

# CALIOPE: A Search for $CP$ -Violation in Positronium

Chelsea Bartram

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## Abstract

We propose to search for  $CP$ -violation in the charged lepton sector by studying ortho-positronium decays. Positronium, a bound state of an electron and positron, occurs in both a singlet and triplet state. The triplet state, ortho-positronium, decays primarily into three gamma rays.  $CP$ -violation could potentially manifest itself in angular correlations between the directions of the three photons and the spin of the ortho-positronium (o-Ps). We propose to use the APEX annular array of NaI detectors for gamma ray detection, combined with a tagged source and a novel, conventional electromagnet. APEX will increase the angular acceptance, hence, statistics, by a factor of 25 over previous experiments. We also implement several experimental improvements to reduce our systematic uncertainties compared to previous experiments. I will present the current status of the experiment, named CALIOPE, which stands for  $CP$  Aberrant Leptons in Ortho-Positronium Experiment, and our future plans.

## 1 Introduction

### 1.1 Fundamental Symmetries

The notion of symmetry permeates our physical theories as well as our intuition. There are three fundamental discrete symmetries that characterize nuclear phenomena and reveal preserved quantities. The charge conjugation operator,  $C$ , flips the charge of a particle. The parity operator,  $P$ , produces a mirror inversion of a system. Finally, the time operator,  $T$ , can be interpreted as a reversal of time, or, identically, a reversal of the momenta. These three operators (and combinations of these operators) form the fundamental discrete symmetries.

The conservation of parity symmetry in particle interactions was not questioned until 1956 when theorists T.D. Lee and C. N. Yang [1] noticed that evidence for its existence in weak interactions was gravely lacking. Parity invariance was so established in the minds of physicists that the subsequent discovery of its violation by Chien-Shiung Wu [2] was received with great skepticism and surprise. Wu's experiment, involving the beta decay of  $^{60}\text{Co}$ , demonstrated that electrons are emitted in a preferential direction relative to the spin orientation of the cobalt nuclei. Not only did it show that parity was violated, it was *maximally* violated. After the fall of parity, physicists assumed that the combination of the charge and parity operators must be conserved, as  $CP$ -invariance seemed to prevail in the vast majority of particle interactions and decays. In 1964, however, it was discovered by Cronin and Fitch [3] that  $CP$ -violation occurred in kaon decay. The discovery of  $CP$ -violation has since prompted physicists to search for other symmetry violating effects, resulting in the discovery of  $CP$ -violation in both B meson [4] and D meson oscillations [5].

In a sense, these discoveries were fortuitous, as Andrei Sakharov [6] pointed out in 1967 that  $CP$ -violation is actually necessary to explain the existing baryon asymmetry in the universe. In fact, there are three so-called *Sakharov conditions* which must have manifested themselves in the early universe in order to produce the matter-antimatter asymmetry that we see today. These are: baryon number violation, violation of  $C$  and  $CP$ -symmetry and departure from thermal equilibrium. The currently observed  $CP$ -violation is not

sufficient to explain the amount of matter versus antimatter in the universe. It is also limited to the quark sector. This provides strong motivation to search for  $CP$ -violation elsewhere, specifically the lepton sector.

Though numerous examples of symmetry violations have been found, the combination of all three symmetry operators,  $CPT$ , is generally believed to be a conserved symmetry. It is difficult to build sensible quantum field theories, such as the standard model, without  $CPT$  conservation and no  $CPT$ -violation has been detected to date.

$CP$ -violation has only been observed in the weak interactions of the quark sector. Its existence in the lepton sector has not been confirmed experimentally. Notably, long-baseline neutrino experiments are being constructed at great cost to look for  $CP$ -violation in the neutrinos. Experiments such as DUNE (Deep Underground Neutrino Experiment) require a baseline which runs from Illinois to South Dakota and a collaboration of more than 525 scientists and engineers [7]. Other groups have been searching for  $CP$ -violation in the charged lepton sector as well [8]. For example, if the electron has an electric dipole moment, this would be a sign of  $T$  violation, which is equivalent to  $CP$ -violation, provided  $CPT$  is conserved.

## 1.2 $CP$ -violation in Positronium

Our experiment will search for  $CP$ -violation in positronium. Positronium consists of two charged leptons: an electron and positron. Unlike experiments such as DUNE, however, our experiment occupies much less space, costs orders of magnitude less, and employs only a few people. CALIOPE is a table-top nuclear experiment which will be assembled at TUNL, the Triangle Universities Nuclear Laboratory. Specifically, we propose to search for  $CP$ -violating angular correlations between the three gamma rays emitted from ortho-positronium decay and the spin of the positronium. Such  $CP$ -violating correlations are predicted within the Standard Model, but at levels far below our detection limit [9]. Any observation of these correlations at our level of sensitivity would be indicative of new physics.

## 1.3 Previous Experiments

Previous experiments studying the angular correlations in ortho-positronium decay did not observe any  $CP$ -violation. The first measurement was made by M. Skalsey and J.V. House in 1991 [10]. This experiment found no  $CP$  amplitude at the 1.5% level. In 2010, Yamazaki, Namba, Asai, and Kobayashi measured an amplitude of a  $CP$ -violating asymmetry consistent with zero at a sensitivity of  $2.2 \times 10^{-3}$  [9]. This result was a factor of 7 improvement in the limit from the previous experiment. Our experiment will use an array with much greater angular coverage, increasing our statistics by a factor of 25. In addition, we will be implementing several features to improve systematic uncertainties over previous experiments. These are described below. Although it is hard to tell what our ultimate sensitivity improvement will be over previous experiments, we can conservatively estimate it to be a factor of 10 or more.

