and torque were taken at 1500 rpm. See figs (7) and (8). By considering the figures 5, 6, 7&8, it can be observed that the asymmetrical single phase fault produces higher oscillating torque and higher peak current compared to the symmetrical three phase fault. Therefore, the asymmetrical single phase fault is worse. According to *Welchko, B.A. et al.,* if there are no other options, a control action must be taken to transit the single phase fault into the symmetrical three phase fault which has less danger [7]. It should be mentioned that there was no significant cross saturation effects in the machine tested. Instead, another 70 KW IPM machine was tested to simulate the torque under short circuit taking the effect of saturation under consideration. The dynamic simulation of torque indicated that the saturation lowers the peak torque [7].

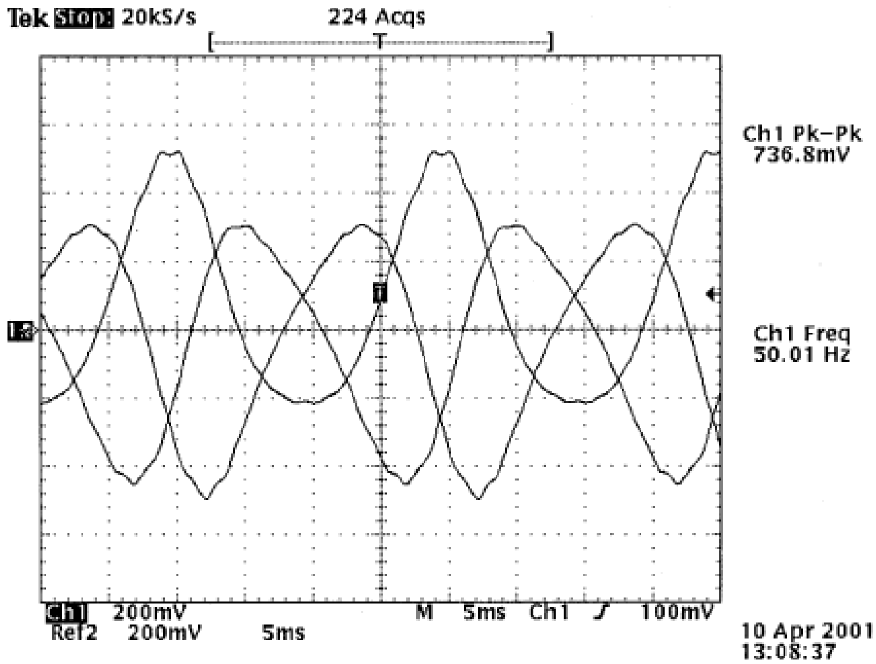


Fig (7): Experimental single phase short circuits currents at V_{bus} = 10V and 1500 rpm with phase (a) shorted to positive dc bus. 2A/div. (Taken from [7]).

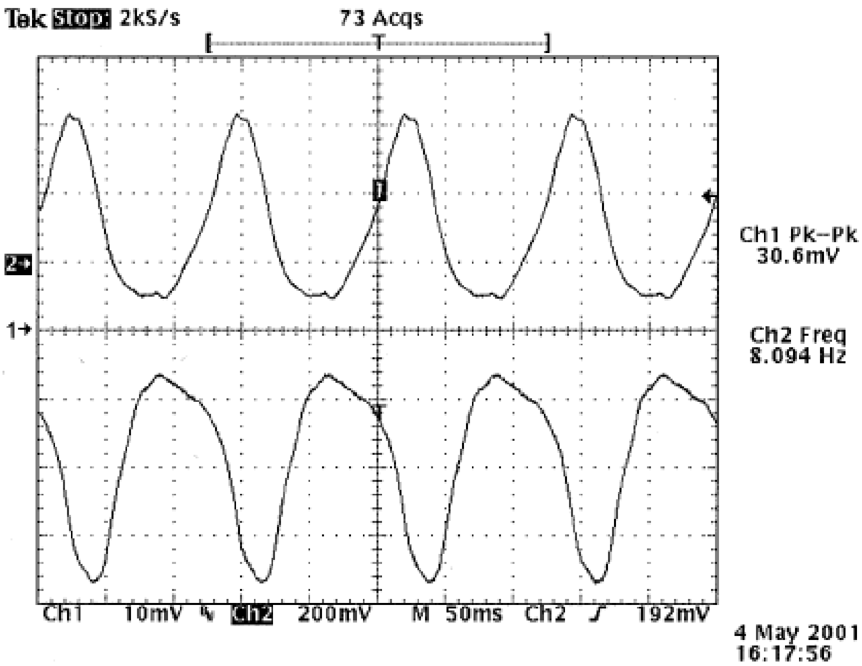


Fig (8): Experimental single phase short circuits at V_{bus} = 10V and 240 rpm
Lower signal is instantaneous torque, 1NM/div. Top signal is i_a, 2A/div.(Taken from [7]).

