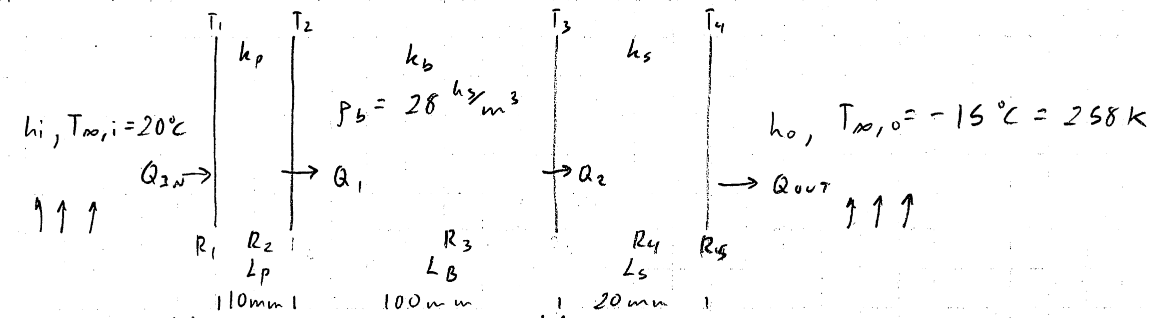
3.13 Problem: A house's wall of wood, fiberglass insulation and plaster board. The wall emits heat to the outside world 7-18-07

69/60

Given:



$h_i = 30\ W/m^2K$ $\quad h_o = 60\ W/m^2K$ $\quad A_T = 350\ m^2$

Find: a) Determine a symbolic expression for the total thermal resistance
b) Determine total heat loss through wall
c) If $h_o = 300\ W/m^2K$ determine percentage increase in heat loss
d) What is the controlling resistance that determines the amount of heat flow through wall

Assume: One-dimensional, steady state heat conduction
No heat generation in wall
No contact heat transfer resistance

Analysis:

a) $$\dot{E}_{IN} + \dot{E}_g^{\,0} - \dot{E}_{OUT} = \dot{E}_{ST}^{\,0} \qquad \dot{E}_{IN} = \dot{E}_{OUT}$$

$$\dot{Q}_{IN} = \dot{Q}_{OUT} \qquad (Q_I = Q_1 = Q_2 = Q_0$$

$$h_i A(T_{\infty i} - T_1) = \frac{k_p A}{L_p}(T_1 - T_2) = \frac{k_B A}{L_B}(T_2 - T_3) = \frac{k_s A}{L_s}(T_3 - T_4) = h_o A(T_4 - T_{\infty o})$$

$$\frac{(T_{\infty i} - T_1)}{R_1} = \frac{(T_1 - T_2)}{R_2} = \frac{T_2 - T_3}{R_3} = \frac{T_3 - T_4}{R_4} = \frac{T_4 - T_\infty}{R_5} = \frac{T_{\infty i} - T_{\infty o}}{R_T}$$

$$R_T = R_1 + R_2 + R_3 + R_4 + R_5$$

$$R_T = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_B}{k_B A} + \frac{L_s}{k_s A} + \frac{1}{h_o A} = \frac{1}{A}\left(\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_B}{k_B} + \frac{L_s}{k_s} + \frac{1}{h_o}\right)$$

b) $$Q = \frac{1}{R_T}(T_{\infty i} - T_{\infty o})$$

$$R_T = \frac{1}{350 m^2}\left((30\ W/m^2K)^{-1} + \frac{0.01m}{0.17\frac{W}{mK}} + \frac{0.1m}{0.038\frac{W}{mK}} + \frac{0.2m}{0.12\frac{W}{mK}} + (60\ \frac{W}{m^2K})^{-1}\right)$$

$$R_T = \underbrace{0.0126\ \frac{K}{W}}_{0.0083}$$

3.13

$Q = \left(0.0126 \frac{K}{W}\right)^{-1}\left(293\,K - 258\,K\right) = 2777.8\,W$ X

c) $h_o = 300\ W/mK$

$R_T = 0.0125 \frac{K}{W}$

$Q' = \left(0.0125 \frac{K}{W}\right)^{-1}\left(293\,K - 258\,K\right) = 2798.7\,W$

