

# Threshold Signatures with Private Accountability

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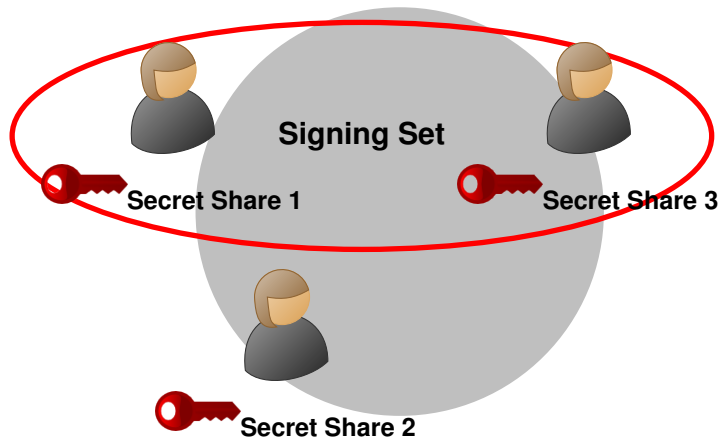
Stanford University

University of Waterloo

CRYPTO, August 13, 2022

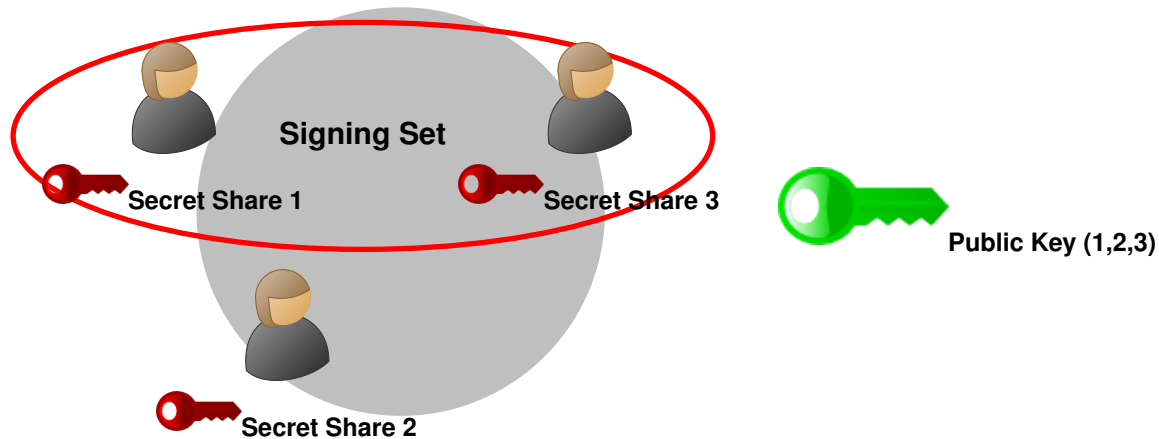
# Threshold Signatures: Joint Public Key, Secret-Shared Private Key

(2, 3) Example



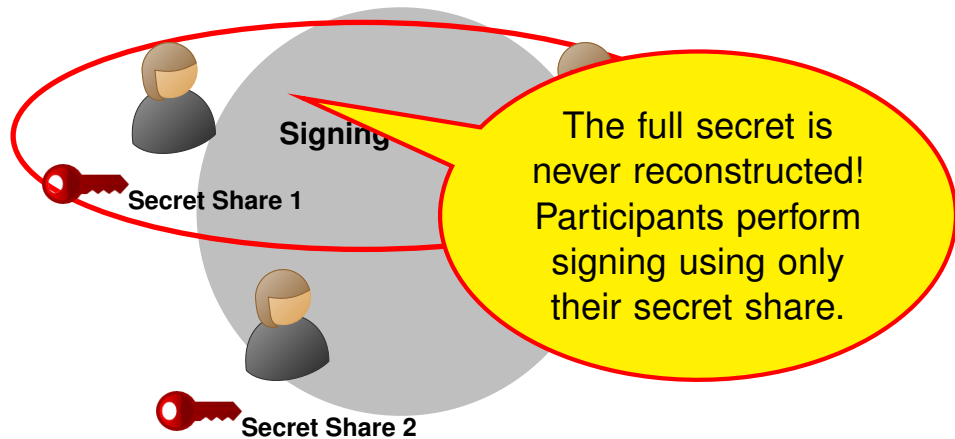
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- ▶ Can be constructed from a multisignature scheme such as MuSig2.