2.2. Open Circuit Faults (within winding or power electronic circuit)

When an open circuit fault occurs, the current drops to zero and less torque can still be produced if the machine is well designed [8]. By comparing the three phase open fault voltage and current with short fault voltages and currents previously shown in fig(3), it can be seen that the short circuit fault causes more sever due to very high current it can produce [3]. *Welchko, B.A. at al.*, have also concluded that open circuit fault is considered to be a quite kind operating mode. Specially, in low back EMF machines [9]. The faults during flux weakening produce more risky effects due to extremely high phase EMF of the machine during such state. An interior PM synchronous motor was considered for open circuit fault during flux weakening operation. The simulation produced zero current and more ripple of torque [10]. See fig (9).

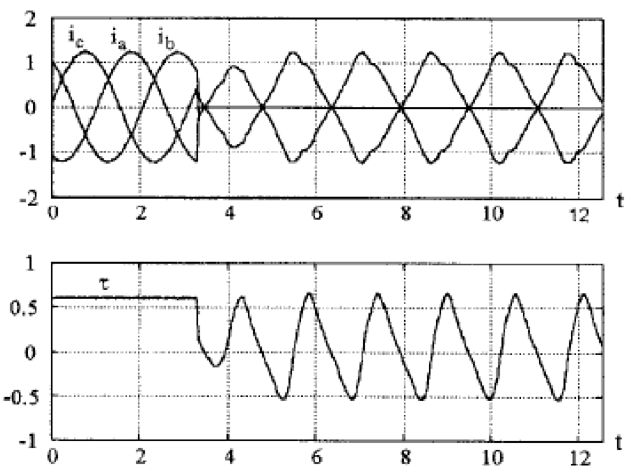


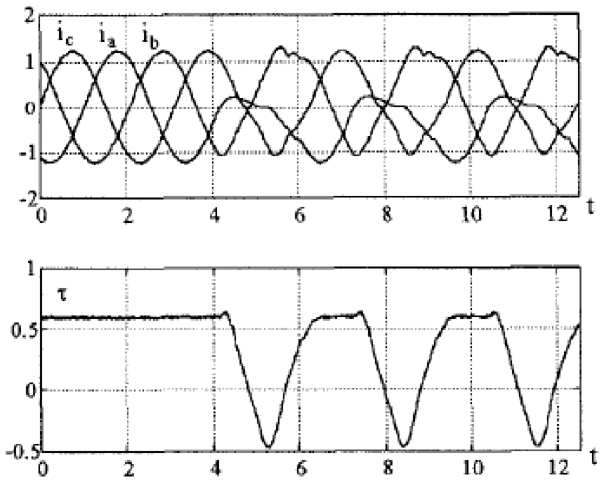
Fig (9): Current (upper) and Torque (lower) under open circuit fault under flux weakening. (Taken from [10]).

Moreover, most of possible open failures that can occur in the power inverter were examined and simulated using the same interior PM synchronous motor under the same condition of flux weakening. Considering the inverter circuit shown in fig (4), the simulation covered the following inverter failures:

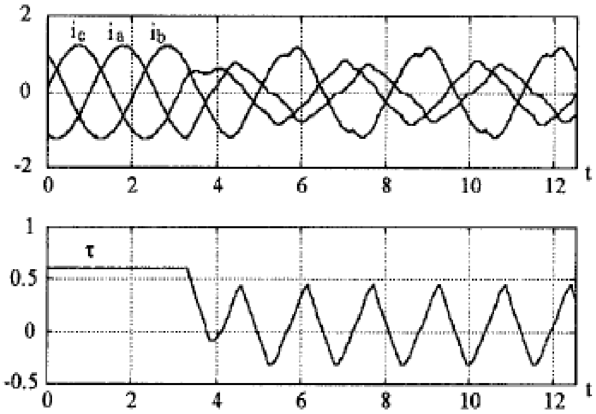
1- Switch-on failure of a transistor (as if switch T1 keeps opened)
2- Switch-on failure of a leg (as if both switches T1&T4 keep opened)
3- Switch-on failure of all the transistors.
4- DC supply failure.

All simulation results are shown in fig (10) for the four cases mentioned above [10].

