

FROST Research Updates

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What is FROST?

- Flexible Round-Optimized Schnorr Threshold Signatures [Komlo & Goldberg, 2020]

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 1. PedPop: Distributed Key Generation (DKG)

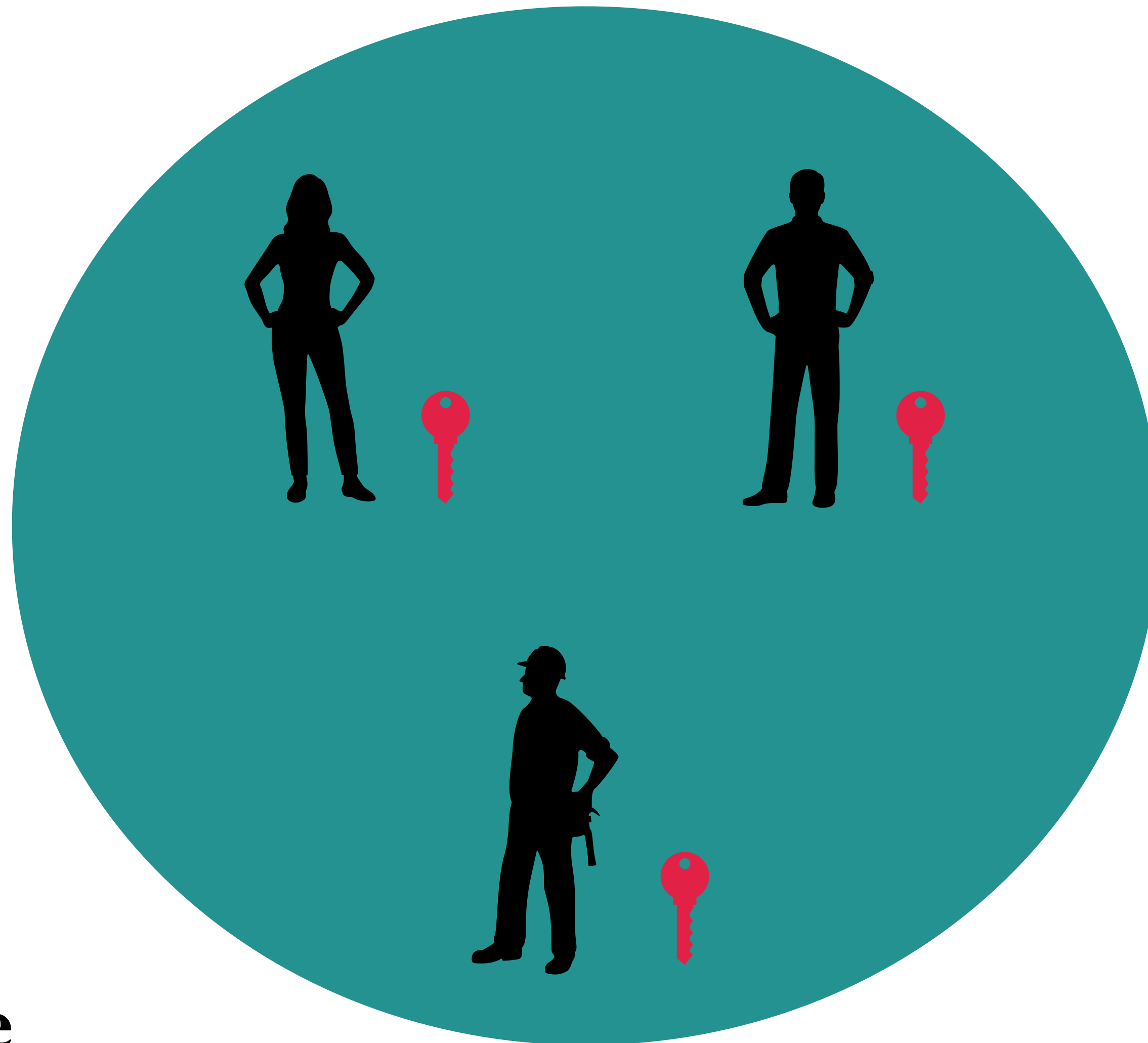
What is FROST?

- **Flexible Round-Optimized Schnorr Threshold Signatures [Komlo & Goldberg, 2020]**
- 1. PedPop: Distributed Key Generation (DKG)
- 2. Two-round threshold signing that is concurrently secure

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- **Flexible Round-Optimized Schnorr Threshold Signatures [Komlo & Goldberg, 2020]**
- 1. PedPop: Distributed Key Generation (DKG)
- 2. Two-round threshold signing that is concurrently secure
- Designed to solve needs in the Zcash ecosystem; now adopted as an industry standard

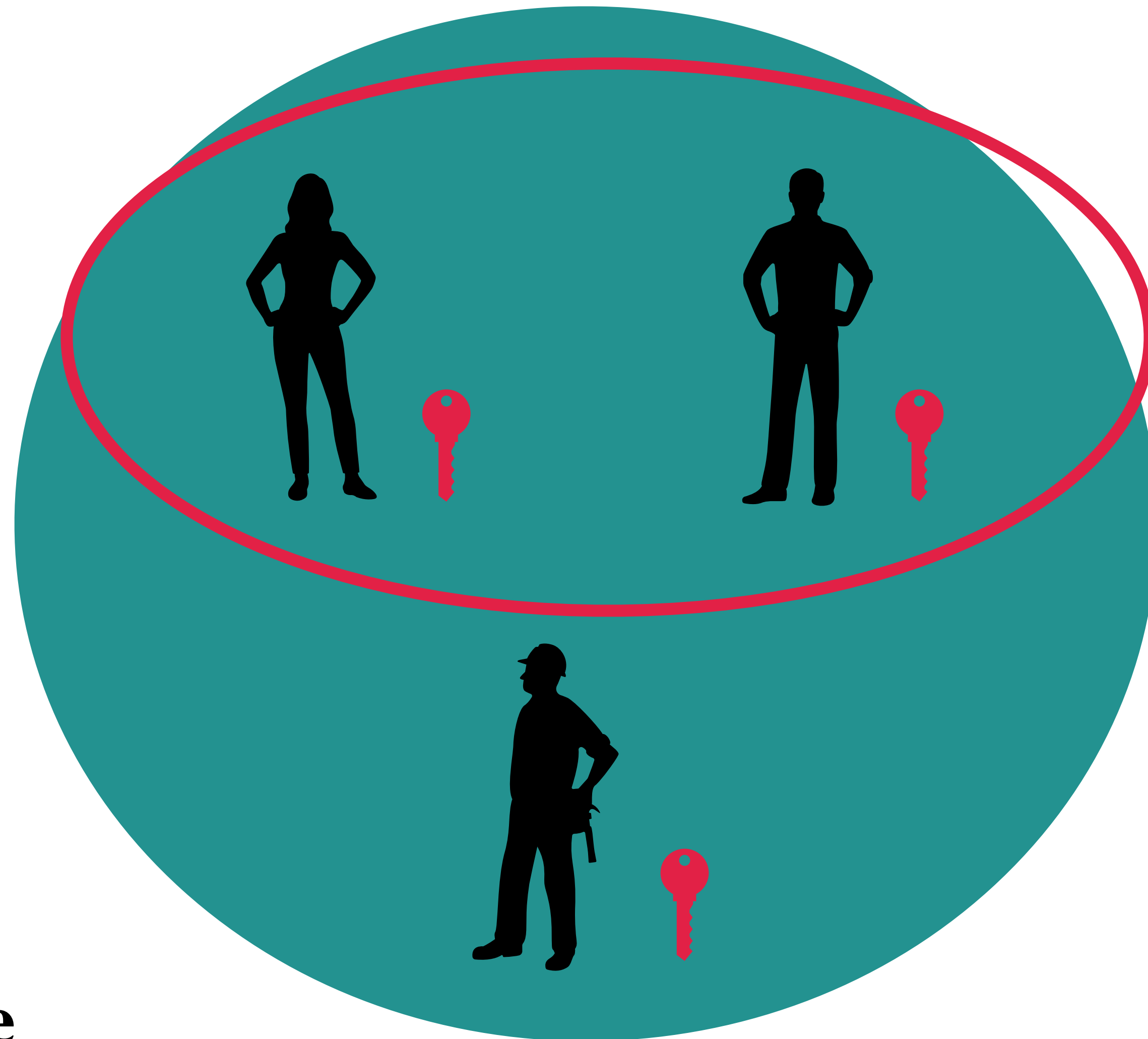
What are Threshold Signatures?



Public Key

(2,3) Example

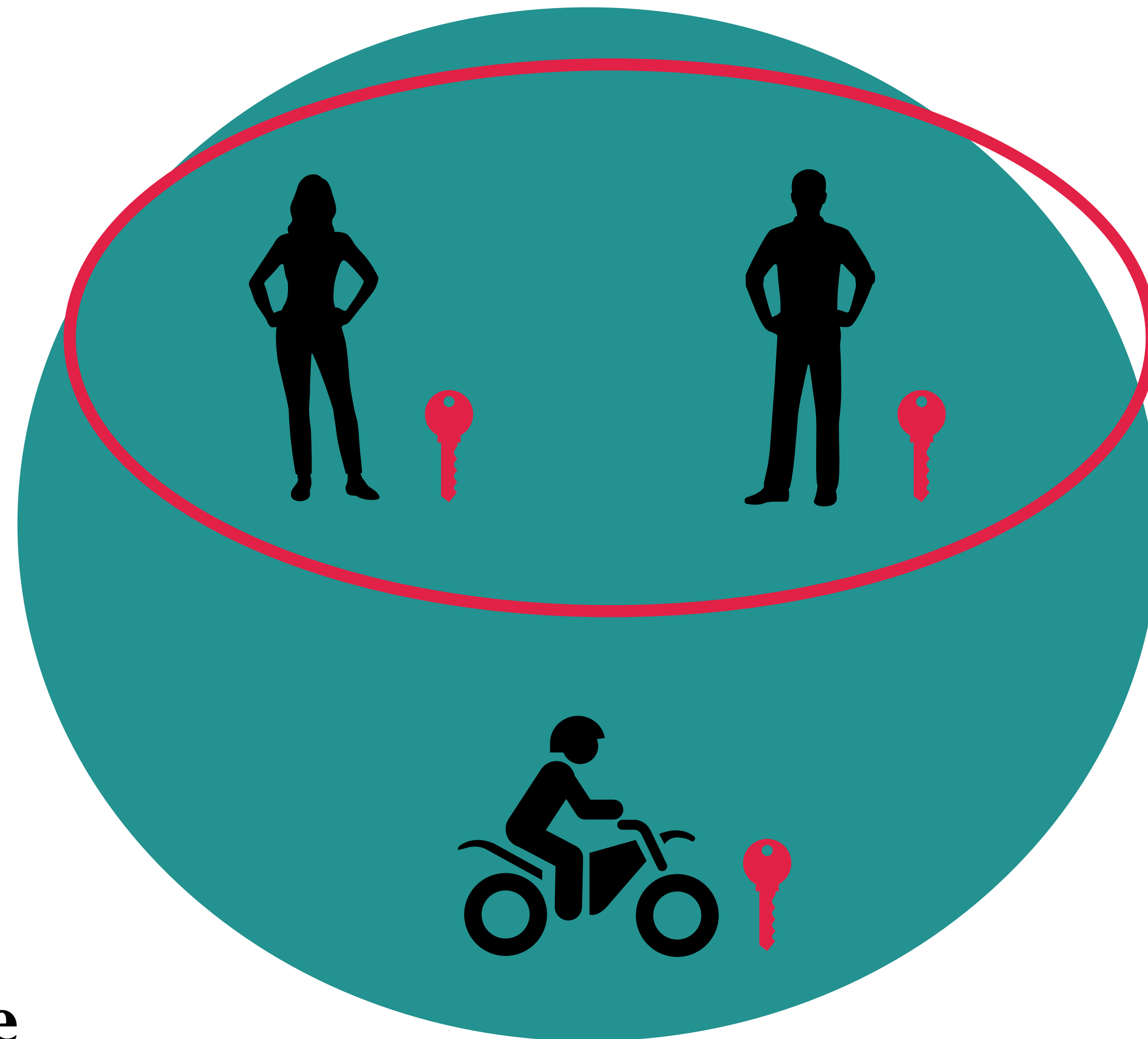
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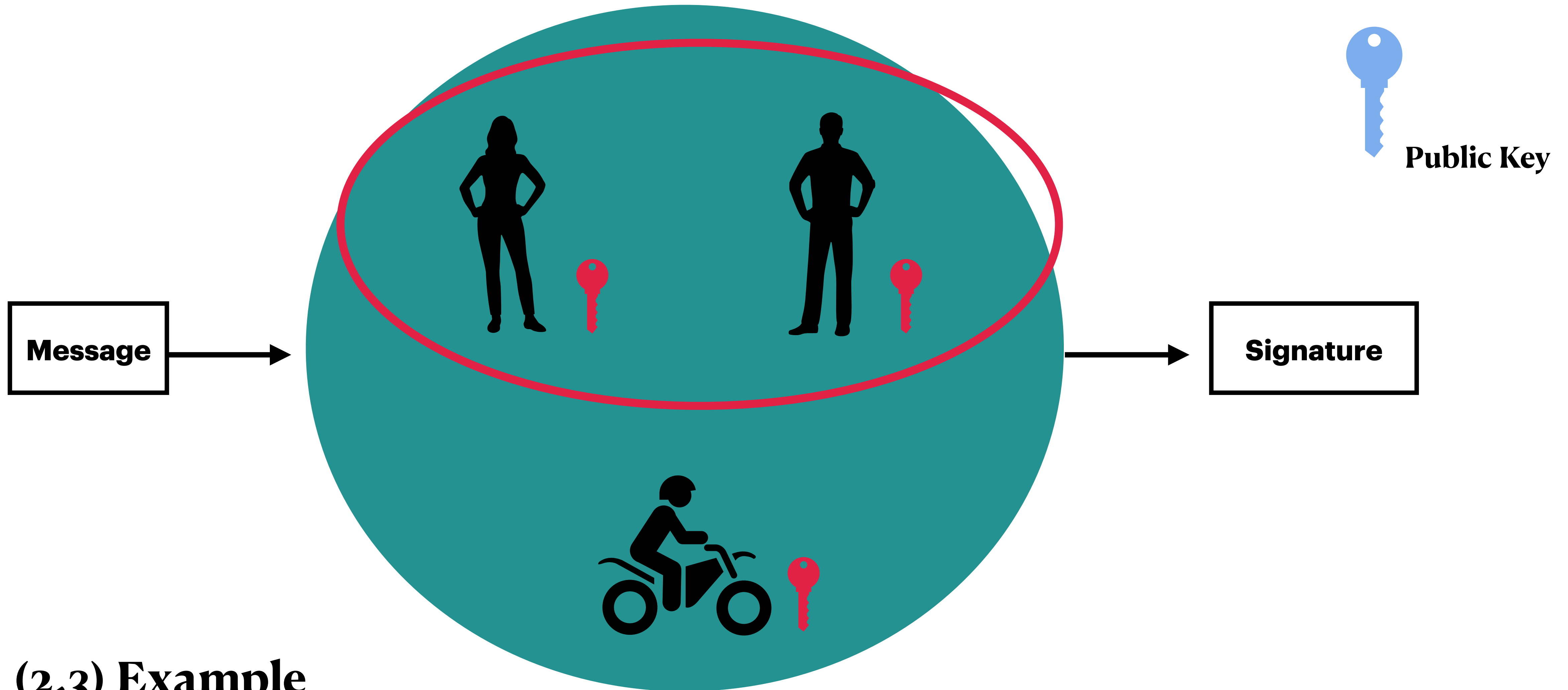
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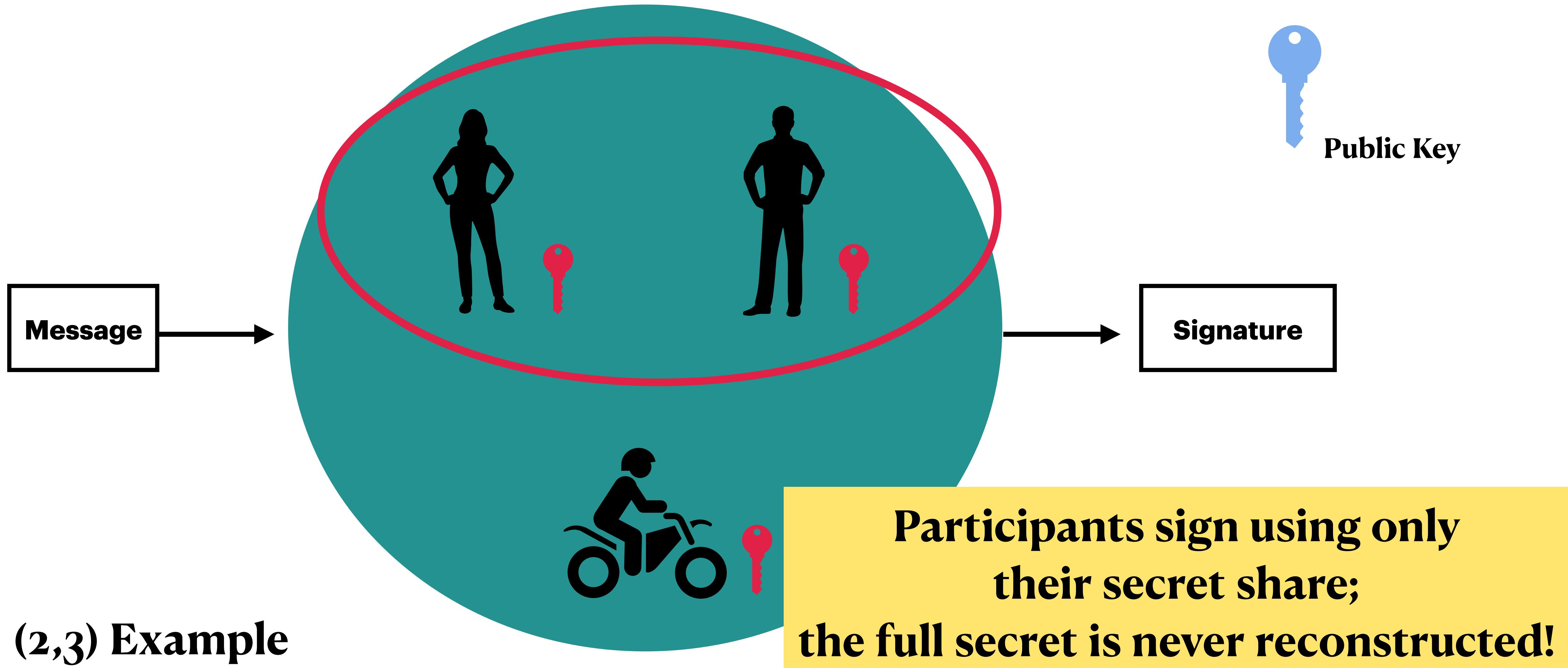
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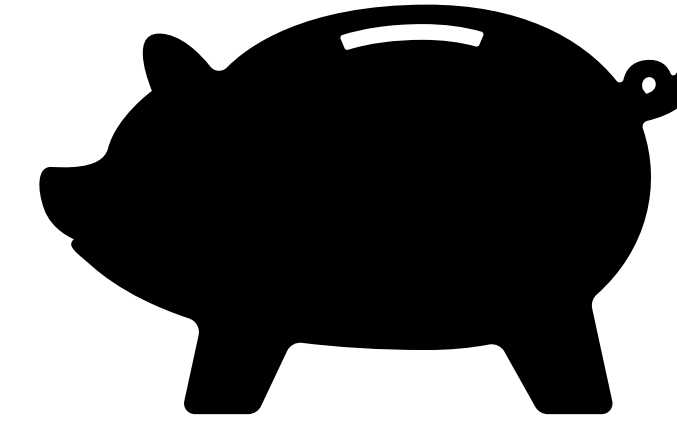
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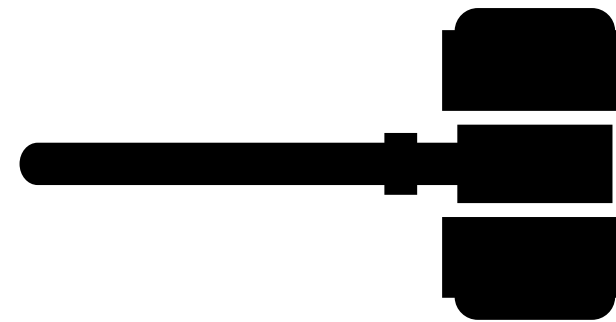
Uses for Threshold Signatures



Banks and Exchanges




Cryptocurrency Wallets



Trust Authorities (CAs)

Where was FROST Last Year?



