MAE 130 C HOMEWORK 2

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MAE-130C 2-25-0

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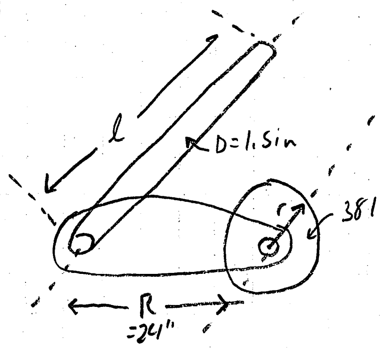
2.20



Determine the natural frequency of the system while locked and unlocked to the arm

$l = 50\,in$
$D = 1.5\,in$
$R = 24\,in$
$r = 9\,in$
$m = 38\,lb$

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$$J\ddot{\theta} + K\theta = 0$$

$$k = \frac{GI}{l} = \frac{\pi G d^4}{32 l} \qquad J_L = J_{wh} + J_{PA} = mr^2 + mR^2$$

$$W_{NL} = \sqrt{K/J_L} = \sqrt{\frac{\pi G d^4}{32 l (mr^2 + mR^2)}}$$

$$K = \frac{\pi(11.2\times10^6\,lb/in^2)(1.5\,in)^4}{32(50\,in)} = 111330.2\ lb\cdot in/rad$$

$$J_L = ((38\,lb_f)(9\,in)^2 + (38\,lb_f)(24\,in)^2)/32.2\,in/s^2 = 775.34\ lb\,in\,s^2$$

$$W_{NL} = \sqrt{K/J_L} = \sqrt{\frac{111330.2\ lbin/rad}{775.34\ lbins^2}} = 11.983\ rad/s$$

$$f_{NL} = 1.907\ Hz$$

$$J_U = mR^2 = (38\,lb_f)(32.2\,in/s^2)^{-1}(24\,in)^2 = 679.75\ lbins^2$$

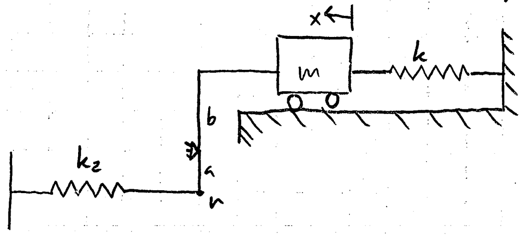
$$W_{NU} = \sqrt{K/J_U} = \sqrt{\frac{111330.2\ lbin/rad}{679.75\ lbins^2}} = 12.798\ rad/s$$

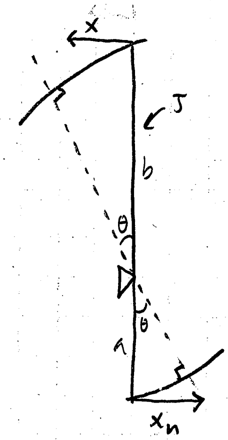
$$f_{NU} = 2.04\ Hz$$

Discussion? -1

2.24 Determine the effective mass at n and find its natural frequency

-Assume $\sin\theta = \theta$ $\cos\theta = 1$, rod has moment J





$$\tan\theta = \frac{x}{b}$$

$$\tan\theta = \frac{x_n}{a}$$

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$$\frac{\sin\theta}{\cos\theta} = \frac{x}{b} \Rightarrow \frac{\theta}{1} = \frac{x}{b} \qquad \frac{x}{b} = \theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{x_n}{a}$$

$$\theta = \frac{x_n}{a}$$

$$\frac{x}{b} = \frac{x_n}{a} \qquad x_n = \frac{a}{b}x \qquad x_n = a\theta \qquad x = \frac{b}{a}x_n$$

$$\dot{x}_n = \frac{a}{b}\dot{x} \qquad \dot{x}_n = a\dot{\theta}$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J\dot{\theta}^2 = \frac{1}{2}m\left(\frac{b}{a}\dot{x}_n\right)^2 + \frac{1}{2}J\left(\frac{\dot{x}_n}{a}\right)^2 = \frac{1}{2}\left[m\left(\frac{b}{a}\right)^2 + J/a^2\right]\dot{x}_n^2$$

