MAE 101B, Spring 2007

Homework 3

Due Thursday, May 3, in class

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.
Please refrain from copying. Refer to the course outline for what constitutes copying

1. The cruise speed and altitude of an aircraft are $150\,m/s$ and $10\,km$, respectively. Assume that the boundary layer on the wing surface can be approximated as that on a flat plate.

a) Obtain the expected length of laminar boundary layer flow on the wing assuming properties at sea level.

b) Repeat part (a) but now accounting for the elevation of $10\,km$.

2. A laboratory wind tunnel has sides of width $30\,cm$. The turbulent boundary layer height is measured at section 1 to be $\delta_1 = 10\,mm$ and at another downstream section 2 to be $\delta_2 = 13\,mm$. The free-stream velocity and pressure at location 1 is measured as $V_1 = 18\,m/s$ and $p_1 = -215\,N/m^2$(gage). Assume that the velocity profile in the turbulent boundary layer can be approximated by

$$\frac{u}{U} = (\frac{y}{\delta})^{1/7}$$

where U is the free-stream velocity.

a) What is the free-stream velocity at section 2?

b) What is the pressure at location 2?

3. SAE 10 oil ($\rho = 890\,kg/m^3$ and $\mu = 0.29\,kg/m-s$ with velocity $5\,m/s$ flows past a flat plate $40\,cm$ by $80\,cm$ that is immersed in the oil.

a) What is the drag force if the flow is parallel to the long side?

b) What is the drag force if the flow is parallel to the short side?

4.

a) Show that the curvature of the velocity profile d^2u/dy^2 must be zero at the wall in a boundary layer with zero pressure gradient.

b) The parabolic profile assumption in a laminar boundary layer can be replaced by the following sinusoidal profile (which satisfies the zero curvature requirement) in the Karman momentum integral analysis:

$$\frac{u}{U} = \sin\frac{\pi y}{2\delta}$$

Obtain the shape factor, H. Obtain an expression that relates δ/x to Re_x.

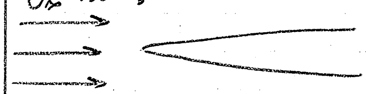
$\cos 2x = 1 - 2\sin^2 x$ $\quad$ $\cancel{2}\sin^2 x = \frac{1-\cos 2x}{2}$

MAE 101B	HOMEWORK 3		Michael Sankur A05812168 MAE 101B

1 Problem: The cruise speed and altitude of an aircraft are $150\ \mathrm{m/s}$ and $10\ \mathrm{km}$. 5-2-07

Given:

$U_\infty = 150\ \mathrm{m/s}$



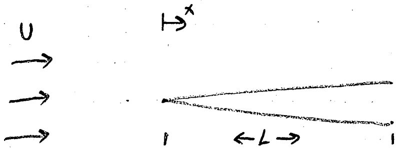
1	10/10
2	7/10
3	10/10
4	7/10
	34/40

Find: a) Obtain the expected length of laminar boundary flow on the wing assuming sea level properties

b) Repeat part a but a 10 km

Assume: BL on wing can be approximated as that of flat plate

Analysis



$$Re_x = \frac{Ux}{\nu} = \rho\frac{Ux}{\mu}$$

$$Re_x < 1\times10^6 \quad \text{Laminar}$$

$$\rho\frac{Ux}{\mu} < 5\times10^5 \qquad x < \frac{5\times10^5\mu}{\rho U}$$

Sea Level $z = 0$

$$x < \frac{5\times10^5\,(1.71\times10^{-5}\ \mathrm{kg/sm})}{(1.225\ \mathrm{kg/m^3})(150\ \mathrm{m/s})}$$

$$x_{tr} = 0.0465\ \mathrm{m}$$

10 km $\quad z = 10\times10^3\ \mathrm{m}$

$$\nu = (1.71\times10^{-5}\ \mathrm{kg/ms})\left(\frac{223.16}{288.16}\right)^{\frac{3}{2}}\left(\frac{273+110.4}{223+110.4}\right)$$

$$\nu = 1.34\times10^{-5}\ \mathrm{kg/ms}$$

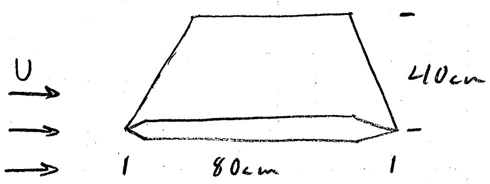
$$x < \frac{5\times10^{-5}\,(1.34\times10^{-5}\ \mathrm{kg/ms})}{(0.4125\ \mathrm{kg/m^3})(150\ \mathrm{m/s})}$$

$$x_{tr} = 0.1083\ \mathrm{m}$$

10/10

3 Problem: SAE 10 oil flows past a flat plate that is immersed

Given:



