MAE 280A | HOMEWORK 1 | $\frac{15}{15}$ great

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MAE 280A
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7. An LTI system obeys

$$\dddot{y} + \ddot{y} + \dot{y} + y = \ddot{u} + \dot{u} + u \qquad y \in \mathbb{R}^2 \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$u \in \mathbb{R}^2 \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} \dddot{y}_1 \\ \dddot{y}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A_2}\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A_1}\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A_0}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B_2}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B_1}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B_0}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$x_1 = y_1 \qquad x_2 = y_2 \qquad \dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4 \qquad \dot{x}_3 = x_5 \qquad \dot{x}_4 = x_6$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

✓

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T$$

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8 A linear system is described by its transfer function

$y(s) = G(s)\,u(s) \qquad G(s) = \dfrac{s^2+s+1}{s^3+s^2+s+1}$

Put this into $\dot{x} = Ax + Bu, \quad y = Cx$

$y(s)\,(s^3+s^2+s+1) = u(s)\,(s^2+s+1)$

$n=3 \qquad A_2 = 1 \quad A_1 = 1 \quad A_0 = 1$

$\qquad\quad B_2 = 1 \quad B_1 = 1 \quad B_0 = 1$

$x_1 = y \quad \dot{x}_1 = x_2 \; \ldots$ ✓ ✓

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$y = [1 \quad 0 \quad 0]\,[x_1 \quad x_2 \quad x_3]^T$ ✓

$\dddot{y} + \ddot{y} + \dot{y} + y = \ddot{u} + \dot{u} + u$

$\dddot{y} = -\ddot{y} - \dot{y} - y + \ddot{u} + \dot{u} + u$

$x_1 = y \qquad x_2 = \dot{x}_1 \qquad x_3 = \dot{x}_2 \qquad u_1 = u \qquad u_2 = \dot{u} \qquad u_3 = \ddot{u}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

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9 Linearize the system $\dot{x} = \sin x + x^2 + xu$ about $\bar{x} = 0, \bar{u} = 1$
Write $\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$ $\tilde{x} = x - \bar{x}$, $\tilde{u} = u - \bar{u}$

$$\dot{x} = f(x, u, t) = \sin x + x^2 + xu$$

$$A(t) = \left[\frac{\partial f}{\partial x}\right]^T_{\bar{x}, \bar{u}} = [\cos x + 2x + u]_{\bar{x}=0, \bar{u}=1} = [2]$$

$$B(t) = \left[\frac{\partial f}{\partial u}\right]^T_{\bar{x}, \bar{u}} = [x] = [0]$$

$$\Rightarrow \quad \dot{\tilde{x}} = [2]\,\tilde{x} + [0]\,\tilde{u}$$ ✓

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