Rumor Propagation in Random and Small-World Networks

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Abstract

In this paper we will study the flow of rumors through graphs representing social networks. In order to determine how to stop rumors from causing damage to societies, we analyze the behavior of rumor propagation through networks and then establish methods to slow the rate at which a rumor propagates. First we discuss basic graph theory necessary for network analysis, introducing matrix representations of graphs along with measures of centrality. Next we introduce random and small-world graph models, discussing their properties, the algorithms used to create them, and the potential shortcomings they have as models for real-world networks. We then introduce two stochastic models for rumor propagation, the Basic and the Maki-Thompson models. Lastly, we perform an analysis of time trials involving each of these models and then propose a method to slow rumor propagation in networks of people. By analyzing the amount of time it takes for a rumor to fully propagate a network under different circumstances, it can be shown that preventing nodes central in the network from becoming spreaders of a rumor slows the rate of that rumor's propagation through that network.

Contents

1 Introduction

Throughout history, rumors have incited riots, swayed elections, and crashed economies, spreading unease and uncertainty through social structures, ultimately leading individuals and groups to make irrational and misinformed decisions. Rumors are spread through networks, such as social networks, media outlets, or internet blogs. Studying rumor propagation helps generate an understanding of rumor flow so that preventative measures may be taken to mitigate the potential harm of a rumor. By modeling rumor propagation we will be able to compare propagation speeds through various types of networks. Then by making adjustments in successive trials, we can find an effective way to slow rumor propagation by creating structural holes that result in a slower rate of rumor spread.[1]

1.1 Definition of Rumor

A rumor is by definition a piece of unverified information. Rumors exist in many shapes and forms, such as those that arise from a misinformed news story published by a media outlet, or even those that become viral from an individual posting online. It is important to note that rumors can be true or false, but what defines them as a rumor is the fact that the information is unverified.[2] In the Information Age, rumors take on a new significance far beyond that previously attributed to them. Incidents such as the Facebook, Google, and Twitter Senate Intelligence hearings, which were called to address the mass propagation of manipulative political advertisements and "fake news" through each company's respective platform, making it clear that understanding rumors has become only more important in the age of technology.

2 Graph Theory

2.1 Terms and Definitions

A graph is a mathematical structure used to model relationships among a set of items representing nodes in a network.[3] In this section, we will introduce some basic components of graph theory necessary for network analysis.

- **Nodes and Edges:** Nodes represent a set of objects, for example people in a social network, or cities in a country. Edges represent connections among these objects. For the previous examples, the respective edges between nodes could represent a relationship between two people, or roads between two cities. Edges can be directed or undirected, weighted or unweighted, and positive or negative. In this paper however, we will only consider positive, unweighted, undirected edges.

- **Paths:** A path between two nodes is defined as a set of edges which connects the nodes. Since we will only consider positive, unweighted edges in this paper, we can define the distance d of any path to be the sum of the number of the number of elements in the edge set. In this instance, each edge between the two nodes represents a single step on the path.

- **Random Networks:** A random network is created using a specific network size determined by the total number of nodes and edge in the network. In this article, we use Erdös-Rényi model[4] to build a random network, in which nodes are connected with a uniform probability and the degree distribution follows a binomial distribution.

- **Small World Networks:** A small world network is similar to a random network, but possesses a clustering property that random networks do not. Clustering between nodes is a common phenomenon in real-world networks, and so we find that small-world networks are better suited to model real-world networks than their random counterparts. In this

1. Nan Lin, Karen S Cook, and Ronald S Burt, *Social capital: Theory and research* (Transaction Publishers, 2001).
2. Pamela Donovan, "How idle is idle talk? One hundred years of rumor research," *Diogenes* 54, no. 1 (2007): 59–82.
3. David Easley and Jon Kleinberg, *Networks, crowds, and markets: Reasoning about a highly connected world* (Cambridge University Press, 2010).
4. P Erdös and A Rényi, "On random graphs I," *Publ. Math. Debrecen* 6 (1959): 290–297.

paper we employ the Watts-Strogatz model[5] in order to construct a small world network by stochastically rewiring edges in a regular ring lattice graph.

- **Adjacency Matrix:** Adjacency matrices are used in graph theory to represents networks. The (i, j)th entry represents the connection between nodes i and j. If the entry is 1, an edge exists between the two nodes, if the entry is 0 no edge exists between the two nodes. Due to the fact that we consider undirected graphs, an edge between i and j indicates an edge between j and i. These are in fact the same edge, but as a result of this we find that adjacency matrices for undirected graphs are necessarily symmetric. To represent the network then, it is only necessary to consider the upper or lower triangular matrix generated from the adjacency matrix.

2.2 Centrality Measures

In a graph, nodes occupying different positions have distinct structural roles. There are some useful measures that we can use to identify central or important nodes in a network. In this section we will discuss these centrality measures.

- **Degree:** The degree of a network node is the number of edges incident to it.[6] Since our networks are undirected, the degree of a node is calculated by the number of edges incident to it. Additionally, the average degree of a network is simply the sum of the degrees of all nodes divided by the total number of nodes in the network:

$$d = \frac{\sum_i^n d_i}{n} \tag{1}$$

Where d_i is the degree of node i and n is the number of nodes in the network. The appendices include an example of degree centrality designated as *Figure 1*.