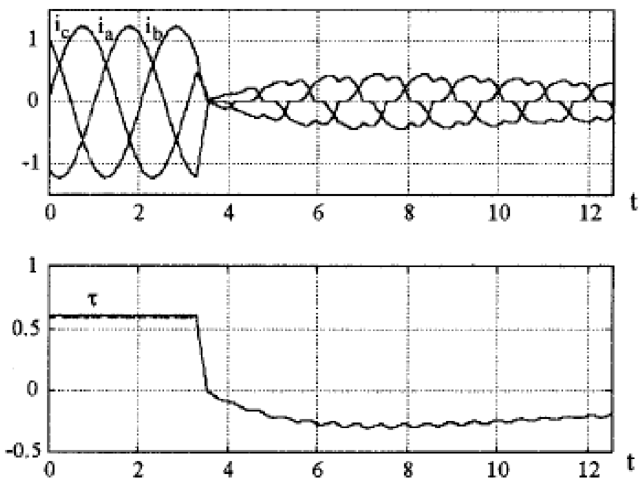
Switch-on failure of a transistor



Switch-on failure of both the transistor



Switch-on failure of all the transistors



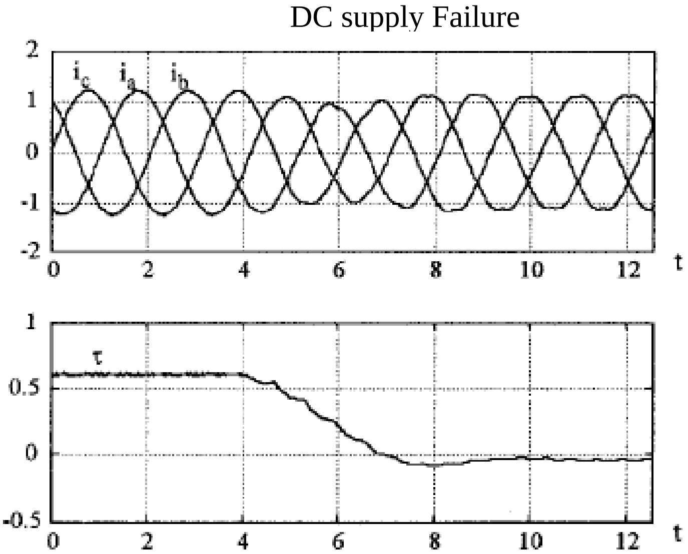


Fig (10): Currents (upper plot) and Torque (lower plot) for all different cases of inverter Faults under flux weakening. All simulations were produced at the same motor speed. (Taken from [10]).

Fig (10) displays that more torque ripples were produced due to leg failure than the one transistor failure. However, there was no big difference in the currents. In case of DC supply failure, the currents remained almost sinusoidal but out of phase while the torque fell quickly. Finally, the currents were highly affected and the torque dropped negatively. It should be mentioned that other simulations for the same circuit were taken at motor higher speed and it could be noticed that the effect of failures got worse at higher speed [10].

According to *Neely, J.D. et al.*, faults in PM machines might result in a damage on the permanent magnets [3]. The level of this damage depends on the magnet material. This damage is simply because of high temperature of the machine under high current faults. Moreover, the bearing high current faults might lead to a quick damage of shaft bearing. However, there are many ways to reduce the bearing current such as using insulated bearings, hybrid or ceramics bearings [3].

3. Detection and Mitigation of Faults

Early detection of faults in electrical machines would eliminate consequential damage and keep the reliability of the machine up to standard. Therefore, the research and development of theoretical and experimental techniques of fault detection and toleration in electrical machines have been in interest of many past and new works and research papers. These techniques have been developed depending on different criteria such as flux distribution, current, voltage, torque and back EMF characteristics under normal and faulty machine's operation. Some of these previous works will be considered in this section. However, the main requirements for a machine to have a continuous operation under fault conditions should be discussed first. In other words, a fault tolerant machine, according to *Mecrow, B.C. at al.*, needs to have the following design properties [2]

1- Phases should be completely electrically isolated. This can be done by driving each phase from a separate single phase bridge.
2- Phases need to be magnetically isolated by designing the machine with minimal phase to phase couple induction.
3- Phases need to be effectively thermally isolated. This can be done by making each slot containing one phase only so that minimum thermal interaction between phases is achieved.
4- Phases need to be physically isolated. This can be performed by placing each winding round a tooth. Thus, there is no possibility for a serious phase to phase fault.
5- For the machine to keep producing the rated power under fault, all phases should be designed overrated by a fault tolerant factor which is equal to n/(n-1) where n is the number of phases in the machine.
6- If possible, The machine should be designed with one per unit armature self inductance in order not to allow the steady state current to go beyond one per unit in case of a phase terminal short circuit [2]. For salient PM synchronous machines, dynamic simulations produced under three phase and single phase faults indicated that a high quadrature inductance relative to direct inductance would increase the peak values of the negative direct current and hence demagnetization stresses would be worse [7].

To achieve these requirements, a machine should be designed with a high leakage inductance and this is done by controlling what is called stator reactance slot. (i.e. the depth and width of slot opening) [2].

A 16KW, 6-phase, fault tolerant PM synchronous machine driven by a power converter was tested under a shorted turn. Figs (11) & (12) show the current signals obtained under both healthy phase and a phase with shorted turn. By considering the current waves only (trace 1), the shorted turn phase current peak was almost double the normal phase current peak. This overshoot current gave an indication of a turn to turn fault. However, it might not be an easy task to detect turn to turn fault because of many factors that can affect the delectability [5]. Once turn to turn fault is detected, a suitable action must be taken depending on the type of machine. For example, in case of PM synchronous machine, the whole winding involved in the fault must be shorted at the machine terminal and thereafter the shorted turn and the remainder turns both carry equal current. Applying this action to the tested machine produced the current waves shown in fig (13). This result can be guaranteed due to the strong mutual coupling between individual turns [5].

