Accountability and Privacy for Threshold Signatures

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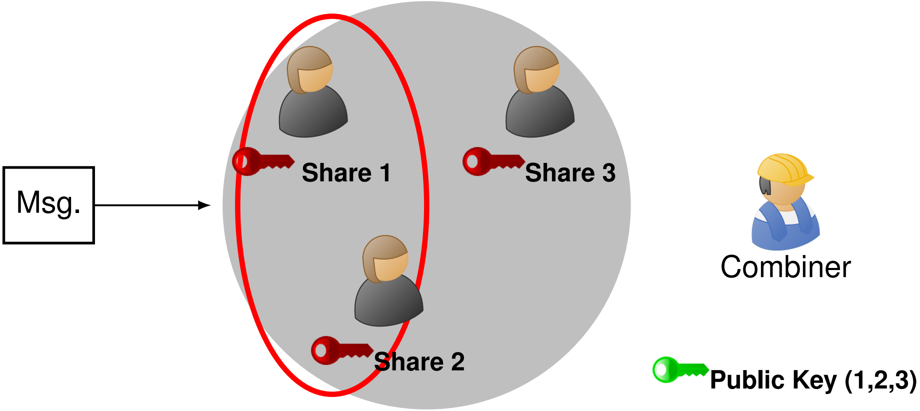
Threshold Signatures: Joint Public Key, Secret-Shared Private Key

$(t = 2, n = 3)$ Example



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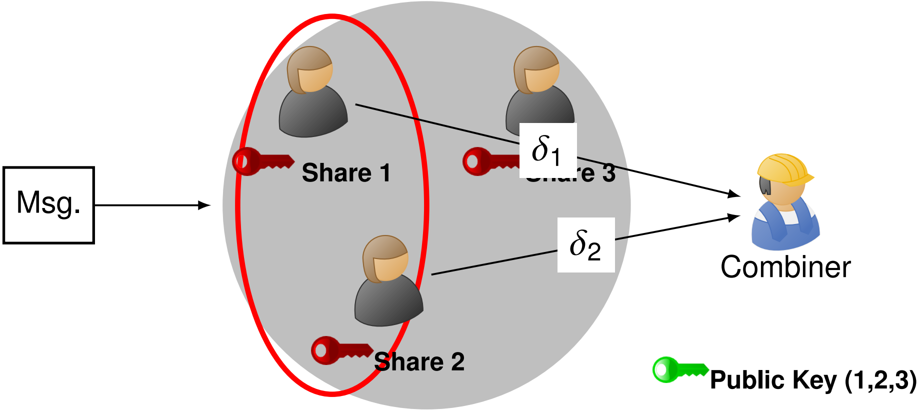

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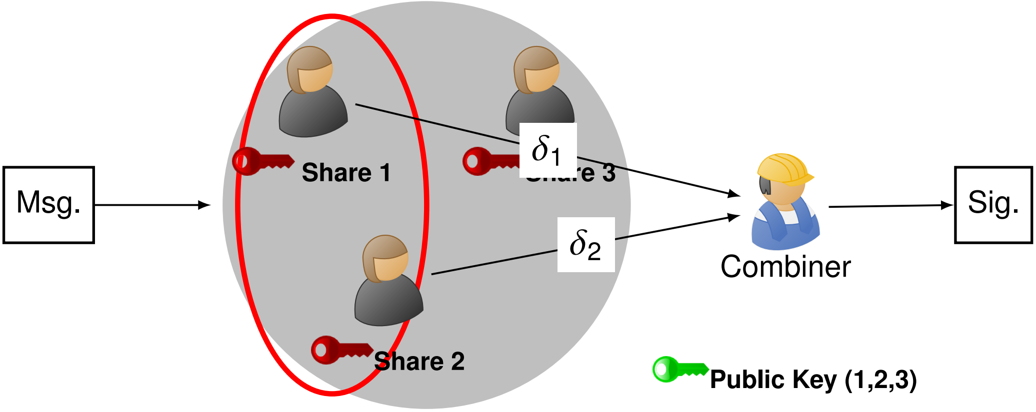

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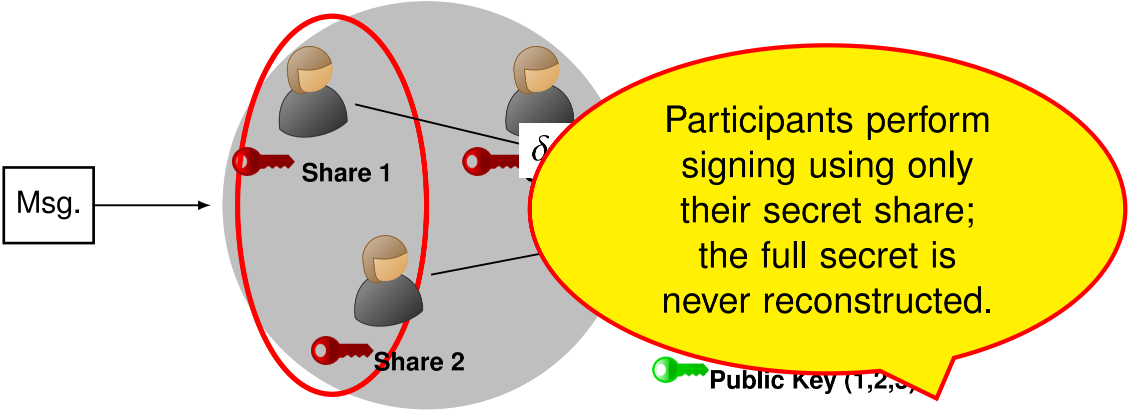

Existing Threshold Schemes

- In the literature, two types of (mutually exclusive) threshold schemes.
- **Full Privacy**: Private Threshold Scheme (PTS)
- **Full Accountability**: Accountable Threshold Scheme (ATS).
  - Also known as an Accountable Subgroup Multisignatures

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Definition

A **private threshold signature** scheme, or **PTS**, is a tuple of four polynomial time algorithms

$$\mathcal{S} = (\mathit{KeyGen}, \mathit{Sign}, \mathit{Combine}, \mathit{Verify})$$

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An **accountable threshold signature** scheme, or **ATS**, is a tuple of five polynomial time algorithms

$$\mathcal{S} = (\mathit{KeyGen}, \mathit{Sign}, \mathit{Combine}, \mathit{Verify}, \mathit{Trace})$$

Such that $\mathit{Trace}(pk, m, \sigma) \rightarrow C \subseteq \{1, \ldots, n\}$

PTS versus ATS

	who learns signer quorum		**who learns threshold**	
	public	signers	public	signers
PTS	X	X	X	✓
ATS	✓	✓	✓	✓

Segway: Schnorr PTS/ATS Schemes

Let's see how existing multi-party Schnorr signature schemes meet these PTS/ATS notions.