# 2 Theory

## 2.1 Positronium Physics

Positronium was first discovered by Martin Deutsch at MIT in 1951 [11]. He was able to show that the time it took for gamma rays emitted from the positronium source to reach the detector was longer than would be expected from ordinary annihilation, implying the existence of a long-lived bound state.

Positronium's energy levels can be obtained by modeling it like the hydrogen atom but replacing the mass of the proton with the reduced mass of the electron and positron. Unlike the hydrogen atom, however, positronium is unstable and decays into gamma rays after a finite amount of time. Like the hydrogen atom, positronium also comes in both a triplet ( $S = 1$ ) and singlet ( $S = 0$ ) state. For this experiment, we are only concerned with positronium in its ground state where  $n = 1$ .

The number of gamma rays emitted is determined by charge conjugation invariance. Charge conjugation parity in positronium is given as  $C = (-1)^{s+l}$ , where  $s$  is the spin quantum number and  $l$  is the orbital

angular momentum number. In our system, positronium decays from the  $l = 0$  state, so the  $C$  parity is  $-1$  for the triplet state and  $1$  for the singlet state.  $C$  parity is multiplicative, and the photon has odd charge conjugation parity. Therefore, the triplet state, also known as ortho-positronium, decays into three photons with odd charge conjugation parity and the singlet state, or para-positronium, decays into two photons with even charge conjugation parity. While it is possible for ortho-positronium to decay into a larger, odd number of photons (and likewise, para-positronium into a larger, even number of photons), this rarely happens because the branching ratios for these decays are greatly suppressed.

Number of photons aside, the triplet state and singlet state can also be easily differentiated by their lifetimes. The triplet state has a much smaller phase space and an extra vertex that contributes an extra factor of the fine structure constant. These two features extend the lifetime of the triplet state (142 ns) to nearly a factor of 1000 greater than that of the singlet state (125 ps).

## 2.2 $CP$ -Violating Correlation

$CP$ -violation in ortho-positronium decay could manifest itself as a  $CP$ -violating angular distributions of the emitted gamma rays. One such  $CP$ -violating correlation, as introduced by Bernreuther [12], can be written in the following way:

$$\langle \vec{S} \cdot \vec{n} \rangle \quad (1)$$

$\vec{S}$  is the spin polarization axis and  $\vec{n}$  is the normal to the ortho-positronium decay plane. A measurement of this alone would not only be conclusive of  $CP$ -violation, but also of  $CPT$ -violation. Though this would be an interesting experimental search, it is not the focus of our proposal.

In our experiment, which searches for  $CP$ -violation exclusively, we measure the following correlation, which is  $CPT$ -conserving:

$$Q = P_2(\vec{S} \cdot \vec{k}_1)(\vec{S} \cdot \vec{k}_1 \times \vec{k}_2) = P_2 \sin 2\theta \sin \psi \cos \phi \quad (2)$$

Here,  $\vec{S}$  is the spin of the ortho-positronium, and  $\vec{k}_i$  is the direction of the  $i^{th}$  most energetic gamma ray from the decay.  $P_2$  is known as the tensor polarization.  $\theta$  is the angle between the normal to the ortho-positronium decay plane and the spin quantization axis.  $\psi$  is the angle between  $\vec{k}_1$  and  $\vec{k}_2$ .  $\phi$  is the angle between  $\vec{k}_1$  and the projection of the spin quantization axis onto the ortho-positronium decay plane.  $\psi$  is the angle between the  $\vec{k}_1$  and  $\vec{k}_2$  vectors (See Figure 1).

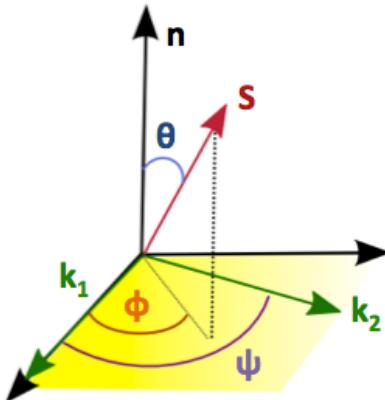


Figure 1: o-Ps Angles

This term,  $Q$ , is sometimes called the *analyzing power* [9], which is scaled by the  $CP$ -violating amplitude in the decay rate.

$$N = N_0[1 + C_{CP} (\vec{S} \cdot \vec{k}_1) (\vec{S} \cdot \vec{k}_1 \times \vec{k}_2)] \quad (3)$$

We will measure the following asymmetry term:

$$A = C_{CP} Q(\theta, \psi, \phi) \quad (4)$$

A positronium system with non-zero asymmetry exhibits  $CP$ -violation.

The tensor polarization (also known as spin-alignment term),  $P_2$ , is defined as follows:

$$P_2 = \frac{N_{+1} - 2N_0 + N_{-1}}{N_{+1} + N_0 + N_{-1}} = 0 \quad (5)$$

$N_m$  stands for the number of o-Ps in the  $m^{th}$  quantum state, where  $m$  is the  $m_s$  quantum number of ortho-positronium. With no magnetic field present, these states have the same half-life and are equally populated at all times, yielding  $P_2 = 0$ . An external magnetic field mixes the triplet  $m = 0$  state with the singlet state and shortens its half-life, yielding a finite value for  $P_2$  that is time-dependent. The  $m = \pm 1$  states are unaffected by the external B-field.

The magnitude of this effect is dependent upon the magnetic field strength. Generally speaking, the greater the magnetic field, the greater the mixing (Figure 2). When  $P_2$  is nonzero, we are able to look for  $CP$ -violation, as the analyzing power would then have nonzero amplitude.

