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MAE 101C
7-11-07

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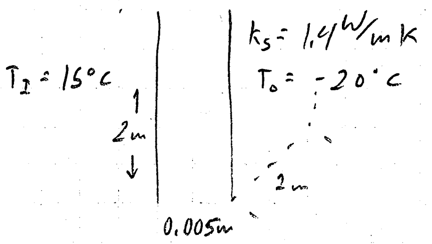
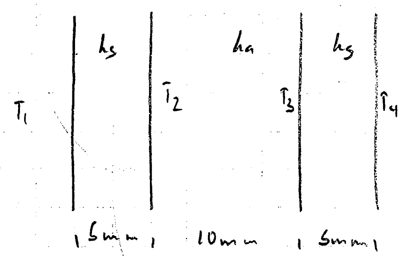
1.6 Problem: A glass window conducts heat from a hot room to the cold outdoors

Given:
single pane | Double pane





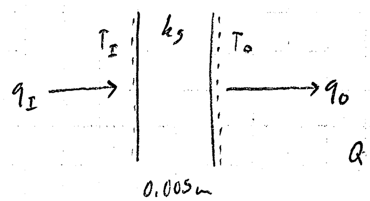
$k_a = 0.024\ W/mK$
$T_1 = 288K$
$T_2 = 283K$
$T_3 = 258K$
$T_4 = 253K$

Find: a) Heat loss through single pane window
b) Heat loss through double pane window

Assume: No energy generated in window panes
Glass surfaces uniform temperature
One dimensional, steady state heat transfer
Constant thermal conductivity

Analysis:

a)

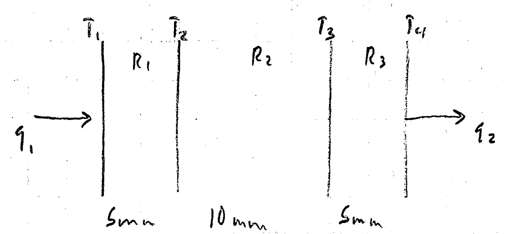


$$q = Aq'' = Ak\frac{\Delta T}{L}$$

$$Q = (2m)(2m)(1.4\ W/mK)(288K - 253K)(0.005m)^{-1}$$

$$Q = 19600\ W = 19.6\ kW$$

b)



$$Q = \frac{T_1 - T_2}{R_T}$$

$$R_T = R_1 + R_2 + R_3 = \frac{L_1}{A_1 k_1} + \frac{L_2}{A_2 k_2} + \frac{L_3}{A_3 k_3} = \frac{0.005m}{(2m^2)(1.4\ W/mK)} + \frac{0.01m}{(2m^2)(0.024\ W/mK)} + \frac{0.005m}{(2m^2)(1.4\ W/mK)}$$

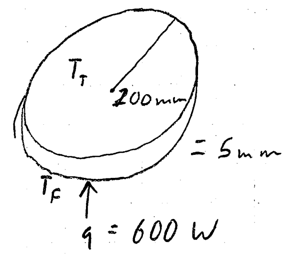
$$R_T = 0.212\ K/W$$

$$Q = \frac{288K - 253K}{0.212\ K/W} = 165\ W$$

+4

1.10 Problem: A pan is to be made from either Al, or Cu

Given:



$k_{Al} = 240\ W/mK$

$k_{Cu} = 390\ W/mK$

$T_T = 110°C = 383\ K$

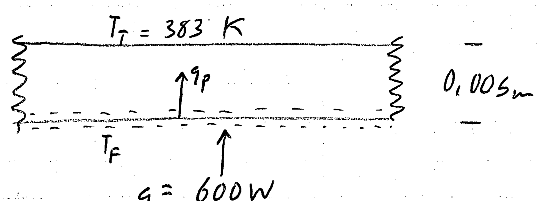
Find: T_F for both materials

Assume: No heat generated in pan
Steady state, one dimensional heat flow
Uniform surface properties, also constant

Analysis:

$$\frac{dE}{dt}_{st} = \dot{E}_{IN} - \dot{E}_{OUT} + \cancel{\dot{E}_g}^{\,0} = 0$$ Energy Balance at stove top surface T_F

$$\dot{E}_{IN} = \dot{E}_{OUT}$$



$$q_s = q_p$$

$$q_s = A q_p'' = \frac{\pi}{4} d^2 k \frac{\Delta T}{L} = \frac{\pi}{4} d^2 \frac{k}{L} (T_F - T_T)$$

$$T_F = q_s \left(\frac{4}{\pi}\right) L/kd^2 + T_T$$

Aluminum

$$T_F = (600\,W)\left(\frac{4}{\pi}\right)\left(\frac{0.005m}{240 W/mK (0.2m)^2}\right) + 383\ K = 383.4\ K$$