- **Closeness:** The closeness measure of a node is defined as the reciprocal of the sum of the distances of that node every other nodes in the network:

$$C_c(v) = \frac{1}{\sum_{t \in N} d_G(v, t)} \tag{2}$$

Where the distance d_G is defined as the number of steps in the shortest path between nodes v and t. A node with a relatively high closeness measure is closer to the rest of the nodes, because of this it can be considered more central in the network than a node with a lower closeness measure.[7]

- **Betweenness:** The betweenness measure of a node reflects its role as a mediator in the connection of any other pair of nodes. A node's betweenness is defined as the sum of the ratio of shortest paths between every pair of nodes and the number of those paths that pass through the node:

$$C_b(v) = \sum_{v \neq s \neq t \in N} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{3}$$

Where N is the set of nodes, the denominator is the number of shortest paths between any two nodes s and t, and the numerator is the number of those shortest paths that pass through node v.[8]

- **Eigenvector centrality:** Unlike degree centrality that treats a node's connections with all its neighbors equally, eigenvector centrality puts different weights on the connections between a node with its different neighbors. The weights are proportional to the neighbors' eigenvector centrality scores.[9] Therefore, connecting to a high-scoring neighbor makes a node

5. Duncan J Watts and Steven H Strogatz, "Collective dynamics of 'small-world' networks," *nature* 393, no. 6684 (1998): 440–442.
6. Linton C Freeman, "A set of measures of centrality based on betweenness," *Sociometry*, 1977, 35–41.
7. Ibid.
8. Ibid.
9. Phillip Bonacich, "Some unique properties of eigenvector centrality," *Social networks* 29, no. 4 (2007): 555–564.

more central than connecting to a low-scoring neighbor. We can use adjacency matrix to calculate the eigenvector centrality of nodes in a network of n number of nodes is:

$$C_e(i) = x_i = \frac{1}{\lambda}\sum_{j=1}^{n} a_{ij}x_j \tag{4}$$

Where a_{ij} are entries of the adjacency matrix A, $a_{ij} = 1$ if node i and node j are connected, otherwise $a_{ij} = 0$. λ is an eigenvalue of A and x_i and x_j are the i_{th} and j_{th} entries of the corresponding eigenvector $\mathbf{x}$ (x_1, x_2, ..., x_n). That is:

$$A\mathbf{x} = \lambda\mathbf{x} \tag{5}$$

Since all entries of $\mathbf{x}$ must be non-negative, according to the Perron-Frobenius theorem, λ is the unique largest eigenvalue of matrix A and $\mathbf{x}$ is the corresponding eigenvector. When normalized, the sum of the eigenvector centrality measures of all nodes in a network equals one. Therefore, networks of the same size have the same average eigenvector centralities, regardless of the graphical model used.

3 Graphical Models for Networks

A network model is a type of data model that is used as a method of representing objects and their respective relationships. Network models can be represented by graphs, which themselves are represented in the form of an adjacency matrix. To analyze the propagation of rumors through networks, we will consider two graph models, the Erdös-Rényi and the Watts-Strogatz Models.

3.1 Random Graphs

The theory of random graphs lies at the junction of graph theory and probability. Random graphs have useful properties for analyzing the structure and behavior of typical graphs, and so they provide a reference point from which to compare the behavior of atypical graphs, such as those that are used to represent social networks. Random networks can be easily represented by random graphs, which are defined as graphs obtained through the uniform sampling of the set of graphs with identical attributes as the network, namely those graphs with n nodes and m edges. This approach is known as the *sampling view* of generating a random graph, and the associated graph chosen is denoted as $G(n, m)$. Alternately, we can consider a *constructive view*, which involves choosing a vertex set V containing n nodes and then selecting uniformly at random one edge from the edges not yet chosen, repeating this m times. This graph can be denoted as $G(n, p)$, where n denotes the number of nodes in the network and $0 <= p <= 1$ represents the probability of existence of any individual edge between two nodes. It is important to note that the term "random graph" refers almost exclusively to the Erdös-Rényi models, $G(n, p)$ and $G(n, m)$, which have just been defined.[10] In this paper we will consider only the $G(n, p)$ graph, which behaves similar to $G(n, m)$ for a sufficiently large n.

3.1.1 The G(n,p) Model

Due to the fact that every possible edge is included in the $G(n, p)$ graph with probability p, all graphs with n nodes and m edges have an equal probability of:

$$p^m(1-p)^{\binom{n}{2}-M} \tag{6}$$

It follows than a $G(n, p)$ graph with n nodes has an expected number of $\binom{n}{2}p$ edges. The probability distribution of the degree of any vertex v is then binomial:

$$P(deg(v) = k) = \binom{n-1}{k}p^k(1-p)^{n-k-1} \tag{7}$$

10. Paul Erdös and Alfréd Rényi, "On the evolution of random graphs," *Publ. Math. Inst. Hung. Acad. Sci* 5, no. 1 (1960): 17–60.

where n is the total number of nodes in the network. By Poisson approximation, as n tends to infinity while the product np remains constant, the Binomial distribution converges towards the Poisson distribution. It follows that

$$P(deg(v) = k) = (np^k e^{-np})/k! \tag{8}$$

as $n \to \infty$ where $\lambda = np$.
Although the $G(n, p)$ Model offers a wide range of applications in network analysis, there are two important properties observed in real-world networks that a random graph does not possess:
1. The $G(n, p)$ Model fails to generate local clustering and triadic closures due to the fact that the probability of two arbitrary nodes being connected is equal for any pair of nodes in the network. In particular, the $G(n, p)$ model fails to represent the "small-world" phenomenon of real-world networks.
2. The $G(n, p)$ Model fails to account for the formation of hubs. In particular, the degree distribution of $G(n, p)$ graphs converge to the Poisson distribution rather than a power law observed in many real-world networks.
To properly model a social network then, it is necessary to consider another graphical model. In this paper we turn to the Watts-Strogatz model, which addresses the first of these two limitations.

