Modeling Brier Scores in Behavioral Data: A Beta Solution

Chelsea Parlett-Pelleriti

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Talk Plan

1. Brier Scores and Bounded Variables
2. Beta Regression
3. Inflated Models (ZOIB)
4. A ZOIB Analysis of Brier Scores in MetaMemory
5. A Simulated IB Analysis of Sliding Scale Data

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Brier Scores

$$BS = \frac{1}{N}\sum_{t=1}^{N}(f_t - o_t)^2$$

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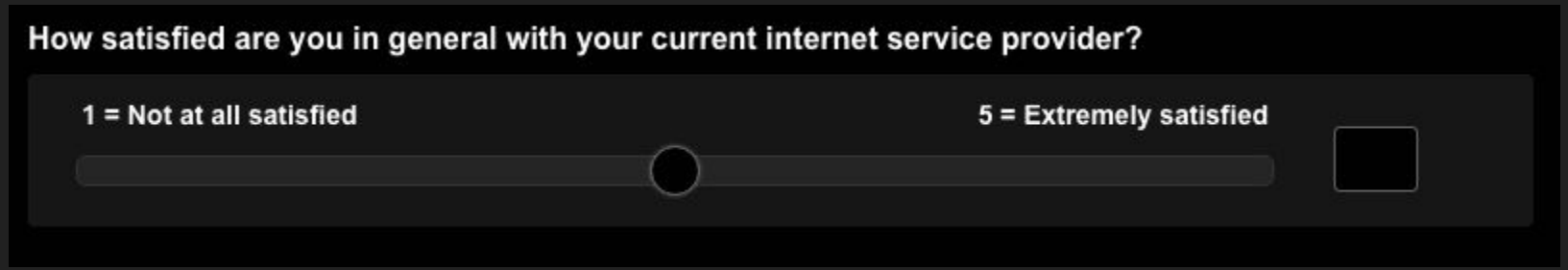
Other Bounded Data



$$G = \frac{n_s - n_d}{n_s + n_d}$$

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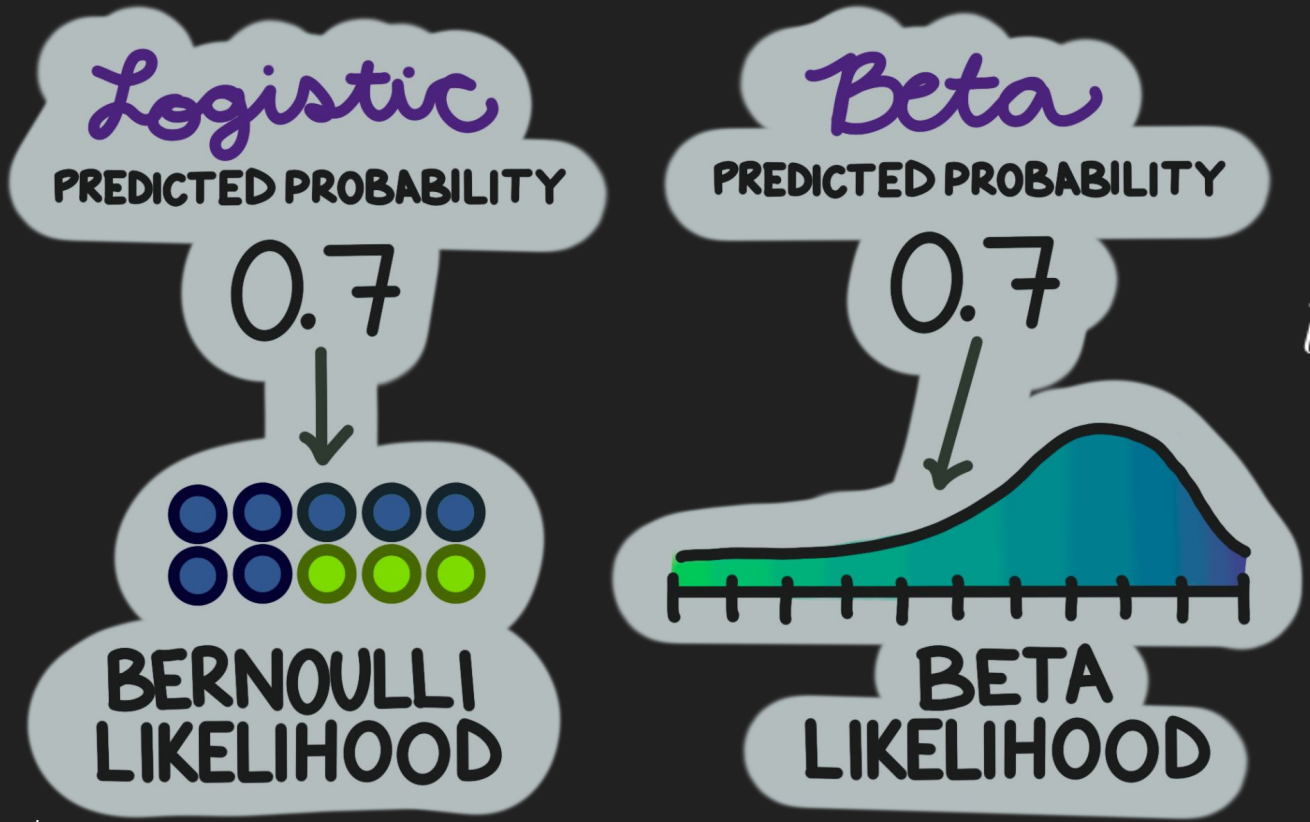
Beta Regression