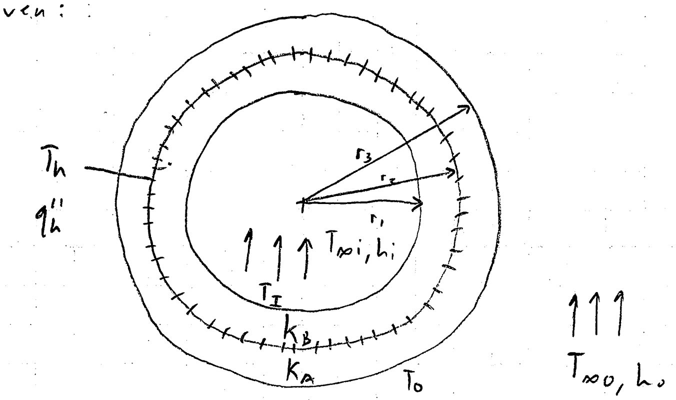
0.75 % increase 0.6%

d) The controlling resistance is the glass fiber insulation
This contributes the most resistance

8/10

3.46 Problem: A composite cylindrical wall is composed of two materials of thermal conductivity.

Given:



Find: a) sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables

b) Obtain an expression that may be used to determine T_h

c) Obtain an expression for the ratio of heat flows to the inner and outer fluids. q_o'/q_i'. How might the variables be adjusted to minimize the ratio

Assume: Steady state, One dimensional heat conduction

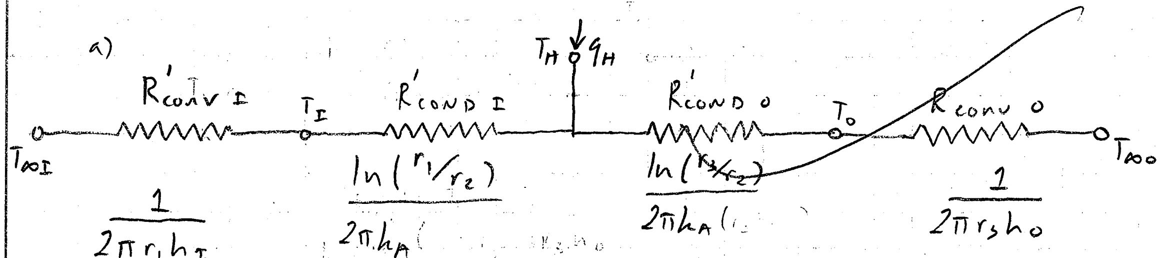
No heat generation in B, A

Negligible contact resistances

Negligible radiation heat loss

Analysis:

a)



Rtotal = ?

b) $\cancel{\dot{E}_{IN}} - \dot{E}_{OUT} + \dot{E}_S = \cancel{\dot{E}_{ST}}$ $\quad \dot{E}_S = \dot{E}_{OUT}$

$$2\pi r_2 q_h'' = \frac{T_H - T_{\infty I}}{R'_{IN}} + \frac{T_H - T_{\infty o}}{R'_{OUT}}$$

$$2\pi r_2 q_h'' = \frac{1}{R'_{IN} R'_{OUT}}\left[R'_{OUT}(T_H - T_{\infty I}) + R'_{IN}(T_H - T_{\infty o})\right]$$

$$2\pi r_2 R'_{IN} R'_{OUT} q_h'' = T_H(R'_{OUT} + R'_{IN}) - R'_{OUT} T_{\infty I} - R'_{IN} T_{\infty o}$$

3.46

$$T_H(R'_{OUT} + R'_{IN}) = 2\pi r_2 R'_{IN} R'_{OUT} q''_h + R'_{OUT} T_{\infty I} + R'_{IN} T_{\infty O}$$

$$T_H = \frac{2\pi r_2 R'_{IN} R'_{OUT} q''_h + R'_{OUT} T_{\infty I} + R'_{IN} T_{\infty O}}{R'_{OUT} + R'_{IN}}$$

$$R'_{IN} = \frac{1}{2\pi r_i h_1} + \frac{\ln(r_1/r_2)}{2\pi k_B} \qquad R'_{OUT} = \frac{\ln(r_3/r_2)}{2\pi k_A} + \frac{1}{2\pi r_3 h_o}$$

c) $$q'_o = \frac{T_H - T_{\infty O}}{\frac{1}{2\pi r_3 h_o} + \frac{\ln(r_3/r_2)}{2\pi k_A}} \qquad q'_I = \frac{T_H - T_{\infty I}}{\frac{1}{2\pi r_1 h_I} + \frac{\ln(r_1/r_2)}{2\pi k_B}}$$