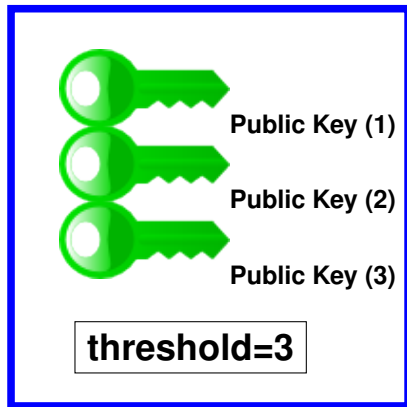
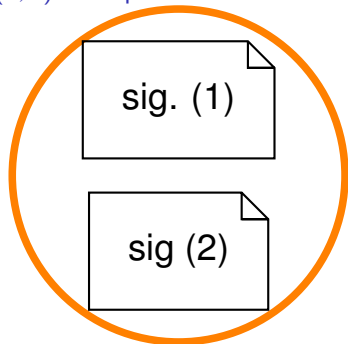
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# Trivial ATS

(2, 3) Example



Verification: Perform one single-party verification for each signer.

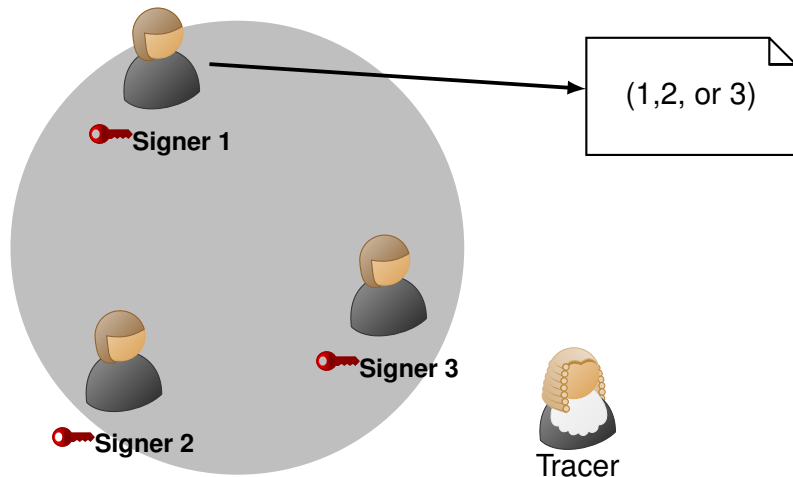
# TAPS

- ▶ Achieves *both* privacy and accountability
- ▶ Applications include:
  - ▶ Financial institutions- to prove or disprove issuance of funds.
  - ▶ Identification of  $t$  misbehaving entities (i.e, TLS servers).