**Paper
Published
(FROST1)**




**Presented to
NIST**



**IETF
Draft Started**

Where is FROST Now?



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Where is FROST Now?



**5+
Implementations**



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**Additional
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**Threshold BLS
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Where is FROST *Now?*

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**Optimized
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**Practical
use!**

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How to Prove Schnorr Assuming Schnorr: Security of Multi- and Threshold Signatures

Elizabeth Crites, Chelsea Komlo, Mary Maller

(Single-Party) Schnorr Signature Scheme



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To generate a key pair:

$$sk \xleftarrow{\$} \mathbb{F} ; PK \leftarrow g^{sk}$$

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To verify (PK, σ, m) :

$$c \leftarrow H(PK, m, R)$$

$$R \cdot PK^c \stackrel{?}{=} g^z$$

output accept/reject

Attempt: Multi-Party Schnorr

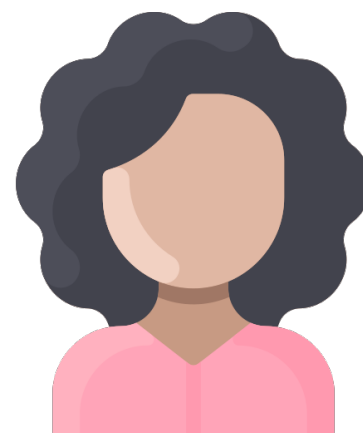
PK_2



PK_1

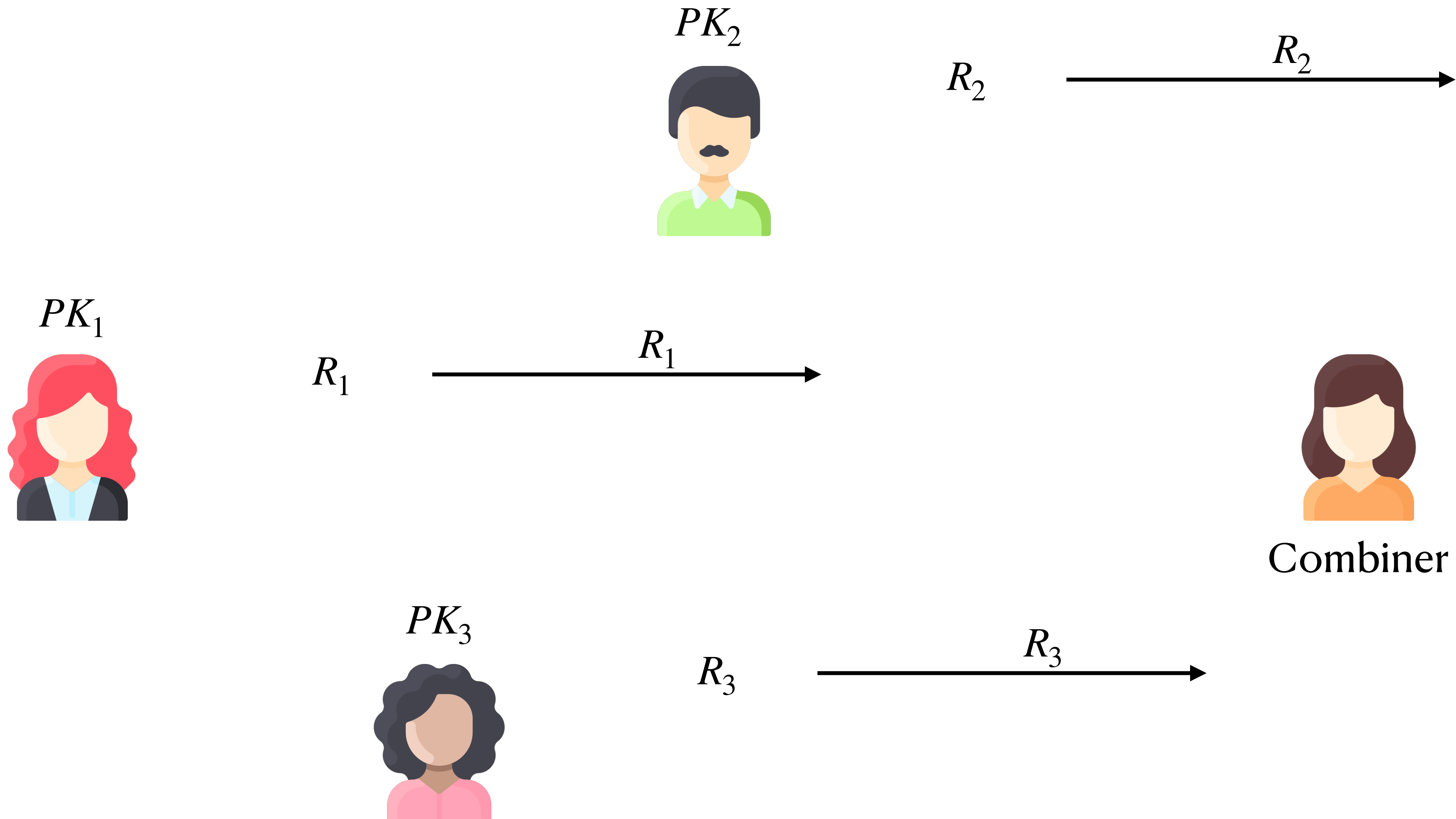


PK_3

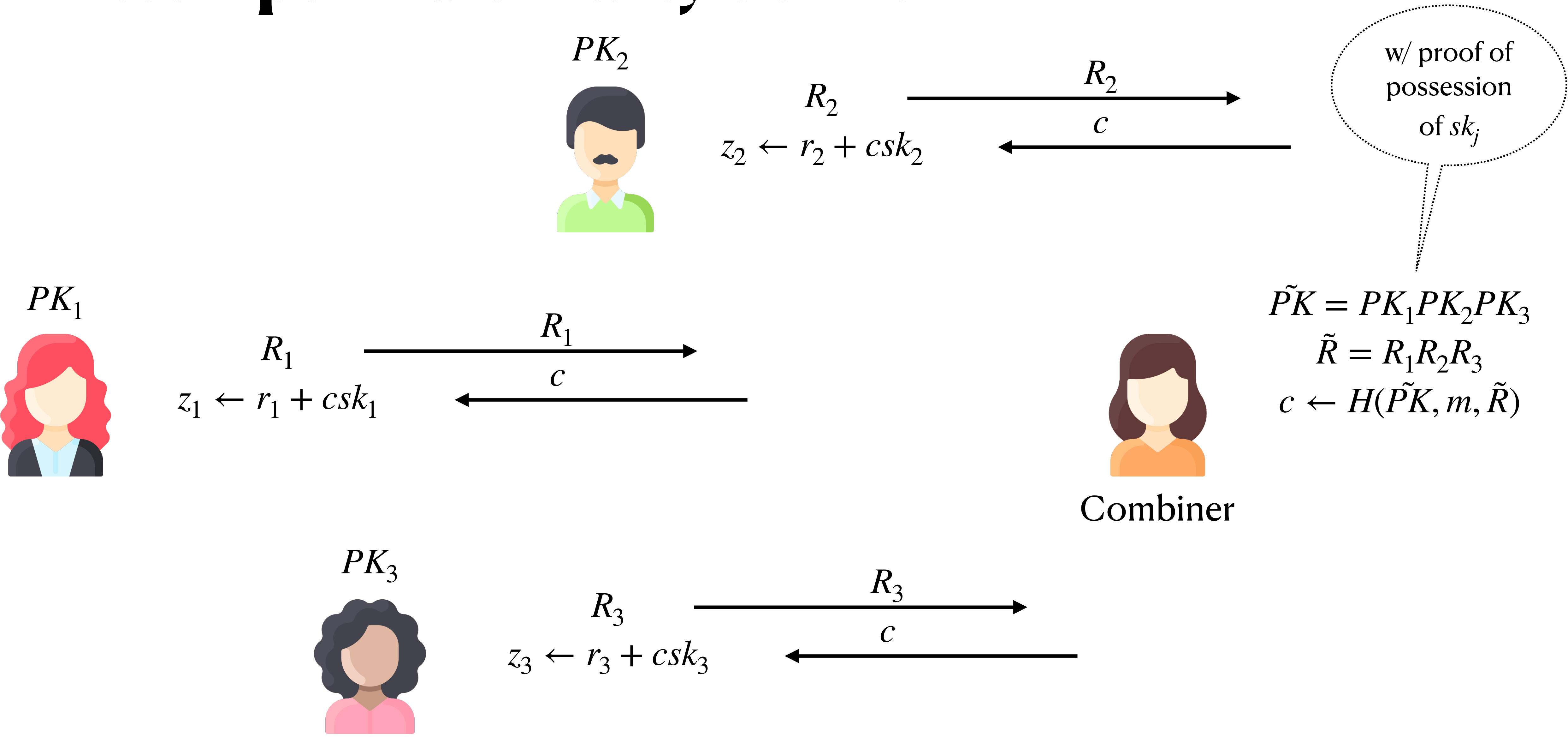


Combiner

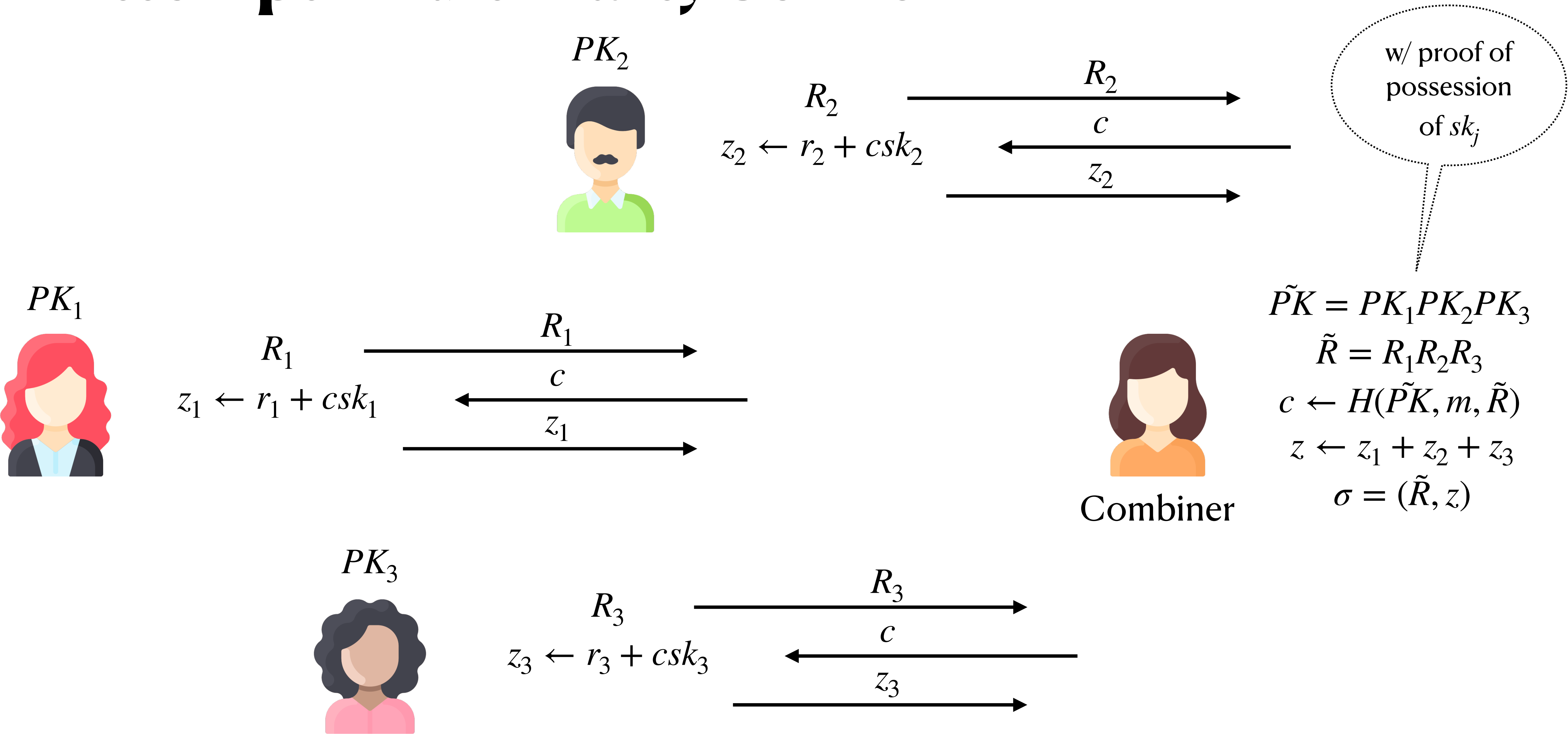
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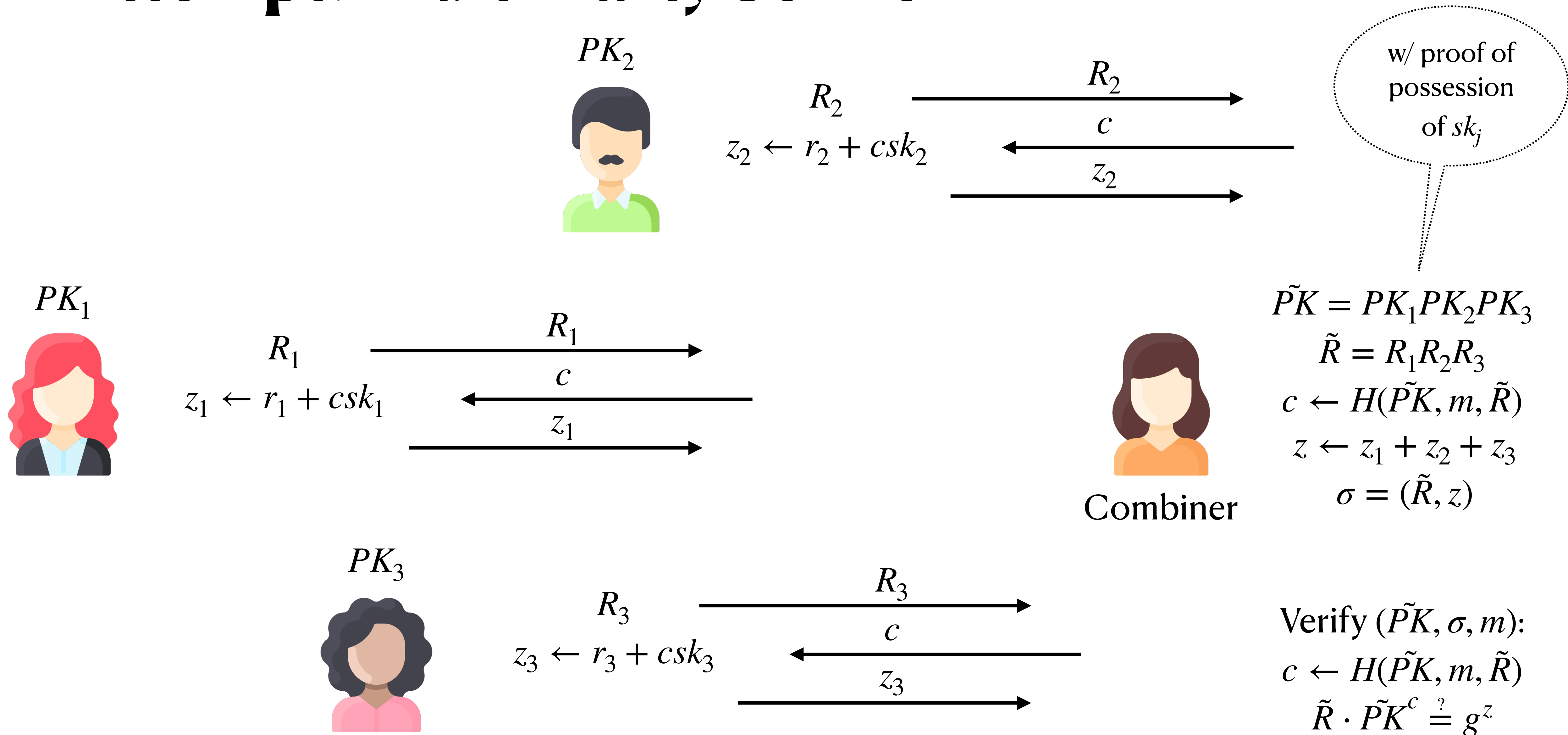
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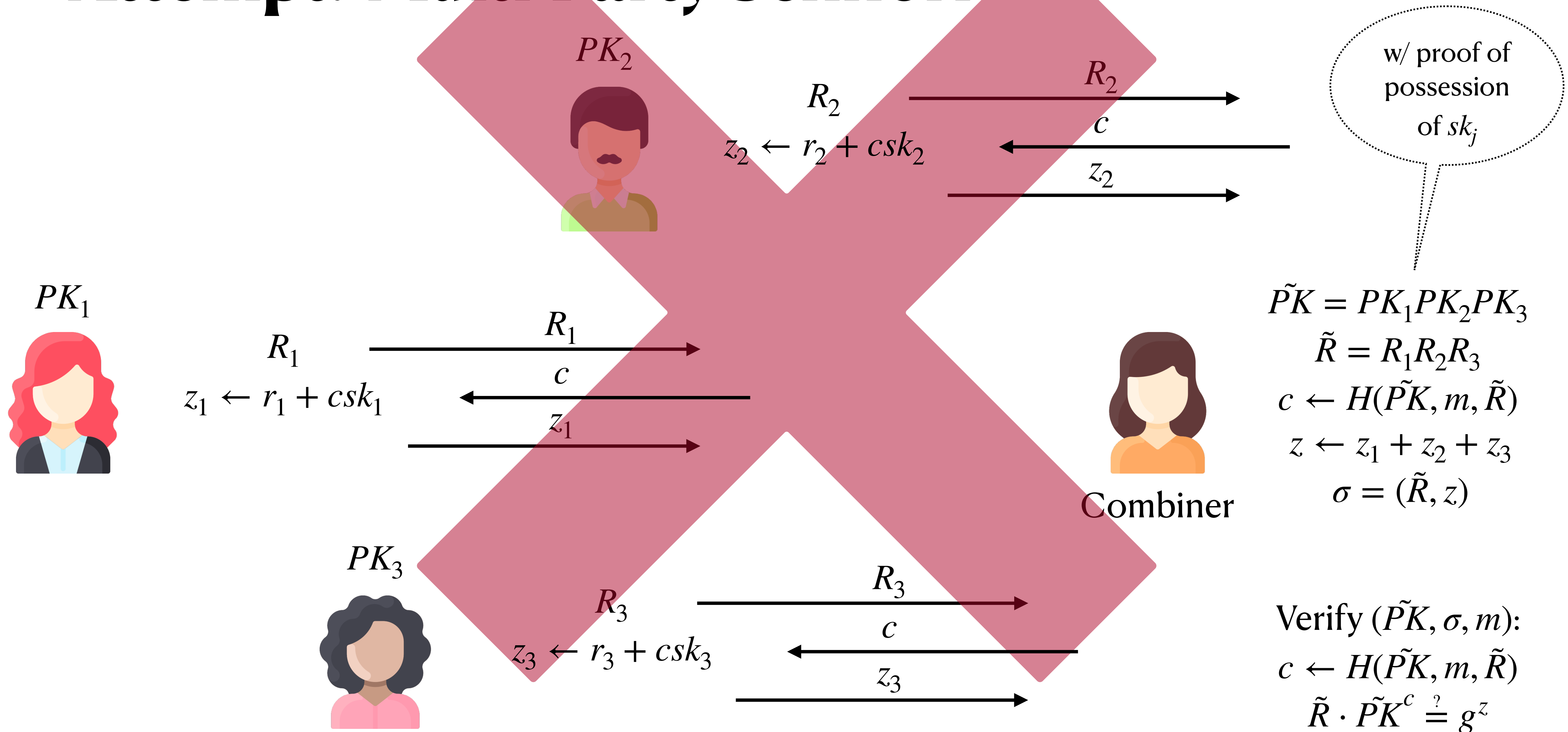
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ROS Attack

