FROST: Flexible Round-Optimized Schnorr Threshold Signatures

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Cryptography, Security, and Privacy
— Research Group @ uWaterloo —

Threshold Secret Sharing

- Partitions a secret among a set of participants, such that recovering/using the secret requires cooperation among a threshold number of participants.
- Shamir secret sharing is the most well-known algorithm and what FROST builds upon.
- n represents the *total* number of allowed participants; t the threshold.

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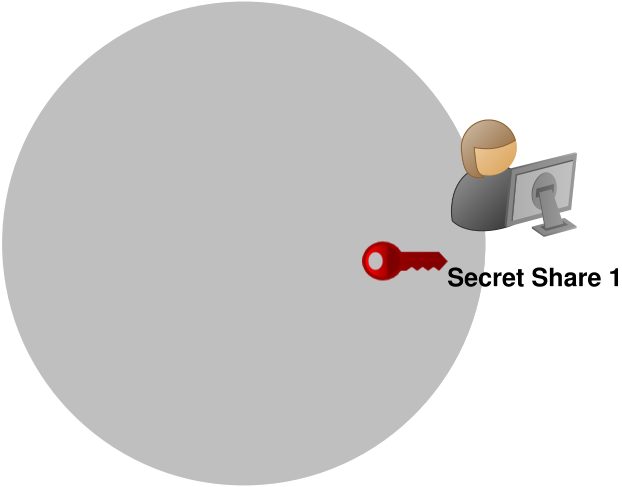
(t, n)-Threshold Secret Sharing

$(2, 3)$-Threshold Scheme Example



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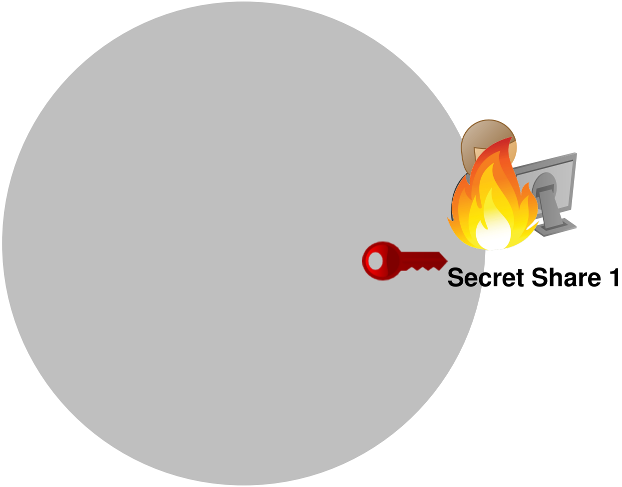

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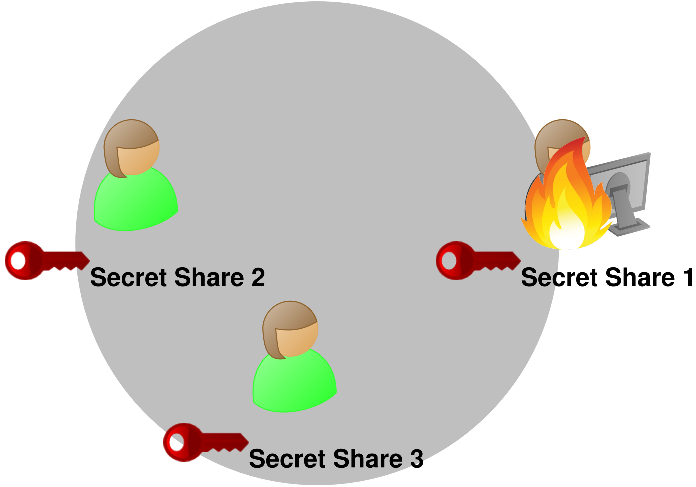

