MAE 104	HOMEWORK 1	83/98	Michael Sanke A05812168

MAE 104
10-9-07

1.2

Problem: Obtain the missing values

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Given: $P = \rho R T$ $\quad R = 287 \, J/kg\,K = 1716 \, lb_f ft / sl\, R°$

Find: a) ρ when $P = 1.9 \times 10^4 \, N/m^2$ $\quad T = 203 \, K$

b) T when $P = 1058 \, lb_f/ft^2$, $\rho = 1.23 \times 10^{-3} \, sl/ft^3$

Analysis:

a) $\rho = P/RT = (1.9 \times 10^4 \, N/m^2) / [(287 \, J/kg\,K)(203\,K)] = 0.326 \, kg/m^3$

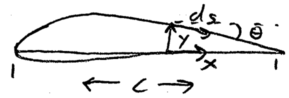
b) $T = P/\rho R = (1058 \, lb_f/ft^2) / (1.23 \times 10^{-3} \, sl/ft^3)(1716 \, lb_f ft/sl\, R°)$

$T = 501.3 \, R°$

1	6
2	10
3	13
4	13
5	10
6	15
7	8
8	8
Tot	83

1.2

Problem: Derive (1.15), (1.16), (1.17)



$ds\cos\theta = dx$
$ds\sin\theta = -dy$

19/11

Analyisis

a) (1.15) $$N' = -\int_{LE}^{TE}(P_u\cos\theta + \tau_u\sin\theta)ds_u + \int_{LE}^{TE}(P_L\cos\theta - \tau_L\sin\theta)ds_L$$

$$N' = -\int_{LE}^{TE} P_u\cos\theta\, ds_u + \int_{LE}^{TE} P_L\cos\theta\, ds_L - \int_{LE}^{TE}\tau_u\sin\theta\, ds_u - \int_{LE}^{TE}\tau_L\sin\theta\, ds_L$$

$$N' = -\int_{LE}^{TE} P_u\, dx + \int_{LE}^{TE} P_L\, dx - \int_{LE}^{TE}\tau_u\, dy_u - \int_{LE}^{TE}\tau_L\, dy_L$$

$$N' = \int_{LE}^{TE}(P_L - P_u)\,dx - \int_{LE}^{TE}(\tau_u\, dy_u) - \int_{LE}^{TE}\tau_L\, dy_L$$

$$N' = \int_{LE}^{TE} P_L - P_\infty - (P_u - P_\infty)\,dx - \int_{LE}\left[\tau_u\frac{dy_u}{dx} + \tau_L\frac{dy_L}{dx}\right]dx$$

$$N' = q_\infty c_n c \qquad c_n = \frac{N'}{c q_\infty}$$

$$c_n = \frac{1}{c}\int_{LE}^{TE}\frac{P_L - P_\infty}{q_\infty} - \frac{P_u - P_\infty}{q_\infty}\,dx - \int_{LE}^{TE}\frac{1}{c}\left[\frac{\tau_u}{q_\infty}\frac{dy_u}{dx} + \frac{\tau_L}{q_\infty}\frac{dy_L}{dx}\right]dx$$

$$c_n = \frac{1}{c}\int_0^c c_{p,L} - c_{p,u}\, dx + \frac{1}{c}\int_0^c\left(c_{f,u}\frac{dy_u}{dx} + c_{f,L}\frac{dy_L}{dx}\right)dx$$

b) 1.16 $$A' = \int_{LE}^{TE}(-P_u\sin\theta + \tau_u\cos\theta)ds_u + \int_{LE}^{TE}(P_L\sin\theta + \tau_L\cos\theta)ds_L$$

$$A' = \int_{LE}^{TE} -P_u\sin\theta\, ds_u + \int_{LE}^{TE} P_L\sin\theta\, ds_L + \int_{LE}^{TE}\tau_u\cos\theta\, ds_u + \int_{LE}^{TE}\tau_L\cos\theta\, ds_L$$