Single-Party Schnorr Signing and Verification

Signer		**Verifier**
$(x, Y) \leftarrow KeyGen()$		
	$\xleftarrow{(m, Y)}$	
$k \xleftarrow{\$} \mathbb{Z}_q$ $R = g^k \in \mathbb{G}$ $c = H(R, Y, m)$ $z = k + c \cdot x$		
	$\xrightarrow{(m, \sigma = (R, z))}$	
		$c = H(R, Y, m)$ $R' = g^z \cdot Y^{-c}$ Output $R \stackrel{?}{=} R'$

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Example PTS: FROST

- Threshold signatures that rely on *polynomial interpolation* (i.e, Shamir's secret sharing) are inherently a PTS.

$$z = \sum_{i \in S} z_i = r + c \cdot \sum_{i \in S} \lambda_i \cdot sk_i$$

- Shamir's secret sharing: each $(i, sk_i), i \in \{1, \ldots, n\}$ is a point on a secret polynomial f, where $(0, f(0))$ is sk. So:

$$sk = \sum_{i \in S} \lambda_i \cdot sk_i$$

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TAPS: Privacy and Accountability

- We introduce a *new* type of threshold signature.
- **Private *and* Accountable Threshold Signatures** (TAPS)
- Like a PTS, the public does not learn 1) the threshold, or 2) the signing quorum
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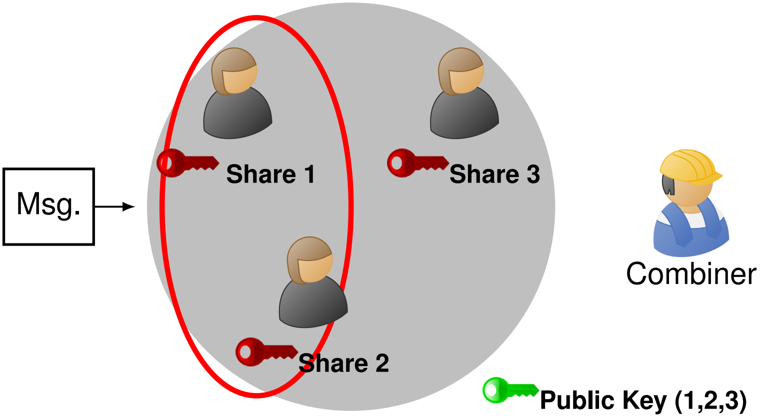

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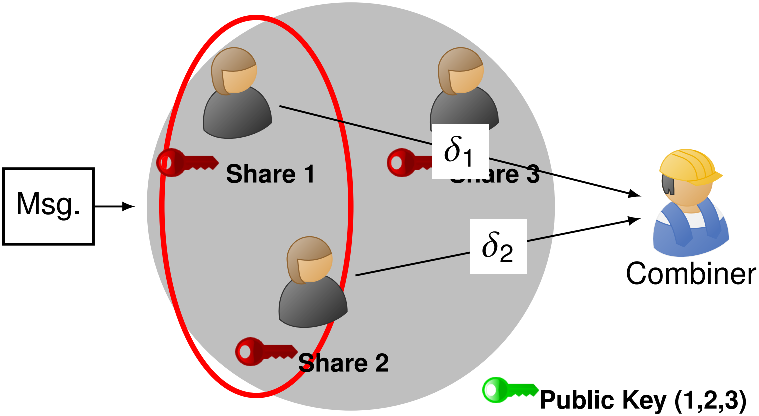

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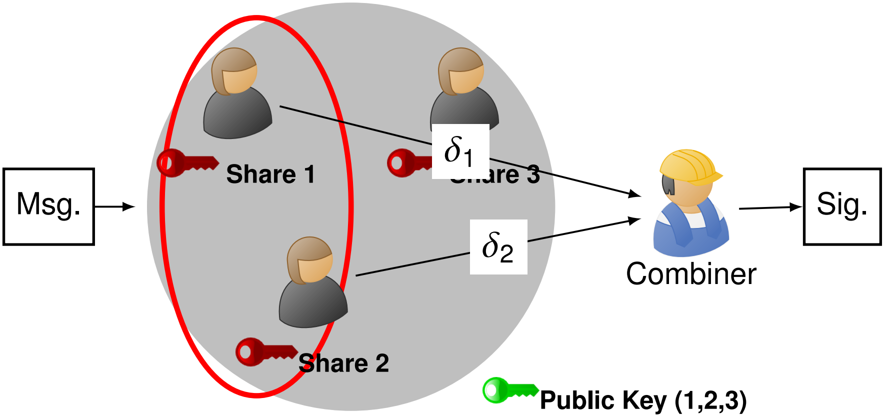

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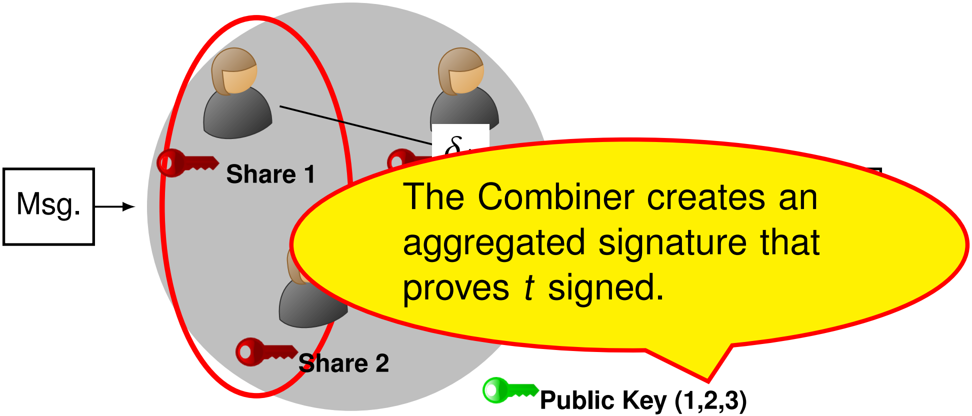