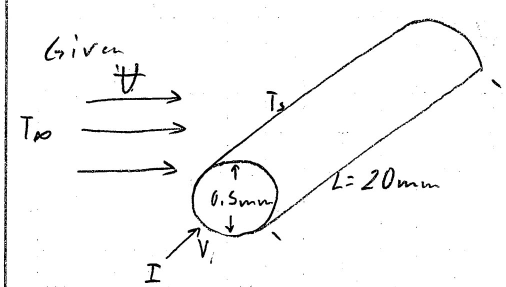
+5

Copper

$$T_F = (600\,W)(4/\pi)\left(\frac{0.005m}{(390\ W/mK)(0.2m)^2}\right) + 383\ K = 383.25\ K$$

1.17 Problem: A velocimeter measures the speed of air by using electrical heating

Given



$V = 6.25 \times 10^{-5} h^2$ $\quad h = W/m^2K$

$T_\infty = 298\ K$ $\quad V_A - V_B = 5V$

$T_s = 348\ K$ $\quad I = 0.1A$

Find: Velocity of the air ϑ

Assume: Constant properties, uniform properties
Steady state, heat conduction negligible
All energy created due to electric current

Analysis: $\frac{dE_{st}}{dt} = \cancel{\dot{E}_{IN}}^{0} - \dot{E}_{OUT} + \dot{E}_S = 0$ constant temperature of wire

$\dot{E}_S = \dot{E}_{OUT}$ $\qquad \dot{E}_S = P = V \cdot I = 5V \times 0.1A = \frac{1}{2} W$

$\dot{E}_{OUT} = A h (T_s - T_\infty) + A \cancel{\varepsilon\sigma}^{0} (T_s^4 - T_{sur}^4)$ $\qquad T_{sur} = T_\infty$

$h = 10.9\ W \cdot s^{0.8}/m^{2.8} K\ \vartheta^{0.8}$? Radiation transfer negligible

$V \cdot I = \pi D L (10.9\ Ws^{0.8}/m^{2.8}) \vartheta^{0.8} (T_s - T_\infty)$

$$\vartheta = \left[\frac{VI}{\pi D L (T_s - T_\infty)(10.9\ Ws^{0.8}/m^{2.8})} \right]^{5/4}$$

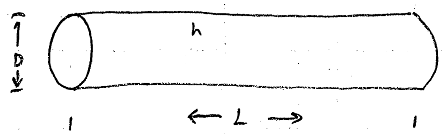
$$\vartheta = \left[\frac{(5\ J/C)(0.1\ C/s)}{\pi (0.0005m)(0.02m)(50°K)(10.9\ Ws^{0.8}/m^{2.8})} \right]^{5/4}$$

$\vartheta = 67.89\ m/s$

+2

1.28 Problem: A steam pipe emits energy and maintains a constant surface temperature

Given:



$T_{sur} = 25°C = 298\ K$

$T_s = 150°C = 423\ K$

$D = 0.1m$

$L = 25m$

$h = 10\ W/m^2 K \qquad \varepsilon = 0.8$

Find: a) rate of heat loss from steam line

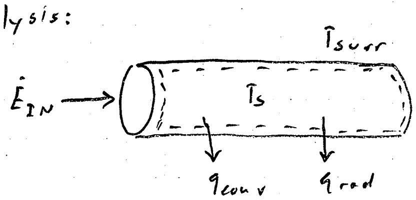
b) If steam if generated in a gas fired boiler withe efficiency $\eta_f = 0.90$ and gas is priced at \$0.01/MJ what is the annual cost of heat loss through the line

Assume: Negligible heat loss due to conduction (wanted)

Steady state, constant uniform properties

One Dimensional heat transfer out from center

Analysis:



$T_{surr} = T_\infty$

a) Heat loss $= A(q''_{conv} + q''_{rad})$

$q''_{conv} = h(T_s - T_{sur})$

$q''_{rad} = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ Assume no absorptivity and spherical atmosphere

$Q_{Loss} = [\pi(0.1m)(25m)][10\ W/m^2K(423K - 298K) + 0.8(5.67\times10^{-8} W/m^2K^4)((423K)^4 - (298)^4)]$