$$A' = \int_{LE}^{TE}\left(-P_u\frac{dy_u}{dx} + P_L\frac{dy_L}{dx}\right)dx + \int_{LE}^{TE}\tau_u + \tau_L\, dx$$

$$A' = \int_0^c\left(P_\infty - P_u\frac{dy_u}{dx} + P_L - P_\infty\frac{dy_L}{dx}\right)dx + \int_0^c(\tau_u + \tau_L)\,dx$$

$$A' = q_\infty c_A c \qquad c_A = \frac{A'}{q_\infty c}$$

$$c_A = \frac{1}{c}\int_0^c\left(\frac{P_\infty - P_u}{q_\infty}\frac{dy_u}{dx} + \frac{P_L - P_\infty}{q_\infty}\frac{dy_L}{dx}\right)dx + \frac{1}{c}\int_0^c\left(\frac{\tau_u + \tau_L}{q_\infty}\right)dx$$

$$c_a = \frac{1}{c}\int_0^c\underbrace{\left(-c_{p,u}\frac{dy_u}{dx} + c_{p,L}\frac{dy_L}{dx}\right)}_{c_{p_u} - c_{p_L}}dx + \frac{1}{c}\int_0^c(c_{f,u} + c_{f,L})\,dx$$

1.2

c)

$$M'_{LE} = \int_{LE}^{TE} [(P_U \cos\theta + \tau_U \sin\theta)x - (P_U \sin\theta - \tau_U \cos\theta)y] ds_U$$

$$+ \int_{LE}^{TE} [(-P_L \cos\theta + \tau_L \sin\theta)x + (P_L \sin\theta + \tau_L \cos\theta)y] ds_L$$

$$= \int_{LE}^{TE} (P_U - P_L)x\,dx + \int_{LE}^{TE} \tau_U x \frac{dy_U}{dx} dx - \int P_U y_U \frac{dy_U}{dx} dx + \int \tau_U y_U dx$$

$$+ \int \tau_L x \frac{dy_L}{dx} dx + \int P_L y_L \frac{dy_L}{dx} dx + \int \tau_L y_L dx$$

$$= \int_0^c (P_U - P_L)x\,dx + \int_0^c \left(\tau_U \frac{dy_U}{dx} + \tau_L \frac{dy_L}{dx}\right)x\,dx + \int_0^c \left[(P_U - P_\infty)\frac{dy_U}{dx} + \tau_U\right] y_U\,dx$$

$$+ \int_0^c \left((P_\infty - P_L)\frac{dy_L}{dx} + \tau_L\right) y_L\,dx$$

$$\frac{M'_{LE}}{q_\infty c} = \frac{1}{c}\int_0^c (C_{p,u} - C_{p,l})x\,dx + \frac{1}{c}\int_0^c \left(C_{f,u}\frac{dy_U}{dx} + C_{f,l}\frac{dy_L}{dx}\right)x\,dx$$

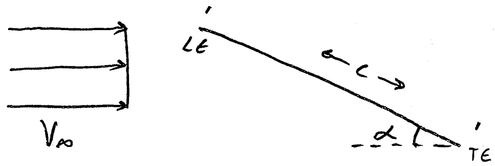
$$+ \frac{1}{c}\int_0^c \left(C_{p,u}\frac{dy_U}{dx} + C_{f,u}\right) y_U\,dx + \frac{1}{c}\int_0^c \left(-C_{p,l}\frac{dy_L}{dx} + C_{f,l}\right) y_L\,dx$$

1.4

13/15

Problem: An infinitely flat plate has a flow over it

Given:



$c = 1m$
$\alpha = 10°$

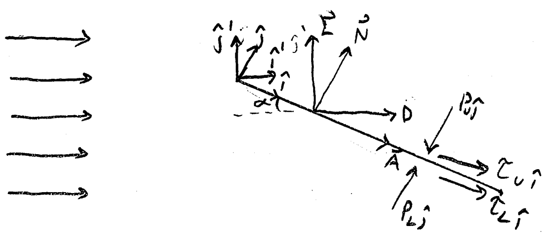
$P_U = [4\times10^4(x-1)^2 + 5.4\times10^4]\,N/m^2$ $\qquad \tau_U = [288x^{-0.2}]\,N/m^2$

