MAE 101B, Spring 2007

Homework 2

Due Thursday, April 19, in class

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.
Please refrain from copying. Refer to the course outline for what constitutes copying

1. A turbine extracts 350 W of power in the configuration shown in the figure. The pipes are made of wrought iron. What is the flow rate Q in m^3/h?

2. Water flows out of a cylindrical tank of diameter D owing to gravitational head.

a) Assume turbulent flow with an average friction factor, f_0. Obtain the time for the water level to decrease from h to $h/2$.

b) Suppose the flow were laminar. Again, obtain the time for the water level to decrease from h to $h/2$.

3. Water flows through a sudden contraction between two pipes of diameter $D_1 = 50\,mm$ and $D_2 = 25\,mm$. The pressure drop between two points upstream and downstream of the contraction, respectively, is measured to be $4\,kPa$. What is the flow rate in m^3/s?

4. The dataset (download it from the 101B web site) gives the mean velocity, U, in turbulent channel flow. It is obtained by averaging the 3 D, unsteady data in a direct numerical simulation of channel flow by Hoyas and Jimenez (2006).

a) Analysis of the data gives $u^* = 0.02\,m/s$. Plot the data and identify the the following regions: viscous sublayer, logarithmic overlap region, and outer law profile. These regions were identified in the class and are also shown in Fig. 6.10 of White.

b) Suppose you were *not* given the value of u^*. Estimate u^* from the data.

Ungraded problems From text, 6.40, 6.148

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4-19-07

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2	8/10
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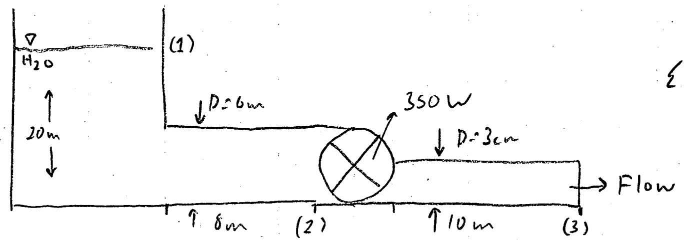
1 Problem: A turbine extracts 350 W of power in the below configuration. The pipes are made of wrough iron.

Given:



$\varepsilon = 0.046$ mm (+1)

Find: Q in $[m^3/h]$

Engr Model: 1 Steady incompressible flow

Analysis:

Eq 6.7 $\left(\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1\right) = \left(\cancel{\frac{P_2}{\rho g}} + \frac{V_3^2}{2g} + \cancel{z_2}\right) + h_{f_2} + h_{f_3} + h_t$

$$z_1 = \frac{V_3^2}{2g} + h_{f_2} + h_{f_3} + h_t$$ 5/5

$$h_{f_2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = \frac{L_2}{2D_2 g} f_2 \left(\frac{Q}{A_2}\right)^2 = \frac{L_2}{2gD_2} f_2 \left(\frac{Q}{\frac{\pi}{4}D_2^2}\right)^2$$

$$h_{f_2} = \frac{16 L_2 f_2 Q^2}{2\pi^2 g D_2^5} = 850070\, f_2 Q^2$$

$$h_{f_3} = \frac{16 L_3 f_3 Q^2}{2\pi^2 g D_3^5} = 34002821\, f_3 Q^2 \qquad h_t = \frac{\dot{W}}{\rho g Q} = \frac{0.03575}{Q}$$

$\frac{\varepsilon}{D_2} = 7.667 \times 10^{-4}$ (+1)

$\frac{\varepsilon}{D_3} = 0.001533$

$Re = \rho \frac{VD}{\mu}$

$Re = 1\times10^6\, VD = \frac{QD}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D} \times 10^6$

$$z_1 = 850070 f_2 Q^2 + 34002821 f_3 Q^2 + 0.03575/Q$$

(+1) Guess $Q = 0.0025\ m^3/s$

$V_2 = 0.884194\ m/s$	$Re_2 = 53051.65$	$f_2 = 0.02$
$V_3 = 3.54\ m/s$	$Re_3 = 106200$	$f_3 = 0.022$ (+1)

$$z = 850070(0.02)(0.0025)^2 + 34002821(0.022)(0.0025)^2 + \frac{0.03575}{0.0025}$$