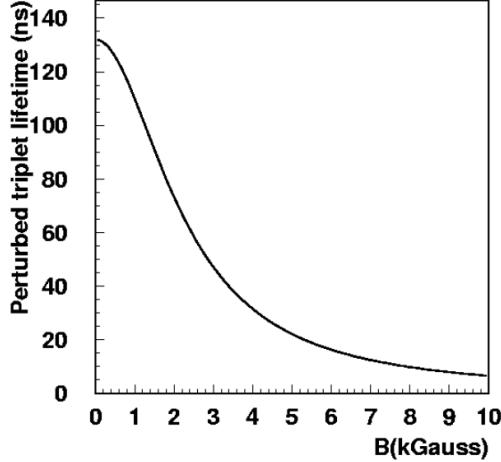


Figure 2: Ortho-positronium lifetime in a magnetic field (in vacuum) [13]

The energies and momenta of the gamma rays must abide by the usual conservation laws, where  $m$  is the rest mass of the electron and  $\vec{k}_i$  are the momenta vectors of the gamma rays:

$$|\vec{k}_1| + |\vec{k}_2| + |\vec{k}_3| = 2m \quad (6)$$

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = \vec{0} \quad (7)$$

The energy spectra is given by the following equation, where  $m$  is the rest mass of the electron, as derived by Ore and Powell [14]:

$$F(k_1) = \int_{m-k_1}^m \left( \frac{m^2(m-k_1)^2}{k_2^2 k_3^2} + \frac{m^2(m-k_2)^2}{k_3^2 k_1^2} + \frac{m^2(m-k_3)^2}{k_1^2 k_2^2} \right) \frac{dk_2}{m} \quad (8)$$

$$= 2 \left( \frac{k_1(m-k_1)}{(2m-k_1)^2} - \frac{2m(m-k_1)^2}{(2m-k_1)^3} \ln \left( \frac{m-k_1}{m} \right) + \frac{2m-k_1}{k_1} + \frac{2m(m-k_1)}{k_1^2} \ln \left( \frac{m-k_1}{m} \right) \right) \quad (9)$$

Due to the symmetry in the distributions, we can use this equation to pick values for the two highest energy photons when we perform Monte Carlo simulations, for example. The energy for  $\vec{k}_3$  is determined by conservation of energy. Figures 3 and 4 show the  $k_1$  and  $k_2$  energy distributions, respectively.

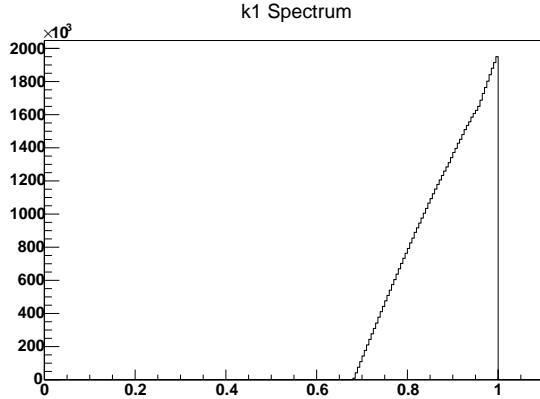


Figure 3:  $k_1$  Energy Spectrum

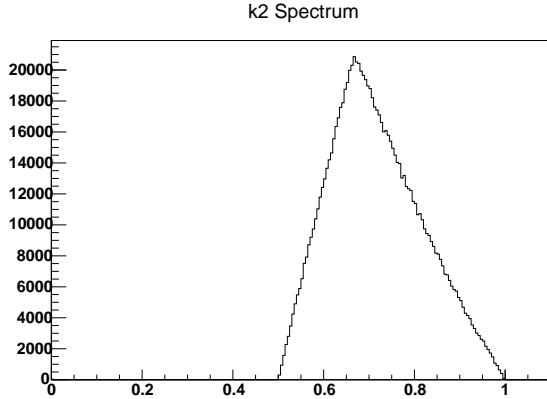


Figure 4:  $k_2$  Energy Spectrum

In the presence of  $CP$ -violation, the energy spectrum for the small fraction of  $CP$ -violating decays will depend on the physics of the  $CP$ -violating process, which is unknown. For our work we will use a simple model that only includes basic phase space considerations, as computed by Ore and Powell. It is easy to check different models using Monte Carlo simulations in the future.

The Standard Model predicts an angular distribution for the gamma rays emitted in ortho-positronium. This distribution was determined by Bernreuther [12] and changes depending on the magnetic quantum number of the ortho-positronium. For  $m = 0$  states, the angle,  $\theta$ , which is defined as the angle between the normal to the decay plane and the spin, the distribution is given as  $P(\theta) = 1 + \cos^2\theta$ . For  $m = \pm 1$ , the distribution is given as  $P(\theta) = \frac{3-\cos^2\theta}{2}$ .

In conclusion, positronium is a well-understood leptonic system, not complicated by effects from quarks and QCD. It serves as a probe for fundamental symmetries, in particular for searches for  $CP$ -violation.

### 3 Experimental Setup

#### 3.1 Positronium Formation, Decay, and Detection

Our experiment can be conceptualized by visualizing the trajectory of a positron which originates in the center of the array and mentally tracking its subsequent interactions. The positron is emitted from a  $^{22}\text{Na}$  source in the middle of our detector.  $^{22}\text{Na}$  decays via the emission of a beta particle (Figure 10) to an excited state of  $^{22}\text{Ne}$ , with a 90% branching ratio. This state is short-lived and decays to the ground state of the  $^{22}\text{Ne}$  via emission of a 1.274 MeV gamma ray. Because the lifetime of the excited state is so short, the 1.274 MeV gamma ray can be used in the event selection process.