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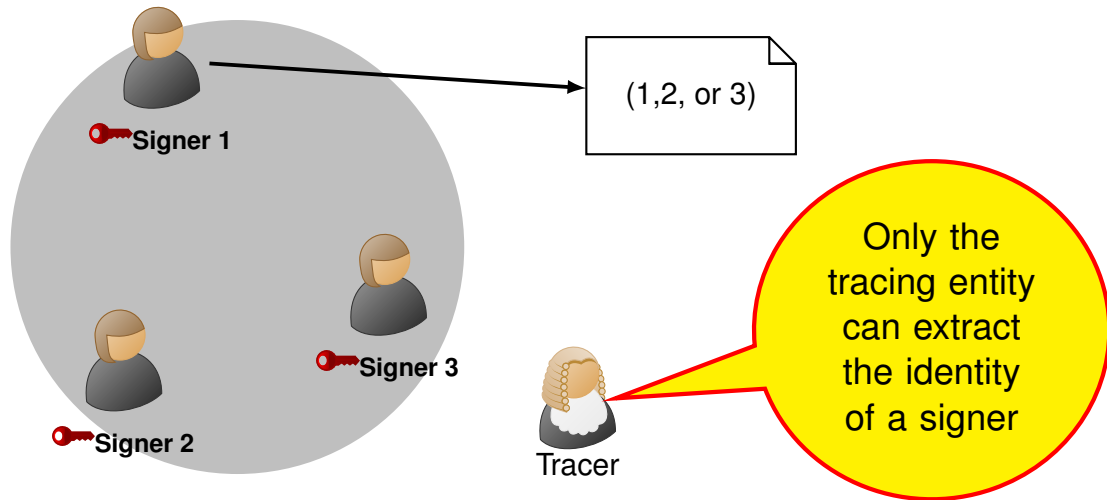
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# Group Signatures

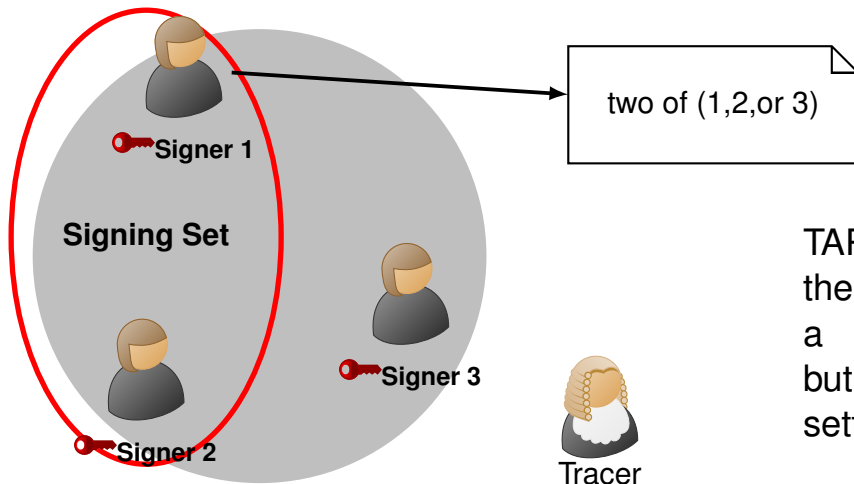


Group signatures prove that a member of a group signed, but not *which* member.

# Group Signatures

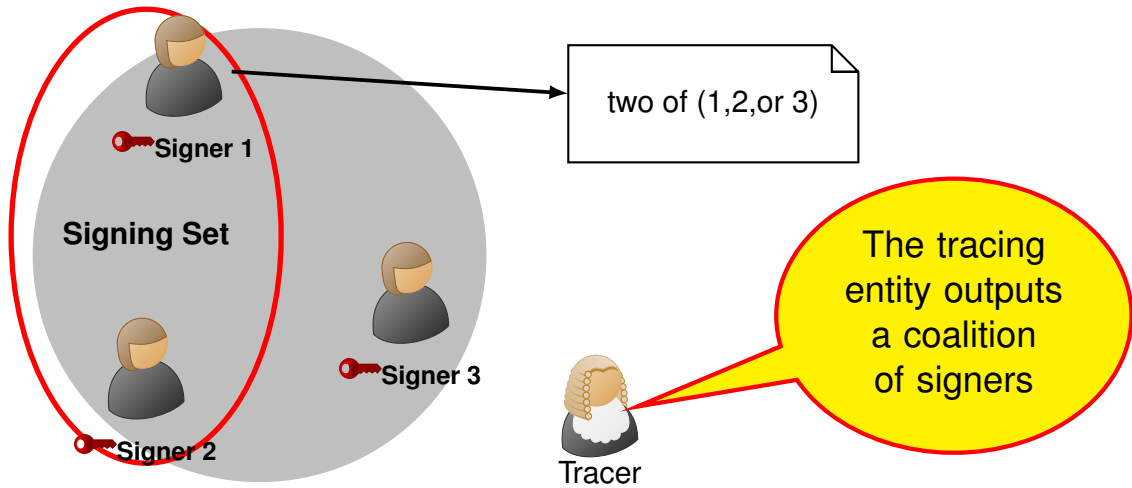


# TAPS: Another Perspective



TAPS generalizes the same notion of a group signature, but to a threshold setting.

