quanmu 2
jianc 2

1. There's a point (x, y, z) and it translated by $(1, 9, 6)$, followed by a roll of 45 deree.

2. $$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & -\frac{1}{2} & -\frac{1}{2}\sqrt{3} & 9 \\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. $(T_2 T_1)^{-1} = T_1^{-1} \cdot T_2^{-1} =$
$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{-19-8\sqrt{3}}{2\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1-8\sqrt{3}}{2\sqrt{2}} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{8\sqrt{3}-4}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. for q_1: $q_{1,0} = \frac{1}{2}\sqrt{2+\sqrt{2}}$ $q_{1,1} = \frac{1}{\sqrt{4+2\sqrt{2}}}$ $q_{1,2} = 0$ $q_{1,3} = \frac{1}{\sqrt{4+2\sqrt{2}}}$

$$q_1 = \left[\frac{1}{2}\sqrt{2+\sqrt{2}}, 0, 0, \frac{1}{\sqrt{4+2\sqrt{2}}}\right]$$

for q_2: $q_{2,0} = \frac{1}{2}$ $q_{2,1} = \frac{1}{2}\sqrt{3}$ $q_{2,2} = 0$ $q_{2,3} = 0$

$$q_2 = \left[\frac{1}{2}, \frac{1}{2}\sqrt{3}, 0, 0\right]$$

5. $q_1 \cdot q_2 = \frac{1}{4}\sqrt{2+\sqrt{2}}$ $(q_0 \cdot q_0 + q_1 \cdot q_1 + q_2 \cdot q_2 + q_3 \cdot q_3)$

6. In the cases that the matrix can only move along wor with each axis at most once.
for example, if you move along x axis, the x axis cannot move during the rotation (must be in the same direction as before).

for 3:

$$T_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{9-8\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{9\sqrt{3}+8}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -5\sqrt{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -4\sqrt{2} \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$