$\rho = 890\ \text{kg/m}^3$

$\mu = 0.29\ \text{kg/ms}$

$U = 5\ \text{m/s}$

Find: a) Drag force when flow is parallel to long side
b) Drag force when flow is parallel to short side

Assume: 1 Laminar flow
2 Flat plate flow

Analysis:

a) $Re_L = \frac{UL_a}{\nu} = \frac{(890\ \text{kg/m}^3)(5\ \text{m/s})(0.8\ \text{m})}{0.29\ \text{kg/ms}} = 12275.86 < 5\times10^5$

Laminar flow +2

$C_D = \frac{1.328}{Re_L^{1/2}} = \frac{1.328}{\sqrt{12275}} = 0.0119593$ +2

$D = \rho b L C_D U^2 = (890\ \text{kg/m}^3)(0.40\text{m})(0.8\text{m})(0.0119593)(5\ \text{m/s})^2$

$D = 85.15\ N$ +2

b) $Re_L = \frac{UL_b}{\nu} = \frac{(890\ \text{kg/m}^3)(5\ \text{m/s})(0.4\ \text{m})}{(0.29\ \text{kg/ms})} = 6137.93 < 5\times10^5$

Laminar +2

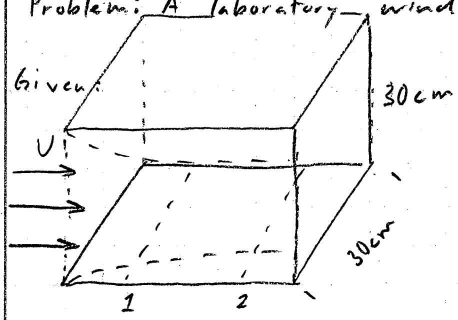
$C_D = \frac{1.328}{Re_L^{1/2}} = \frac{1.328}{\sqrt{6137.93}} = 0.01695$ +2

$D = \rho b L C_D U^2 = (890\ \text{kg/m}^3)(0.80\text{m})(0.40\text{m})(0.01695)(5\ \text{m/s})^2$

$D = 120.684\ N$ +2

2 Problem: A laboratory wind tunnel has air flowing through it

Given:



$\delta_1 = 10\text{ mm} = 0.01\text{ m}$ $\delta_2 = 13\text{ mm} = 0.013\text{ m}$

$V_1 = 18\text{ m/s}$

$P_1 = -215\text{ N/m}^2$ (gage)

$\frac{u}{U_i} = \left(\frac{y}{\delta}\right)^{1/7}$

Find: a) free stream velocity at section 2

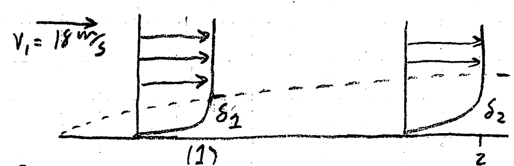
b) Pressure at location 2

Assume: Turbulent BL

Steady, incompressible flow, negligible flow within BL

$\frac{u}{U} = (y/\delta)^{1/7}$

Analysis



$\frac{\delta_1}{x_1} = \frac{0.16}{\left(\frac{\rho U_1 x_1}{\mu}\right)^{1/7}}$

$\frac{\delta_1^7}{x_1^7} = \frac{(0.16)^7 \mu}{\rho U_1 x_1}$

a) $P_1 = -215\text{ N/m}^2$

$x_1 = \left[\frac{\rho \delta_1^7 U_1}{(0.16)^7(\mu)}\right]^{1/6} = \left[\frac{(1.29\text{ kg/m}^3)(0.01\text{ m})^7(18\text{ m/s})}{(0.16)^7(1.71\times10^{-5}\text{ kg/ms})}\right]^{1/6} = 0.4143\text{ m}$

$Re_1 = \frac{(1.29\text{ kg/m}^3)(18\text{ m/s})(0.4143\text{ m})}{1.71\times10^{-5}\text{ kg/ms}} > 5\times10^5$

$\cancel{\rho_1} A_1 V_1 = \cancel{\rho_2} A_2 V_2$

8 = 8/8 (-3)

$V_2 = \frac{A_1}{A_2} V_1 = \frac{(h_1 b_1)}{h_2 b_2} V_1 = \frac{[0.3\text{m} - 2(0.01\text{m})][0.3\text{m} - 2\cancel{(0.01\text{m})}]}{[0.3\text{m} - 2(0.013\text{m})][0.3\text{m} - 2(0.013\text{m})]}(18\text{ m/s})$

$V_2 = \cancel{18.797}\text{ m/s}$ 18.09

b) Applying Energy equation to center streamline, no head loss

$P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g z_1} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g z_2}$

$P_2 = P_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) = (101350\text{ Pa} - 215\text{ Pa}) + \frac{1}{2}(1.225\text{ kg/m}^3)(324\text{ m}^2/\text{s}^2 - 353.33\text{ m}^2/\text{s}^2)$