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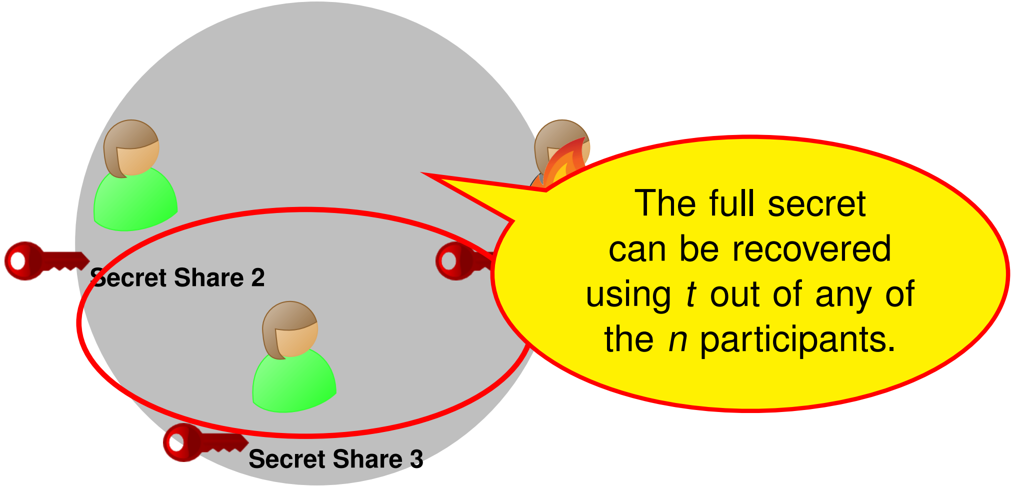

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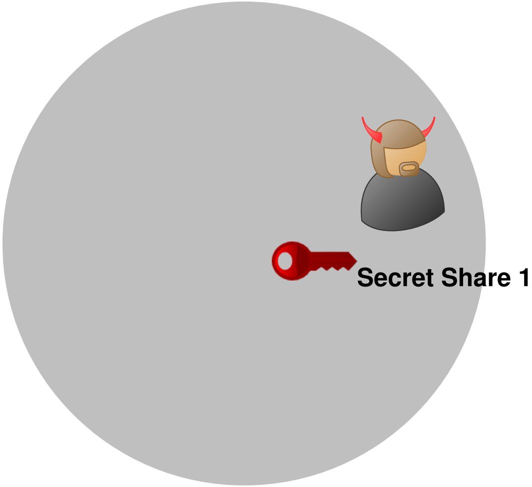

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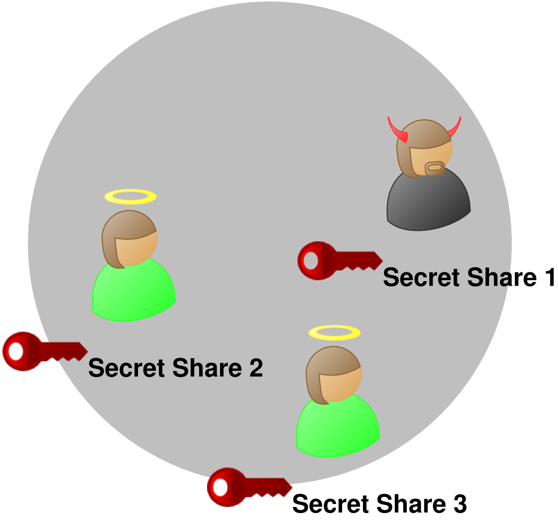

