

Now merge G_1 and G_2 to get G

Then Minimum Spanning Tree is $A^{10}-B^1-C^{10}-D$ Cost = 61
 $\begin{array}{ccc} 10 & & 10 \\ | & & | \\ E-F & & G-H \\ 10 & & 10 \end{array}$

However, this is not minimum spanning tree

Minimum spanning tree of G is $A^{10}-B^1-C^{10}-D$ Cost = 53
 $\begin{array}{ccc} 10 & & 1 \\ | & & | \\ E-F-G-H \\ 10 & 2 & 10 \end{array}$

Thus, this algorithm fails

2. RANDOMIZED QUICKSORT

In the worst case, randomly picked pivots are always the smallest / largest. Thus each subsequence has size $n-1$.

Since the array is of size n , worst case has $\Theta(n^2)$

In the best case, randomly picked pivots are always median of the sequence. Thus, each subsequence has size $\leq n/2$

Since the array is of size n , best case has $\Theta(n \log n)$.

3. $A[1 \dots n]$ - array of n distinct numbers

$i < j$ and $A[i] > A[j] \Rightarrow (i, j)$ is inversion of A

Elements of A form uniform random permutation of $\langle 1, 2, \dots, n \rangle$

Let X_{ij} be indicator random variable. for event that $A[i] > A[j]$ for $i < j$.

Then we have $P[X_{ij} = 1] = \frac{1}{2}$ because given two distinct random numbers, probability that first is greater than the second is $1/2$.

$$E[X_{ij}] = 1/2$$

$$X = \sum_{i=1}^{n-1} \sum_{j=i+1}^n X_{ij}$$