$$ln(\frac{\hat{y}}{1-\hat{y}}) \sim \beta X$$

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Beta Regression

Distribution		*Canonical Link:* $\theta = g(\mu)$	*Inverse Link:* $\mu = g^{-1}(\theta)$
Poisson		$\log(\mu)$	$\exp(\theta)$
Binomial	*logit link:*	$\log\left(\frac{\mu}{1-\mu}\right)$	$\frac{\exp(\theta)}{1+\exp(\theta)}$
	probit link:	$\Phi^{-1}(\mu)$	$\Phi(\theta)$
	cloglog link:	$\log(-\log(1-\mu))$	$1-\exp(-\exp(\theta))$
Normal		μ	θ

$$[0, 1] \longrightarrow \mathbb{R}$$

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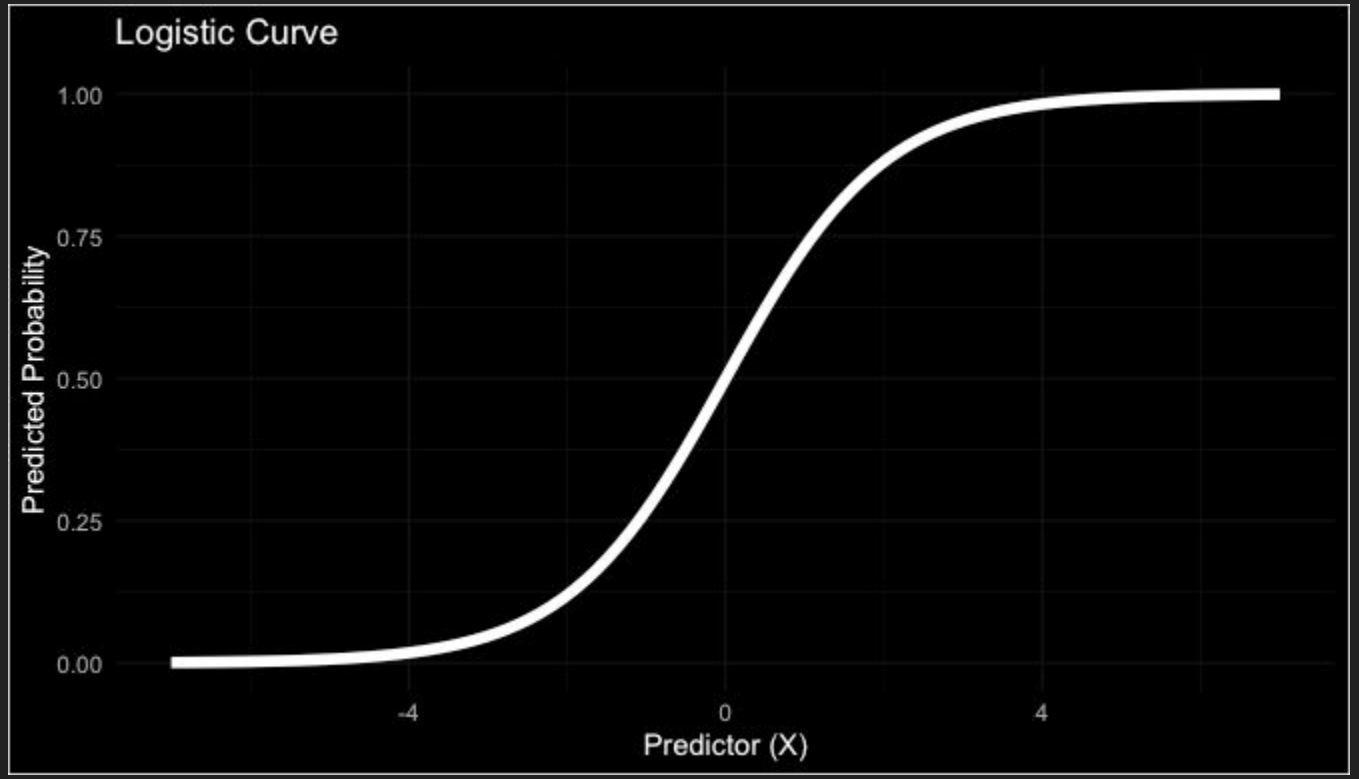