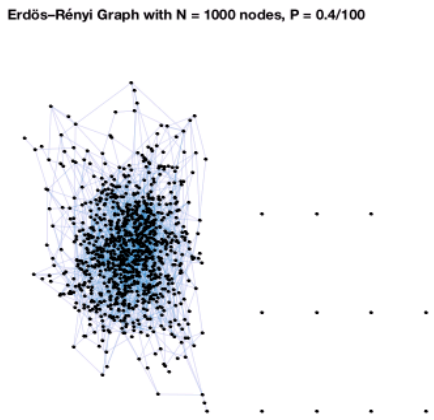

Figure 1: A random G(n,p) graph of 1000 nodes

3.2 Small-World Graphs

A small-world graph is one in which most nodes are not neighbors of one another, but the neighbors of an arbitrary node are likely to be neighbors of each other. This property accounts for the triadic closure present in the graph, which is the property that among three nodes A, B, and C, if there is a strong relationship between AB and AC, then there is a strong or weak relationship between BC. In particular, a small-world network is defines as a network where the expected distance $E(d)$ between two arbitrary nodes grows proportionally to the logarithm of the number of nodes in the network:

$$E(d) \propto log(N) \tag{9}$$

This property results in a "small-world phenomenon" where strangers are linked by a short chain of acquaintances. This phenomenon has been popularized by the phrase "six degrees of separation", which proposes that any arbitrarily chosen pair of people can reach one another through no more than six intermediary nodes. Colloquially this can be interpreted as a chain of "a friend of a friend" statements.

3.2.1 The Watts-Strogatz Model

In 1998, Watts and Strogatz proposed a graphical model that addressed the first of the two limitations present in the Erdös-Rényi model.[11] The Watts-Strogatz model accounts for the clustering

11. Watts and Strogatz, “Collective dynamics of ‘small-world’ networks.”

present in real-world networks by forming a graph that interpolates between a random graph and a regular ring lattice. As a result, the model is able to partially explain the small-world phenomenon present in real-world networks.
To construct a Watts-Strogatz graph, we again consider a vertex set V that contains n nodes. We define the variable k as the expected degree of an individual node, and the variable $0 \leq \beta \leq 1$ as a geometric parameter of the model. An undirected graph with n nodes and $nk/2$ edges is then constructed through the following process:
1. Construct a regular ring lattice of n nodes each connected to k neighbors, where each node has $k/2$ neighbors on each of their sides. Then two arbitrarily chosen nodes i and j have an edge between them if and only if:

$$0 < |i - j| mod(n - 1 - k/2) <= k/2 \tag{10}$$

2. For every node i take every edge (n_i, n_j) where $i < j$ and rewire it with probability β. This is done by replacing (n_i, n_j) with (n_i, n_k) where k is a uniformly sampled node and $k \neq i$ to prevent self-loops and link duplication from occurring.
The expected number of non-lattice edges is then $\beta\frac{nk}{2}$, and the introduction of the β variable allows for the interpolation between a regular ring lattice ($\beta = 0$) and a random graph ($\beta = 1$) that approaches the Erdös-Rényi Model G(n,p) with $p = \frac{np}{2\binom{n}{2}}$
In the case of $\beta = 0$ the degree distribution is Dirac delta function centered at k. If $\beta = 1$ the degree distribution approaches a Poisson distribution in the same fashion as the Erdös-Rényi model. Of particular interest is the degree distribution for $0 < \beta < 1$. This can be written as

$$P(k) = \sum_{n=0}^{f(k,K)} \binom{n}{K/2} (1-\beta)^n \beta^{K/(2-n)} \frac{\beta K/2^{(k-K)/(2-n)}}{((k-K)/(2-n))!} e^{-\beta K/2} \tag{11}$$

where $f(k, K) = min(k - K/2, K/2)$ and k_i denotes the degree of node i.
The lattice structure of the Watts-Strogatz Model produces a locally clustered network, while the randomly rewired edges reduce the average path lengths to ones similar to those in the Erdös-Rényi Model.
Although the Watts-Strogatz Model accounts for the presence of clustering in real-world networks, like the Erdös-Rényi Model it does not account for the formation of hubs, that is, the degree distribution does not follow a power law. Conversely, Real-world networks are often scale-free networks that have hubs and a scale-free degree distribution. These types of networks are better described by the class of preferential attachment models, the most well-known of which is the Barabási-Albert model. In this paper however, we will only consider the random and small-world models previously mentioned.

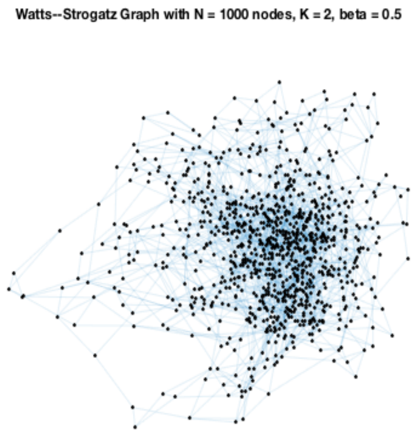

Figure 2: A small-world graph of 1000 nodes

4 Stochastic Models for Rumor Propagation

To examine the flow of rumors through a network we propose two stochastic propagation models, the Basic Model and the Maki-Thompson Model.[12] Wattz-Strogatz will be able to generate random probabilities for each of the edges a node has, in turn making it so that if that probability is less than the beta value that the edge will be adjusted, a stochastic feature of the model.[13] On the other hand, the Erdös-Rényi model is a simplistic version of a randomly generated adjacency matrix, which outputs a graph representation. From there, the propagation models, such as the basic model and Maki-Thompson model, can be applied to analyze propagation.

4.1 Basic Model

The Basic Model for rumor propagation involves nodes with one predefined role, that of a spreader, which attempt to convert its all of its neighbors to spreaders informed of the rumor. A key trait of the Basic Model is that every node will eventually become a spreader. This is a highly assumptive model, but it serves as a reference point from which to compare the more realistic Maki-Thompson Model. The Basic Model consists of first choosing a transmission rate t which represents the percent chance of a spreader successfully converting one of its neighbor nodes to a spreader in one time period. A node r is then randomly chosen to seed the rumor to. For every number of neighbors n_r that node r has, a random probability p is generated and stored in a vector $\vec{\mathbf{r}}$. If $\mathbf{r_i} < t, i \in n$ the ith neighbor of node r becomes a spreader and in turn beings to convert its neighbors to spreaders. In the successive time periods, all nodes that have assumed the role of spreader attempt to convert their neighbors to spreaders. This continues every time period until every node in the network has assumed the role of spreader.

