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MAE 104
10-16-07

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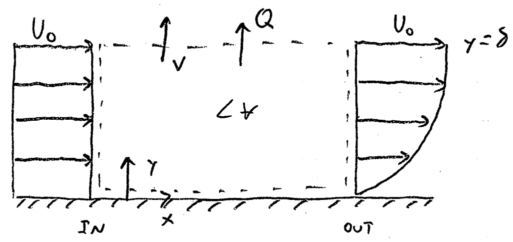
P1

Problem: An incompressible fluid flows past an impermeable flat plate

Given:



$\eta = \frac{y}{\delta}$

$u_2 = U_0\left(\frac{3\eta - \eta^3}{2}\right)$

Find: Volume Flow Q across surface

Assume: Steady flow $\partial/\partial t = 0$
Incompressible $\partial\rho = 0$
2D flow $w=0,\ \partial/\partial z = 0$

Analysis

Conservation of Mass: $\cancel{\frac{\partial}{\partial t}\iiint_{CV}\rho\, d\forall}^{0} + \oiint \rho \vec{V}\cdot d\vec{s} = 0 \Rightarrow \oiint_{CS} \vec{V}\cdot d\vec{s}$

$$\oiint_{CS_{IN}} U_1 \hat{\imath}(-\hat{\imath})\, dA_1 + \iint_{CS_{OUT}} U_2 \hat{\imath}(\hat{\imath})\, dA_2 + \iint_{CS_{UP}} V\hat{j}(\hat{j})\, dA_3 = 0$$

$$Q_{OUT} = \iint_{CS_1} U_1\, dA_1 - \iint_{CS_2} U_2\, dA_2 = \iint_0^{\eta=1} b\delta U_0\, d\eta - \iint_0^{\eta=1} b\delta U_0\left(\frac{3\eta - \eta^3}{2}\right)d\eta \quad +4$$

$$= b\delta U_0(1) - b\delta U_0 \tfrac{1}{2}\left(\tfrac{3}{2}\eta^2 - \tfrac{1}{4}\eta^4\right)\Big|_0^{\eta=1}$$

$$= b\delta U_0 - \tfrac{1}{2}b\delta U_0\left(\tfrac{3}{2} - \tfrac{1}{4}\right) = b\delta U_0\left[1 - \tfrac{1}{2}\left(\tfrac{6}{4} - \tfrac{1}{4}\right)\right]$$

$$= b\delta U_0\left[1 - \tfrac{1}{2}\left(\tfrac{5}{4}\right)\right] = b\delta U_0\left[1 - \tfrac{5}{8}\right] = \tfrac{3}{8}\, b\delta U_0 \quad +2$$

1	6
2	7
3	7
4	8
5	6
6	10
7	6
8	4
tot	54

P2

Problem: A cylinder immersed in a flow creates a broad wake

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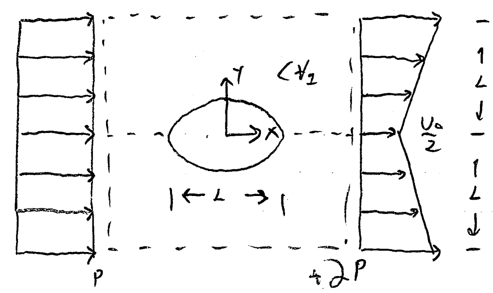
Given:



$U_0 = 4\,m/s \quad L = 0.8\,m$

$\rho = 998\,kg/m^3$

Find: Drag Force on cylinder

$C_D = 2F/\rho U_0^2 bL$

Need steady-flow continuity to find $H = \frac{3}{4}L$

Assume: Incompressible flow $\partial\rho = 0$
Steady flow $\partial/\partial t = 0$
Inviscid flow $\mu = 0$

$U_0 = mL + \frac{U_0}{2}$

$L \quad U_0 \qquad 0 \quad \frac{U_0}{2}$

$y = mx + b \qquad \frac{U_0}{2} = b \qquad \frac{U_0}{2L} = m$

Analysis:

$$V_2 = \frac{U_0}{2} + \frac{U_0 y}{2L}$$

Conservation of Linear Momentum for half CV

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$$CV_2 \quad \cancel{\frac{\partial}{\partial t}\iiint \rho \vec{V}\cdot\vec{V}\,d\forall} + \oiint (\rho\vec{V}\cdot d\vec{S})\cdot\vec{V} = -\cancel{\oiint P\,d\vec{S}} + \cancel{\iiint \rho f\,d\forall} + \cancel{\vec{F}_{VISCOUS}} + F_{DRAG}$$

$$\iint_{CS_1} \rho\vec{V}_1\cdot\vec{V}_1 d\vec{S}_1 + \iint_{CS_2} \rho\vec{V}_2\cdot\vec{V}_2\,dS_2 = \frac{1}{2}F_{DRAG}$$