$$ln(\frac{\hat{y}}{1-\hat{y}}) \sim \beta X$$

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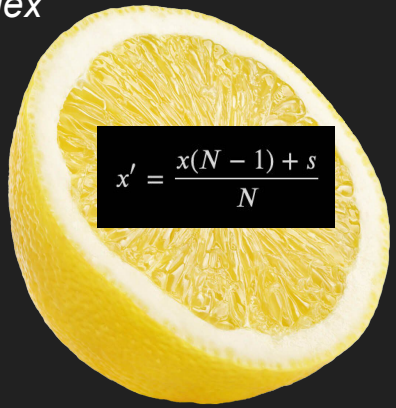
Inflated Models

Probabilities:
[P1 P2 ... Pn]

simplex



K_1 K_2 ... K_n

components

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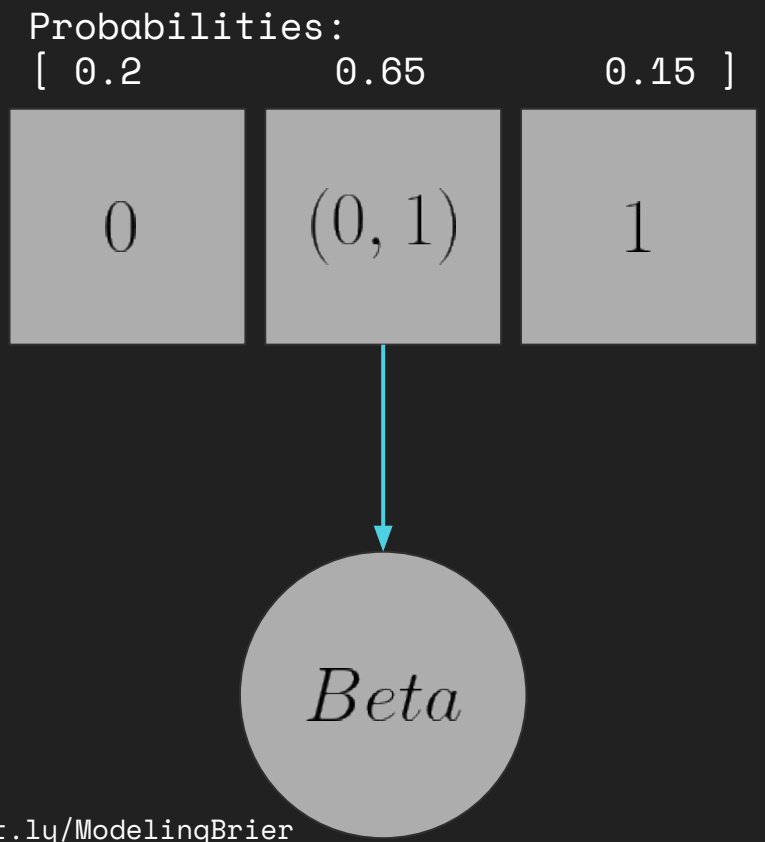
ZOIB



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ZOIB

```
// likelihood
for (i in 1:N) {

  if (brier[i] == 0) {
    // likelihood when score is exactly 0
    target += log(lambda[1]);

  } else if (brier[i] == 1) {
    // likelihood when score is exactly 1
    target += log(lambda[3]);

  } else {
     // likelihood when score is between 0-1

    mu = intercept +
     binary_pred1_b*binary_pred1[i] +
     continuous_pred1_b*continuous_pred1[i] +
     continuous_pred2_b*continuous_pred2[i] +
     u[id[i]] +
     w[word[i]];

     mu_p = inv_logit(mu); // predicted value

    target += log(lambda[2]) + beta_proportion_lpdf(brier[i] | mu_p, kappa); // using proportion parameterization
  }
}
```

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A ZOIB Analysis of Brier Scores in MetaMemory