$$m_{eff} = \left(\frac{b}{a}\right)^2 m + J/a^2$$

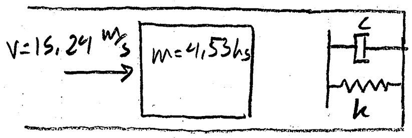
$$U = \frac{1}{2}k_2 x_n^2 + \frac{1}{2}kx^2 = \frac{1}{2}k_2 x_n^2 + \frac{1}{2}k\left(\frac{b}{a}x_n\right)^2 = \frac{1}{2}\left[k_2 + \left(\frac{b}{a}\right)^2 k\right]x_n^2$$

$$U_{max} = T_{max} \qquad \cancel{\tfrac{1}{2}}\left[k_2 + \left(\frac{b}{a}\right)^2 k\right]x_{n\,max}^2 = \cancel{\tfrac{1}{2}}\left[\left(\frac{b}{a}\right)^2 m + J/a^2\right]\dot{x}_{n\,max}^2$$

$$\dot{x}_{N\,max}^2 = \frac{k_2 + \left(\frac{b}{a}\right)^2 k}{\left(\frac{b}{a}\right)^2 + \frac{J}{a^2}}\, x_{N\,max}^2$$

$$W_N = \sqrt{\frac{k_2 + \left(\frac{b}{a}\right)^2 k}{\left(\frac{b}{a}\right)^2 + J/a^2}}$$

2.47 A piston travels at 15.24 m/s and engages a spring and damper. Determine the maximum displacement of the piston after engaging the spring-damper. How many seconds does it take



$$c = (1.75\ \text{Ns/cm})(100\ \text{cm/m}) = 175\ \text{Ns/m}$$

$$k = (350\ \text{N/cm})(100\ \text{cm/m}) = 35000\ \text{N/m}$$

X 10/10

$$\Sigma F_x = m\ddot{x} = -c\dot{x} - kx \qquad m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \qquad \ddot{x} + 2\zeta w_N \dot{x} + w_N^2 x = 0$$

$$2\zeta w_N = \frac{c}{m} \qquad 2\frac{\cancel{c}}{c_c}\sqrt{\frac{k}{m}} = \frac{\cancel{c}}{m} \qquad c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{km}$$

$$c_c = 2\sqrt{(35000\ \text{N/m})(4.53\ \text{kg})} = 796.367\ \text{kg/s}\ \text{Ns/m}$$

$$\zeta = \frac{c}{c_c} = \frac{175\ \text{Ns/m}}{796.367\ \text{Ns/m}} = 0.21974$$

$$x(t) = Xe^{-\zeta w_N t}\sin(\sqrt{1-\zeta^2}\,w_N t + \phi) \qquad x(0) = 0 \qquad w_N = \sqrt{k/m} \qquad \phi = 0$$

$$x(t) = Xe^{-\zeta w_N t}\sin(\sqrt{1-\zeta^2}\,w_N t)$$

$$\dot{x}(t) = X\left[e^{-\zeta w_N t}\sqrt{1-\zeta^2}\,w_N\cos(\sqrt{1-\zeta^2}\,w_N t) + \sin(\sqrt{1-\zeta^2}\,w_N t)(-\zeta w_N)(e^{-\zeta w_N t})\right]$$

$$15.24\ \text{m/s} = \dot{x}(0) = X(\sqrt{1-\zeta^2}\,w_N) \qquad w_N = \sqrt{\frac{35000\ \text{N/m}}{4.53\ \text{kg}}} = 87.899\ \text{rad/s}$$

$$X = 15.24\ \text{m/s} / 87.899\ \text{rad/s} = 0.1734\ \text{m}$$

$$\dot{x}(t_1) = 0 = 0.1734\text{m}\left[\cancel{e^{-\zeta w_N t_1}}\sqrt{1-\zeta^2}\,\cancel{w_N}\cos(\sqrt{1-\zeta^2}\,w_N t_1) - \zeta\cancel{w_N}\cancel{e^{-\zeta w_N t_1}}\sin(\sqrt{1-\zeta^2}\,w_N t_1)\right]$$

$$\tan(\sqrt{1-\zeta^2}\,w_N t_1) = \frac{\sqrt{1-\zeta^2}}{\zeta} = 4.4396$$

$$\sqrt{1-\zeta^2}\,w_N t_1 = 1.34924821\ \text{rad}$$

$$\boxed{t_1 = 0.0157346\ \text{s}}$$

$$x(t_1) = 0.1734\text{m}\left[e^{-(0.21974)(87.899)(0.0157346)}\sin(\sqrt{1-0.21974^2}(87.899\ \text{rad/s}(0.0157346\text{s})))\right]$$

$$\boxed{x(t_1) = 0.12483\ \text{m}}$$