4.2 Maki-Thompson Model

The Maki-Thompson Model expands on the Basic by adding a new role for nodes in the network, the stifler.[14] The stifler represents a neutral node, or one that will not spread the rumor after being informed. This will in turn affect the rate of the rumor's propagation through the network. In order to establish a model, each node is first randomly assigned as either a spreader or stifler. A transmission rate t is then chosen in the same fashion as that for the Basic Model. A random node r is then chosen to initially seed the rumor to. If this node is a stifler, the rumor immediately stops and the network is not infected. If it is a spreader, a random probability p is generated for each neighbor and stored in a vector $\vec{\mathbf{r}}$. If $\mathbf{r_i} < t, i \in n$ and the ith neighbor of node r has been designated as a spreader, it then assumes that role and beings to convert its neighbors to spreaders. If the ith neighbor of node r is a stifler, it does not assume the role of spreader, and in turn does not attempt to spread the rumor to its neighbors. In the successive time period, all nodes that have assumed the role of spreader attempt to convert their neighbors to spreaders, as in the Basic Model. This continues until all nodes who have been predesignated as spreaders and who have a path without any stifler nodes between themselves and the initially informed node have assumed the role of spreader. The introduction of the new stifler role slows the rate of the rumor's propagation through the network, and unlike the Basic Model, not all nodes will become spreaders. In fact, it is entirely possible that some nodes predesignated as spreaders will never assume the role of one, if there are no paths from those nodes to the initially informed node that does not have a stifler. This elaboration of the Basic Model serves to represent the flow of rumors through real social networks, as it is not realistic that all members of a network would become spreaders of a rumor.

12. Alan Demers et al., "Epidemic algorithms for replicated database maintenance," in *Proceedings of the sixth annual ACM Symposium on Principles of distributed computing* (ACM, 1987), 1–12.
13. Watts and Strogatz, "Collective dynamics of 'small-world' networks."
14. Daniel P Thompson Maki et al., *Mathematical models and applications: with emphasis on the social life, and management sciences*, technical report (1973).

5 Numerical Results

5.1 Methodology

We assume the Watts-Strogatz model is more representative of real-world networks due to the attributes previously discussed and use the results of the Erdös-Rényi Model as a reference point from which to compare those of the Watts-Strogatz Model.
To run time trials we first generate a Watt-Strogatz graph with a β that optimizes the centrality measures of the graph. The methodology behind this choice is discussed in Appendix *7.2*. This serves to create a small-world graph in which the nodes have relatively high closeness and betweenness centrality measures when compared to a graph generated with a randomly chosen β. Since our concern is modeling real-world networks, we will focus primarily on the behavior of Watts-Strogatz graphs, using results from the Erdös-Rényi model as a reference point.
Once the models have been created, we will calculate the respective centrality measures of the nodes in each graph and note the number of time periods needed until the rumor has fully spread through both networks using both propagation models. We will then compare these time periods for each network and propagation model. After this we will turn specifically to the Maki-Thompson model and compare the rate of rumor propagation through the Watts-Strogatz model when stiflers are randomly chosen and when they are assigned based on high centrality measures.

5.2 Comparable Models

Keeping the number of edges and nodes constant for both the Watts-Strogatz and Erdös-Rényi allows for a comparison of the structural behavior between the graphs due to the fact that they are of equal size.
To compare the Maki-Thompson method of rumor propagation in both models, it is necessary to set the probability of being a stifler equal in both network models. The transmission rate t must also be equal in both models. In the following trials, the transmission rate has been set to 1, indicating that the percent chance a spreader converts one of its neighbors to a spreader is 100, assuming that the neighbor is not a stifler. The probability p of being a stifler is also held constant in both models at $p = 0.2$.
To compare propagation through the same network with different stifling nodes a node r is randomly chosen, as before, however after this it is used as the initial node in each time trial, so that the difference in the number of time periods needed to propagate the network can be seen as a direct result of the reassigned stiflers.

5.3 Time Trial Results

5.3.1 Propagation rate comparison of random and small world networks

We calculate the number of time periods needed until the rumor fully propagates through both network models for network sizes $n = 50, n = 100, n = 1000$ using both the Basic and Maki-Thompson Model. Our data shows that the number of time periods needed for a rumor to propagate a small-world network with an optimal β is, on average, less than or equal to that needed to propagate through a random network regardless of which propagation model is used. Discrepancies in this finding can be attributed to the stochastic nature of both the rumor flow and the network structures. This indicates that rumors tend to flow faster through small-world networks than through random networks. This result is expected due to the small-world phenomenon present in small-world graphs.

Table 1: Propagation rate of random and small world networks using Basic Model

	n=50	n=100	n=1000
	time trials	time trials	time trials
Erdös-Rényi	5	7	10
Watts-Strogatz (β=0.4)	5	6	9

Of the two models we consider for rumor propagation, the Maki-Thompson is much more realistic

Table 2: Propagation rate of random and small world networks using Maki-Thompson Model

	n=50	n=100	n=1000
	time trials	time trials	time trials
random network	6	7	10
small world network (β=0.4)	5	7	10

than the Basic due to presence of the stifler role. Additionally, small-world networks represent real-world networks more accurately than random networks due to the presence of clustering. In the next section, then, we will focus on reducing the Maki-Thompson propagation rate in small-world networks by designating nodes with high centrality measures as stiflers.