# TAPS: Another Perspective



# TAPS

## Definition

A **private and accountable threshold signature** scheme, or **TAPS**, is a tuple of five polynomial time algorithms

$$S = (\textit{KeyGen}, \textit{Sign}, \textit{Combine}, \textit{Verify}, \textit{Trace})$$



# TAPS

$$\text{KeyGen}(1^\lambda, n, t) \rightarrow (pk, (sk_1, \dots, sk_n), sk_c, sk_t)$$

- ▶  $pk$ : The group's public key
- ▶  $(sk_1, \dots, sk_n)$ : Secret keys for each of the  $n$  participants.
- ▶  $sk_c$ : Secret key for the combiner
- ▶  $sk_t$ : Secret key for the tracer

# TAPS

$$\text{Sign}(sk_i, m, C) \rightarrow \delta_i$$

- ▶  $m$ : Message to be signed
- ▶  $C$ : Coalition of signers
- ▶  $\delta_i$ : Partial signature for participant  $i$

$$\text{Combine}(sk_c, m, C, \{\delta_i\}_{i \in C}) \rightarrow \sigma$$

- ▶ Outputs  $\sigma$ , a TAPS signature

# TAPS

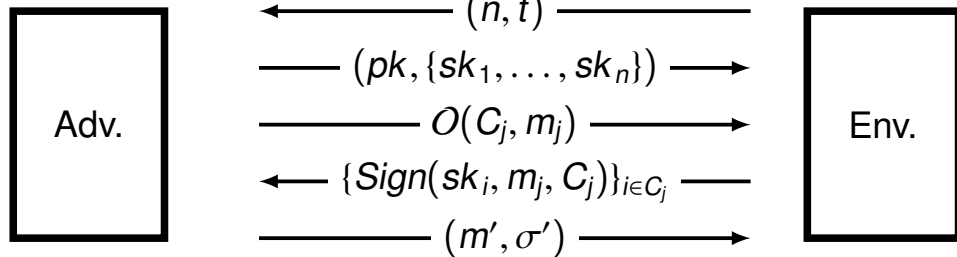
$Verify(pk, m, \sigma) \rightarrow 0/1$ :

- ▶ Outputs a bit indicating if  $\sigma$  is valid for  $pk, m$

$Trace(sk_t, m, \sigma) \rightarrow C/fail$ :

- ▶ Outputs either the coalition of signers or fails.

# Unforgeability and Accountability



Adv wins if:

- (1) It produces a valid signature and controls fewer than  $t$  parties (**unforgeability**)
- (2) It controls more than  $t$  parties, and outputs a valid signature that traces to an honest non-signer (**accountability**)

---

TAPS is *unforgeable* and *accountable* if  $\Pr[\text{Adv wins}]$  is negligible.

# Privacy

- ▶ Privacy against the public
- ▶ Privacy against (non)-signers
- ▶ Privacy against other signers (which we don't consider)