Figure 5 shows a schematic of the proposed experiment. The  $^{22}\text{Na}$  is deposited on two thin sheets of kapton foil and sandwiched between scintillator. The scintillator, which is orthogonal to the  $z$ -axis of our

cylindrical detector, tags the positron as it leaves the source. An optical fiber fed through a small hole in the source holder propagates the scintillation light to a nearby photomultiplier tube. This is the “start signal” for an event. The entire source holder is designed to be suspended in the center of the experiment via two long, hollow rods which fit into bore holes in the electromagnet.

Still in the source holder, the positron exits the scintillator and into an adjacent disk of aerogel. Aerogel is comprised of silica dioxide chains folded in such a way that the resulting material is highly porous. The aerogel borders the scintillator on both sides. Once the positron loses nearly all of its energy via scattering, it interacts with an electron in the  $\text{SiO}_2$  to form positronium. The positronium travels almost freely in the interstitial spaces of the aerogel until it decays, minimizing matter-interaction effects. The average lifetime of the  $m = 0$  states of positronium ( $\sim 22$  ns) is dictated by the 5 kGauss (0.5 T) magnetic field generated by our C-shaped electromagnet. We selected this field strength based on the optimum fields determined by previous groups. We will likely adjust this field strength as our studies of systematics mature. The electromagnet serves the dual purpose of separating our the spin states of ortho-positronium and providing spin-alignment. Based on the modeling described below, we have determined that our field inhomogeneity will be less than 2%, a significant improvement over the 10% inhomogeneity of past experiments. Furthermore, we have the ability to switch the magnet’s polarity, which will further reduce our systematics. This improvement has been suggested but not implemented by those who worked on previous  $CP$ -violating searches in positronium. The magnet is tapered in such a way that it cannot interfere with the trajectory of any positronium gamma ray which could be counted in our detector.

Gamma rays from the positronium decay propagate outwards, leaving the source holder and eventually reaching the sodium iodide segments of our detector, where they interact via the photoelectric effect and compton scattering. We use the APEX (ATLAS Positron Experiment [15]) array, a detector which consists of 24 trapezoidal sodium iodide segments (Figure 6). The dimensions of each segment is  $55 \times 6 \times 5.5(7.0)$  cm $^3$ . Each sodium iodide bar is bookended by two photomultiplier tubes, which pick up the resulting scintillation light in the crystal. The photomultiplier tubes run to the DAQ, situated in an electronics rack. Both the APEX detector and the electronics already exist at TUNL.

The cross section of the experiment is shown below.

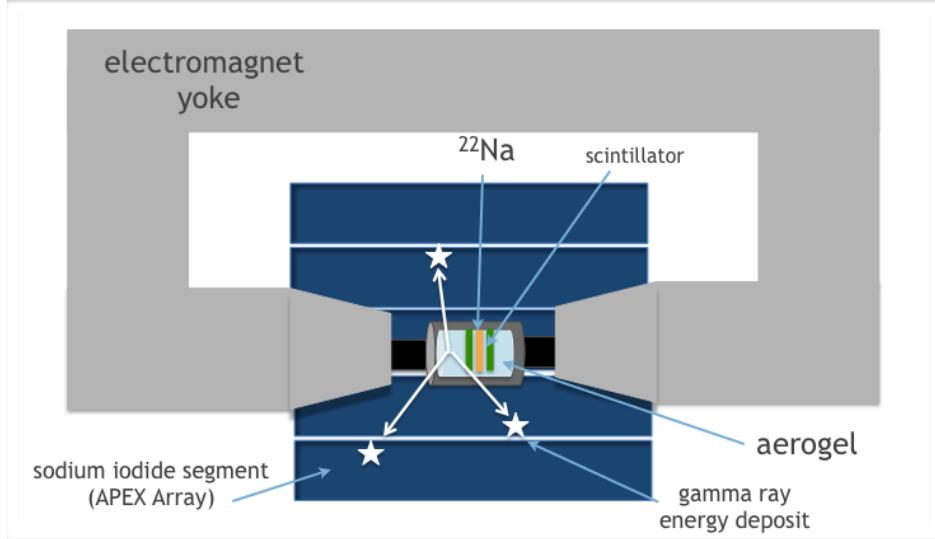


Figure 5: CALIOPE Cross Section (Bird’s Eye View)

### 3.2 Event Reconstruction

We calculate the position and energy of the hits in our detector by using the charge amplitudes measured by the photomultiplier tubes and a technique developed by a former graduate student, Stephen Daigle [16]. The amplitude of the signal from the first photomultiplier tube is

$$A_1 = \frac{E_\gamma P}{E_0} \exp[-\mu(L/2 + z)] \quad (10)$$

where  $z$  is the position of a hit,  $E_\gamma$  is the energy deposited by the gamma ray,  $P$  is the quantum efficiency of the photomultiplier tubes,  $E_0$  is the energy deposited per light photon created in the scintillator and  $\mu$  is the light attenuation coefficient. The attenuation coefficients were all measured by Stephen Daigle [16]. For the second photomultiplier tube, we have a similar equation:

$$A_2 = \frac{E_\gamma P}{E_0} \exp[-\mu(L/2 - z)] \quad (11)$$

We can combine these equations to find the position in the bar from the charge amplitudes.

