MAE 101B, Spring 2007
Homework 1
Due Thursday, April 12, in class

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.
Please refrain from copying. Refer to the course outline for what constitutes copying

1. The laminar fully-developed flow in a channel between two infinite plates at $y = \pm h$ is:

$$u = \frac{h^2 \Delta p}{2\mu l}\left(1 - \frac{y^2}{h^2}\right) . \tag{1}$$

Flow is in the $+x$ direction, and the quantity, Δp, is defined to be positive.

a) Start from the incompressible Navier-Stokes equations to derive Eq. (1).

b) Start from Eq. (1) to derive a relationship between the friction factor, f, and the Reynolds number, $Re_h = Vh/\nu$.

2.

a) Glycerin flows with a given flow rate, Q, through a pipe of diameter D_1 that connects to a pipe of diameter $D_2 = D/2$. The pressure gradient measured in the section with diameter D_1 is measured to be $1 N/m^3$. Assume laminar flow. What is the pressure gradient in the section with diameter D_2? Does your answer change if the fluid is water instead of glycerin?

b) Consider the same geometry as in part (a) and let $Q = 3m^3/s$. It is desired that the flow be laminar. What is the minimum value of D_1 and the corresponding $D_2 = D_1/2$. What is the pressure drop under these conditions? For glycerin, take $\rho = 1260\, kg/m^3$ and $\mu = 1.49\, kg/m - s$.

3. Water flows through a horizontal pipe of length $10\, m$. The power delivered to the pipe is $1\, hp$. Take $\rho = 1000\, kg/m^3$, $\nu = 10^{-6}\, m^2/s$

a) The flow is at the laminar transition point. What is the pipe diameter?

b) What is the wall shear stress?

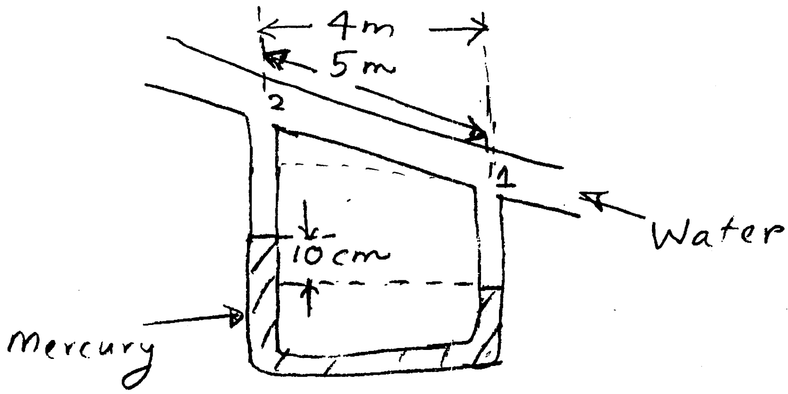
4. Water flows upward at $10\ m/s$ in a $5\ cm$ diameter pipe as shown in the figure on the next page. The mercury manometer has a reading of $h = 10\, cm$.

a) What is the pressure differential, $\Delta p = p_1 - p_2$.

b) What is the head loss?

c) What is the friction factor?

Ungraded problems From text. 6.27, 6.36,



Problem 4

MAE 101B | HOMEWORK 1

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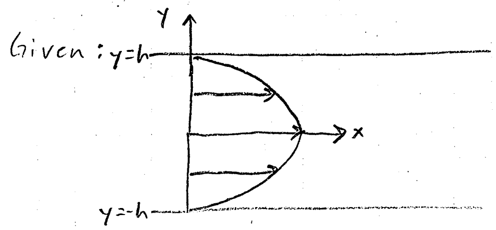
1 Problem: The laminar fully developed flow in a channel between infinite plates at $y=\pm h$ is $u = \frac{h^2 \Delta P}{2\mu L}\left(1 - y^2/h^2\right)$