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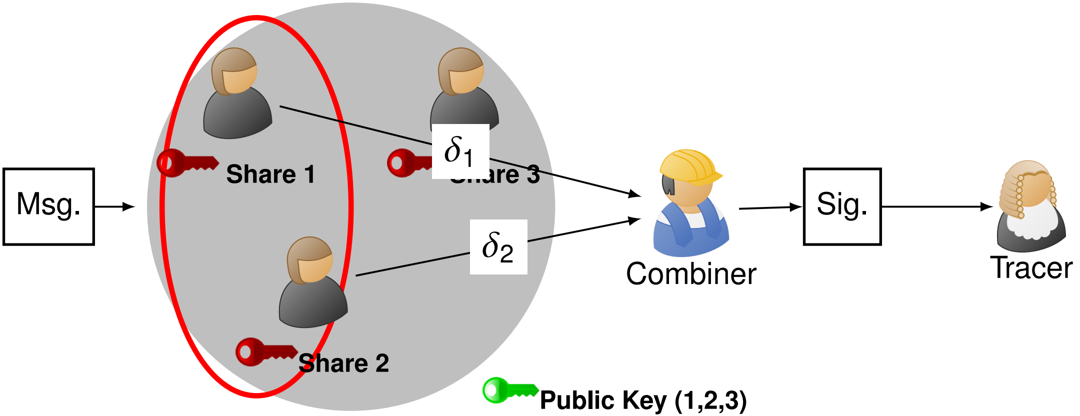

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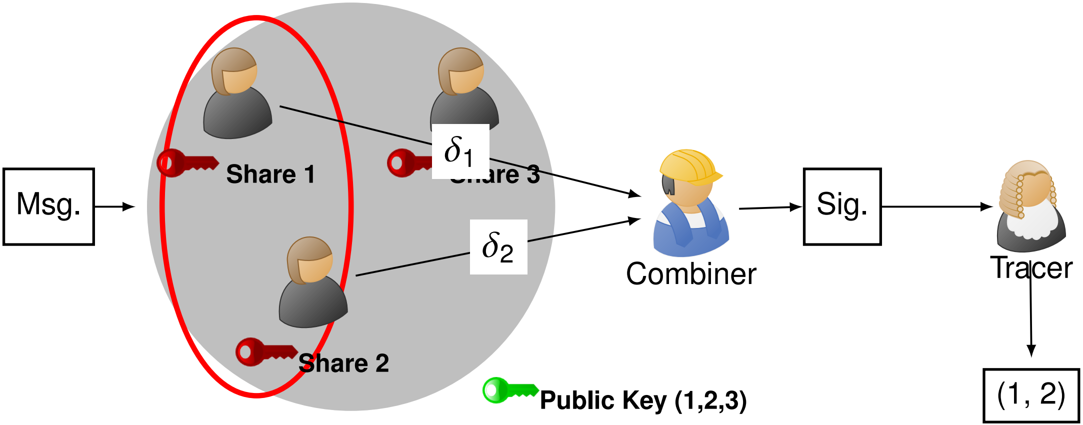

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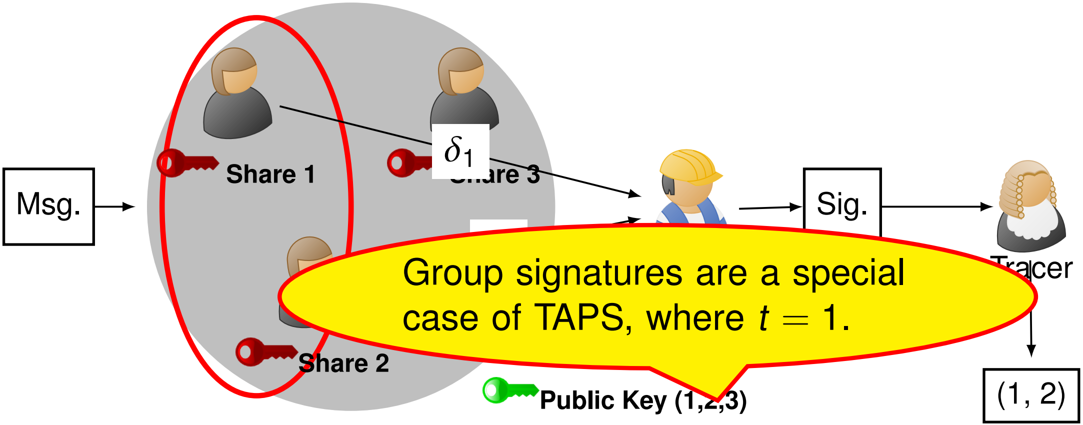

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$$\mathcal{S} = (\mathit{KeyGen}, \mathit{Sign}, \mathit{Combine}, \mathit{Verify}, \mathit{Trace})$$

Where *Trace* can be performed only by a designated entity.

A TAPS must be secure (unforgeable), private, and accountable.

TAPS

$$KeyGen(1^\lambda, n, t) \rightarrow \left(pk, (sk_1, \ldots, sk_n), sk_c, sk_t\right)$$

- pk: The group's public key
- $(sk_1, \ldots, sk_n)$: Secret keys for each of the n participants.
- sk_c: Secret key for the combiner
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- Could be performed in a distributed manner.