$Q_{Loss} = 18413.75\ W$

b) $1\ yr = (365\ day)(86400\ s/day) = 31536000\ sec$

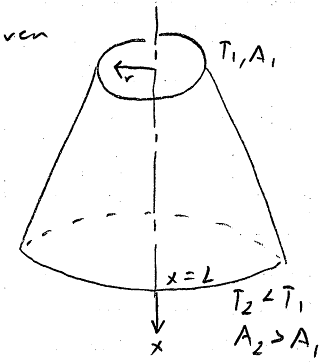
$TC = \dfrac{Q(1\ yr)(C_g)}{(0.9)(\ }= \dfrac{(18413.75\ W)(31536000\ sec)(\$0.01/10^6 J)}{0.9} =$

$TC = \$6452.20$

+5

2.5 Problem: A solid cone serves as a support

Given



T_1 constant

$k = k_0 - aT \quad a>0$

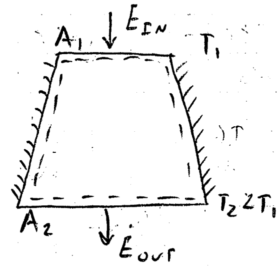
Find: Do q_x, q''_x, k, dT/dx increase or decrease with x

Assume: One dimensional heat flow
Steady state constant uniform properties
No heat generation

Analysis

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k(T)\, r\cancelto{0}{\frac{\partial T}{\partial r}}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k \cancelto{0}{\frac{\partial T}{\partial \theta}}\right) + \frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) + \cancelto{0}{\dot{q}} = \rho c_p \cancelto{0}{\frac{\partial T}{\partial t}}$$

BCs 1 $T(0,t) = T_1$ One dimensional (x) heat conduction
2 $T(L,t) = T_2$



$$\dot{E}_{IN} - \dot{E}_{OUT} + \cancel{\dot{E}_g} = 0$$

$$q_1 = q_2 \qquad q = q'' A$$

$$\frac{\partial q}{\partial x} = q''\frac{\partial A}{\partial x} + A\frac{\partial q''}{\partial x} = 0$$

a) q remains constants with x by 1st law of thermo

$$\frac{\partial q''}{\partial x} = -\frac{q''}{A}\frac{\partial A}{\partial x} \qquad \frac{\partial A}{\partial x} > 0 \Rightarrow \frac{\partial q''}{\partial x} < 0$$

b) q''_x decreases with x

c) $k = k_0 - aT$ increases with x as T decreases with x

d) $q = -kA\frac{dT}{dx} = \text{constant} \qquad \frac{\partial q}{\partial x} = -k\left(A\frac{\partial^2 T}{\partial x^2} + \frac{\partial A}{\partial x}\frac{\partial T}{\partial x}\right) + A\frac{dT}{dx}\frac{\partial k}{\partial x} = 0$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} = 0 \Rightarrow \frac{\partial T}{\partial x} \text{ constant}$$

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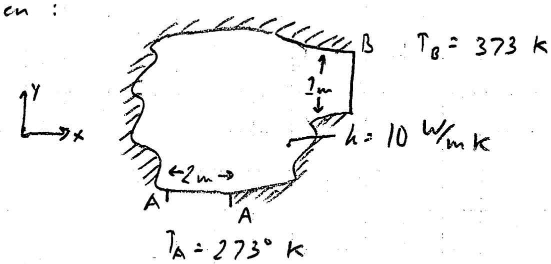
2.11 Problem: The two dimensional body has two temperature gradients

Given:



$$\left(\frac{\partial T}{\partial y}\right)_A = 30\ \text{K/m}$$

Find: $\frac{\partial T}{\partial y}$, $\frac{\partial T}{\partial x}$ at B

Assume: Constant, uniform properties k
Steady state
No heat generation, steady state value