$P_L = [2\times10^4(x-1)^2 + 1.73\times10^5]\,N/m^2$ $\qquad \tau_L = [731x^{-0.2}]\,N/m^2$

Find: $\vec{N}', \vec{A}', \vec{L}', \vec{D}', \vec{M}'_{LE}, \vec{M}'_{QC}$, AC

Assume: Steady flow, plate uniform over span

Analysis: Flat plate $\Rightarrow$ $\theta = 0$



$$\vec{N}' = \hat{j}\int_{LE}^{TE}(P_L\cos\theta - P_U\cos\theta + \tau_U\sin\overset{0}{\theta} - \tau_L\sin\overset{0}{\theta})\,ds_L$$

$$\vec{N}' = \hat{j}\int_0^{1m}[2\times10^4(x-1)^2 + 1.773\times10^5]\tfrac{N}{m^2} - (4\times10^4(x-1)^2 + 5.4\times10^4)\tfrac{N}{m^2}\,dx$$

$$\vec{N}' = \int_0^{1m}(1.233\times10^5 - 2\times10^4(x-1)^2)\tfrac{N}{m^2}\,\hat{j}\,dx \qquad x^2-2x+1$$

$$\vec{N}' = \left[1.223\times10^5\tfrac{N}{m^2}x - 2\times10^4\left(\tfrac{x^3}{3} - x^2 + x\right)\tfrac{N}{m^2}\right]\hat{j}\,\Big|_0^{1m}$$

$$\vec{N}' = \left(1.223\times10^5\tfrac{N}{m} - 2\times10^4\left(\tfrac{1}{3}\right)\tfrac{N}{m}\right)\hat{j} = 1.023\times10^5\,\hat{j}\,N/m$$

$$\vec{A}' = \hat{i}\int_{LE}^{TE}(-P_U\sin\overset{0}{\theta} + \tau_U\cos\theta) + (P_L\sin\overset{0}{\theta} + \tau_L\cos\theta)\,dx$$

$$\vec{A}' = \hat{i}\int_0^{1m}[(288+731)x^{-0.2}]\,N/m^2\,dx = 1019\,(N/m^2)\,\hat{i}\,\tfrac{1}{0.8}x^{0.8}\Big|_0^{1m}$$

$$\vec{A} = 1273.15\,N/m\,\hat{i}$$

$$\vec{L}' = \vec{N}'\cos\alpha - \vec{A}\sin\alpha = [1.023\times10^5\cos 10\,N/m - 1273.15\sin 10\,N/m]\,\hat{j}'$$

$$\vec{L}' = 100524\,N/m\,\hat{j}'$$

$$\vec{D}' = [1.023\times10^5\,N/m\sin 10 + 1273.15\,N/m]\,\hat{i}'$$

$$\vec{D}' = 19018\,N/m\,\hat{i}'$$

2.4

$$M'_{LE} = \int_{LE}^{TE} P_U \cos\theta\, x - P_L \cos\theta\, x \; dx$$

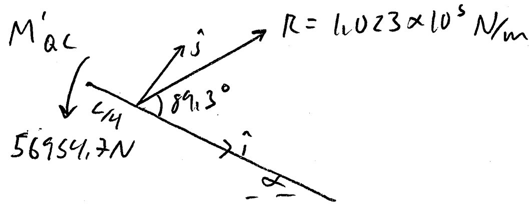
$$= \int_0^{2m} \cos 10\left[4\times10^4 (x-1)^2 + 5.4\times10^4\right] x \frac{N}{m^2} - \cos 10\left[2\times10^4 (x-1)^2 + 1.73\times10^5\right] x \frac{N}{m^2}\, dx$$

$$= \int_0^{2m} -2\times10^4 (x^3 - 2x^2 + x)\cos 10 \frac{N}{m^2} - 119000x \cos 10 \; dx$$