Given: $y=h$, $y=-h$



$\Delta P > 0$

1	7/10
2	9.5/10
3	10/10
4	10/10
	36.5/40

Find a) Derive the flow profile
b) Derive a relationship between f, $Re_h = Vh/\nu$

Engr Model 1 Laminar flow $Re_h < 2300$
2 Steady, fully developed flow, Incompressible

Analysis

N-S for x $\quad \rho g_x^{\,0} - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2}^{0} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}^{0}\right) = \rho \frac{du}{dt}^{0}$

$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x} \qquad u(y) = \frac{1}{2\mu}\frac{\partial P}{\partial x} y^2 + C_1 y + C_2 \qquad$ BCs $u(y=\pm h) = 0$

$0 = \cancel{\frac{1}{2\mu}\frac{\partial P}{\partial x}h^2} + C_1 h + \cancel{C_2} = \cancel{\frac{1}{2\mu}\frac{\partial P}{\partial x}h^2} + C_1 h + \cancel{C_2} \qquad C_1 = 0$

$0 = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)h^2 + C_2 \qquad C_2 = -\frac{h^2}{2\mu}\left(\frac{\partial P}{\partial x}\right) \qquad \frac{\partial P}{\partial x} = \frac{P_1 - P_2}{x} = -\frac{\Delta P}{L}$

$u(y) = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left[y^2 - h^2\right] = \frac{h^2}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(\frac{y^2}{h^2} - 1\right)$

$u(y) = \frac{h^2 \Delta P}{2\mu L}\left[1 - y^2/h^2\right] \qquad U_{MAX} \; \frac{h^2 \Delta P}{2\mu L}$

b) $Re_h = Vh/\nu = \frac{\rho V h}{\mu} \qquad f = \frac{8\tau_w}{\rho V^2} \qquad V = \cancel{\tfrac{1}{2} U_{MAX}} = \frac{h^2 \Delta P}{4\mu L} \quad \frac{h^2}{3\mu}\frac{\Delta P}{L}$

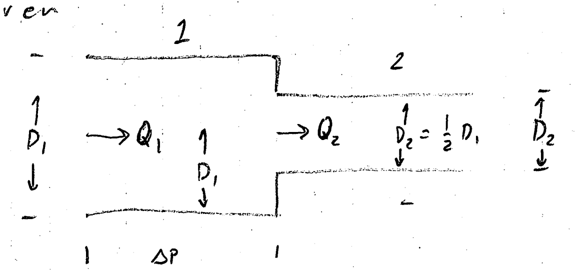
$\tau_w = \mu \frac{du}{dy}\Big|_{y=-h} = \frac{h^2 \Delta P}{2L}\left[-\frac{2y}{h^2}\right|_{y=-h} = \frac{h \Delta P}{L}$

$f = \frac{8\frac{h\Delta P}{L}}{\rho\left(\frac{h^2 \Delta P}{4\mu L}\right)^2} = \frac{8 h \Delta P / L}{\frac{\rho h^4 \Delta P^2}{16\mu^2 L^2}} \quad \frac{(8 h \Delta P)(16\mu^2 L^{\cancel{2}})}{\rho h^4 \Delta P^{\cancel{2}} \cancel{L}} = \frac{128 \mu^2 L}{\rho h^3 \Delta P}$

$= \frac{32\mu}{\rho h}\left(\frac{4\mu L}{h^2 \Delta P}\right) = \frac{32\nu}{h}\left(\frac{1}{V}\right) = \frac{32\nu}{Vh} = \cancel{32/Re_h} \quad \frac{24}{Re} \quad -3$

2 Problem: Glycerin flows from pipe 1 to pipe 2

Given



$D_2 = D_1/2$

$Q_1 = Q_2$

$\Delta P_1 = 1 \, N/m^3$

$\rho_s = 1260 \, kg/m^3$

$\mu = 1.49 \, kg/ms$

Find: a) pressure gradient in pipe 2

Eng Model: 1 Incompressible, steady flow

2 Laminar flow

Analysis

a) $\frac{\Delta P}{L} = 1 \, N/m^3$ Eq 6.12 $h_f = \cancel{\Delta z}^{\,0} + \frac{\Delta P}{\rho g} = \frac{128 \mu L Q}{\pi \rho g d^4}$