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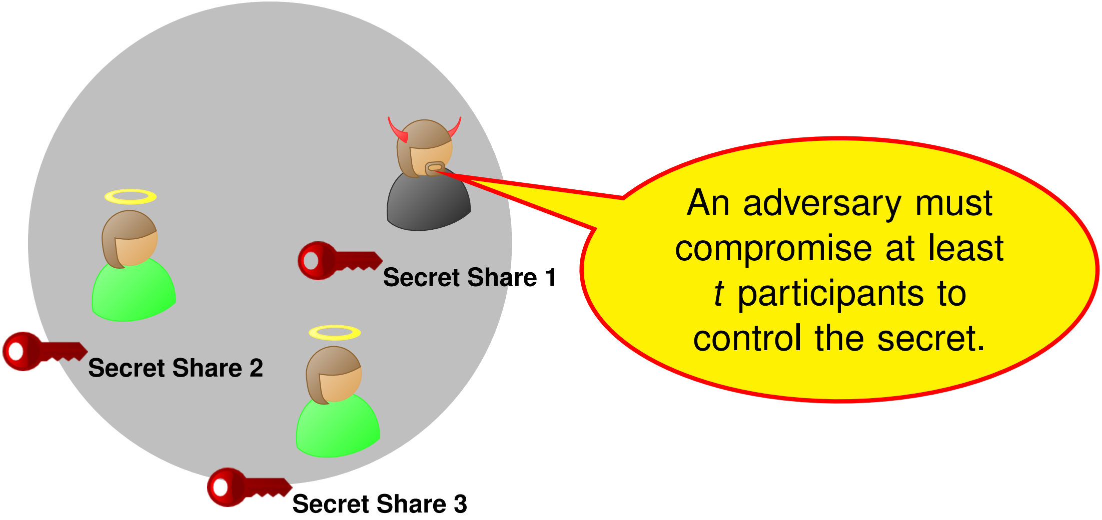

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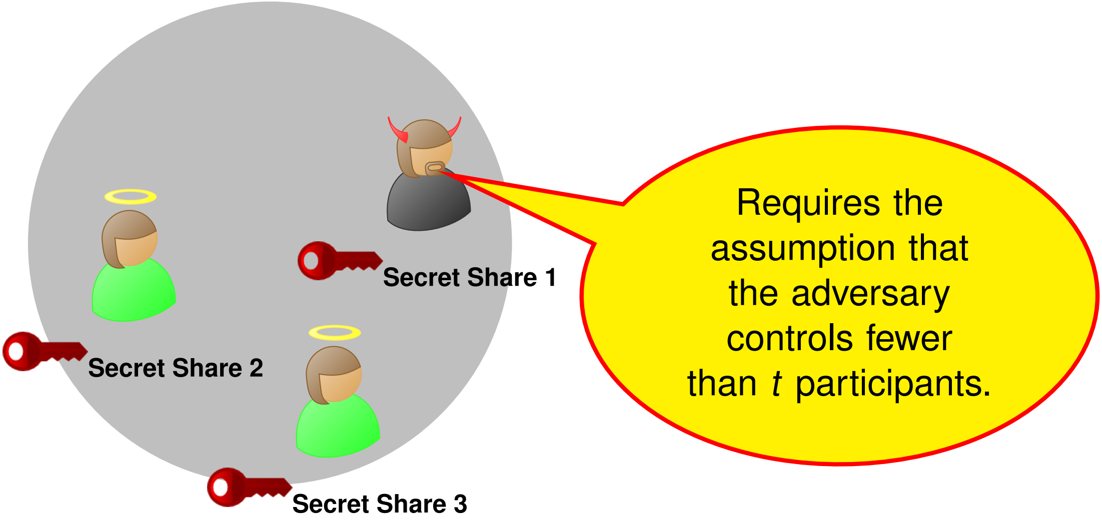
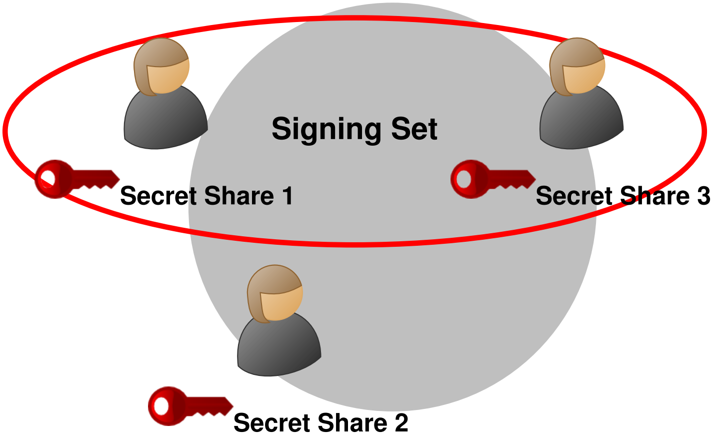