$$= \left[2\times10^4\left(\frac{x^4}{4} - \frac{2}{3}x^3 + \frac{x^2}{2}\right)\cos 10 - 119000\frac{x^2}{2}\cos 10\right]\Bigg|_0^{2m} \hat{k}$$

$$= \left[\frac{1}{12}(2\times10^4) - 59500\right]\cos 10 \; N\,\hat{k} = +56954.72 \; N\,\hat{k}$$

$$\vec{M}'_{LE} = +56954.72 \; N\,\hat{k} \Rightarrow -56954.7 \; N$$



$M'_{c/4}$, x_{cp} ?

1.7

Problem: The drag on a ship depends on the height of wake

Given:

$$D = f(\rho_\infty, V_\infty, c, g)$$

13/13

Find: Define $C_D = D/q_\infty c^2$

$$Fr = V/\sqrt{g c}$$

$$C_D = f(Fr)$$

Analysis:

$$D = f(\rho_\infty, V_\infty, c, g) \qquad N = 5 \quad K = 3$$

$$f_1(\Pi_1, \Pi_2) = 0$$

$$\begin{array}{c|ccccc} & \overset{N_1}{\rho_\infty} & \overset{N_2}{V_\infty} & \overset{N_3}{c} & g & D \\ M & 1 & 0 & 0 & 0 & 1 \\ L & -3 & 1 & 1 & 1 & 1 \\ T & 0 & -1 & 0 & -2 & -2 \end{array}$$

$$\Pi_1 = \rho_\infty^A V_\infty^B c^E D \qquad M^A L^{-3A} L^B T^{-B} L^E M L T^{-2}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ E \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ +2 \end{bmatrix} \Rightarrow \begin{array}{l} A = -1 \\ B = -2 \\ E = -2 \end{array}$$

$$\Pi_1 = \rho_\infty^{-1} V_\infty^{-2} c^{-2} D \qquad q_\infty = \tfrac{1}{2}\rho_\infty V_\infty^2$$

$$\Pi_1 = \frac{D}{q_\infty c^2} \Rightarrow C_D = D/q_\infty c^2$$

$$\Pi_2 = \rho_\infty^A V_\infty^B c^E g \qquad M^A L^{-3A} L^B T^{-B} L^E L T^{-2}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \Rightarrow \begin{array}{l} A = 0 \\ B = -2 \\ E = 1 \end{array}$$

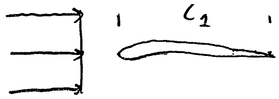
$$\Pi_2 = V_\infty^{-2} c g \qquad Fr^{-2} = V_\infty^{-2} c g \Rightarrow Fr = V_\infty/\sqrt{cg}$$

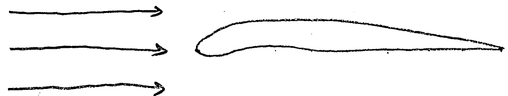
$$f_1(\Pi_1, \Pi_2) = 0 \Rightarrow f_1(C_D, Fr) = 0 \Rightarrow C_D = f(Fr)$$

1.9 10/10

Problem: Two Airfoils of the same shape have flows over them with one being twice the size of the other

Given:





$V_{\infty 1} = 100\ m/s$ $\quad$ $V_{\infty 2} = 200\ m/s$

$\rho_{\infty 1} = 1.23\ kg/m^3$ $\quad$ $\rho_{\infty 2} = 1.739\ kg/m^3$

$T_{\infty 1} = 200\ K$ $\quad$ $T_{\infty 2} = 800\ K$

Find: Are flows dynamically similar

Assume: $a, \mu \propto T^{\frac{1}{2}}$ $\quad$ +2 +2

Analysis

$$M_2 \overset{+2}{=} \frac{V_{\infty 2}}{a_2} \qquad M_1 = \frac{V_{\infty 1}}{a_1} \qquad \frac{a_2}{a_1} \propto \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{800}{200}} = 2$$