$$\frac{1}{2}F_D\,\hat{\imath} = \int_0^L \rho b\,U_0\hat{\imath}\cdot U_0\hat{\imath}\cdot(-\hat{\imath})\,dy + \int_0^L \rho b\left(\frac{U_0}{2}+\frac{U_0 y}{2L}\right)\hat{\imath}\left(\frac{U_0}{2}+\frac{U_0 y}{2L}\right)\hat{\imath}\,(\hat{\imath})\,dy$$

$$= -\rho b U_0^2 L\hat{\imath} + \hat{\imath}\rho b\int_0^L\left(\frac{U_0^2}{4} + \frac{U_0^2 y}{2L} + \frac{U_0^2 y^2}{4L^2}\right)dy$$

$$= -\rho b U_0^2 L\hat{\imath} + \rho b\left[\frac{U_0^2}{4}y + \frac{U_0^2 y^2}{4L} + \frac{U_0^2 y^3}{12L^2}\right]\Bigg|_0^L$$

$$= -\rho b U_0^2 L\hat{\imath} + \rho b U_0^2 L\left(\frac{7}{12}\right)\hat{\imath}$$

$$F_D = -\frac{5}{6}\rho b U_0^2 L\,\hat{\imath}$$

$$F_{D,C} \neq \frac{5}{6}\rho b U_0^2 L\,\hat{\imath} \qquad F_D = \frac{1}{3}\rho U^2 L b$$

$$F'_{D,C} = \frac{5}{6}\rho\,U_0^2 L\,\hat{\imath} = \frac{5}{6}(998\,kg/m^3)(4\,m/s)^2(0.8\,m)\,\hat{\imath}$$

$$F'_{D,C} = 10645.33\ N/m\ \hat{\imath}$$ -1 continuous

P2

$C_D = 2F / \rho U_0^2 b L \qquad C_D' = \frac{2F'}{\rho U_0^2 L}$

$C_D' = \frac{2(10645.33 \text{ N/m})}{(998 \text{ kg/m}^3)(4 \text{ m/s})^2 (0.8 \text{ m})} = \frac{5}{3}$ +1 contrib

P3

Problem: An idealized incompressible flow has a velocity
$\vec{V} = 4xy^2\hat{i} + f(y)\hat{j} - zy^2\hat{k}$

Given: $\vec{V} = 4xy^2\hat{i} + f(y)\hat{j} - zy^2\hat{k}$

Find $f(y)$ that satisfies the continuity equation

Analysis

Incompressible, steady state

$\Rightarrow \nabla \cdot \vec{V} = 0 \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ +4

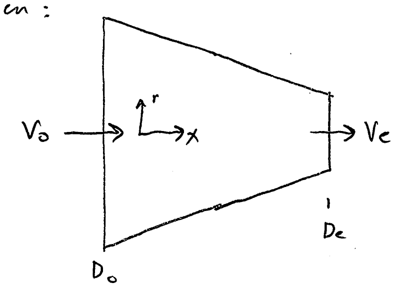
$4y^2 + \frac{\partial v}{\partial y} + (-y^2) = 0$ $\quad \frac{\partial v}{\partial y} = -3y^2$ +2 $\quad f(y) = -y^3 + C$ +1/2

$\vec{V} = 4xy^2\hat{i} - y^3\hat{j} - zy^2\hat{k}$

P4

Problem: Air flows under steady ~ 1D conditions through a conical nozzle

Given:



$a \sim 340 \ m/s$

Find: Minimum D_e/D_o for which we can safely neglect compressibility effects if

a) $V_o = 10 \ m/s$

b) $V_o = 30 \ m/s$

Assume: Steady flow

Compressibility pertinent at $M \geq 0.3$ +2

Uniform flow profile in r

Analysis:

$$\oiint \rho \vec{V} \cdot d\vec{s} = 0 \Rightarrow -\int V_o \, dA_1 + \int V_e \, dA_2 = 0$$

+2

$$\frac{\pi}{4} D_o^2 V_o = \frac{\pi}{4} D_e^2 V_e \qquad V_e = \left(\frac{D_o}{D_e}\right)^2 V_o$$

$$V_e \ Max = 102 \ m/s$$

+1 +1

$$102 \ m/s \geq \left(\frac{D_o}{D_e}\right)^2 V_o$$

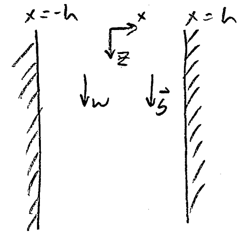
problem asks for $\frac{D_e}{D_o}$

a) $$\frac{102 \ m/s}{10 \ m/s} \geq \left(\frac{D_o}{D_e}\right)^2 \qquad \frac{D_o}{D_e} \leq 3.194 \ +1$$

b) $$\frac{102 \ m/s}{30 \ m/s} \geq \left(\frac{D_o}{D_e}\right)^2 \qquad \frac{D_o}{D_e} \leq 1.84 \ +1$$

Problem: A viscous fluid falls due to gravity between two pletes

Given:



