ρadmap to Analysis

Christopher Helmes

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1 Prerequisities

1.1 Real World

In principal we want to analyse the mass m_ρ and the decay width Γ_ρ of the ρ-resonance. For this we utilize in a first approach a 2×2 Correlation Matrix in the center of mass frame and two different moving frames. The entries of the correlation matrix involve the two different For each frame under consideration the analysis follows the same steps operators specified in the paper by Feng, Jansen and Renner:

$$\mathcal{O}_{\pi\pi}(t) = \pi^+(\vec{P}, t)\pi^-(0, t) - \pi^+(0, t)\pi^+(\vec{P}, t) \tag{1}$$

$$\mathcal{O}_\rho(t) = \rho^0(\vec{P}, t) \tag{2}$$

For each frame under consideration the analysis follows the same steps. The scattering amplitude in inelastically interacting systems is given by the relativistic Breit-Wigner formula:

$$a_l = \frac{-\sqrt(s)\Gamma_R(s)}{s - M_R^2 + i\sqrt{s}}$$

with the CM energy $E_{CM}^2 = s$ and the mass of the resonance M_R. The relation between scattering amplitude and scattering phaseshift δ_l yields:

$$\tan\delta_l = \frac{\sqrt{s}\Gamma_R(s)}{M_R^2 - s} \tag{3}$$

Taking into account p-wave scattering ($l = 1$), the effective coupling constant $g_{\rho\pi\pi}$, the ρ-mass, the π-mass m_π and the center of mass energy give rise to the effective range formula

$$\tan\delta_1 = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E_{CM}(m_\rho^2 - E_{CM}^2)} \quad , \qquad p = \sqrt{\frac{E_{CM}^2}{4} - m_\pi^2}$$

1.2 Lattice

2 Analysis

Given the contractions have been made the resulting Correlation functions involving every combination of the operators $\mathcal{O}_{\pi\pi}$ and $\mathcal{O}_\rho$ are combined in a bootstrap analysis to extract the two energy-levels from the correlation matrix. In a first step the N_C configurations are bootstrapped according to the following pattern: View the (symmetrized) correlation functions as a 2d array in $n_c = 1, \ldots, N_C$ and $t = 1, \ldots, {}^T/_2$, where T is the lattice time extent. One obtains N_{BS} Bootstrapsamples of the N_C correlation functions at every value for t as a sum over N_C arbitrarily chosen values of $C(t)$ the sample with $n_{BS} = 1$ is taken as the sum (average) of the original dataset. In the end there exists a 2d array of size $N_{BS} \times {}^T/_2$ A short illustration may clarify things.

On every bootstrap sample the correlation matrix $C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle$ can be built. Cast into a generalized eigenvalue problem this yields:

$$C(t)v_n(t,t_0) = \begin{pmatrix} \langle \mathcal{O}_{\pi\pi}\mathcal{O}_{\pi\pi}^\dagger \rangle (t) & \langle \mathcal{O}_{\pi\pi}\mathcal{O}_{\rho}^\dagger \rangle (t) \\ \langle \mathcal{O}_{\rho}\mathcal{O}_{\pi\pi}^\dagger \rangle (t) & \langle \mathcal{O}_{\rho}\mathcal{O}_{\rho}^\dagger \rangle (t) \end{pmatrix} \begin{pmatrix} v_{n,1}(t,t_0) \\ v_{n,2}(t,t_0) \end{pmatrix} = \lambda_n(t,t_0) C(t_0) v_n(t,t_0) \quad (4)$$

The eigenvalues λ_n are given by:

$$\lambda_n(t,t_0) = \exp[-E_n(t-t_0)] \quad (5)$$

corresponding to the energies of interest (in our case $n < 2$). To prevent contamination stemming from states with $n > 2$ we use the correlation matrix $R(t,t_R)$ composed of:

$$R(t,t_R) = C_{2\times 2}(t) C_{2\times 2}^{-1}(t_R) \quad (6)$$

In this approach the eigenvalues behave as

$$\tilde{\lambda}_n(t,t_0) = A_n \cosh\left[-E_n\left(t - \frac{T}{2}\right)\right] \quad (7)$$