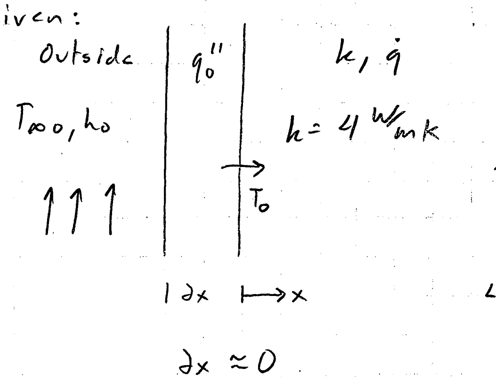
$$\frac{q'_o}{q'_I} = \frac{(T_H - T_{\infty O})}{(T_H - T_{\infty I})}\left[\frac{\frac{1}{2\pi r_1 h_I} + \frac{\ln(r_1/r_2)}{2\pi k_B}}{\frac{1}{2\pi r_3 h_o} + \frac{\ln(r_3/r_2)}{2\pi k_A}}\right]$$

- One can minimize h_o, and increase h_I
- Increase k_B and decrease k_A
- Increase r_3, decrease r_1

9/10

3.79 Problem: The air inside the chamber is heated convectively by a wall with uniform heat generation. A strip heater is placed on the outside of the wall to prevent heat loss

Given:



$T_{\infty i} = 50\,°C$

$T_{\infty o} = 25\,°C$

$h_i = 20\ W/m^2 K$

$h_o = 5\ W/m^2 K$

$\dot{q} = 1000\ W/m^3$

Find: a) Sketch the temperature distribution in the wall on T-x coordinates for the condition where no heat generated in wall is lost to the outside

b) What are the temperatures at the wall boundaries $T(0)$, $T(L)$ for part (a)

c) Determine q_o'' so that all wall heat is transferred to inside chamber

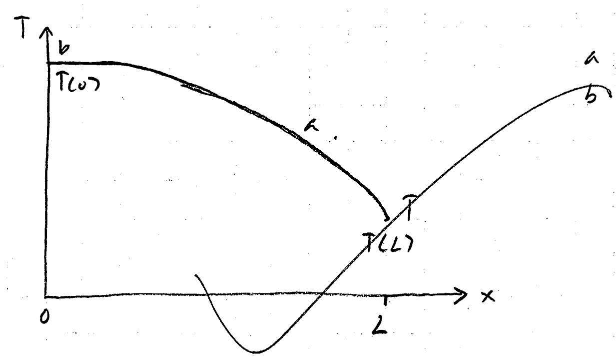
d) If $\dot{q} = 0$, q_o'' = constant, what is $T(0)$

Assume: One dimensional, steady state heat conduction

Constant thermal properties

Analysis:

a) No heat lost to outside: $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$

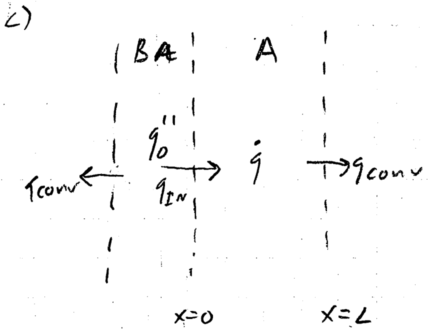


a parabolic within $0 < x < L$

b $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$

3.79

c)



A $\quad \dot{E}_{IN} + \dot{E}_g - \dot{E}_{OUT} = \dot{E}_{ST}^{\nearrow 0}$

$$q_o'' A + \dot{q} A L - h_i (T_L - T_\infty) = 0$$

B $\quad q_o'' = -k \frac{\partial T}{\partial x}\Big|_{x=0} + h(T_o - T_{\infty o})$

A $\quad \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$

1 $\quad \frac{\partial T}{\partial x}\Big|_{x=0} = 0$

2 $\quad -k \frac{\partial T}{\partial x}\Big|_{x=L} = h_i (T_L - T_\infty)$

A).
temp
disteribution graph

? $T(x) = \frac{-\dot{q}}{2k} x^2 + C_1 x + C_2$ $\qquad \frac{\partial T}{\partial x} = -\frac{\dot{q}}{k} x + C_1 \Big|_{x=0} = 0 \Rightarrow C_1 = 0$

$T(x) = C_2 - \frac{\dot{q}}{2k} x^2$ $\qquad \frac{\partial T}{\partial x} = \frac{-\dot{q} x}{k}$

$-k\left(\frac{-\dot{q}L}{k}\right) = h_i (T_L - T_{\infty i})$ $\qquad \dot{q}L = h_i (T_L - T_{\infty i})$