Analysis

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\cancel{\frac{\partial T}{\partial z}}^{0}\right) + \cancel{\dot{q}}^{0} = \rho c_p \cancel{\frac{\partial T}{\partial t}}^{0}$$

$$k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial y^2} = 0 \qquad \left(\frac{\partial T}{\partial y}\right)_B = 0 \quad T_B \text{ constant}$$

$$\dot{E}_{IN} - \dot{E}_{OUT} + \cancel{\dot{E}_g}^{0} = \cancel{\dot{E}_{st}}^{0} \qquad Q_A - Q_B = 0$$

$$Q_A = Q_B \qquad Q_A = -kA_A\left(\frac{\partial T}{\partial y}\right)_A$$

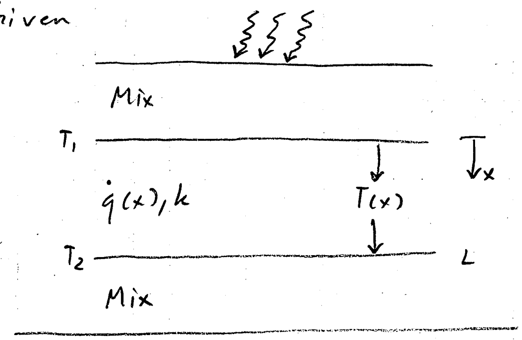
$$Q_B = -kA_B\left(\frac{\partial T}{\partial x}\right)_B$$

$$\frac{\partial T}{\partial x} = \frac{A_A}{A_B}\left(\frac{\partial T}{\partial y}\right)_A = \frac{(2\,m)b}{(1\,m)b}\left(30\ \text{K/m}\right) = 60\ \text{K/m}$$

t/0

2.27 Problem: A salt-gradient solar pond is heated by sun radiation

Given



$T_2 > T_1$

$$\dot{q} = Ae^{-ax}$$

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

$A\,[W/m^3]$ $\quad a\,[1/m]$
$B\,[K/m]$ $\quad C\,[K]$
$k\,[W/mK]$

Find: a) Obtain expressions for the rate at which heat is transferred per unit area from the lower mixed layer to the central layer and from central layer to upper layer
b) Determine whether conditions are steady or transient
c) Obtain an expression for the rate at which thermal energy in the entire central layer is generated per unit surface area

Assume: One dimensional heat flow by conduction
A, k, a, B, C constant

Analysis:

a) $x = 0$ at upper layer Boundary

$$q = -k\frac{\partial T}{\partial x}\Big|_{x=0} = -k\left(+\frac{A}{ka}e^{-ax} + B\right)\Big|_{x=0}$$

$$q = -Bk - \frac{A}{a}$$

$x = L$ at lower layer boundary

+3

$$q = -k\frac{\partial T}{\partial x}\Big|_{x=L} = -k\left(\frac{A}{ka}e^{-ax} + B\right)\Big|_{x=L}$$

$$q = -Bk - \frac{A}{a}e^{-aL}$$

b) $$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial x} = \frac{A}{ka}e^{-ax} + B \qquad \frac{\partial^2 T}{\partial x^2} = \frac{-A}{k}e^{-ax}$$

+4

$$\cancel{\frac{-A}{k}e^{-ax}}^{\,0} + \cancel{\frac{A}{k}e^{-ax}}^{\,0} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \frac{\partial T}{\partial t} = 0 \quad \underline{\text{steady}}$$

2.27

c)

$\dot{q} = Ae^{-ax}$

+0

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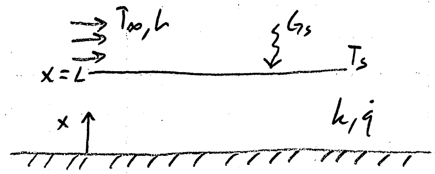
2.31 Problem: A layer of coal sits in the sun

Given:



$L = 1\,m$ $\quad \dot{q} = 20\ W/m^3$
$h = 5\ W/m^2K$ $\quad T_\infty = 298\ K$
$G_s = 400\ W/m^2$ $\quad \alpha_s = \varepsilon = 0.95$

Find a) Write the steady state heat diffusion equation

$$T(x) = T_s + \frac{\dot{q}L^2}{2k}\left(1 - x^2/L^2\right)$$

b) Obtain an expression for the rate of heat transfer by conduction per unit area at $x=L$. Applying an energy balence for the top layer, find T_s, $T(0)$

c) For $h = 5\ W/m^2K$ compute and plot T_s, $T(0)$ as a function of G_s for $50 \le G_s \le 500\ W/m^2$.
For $G_s = 400\ W/m^2$ compute/plot T_s, $T(0)$ as a function of h for $5 \le h \le 50\ W/m^2K$

Assume: One dimensional steady state heat conduction through coal

Analysis:

a) Eq 2.19 $\quad \frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \frac{\dot{q}}{k} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0 \qquad BCs \quad 1 \quad x = L \quad T = T_s$$
$$\qquad\qquad\qquad 2 \quad x = 0 \quad \frac{\partial T}{\partial x} = 0$$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 \quad \Rightarrow \quad T(x) = T_s$$

$$\frac{\dot{q}}{k}x + C_1\Big|_{x=0} = 0 \quad \Rightarrow \quad C_1 = 0$$

f4

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_2 \qquad \frac{\dot{q}L^2}{2k} + C_2 = T_s \qquad C_2 = T_s - \frac{\dot{q}L^2}{2k}$$

temperature distribution graph ?