$$\frac{V_{\infty 2}}{V_{\infty 1}} = 2 \qquad M_2 = \frac{V_{\infty 2}}{a_2} = \frac{2V_{\infty 1}}{2a_1} = M_1^{+1}$$

$$Re_2 \overset{+2}{=} \frac{\rho_{\infty 2} V_{\infty 2} c_2}{\mu_2} \qquad Re_1 = \frac{\rho_{\infty 1} V_{\infty 1} c_1}{\mu_1} \qquad \frac{\mu_2}{\mu_1} = \sqrt{\frac{T_2}{T_1}} = 2$$

$$\frac{\rho_{\infty 2}}{\rho_{\infty 1}} = 1.414$$

$$Re_2 = \frac{\rho_{\infty 2} V_{\infty 2} c_2}{\mu_2} = \frac{1.414\,\rho_{\infty 1}(2V_{\infty 1})(2c_1)}{2\mu_1} = 2.818\,\frac{\rho_{\infty 1} V_{\infty 1} c_1}{\mu_1}$$

$Re_2 \overset{+1}{\neq} Re_1$ $\quad$ Flows not dynamically similar

15/15

1.20

Problem: A real Lear-Jet and a model are experiencing different flows, but the same C_L, C_D are needed

Given: Real = 1
$V_{\infty R} = 250$ m/s
$\rho_{\infty R} = 0.414$ kg/m³
$T_{\infty R} = 223$ K

Model 1/5 scale ⇒ 2
$V_{\infty M} =$
$\rho_{\infty M} =$
$T_{\infty M} =$
$P_{\infty M} = 1.01 \times 10^5$ N/m²

$P = \rho R T$

Find: Velocity, Temperature, density of model airflow so that C_L, C_D are same for both real and model

Assume: Steady flow

Analysis: For C_L, C_D to be equal the flows must be dynamically similar +2

$M_1 \overset{+1}{=} M_2$ $\qquad Re_1 \overset{+1}{=} Re_2$ $\qquad a_1 \overset{+1}{\propto} \sqrt{T_1}$

$\mu_1 \propto \sqrt{T_1}$ +1

+1 $\dfrac{V_1}{a_1} \overset{+1}{=} \dfrac{V_2}{a_2}$ $\qquad$ +1 $\dfrac{\rho_1 V_1 c_1}{\mu_1} \overset{+1}{=} \dfrac{\rho_2 V_2 c_2}{\mu_2}$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}} \qquad \frac{\mu_2}{\mu_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2}\frac{c_1}{c_2}\frac{\mu_2}{\mu_1} \qquad \frac{\rho_2}{\rho_1} = \sqrt{\frac{T_1}{T_2}}\,\frac{c_1}{c_2}\sqrt{\frac{T_2}{T_1}} \qquad \frac{\rho_2}{\rho_1} = \frac{c_1}{c_2}$$

$\rho_2 = (0.414 \text{ kg/m}^3)(5) = 2.07 \text{ kg/m}^3$ +1

$T_2 \overset{+2}{=} P_2/\rho_2 R = (1.01 \times 10^5 \text{ N/m}^2) / (287 \text{ J/kg K})(2.07 \text{ kg/m}^3) =$

$T_2 = 170$ K

$V_2 = 250 \text{ m/s} \sqrt{\dfrac{170\text{ K}}{223\text{ K}}} = 218.3$ m/s +1

$V_{\infty 2} = 218.3$ m/s $\qquad T_2 = 170$ K $\qquad \rho_2 = 2.07$ kg/m³ +1

1.22 Problem:

Given:

$\forall = 15{,}000\ m^3$ $\quad d_M = 14\ m$

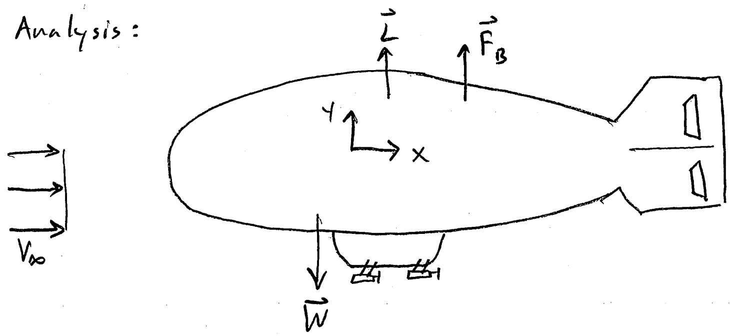
$V_\infty = 30\ m/s$ $\quad z = 1000\ m$ $\quad C_L = 0.05$