$\Delta P = \frac{128 \mu L Q}{\pi d^4}$ $Q = \frac{\pi d^4 \Delta P}{128 \mu L}$ $Q_i = \frac{\pi d_i^4 \Delta P_i}{128 \mu L_i}$

$Q_1 = Q_2$ $\frac{\cancel{\pi} d_1^4 \Delta P_1}{\cancel{128 \mu} L_1} = \frac{\cancel{\pi} d_2^4 \Delta P_2}{\cancel{128 \mu} L_2}$

$\left(\frac{\Delta P}{L}\right)_2 = \left(\frac{d_1}{d_2}\right)^4 \left(\frac{\Delta P}{L}\right)_1 = \left(\frac{D_1}{D_1/2}\right)^4 (1 \, N/m^3) = 16 \, N/m^3$

$\left(\frac{\Delta P}{L}\right)_2 = 16 \, N/m^3$ Not dependent on μ, so same for all liquids

b) $Q = 3 \, m^3/s$ Laminar flow needed

$\rho = 1260 \, kg/m^3$ $\mu = 1.49 \, kg/ms$ Find min D_1

$Re_d = \frac{\rho V D}{\mu} < 2300$ $VD < 2300 \mu/\rho = 2300(1.49 \, kg/ms / 1260 \, kg/m^3)$

$= 2.7199 \, m^2/s$

$\frac{Q}{VD} = \frac{\pi D^2 V}{VD} = \pi D$ $D = \frac{Q}{\pi V D}$ $D = \frac{3 \, m^3/s}{\pi (2.7199 \, m^2/s)} = 0.3511 \, m$

$D_1 = 0.3511 \, m$ $D_2 = 0.175545 \, m$ $\frac{\Delta P}{L} = \frac{128 \mu Q}{\pi D^4}$

$V_1 = 7.746796 \, m/s$ $V_2 = 15.4937 \, m/s$

$\left(\frac{\Delta P}{L}\right)_1 = 11985 \, N/m^3$ $\left(\frac{\Delta P}{L}\right)_2 = 191784 \, N/m^3$

b) $Q = 3\ m/s$ $\quad Re < 2300$

$\rho = 1260\ kg/m^3$ $\quad \mu = 1.49\ kg/ms$

Laminar $Re \leq 2300$

$$Re = \frac{\rho V_2 D_2}{\mu} \qquad V_2 D_2 = \frac{\mu Re}{\rho} =$$

$$V_2 D_2 = \frac{(1.49\ kg/ms)(2300)}{1260\ kg/m^3} = 2.7194\ m^2/s$$

$$\frac{Q = \frac{\pi}{4} D_2^2 V_2}{D_2 V_2 \quad V_2 D_2} \qquad D_2 = \frac{4Q}{\pi D_2 V_2} \qquad D_2 = \frac{4(3\ m^3/s)}{\pi(2.7198\ m^2/s)} = 1.4044\ m$$

$D_2 = \quad 4(3\ m/s)$

$\boxed{D_1\ min = 2.8088\ m}$ ✓ $\Rightarrow V_1 = 0.48416\ m/s$

$\boxed{D_2 = 1.4044\ m}$ ✓ $\Rightarrow V_2 = 1.936\ m/s$

$$h_f = \Delta \cancel{z}^{0} + \frac{\Delta P}{\rho g} = \frac{32 \mu L V}{\rho g d^2} \qquad \frac{\Delta P}{L} = \frac{32 \mu V}{D^2}$$

$$\left(\frac{\Delta P}{L}\right)_1 = \frac{32(1.49\ kg/ms)(0.48416\ m/s)}{(2.8088\ m)} = 2.926\ N/m^3$$ ✓