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$Sign(sk_i, m, C) \rightarrow \delta_i$

- m: Message to be signed
- C: Coalition of signers
- δ_i: Partial signature for participant i

$Combine(sk_c, m, C, \{\delta_i\}_{i \in C}) \rightarrow \sigma$

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$Verify(pk, m, \sigma) \rightarrow 0/1$:

- Outputs a bit indicating if σ is valid for pk, m

$Trace(sk_t, m, \sigma) \rightarrow C/\text{fail}$:

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Unforgeability and Accountability

- **Unforgeability.** Adversary cannot output a valid signature when controlling fewer than t parties.
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Adv.

$\xrightarrow{(n, t, C)}$

$\xleftarrow{(pk, \{sk_i\}_{i \in C})}$

$\xrightarrow{\mathcal{O}(C_j, m_j)}$

$\xleftarrow{\{Sign(sk_i, m_j, C_j)\}_{i \in C_j}}$

$\xrightarrow{(m', \sigma')}$

Env.

$(pk, \{sk_i\}_{i \in [n]}, sk_{\mathrm{c}}, pk_{\mathrm{t}}) \xleftarrow{\$} KeyGen(1^\lambda, n, t)$

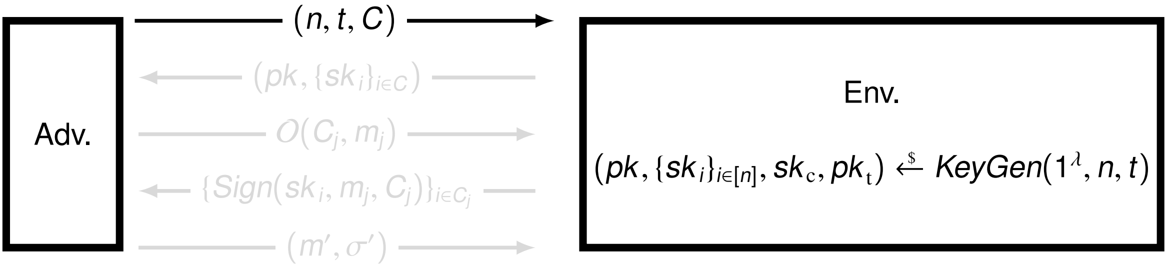
Adv wins if:

(1) It produces a valid signature and $|C| < t$ **(unforgeability)**, or

(2) It outputs a valid signature that traces to an honest non-signer $i \notin C$ **(accountability)**.

TAPS is *unforgable* and *accountable* if Pr[Adv wins] is negligible.

Unforgeability and Accountability



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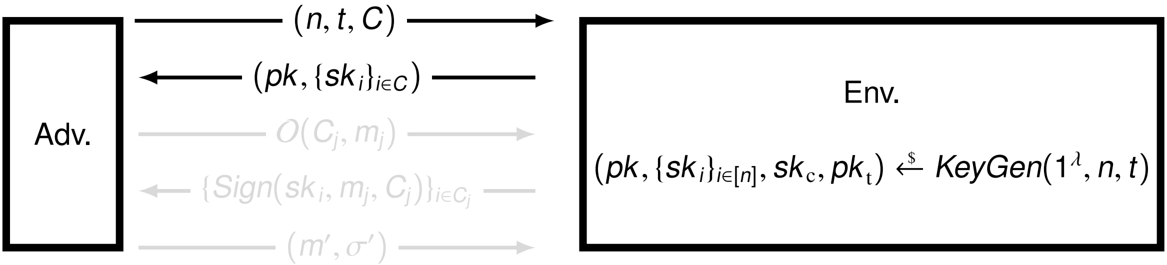

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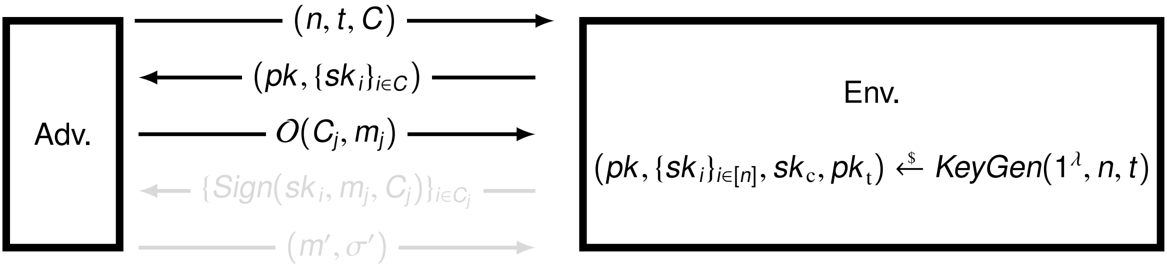

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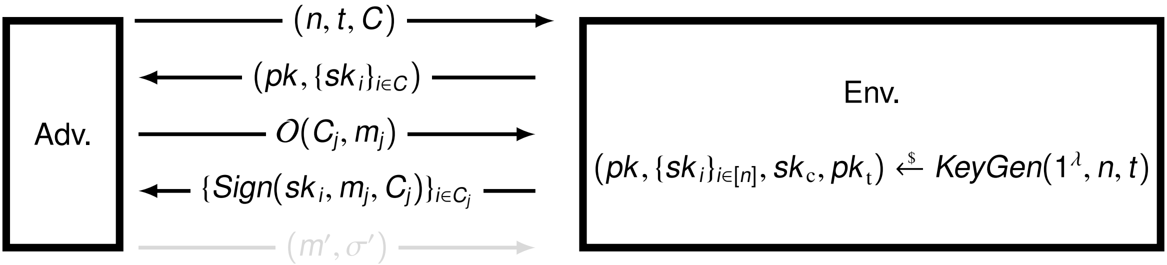

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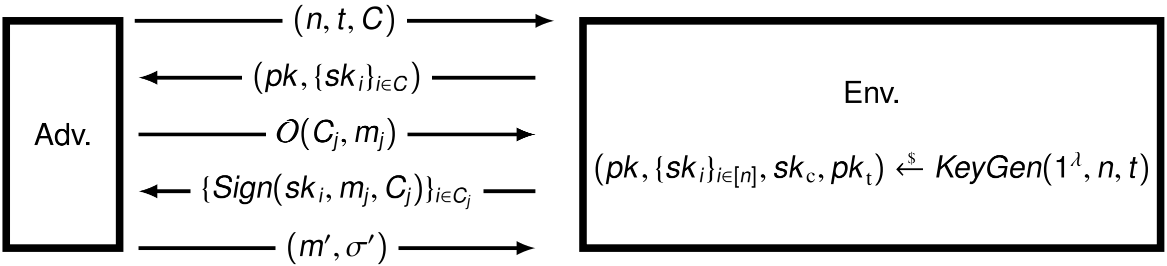
Unforgeability and Accountability