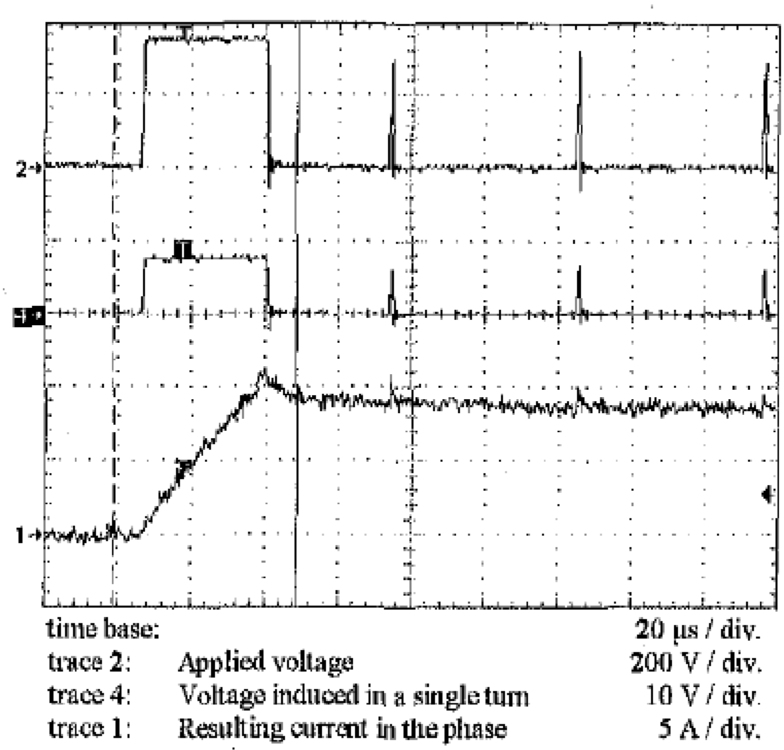


Fig (11): Applied volts and resulting current in a normal phase. (Taken from [5]).

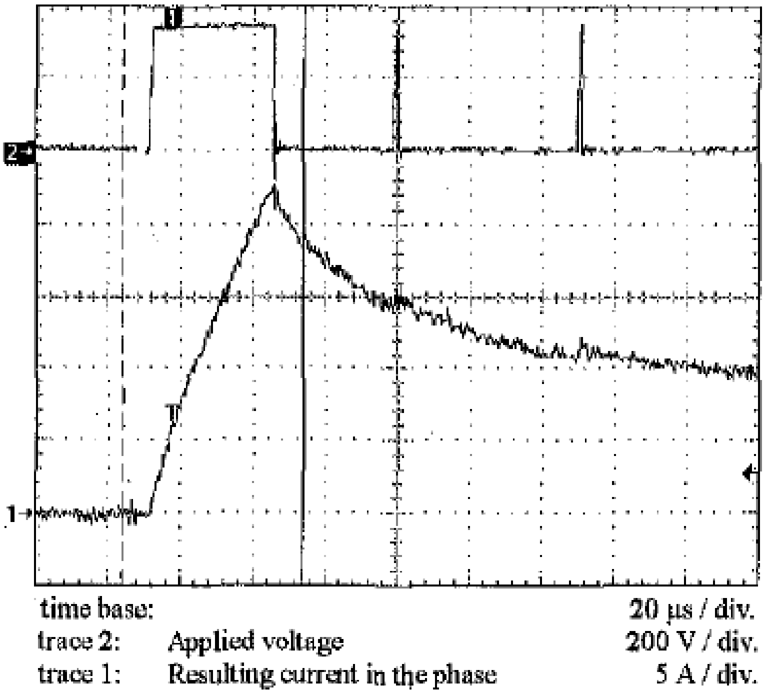


Fig (12): Applied volts and resulting current in a shorted turn phase. (Taken from [5]).