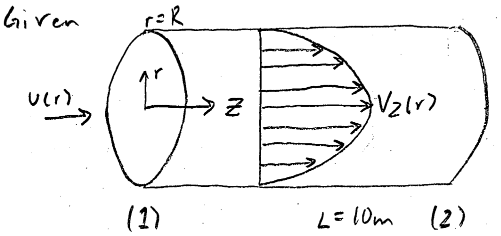
$$\left(\frac{\Delta P}{L}\right)_2 = \frac{32(1.49\ kg/ms)(1.936\ m/s)}{(1.4044\ m)^4} = 46.8\ N/m^3$$ ✓ −0.5 check math

$$\Delta P_T = -2.926\ N/m^3 (L_1) - 46.8\ N/m^3 (L_2)$$

9.5 / 10

3 Problem: Water flows through a pipe delivering 1 hp of work

Given



$L = 10m$
$\rho = 1000\ ^{kg}/_{m^3}$
$\nu = 10^{-6}\ ^{m^2}/_s$
$\dot{W} = 1\ hp = 746\ W$

Find: a) pipe diameter for laminar flow
b) wall shear stress τ_w

Engr Model: 1 Flow at laminar transition $\Rightarrow$ $Re_d = 2300$
2 Smooth pipe walls
3 Fully developed, steady flow $v = v(r)$ $\frac{\partial v}{\partial t} = 0$

Analysis:

$$\cancelto{0}{\frac{\partial V_z}{\partial t}} + \left(\cancelto{0}{V_r}\frac{\partial V_z}{\partial r} + \frac{1}{r}\cancelto{0}{V_\theta}\frac{\partial V_z}{\partial \theta} + \cancelto{0}{V_z}\frac{\partial V_z}{\partial z}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \cancelto{0}{g_z} + \nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 V_z}{\partial \theta^2}} + \cancelto{0}{\frac{\partial^2 V_z}{\partial z^2}}\right)$$

$$\nu\int_{r=0}^{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) = \int_{r=0}^{r}\left(\frac{1}{\rho}\frac{\partial P}{\partial z}\right)r \qquad r\frac{\partial V_z}{\partial r}\Big|_{r=0}^{r} = \frac{1}{\rho}\frac{\partial P}{\partial z}\frac{r^2}{2}\Big|_{r=0}^{r} \qquad r\frac{\partial V_z}{\partial r} = \frac{1}{\nu\rho}\frac{\partial P}{\partial z}\frac{r^2}{2}$$

$$\int_{r}^{r=R}\frac{\partial V_z}{\partial r} = \left(\frac{1}{\nu 2\rho}\frac{\partial P}{\partial z}\right)\int_{r}^{r=R} r \qquad V_z(r)\Big|_{r}^{r=R} = \left(\frac{1}{2\rho\nu}\frac{\partial P}{\partial z}\right)\frac{r^2}{2}\Big|_{r}^{r=R}$$

$$\cancelto{0}{V_z(R)} - V_z(r) = \left(\frac{1}{4\rho}\frac{\partial P}{\partial z}\right)(R^2 - r^2) \qquad V_z(r) = \left(\frac{R^2}{4\rho}\right)\left(\frac{\partial P}{\partial z}\right)\left(\frac{r^2}{R^2} - 1\right)$$

$$\frac{\partial P}{\partial z} = -\frac{\Delta P}{L} \qquad V_z(r) = \left(\frac{R^2}{4\mu}\right)\left(\frac{\Delta P}{L}\right)\left(1 - \frac{r^2}{R^2}\right) = U_{MAX}\left(1 - \frac{r^2}{R^2}\right)$$

$$\dot{W} = Q\Delta P = VA\Delta P \qquad h_f = \cancelto{0}{\Delta z} + \frac{\Delta P}{\rho g} = \frac{128\mu L Q}{\pi\rho g d^4} \quad Eq\ 6.13$$