5.3.2 Reducing Maki-Thompson propagation rates in small-world networks

Since nodes with high centrality measures play an important role in mediating information in real-world networks, preventing these nodes from becoming spreaders should slow or stop rumor propagation through the network. We first identify the hub nodes in the small world network using either a betweenness, a closeness, or an eigenvalue centrality measure, we then designate these nodes as stiflers and run the trial. Lastly, we note the number of time periods needed to propagate through the same network in four cases:

1. Stiflers are randomly assigned.
2. Stiflers are assigned based on high betweenness centrality measures.
3. Stiflers are assigned based on high closeness centrality measures.
4. Stiflers are assigned based on high eigenvalue centrality measures.

If our assumption that removing nodes with high centrality measures slows the rate of rumor propagation is true, there should be a statistical discrepancy between the number of trials in case (1) and those in cases (2),(3), and (4).

Table 3: Propagation rate of small world network (n=1000, β=0.4) after designating hubs as stiflers

Stiflers	Time Periods	increasing rate
Random	13	1.0
Closeness Hubs	15	1.15
Betweenness Hubs	18	1.38
Eigenvector Hubs	14	1.1

From the tables above it is clear that the total number of time periods needed for a rumor to fully propagate the network increases after designating nodes with high centrality measures as stiflers. This is true regardless of which centrality measure is chosen. The betweenness measure exhibits the best performance in slowing the rumor, but this is to be expected considering the underlying lattice structure of the Watts-Strogatz Model. Therefore, designating nodes with high betweenness or closeness centrality measures as stiflers is an effective way to suppress rumor propagation in small-world networks.

6 Conclusion

From our analysis it is clear that rumors in small-world networks will tend to propagate faster than those in random networks. This is a direct result of the absence of clustering in random networks. To effectively slow the spread of a rumor it is first necessary to identify the nodes with high centrality measures, these nodes should then be designated as stiflers. Designating hub nodes as stiflers slows the rate of rumor propagation when compared to trials where the stiflers are randomly chosen. Models where nodes with high betweenness centrality measures are designated as spreaders run slowest, this reflects the underlying ring lattice structure of the Watts-Strogatz graph. By preventing nodes central in the network from becoming spreaders, it is possible to slow the spread of a rumor. The final numerical results show that rumor following the Maki-Thompson propagation model in small-world networks can be slowed but not completely stopped by designating nodes with high centrality measures as stiflers.

6.1 Further Analysis of Rumor Propagation

Further work would involve analyzing rumor propagation in a preferential attachment model, which accounts for the scale-free property of real-world networks. The Barabasi-Albert Model is the most well-known of these models.[15] Additionally, we could expand the Maki-Thompson Model into the Daley-Kendall Model, which introduce a third role for nodes, the ignorant. Exploration on the possible formation of a matrix dynamical system with Markov processes would have provided the means to predict rumors, which in turn could be used to propose methods to slow or stop rumor propagation.[16] Improvements in the project would also include time trials with significantly larger graphs that have weighted edges. This requires more time for computation and so for large-scale analysis a more powerful computer would be needed.

15. Réka Albert and Albert-László Barabási, "Statistical mechanics of complex networks," *Reviews of modern physics* 74, no. 1 (2002): 47.

16. Daryl J Daley and David G Kendall, "Stochastic rumours," *IMA Journal of Applied Mathematics* 1, no. 1 (1965): 42–55.

7 Appendices

7.1 Variables

Variable Name	Variable Type	Variable Description
n	scalar	number of nodes (individuals)
m	scalar	number of edges
β	integer	geometric operator
A	matrix	network representation
p	scalar	probability of spreader

7.2 Optimal Beta

In order to find the optimal β which maximizes closeness and betweenness centrality, we ran a series of trials on Watt-Strogatz graphs of various size for decimal values of β, we then calculated the centrality measures for each respective graph. We found regardless of average degree, the β that maximizes centrality for the Watts-Strogatz lies around 0.4, although the optimal centrality values change with average degree. As the number of nodes increases, the centrality measures experience an asymptotic smoothing towards the x-axis.