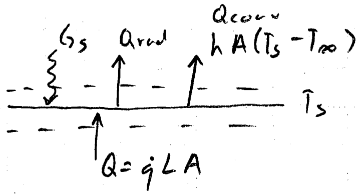
$$T(x) = T_s - \frac{\dot{q}L^2}{2k} + \frac{\dot{q}}{2k}x^2 = T_s + \frac{\dot{q}L^2}{2k}\left(1 - x^2/L^2\right)$$

$$\text{At } x = 0 \quad T(0) = T_s + \frac{\dot{q}L^2}{2k} \qquad \frac{\partial T}{\partial x}\Big|_{x=0} = 0$$

b) $$Q = -kA\frac{\partial T}{\partial x}\Big|_{x=L} = -kA\frac{\partial}{\partial x}\left(-\frac{\dot{q}}{2k}x^2\right) = +kA\frac{\dot{q}}{k}x\Big|_{x=L}$$

$$Q = +\cancel{k}A\frac{\dot{q}L}{\cancel{k}} = \dot{q}LA \qquad q'' = \dot{q}L$$

2.31



$$\dot{E}_{IN} + \cancel{\dot{E}_g}^{\,0} - \dot{E}_{OUT} = \cancel{\dot{E}_{ST}}^{\,0}$$

$$q_{x=L} = \dot{q}L$$

$$Q_{COND} + Q_{AB} - Q_{RAD} - Q_{CONV} = 0$$

$$\dot{q}L + \alpha G_s - \varepsilon\sigma(T_s^4) - h(T_s - T_\infty) = 0$$

$$(20\,W/m^3)(1\,m) + 0.15(400\,W/m^3) - 0.95(5.67\times10^{-8}\,W/m^2K^4)(T_s^4) - (5\,W/m^2K)(T_s - 298\,K) = 0$$

$$(1890\,W/m^2) - (5.3865\times10^{-8}\,W/m^2K^4)T_s^4 - (5\,W/m^2K)T_s = 0$$

$$T_s = 295.67\,K$$

$$T(0) = T_s + \frac{\dot{q}L^2}{2k} = 295.67\,K + \frac{(1\,m)^2(20\,W/m^3)}{2(0.26\,W/mK)} = 334.13\,K$$

+5

c) $h = 5\,W/m^2K$ $\quad 50 \le G_s \le 500\,W/m^2$

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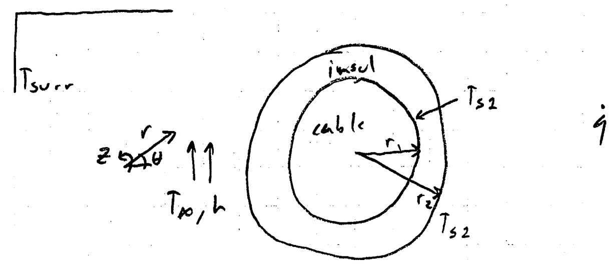
2.41 Problem: An electric cable of radius r_1 and thermal conductivity k_c is enclosed by an insulating sleeve of outer radius r_2 and experiences convection heat transfer and radiation exchange with the adjoining air and large surroundings

Given:



Find: a) Write ss forms of heat diffusion equation for the insulation and cable.

$$T(r) = T_{s2} + (T_{s1} - T_{s2}) \frac{\ln(r/r_2)}{\ln(r_1/r_2)} \quad \text{Insul}$$

$$T(r) = T_{s,1} + \frac{\dot{q} r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right) \quad \text{Cable}$$

b) Apply Fouriers law, find q'_r, find $q_r(\dot{q}, r_1)$

c) Apply energy balance to control surface on outer surface of sleeve, find $T_{s2}(\dot{q}, r_1, h, T_\infty, \varepsilon, T_{sur})$

d) Consider 250 A with $R_e = 0.005\ \Omega/m$, $r_1 = 15$ mm
$k_c = 200$ W/mK. $k_s = 0.15$ W/mK, $r_2 = 15.5$ mm, $h = 25$ W/m²K
$\varepsilon = 0.9$, $T_\infty = 25°C$, $T_{sur} = 35°C$, Find T_{s1}, T_{s2}, T_o

e) With all conditions the same, compute and plot T_o, T_{s1}, T_{s2} as a $f(r_2)$ for $15.5\text{mm} \le r_2 \le 20\text{mm}$

