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MAE 101C
7-25-07

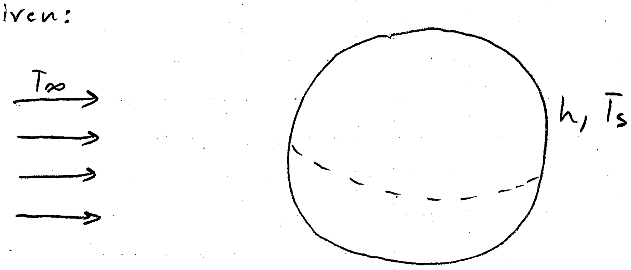
95/100

MAE 101C HOMEWORK 3

5.7

Problem: The heat transfer coefficient is to be determined by placing it in a uniform, constant temperature airstream.

Given:



$d = 0.0127\,m$

$T_\infty = 300\,K$

$T_s(r, \theta, \phi, 0) = 339\,K$

$T(69\,s) = 328\,K$

Find: Justify the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient h.

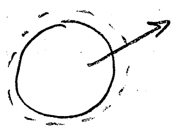
Assume: Sphere behaves as isothermal object
Constant thermal properties

Analysis:

For spacewise isothermal object $T(r, \theta, \phi, t) = T(t)$

$$Bi = \frac{h\forall}{A_s k} = \frac{hr}{3k} < 0.1$$

$r = 0.00635\,m \qquad k = 401\ W/mK$



$$-hA_s(T - T_\infty) = \rho \forall c_p \frac{dT}{dt}$$

$$\theta = T - T_\infty$$

$$\frac{\rho\forall c}{hA_s}\frac{d\theta}{dt} = -\theta \qquad \frac{\rho\forall c}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

$$\frac{\rho\forall c}{hA_s}\ln\left(\frac{\theta}{\theta_i}\right) = -t \qquad \frac{\theta}{\theta_i} = \exp\left[-\frac{hA_s t}{\rho\forall c}\right]$$

$$h = -\frac{\rho\forall c}{A_s t}\ln\left(\frac{\theta}{\theta_i}\right)$$

5.7

$$h = -\frac{(8933\ \mathrm{kg/m^3})(\frac{4}{3}\pi(0.00635\mathrm{m})^3)(385\ \mathrm{J/kg\,K})}{4\pi(0.00635\mathrm{m})^2(69\,\mathrm{s})} \ln\left[\frac{328\mathrm{K} - 300\mathrm{K}}{331\mathrm{K} - 300\mathrm{K}}\right]$$

$$h = +34.96\ \mathrm{W/m^2K}$$

$$Bi = \frac{(34.96\ \mathrm{W/m^2K})(0.00635\mathrm{m})}{3(401\ \mathrm{W/mK})} = 1.84 \times 10^{-4}$$

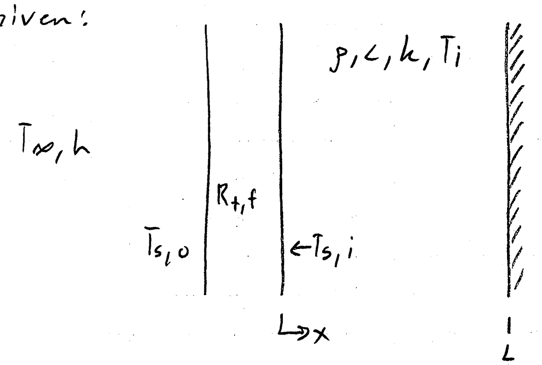
Since the Biot number is so low, the sphere can be modeled as a isothermal object.

30/30

5.16

Problem: A furnace wall is made from carbon steel and has a ceramic coating to prevent corrosion

Given:



$k = 60 \ W/mK$
$\rho = 7850 \ kg/m^3$
$c = 430 \ J/kgK$
$L = 0.01 \ m$

$R_{tf} = 0.01 \ m^2K/W$

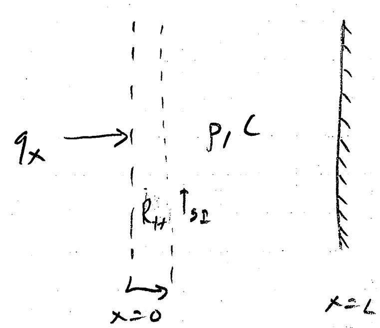
$h = 25 \ W/m^2K$

At $t=0$ $\quad T_i = 300K \ , \ T_\infty = 1300 \ K$

Find a) How long it takes for $T_{s,i}$ to be $1200K$
b) What is $T_{s,o}$ at this time?

Assume: Constant thermal properties,
Ceramic film has negligible thermal capicitance
Negligible radiation exchange, Negligible contact resistance
Isothermal furnace walls - lumped capacitance

Analysis:



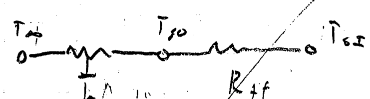
$Bi = \frac{hL}{k} = \frac{(25 \ W/m^2K)(0.01m)}{(60 \ W/mK)} = 0.00417$

$\dot{E}_{IN} = \dot{E}_{ST}$ $\quad$ Isothermal assumption valid

$\frac{T_\infty - T_{s,i}(t)}{R_{TOT}} = \rho \forall c \frac{dT}{dt}$

$\theta = T_\infty - T_{si}$ $\quad \frac{dT}{dt} = \frac{d\theta}{dt}\frac{dT}{d\theta} = -\frac{d\theta}{dt}$