$$z = \frac{1}{2\mu} \ln \frac{A_2}{A_1} \quad (12)$$

The resolution for the  $z$  position was measured to be 3.5 cm FWHM. In a similar manner, the energy can also be calculated from the charge amplitudes using the following equation:

$$E_\gamma^2 = A_1 A_2 \left( \frac{E_0}{P} \right)^2 e^{\mu L} \quad (13)$$

$$E_\gamma = \sqrt{A_1 A_2} \frac{E_0}{P} e^{\mu L/2} \quad (14)$$

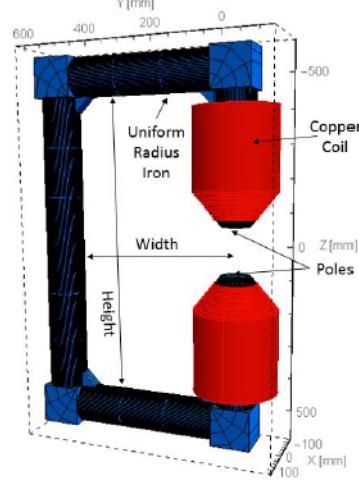


Figure 6: Magnet



Figure 7: Side View of APEX at TUNL

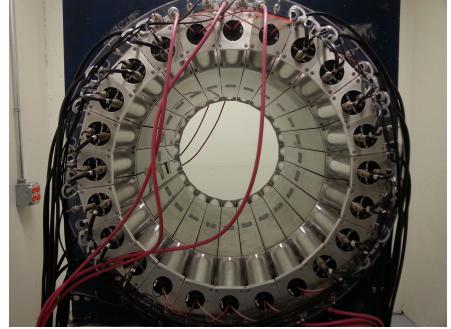


Figure 8: Front View of APEX at TUNL

The resolution for the energy was measured to be 15% for the 662 keV gamma ray line in  $^{137}\text{Cs}$ . The large solid-angle coverage (75%) and high intrinsic efficiency result in a total photopeak detection probability of 17% at 1332 keV, as measured by Perry et al [17].

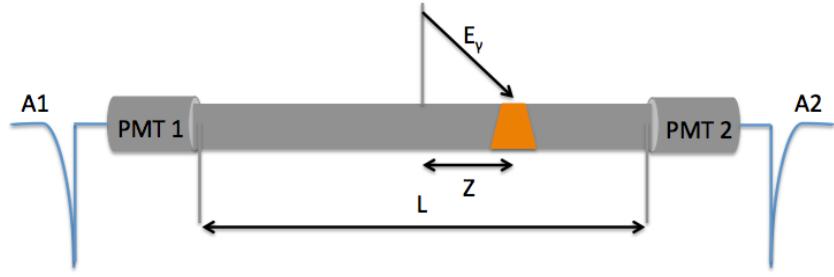


Figure 9: NaI Segment

This concludes our overview of the CALIOPE experiment. In the following section, we once again work from the inside out to present our experimental progress. We describe all facets of the construction, from the design phase up to the commissioning and integration of all major components of the experiment.

## 4 Simulation and Experimental Progress

### 4.1 Monte Carlo

Using Geant4 [18], we have created a replica of the APEX geometry for our simulation. Most of the code for the detector construction was written by Stephen Daigle. Some improvements have recently been made by an undergraduate, Chiara Salemi, who is working with our group. We have added additional components to the simulation, specifically the electromagnet and a holder containing the source and aerogel. I have implemented one possible source holder in the simulation (Figure 11), complete with the positron source ( $^{22}\text{Na}$ ) deposited on kapton foil and sandwiched between pieces of scintillator and aerogel. In the simulation, as in the physical world, a positron exiting the source is tagged and then scatters within the aerogel until it loses enough kinetic energy to form positronium. The aluminium source and all of its constituents were incorporated into the simulation this past year.

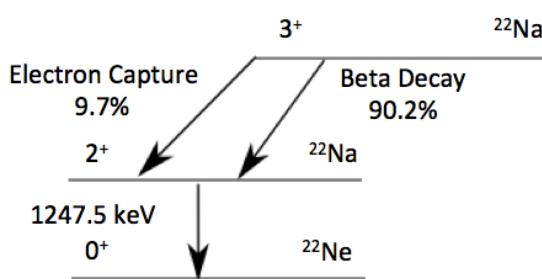


Figure 10:  $^{22}\text{Na}$  Decay Scheme

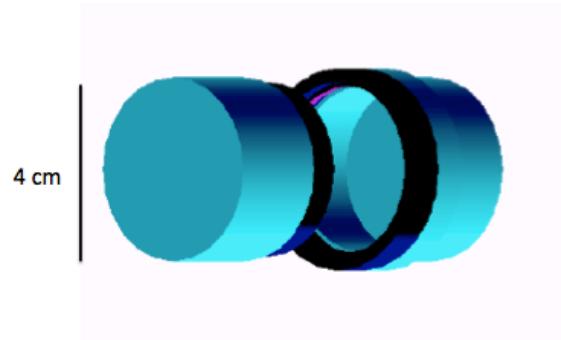


Figure 11: Source Holder Aluminium Exterior  
(Geant4)

The Geant4 software package does not provide an easy means to simulate positronium physics to suit our purposes. Hence, it was necessary to program it on our own. In our simulation, Geant4 tracks the positron as it loses its kinetic energy in the aerogel via scattering. We have modified the Geant4 prompt annihilation process by adding a custom positronium decay generator. Once the positron slows to a stop, we model the positronium lifetime in our simulation by inserting the appropriate time delay at the point where the positron has lost all of its energy. Once the appropriate time has past, either two or three gamma

rays are created. The suitable probabilities for the various positronium states are similarly folded into the simulation. In addition to the ordinary constraints of energy and momentum conservation, we must also consider the orientation of the decay plane, which depends on the spin state of ortho-positronium.