$T_L = \frac{\dot{q}L}{h_i} + T_{\infty i}$

$\frac{\dot{q}L}{h_i} + T_{\infty i} = C_2 - \frac{\dot{q}L^2}{2k}$ $\qquad C_2 = \dot{q}\left(\frac{L^2}{2k} + \frac{L}{h_i}\right) + T_{\infty i}$

$T(x) = T_{\infty i} + \dot{q}\left(\frac{L^2}{2k} + \frac{L}{h_i}\right) - \frac{\dot{q} x^2}{2k}$

$T(0) = 323K + 1000 \frac{W}{m^3}\left(\frac{(0.2m)^2}{2(4 \, W/mK)} + \frac{0.2m}{20 \, W/mK}\right) = 413K$

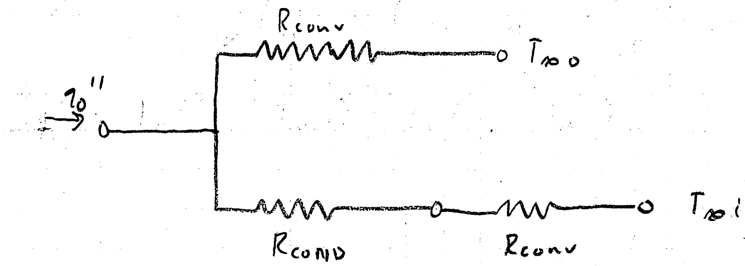
$q_o'' = -k \frac{\partial T}{\partial x}^{\nearrow 0}\Big|_{x=0} + h(T_o - T_{\infty o})$

$q_o'' = 5 W/m^2K \, (413K - 298K) = 575 \, W/m^2$

d) $\quad q_o'' - h_o (T_o - T_{\infty o}) - h_i (T_L - T_{\infty i}) = 0$ $\qquad \frac{\partial T}{\partial x}\Big|_{x=0} = \frac{1}{k}\left(q_o'' - h_o (T_o - T_{\infty o})\right)$

$\frac{\partial^2 T}{\partial x^2} = 0$ $\qquad T = C_1 x + C_2$ $\qquad \frac{\partial T}{\partial x}\Big|_{x=2L} = \frac{h}{k}(T_L - T_{\infty i})$

3.79



$q_o'' = 575 \ W/m^2$

$$q_o'' = \frac{T_o - T_{\infty o}}{R_o} + \frac{T_o - T_{\infty i}}{R_{IN}}$$

$$R_o = \frac{1}{h_o} = \frac{1}{5} \frac{K m^2}{W}$$

$$R_{IN} = R_{COND\,I} + R_{CONV\,I} = \frac{0.2 m}{4 \ W/m \ K} + \frac{1}{20} \frac{K m^2}{W} = 0.1 \frac{K m^2}{W}$$

$$q_o'' = \frac{R_{IN}(T_o - T_{\infty o}) + R_o(T_o - T_{\infty I})}{R_o R_{IN}}$$

$$R_o R_{IN} q_o'' = T_o(R_{IN} + R_o) - R_{IN} T_{\infty o} - R_o T_{\infty I}$$

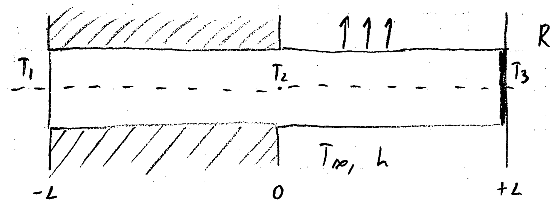
$$T_o = \frac{(R_o R_{IN} q_o'') + (R_{IN} T_{\infty o} + R_o T_{\infty I})}{R_{IN} + R_o}$$

$T_o = 353 \ K$

5/10

3.108 Problem: A rod of diameter D is insulated over one half and has convection over the other

Given:



$R''_{t,c} = 4\times 10^{-4}\ m^2K/W$

$D = 5\ mm$ $\quad T_\infty = 20°C$

$L = 50\ mm$ $\quad h = 500\ W/m^2K$

$k = 100\ W/mK$

insulated $-L \le x \le 0$ $\quad$ convection $0 \le x \le L$

$T(-L, t) = T_1$

Find: a) Sketch the temperature distribution on T-x coordinates. Assume $T_1 > T_3 > T_\infty$