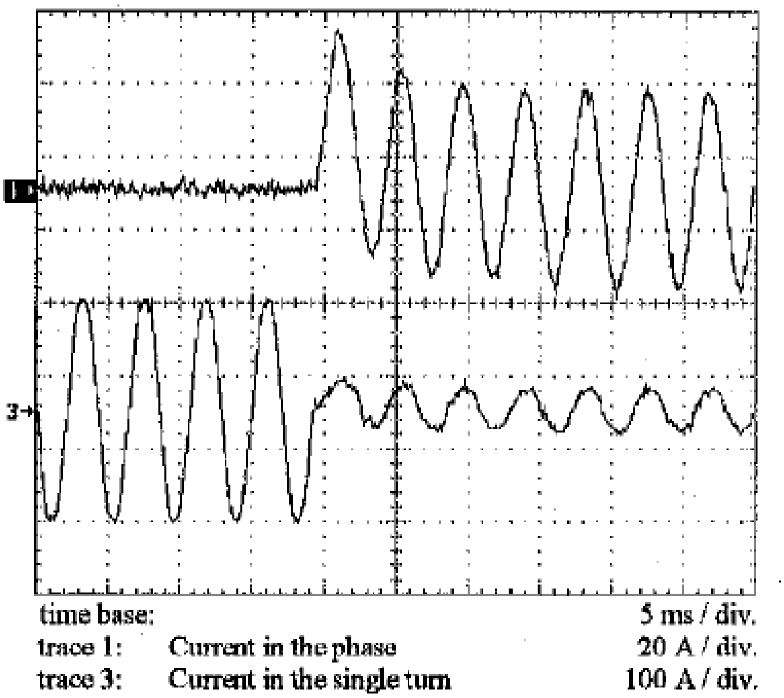
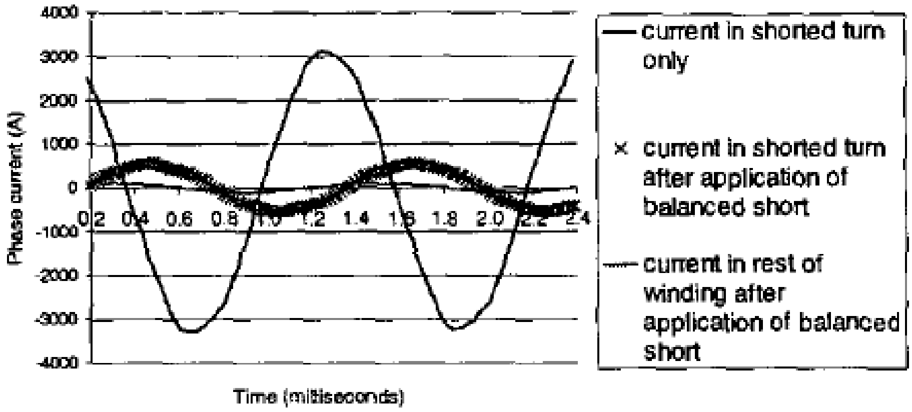
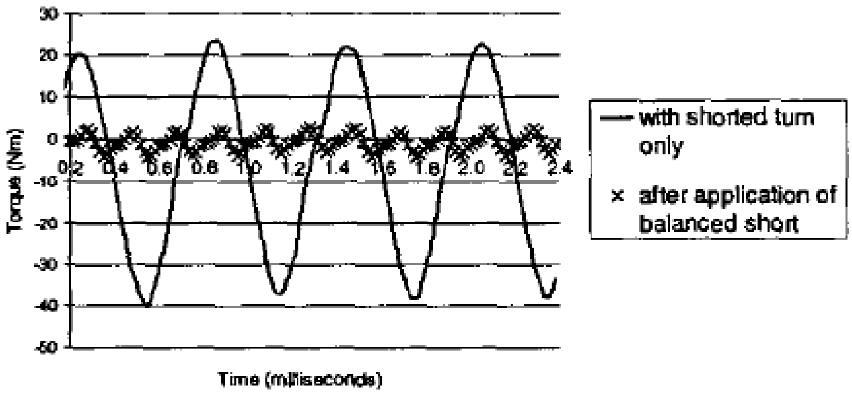


Fig (13): Currents in the shorted and remainder turns of the phase before and after the phase is Shorted at the machine terminal. (Taken from [5]).

An agreement to the idea of applying balanced short was obtained in a single shorted turn test done on a non salient (surface magnet) prototype PM motor designed for car applications [6]. See fig (14).



A) Current in shorted turn and remainder of winding



B) Torque

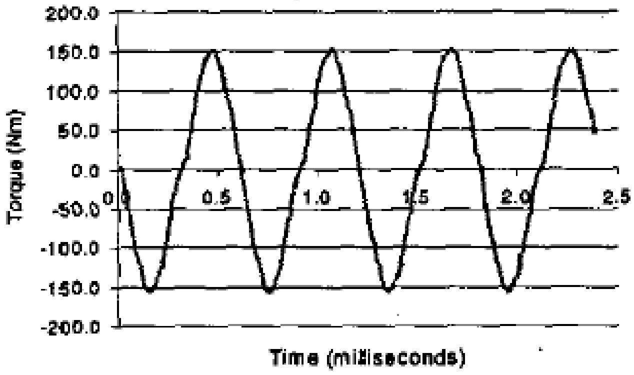
Fig (14): A simulation of single turn short characteristics before and after the application of a balanced short circuit to the remainder windings. (Taken from [6]).

However, the method of shorting the affected phase doesn't succeed with bar wound PM machines under single shorted turn. Instead, a current greater than the terminal short circuit current is injected into the faulted windings at 90° lagging with phase EMF [11].