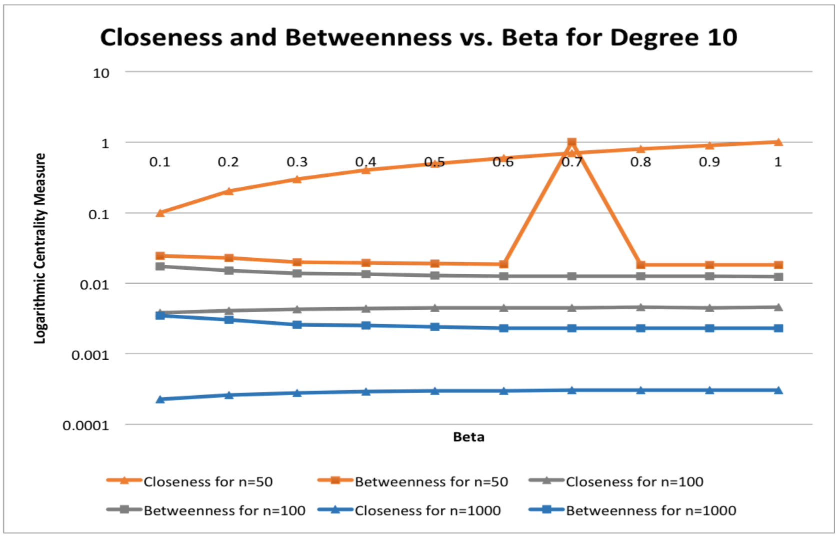

Figure 3: Centrality Measurements for Degree 10, Optimal Beta=0.7

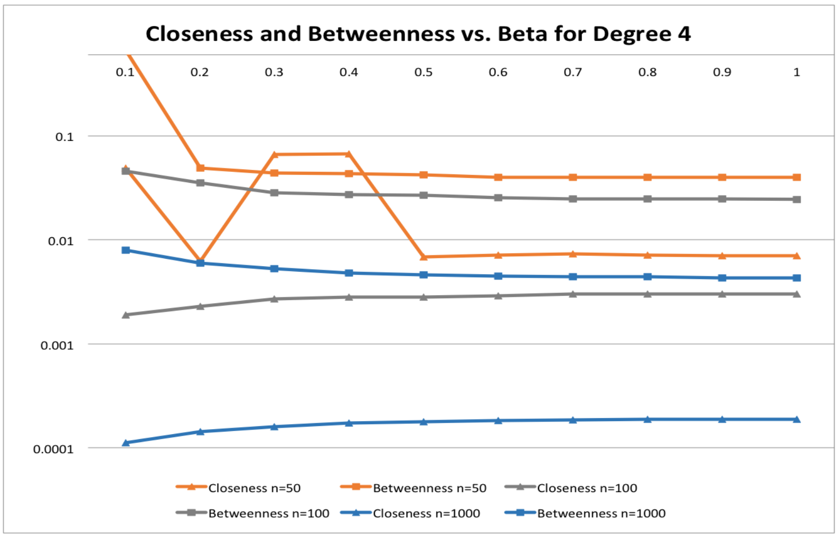

Figure 4: Centrality Measurements for Degree 4, Optimal Beta=0.3

7.3 Graphical Visualizations of Centrality

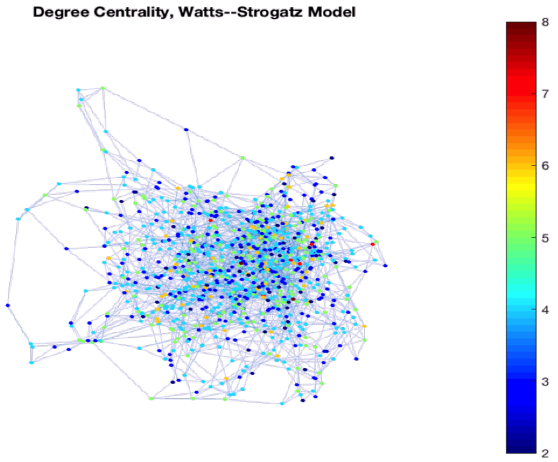

Figure 5: Degree Centrality in a small-world graph

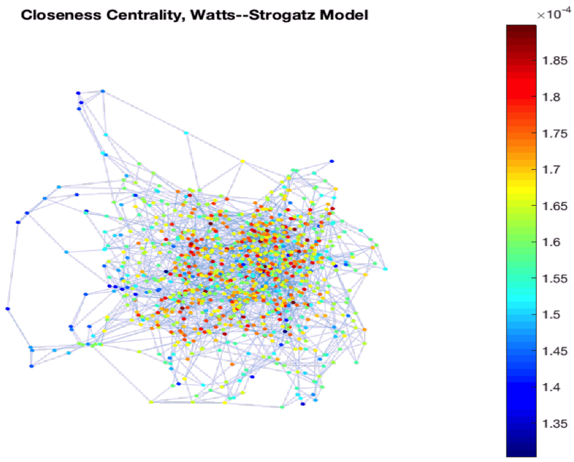

Figure 6: Closeness Centrality in a small-world graph

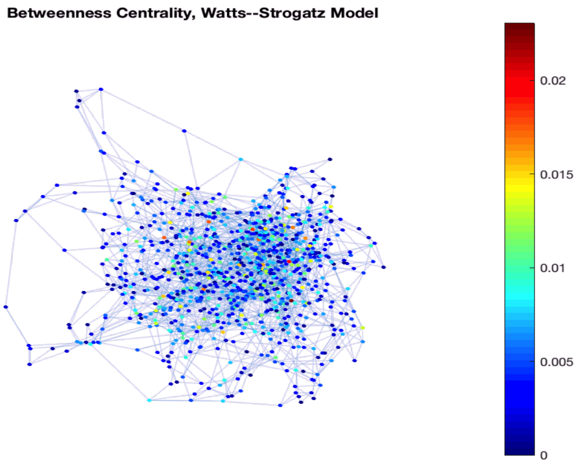

Figure 7: Betweenness Centrality in a small-world graph

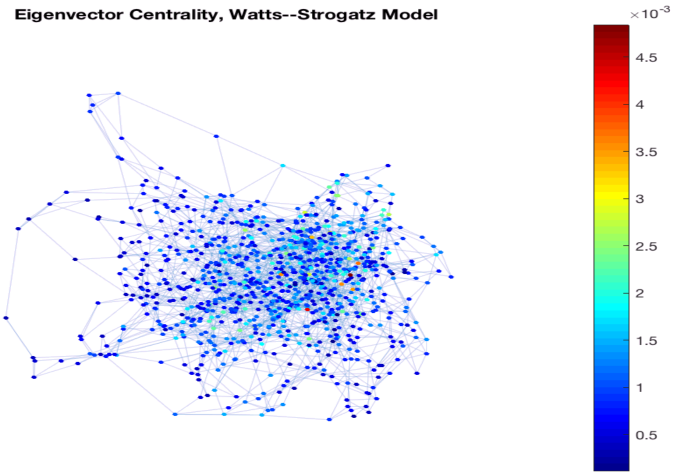

Figure 8: Eignevalue Centrality in a small-world graph

7.4 MATLAB Code

Rumor Propagation Through Random and Small World Networks

Erdös–Rényi Network Model

```
n = 1000;
p = (0.4/100);
randomMatrix= rand(n);
A = triu(randomMatrix<p,1);
A = sparse(A);
A = A+A';
G = graph(A);
plot(G,'NodeColor', 'k', 'EdgeAlpha', 0.2)
axis off
title('Erdös–Rényi Graph with N = 1000 nodes, P = 0.4/100')
```

Degree Centrality, Erdös–Rényi Random Graph

```
h = plot(G,'NodeColor', 'r', 'EdgeAlpha', 0.2);
degree = centrality(G, 'degree');
h.NodeCData = degree;
axis off
colormap jet
colorbar
title('Degree Centrality, Erdös–Rényi model')

S = sum(degree);
avgDeg = S/n
NN = numedges(G)
```

