# $\rho$admap to Analysis

Christopher Helmes

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## 1 Prerequisities

### 1.1 Real World

In principal we want to analyse the mass $m_\rho$ and the decay width $\Gamma_\rho$ of the $\rho$-resonance. For this we utilize in a first approach a $2 \times 2$ Correlation Matrix in the center of mass frame and two different moving frames. The entries of the correlation matrix involve the two different For each frame under consideration the analysis follows the same steps operators specified in the paper by Feng, Jansen and Renner:

$$\mathcal{O}_{\pi\pi}(t) = \pi^+(\vec{P}, t)\pi^-(0, t) - \pi^+(0, t)\pi^+(\vec{P}, t) \tag{1}$$

$$\mathcal{O}_\rho(t) = \rho^0(\vec{P}, t) \tag{2}$$

For each frame under consideration the analysis follows the same steps. The scattering amplitude in inelastically interacting systems is given by the relativistic Breit-Wigner formula:

$$a_l = \frac{-\sqrt(s)\Gamma_R(s)}{s - M_R^2 + i\sqrt{s}}$$

with the CM energy $E_{CM}^2 = s$ and the mass of the resonance $M_R$. The relation between scattering amplitude and scattering phaseshift $\delta_l$ yields:

$$\tan\delta_l = \frac{\sqrt{s}\Gamma_R(s)}{M_R^2 - s} \tag{3}$$

Taking into account p-wave scattering ($l = 1$), the effective coupling constant $g_{\rho\pi\pi}$, the $\rho$-mass, the $\pi$-mass $m_\pi$ and the center of mass energy give rise to the effective range formula

$$\tan\delta_1 = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E_{CM}(m_\rho^2 - E_{CM}^2)} \quad , \qquad p = \sqrt{\frac{E_{CM}^2}{4} - m_\pi^2}$$

### 1.2 Lattice

## 2 Analysis

Given the contractions have been made the resulting Correlation functions involving every combination of the operators $\mathcal{O}_{\pi\pi}$ and $\mathcal{O}_\rho$ are combined in a bootstrap analysis to extract the two energy-levels from the correlation matrix. In a first step the $N_C$ configurations are bootstrapped according to the following pattern: View the (symmetrized) correlation functions as a 2d array in $n_c = 1, \ldots, N_C$ and $t = 1, \ldots, T/2$, where $T$ is the lattice time extent. One obtains $N_{BS}$ Bootstrapsamples of the $N_C$ correlation functions at every value for $t$ as a sum over $N_C$ arbitrarily chosen values of $C(t)$ the sample with $n_{BS} = 1$ is taken as the sum (average) of the original dataset. In the end there exists a 2d array of size $N_{BS} \times T/2$ A short illustration may clarify things.

On every bootstrap sample the correlation matrix $C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle$ can be built. Cast into a generalized eigenvalue problem this yields:

$$C(t)v_n(t,t_0) = \begin{pmatrix} \langle \mathcal{O}_{\pi\pi}\mathcal{O}_{\pi\pi}^\dagger \rangle (t) & \langle \mathcal{O}_{\pi\pi}\mathcal{O}_{\rho}^\dagger \rangle (t) \\ \langle \mathcal{O}_{\rho}\mathcal{O}_{\pi\pi}^\dagger \rangle (t) & \langle \mathcal{O}_{\rho}\mathcal{O}_{\rho}^\dagger \rangle (t) \end{pmatrix} \begin{pmatrix} v_{n,1}(t,t_0) \\ v_{n,2}(t,t_0) \end{pmatrix} = \lambda_n(t,t_0) C(t_0) v_n(t,t_0) \quad (4)$$

The eigenvalues $\lambda_n$ are given by:

$$\lambda_n(t,t_0) = \exp[-E_n(t-t_0)] \quad (5)$$

corresponding to the energies of interest (in our case $n < 2$). To prevent contamination stemming from states with $n > 2$ we use the correlation matrix $R(t,t_R)$ composed of:

$$R(t,t_R) = C_{2\times 2}(t) C_{2\times 2}^{-1}(t_R) \quad (6)$$

In this approach the eigenvalues behave as

$$\tilde{\lambda}_n(t) = A_n \cosh\left[-E_n\left(t - \frac{T}{2}\right)\right] \quad (7)$$

The derivative method used in the case of $\pi\pi$-scattering to cancel out contributions to the correlation matrix which are constant in $t$ is spoiled here by the time dependence of these contributions. A general correlation function can be written as:

$$\langle \mathcal{O}_i \mathcal{O}_j^\dagger \rangle (t) = \frac{1}{Z} \mathrm{tr}\left[\exp[-\hat{H}(T-t)] \mathcal{O}_i(0) \exp[-\hat{H}t] \mathcal{O}_j^\dagger(0)\right] \quad (8)$$

$$= \sum_{m,n} V_{imn} V_{jmn}^\dagger \exp\left[-(E_m + E_n)T/2\right] \cosh\left[(E_m - E_n)(t - T/2)\right] \quad (9)$$

with the matrix elements $V_{imn}$

$$V_{imn} = \langle m | \mathcal{O}_i(t) | n \rangle \quad (10)$$

Taking $E_m > 0$ and $E_n > 0$ the thermally suppressed states also contribute with a time dependant factor of the cosh and therefore influence the Correlation functions. Because of the nonconstant contribution the derivative method fails.

## 2.1 Uncertainties

The uncertainties in the approach proposed above are divided into statistical and systematical ones. Statistical uncertainties can be assessed by the standard deviation of the mean value on each bootstrap sample. The difference between the sample means and the mean value obtained from the original data serves as an estimator for the bias. Systematic uncertainties can be inferred from a distribution of fit results where each fit result is obtained using a varying fit range. The median of the distribution of the values gives an estimate on the systematic error.