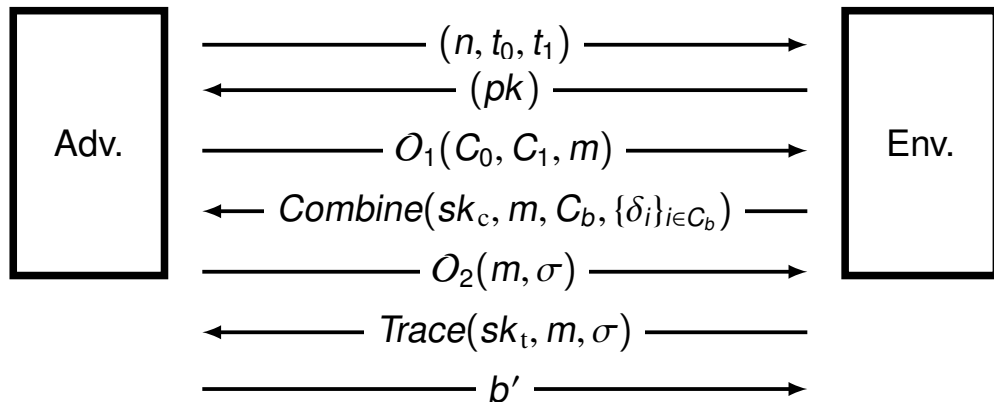
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# Privacy Against The Public



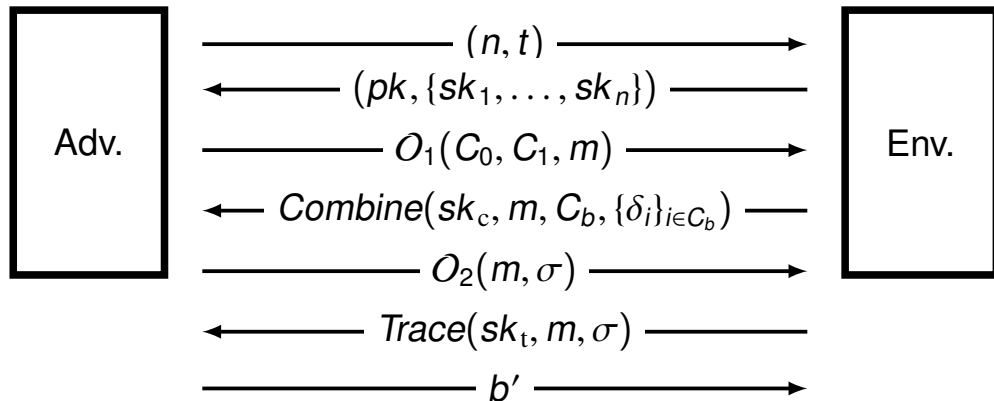
Restriction: Inputs to  $O_2$  cannot be outputs from  $O_1$ .

Adv wins if it can gain information about  $t$  or the set of signers.

TAPS is *private against the public* if  $\Pr[\text{Adv wins}]$  is negligible.



# Privacy Against Signers



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TAPS is *private against signers* if  $\Pr[\text{Adv wins}]$  is negligible.

# Generic TAPS

Instantiated with:

- ▶ An ATS
- ▶ A public-key encryption scheme
- ▶ A commitment scheme
- ▶ A signature scheme
- ▶ A non-interactive zero-knowledge argument of knowledge

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- ▶ Requires a Schnorr ATS, such as MuSig2
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# Performance

	Public Key Size		Signature Size	
	$\mathbb{G}$	$\mathbb{Z}_q$	$\mathbb{G}$	$\mathbb{Z}_q$
Sigma	$2n + 4$	0	$n + 4$	$2n + 5$
Bulletproofs	$n + \frac{n}{e} + O(1)$	0	$\frac{n}{e} + O(\log n)$	4

- ▶ Bulletproofs TAPS is shorter by a factor of about  $e$ .

# Performance

	Verify Time	Trace Time
Sigma	$O(n)$	$O(n)$
Bulletproofs	$O(n)$	$O(n \cdot 2^{e/2})$

- Measured in number of group operations.

# Takeaways

- ▶ TAPS are a new type of threshold signature with both privacy and accountability.
- ▶ We define a generic construction that employs an ATS and other standard building blocks.
- ▶ We then define a Schnorr construction with both sigma and bulletproofs as the zero-knowledge argument system.

Thank You!

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<https://eprint.iacr.org/2022/TODO>

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