Closeness Centrality, Erdös–Rényi Random Graph

```
h = plot(G,'NodeColor', 'k', 'EdgeAlpha', 0.2);
closeness = centrality(G,'closeness');
h.NodeCData = closeness;
colormap jet
colorbar
axis off
title('Closeness Centrality, Erdös–Rényi model')

S = sum(closeness);
avgClose = S/n
```

Betweenness Centrality, Erdös–Rényi Random Graph

```
h = plot(G,'NodeColor', 'k', 'EdgeAlpha', 0.2);
G.Nodes.betweenness = zeros(n,1);
BC = centrality(G,'betweenness');
h.NodeCData = (2*BC)./((n-2)*(n-1));
colormap(jet)
```

```
colorbar
axis off
title('Betweenness Centrality, Erdös–Rényi model')

S = sum((2*BC)./((n-2)*(n-1)));
avgBtwn = S/n
```

Eigenvector Centrality, Erdös–Rényi Random Graph

```
h = plot(G, 'NodeColor', 'k', 'EdgeAlpha', 0.2);
eig = centrality(G,'eigenvector');
h.NodeCData = eig;
colormap jet
colorbar
axis off
title ('Eigenvector Centrality, Erdös–Rényi model')

S = sum(eig);
avgEig = S/n
```

Basic Propagation Model, Erdös–Rényi Random Graph

```
BasicERTrial = 1;
r = randi([1,n]);
G.Nodes.status = zeros(n,1);
G.Nodes.status(r,1) = 1;
array = find(G.Nodes.status);

c = 1;
t = 1;

h = plot(G,'NodeColor', 'k', 'EdgeAlpha', 0.2);
highlight(h, array, 'NodeColor', 'r');
axis off
BasicER(c) = getframe;
c = c+1;

for i = 1:n
    if isempty(neighbors(G, i)) == 1
        G.Nodes.status(i) = 1;
    end
end


while size(find(G.Nodes.status)) ~=n
    for k = 1:size(array)
        if G.Nodes.status(array(k,1),1) == 1
            N = neighbors(G, array(k,1));
            if (N ~= 0)
                P = rand(size(N,1),1);
                for v = 1:size(N)
                    if (G.Nodes.status(N(v,1),1) == 1)
                        continue
                    else
                        if t > P(v,1)
                            G.Nodes.status(N(v,1),1) = 1;
```

```
                            newarray = find(G.Nodes.status);
                        end
                    end
                end
            end
        end
    end
    array = newarray;
    BasicERTrial = BasicERTrial + 1;

    highlight(h, array, 'NodeColor', 'r');
    drawnow
    axis off
    BasicER(c) = getframe;
    c = c+1;
end
axis off
title('Basic Rumor Propogation in the Erdös–Rényi Model')
```

Maki-Thompson Propagation Model, Erdös–Rényi Random Graph

```
MakiERTrial = 0;
r = randi([1,n]);
G.Nodes.role = zeros(n,1);
G.Nodes.status = zeros(n,1);
G.Nodes.status(r,1) = 1;
array = find(G.Nodes.status);
b = 0.8;                        % probability of being spreader
t = 1;
c = 1;

for x = 1:n
    p = rand;
    if p < b                    % if random prob < chosen prob, designate node as spreader
        G.Nodes.role(x,1) = 1;
    end
end

h = plot(G,'NodeColor', 'k', 'EdgeAlpha', 0.35);
highlight(h, array, 'NodeColor', 'r');
axis off;
MakiER(c) = getframe;
c = c+1;
trial = 0;

count = 1;
while count ~= 0
    trial = trial +1;
    count = 0;
    for k = 1:size(array)
        N = neighbors(G,array(k,1));
        if N~=0
            P = rand(size(N,1),1);
            for v = 1:size(N)
                if (G.Nodes.status(N(v,1),1) == 1)
                    continue
                else
```

```
                    if t > P(v,1)
                        if G.Nodes.role(N(v,1),1) == 1
                            G.Nodes.status(N(v,1),1) = 1;
                            newarray = find(G.Nodes.status);
                        end
                    end
                end
            end
        end
    end
    array = newarray;
    MakiERTrial = MakiERTrial + 1;

    highlight(h, array, 'NodeColor', 'r');
    drawnow
    axis off
    MakiER(c) = getframe;
    c = c+1;

    for i = 1:size(array)
        X = neighbors(G,array(i,1));
            for v = 1:size(X)
                if (G.Nodes.role(X(v,1),1) == 1) && (G.Nodes.status(X(v,1),1)  == 0)
                    count = count + 1;
                    break;
                end
                if count~= 0
                    break;
            end
        end
    end

end
axis off
title('Maki-Thompson Rumor Propogation in the Erdös–Rényi Model')
```

Watts-Strogatz Network Model

```
k = 2;
beta = 0.5;
s = repelem((1:n)',1,k);
t = s + repmat(1:k,n,1);
t = mod(t-1,n)+1;
for source=1:n
    switchEdge = rand(k, 1) < beta;

    newTargets = rand(n, 1);
    newTargets(source) = 0;
    newTargets(s(t==source)) = 0;
    newTargets(t(source, ~switchEdge)) = 0;

    [~, ind] = sort(newTargets, 'descend');
    t(source, switchEdge) = ind(1:nnz(switchEdge));
end

H = graph(s,t);
plot(H,'NodeColor', 'k', 'EdgeAlpha', 0.1)
axis off
```

```
title('Watts--Strogatz Graph with N = 100 nodes, K = 2, beta = 0.5')
```

Degree Centrality, Watts-Strogatz Small-World Graph

```
h = plot(H,'NodeColor', 'k', 'EdgeAlpha', 0.2);
degree = centrality(H, 'degree');
h.NodeCData = degree;
colormap jet
colorbar
axis off
title('Degree Centrality, Watts--Strogatz Model')

S = sum(degree);
avgDeg2 = S/n

HH = numedges(H)
```