- ROS problem originally stated in [Schnorr91]
- Drijvers et al. [DEFKLNS19] show how to break unforgeability
- confirmed polynomial-time attack by Benhamouda et al. [BLOR20]
- concurrent attack:
 - adversary opens multiple signing sessions at once
 - sees honest nonces first and makes its nonce a function of them
 - forges signature

Fix #1: 3 Rounds

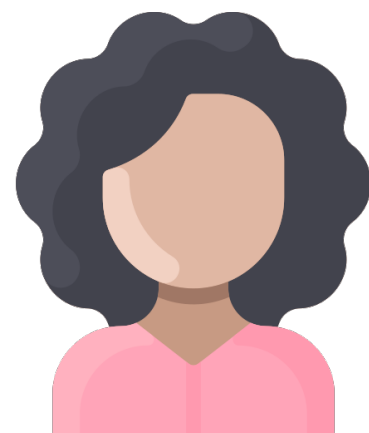
PK_2



PK_1

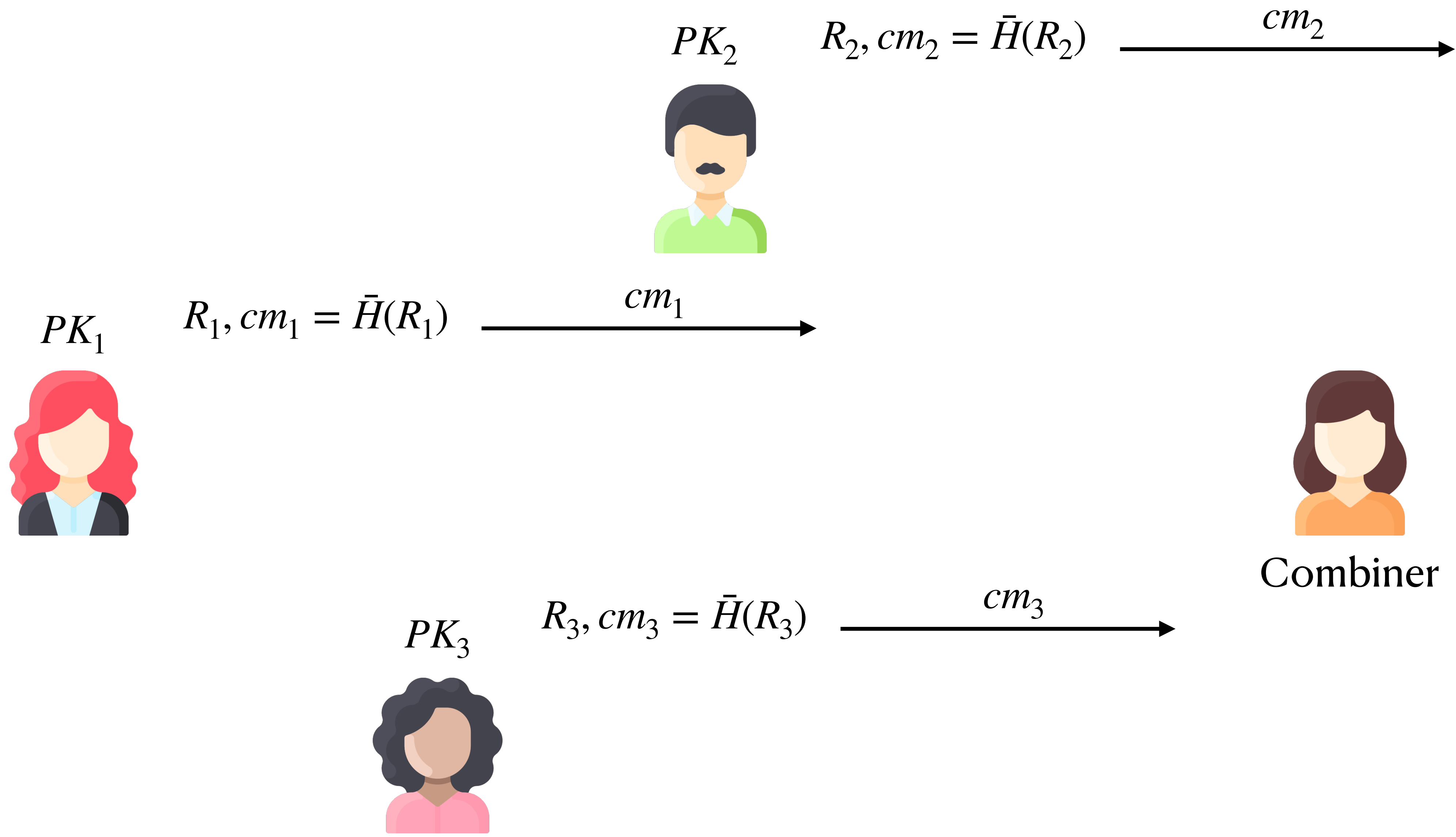


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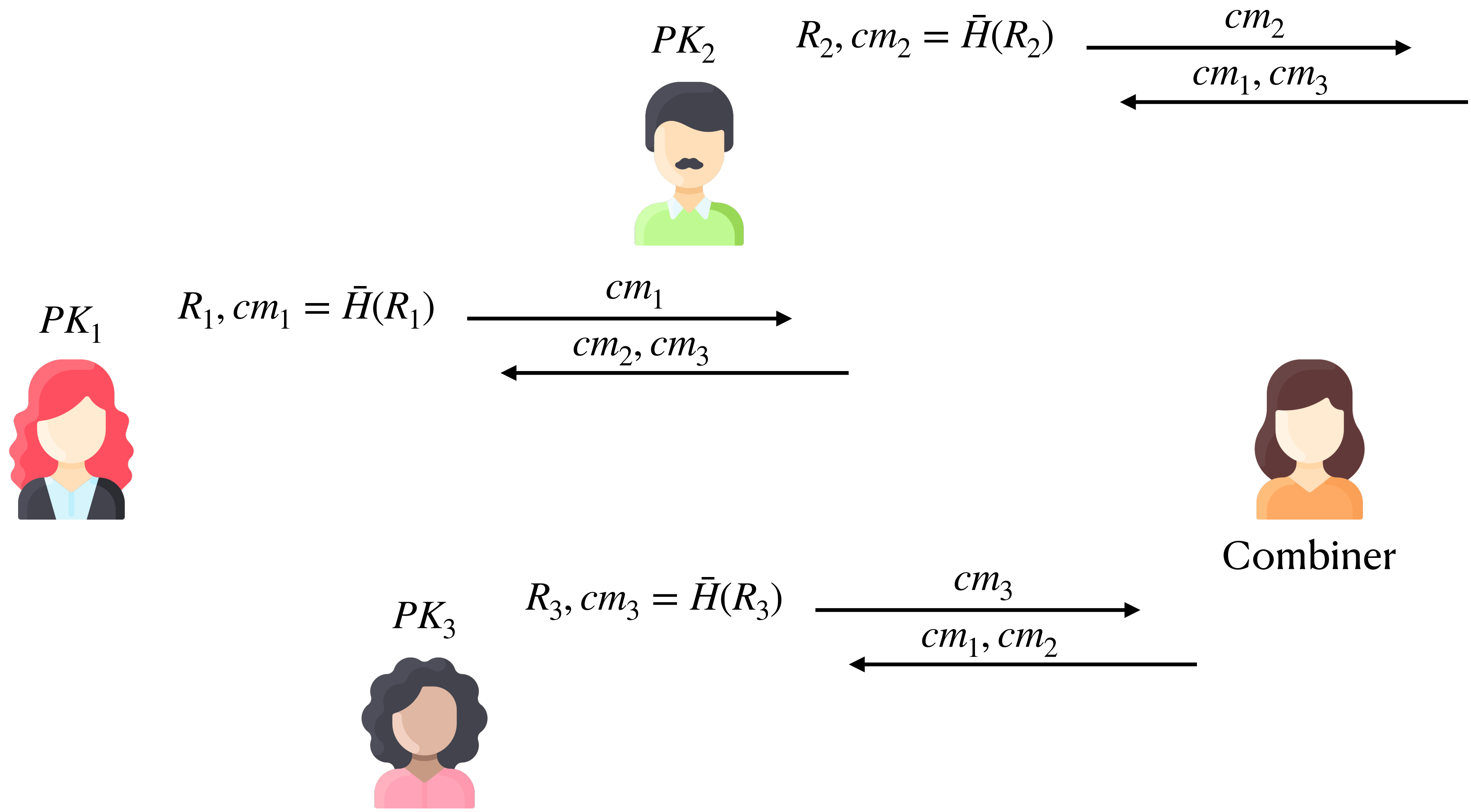


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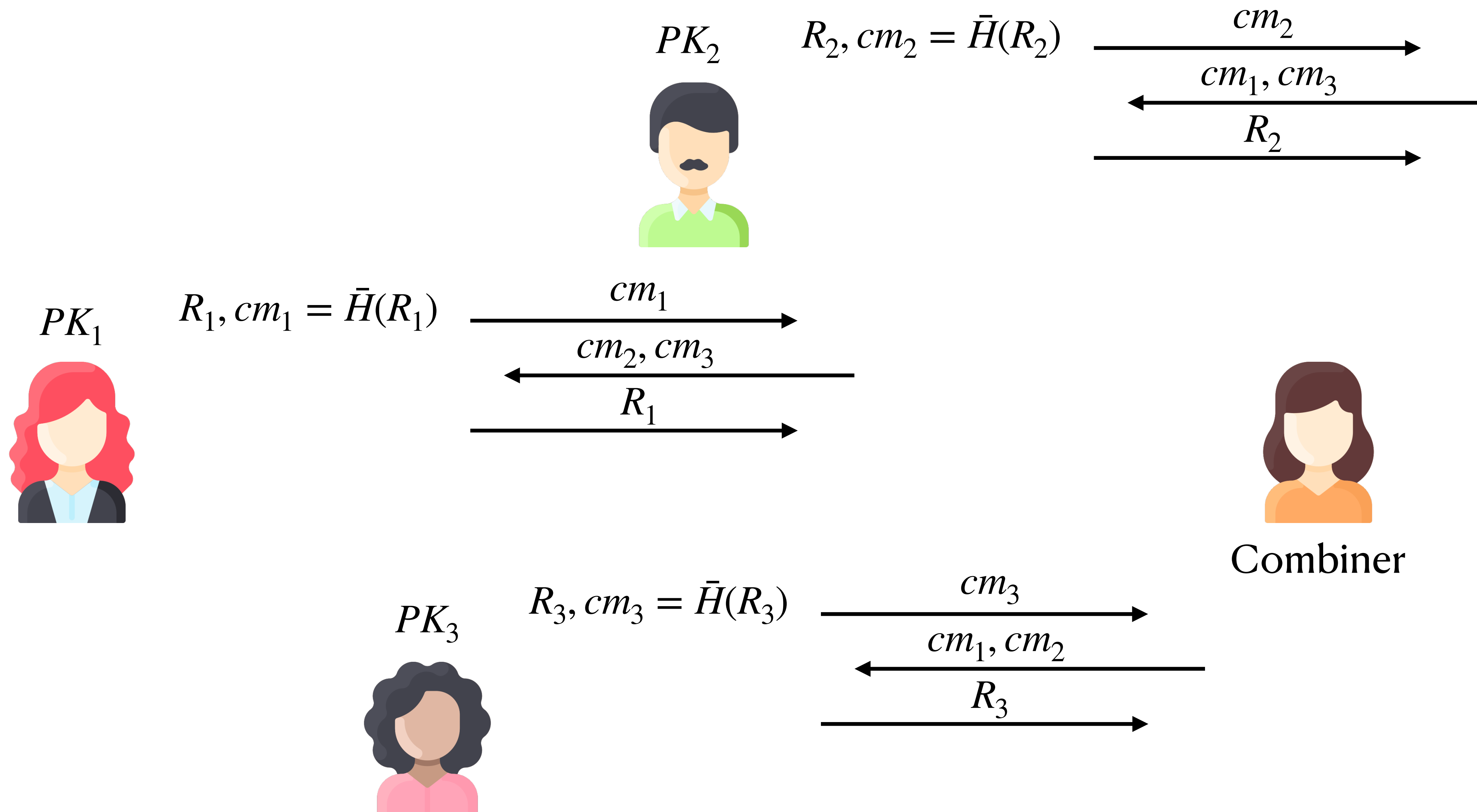
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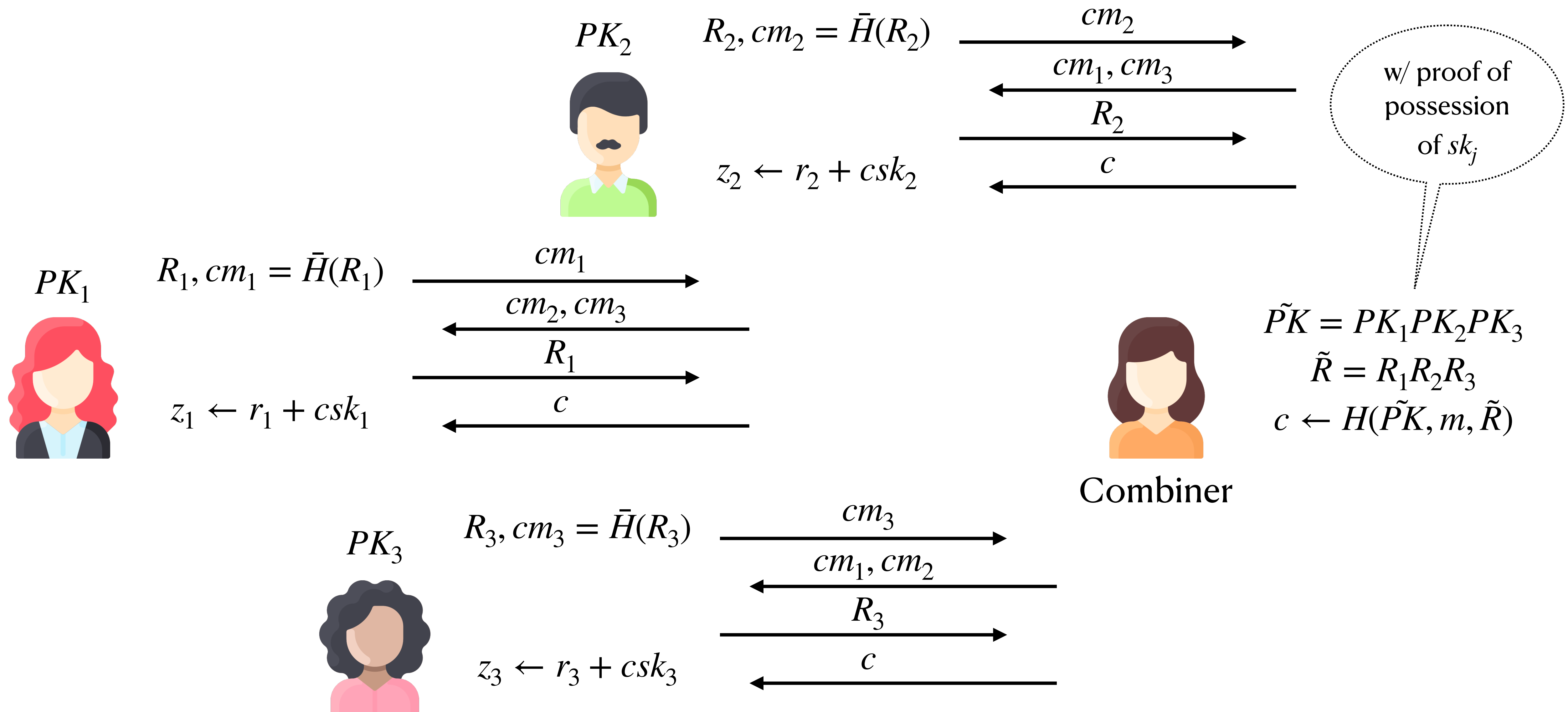
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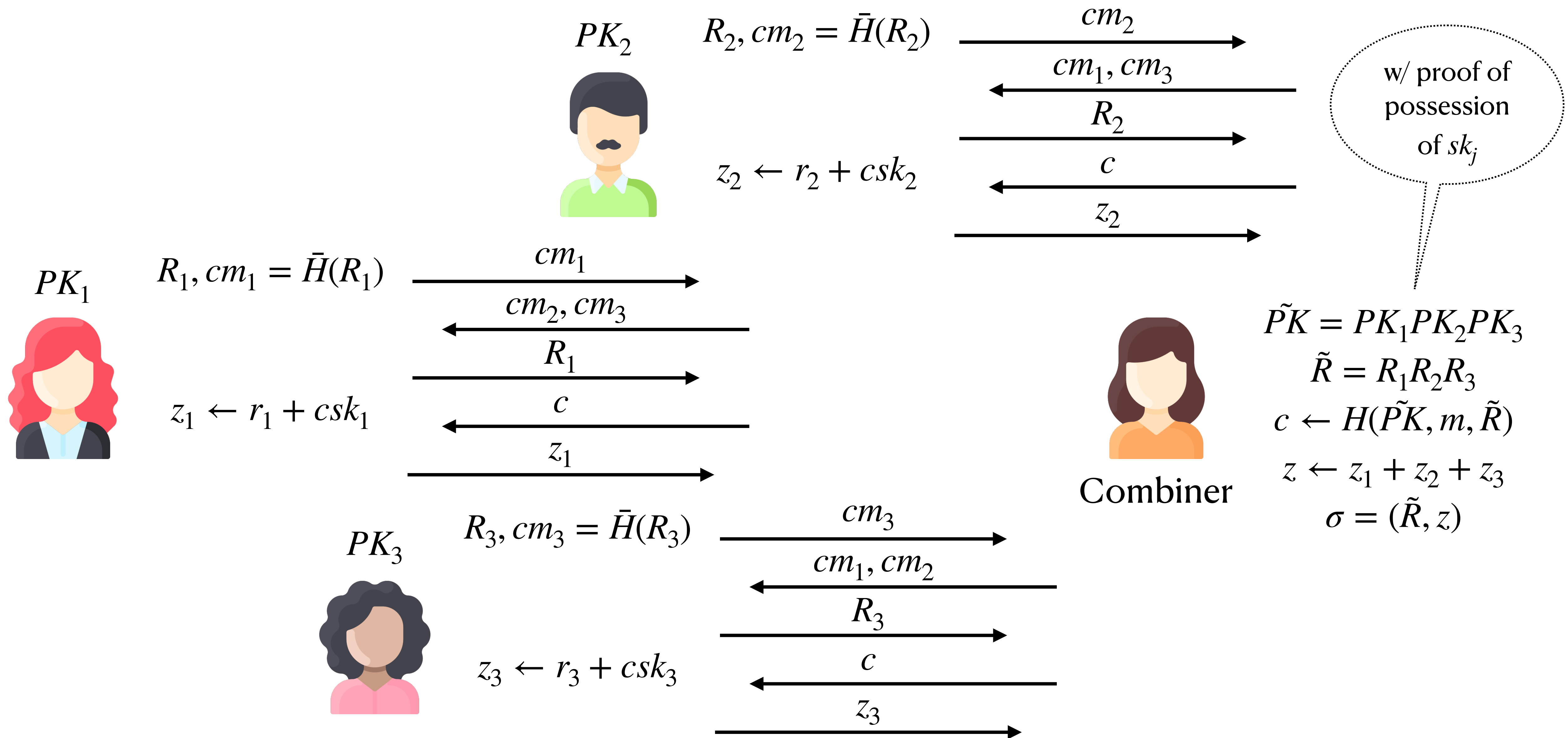
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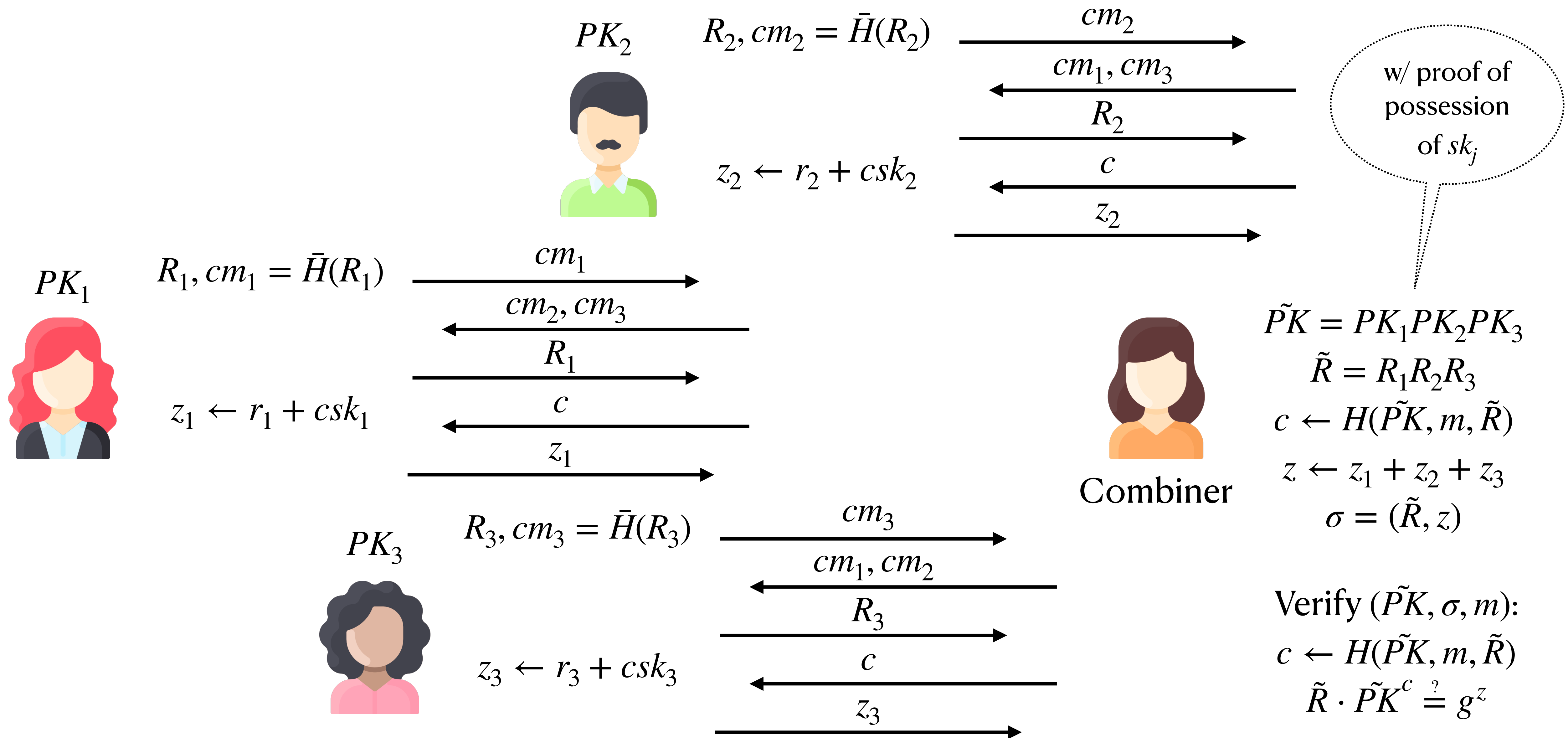
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Fix #1: 3 Rounds