Threshold Signatures: Joint Public Key, Secret-Shared Private Key



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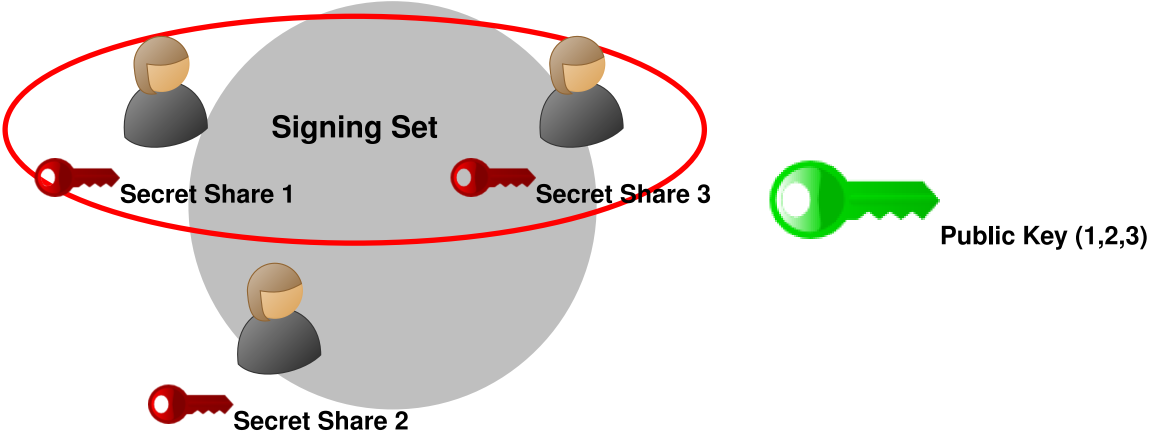

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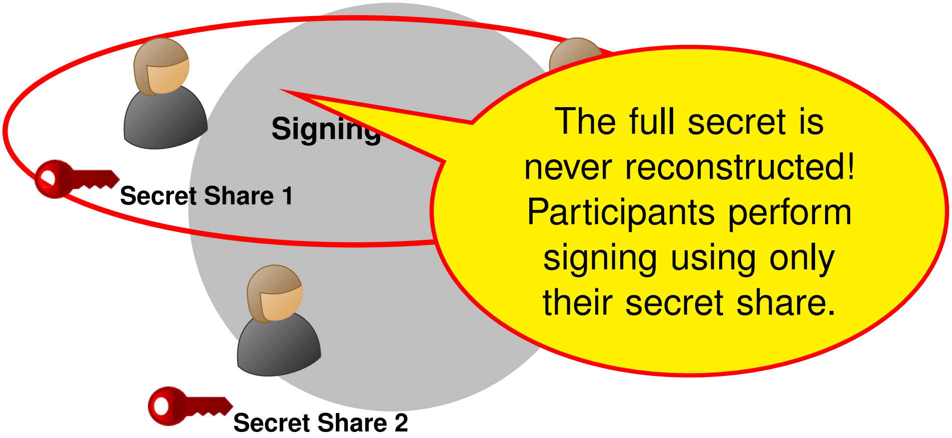

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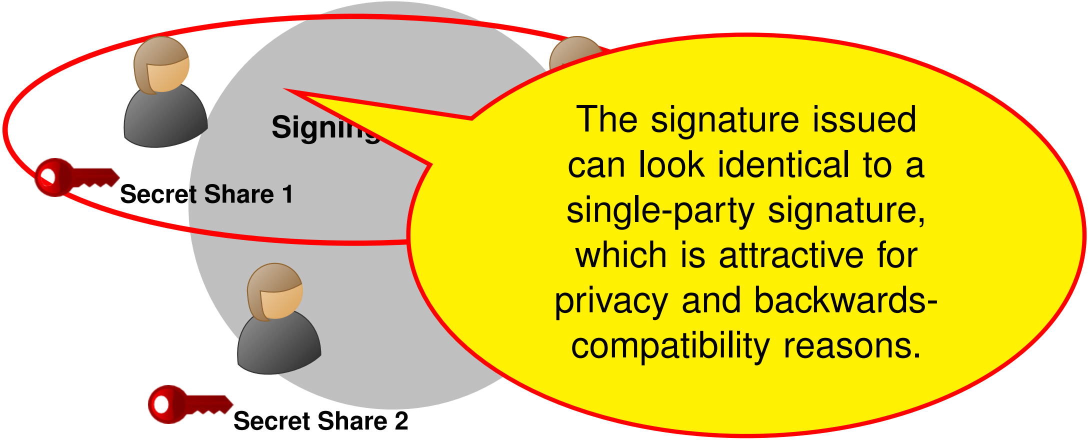