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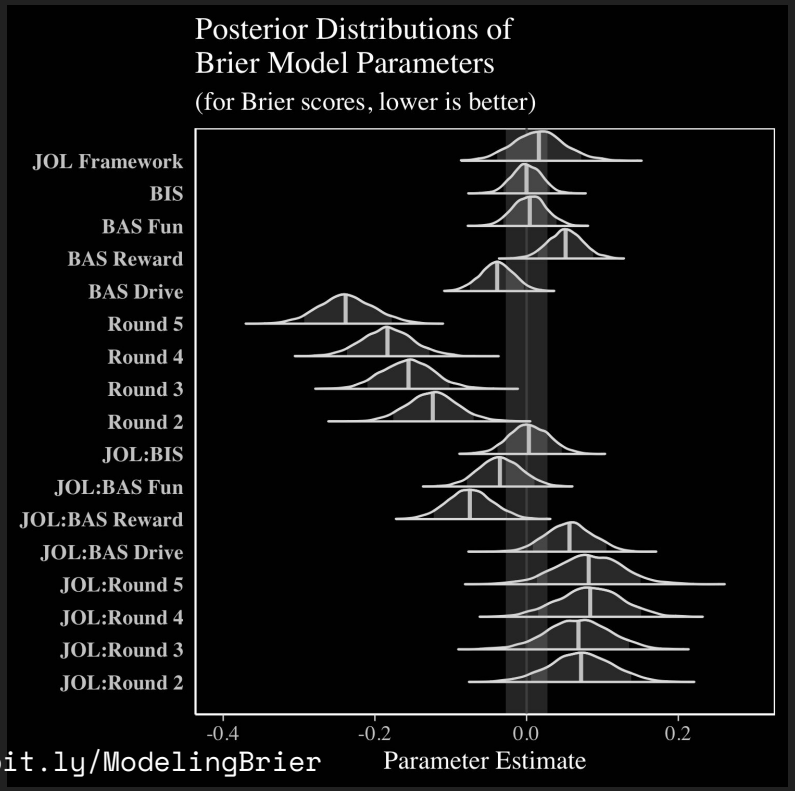

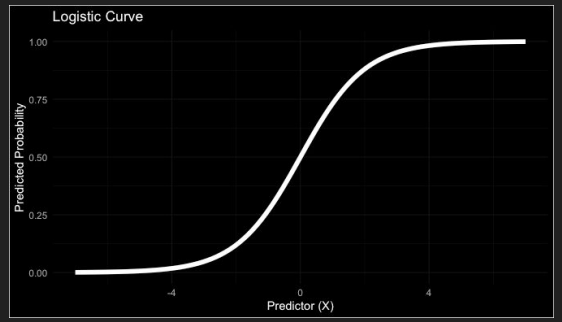

$$ln(\frac{Brier}{1-Brier}) \sim \beta_0 + \beta_1 X_1 + ... + \beta_n X_n$$

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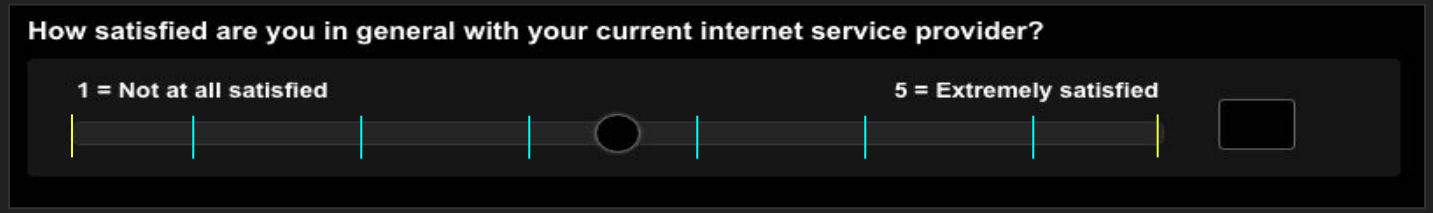
An IB Analysis of Sliding Scale Data