“How to Prove Schnorr Assuming Schnorr”

Elizabeth Crites, Chelsea Komlo, Mary Maller

- Our Contributions:
 - 3-round (n,n) multisignature SimpleMuSig (with PoP of keys)
 - 3-round (t,n) threshold signature SimpleTSig (with PedPoP)

Fix #2: 2 Nonces (“FROST”)

PK_2

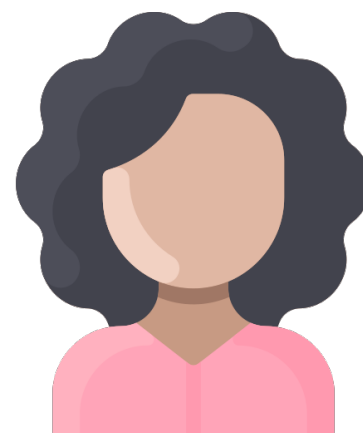


PK_1

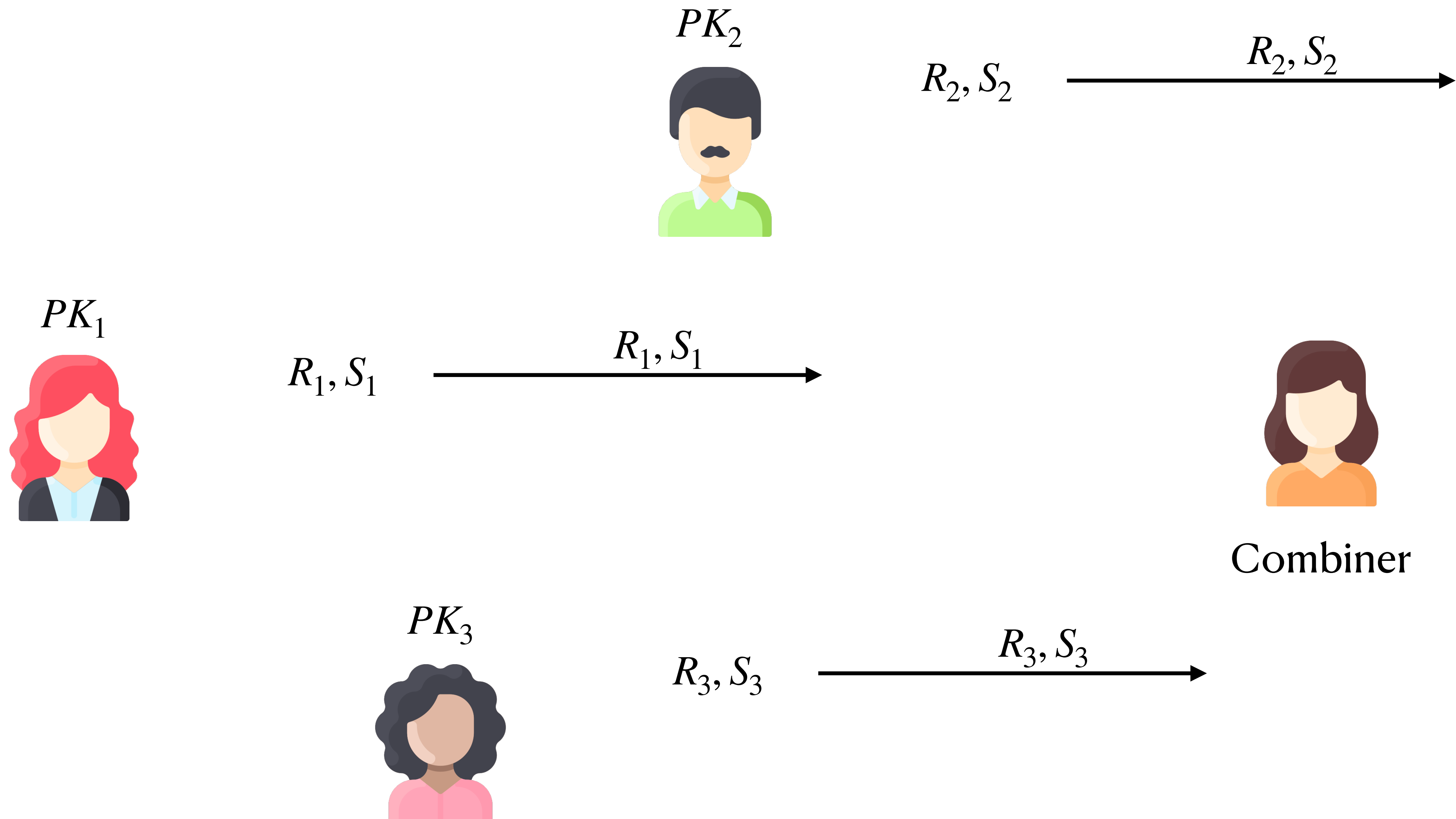


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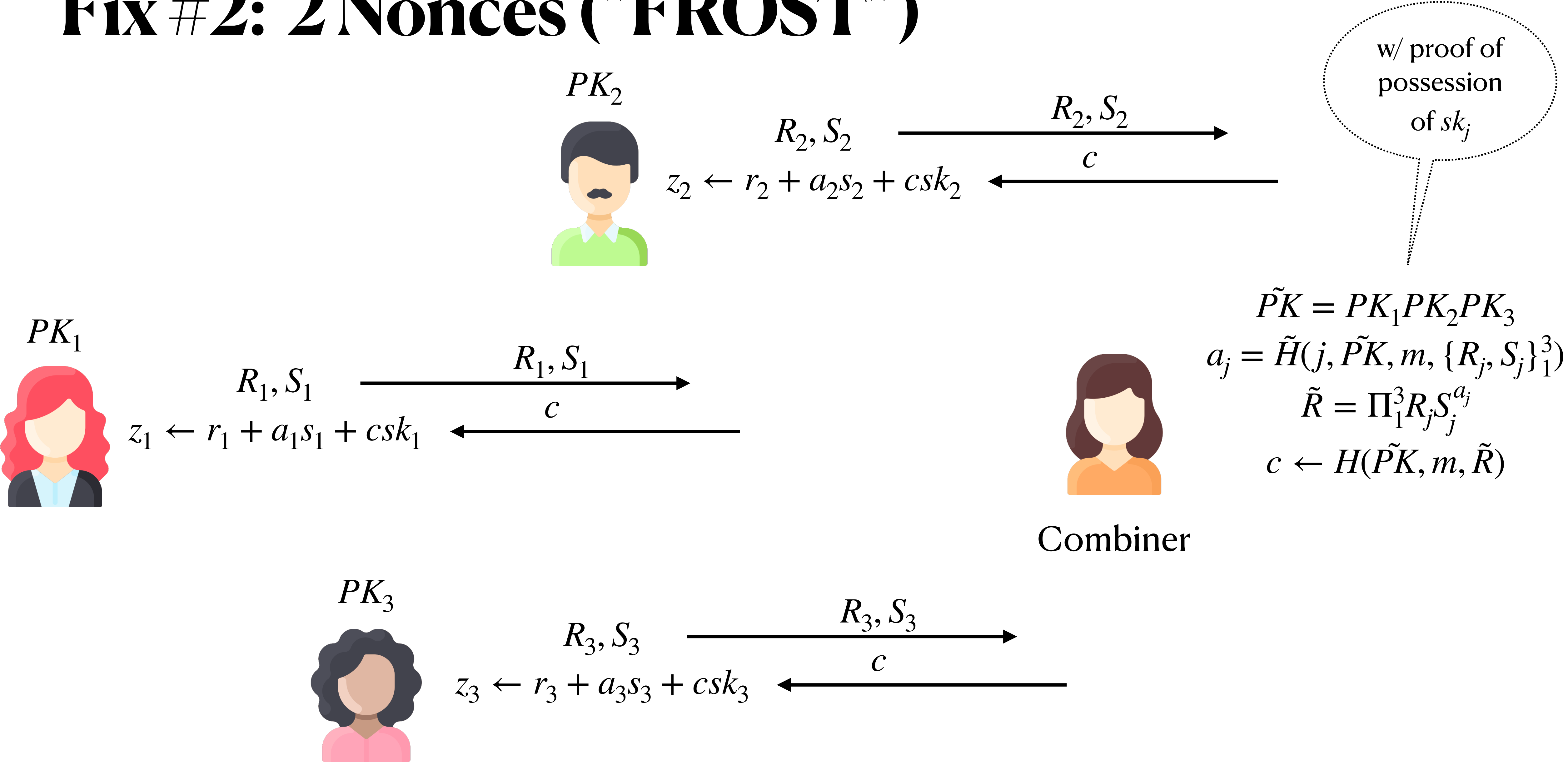
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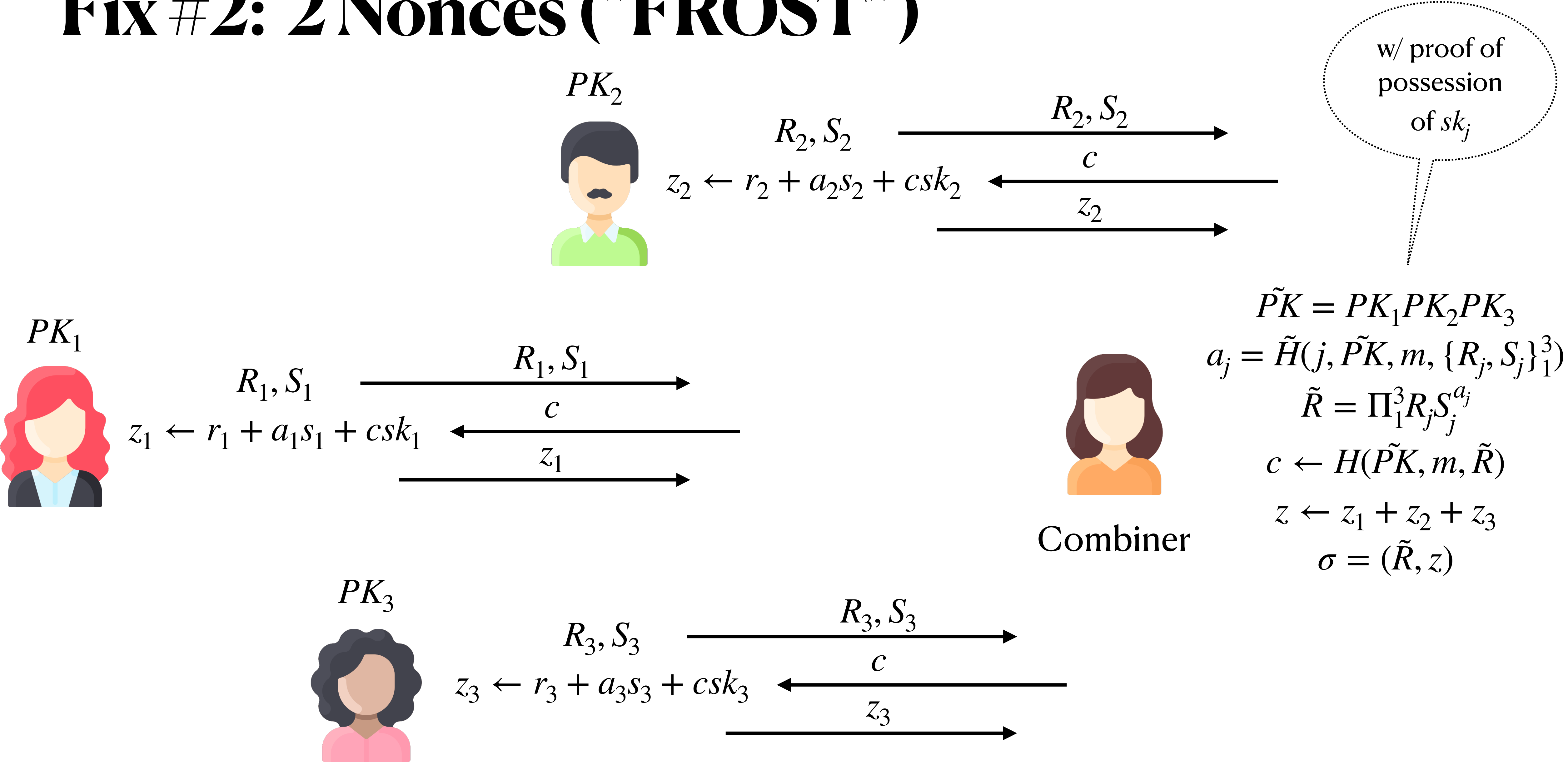
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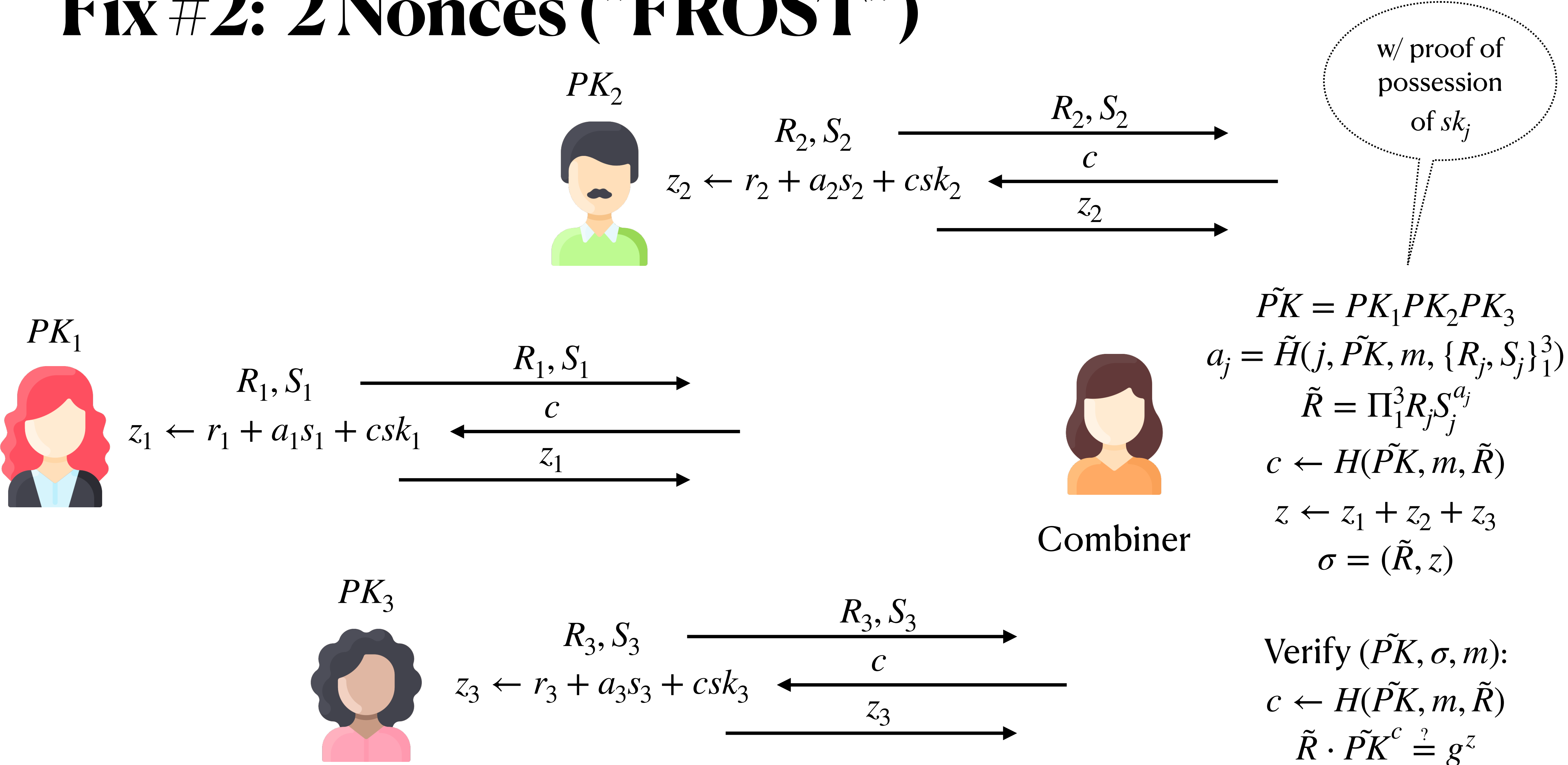
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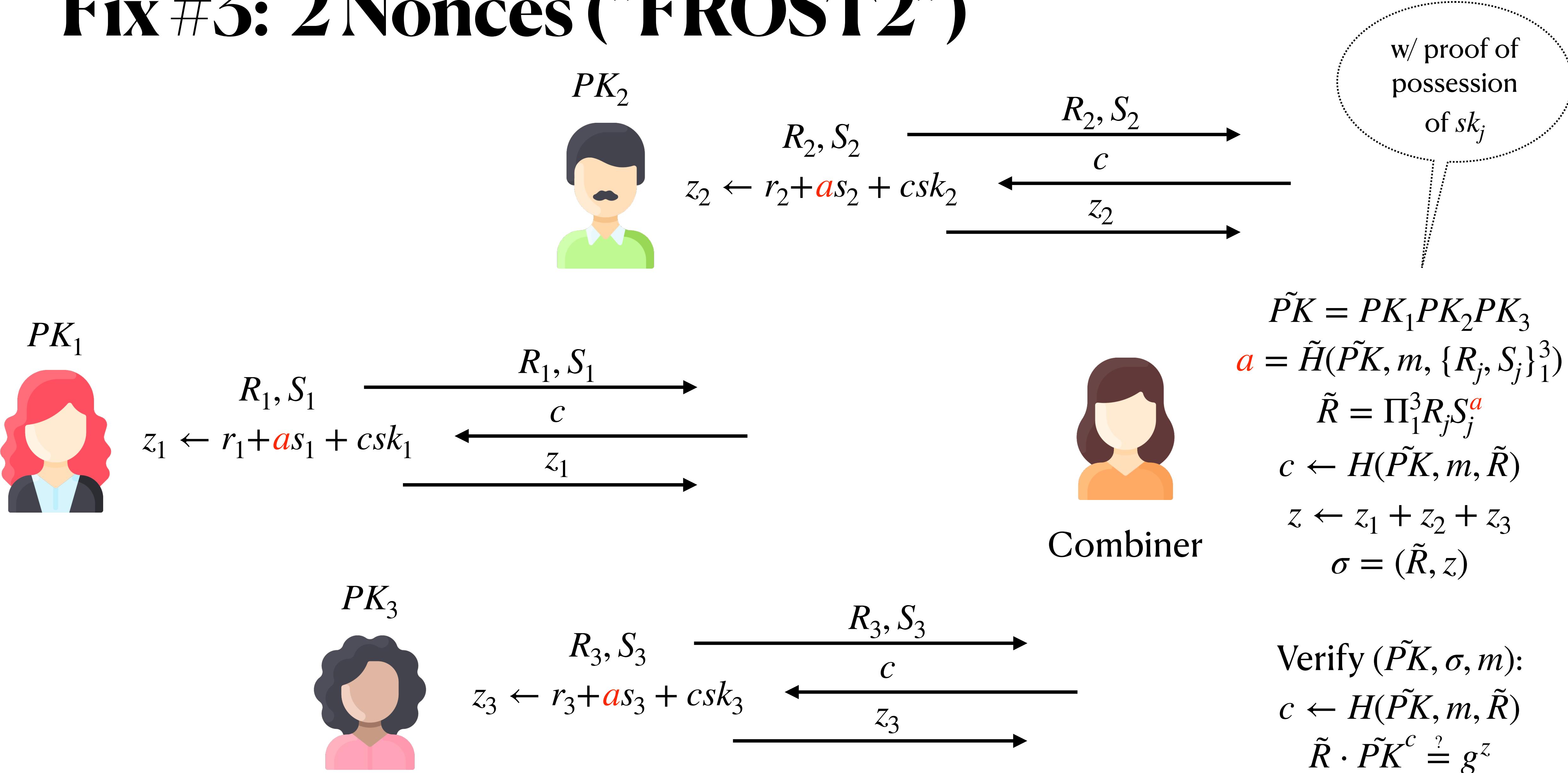
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Fix #3: 2 Nonces (“FROST2”)