$z = 19.082$ m

(+1) for iterate

Guess $Q = 0.0022$

$V_2 = 0.778$	$Re_2 = 46680$	$f_2 \approx 0.02$
$V_3 = 3.112$	$Re_3 = 93360$	$f_3 \approx 0.022$

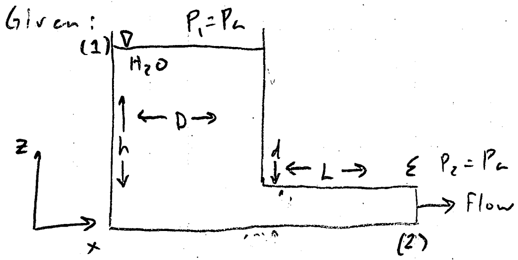
$$z = 850070(0.02)(0.0022)^2 + 34002821(0.022)(0.0022)^2 + \frac{0.03575}{0.0022}$$

$z = 19.95\,m \approx 20\,m$

$$Q = (0.0022\ m^3/s)(3600\ s/h) = 7.92\ m^3/h$$

2 Problem: Water flows out of a cylindrical tank of diameter D owing to gravitational head

Given:



$$\frac{\pi}{4} D^2 \frac{dh}{dt} = \frac{\pi}{4} d^2 V$$

Find: a) Assuming turbulent flow with an average friction factor f_0 obtain the time for the water level to decrease from h to $h/2$

b) Suppose the flow were laminar. Again obtain time to go from h to $h/2$

Engr Model: 1 Incompressible flow

2 Fully developed flow between 2, 3

Analysis:

a)

$$\left(\cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} + z_1\right) = \left(\cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2}\right) + h_f$$

$$V_1 = dh/dt \qquad z_1 = h(t)$$

$$z_1 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + f_0 \frac{L}{D}\frac{V_2^2}{2g} \qquad \frac{\pi}{4} D^2 \frac{dh}{dt} = -\frac{\pi}{4} d^2 V \qquad \frac{dh}{dt} = \left(\frac{d}{D}\right)^2 V_2$$

+1

$$h(t) = \frac{1}{2g}\left[V_2^2 - \left(\frac{dh}{dt}\right)^2 + \frac{f_0 L}{D} V_2^2\right] = \frac{1}{2g}\left[\left(\frac{D}{d}\right)^4\left(\frac{dh}{dt}\right)^2 - \left(\frac{dh}{dt}\right)^2 + \frac{f_0 L}{d}\left(\frac{D}{d}\right)^4\left(\frac{dh}{dt}\right)^2\right]$$

$$h(t) = \left(\frac{dh}{dt}\right)^2\left[\frac{1}{2g}\left(\left(\frac{D}{d}\right)^4 - 1 + \frac{f_0 L}{d}\left(\frac{D}{d}\right)^4\right)\right] \qquad B = \left[\frac{1}{2g}\left(\left(\frac{D}{d}\right)^4 - 1 + \frac{f_0 L}{d}\left(\frac{D}{d}\right)^4\right)\right]$$

$$h(t) = B\left(\frac{dh}{dt}\right)^2 \qquad \frac{dh}{dt} = B^{-\frac{1}{2}} h^{\frac{1}{2}}$$

+1 $$\int_{h}^{h/2} \frac{dh}{h^{1/2}} = \int_{0}^{t_A} B^{-\frac{1}{2}}\, dt \Rightarrow 2h^{\frac{1}{2}}\Big|_{h}^{h/2} = B^{-\frac{1}{2}} t_A$$

$$2\left(\sqrt{\frac{h}{2}} - \sqrt{h}\right) = B^{-\frac{1}{2}} t_A \qquad t_A = \left(2\sqrt{\frac{h}{2}} - \sqrt{h}\right) B^{\frac{1}{2}}$$

