

3.  $T_A$  - function returning running time of algorithm A

$T_B$  - function returning running time of algorithm B

$$T_A(n) = 7T_A\left(\frac{n}{2}\right) + n^2$$

$$T_B(n) = \alpha T_B\left(\frac{n}{4}\right) + n^2$$

$$k = \log_2 7 \quad 2^2 < 7 < 2^3$$

$$k = \log_4 \alpha \quad \frac{4^2}{16} < \alpha < \frac{4^3}{64}$$

$$= 2.80735$$

$$f(n) = n^2$$

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$\Rightarrow n^{\log_2 7}$  grows faster than  $f(n)$

$$\text{Thus } T_A(n) = \Theta(n^{\log_2 7})$$

$$\Rightarrow \log_2 7 = \log_4 49$$

Thus if algorithm B is faster than A,  $\alpha$  must be smaller than 49.

Therefore, the largest integer value of  $\alpha$  is 48.