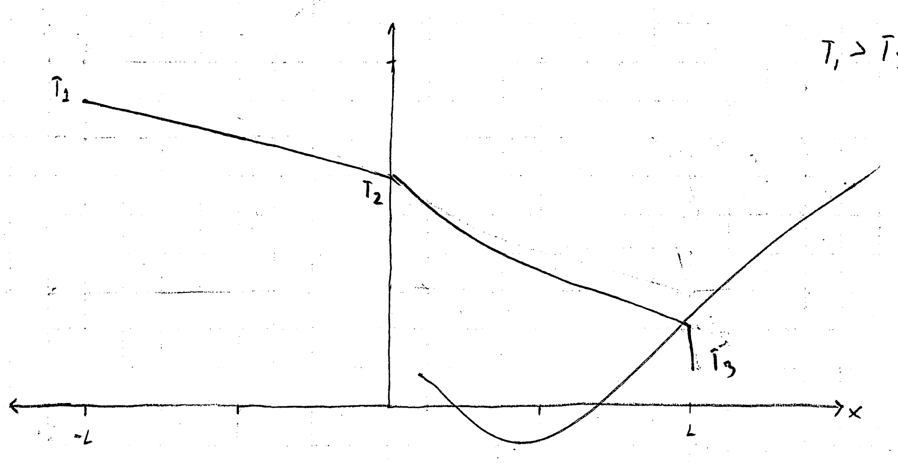
b) Derive an expression for the midpoint temperature T_2 in terms of thermal/geometric parameters

c) $T_1 = 200°C$ $\quad T_3 = 100°C$ find T_2, plot the temperature distribution.

Assume: One dimensional steady state heat conduction
temperature function only of x
Negligible radiation from exposed surfaces
No heat generation in rod

Analysis

a) $T_1 > T_3 > T_\infty$



$$\frac{\partial^2 T}{\partial x^2} = 0 \qquad x < 0 \qquad x = -L \quad T = T_1$$
$$x = 0 \quad T = T_2$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

3.108

b $x < 0$ $\frac{\partial^2 T}{\partial x^2} = 0$ $x = 0 \quad T = T_2$
$x = -L \quad T = T_1$

$T(x) = C_1 x + C_2$ $C_2 = T_2$ $T_1 = -C_1 L + T_2$ $C_1 = \frac{T_2 - T_1}{L}$

$T(x) = \frac{T_2 - T_1}{L} x + T_1$ $\left.\frac{\partial T}{\partial x}\right|_{x=L} = \frac{T_2 - T_1}{L}$

$x > 0$ $\frac{d^2 T}{dx^2} - \frac{hP}{A_c k}(T - T_\infty) = 0$ $x = 0 \quad T = T_2$
$x = L \quad \frac{\partial T}{\partial x} = \frac{T_L - T_3}{k R_{tc}}$

$\theta = T - T_\infty$ $m^2 = \frac{hP}{A_c k}$

$\frac{d}{dx}\left(\frac{dT}{dx}\right) - m^2 \theta = 0$

$\frac{d}{dx}\left(\frac{d\theta}{dx}\frac{dT}{d\theta}\right) - m^2 \theta = 0$ $\frac{d^2\theta}{dx^2} - m^2 \theta = 0$

$x = 0 \quad \theta = \theta_b$
1 $x = L \quad \frac{\partial \theta}{\partial x} = -\frac{T_L - T_3}{k R_{tc}}$

2 $x = 0 \quad \frac{\partial \theta}{\partial x} = \frac{T_2 - T_1}{L}$

$\theta = C_1 e^{mx} + C_2 e^{-mx}$

$\frac{T_2 - T_1}{L} = m(C_1 - C_2)$ $\frac{T_3 - T_L}{k R_{tc}} = m(C_1 e^{mL} - C_2 e^{-mL})$

$C_1 = \frac{T_2 - T_1}{mL} + C_2$ $\frac{T_3 - T_L}{m k R_{tc}} = \left(\frac{T_2 - T_1}{L} - C_2\right) e^{mL} - C_2 e^{-mL}$

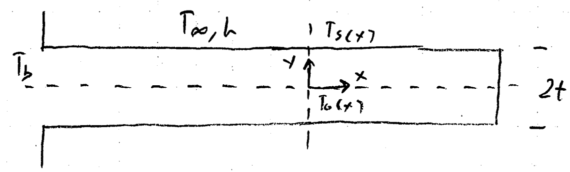
$\frac{T_3 - T_L}{m k R_{tc}} + \frac{T_1 - T_2}{L} e^{mL}$

part C ?