$$\Delta P = 128\mu L Q/\pi d^4$$

$$\dot{W} = \frac{128\mu L V^2\left(\frac{\pi}{4}d^2\right)^2}{\pi d^4} = \frac{128}{16}\pi\mu L V^2$$

$$\Rightarrow V = \sqrt{\frac{16\dot{W}}{128\mu L\pi}} = \sqrt{\frac{16(746W)}{128(1\times10^{-3}\ ^{kg}/_{m\,s})(10\,m)\pi}} = 54.482\ ^{m}/_{s}$$

$$d = \frac{\mu Re}{\rho V} = \frac{(1\times10^{-3}\ ^{kg}/_{m\,s})(2300)}{(1000\ ^{kg}/_{m^3})(54.482\ ^{m}/_{s})} = 4.222\times10^{-5}\,m = 42.22\,\mu m$$

b)

$\Sigma F = \Delta P A - \tau_w A_w$

$\tau_w = \dfrac{\Delta P A_c}{A_w}$ ✓

$\Delta P = \dfrac{128 \mu L V (\frac{\pi}{4} D^2)}{\pi D^4 \; D^2} = \dfrac{32 \mu L V}{D^2}$ ✓

$= \dfrac{32 (1\times10^{-3}\ kg/ms)(10\ m)(54.482\ m/s)}{(42.22\times10^{-6}\ m)^2}$

$= 9.78\ GPa$

$\tau_w = \dfrac{\Delta P \frac{\pi}{4} D^2}{\pi D L} = \dfrac{\Delta P D}{4L} = \dfrac{(9.78\ GPa)(42.22\times10^{-6}\ m)}{4(10\ m)}$ ✓

$\tau_w = 10322.79\ N/m^2$ ✓

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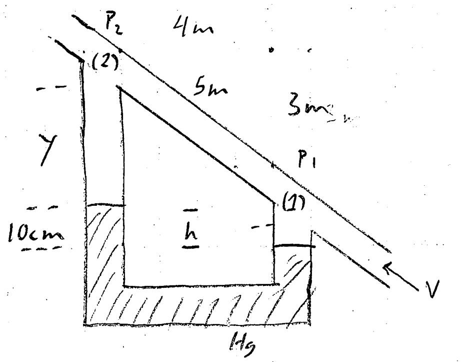
4 Problem: Water flows upward in a pipe

Given



$V = 10\ m/s$
$d = 5cm$
$h = 10cm$

Find a) Pressure differential $P_1 - P_2$
b) head loss h_f
c) friction factor f

Engr Model: 1 Smooth pipe
2 Incompressible, steady, fully developed flow

Analysis

a) $P_2 + \gamma_w Y + \gamma_{Hg} h = P_1 \qquad P_1 - P_2 = \gamma_w Y + \gamma_{Hg} h$

$P_1 - P_2 = 9.81\ m/s^2 \left[(998\ kg/m^3)(3\ m) + (13550\ kg/m^3)(0.10\ m)\right] =$

$P_1 - P_2 = 42663.7\ Pa = 42.664\ kPa$

b) $HGL_1 = z_1 + \frac{P_1}{\rho g} = 0 + \frac{42.664\ kPa}{(998\ kg/m^3)(9.81\ m/s^2)} = 4.35775\ m$

$HGL_2 = z_2 + \frac{P_2}{\rho g} = 3m + 0 = 3m$

$h_f = HGL_1 - HGL_2 = 1.35775\ m$

c) $Re_d = \frac{\rho V d}{\mu} = \frac{(998\ kg/m^3)(10\ m/s)(0.05\ m)}{1 \times 10^{-3}\ kg/ms} = 499000$ turbulent

$h_f = f \frac{L}{d} \frac{V^2}{2g} \qquad f = \frac{2gdh_f}{LV^2} = \frac{2(0.05m)(9.81\ m/s^2)(1.35775\ m)}{(5m)(10\ m/s)^2} = 2.664 \times 10^{-3}$

10/10