Zeppelin in straight, level flight

Find: Total Weight of Zeppelin

Assume: Steady flow, no vertical acceleration
Zeppelin can be modeled as cylinder of d_M

Analysis:



$\forall = \frac{\pi}{4} d^2 \ell$

$\ell = \dfrac{15000\ m^3}{\left(\frac{\pi}{4}\right)(14m)^2} = 97m$

$S = \frac{\pi d^3}{4}$

+3 $\quad$ -1

$L = \frac{1}{2} C_L \rho_\infty V_\infty^2 A \qquad \rho_\infty = 1.1117\ kg/m^3$

$L = \frac{1}{2}(0.05)(1.117\ kg/m^3)(30\ m/s)^2 \pi (14m)(97m) = 107\,222\ N$

$F_B = \overset{+3}{\rho_A} \forall g = (1.117\ kg/m^3)(15000\ m^3)(9.8\ m/s^2) = 164199\ N$

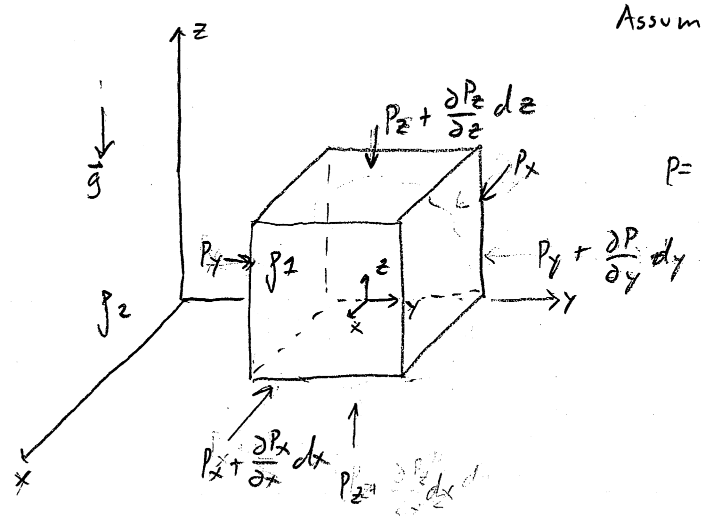
$\Sigma \vec{F}_y = 0 = \overset{-1}{L} + F_B - W \qquad W = L + F_B$

$W = 107\,222N + 164199N = 271421\ N$

1.14

Problem: Derive Archimedes Principle

Assume: Body in equilibrium



$P = P(x, y, z)$

$P = P_a + \rho g z$

$\frac{\partial P}{\partial z} = -\rho_2 g$

$$dF_x = \left(P_x - \left(P_x + \frac{\partial P_x}{\partial x} dx\right)\right) dy\, dz = -\frac{\partial P_x}{\partial x} d\forall = 0$$

$$dF_y = \left(P_y - \left(P_y + \frac{\partial P_y}{\partial y} dy\right)\right) dx\, dz = -\frac{\partial P_y}{\partial y} d\forall = 0$$

$$dF_z = \left(P_z - \left(P_z + \frac{\partial P_z}{\partial z} dz\right)\right) dx\, dy - \rho_1 g\, d\forall$$

$$= -\frac{\partial P_z}{\partial z} d\forall - \rho_1 g\, d\forall = (\rho_2 g - \rho_1 g)\, d\forall$$

$$dF_z = (\rho_2 - \rho_1) g\, d\forall\, \hat{k}$$

$$F_z = (\rho_2 - \rho_1) g \forall \hat{k}$$

not general body