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 - 3-round (n,n) multisignature SimpleMuSig (with PoP of keys)
 - 3-round (t,n) threshold signature SimpleTSig (with PedPoP)
 - optimized 2-round (n,n) multisignature SpeedyMuSig (with PoP of keys)
 - optimized 2-round (t,n) threshold signature FROST2 (with PedPoP)
 - reduces exponentiations from at least t to one
 - new proving framework

Proving the Security of Multi-Party Schnorr

- Security reductions for multi-party signatures have two moving parts:
 1. Simulating honest users interacting with the adversary
 2. Extracting a solution to some hard problem from the adversary's responses
- Idea: Separate the two parts for a more modular reduction

Proving the Security of Multi-Party Schnorr

multi-party

SimpleTSig
SimpleMuSig

single party

Schnorr
Assumption

Discrete Log

FROST2
SpeedyMuSig

Bischnorr
Assumption

One-More
Discrete Log

“Stronger Security for Non-Interactive Threshold Signatures”

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- Merged with our paper for CRYPTO 2022

eprint.iacr.org/2022/833

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“Stronger Security for Non-Interactive Threshold Signatures”

Mihir Bellare, Stefano Tessaro, Chenzhi Zhu

- Merged with our paper for CRYPTO 2022
- Hierarchy of notions of unforgeability for threshold signatures
- Independent proofs for unforgeability of FROST₁ and FROST₂ in ROM/OMDL
- Finds FROST₂ to be malleable with respect to the signing set

- γ = Lagrange coefficient for signing set (1, 3, 4),
- δ = Lagrange coefficient for signing set (1, 2, 3) / γ

PK_2

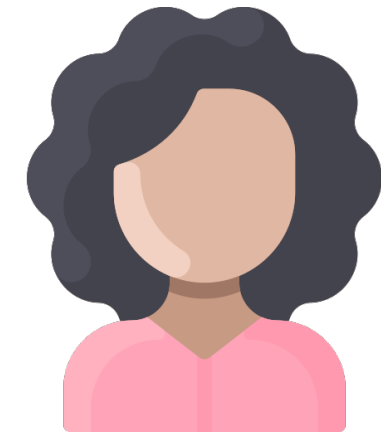


PK_1



Honest

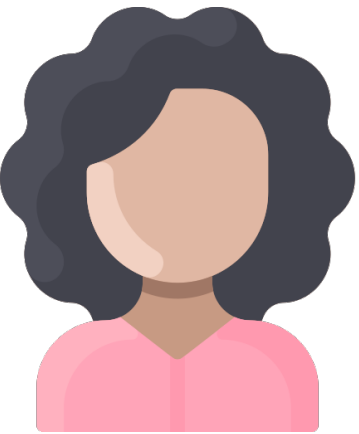
PK_3, PK_4



Corrupt


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PK_3, PK_4



Corrupt

PK_2




$R_2 = g^{r_2}, S_2 = g^{s_2}$

R_2, S_2

Honest

PK_1

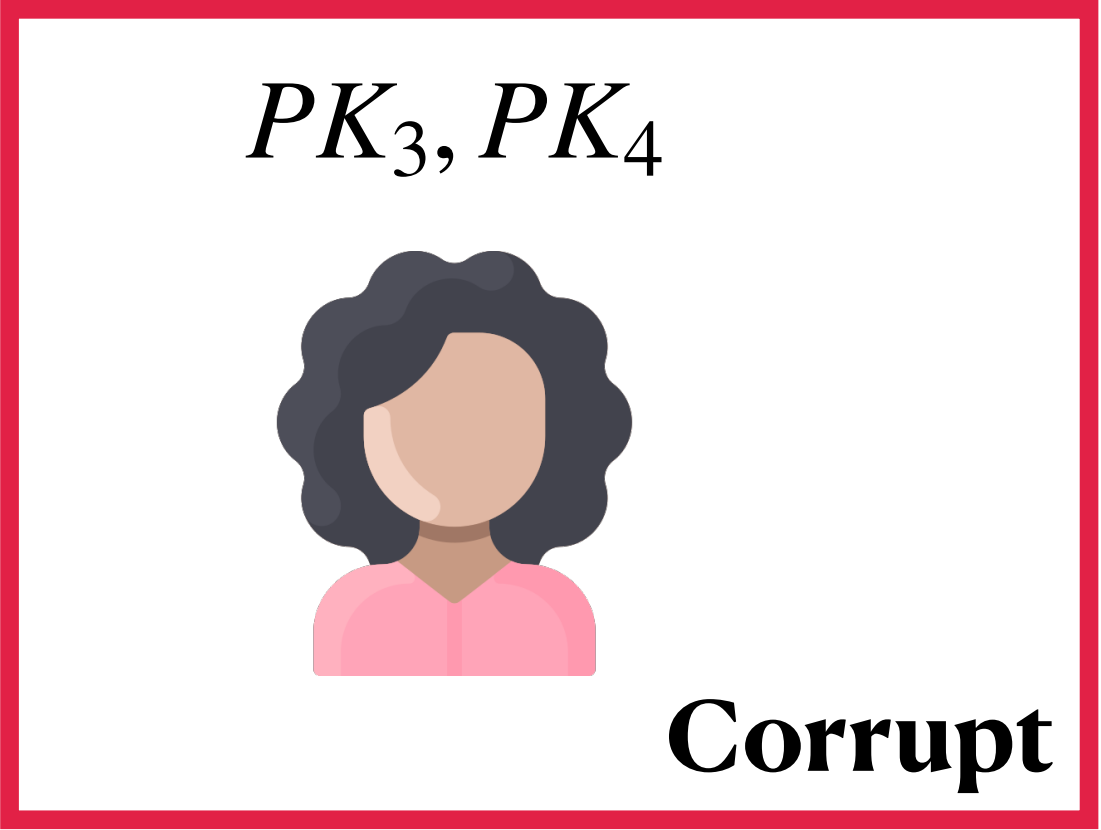
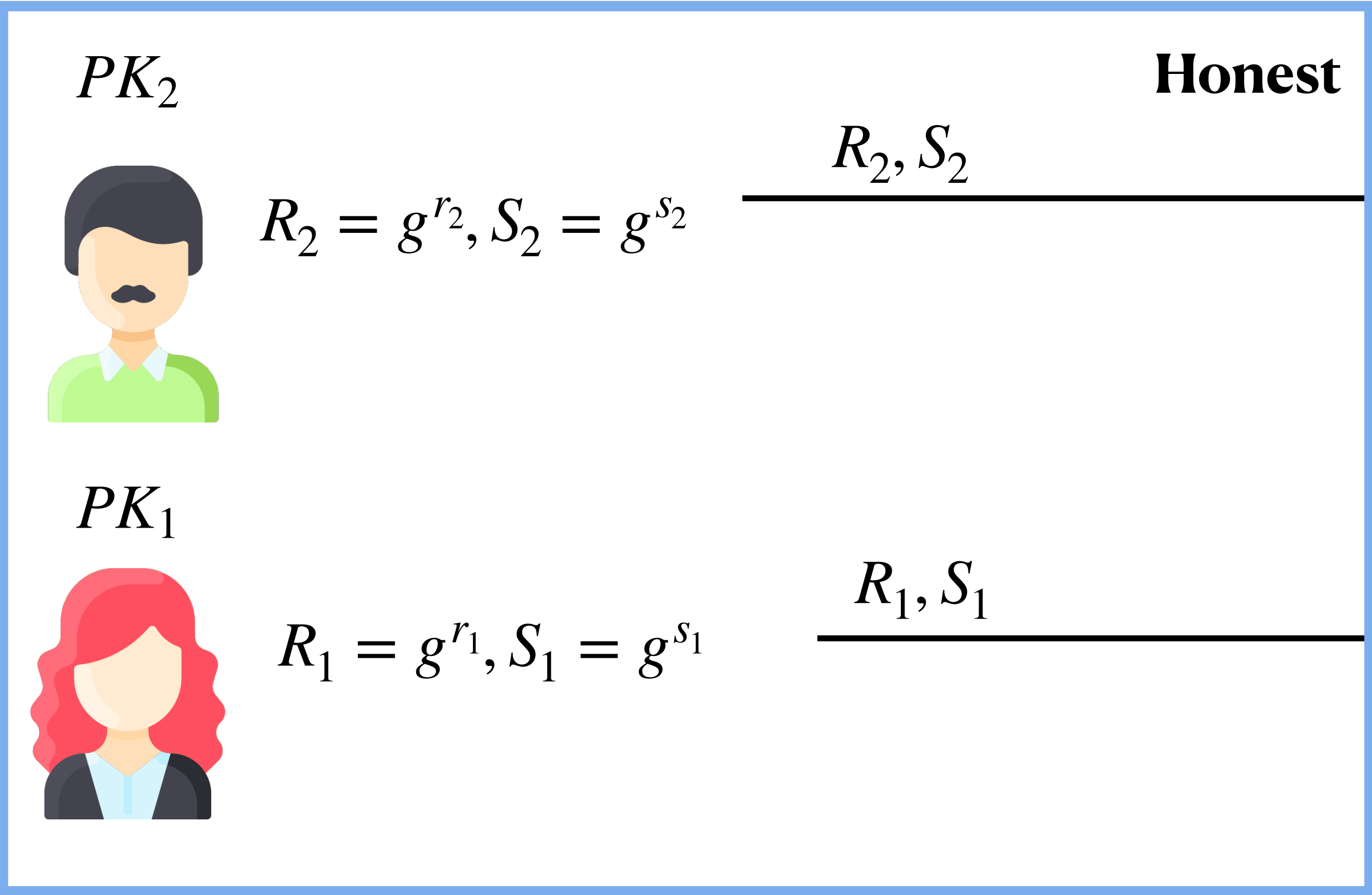


$R_1 = g^{r_1}, S_1 = g^{s_1}$

R_1, S_1

Begin signing protocol with signers (1, 2, 3)

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$$R_3 = (R_1)^{\delta-1} \cdot (R_2)^{-1}$$

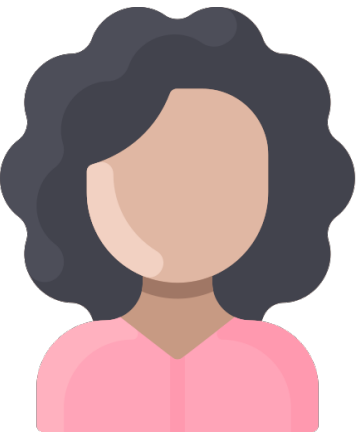
$$S_3 = (S_1)^{\delta-1} \cdot (S_2)^{-1}$$

$$\xleftarrow{R_3, S_3}$$

$$R = \prod_{i=1}^3 R_i \cdot (S_i)^a$$


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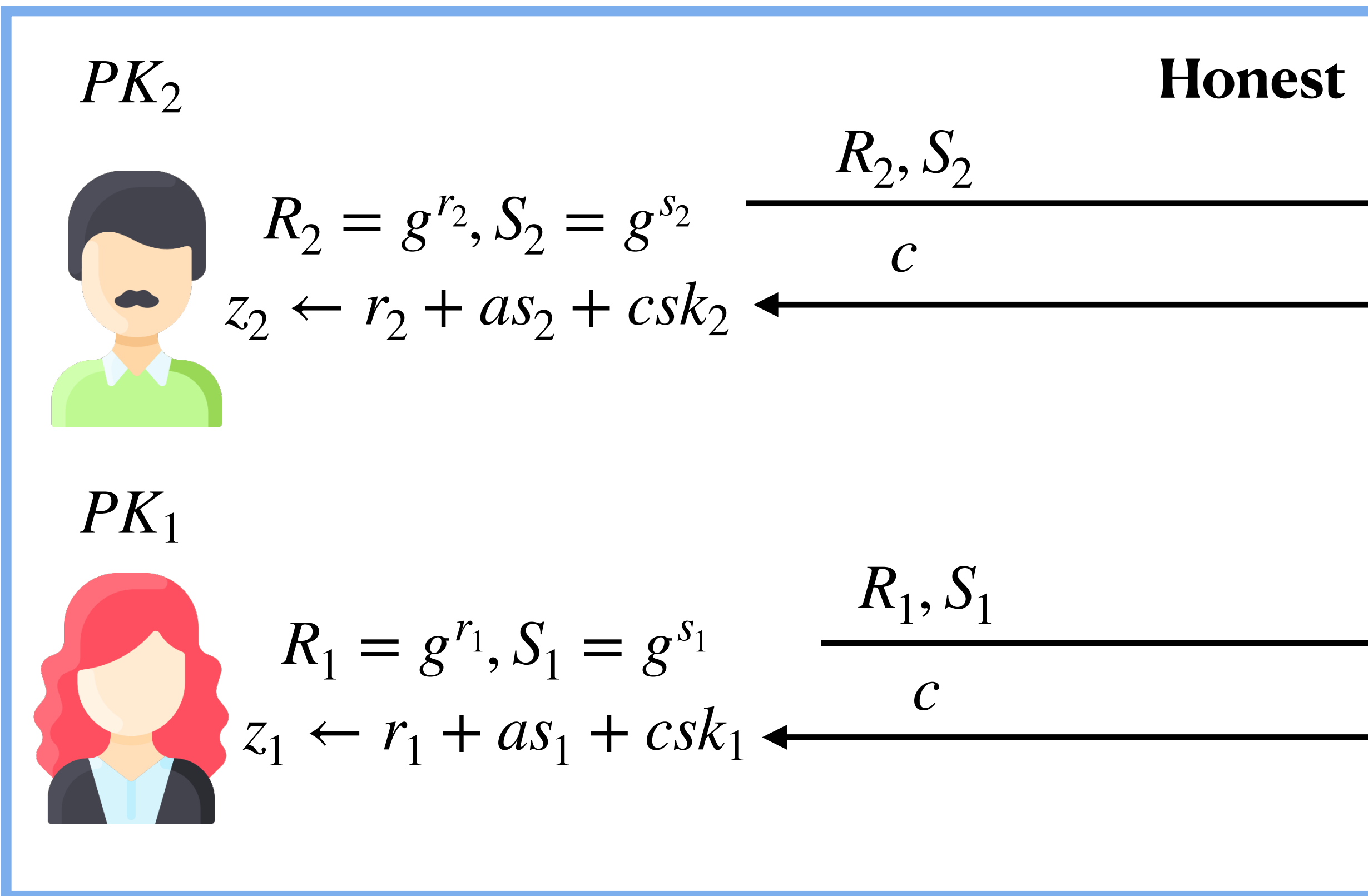
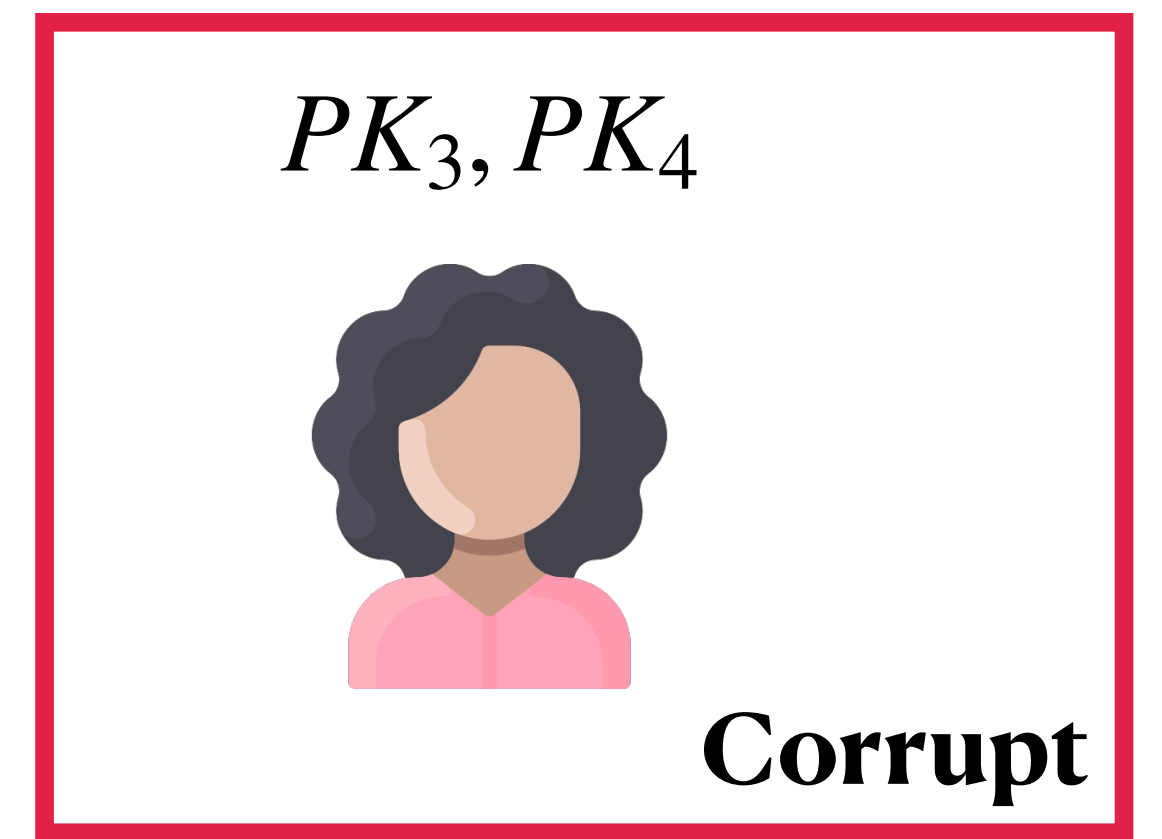