Adv wins if:

(1) It produces a valid signature and $|C| < t$ **(unforgeability)**, or

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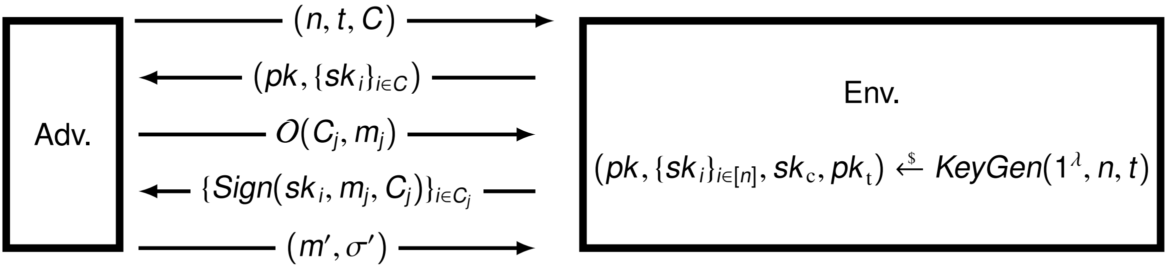

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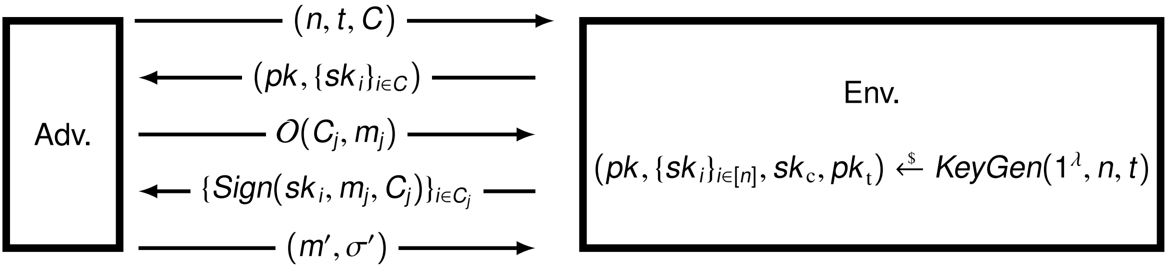

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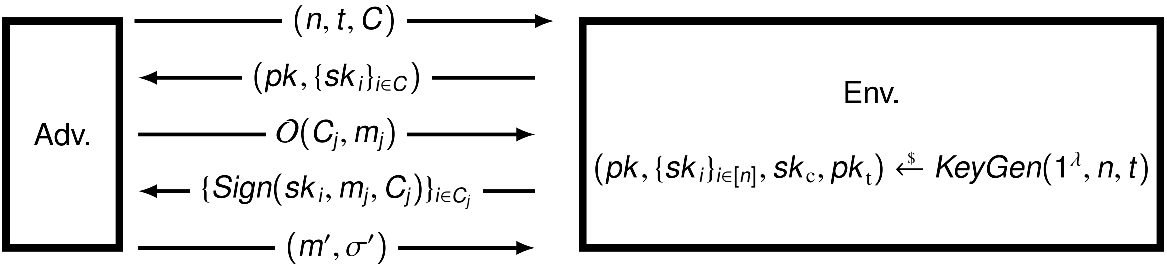

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Privacy

- **Privacy against the public.** The TAPS signature reveals nothing about the threshold or quorum of signers to the *public*.
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Privacy Against The Public

Adv.		Env.
	$\xrightarrow{(n, t_0, t_1)}$	
	$\xleftarrow{(pk)}$	$(pk, \{sk_i\}_{i\in[n]}, sk_c, pk_t) \xleftarrow{\$} KeyGen(1^\lambda, n, t_b)$
	$\xrightarrow{O_1(C_0, C_1, m)}$	
	$\xleftarrow{Combine(sk_c, m, C_b, \{\delta_i\}_{i\in C_b})}$	
	$\xrightarrow{O_2(m, \sigma)}$	
	$\xleftarrow{Trace(sk_t, m, \sigma)}$	
	$\xrightarrow{b'}$	

Restriction: Inputs to O_2 cannot be outputs from O_1.

Privacy against the public. Adversary wins if it can gain any information about t.

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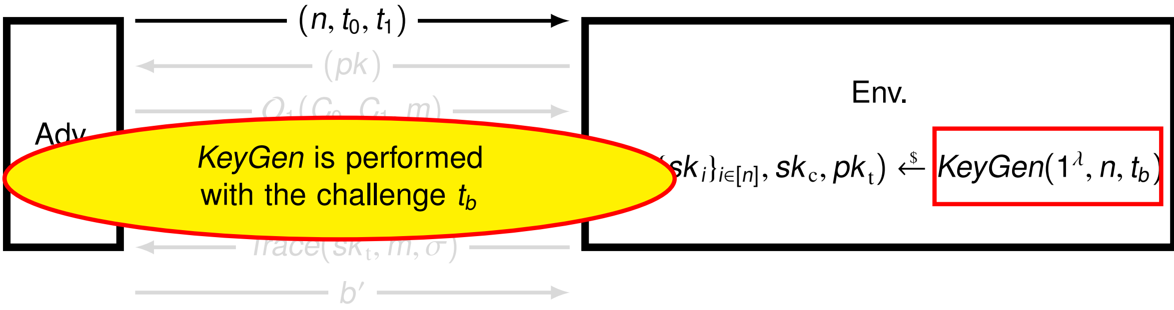