Assume: Constant thermal properties
1 dimensional heat conduction

Analysis:

a) Insulation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)} + \cancel{\dot{q}} = \cancel{\rho c_p \frac{\partial T}{\partial t}}$$

$$\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) = 0 \qquad \text{BC} \quad 1 \quad x = r_1 \quad T = T_{s1} \qquad -k\frac{\partial T}{\partial r}\Big|_{r=r_1} = q$$
$$\qquad\qquad 2 \quad x = r_2 \quad T = T_{s2}$$

$$kr\frac{\partial T}{\partial r} = C_1 \qquad \frac{\partial T}{\partial r} = \frac{C_1}{kr} \qquad T = \frac{C_1}{k}\ln r + C_2$$

$$T_{s1} = \frac{C_1}{k}\ln r_1 + \cancel{C_2} \qquad T_{s2} = \frac{C_1}{k}\ln r_2 + C_2$$

$$-\left(T_{s2} = \frac{C_1}{k}\ln r_2 + \cancel{C_2}\right)$$

$$T_{s1} - T_{s2} = \frac{C_1}{k}(\ln r_1 - \ln r_2) \qquad C_1 = \frac{k(T_{s1} - T_{s2})}{\ln(r_1/r_2)}$$

2.41 $C_2 = T_{s2} - \frac{C_1}{k}\ln r_2$

$$T(r) = \frac{1}{\cancel{k}}\frac{\cancel{k}(T_{s1}-T_{s2})}{\ln(r_1/r_2)}\ln r + \left(T_{s2} - \frac{1}{k}\frac{k(T_{s1}-T_{s2})}{\ln(r_1/r_2)}\ln r_2\right)$$

$$= \frac{(T_{s1}-T_{s2})}{\ln(r_1/r_2)}(\ln r - \ln r_2) + T_{s2}$$

$$T(r) = T_{s2} + (T_{s1}-T_{s2})\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$$

Cable

$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot q = 0 \qquad \frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) = -\dot q r$

1 $\quad r = r_1 \quad T = T_{s1}$

2 $\quad r = 0 \quad \frac{\partial T}{\partial r}\Big|_{r=0}$ finite

$$kr\frac{\partial T}{\partial r} = -\frac{\dot q r^2}{2} + C_1 \qquad \frac{\partial T}{\partial r} = -\frac{\dot q r}{2k} + \frac{C_1}{kr}$$

$$T = -\frac{\dot q r^2}{4k} + \frac{C_1}{k}\ln r + C_2$$

$$\frac{\partial T}{\partial r} = -\frac{\dot q r}{2k} + \frac{C_1}{kr}\Big|_{r=0} \Rightarrow C_1 = 0$$

+4

$$T = -\frac{\dot q r^2}{4k} + C_2 \qquad T_{s1} = -\frac{\dot q r_1^2}{4k} + C_2 \qquad C_2 = T_{s1} + \frac{\dot q r_1^2}{4k}$$

$$T = T_{s1} + \frac{\dot q}{4k}(r_1^2 - r^2) = T_{s1} + \frac{\dot q r_1^2}{4k}\left(1 - \frac{r^2}{r_1^2}\right)$$

Temp distribution graph?

b) Fouriers law $q' = -2\pi r k\frac{\partial T}{\partial r}\Big|$ sleeve

$$\frac{\partial T}{\partial r} = \frac{(T_{s1}-T_{s2})\,\frac{1}{r_2}}{\ln(r_1/r_2)\,(r/r_2)} = \frac{T_{s1}-T_{s2}}{r\ln(r_1/r_2)}$$

$$q'_r = \frac{-2\pi k_s \cancel{r}(T_{s1}-T_{s2})}{\cancel{r}\ln(r_1/r_2)} = \frac{2\pi k_s(T_{s1}-T_{s2})}{\ln(r_2/r_1)}$$