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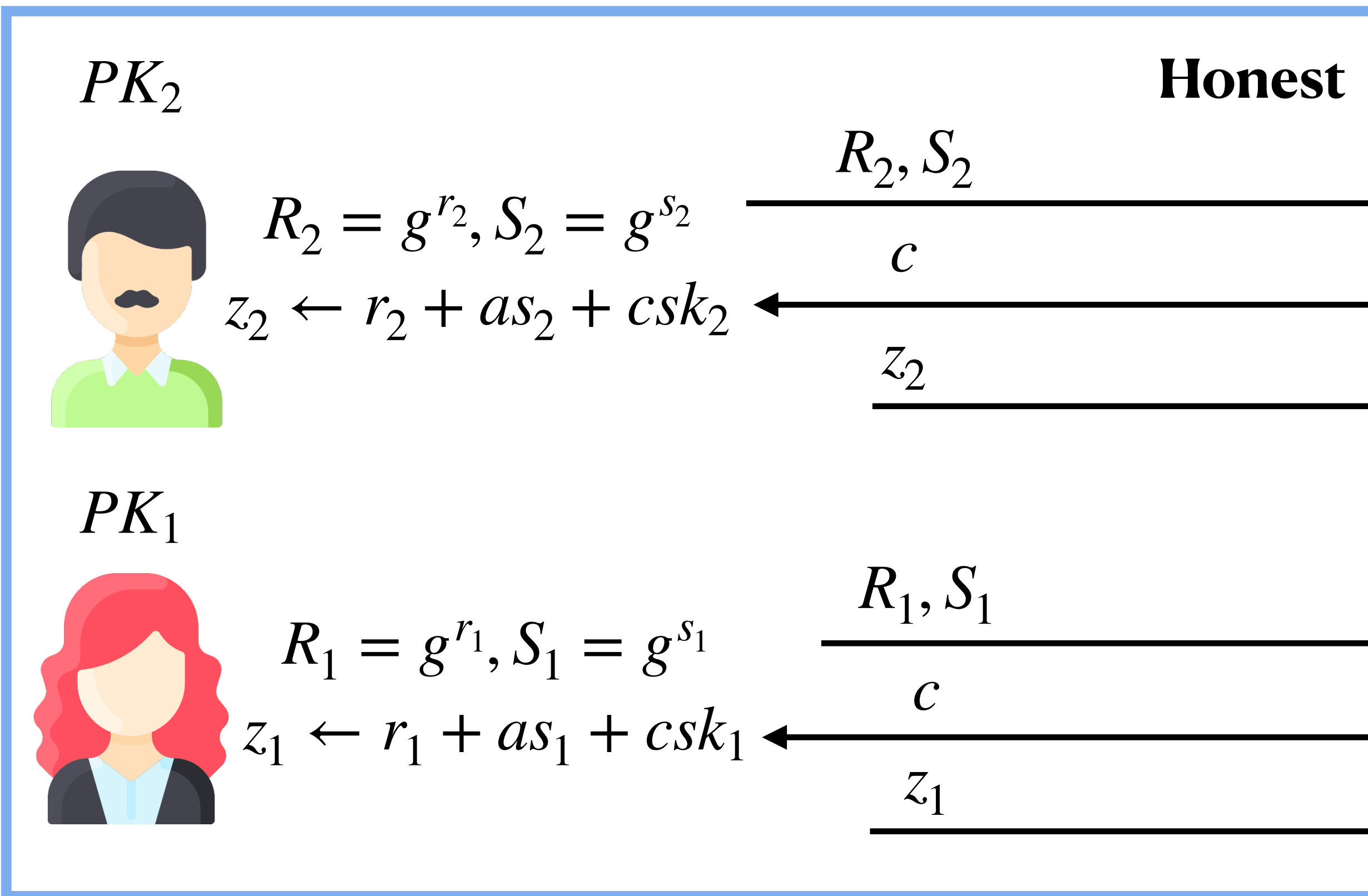
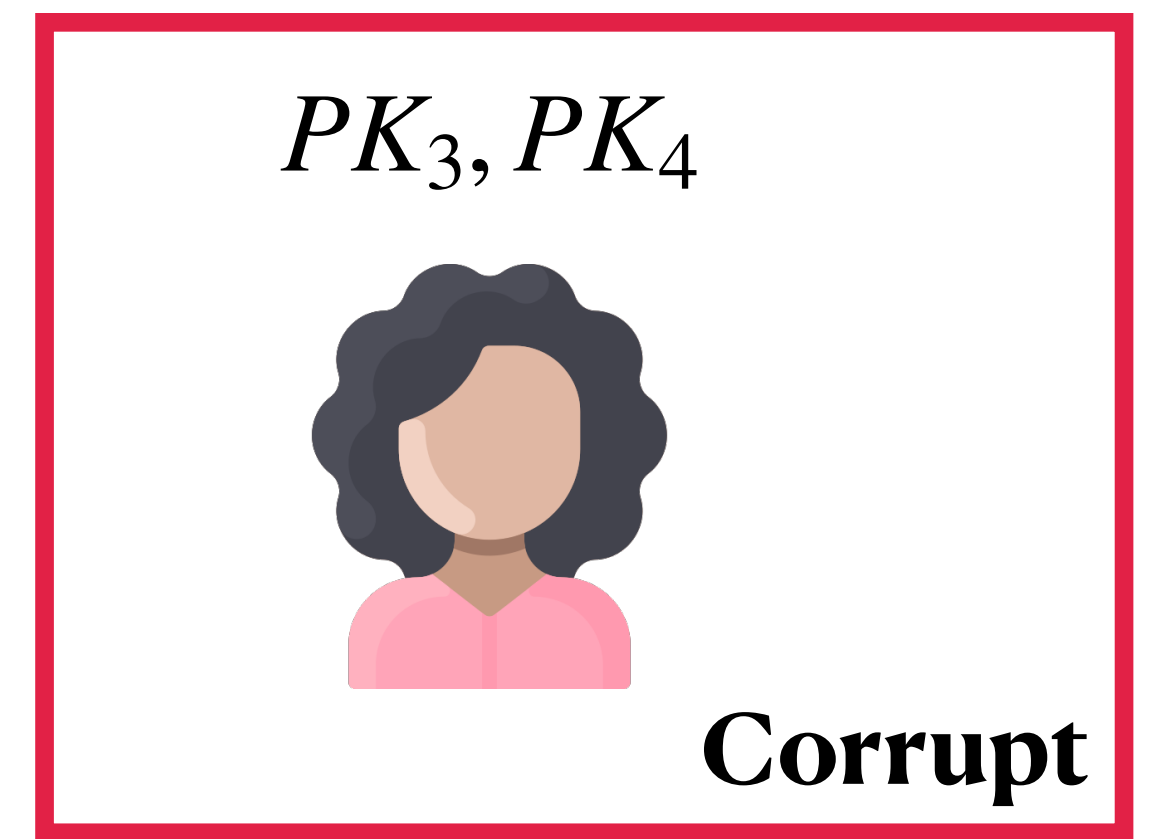


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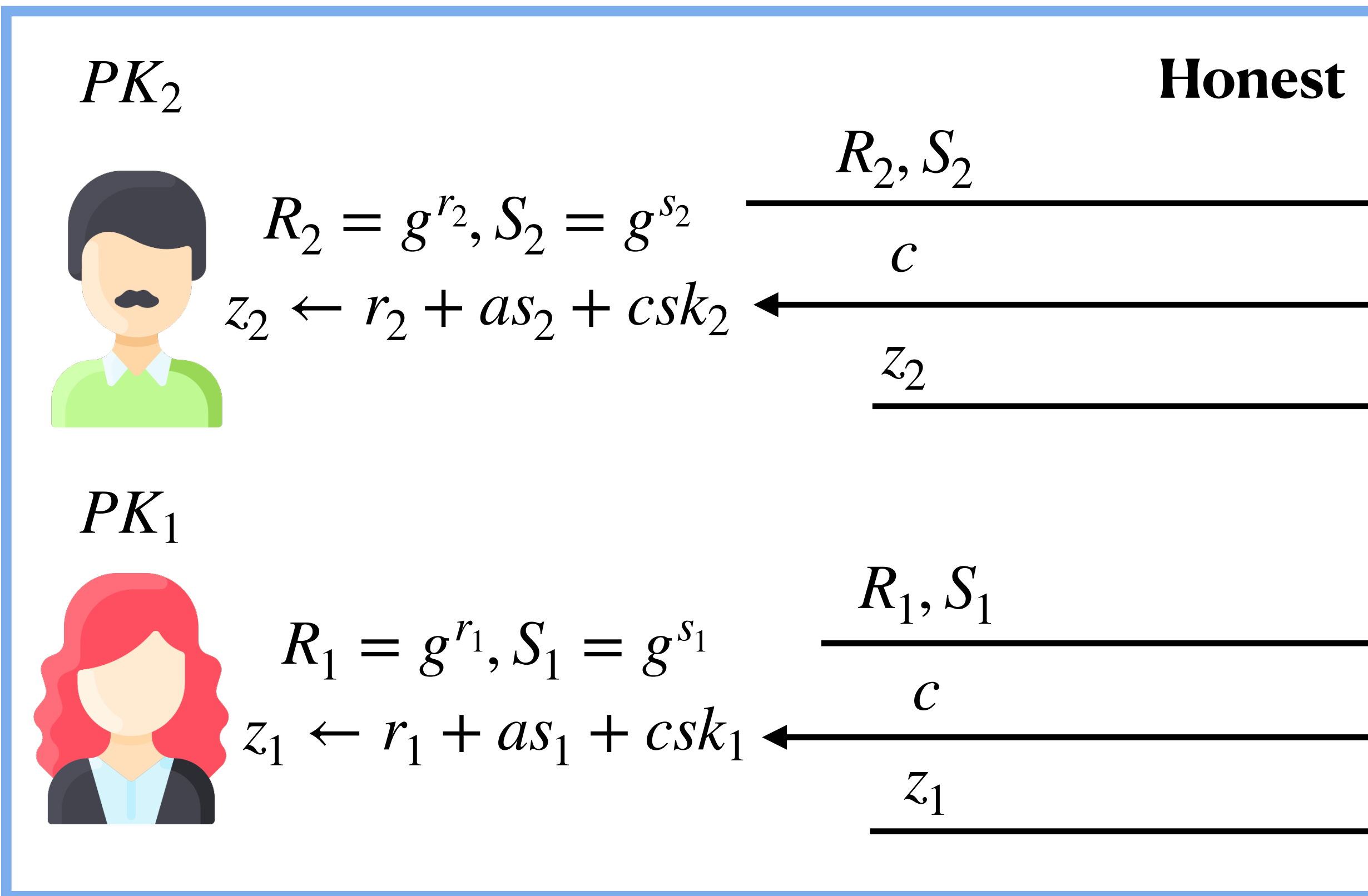
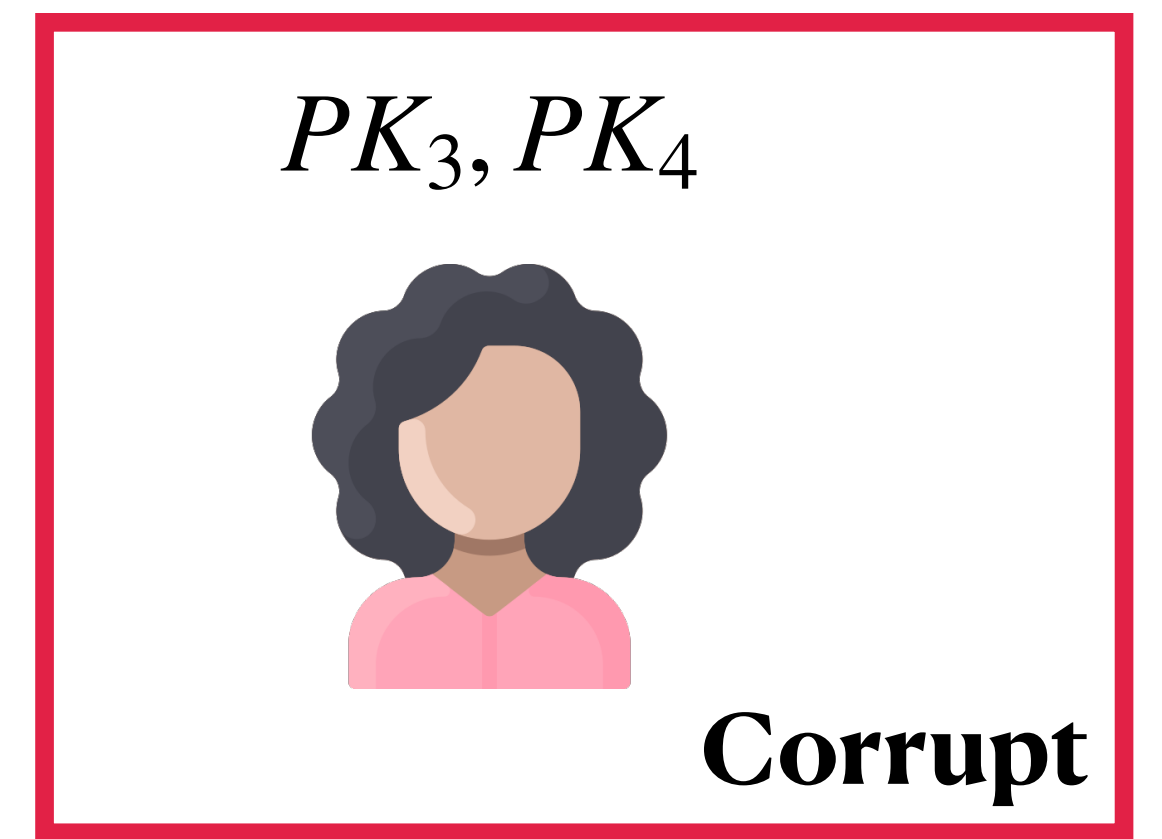


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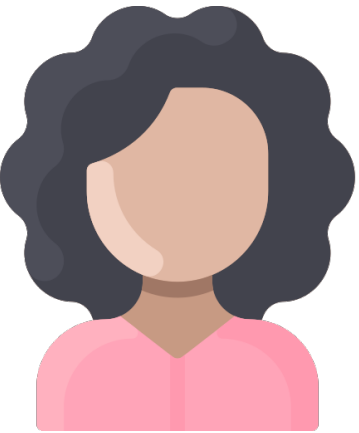
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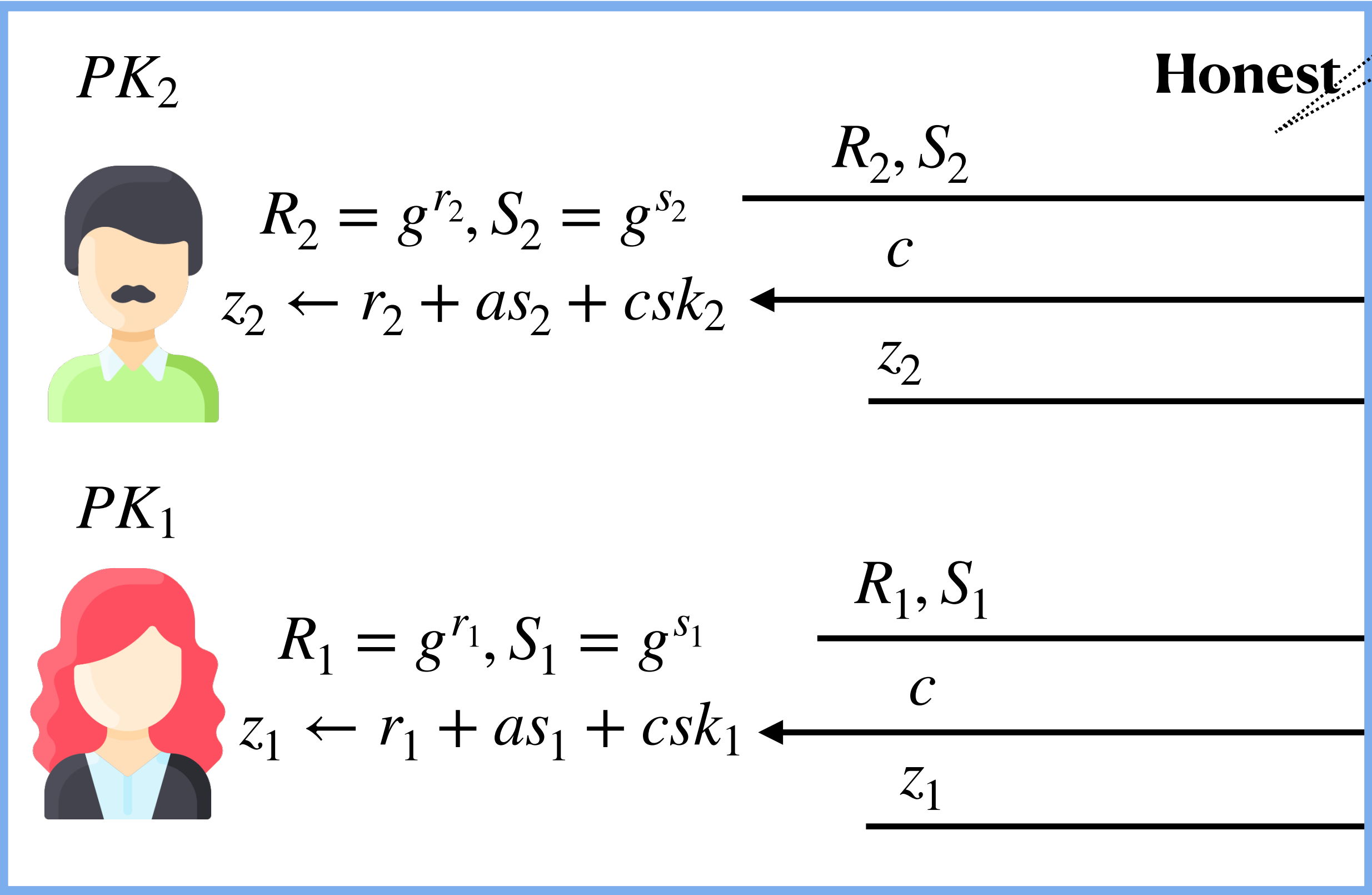
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Signers (1,2) think they are contributing to signing...