12/15

3.118 Problem: An extended surface of retangular cross-section with heat flow in the longitudinal direction

Given



Find a) Assuming $\dfrac{T(y) - T(x)}{T_s(x) - T_o(x)} = \left(\dfrac{y}{t}\right)^2$

Using Fouriers law, write an expression for the conduction heat flux at the surface $q_y''(t)$ in terms of T_s, T_o

b) Write an expression for the convection heat flux at the surface for the x-location. Equating the two expressions for the heat flux by conduction and convection identity the parameter that determines $(T_o - T_s)/(T_s - T_\infty)$

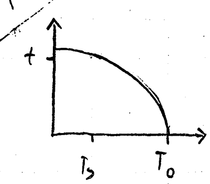
c) From the foregoing analysis, develop a criterion for establishing the validity of the 1D assumption

Assume: One dimensional heat conduction
Constant thermal properties
Steady state
Negligible radiation heat exchange

Analysis

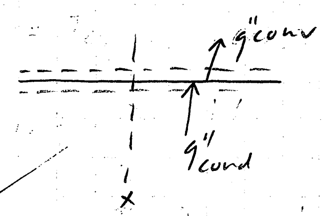
a) $q_y''(t) = -k \left.\dfrac{\partial T}{\partial y}\right|_{y=t}$ $\qquad T(y) = (T_s - T_o)\left(\dfrac{y}{t}\right)^2 + T_o$

$$\frac{\partial T}{\partial y} = (T_s - T_o)\,2\left(\frac{y}{t}\right)\left(\frac{1}{t}\right) = \frac{2}{t^2}(T_s - T_o)\,y$$



$$q_y''(t) = -\frac{2k}{t}(T_s - T_o) = \frac{2k}{t}(T_o - T_s)$$

b) $q''_{conv} = h(T_s(x) - T_\infty)$



$$q''_{conv} = q''_{cond}$$

$$h(T_s - T_\infty) = \frac{2k}{t}(T_o - T_s)$$

$$\frac{T_o - T_s}{T_s - T_\infty} = \frac{ht}{2k}$$

3.118

c) $$\frac{T_0 - T_s}{T_s - T_\infty} = \frac{ht}{2k}$$ $T_0 \approx T_s$ for One dimensional

We want $T_s - T_\infty$ to be as large as possible to maximize convective heat transfer. We also want $T_0 = T_s$ so that T_s is at a maximum and the 2D model holds.

For a $(T_s - T_\infty) >>> (T_0 - T_s)$ $ht << 2k$.

For these conditions, we want a large h value, so t must be small and k large.
For the One dimensional model to be accurate

$$(T_s - T_\infty) >> (T_0 - T_s)$$

$$(ht) <<< 2k$$

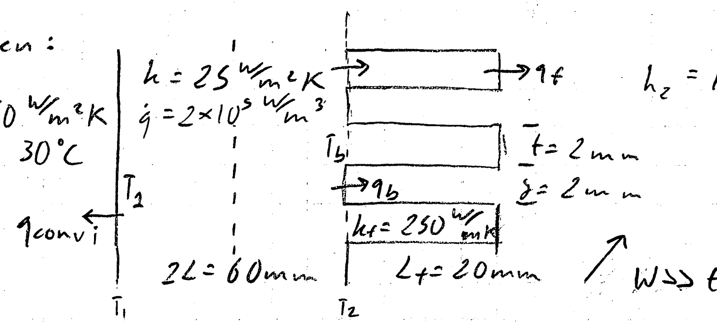
10/10

3.142 Problem: Heat is generated inside a wall with fins attached

Given:

$h_1 = 50\ \frac{W}{m^2K}$

$T_{\infty 1} = 30°C$



$h_2 = 12\ \frac{W}{m^2K}$ $\quad T_{\infty 2} = 15°C$

Find: The maximum temperature that occurs in wall

Assume: One dimensional heat transfer by conduction
Constant thermal properties
Steady state conditions
Negligible radiation exchange with surroundings

Analysis

$$\dot{q}(2L) - h_1(T_1 - T_\infty) - q_{tf} = 0$$

$$q_1 = -k\frac{\partial T}{\partial x}\bigg|_{x=0} = hA(T_1 - T_\infty)$$

$$q_2 = -kA\frac{\partial T}{\partial x}\bigg|_{x=2L} = \frac{T_2 - T_{\infty 2}}{R_o}$$

$$R_o = \frac{1}{hA_t\eta_o} \qquad \eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Assume adiabatic tip $\quad \frac{\partial T}{\partial x}\bigg|_{x=L} = 0$

$$L_c = L + \frac{t}{2} = 21\ mm$$

$$m = \sqrt{\frac{h(2W+2t)}{k(Wt)}} \qquad W \gg t$$

$$m = \sqrt{\frac{2h\cancel{W}}{k\cancel{W}t}} = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(12\ \frac{W}{m^2K})}{(250\ \frac{W}{mK})(0.002m)}} = 6.928\ m^{-1}$$