$P_2 = 101117\text{ Pa (abs)} = -\cancel{232}.97\text{ Pa (gage)}$ ✓

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4 Problem: Derive some stuff

a) Show that the curvature of the velocity profile $\frac{d^2 u}{dy^2}$ must be zero at the wall in a boundary layer with zero pressure gradient

$$\frac{dP}{dx} = 0 \qquad \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dP}{dx} \qquad \frac{dP}{dx} = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = 0$$

N-S eq: For Incompressible, steady

$$\rho \cancel{g_x} - \frac{\partial P}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right) = \rho \cancel{\frac{du}{dt}}^{\,0} \text{ steady}$$

+3

Fully developed

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x} = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = 0$$

b)

$\frac{u}{U} = \sin(\pi y / 2\delta)$ obtain H, obtain how δ/x relates to Re

$$H = \frac{\delta^*}{\theta} = \frac{\int_0^\delta (1 - u/U)\,dy}{\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy} = \frac{\int_0^\delta 1 - \sin(\pi y/2\delta)\,dy}{\int_0^\delta \sin(\pi y/2\delta)\left(1 - \sin(\pi y/2\delta)\right)dy}$$

$$= \frac{\int_0^\delta 1 - \sin(\pi y/2\delta)\,dy}{\int_0^\delta \sin\left(\frac{\pi y}{2\delta}\right) - \frac{1 - \cos(\pi y/\delta)}{2}\,dy} = \frac{y + \frac{2\delta}{\pi}\cos(\pi y/2\delta)\Big|_0^\delta}{-\frac{2\delta}{\pi}\cos\left(\frac{\pi y}{2\delta}\right) - \frac{y}{2} + \frac{1}{2}\left(\frac{\delta}{\pi}\right)\sin\left(\frac{\pi y}{\delta}\right)\Big|_0^\delta}$$ +3

$$= \frac{(\delta + 0) - (0 + 2\delta/\pi)}{(0 - \frac{\delta}{2} +) - (-\frac{2\delta}{\pi})} = \frac{\delta - \frac{2\delta}{\pi}}{\frac{2\delta}{\pi} - \frac{\delta}{2}} = \frac{\delta(1 - 2/\pi)}{\delta\left(\frac{2}{\pi} - \frac{1}{2}\right)} = 2.66$$

$\frac{\delta}{x} \Rightarrow Re_x \qquad Re_x = \frac{\rho U x}{\mu}$ use $u = U\sin\frac{\pi y}{2\delta}$

$$\tau_w = \rho U^2 \frac{d\theta}{dx} \qquad \theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + y^2/\delta^2\right)dy$$

$$\tau_w = \rho U^2 \frac{d}{dx}\int_0^\delta \left(\frac{2y}{\delta} - y^2/\delta^2\right)\left(1 - \frac{2y}{\delta} + y^2/\delta^2\right)dy$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu U\left(\frac{2}{\delta} - \frac{2y}{\delta^2}\right)\Big|_{y=0} = \frac{2\mu U}{\delta}$$ +1

$$\frac{2\mu U}{\delta} = \rho U^2 \frac{d}{dx}\int_0^\delta \left(\frac{2y}{\delta} - y^2/\delta^2\right)\left(1 - 2y/\delta + y^2/\delta^2\right)dy$$

$$\frac{2\mu}{\rho U} = \delta \frac{d}{d\delta}\frac{d\delta}{dx}\int_0^\delta \left(\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + 2y^3/\delta^3 - y^2/\delta^2 + 2y^3/\delta^3 - y^4/\delta^4\right)dy$$

$$\frac{2\mu}{\rho\theta}\, dx = \delta d\delta \frac{d}{d\delta}\left[\frac{y^2}{\delta} - \frac{4y^3}{3\delta^2} + \frac{2y^4}{4\delta^3} - \frac{y^3}{3\delta^2} + \frac{2y^4}{4\delta^3} - \frac{y^5}{5\delta^4} \right)\Bigg|_0^{\delta}$$

$$\frac{2\mu}{\rho\theta}\, dx = \delta d\delta \frac{d}{d\delta}\left(\frac{\delta}{7{,}5}\right) \Rightarrow \int \frac{15\mu}{\rho\theta}\, dx = \int \delta d\delta$$

$$\frac{15\mu x}{\rho\theta} = \frac{\delta^2}{2} \Rightarrow \frac{30\mu x}{\rho\theta x^2} = \delta^2/x^2 \qquad \frac{30}{\frac{\rho\theta x}{\mu}} = \frac{\delta^2}{x^2}$$

$$\frac{\sqrt{30}}{\sqrt{\frac{\rho\theta x}{\mu}}} = \frac{\delta}{x} \qquad \frac{\delta}{x} = \frac{5{,}477}{\sqrt{Re_x}}$$

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