There are two geometries which have been recently included in the detector construction: the source holder and the electromagnet. An optimal source holder would not interact with gamma rays at all. This is unrealistic, so instead we run tests to analyze the effect of the source holder on our asymmetry measurement. We are undertaking a thorough examination of the source holder's effects by calculating the asymmetry both with and without the source holder geometry. We think that by minimizing the thickness of the aluminum exterior and maximizing the symmetry of the design, we can reduce the any impact the holder might have. In addition to swapping out the source holder geometries in Geant4, we anticipate manufacturing a few alternatives, as the holder should be inexpensive and relatively easy to build. We are also starting to characterize the effect of moving the  $^{22}\text{Na}$  source slightly off-center. Obtaining values for the asymmetry under these circumstances will help us determine the impact of various systematic effects and guide the tolerances required for fabrication of parts. The finalized design of the electromagnet will also soon be assimilated into the Geant4 detector construction now that the design is near completion.

Finally, we have instantiated elements of the event reconstruction and have the ability to read out energy and hit positions from the APEX array so that we include the effect of a realistic detector response.

## 4.2 Experimental Infrastructure

Our simulation results for the source holder will factor into our considerations regarding its exact dimensions and shape. We have already rendered some preliminary blueprints of the source holder using an open source CAD software. CAD drawings of the source holder are in the process of being reviewed by our machine shop.

Moreover, supplies such as the source material and scintillator have been ordered. A standard operating procedure, which will enable us to test the uniformity of the  $^{22}\text{Na}$  deposition and test the positronium, is being developed in consultation with TUNL staff. The source fabrication will follow a typical protocol for Positron Annihilation Spectroscopy, or PALS [19], which entails using a radioactive positron source sandwiched between scintillator. The procedure serves two purposes. First, one can check that the source is well characterized by analyzing the  $^{22}\text{Na}$  deposition with a camera. Second, one can run a preliminary test of the setup using photomultiplier tubes.

As mentioned previously, CALIOPE requires a large electromagnet. Undergraduate Ryan Petersberg is working on its simulation and design using the Radia software [20](Figure 6). The results of the magnet simulation will be used to create CAD drawing for the magnet and inform our decisions regarding the procurement of copper coils. We are also in possession of a few 40 amp, 50 volt power supplies which will drive the electromagnet. Sufficient cooling will be provided by a water jacket. Credit is due to Stepan Mikhailov as his extensive expertise greatly facilitated us with our design efforts. The magnet will be constructed using equipment in the machine shop here at UNC.

Figure 12: PALS source [19]

In preparation for the construction phase, the lab has been cleaned and rearranged. Once all calibrations have been finished, we can start a run. The first run is anticipated to last for six months. Because of the high angular acceptance of our detector, the experiment is not statistics limited. Rather, the experiment will be limited by systematics. We are working on a comprehensive study to quantify the systematic uncertainties in the experiment.



## 5 Analysis

### 5.1 Systematics

We have identified a number of events which might induce a false asymmetry in our detector. These include but are not limited to scattering events, inhomogeneities in the magnetic field, nonuniformity of the source holder, small displacements of the source from the center of the array and variations in the uniformity of the aerogel.

First, para-positronium could decay into two gamma rays and one could scatter into a third detector. This would look just like a three gamma event. Fortunately, para-positronium has a lifetime which is three orders of magnitude smaller than that of ortho-positronium. We can effectively eliminate this issue by implementing a timing cut. Figure 13 shows a histogram of detected positronium events at various hit times. Para-positronium events occur at the low end of the spectrum, so we can remove these events by excluding all events below a few nanoseconds.

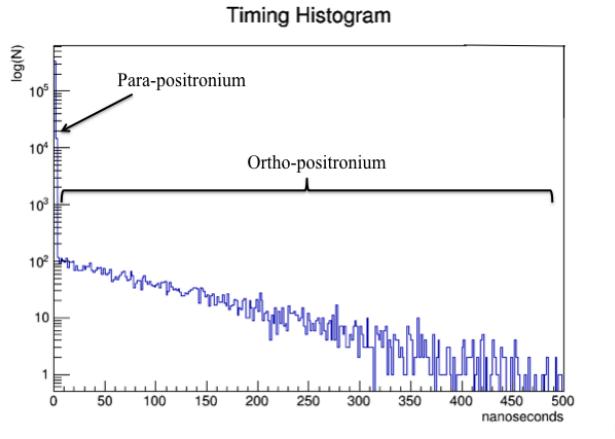


Figure 13: Timing Cut

Next, asymmetries in the magnetic field could induce effects in our ability to distinguish the states. If a small region of the field is stronger, the lifetime of the  $m = 0$  state could be shifted into the timing region of the other states. This introduces an uncertainty in the tensor polarization which propagates into the uncertainty of the  $CP$ -violating term.

There is an effect called pickoff annihilation which we must consider. For a long time, the lifetime of ortho-positronium did not correlate with the theory. The issue was finally resolved when pickoff annihilation was discovered [21]. Pickoff annihilation occurs when the positron in positronium overlaps with an electron orbital from the surrounding material, namely silica aerogel [22]. The positron and external electron will annihilate normally into two gamma rays.

Another situation which can occur is called *spin exchange*, in which the electron from the positron exchanges spins with an electron in the material. In this case, the positronium can “downgrade” to para-positronium, which once again decays into two gamma rays. Note that because the positronium was initially in the triplet state, the lifetime of the para-positronium will seem to be much longer. If this occurs in conjunction with a scattering event, it would be easy to mistake this for a false signal. A significant fraction of the proposed work will be dedicated to quantifying the effect of these systematics on the proposed measurement using a combination of calibration data and Monte Carlo simulations.