3.142

$$\eta_f = \frac{\tanh[(6.928\,m^{-1})(0.021\,m)]}{(6.928\,m^{-1})(0.021\,m)} = 0.993$$

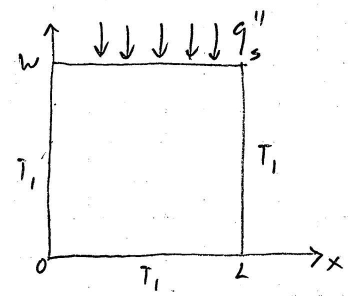
$$\eta_o = 1 - \frac{N(2\cancel{W} + \cancel{2t})L}{\cancel{W}P}(1 - \eta_f) = 1 - \frac{2NL}{P}(1 - \eta_f)$$

5/20

3,142

4.5 Problem: A uniform rectangular plate is subject to constant temperature around three sides and a uniform heat flux into on the top surface

Given:



Find: The temperature distribution for the plate

Assume: Steady state, constant properties
No heat generation in plane

Analysis:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

$x=0$	$0<y<W$	$T=T_1$
$x=L$	$0<y<W$	$T=T_1$
$y=0$	$0<x<L$	$T=T_1$
$y=W$	$0<x<1$	$\frac{\partial T}{\partial y} = \frac{q_s''}{k}$

$$\theta = \frac{T-T_1}{T_c} \qquad \eta = \frac{x}{L} \qquad \xi = \frac{y}{L}$$

$$T_c = \frac{q_s'' L}{k}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial x}\left[\frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial x}\frac{\partial T}{\partial \theta}\right] + \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial y}\left[\frac{\partial \theta}{\partial \xi}\frac{\partial \xi}{\partial y}\frac{\partial T}{\partial \theta}\right] = 0$$

$$\frac{1}{L}\frac{\partial}{\partial \eta}\left(\frac{\partial \theta}{\partial \eta}\frac{1}{L}\frac{1}{T_c}\right) + \frac{1}{L}\frac{\partial}{\partial \xi}\left(\frac{\partial \theta}{\partial \xi}\frac{1}{L}\frac{1}{T_c}\right) = 0$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial^2 \theta}{\partial \xi^2} = 0$$

1	$\eta=0$	$0<\xi<W/L$	$\theta=0$
2	$\eta=1$	$0<\xi<W/L$	$\theta=0$
3	$\xi=0$	$0<\eta<1$	$\theta=0$
4	$\xi=\frac{W}{L}$	$0<\eta<1$	$\frac{\partial \theta}{\partial \xi}\frac{\partial T}{\partial \theta}\frac{\partial \xi}{\partial y} = \frac{q''}{k}$

$$\frac{\partial \theta}{\partial \xi} = \frac{L q''}{T_c k} \qquad \frac{\partial \theta}{\partial \xi}\left(T_c \frac{1}{L}\right) = \frac{q''}{k}$$

$$\theta = w(\eta)\psi(\xi) \qquad \psi\frac{\partial^2 w}{\partial \eta^2} + w\frac{\partial^2 \psi}{\partial \xi^2} = 0$$

$$\frac{1}{w}\frac{\partial^2 w}{\partial \eta^2} = -\frac{1}{\psi}\frac{\partial^2 \psi}{\partial \xi^2} = -\lambda^2$$

$$\frac{\partial^2 w}{\partial \eta^2} + \lambda^2 w = 0 \qquad \frac{\partial^2 \psi}{\partial \xi^2} - \lambda^2 \psi = 0$$

4.5

$$w = A\cos\lambda\eta + B\sin\lambda\eta \qquad \psi = D\cosh\lambda\xi + E\cosh\lambda\xi$$

$$\theta = (\cancel{A\cos\lambda\eta} + B\sin\lambda\eta)(\cancel{D\cosh\lambda\xi} + E\sinh\lambda\xi)$$