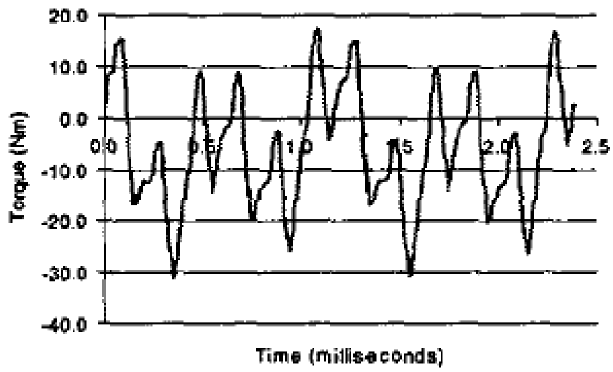
In case of open circuit faults, the concept of shorting the unfalted phases can also be a good control strategy of IPM machines. Alternatively, a technique called uncontrolled generation can be used. This strategy, in case of a single phase open circuit, is done by removing the gate excitation from all of the inverter switches and hence the machine operation is uncontrolled generation mode. This strategy has been suggested to be better than the phase shorting strategy [9].

This work has also investigated the effect of a winding open circuit on the surface magnet PM motor with both cases of Star and Delta three phase connections. For the star connection and after shorting the unfaulted phases, their currents were limited to the rated values but not balanced and hence the resultant torque ripple was high. For the delta connection, in contrast, the effects of torque ripples and unbalanced currents were less as the two unfaulted phases were separately shorted. Fig (15) shows the torque signals for both cases Star and Delta after shorting the healthy phases under a winding open circuit. It can be noticed that the ripples peaks are much smaller in Delta connection [6].

Resultant Torque (Star Connection)



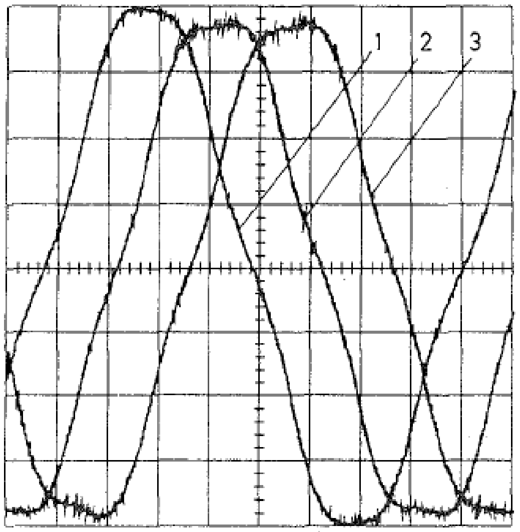
Resultant Torque (Delta Connection)



Fig(15): Analysis of torque after applying a balanced short circuit to an open winding circuit fault for Star and Delta connected surface magnet traction motor running at 12500rpm. (Taken from [6]).

To demonstrate the ability of the tolerant machines to withstand a single phase short fault, a six phase prototype machine designed to supply a per phase no load voltage of 55 Vrms at 12000rpm was tested under a single phase short circuit [2]. The results obtained for this test are shown in fig (16) for both normal and fault condition. It might be good to show a typical arrangement of the prototype machine developed to meet all requirements for fault tolerance. See fig (17). Note that the each phase is wound per slot and each coil is wound around a single tooth.

Normal operation



Fault operation

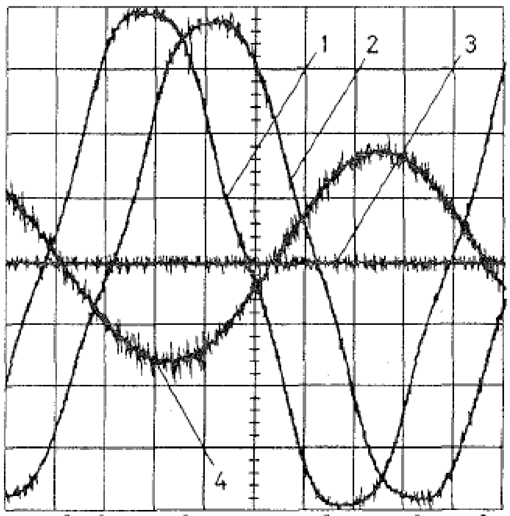


Fig (16): Measurements under no-load (upper) and C shorted phase (lower). (1) Phase A voltage (2) phase B voltage (3) Phase C voltage (4) Phase C current. Vertical sale :20V/div and 5A/div. (Taken from [2]).

By comparing the two plots, it is clearly seen that the voltage of unfaulted phases (a, b) were not affected by the faulted phase (c) and that was because of the efficient physical isolation between phases in the fault tolerant machine. The fault current was limited by the phase inductance to the rated value .Therefore, the machine could continue in operation with the other phases [2].