$-\frac{\theta}{AR_{TOT}} = \rho \frac{\forall}{A} c \frac{d\theta}{dt}$



$AR_{TOT} = R' = \frac{1}{h} + R_{tf} = \frac{1}{25 \ W/m^2K} + 0.01 \frac{m^2K}{W} = 0.05 \frac{m^2K}{W}$

$\int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = \int_0^{\Delta t} (-\rho L c R')^{-1} dt$ $\qquad \ln\left[\frac{\theta}{\theta_i}\right] = (-\rho L c' R)^{-1} \Delta t$

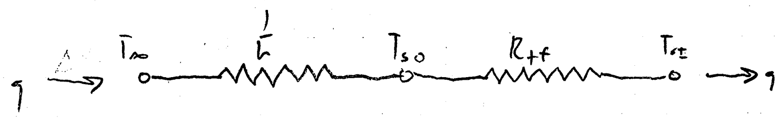
$\Delta t = \rho L c R' \ln\left[\frac{\theta_i}{\theta}\right]$

$\Delta t = (7850 \ kg/m^3)(0.01m)(430 \ J/kgK)(0.05 \ m^2K/W) \ln\left[\frac{1300K - 300K}{1300K - 1200K}\right]$

$\Delta t = 3886.18 \ s = 1 \ hr \ 4 \ min \ 46 \ sec$

5.16

b)



$$h(T_\infty - T_{so}) = \frac{1}{R_{tf}}(T_{so} - T_{sI})$$

$$T_{so}\left(\frac{1}{R_{tf}} + h\right) = hT_\infty + \frac{1}{R_{tf}}T_{sI}$$

$$T_{so} = \left(\frac{1}{R_{tf}} \quad h\right)^{-1}\left[hT_\infty + \frac{1}{R_{tf}}T_{sI}\right]$$

$$T_{so} = \left(100\ \mathrm{W/m^2K} + 25\ \mathrm{W/m^2K}\right)^{-1}\left[25\ \frac{\mathrm{W}}{\mathrm{m^2K}}(1300\mathrm{K}) + \left(100\ \frac{\mathrm{W}}{\mathrm{m^2K}}\right)(1200\mathrm{K})\right]$$

$$T_{so} = 1220\ \mathrm{K}$$

$$q = 25\ \mathrm{W/m^2K}(1300 - 1220)\mathrm{K} = 2000\ \mathrm{W/m^2}$$

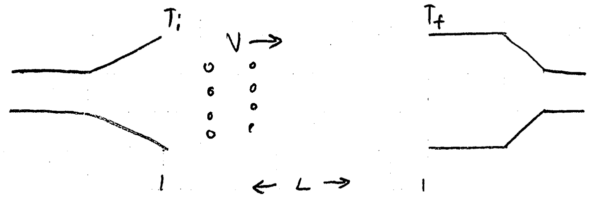
$$q = \frac{1220\mathrm{K} - 1200\mathrm{K}}{0.01\ \mathrm{m^2K/W}} = 2000\ \mathrm{W/m^2}$$

35/35

5.23

Problem: A space station uses a liquid droplet radiator to dissipate excess energy

Given



$T_i = 500K$ $\quad$ $T_f = 300K$

$T_{sur} = 0\ K$

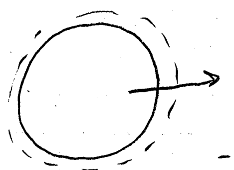
$D = 0.0005\,m$
$\varepsilon = 0.95$
$V = 0.1\ m/s$
$\rho = 885\ kg/m^3$
$c = 1900\ J/kg\,K$
$k = 0.145\ W/mK$

Find: Distance required for droplets to reach collection point at T_f. Find thermal energy rejected by each droplet

Assume: Constant thermal properties
Droplets do not receive any irradiation
Droplets radiate to large surroundings
Droplets are isothermal mass - lumped capacitance valid

Analysis:

$T = T(t)$



$$-\dot{E}_{OUT} = \dot{E}_{ST}$$

$$-\varepsilon\sigma A(T_s^4 - \overset{0}{T_{sur}^4}) = \rho \forall c \frac{dT}{dt}$$

$$-\varepsilon A_s \sigma T_s^4 = \rho \forall c \frac{dT}{dt}$$

$$\int_{T_i}^{T_f} -\frac{dT}{T^4} = \int_0^{\Delta t} \frac{\varepsilon A_s \sigma}{\rho \forall c}\,dt \qquad \frac{1}{3} T^{-3}\Big|_{T_f}^{T_i} = \frac{\varepsilon A_s \sigma t}{\rho \forall c}\Big|_0^{\Delta t}$$

$$\Delta t = \frac{\rho \forall c}{3\varepsilon\sigma A_s}\left[T_i^{-3} - T_f^{-3}\right]$$

prove $Bi<1$ before apply the method.

$$\Delta t = \frac{(885\,kg/m^3)(\frac{4\pi}{3})(0.00025m)^3(1900\,J/kg\,K)}{3(0.95)(5.67\times10^{-8}\,W/m\,K^4)(4\pi)(0.00025m)^2}\left[(300K)^{-3} - (500K)^{-3}\right]$$

$\Delta t = 25.18\,s$ $\qquad$ $\Delta t = L/V$ $\qquad$ $L = V\Delta t$

30/35

$L = (0.1\,m/s)(25.18s) = 2.518m$

$Q = \rho \forall c \Delta T = (885\,kg/m^3)(\frac{4}{3}\pi)(0.00025m)^3(1900\,J/kg\,K)(200K)$

$Q = 0.022\,J = 22\,mJ$ for each droplet