### 5.2 Possible Analyses

Since our detector geometry imposes less rigid constraints than those of previous experiments, we are afforded the luxury of testing alternative coordinate systems. So far, we have focused on using the coordinate system

defined by Yamazaki for our initial simulation tests, though it is possible there are other coordinate systems more conducive to our geometry. The Yamazaki coordinates can be written as follows:

$$P_2(\vec{S} \cdot \vec{k}_1)(\vec{S} \cdot \vec{k}_1 \times \vec{k}_2) = P_2 \sin 2\theta \sin \psi \cos \phi \quad (15)$$

If we define  $N_+$  as the number of events for which this term is positive and  $N_-$  as the number of events for which this term is negative, we can write the asymmetry in the following way:

$$A = \frac{N_+ - N_-}{N_+ + N_-} \quad (16)$$

In our simulation, we calculate the asymmetry both with and without  $CP$ -violating effects. The following histograms show some initial tests using our simulation:

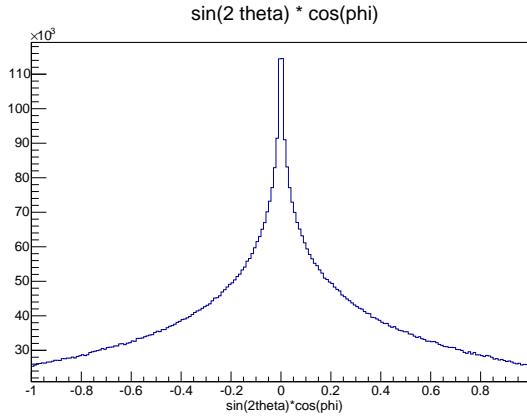


Figure 14: Asymmetry with no  $CP$ -Violation

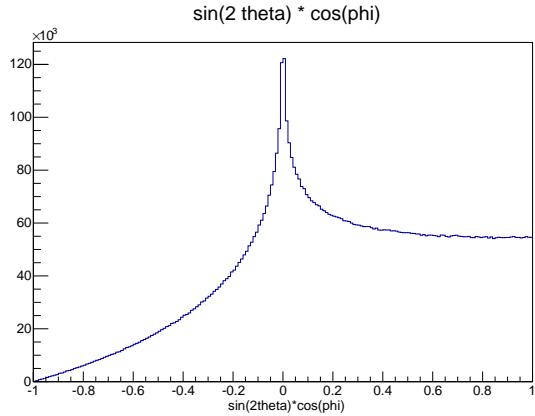


Figure 15: Asymmetry with  $CP$ -Violation

A two-dimensional histogram in theta and phi also demonstrates the effect of  $CP$ -violation on the asymmetry:

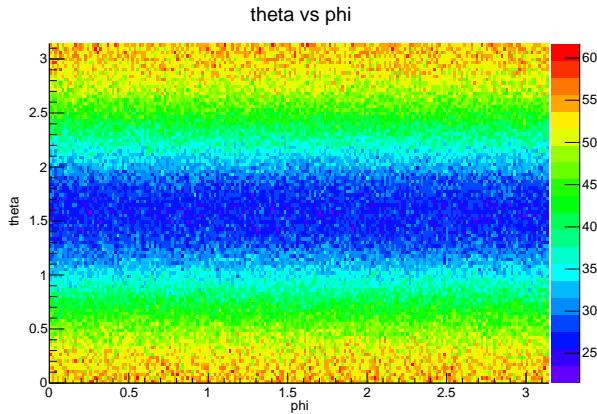


Figure 16: Asymmetry with no  $CP$ -Violation

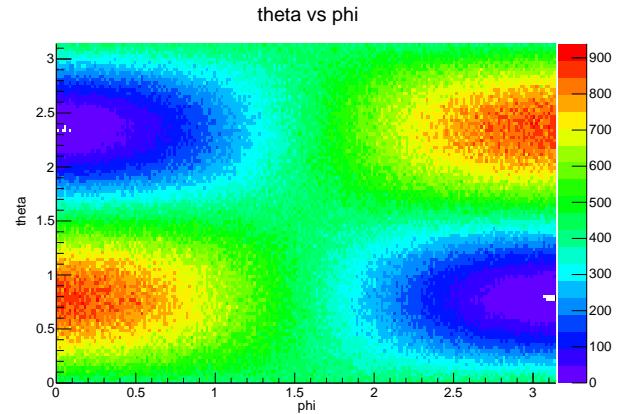


Figure 17: Asymmetry with  $CP$ -Violation

One possible alternative to the framework set forth by Yamazaki is an event-centric cylindrical coordinate system suggested by our undergraduate Ryan Petersburg. The motivation for this is to use variables that are directly measured by the APEX experiment, avoiding the requirement of calculating covariance matrices. In this system, one can define the  $S$  and  $k_i$  as follows:

$$\hat{S} = (0, 0, z_s) \quad (17)$$

$$\hat{k}_i = (r_i, \delta_i, z_i) \quad (18)$$

The radial and z components can be expressed as the following:

$$r_i = \frac{R}{\sqrt{R^2 + z_i^2}} = \sin \alpha_i \quad (19)$$

$$z_i = \frac{z_i}{\sqrt{R^2 + z_i^2}} = \cos \alpha_i \quad (20)$$

where  $R$  is the average radius of the APEX detector,  $z_i$  is the longitudinal hit position in a NaI bar and  $\alpha_i$  are the angles between  $k_i$  and the plane defined by the  $z$ -axis as the normal ( $-180^\circ < \alpha_i < 180^\circ$ ).