$$\dot E_{IN} + \cancel{\dot E_g} - \dot E_{OUT} = \cancel{\dot E_{st}}$$

$$\dot Q_{IN} = Q_{OUT}$$

+5

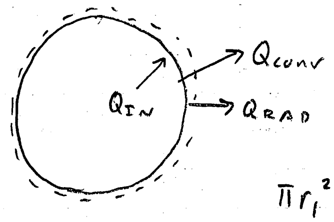
$$\dot Q_{IN} = -kA\frac{\partial T_c}{\partial r}\Big|_{r=r_1} \qquad \dot Q_{OUT} = q'_r(L)$$

$$-2\pi r_1 L\cancel{k_c}\frac{\partial}{\partial r}\frac{-\dot q r^2}{4\cancel{k_c}}\Big|_{r=r_1} = \frac{2\pi r_1 L\dot q\,2r}{4}\Big|_{r=r_1} = \pi r_1^2 L\dot q$$

$$\cancel{L}\pi r_1^2\dot q = q'_r\cancel{L} \qquad q'_r = \pi r_1^2\dot q$$

2.41 c)

$Q_{IN} - Q_{CONV} - Q_{RAD} = 0$



$L q_r' - h 2\pi r_2 L (T_{s2} - T_\infty) - \varepsilon\sigma 2\pi r_2 L (T_{s2}^4 - T_{sur}^4) = 0$

$\pi r_1^2 \cancel{L} \dot{q} - 2\pi r_2 \cancel{L} h (T_{s2} - T_\infty) - 2\pi r_2 \cancel{L} \varepsilon\sigma (T_{s2}^4 - T_{sur}^4) = 0$

$\pi r_1^2 \dot{q} - 2\pi r_2 h (T_{s2} - T_\infty) - 2\pi r_2 \varepsilon\sigma (T_{s2}^4 - T_{sur}^4) = 0$

+5

d) $\pi r_1^2 \dot{q} = I^2 R_e' = (250\ C/s)^2 (0.005\ Js/C^2 m) = 312.5\ W/m = \dot{q}'$

$2\pi r_2 h (T_{s2} - T_\infty) + 2\pi r_2 \varepsilon\sigma (T_{s2}^4 - T_{surr}^4) = \pi r_1^2 \dot{q}$

$2.435\ W/mK (T_{s2} - 298K) + 4.97 \times 10^{-9}\ W/mK^4 (T_{s2}^4 - (308K)^4) = 312.5\ W/m$

$2.435\ W/mK (T_{s2}) + 4.97 \times 10^{-9}\ W/mK^4 (T_{s2}^4) = 1081.96\ W/m$

$T_{s2} = 394.77\ K$

$q_r' = \pi r_1^2 \dot{q} \qquad q_r' = \dfrac{2\pi k_s}{\ln(r_2/r_1)} (T_{s1} - T_{s2})$

+2

$T_{s1} = T_{s2} + \ln\left(\frac{r_2}{r_1}\right) \dfrac{r_1^2 \dot{q}}{2k_s}$

$= 394.77K + \ln\left(\frac{15.5}{15}\right) \dfrac{\left(\frac{15}{1000} m\right)^2 (99.4718\ W/m)}{2(0.15\ W/mK)}$

$= ?$

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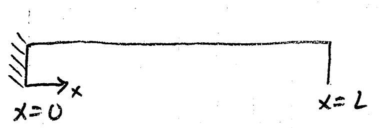
2.52 Problem: A plane wall is insulated on one side $x=0$

Given:



$t=0 \qquad T(x,0) = T_i$

$t=0^+ \qquad T(L,0) = T_s$

Find a) Verify that the following equation satisfies the heat equation and boundary conditions

$$\frac{T(x,t) - T_s}{T_i - T_s} = C_1 \exp\left(-\frac{\pi^2 \alpha t}{4L^2}\right) \cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

b) Obtain an expressions for heat flux at $x=0$, $x=L$

c) Sketch the temperature distribution $T(x)$ at $t=0$, $t \to \infty$ and at an intermediate time. Sketch the variation with time of the heat flux $x=L$, $q_L''(t)$

d) What effect does α have on the thermal response of the material to a change in surface temperature

Assume: One-dimensional heat conduction
Constant thermal properties
No heat generation

Analysis

a) $$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \cancel{\frac{\dot{q}}{k}} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

IC $T(x, t=0) = T_I$
$T(L, 0^+) = T_s$

BC $x=0 \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$

$x=L$

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