$$\frac{2}{10}$$

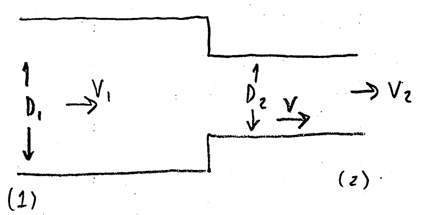
2 b)

$$z_1 = \frac{V_2^2}{2g} + \frac{V_1^2}{2g} + \frac{32\mu L V_2}{\rho g d^2} \qquad V_2 = \left(\frac{D}{d}\right)^2 V_1$$

$$h(t) = \left(\frac{D}{d}\right)^4 \frac{1}{2g}\left(\frac{dh}{dt}\right)^2 + \frac{1}{2g}\left(\frac{dh}{dt}\right)^2 + \frac{32\mu L}{\rho g d}\left(\frac{D}{d}\right)^2\left[\frac{dh}{dt}\right]$$

3 Problem: Water flows through a sudden contraction between two pipes

Given



$D_1 = 50$ mm
$D_2 = 25$ mm
$P_1 - P_2 = 4$ kPa

Find: $Q [m^3/s]$

Eng r Model: 1 Steady, incompressible flow
2 friction loss negligible

Analysis:

$$\underline{Q_1 = Q_2} \qquad \underline{\tfrac{\pi}{4} D_1^2 V_1 = \tfrac{\pi}{4} D_2^2 V_2} \quad \checkmark +1 \qquad V_2 = 4V_1$$

+1

$$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}\right) = \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}\right) + \cancel{h_f}^{\,0} + h_m \qquad h_m = \frac{V_2^2}{2g}\left(0.42\left(1 - \left(\frac{D_2}{D_1}\right)^2\right)\right)$$

+1 +1 +1/2 +2

$$V_1 = V_2/4$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + \frac{V_2^2}{2g}\left[0.42\left(1 - (D_2/D_1)^2\right)\right] \quad \checkmark +1$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - \frac{1}{16}V_2^2}{2g} + \frac{V_2^2}{2g}\left[0.42\left(1 - (D_2/D_1)^2\right)\right]$$

$$\frac{P_1 - P_2}{\rho \cancel{g}} = \frac{1.2525}{2\cancel{g}} V_2^2 \qquad V_2 = \sqrt{\frac{2(4 \times 10^3 \ kg\,m/s^2)}{1.2525(998 \ kg/m^3)}}$$

$V_2 = 2.529$ m/s

$V_1 = 0.63225$ m/s

$$Q = \frac{\pi}{4} D_2^2 V_2$$

$$Q = \frac{\pi}{4}(0.05m)^2 (2.529 \ m/s) = 0.004966 \ m^3/s$$

$Q = 4.966 \times 10^{-3} \ m^3/s$

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4 a

Blue is $u^+ = u/u^*$

Pink is $u^+ = \frac{u}{u^*} = \frac{1}{\kappa} \ln\left(\frac{y u^*}{\nu}\right) + B$

$\kappa = 0.41$
$B = 5$
$\nu = 1 \times 10^{-6}$
$u^* = 0.02\ m/s$

b If not given u^*, I would select a few adata points. Then using Eq 6.28

(1) $u = u^* \left[\frac{1}{0.41} \ln\left(\frac{y u^*}{\nu}\right) + B\right]$ I would see what u^* best matches the data ✓ +1

1 $y = 0.085669\,m$ $u = 0.38379\ m/s$

Plotting eq 1

$u^* = 0.016875\ m/s$

✓

2 $y = 0.058033\,m$ $u = 0.44203\ m/s$ +1

$u^* = 0.01991\ m/s$

3 $y = 0.033708\,m$ $u = 0.46982\ m/s$

$u^* = 0.022221$

4 $y = 0.006066\,m$ $u = 0.48545\ m/s$

$u^* = 0.02775$

$\bar{u}^* = 0.021689$ ✓ +1

I would actual write a program doing this for all data points, and then average out the u^* values to get a good one