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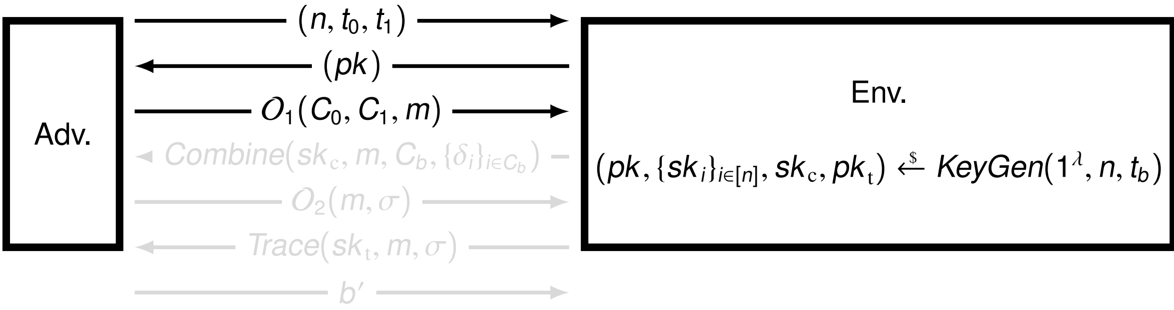

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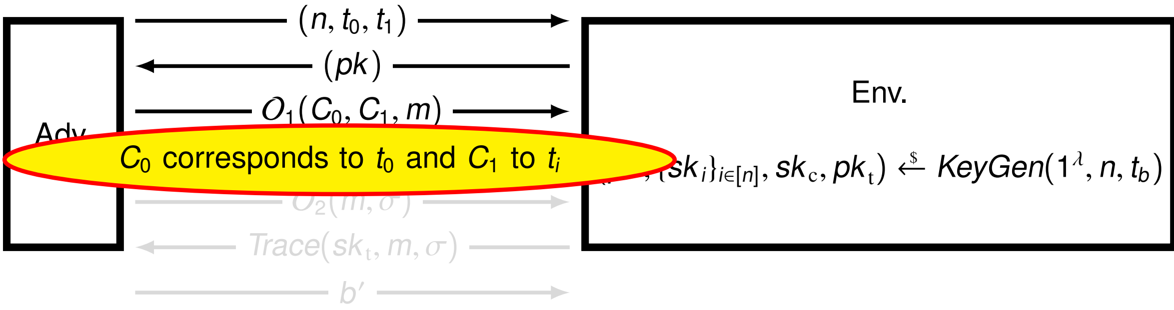

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Adv.

$(n, t_0, t_1) \longrightarrow$

$\longleftarrow (pk)$

$\mathcal{O}_1(C_0, C_1, m) \longrightarrow$

$\longleftarrow \mathit{Combine}(sk_{\mathrm{c}}, m, C_b, \{\delta_i\}_{i \in C_b})$

$\mathcal{O}_2(m, \sigma) \longrightarrow$

$\longleftarrow \mathit{Trace}(sk_{\mathrm{t}}, m, \sigma)$

$b' \longrightarrow$

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$(pk, \{sk_i\}_{i \in [n]}, sk_{\mathrm{c}}, pk_{\mathrm{t}}) \xleftarrow{\$} \mathit{KeyGen}(1^\lambda, n, t_b)$

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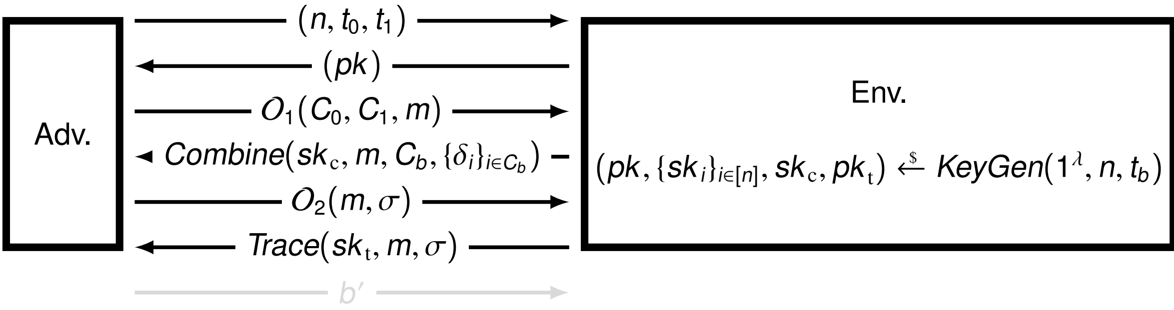

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$$\text{Adv.} \xrightarrow{(n, t_0, t_1)} \text{Env.}$$

$$\text{Adv.} \xleftarrow{(pk)} \text{Env.}$$

$$\text{Adv.} \xrightarrow{\mathcal{O}_1(C_0, C_1, m)} \text{Env.}$$

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Privacy Against Signers

Adv.

$(n, t) \longrightarrow$

$\longleftarrow (pk, \{sk_1, \ldots, sk_n\})$

$\mathcal{O}_1(C_0, C_1, m) \longrightarrow$

$\longleftarrow Combine(sk_c, m, C_b, \{\delta_i\}_{i \in C_b})$

$\mathcal{O}_2(m, \sigma) \longrightarrow$

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$b' \longrightarrow$

Env.

$$(pk, \{sk_i\}_{i \in [n]}, sk_c, pk_t) \xleftarrow{\$} KeyGen(1^\lambda, n, t)$$

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Privacy Against Signers

Adv. | **Env.**

$\xrightarrow{(n, t)}$

$(pk, \{sk_i\}_{i \in [n]}, sk_c, pk_t) \xleftarrow{\$} KeyGen(1^\lambda, n, t)$

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Adv. $\xrightarrow{(n, t)}$ **Env.**

