

5. Bipartite Graph

Graph is bipartite \Leftrightarrow It doesn't have an odd cycle.

① Graph is bipartite \Rightarrow It doesn't have an odd cycle.

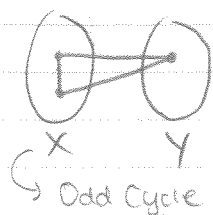
Create a bipartite graph $G=(X \cup Y, E)$

By Contradiction

Suppose we have an odd cycle

Then we have two vertices in the same set. (ie. X)

\Rightarrow This is not a bipartite graph



Odd Cycle

Thus, if a graph is bipartite, then it doesn't have an odd cycle.

② Graph doesn't have an odd cycle \Rightarrow Graph is bipartite.

Suppose we have a graph that doesn't have an odd cycle.

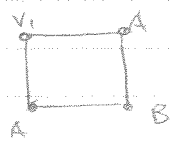
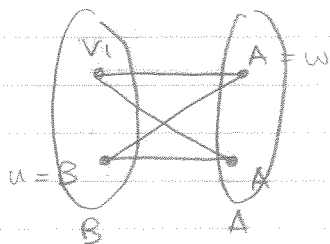
Take any connected component of the graph G_i .

Take any vertex v_i in the connected component.

Put v_i in set A

Put all the nodes whose shortest path is of odd length from v_i in set B.

Put all the nodes whose shortest path is of even length from v_i in set A.



Odd cycle can't be formed because there is no direct edge from v_i to B.

Vertices alternate between two classes: A and B.

Thus A and B lie in two different classes with $A \cap B = \emptyset$ such that

$e = uv \in E$ if $u \in A$ and $v \in B$. This is a bipartite graph.

Thus, if graph doesn't have an odd cycle, then it is a bipartite.