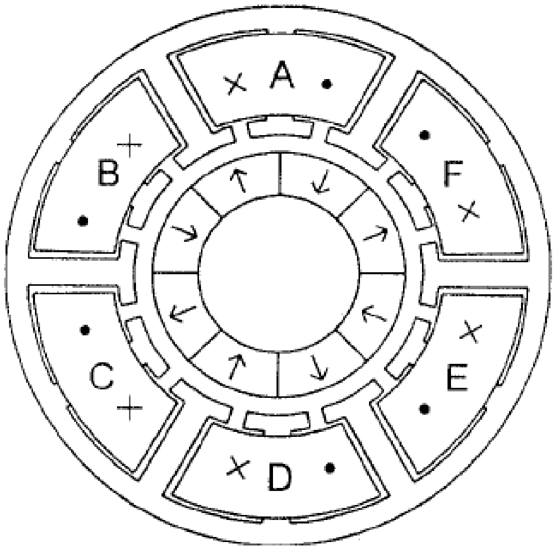
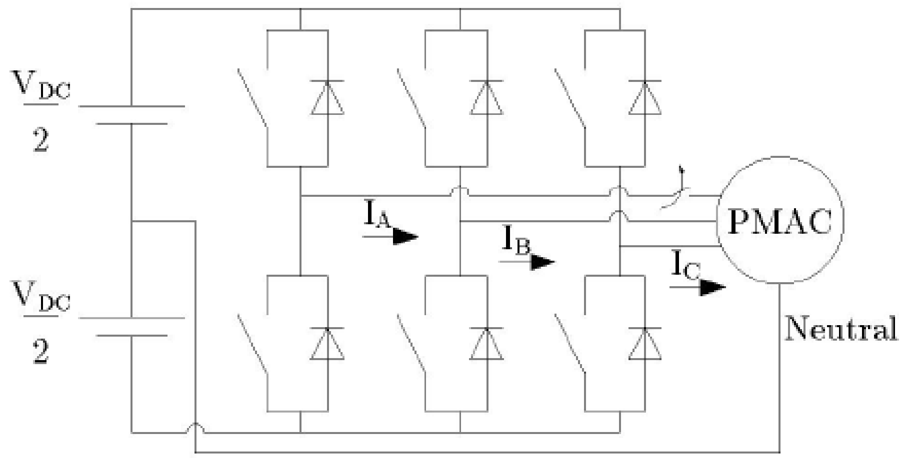
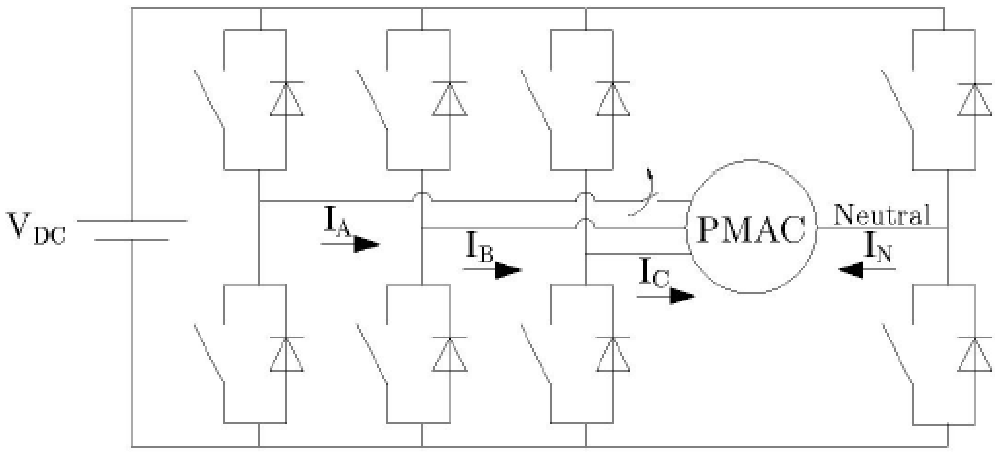


Fig (17): A prototype Machine Topology. (Taken from [12]).

In general, it is possible for a three phase motor to run only on two phases [13]. Therefore, *Neely. J.D. et., al* considered two mitigation techniques to keep continuous machine operation in case of power inverter faults but with less level of power. The two methods considered were the Central Tapped DC bus and the Hardware Redundancy in controller. Fig (18) shows the inverter circuit used for both techniques. In the first one, three phase currents and torque were simulated with phase (A) opened and the motor could still run on the other two phases. However, in the second method, an extra phase leg was connected to the controller in order to act as a return path in case of one phase failure. Simulation results for both techniques are illustrated in fig (19). The 1st and 3rd plots show the three phase currents with phase a failure at around 0.033 s. Torques are displayed in the 2nd and 4th plots and it can be clearly seen that they had slight ripples which disappeared after fault recovery [3].