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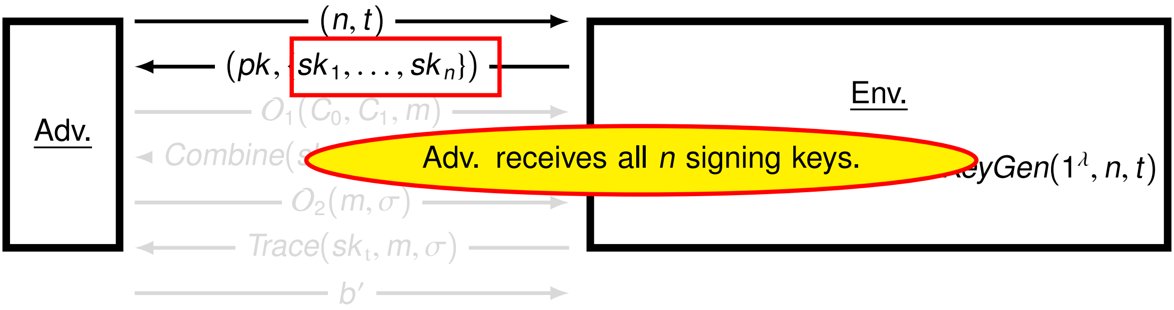

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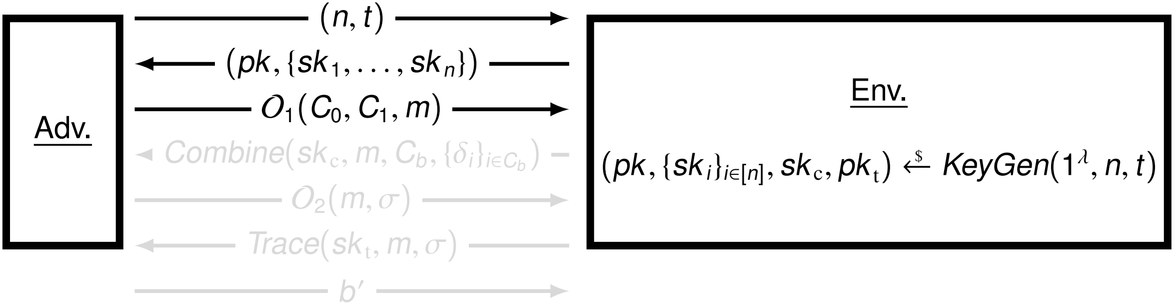

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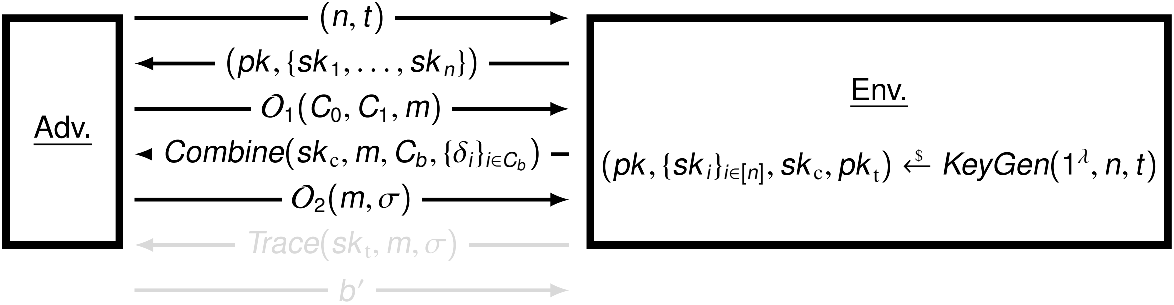

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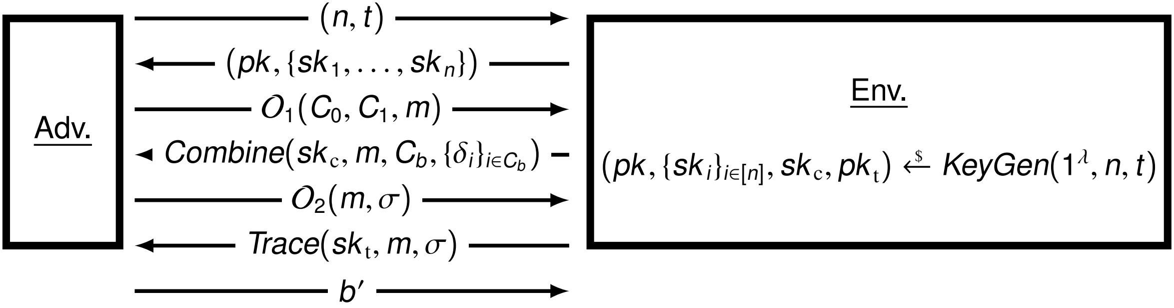

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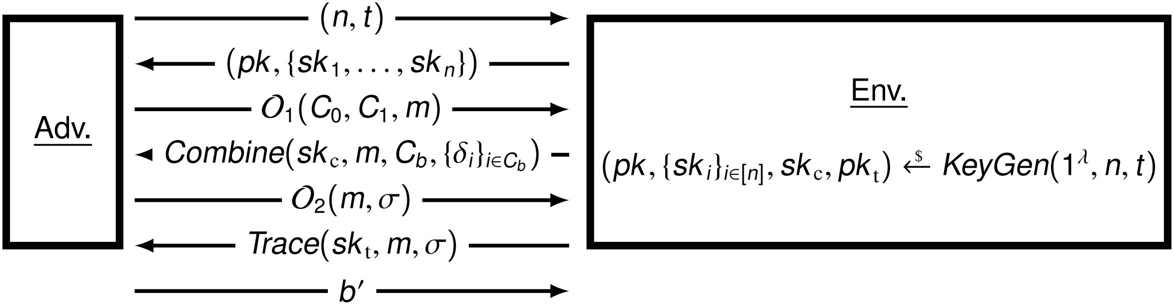

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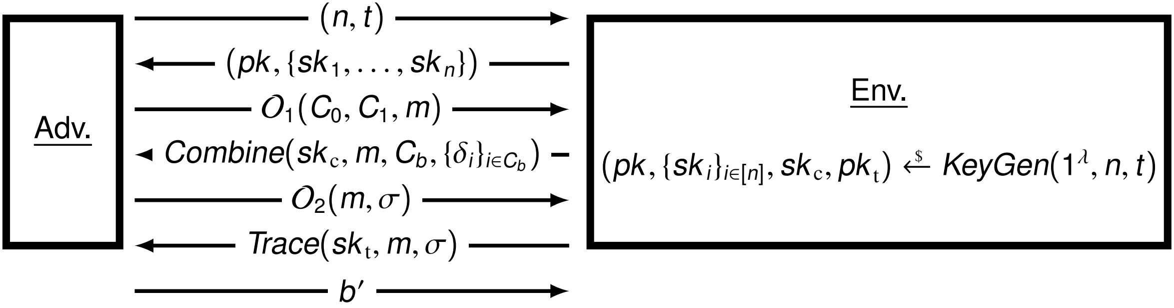