1	$\eta = 0$	$\theta = 0$	$\Rightarrow$	$A = 0$	
3	$\xi = 0$	$\theta = 0$	$\Rightarrow$	$D = 0$	
2	$\eta = 1$	$\theta = 0$	$\Rightarrow$	$\lambda = n\pi$	$n = 1, 2, 3, 4$

$$\theta = \sum_{n=1}^{\infty} C_n \sin(n\pi\eta)\sinh(n\pi\xi)$$

$$\frac{\partial\theta}{\partial\xi} = \sum_{n=1}^{\infty} C_n \sin(n\pi\eta)\sinh(n\pi\xi)$$

$$\int_0^1 \left[\frac{Lq''}{kT_c} = \sum_{n=1}^{\infty} C_n \sin(n\pi\eta)\sinh(n\pi\tfrac{W}{L})\right]\sin(m\pi\eta)$$

$$\int_0^1 \frac{Lq''}{kT_c}\sin(m\pi\eta) = C_m \sinh(m\pi\tfrac{W}{L})\tfrac{1}{2}$$

$$C_m = \frac{2Lq''}{kT_c\sinh(m\pi\frac{W}{L})}\int_0^1 \sin(m\pi\eta)\,d\eta$$

$$= \frac{2Lq''}{kT_c\sinh(m\pi\frac{W}{L})}\left(-\frac{1}{m\pi}\right)\cos(m\pi\eta)\Big|_0^1 =$$

$$= \frac{-2Lq''m\pi}{kT_c\sinh(m\pi\frac{W}{L})}\left[\cos m\pi - 1\right] = \begin{cases} \dfrac{4Lq''m\pi}{kT_c\sinh(m\pi\frac{W}{L})} & m \text{ odd} \\ 0 & m \text{ even} \end{cases}$$

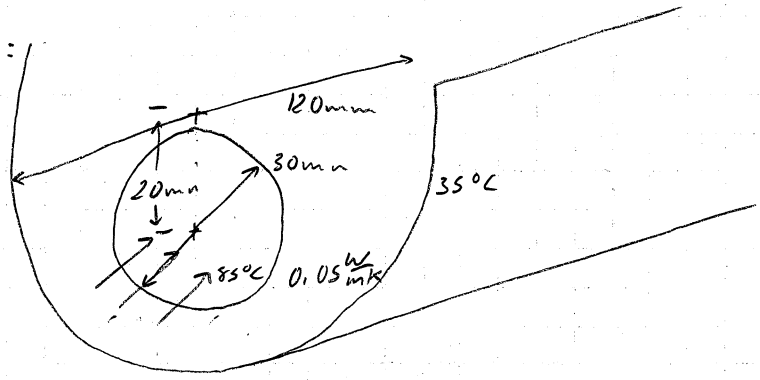
$$\frac{T - T_1}{T_c} = \sum_{m=1}^{\infty} \frac{4Lq''m\pi}{kT_c\sinh(m\pi\frac{W}{L})}\sin(m\pi\eta)\sinh(m\pi\xi)$$

$$T(\eta,\xi) = T_1 + \sum_{m=1}^{\infty}\frac{4Lq''m\pi}{k\sin(m\pi\frac{W}{L})}\sin(m\pi\eta)\sinh(m\pi\xi) \qquad m = 1, 3, 5, 7, \ldots$$

13/15

4.16 Problem: Hot water flows through a thin walled copper tube of 30 mm diameter. The tube is enclosed by a eccentric cylindrical shell and has a diameter of 120mm. The eccentricity is 20 mm. Space between tube/shell is filled with an insulating material of $k = 0.05 \, W/mK$

Given:



Find: Calculate the heat loss per unit length of tube and compare the result for a concentric arrangement

Assume: No contact resistance
Steady state, one dimensional heat conduction
constant, uniform thermal properties

Analysis:

$z = 20mm \quad q = Sk(T_1 - T_2) \quad q' = \frac{S}{L} k (T_1 - T_2)$

$$\frac{S}{L} = \frac{2\pi}{\cosh^{-1}\left(\frac{D^2 + d^2 - 4z^2}{2Dd}\right)} = 4.911$$

$$q' = 4.911 (0.05 \, W/mK)(85 - 35K) = 12.47 \, \frac{W}{m}$$

Compare results

$z = 0$ Concentric

$$LR = \frac{\ln(r_2/r_1)}{2\pi k} = \frac{\ln(120/30)}{2\pi (0.05 \, W/mK)} = 4.413 \, \frac{Km}{W}$$

$$q' = \frac{T_1 - T_2}{RL} = \frac{50K}{4.413 \, \frac{Km}{W}} = 11.33 \, \frac{W}{m}$$

9/10