$u = v = 0$

$w = w(x)$

Find: Velocity profile $w(x)$

Assume: Steady, incompressible $\frac{\partial}{\partial t} = \partial \rho = 0$
No slip condition $w(h) = w(-h) = 0$
No flow in x, y directions, No pressure gradients

Analysis

$$\rho\left[\cancel{u}\frac{\partial w}{\partial x} + \cancel{v}\frac{\partial w}{\partial y} + w\frac{\partial \cancel{w}}{\partial z}\right] = -\cancel{\nabla P} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \cancel{w}}{\partial y^2} + \frac{\partial^2 \cancel{w}}{\partial z^2}\right) + \rho \vec{g}$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{\rho}{\mu} g \qquad w = -\frac{\rho}{2\mu} g x^2 + C_1 x + C_2$$

$$\frac{\partial w}{\partial x}(0) = 0 \qquad -\frac{\rho g}{\mu} x + C_1 = 0 \Rightarrow C_1 = 0$$

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$$w(x) = C_2 - \frac{1}{2}\frac{\rho}{\mu} g x^2$$

$$w(h) = 0 = C_2 - \frac{1}{2}\frac{\rho}{\mu} g h^2 \qquad C_2 = \frac{1}{2}\frac{\rho g}{\mu} h^2$$

$$w(x) = \frac{1}{2}\frac{\rho g}{\mu} h^2 - \frac{1}{2}\frac{\rho g}{\mu} x^2 = \frac{1}{2}\frac{\rho g}{\mu}(h^2 - x^2)$$

$$w(x) = \frac{\rho g h^2}{2\mu}\left(1 - \left(\frac{x}{h}\right)^2\right)$$

P7

Problem: Consider a velocity field where
$u = cx/(x^2+y^2)$, $v = cy/(x^2+y^2)$ c = constant

Given:
$$\vec{V} = \frac{cx}{x^2+y^2}\hat{\imath} + \frac{cy}{x^2+y^2}\hat{\jmath}$$

Find: a) The time rate of change of the volume of a fluid element per unit volume
b) The vorticity

Assume: Incompressible

Analysis:

a) Eq 2.32 $\nabla \cdot \vec{V} = \frac{1}{\forall}\frac{D\forall}{Dt}$

$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v$$

$$= \frac{(x^2+y^2)c - cx(2x)}{(x^2+y^2)^2} + \frac{(x^2+y^2)c - cy(2y)}{(x^2+y^2)^2}$$

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$$= \frac{2c(x^2+y^2) - 2x^2c - 2y^2c}{(x^2+y^2)^2} = 0$$

$$\frac{1}{\forall}\frac{D\forall}{Dt} = 0$$

b) $$\vec{\xi} = \nabla \times \vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\cancel{\frac{\partial w}{\partial y}}^{0} - \cancel{\frac{\partial v}{\partial z}}^{0}\right)\hat{\imath} + \left(\cancel{\frac{\partial u}{\partial z}}^{0} + \cancel{\frac{\partial w}{\partial z}}^{0}\right)\hat{\jmath} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$$= \left[\frac{(x^2+y^2)0 - cy(2x)}{(x^2+y^2)^2} - \frac{(x^2+y^2)0 - cx(2y)}{(x^2+y^2)^2}\right]\hat{k} = 0$$

Problem: Consider a velocity field where the radial and tangential components of the velocity are $V_r = 0$ $V_\theta = cr$

Given

$V_r = 0$ $V_\theta = cr$ $c = \text{constant}$

Find: Is the flow field irrotational?

Assume: Flow is cylindrical

Analysis

$$\nabla \times \vec{V} = \left(\frac{1}{r}\frac{\partial \cancel{V_z}^{\,0}}{\cancel{\partial \theta}} - \frac{\partial \cancel{V_\theta}^{\,0}}{\cancel{\partial z}}\right)\hat{r} + \left(\frac{\partial \cancel{V_r}^{\,0}}{\cancel{\partial z}} - \frac{\partial \cancel{V_z}^{\,0}}{\cancel{\partial r}}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r V_\theta)}{\partial r} - \frac{\partial \cancel{V_r}^{\,0}}{\cancel{\partial \theta}}\right)\hat{k}$$

$$\nabla \times \vec{V} = \frac{1}{r}\left(\frac{\partial}{\partial r}(cr^2)\right) = \frac{1}{r}(2cr) = 2c$$

$\boxed{\nabla \times \vec{V} = 2c}$ Rotational

$$\nabla \times \vec{V} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(\cancel{V_\phi}^{\,0}\sin\theta) - \frac{\partial \cancel{V_\theta}^{\,0}}{\cancel{\partial \phi}}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\cancel{\sin\theta}}\frac{\partial V_r}{\partial \phi} - \frac{\cancel{\partial}}{\cancel{\partial r}}(r V_\phi)^{0}\right)\hat{\theta}$$

$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}(r V_\theta) - \frac{\partial \cancel{V_r}^{\,0}}{\cancel{\partial \theta}}\right)\hat{\phi}$$

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$$\nabla \times \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(cr^2) = 2c$$

$$\nabla \times \vec{V} = 2c$$