Applications of Threshold Signing

- Issuance of certificates by certificate authorities.
- Distribution of Tor's consensus by directory authorities.
- Authentication of blockchain transactions.
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Comparison to Multisignature Schemes

- A multisignature is a compact representation of n signatures over some message.
- Each signer has their own public/private keypair.
- No enforced access structure in the primitive itself.

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Multisignature	No	Yes	Yes
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Contributions of FROST

- Two-round threshold signing protocol, or single-round protocol with preprocessing
- Signing operations are secure when performed concurrently, improving upon prior similar schemes.
- Signing can be performed with a threshold t number of signers, where t can be less than the number of possible signers n.
- Secure against an adversary that controls up to $t - 1$ signers.

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- **Number of Signing Rounds:** Required network rounds to generate one signature.
- **Robust:** Can the protocol complete when participants misbehave?
- **Required Number of Signers:** Can a signature be created by just t participants, or are all n needed?
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Single-Party Schnorr Signing and Verification

Signer

$(x, Y) \leftarrow KeyGen()$

Verifier

$$\xleftarrow{(m, Y)}$$

$k \xleftarrow{\$} \mathbb{Z}_q$

$R = g^k \in \mathbb{G}$

$c = H(R, Y, m)$

$z = k + c \cdot x$

$$\xrightarrow{(m, \sigma = (R, z))}$$

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$R' = g^z \cdot Y^{-c}$

Output $R \stackrel{?}{=} R'$

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FROST Keygen

- Can be performed by either a trusted dealer or a Distributed Key Generation (DKG) Protocol
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FROST Preprocess

Participant i | **Commitment Server**

$((d_{ij}, e_{ij}), \dots) \xleftarrow{\$} \mathbb{Z}_q^* \times \mathbb{Z}_q^*$

$(D_{ij}, E_{ij}) = (g^{d_{ij}}, g^{e_{ij}})$

Store $((d_{ij}, D_{ij}), (e_{ij}, E_{ij}), \dots)$

$\xrightarrow{((D_{ij}, E_{ij}), \dots)}$

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In the two-round variant, this step is performed immediately before signing with only one commitment.

Store $((D_{ij}, E_{ij}), \dots)$

FROST Sign

Signer i

Signature Aggregator

$B = ((1, D_1, E_1), \ldots, (t, D_t, E_t))$

$$\xleftarrow{\quad (m, B) \quad}$$

$\rho_\ell = H_1(\ell, m, B), \ell \in S$

$$R = \prod_{\ell \in S} D_\ell \cdot (E_\ell)^{\rho_\ell}$$

$c = H_2(R, Y, m)$

$z_i = d_i + (e_i \cdot \rho_i) + \lambda_i \cdot s_i \cdot c$

$$\xrightarrow{\quad z_i \quad}$$

Publish $\sigma = (R, z = \sum z_i)$

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“binding value” to
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This step cannot be inverted by anyone who does not know (d_i, e_i).

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Signature format and verification are identical to single-party Schnorr.

Security against Drijvers

Without $\rho_\ell = H_1(\ell, m, B)$, an adversary could produce a c^* such that:

$$c^* = H(R^*, Y, m^*) = \sum_{i=1}^{k} H(R_i, Y, m_i) = \sum c_i \text{ for some } (R_i, m_i), \ldots$$

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After sending receiving the victim's z_i for each (R_i, m_i), the adversary can produce a valid forgery $\sigma^* = (R^*, z)$, as

$$z = \sum d_i + e_i + \lambda_t \cdot s_t \cdot \sum c_i = \sum d_i + e_i + \lambda_t \cdot s_t \cdot c^*$$

The binding factor in FROST makes each z_i strongly tied to (m_i, R_i).

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