PK_3, PK_4


Corrupt



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$$R_3, S_3$$

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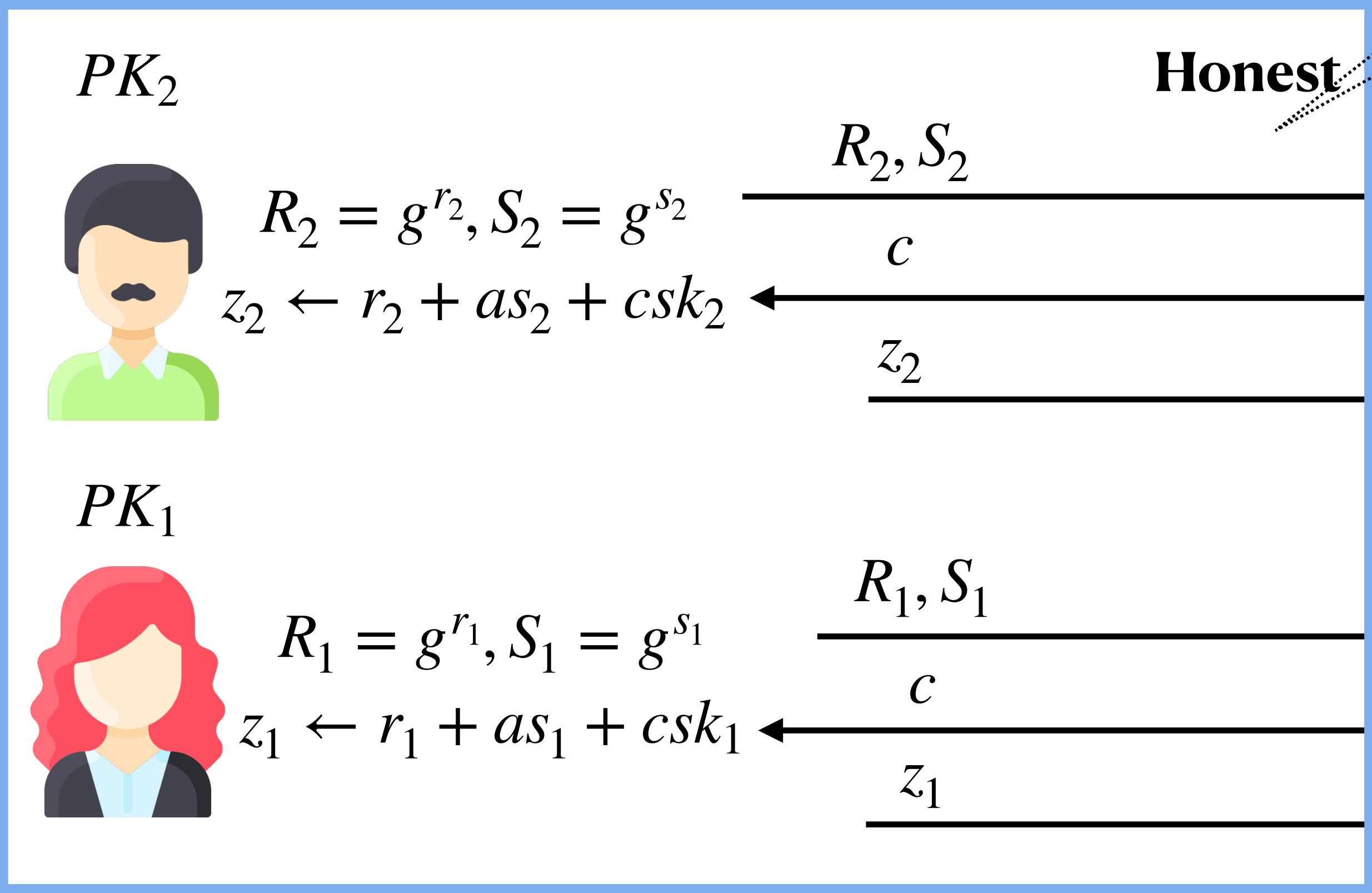
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But the signature represents contributions from only (1, 3, 4) !

Signing Set Malleability

- This is a new notion for threshold signatures - is it actually needed in practice?

Signing Set Malleability

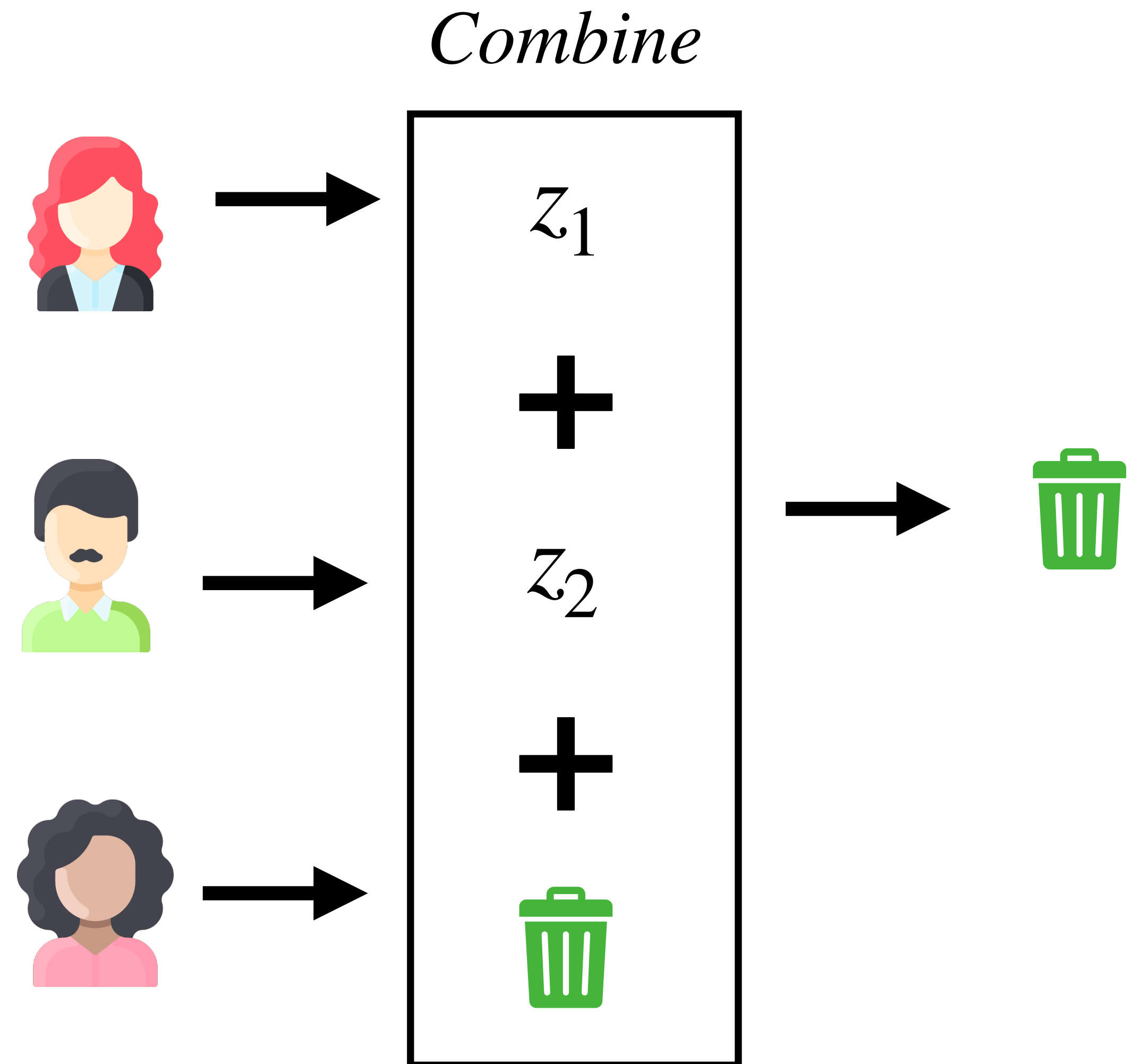
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Signing Set Malleability

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- IETF draft is now back to FROST1.
- FROST2 may be of interest for performance critical settings.

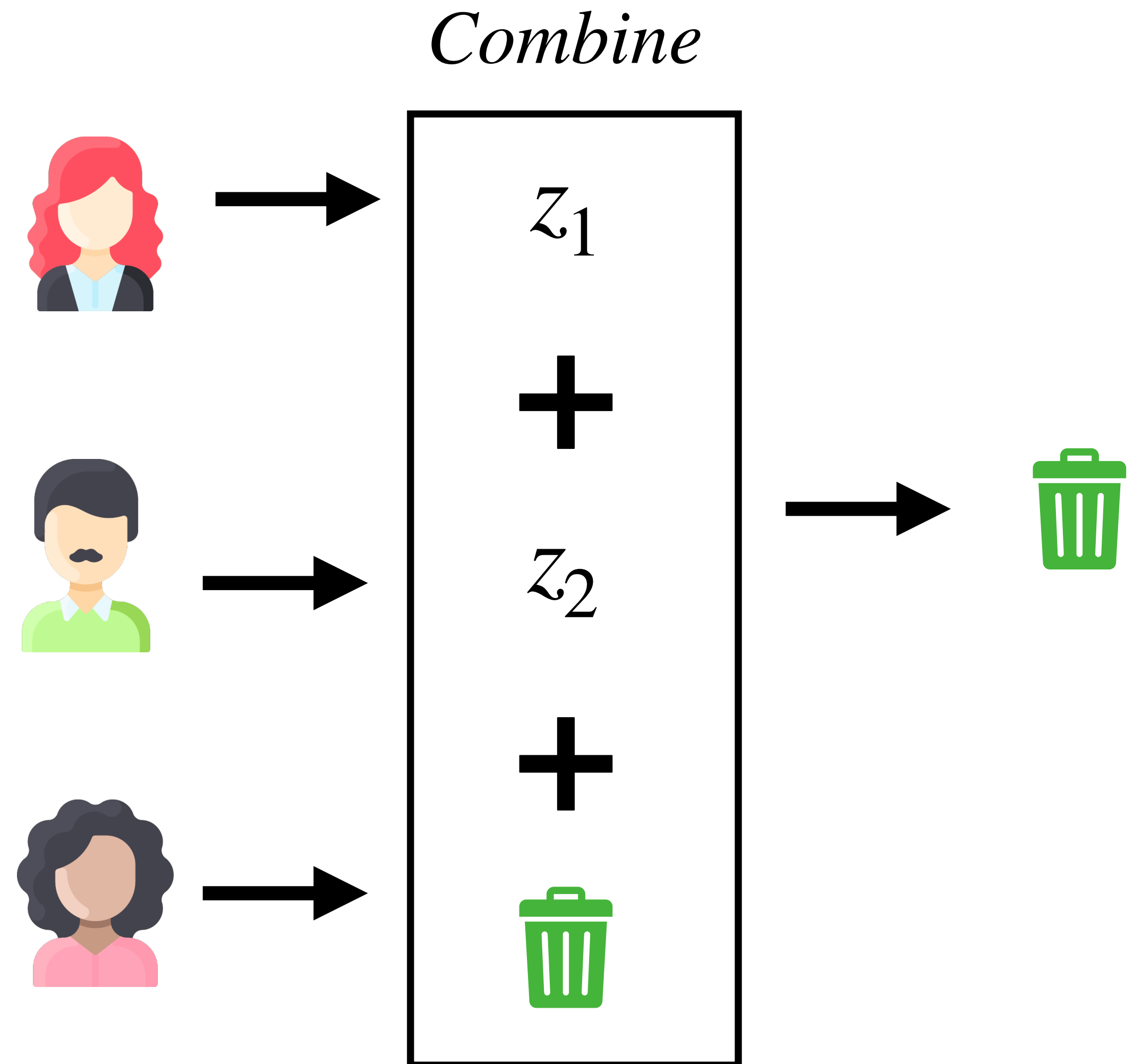
FROST & Robustness

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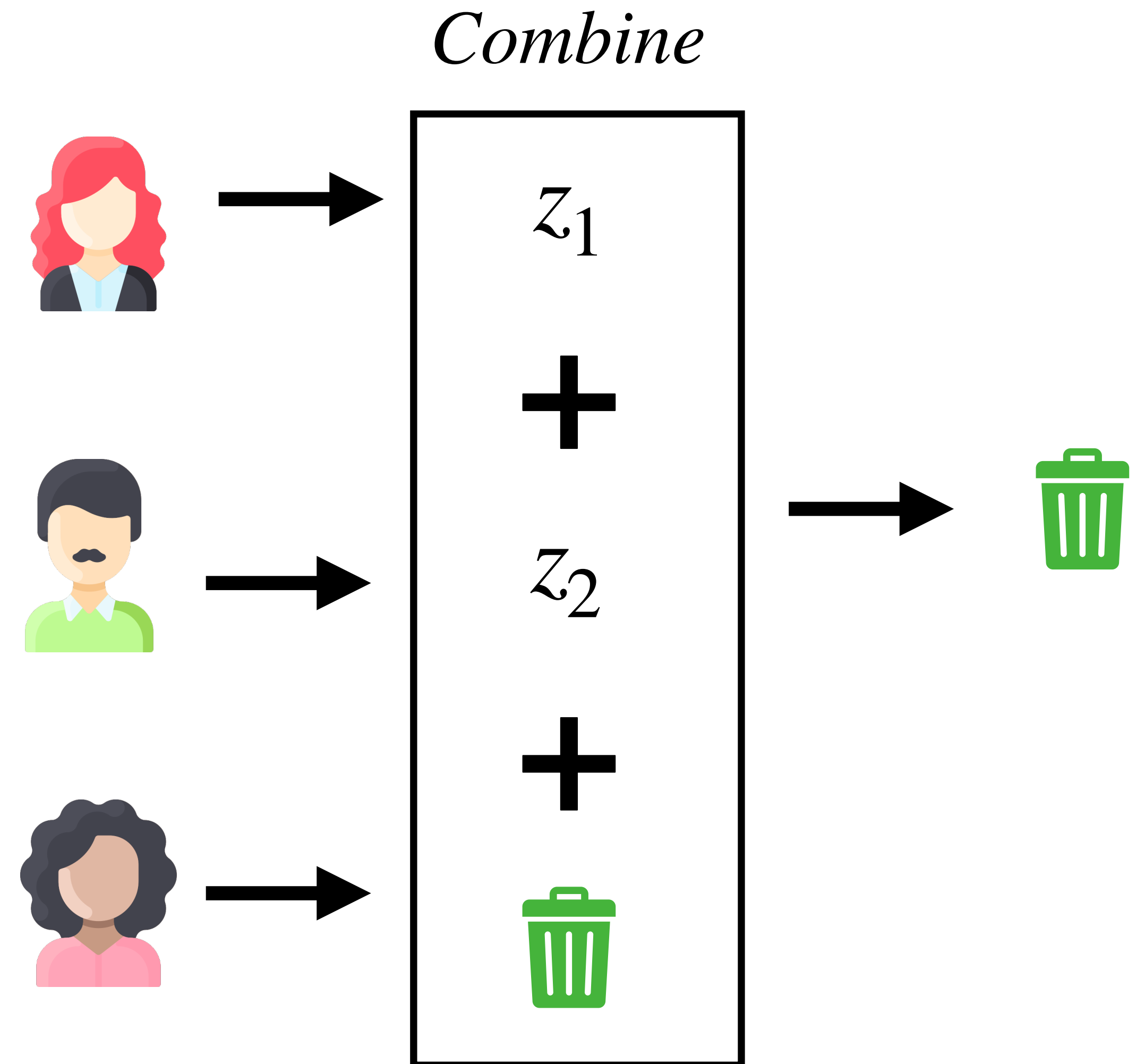
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- FROST is **not** robust.



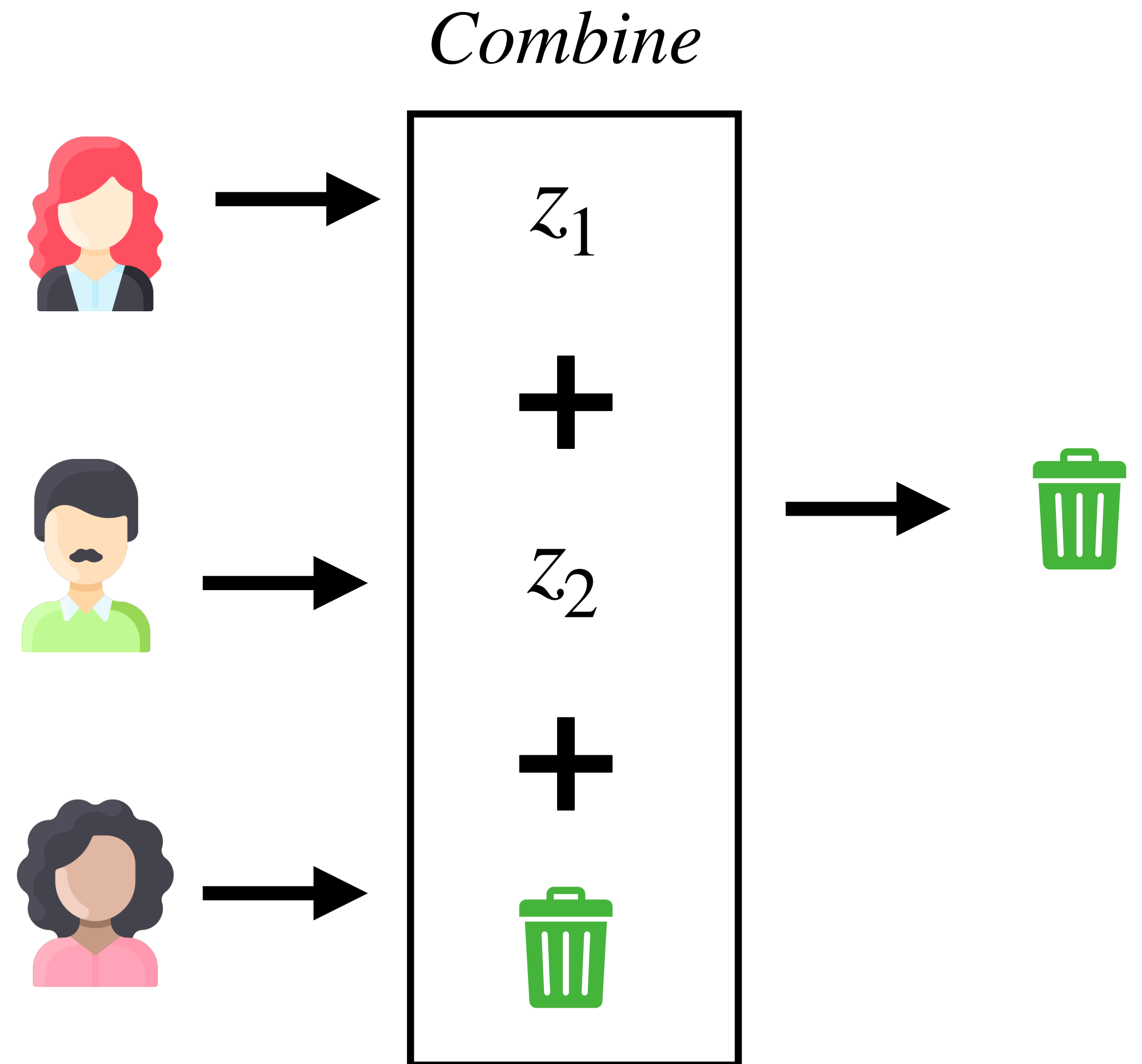
FROST & Robustness

- **Robustness:** the protocol succeeds so long as at least t players participate honestly.
- FROST is **not** robust.
- Example: $n=3$, $t=2$