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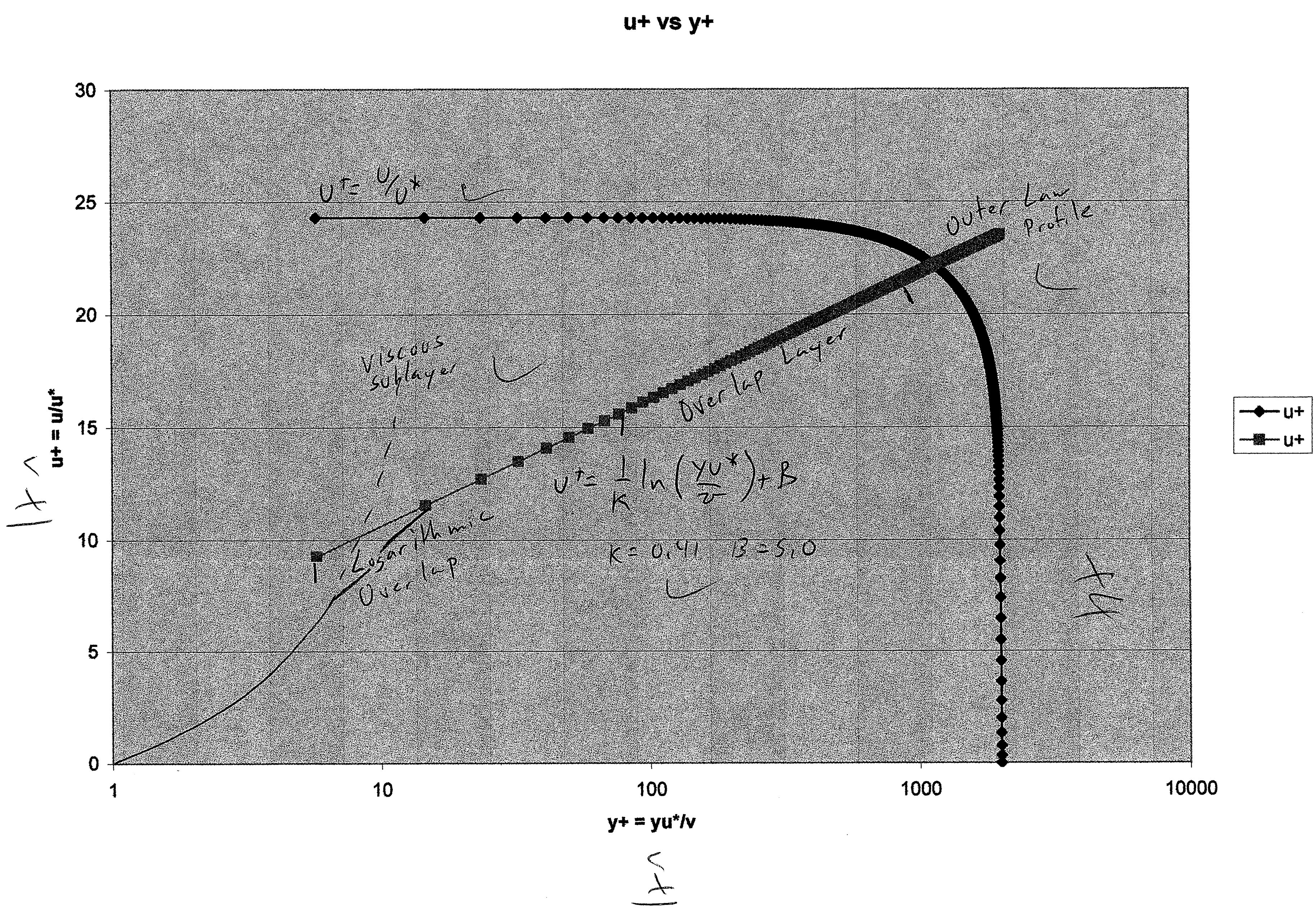

u+ vs y+
30
25
20
15
10
5
0
1
10
100
1000
10000
u+ = u/u*
y+ = yu*/v
u+
u+
$u^+ = U/U^*$
Outer Law Profile
Viscous sublayer
Overlap Layer
$u^+ = \frac{1}{K} \ln\left(\frac{yU^*}{\nu}\right) + B$
Logarithmic Overlap
K = 0.41 B = 5.0
+4
+1
+1