Closeness Centrality, Watts-Strogatz Small-World Graph

```
h = plot(H,'NodeColor', 'k', 'EdgeAlpha', 0.2);
closeness = centrality(H,'closeness');
h.NodeCData = closeness;
colormap jet
colorbar
axis off
title('Closeness Centrality, Watts--Strogatz Model')

S = sum(closeness);
avgClose2 = S/n
```

Betweenness Centrality, Watts-Strogatz Small-World Graph

```
h = plot(H,'NodeColor', 'k', 'EdgeAlpha', 0.2);
BC = centrality(H,'betweenness');
h.NodeCData = (2*BC)./((n-2)*(n-1));
colormap(jet)
colorbar
axis off
title('Betweenness Centrality, Watts--Strogatz Model')

S = sum((2*BC)./((n-2)*(n-1)));
avgBtwn2 = S/n
```

Hubs Watts-Strogatz Model

```
H.Nodes.role = ones(n,1);
[~,sortIndex] = sort(closeness(:),'descend');
maxCloseIndex = sortIndex(1:220);
[~,sortIndex] = sort(BC(:),'descend');
maxBCIndex = sortIndex(1:220);
choice = 0; %Nodes w/ large (1)closeness/(2)betweenness measures designated as stiflers
if choice == 1
    for i = 1:220
```

```
        H.Nodes.role(maxCloseIndex(i)) = 0;
    end
elseif choice == 2
    for i = 1:220
        H.Nodes.role(maxBCIndex(i)) = 0;
    end
end


h = plot(H, 'NodeColor', 'r', 'EdgeAlpha', 0.2);
edges = linspace(min(BC),max(BC), 10);
bins = discretize(BC,edges);
h.MarkerSize = bins;

axis off
title ('Hub Centrality, Betweenness, Watts--Strogatz model')

h = plot(H, 'NodeColor', 'r', 'EdgeAlpha', 0.2);
edges = linspace(min(closeness),max(closeness), 10);
bins = discretize(closeness,edges);
h.MarkerSize = bins;

axis off
title ('Hub Centrality, Closeness, Watts--Strogatz model')
```

Eigenvector Centrality, Watts- Strogatz Small-World Graph

```
h = plot(H, 'NodeColor', 'k', 'EdgeAlpha', 0.2);
eig2 = centrality(H,'eigenvector');
h.NodeCData = eig2;
colormap jet
colorbar
axis off
title ('Eigenvector Centrality, Watts--Strogatz Model')

S = sum(eig2);
avgEig2 = S/n
```

Basic Propogation Model, Watts-Strogatz Small-World Graph

```
H.Nodes.status = zeros(n,1);
BasicWSTrial = 0;
r = randi([1,n]);
H.Nodes.status(r,1) = 1;
array = find(H.Nodes.status);
t = 1;
c = 1;

h = plot(H,'NodeColor', 'k', 'EdgeAlpha', 0.2);
highlight(h, array, 'NodeColor', 'r');
axis off
BasicWS(c) = getframe;
c = c+1;

while size(find(H.Nodes.status),1) ~= n
    for k = 1:size(array)
```

```
        if H.Nodes.status(array(k,1),1) == 1
            N = neighbors(H, array(k,1));
            if (N ~= 0)
                P = rand(size(N,1),1);
                for v = 1:size(N)
                    if (H.Nodes.status(N(v,1),1) == 1)
                        continue
                    else
                        if t > P(v,1)
                            H.Nodes.status(N(v,1),1) = 1;
                            newarray = find(H.Nodes.status);
                        end
                    end
                end
            end
        end
    end
    array = newarray;
    BasicWSTrial = BasicWSTrial + 1;

    highlight(h, array, 'NodeColor', 'r');
    drawnow
    axis off
    BasicWS(c) = getframe;
    c = c+1;
end

axis off
title('Basic Rumor Propogation in the Watts--Strogatz Model')
```

Maki-Thompson Propagation Model, Watts-Strogatz Graph

```
r = randi([1,n]);
MakiWSTrial = 0;
t = 1;
H.Nodes.role = zeros(n,1);
H.Nodes.status = zeros(n,1);
H.Nodes.status(r,1) = 1;
array = find(H.Nodes.status);
b = 0.8;                            % probability of being spreader
c = 1;

h = plot(H,'NodeColor', 'k', 'EdgeAlpha', 0.35);
highlight(h, array, 'NodeColor', 'r') ;
axis off;
MakiWS(c) = getframe;
c = c+1;

% Random designation of stiflers
for x = 1:n
    p = rand;
    if p < b
        H.Nodes.role(x,1) = 1;
    end
end

count = 1;
while count~=0
    count = 0;
```

```
    for k = 1:size(array)
        if H.Nodes.status(array(k,1),1) == 1
            N = neighbors(H,array(k,1));
            if N~=0
                P = rand(size(N,1),1);
                for v = 1:size(N)
                    if (H.Nodes.status(N(v,1),1) == 1)
                        continue
                    else
                        if t > P(v,1)
                            if H.Nodes.role(N(v,1),1) ==1
                                H.Nodes.status(N(v,1),1) = 1;
                                newarray = find(H.Nodes.status);
                            end
                        end
                    end
                end
            end
        end
    end
    array = newarray;
    MakiWSTrial = MakiWSTrial + 1;

    highlight(h, array, 'NodeColor', 'r') ;
    drawnow
    MakiWS(c) = getframe;
    c = c+1;

    for i = 1:size(array)
        X = neighbors(H,array(i,1));
        for v = 1:size(X)
            if ((H.Nodes.role(X(v,1),1) == 1) && (H.Nodes.status(X(v,1),1)  == 0))
                count = count + 1;
                break;
            end
        end
        if count ~= 0
            break
        end
    end

end
axis off
title('Maki-Thompson Rumor Propogation in the Erdös–Rényi Model')
```

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