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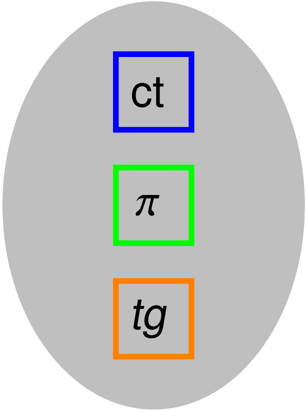

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- π: zero-knowledge proof that the ATS is a valid signature on m
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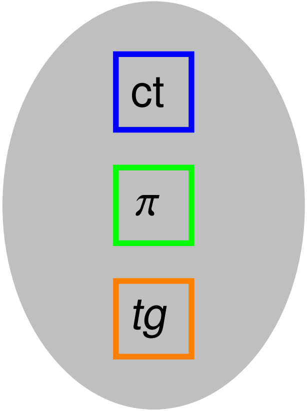

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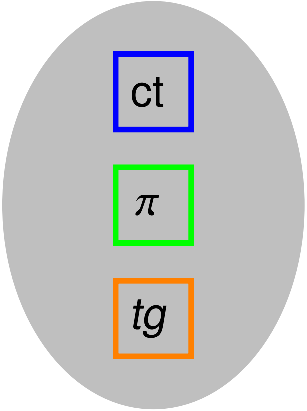

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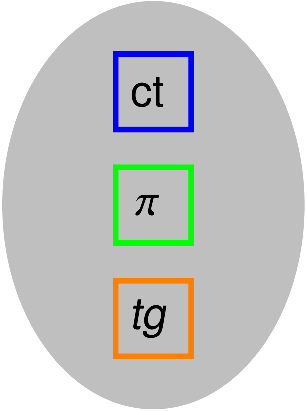

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- The generic scheme requires proving the signature is valid in generic zero knowledge.
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- Recall a Schnorr signature σ:

$$\sigma = \begin{array}{l}(R, \quad \text{// This is a commitment to a nonce} \\ z) \quad \text{// This is the response}\end{array}$$

- The verifier derives $c = H(R, m)$, and checks $R \stackrel{?}{=} g^z \cdot pk^{-c}$.

- **Insight**: Publishing R in the clear does not hurt privacy! Only z must be protected.

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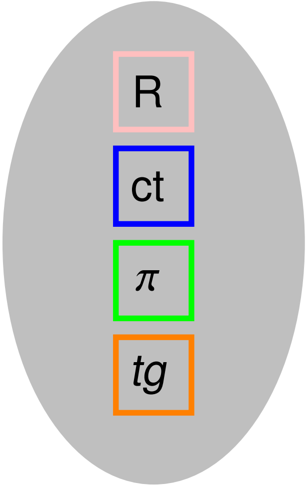

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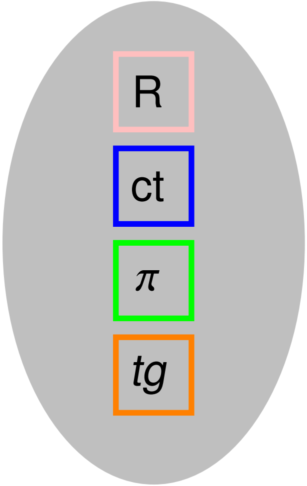

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- *KeyGen*: Generate ATS keypairs for each signer, and keypairs for the combiner (signing) and tracer (encryption).
- *Sign*: Each signer performs an ATS signing operation.
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- *Combine*: Perform the ATS combine to generate σ. Encrypt σ to the combiner's public key, then generate a ZKP π that the encryption is correct, authenticated using the tracer's signing key.
- *Verify*: Check the correctness of π relative to the combiner's verification key and the ATS public key.
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$$\mathcal{R}_S = \left\{ (g, h, pk' = (pk_1, \ldots, pk_n), pk_t, T_0, T_1, R, c, ct = (c_0, c_1)) \ ; \ (z, \rho, \psi, b_1, \ldots, b_n) \right\}$$

- g, h are generators whose dlog relation is unknown
- (T_0, T_1) is an ElGamal commitment to t
- R is the Schnorr commitment
- c is the Schnorr challenge
- $ct = (c_0, c_1)$ is the encrypted ATS signature z
- ρ, ψ are randomizers
- $b_1, \ldots, b_n$ are bits indicating which signers participated

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$$(1) \quad g^z = \Big[\prod_{i=1}^{n} (pk_i)^{b_i}\Big]^c \cdot R$$

// Proves z is valid for a subset of public keys in PK

$$(2) \quad c_0 = g^\rho \quad \text{and} \quad c_1 = g^z \cdot pk_t^\rho$$

// Proves the ciphertext is a valid encryption to tracer's public key

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// Proves signing quorum contains t signers

$$(4) \quad b_i(1 - b_i) = 0 \qquad \text{for } i = 1, \ldots, n \quad \text{// Proves each } b_i \in \{0, 1\}$$

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Performance of Schnorr TAPS

	Public Key Size		Signature Size	
	$\mathbb{G}$	$\mathbb{Z}_q$	$\mathbb{G}$	$\mathbb{Z}_q$
Sigma	$\approx 2n$	0	$\approx n$	$\approx 2n$
Bulletproofs	$\approx n + \frac{n}{40}$	0	$\approx \frac{n}{40}$	4

- Both constructions reduce to discrete logarithm assumptions.

Takeaways

- TAPS are a new type of threshold signature with both privacy and accountability.
- We define a generic construction employing an encrypted ATS.
- We define a Schnorr construction that leverages the structure of Schnorr to simplify the zero-knowledge statement.
- We define sigma and bulletproofs instantiations of the zero-knowledge argument.

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