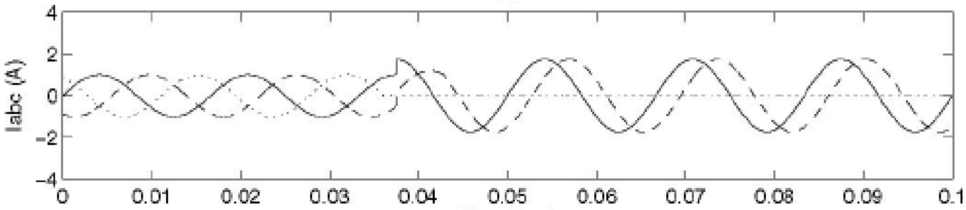


Centre Tapped DC-bus Inverter Topology

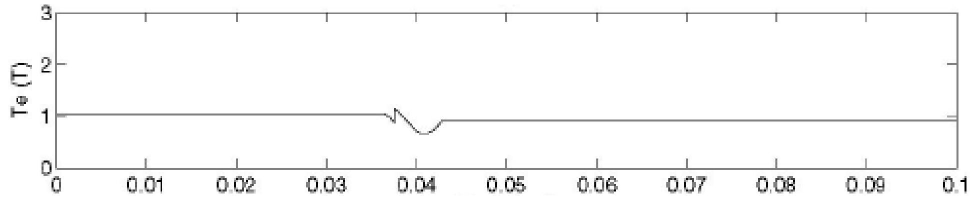


Redundant Phase Leg Inverter Topology

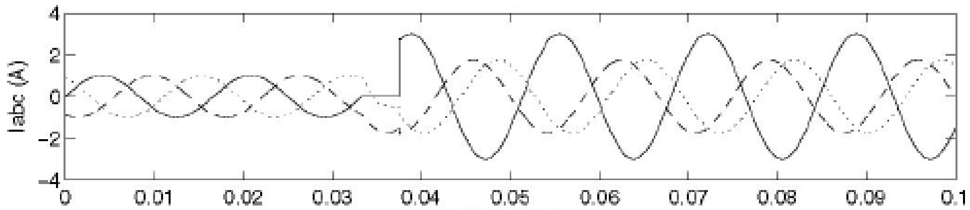
Fig (18): Mitigation Techniques Topologies. (Taken from [3]).



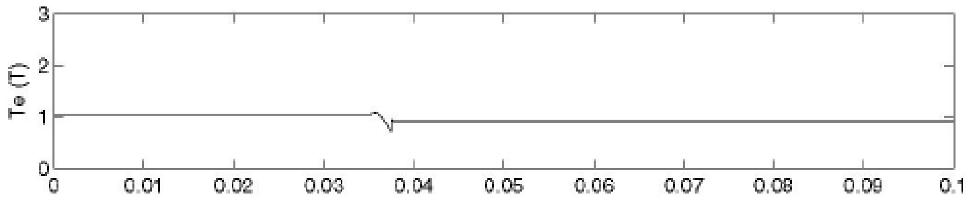
Three phase currents (Centre Tapped DC)



Torque (Centre Tapped DC)



Three phase currents (Redundant Phase Leg Inverter)



Torque (Redundant Phase Leg Inverter)

Fig (19): Mitigation techniques simulation results. (Taken from [3]).

In safety critical application, it is definitely more important to quickly detect and implement a recovery. One of the efficient detection techniques is to use of multiple single phase bridges with help of some current sensors. Such technique provides high protection of a single bridge from device open and short circuit failures [14].

In fact, there have been plenty of papers and works which have studied and developed different control strategies for PM machines operating under faults. However, understanding the operation of PM machines under faults is an essential background for any development process. Either in fault studies or development research of PM machines, modelling methods can offer a great help. One of the most familiar modelling methods is known as Finite element. This modelling technique will be discussed and covered in more details in later sections. But, in general, it has been suggested that the finite element model offers more advantageous over other modelling techniques [15].

Further work will be done in this project to study and demonstrate the properties of the finite element based phase variable model and how useful it is in modelling of PM machines under faults.

CONCLUSION

This review has examined some past and recent works and paper published in the field of permanent magnet machines operating under fault conditions. The main fault modes that can occur within the machine itself or the power converter circuit are :

1- Winding short circuit. 2- Winding open circuit. 3- Power device short circuit. 4- Power device open circuit and 5- DC supply failure.

The main observations and conclusions can be outlined as following

- Although the damper cage can minimize the flux demagnetization under single phase short circuit, demagnetization cannot be completely avoided [4].
- Short circuit fault is more dangerous than open circuit fault due to the large fault current it can produce [3]. In general, open circuit fault is relatively benign mode [9] .
- Turn to Turn fault can result in most of phase to frame faults [5].
- The single shorted turn fault is very serious since it produces a huge shorted turn current[6].
- For Interior (IPM) machine, the single phase asymmetrical short circuit faults produce more sever fault responses than the three phase symmetrical short circuit [7].
- For IPM machine, the saturation lowers the peak torque during short circuit fault [7].
- Under open circuit faults, current drops to nothing while torque can still be produced with less values if the machine is well designed [8].
- The faults during flux weakening operation mode produce more risky effects [10].
- For tolerant PM machines, phases should be electrically, magnetically, thermally and physically isolated [2].
- For PM synchronous machines, a technique of shorting all windings at the machine terminal can be applied in case of short or open circuit faults in order to minimize the effects of faults [5,9 &14]. However, this technique is not useful for bar wound PM machines [11].
- Finite element has been suggested to be one of most efficient modelling techniques used for machines analysis.

Further work will be done in this project to study and demonstrate the properties of the finite element based phase variable model and how useful it is in modelling of PM machines under faults.

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