The angular correlation  $Q$  can then be written as follows:

$$(\hat{S} \cdot \hat{k}_1) = z_s z_1 \quad (21)$$

$$(\hat{S} \cdot \hat{k}_1 \times \hat{k}_2) = z_s r_1 r_2 (\cos \delta_1 \sin \delta_2 - \sin \delta_1 \cos \delta_2) = z_s r_1 r_2 \sin(\delta_2 - \delta_1) \quad (22)$$

$$Q = (\hat{S} \cdot \hat{k}_1)(\hat{S} \cdot \hat{k}_1 \times \hat{k}_2) = z_s^2 r_1 z_1 r_2 \sin \delta \quad (23)$$

where  $\delta = \delta_2 - \delta_1$  is the azimuthal angle between  $k_1$  and  $k_2$ . This can then be simplified into observable quantities for our experiment.

$$Q = \frac{R z_1}{R^2 + z_1^2} \frac{R}{\sqrt{R^2 + z_2^2}} \sin \delta = \frac{1}{2} \sin 2\alpha_1 \cos \alpha_2 \sin \delta \quad (24)$$

This asymmetry measurement would then rely on four parameters which can be recovered easily from the APEX event reconstruction:  $z_1$ ,  $z_2$ , and  $\delta$ , the difference between  $\delta_1$  and  $\delta_2$ .

One can see that there are many possible options for the analysis. We will strive to devise and select the best method(s) as the experiment unfolds.

## 6 Conclusion and Implications

If  $CP$ -violation in positronium is discovered, we will have evidence for new physics beyond the Standard Model. In order for a nonzero  $CP$ -violating amplitude to appear in positronium at our proposed sensitivity, there would need to be some as-yet unknown mechanism through which this  $CP$ -violation could occur [9]. Our experiment will push the boundaries regarding  $CP$ -violation in the charged lepton sector.

# Appendices

## CALIOPE'S Schedule

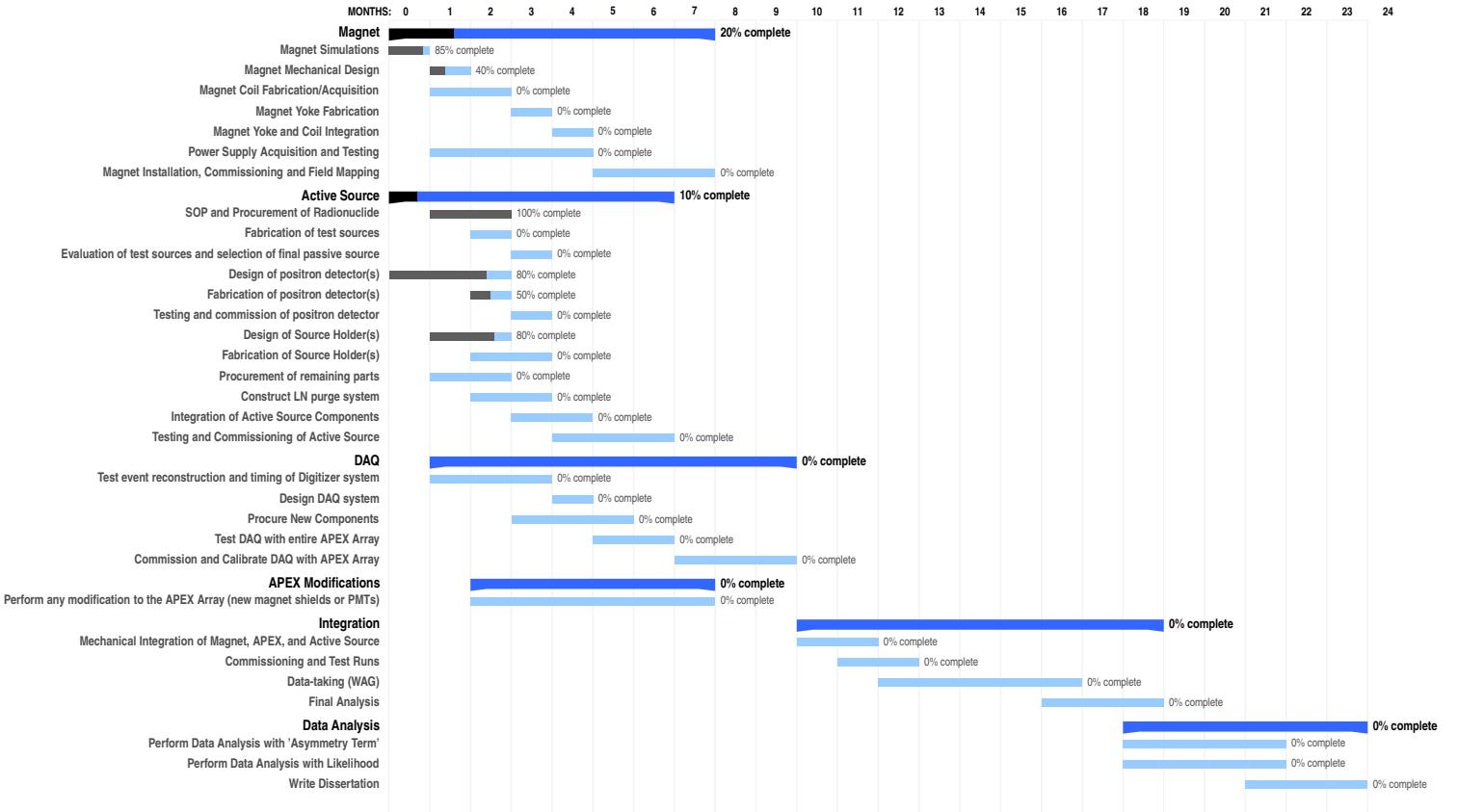


Figure 18: CALIOPE Schedule

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