FROST & Robustness

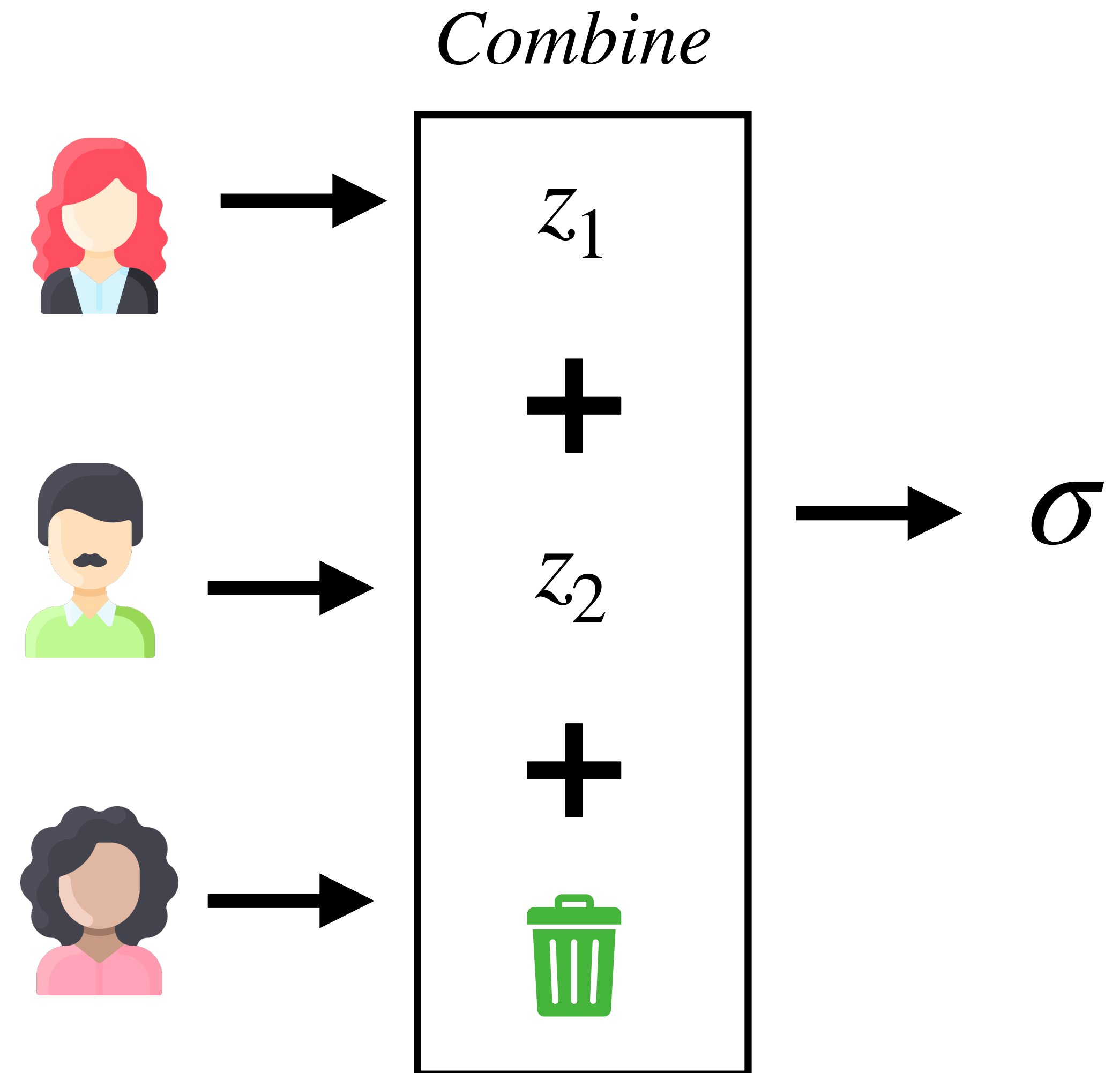
- **Robustness:** the protocol succeeds so long as at least t players participate honestly.
- FROST is **not** robust.
- Example: $n=3$, $t=2$
- If even one FROST signer issues garbage, the resulting signature is garbage and the protocol must be re-run (even if more than two signed).



“ROAST: Robust Asynchronous Schnorr Threshold Signatures”

Tim Ruffing, Viktoria Ronge, Elliott Jin, Jonas Schneider-Bensch, Dominique Schröder

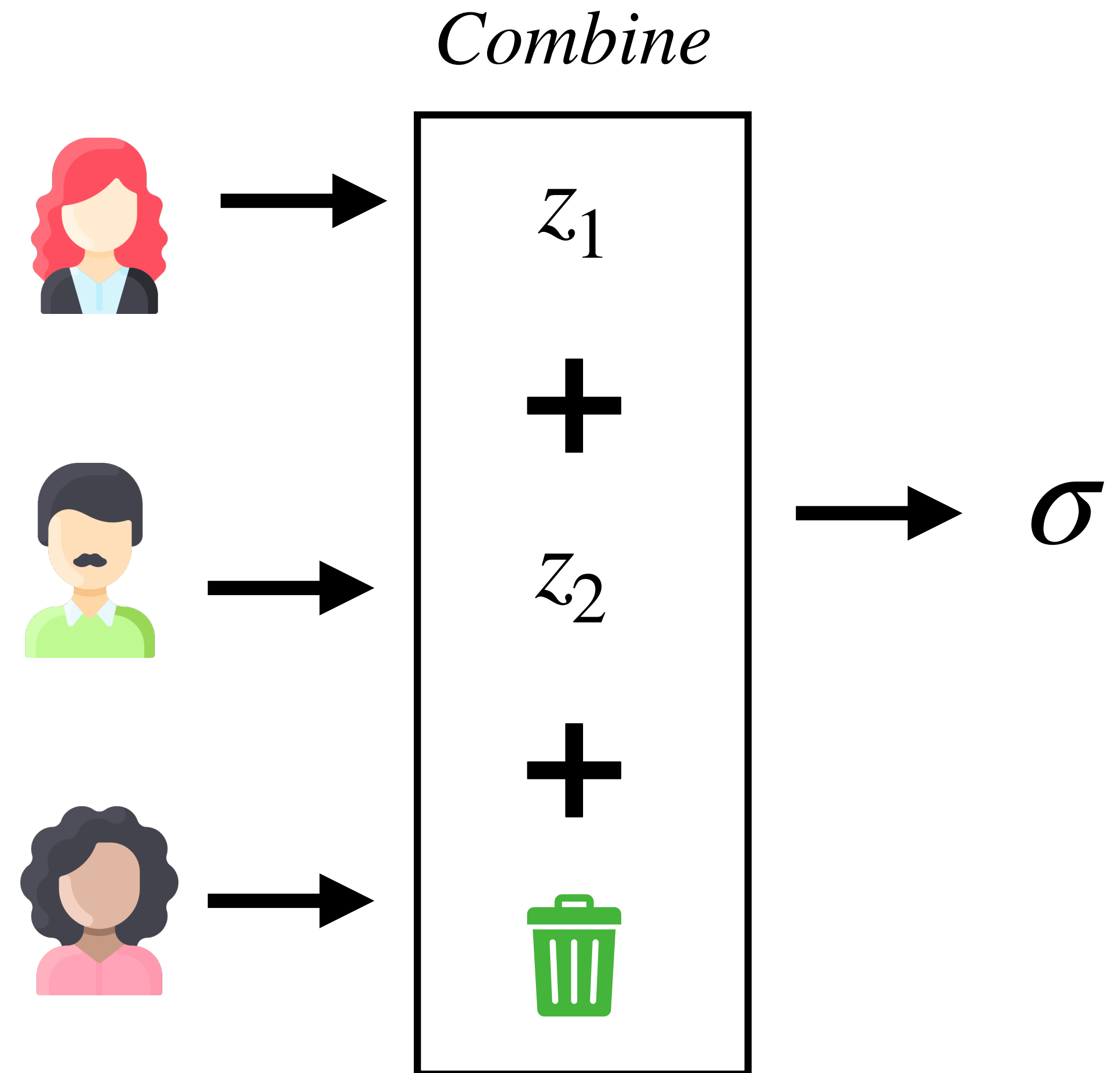
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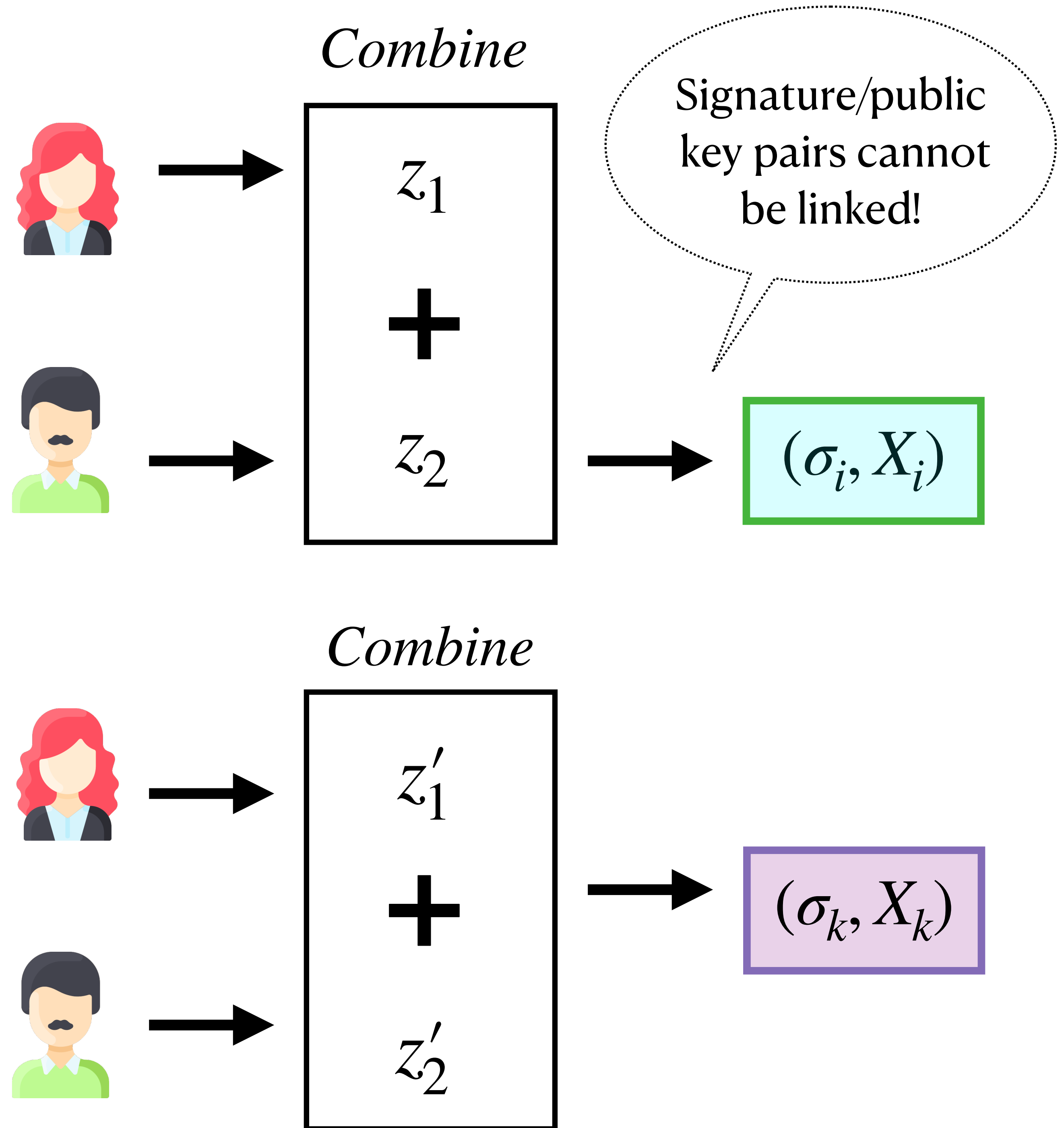
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- ROAST defines a wrapper protocol to make FROST robust.
- Improves on the trivial solution of maintaining $\binom{n}{t}$ concurrent sessions to $n - t + 1$



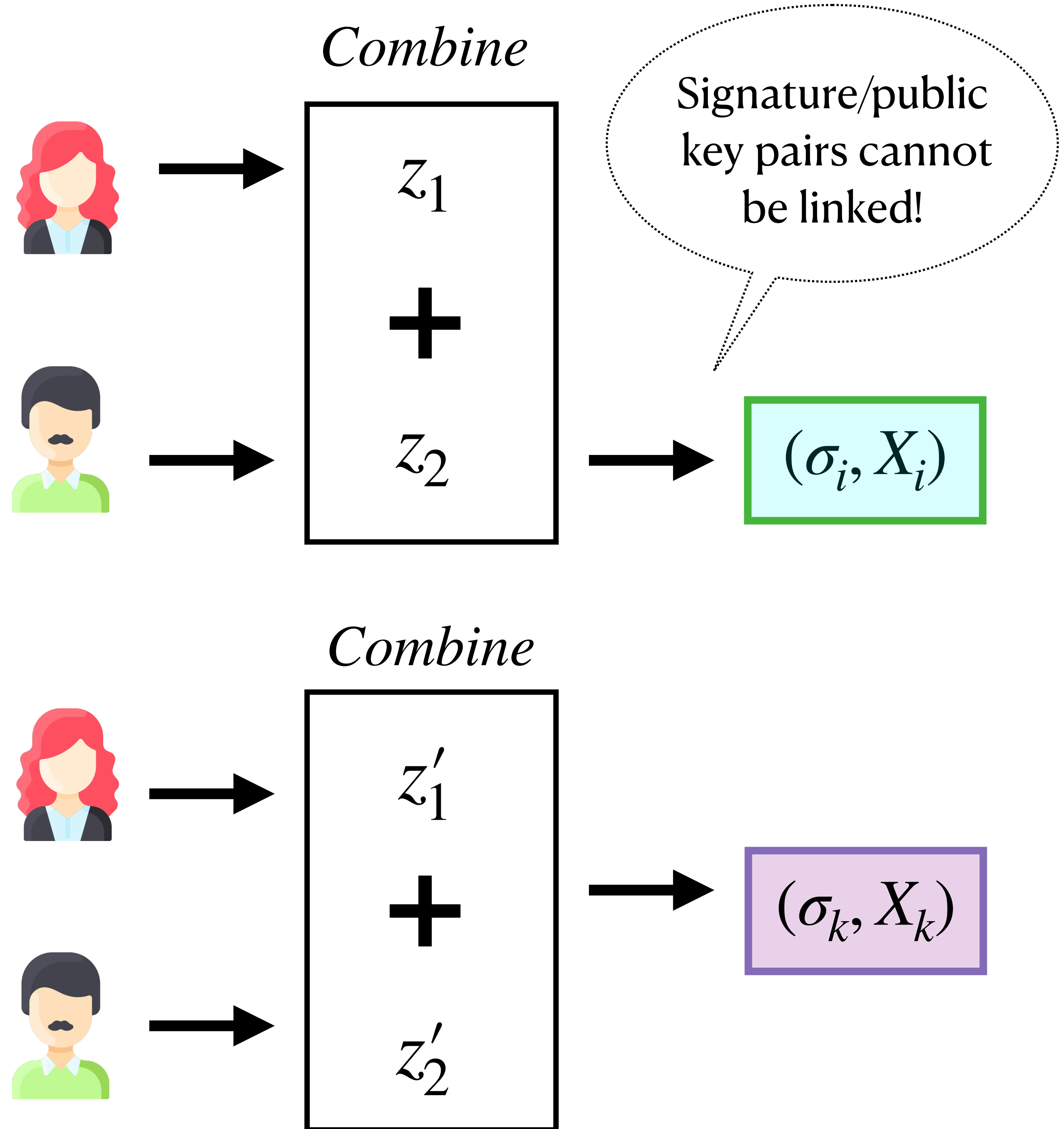
What's Next: Unlinkable FROST

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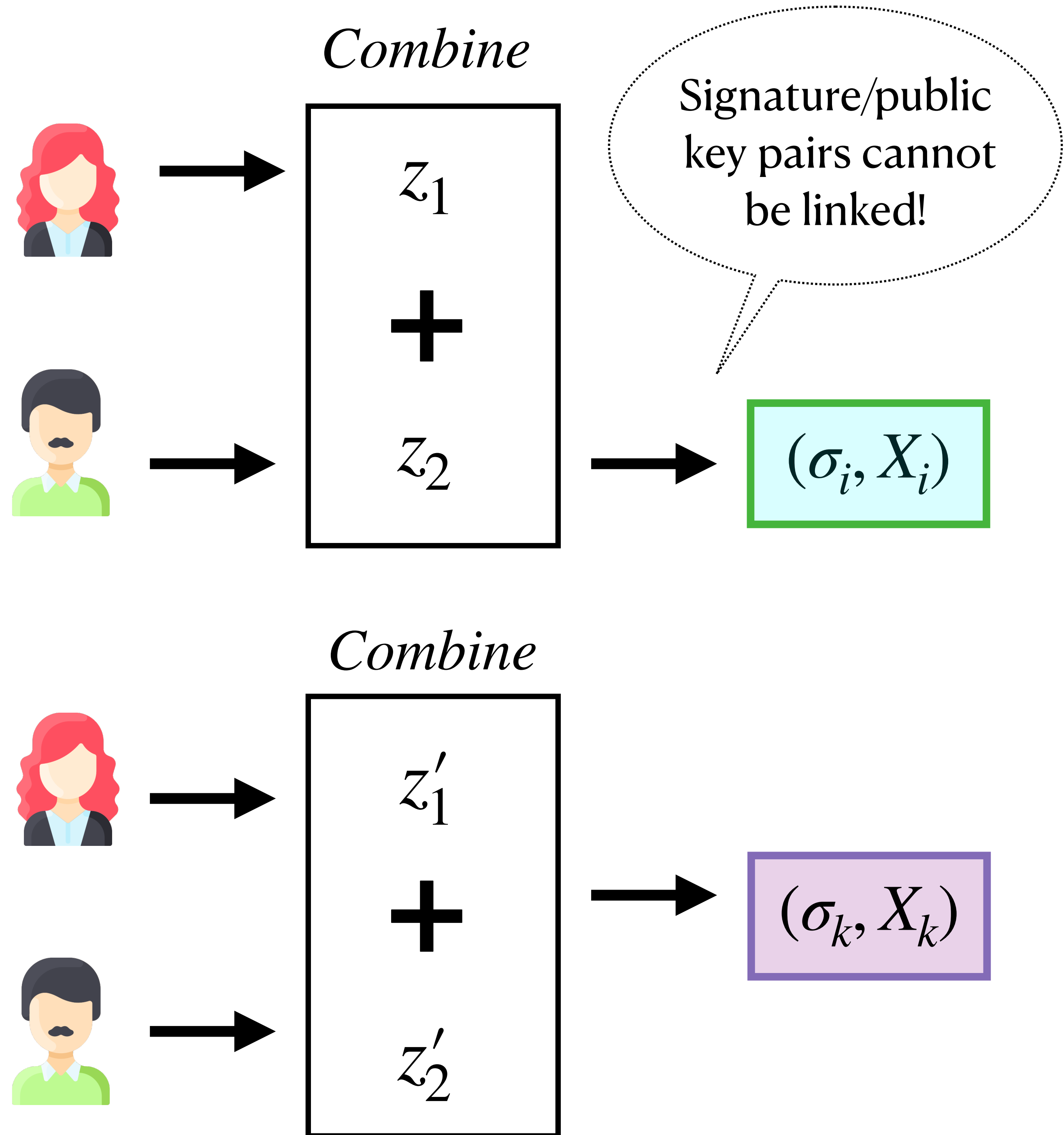
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- This is an ongoing effort, we have a candidate scheme
- We are investigating the various trust and privacy tradeoffs
- If you are interested in this work or have use cases, come talk to me!



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Thank you! ❄️