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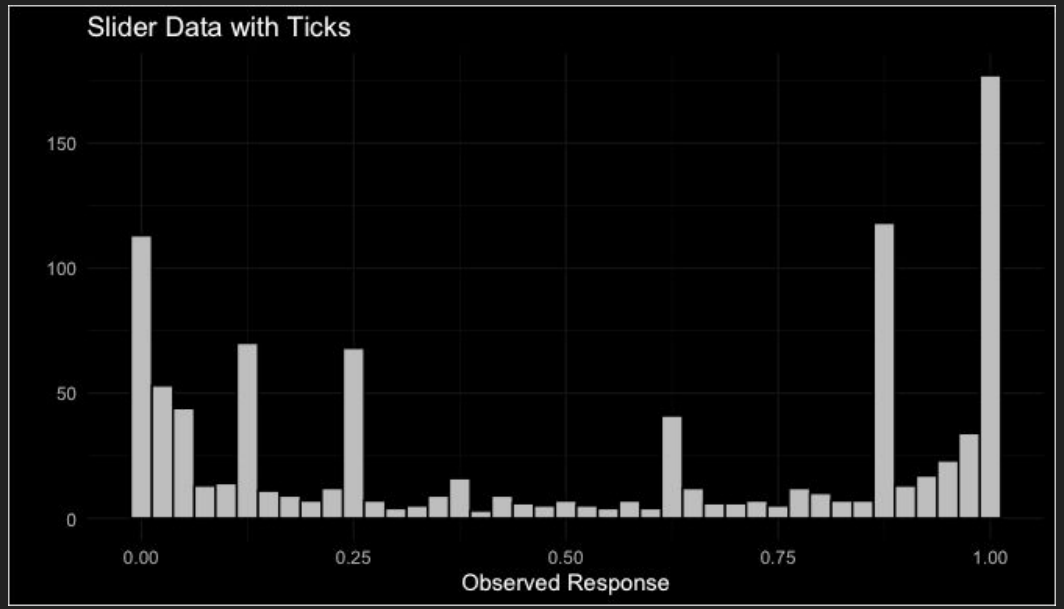

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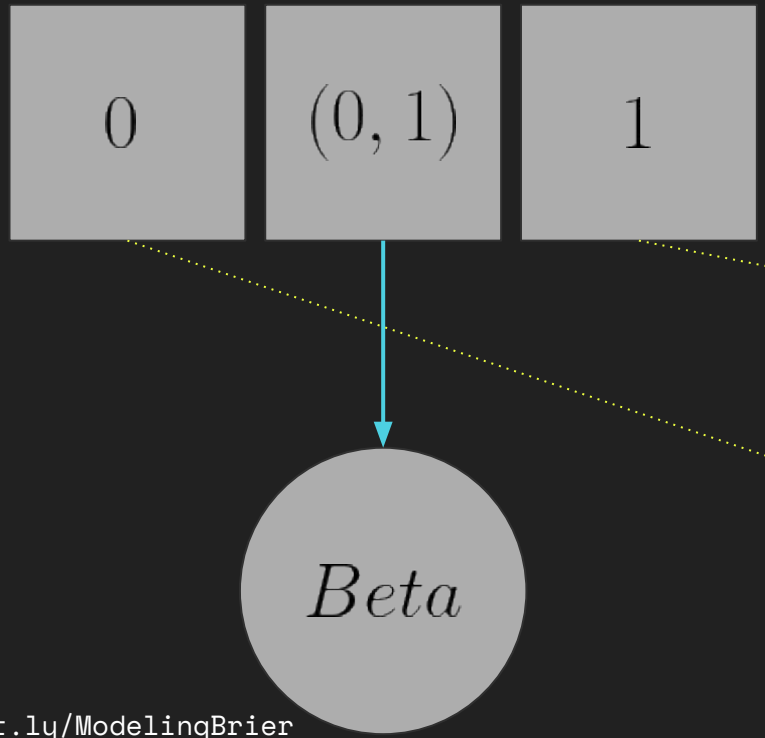
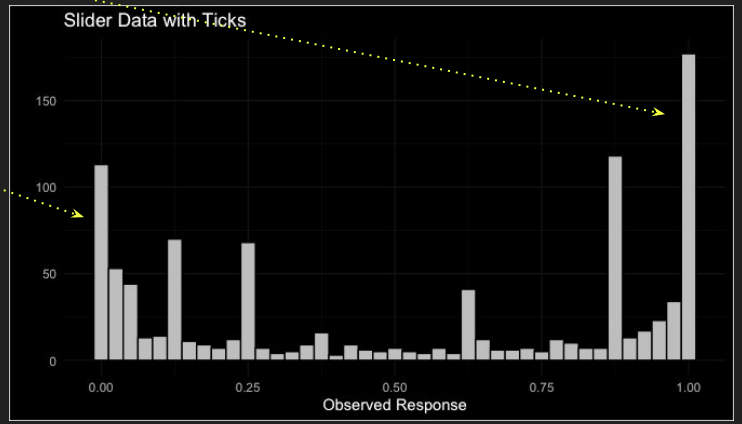
An IB Analysis of Sliding Scale Data

Probabilities:
[0.2 0.65 0.15]





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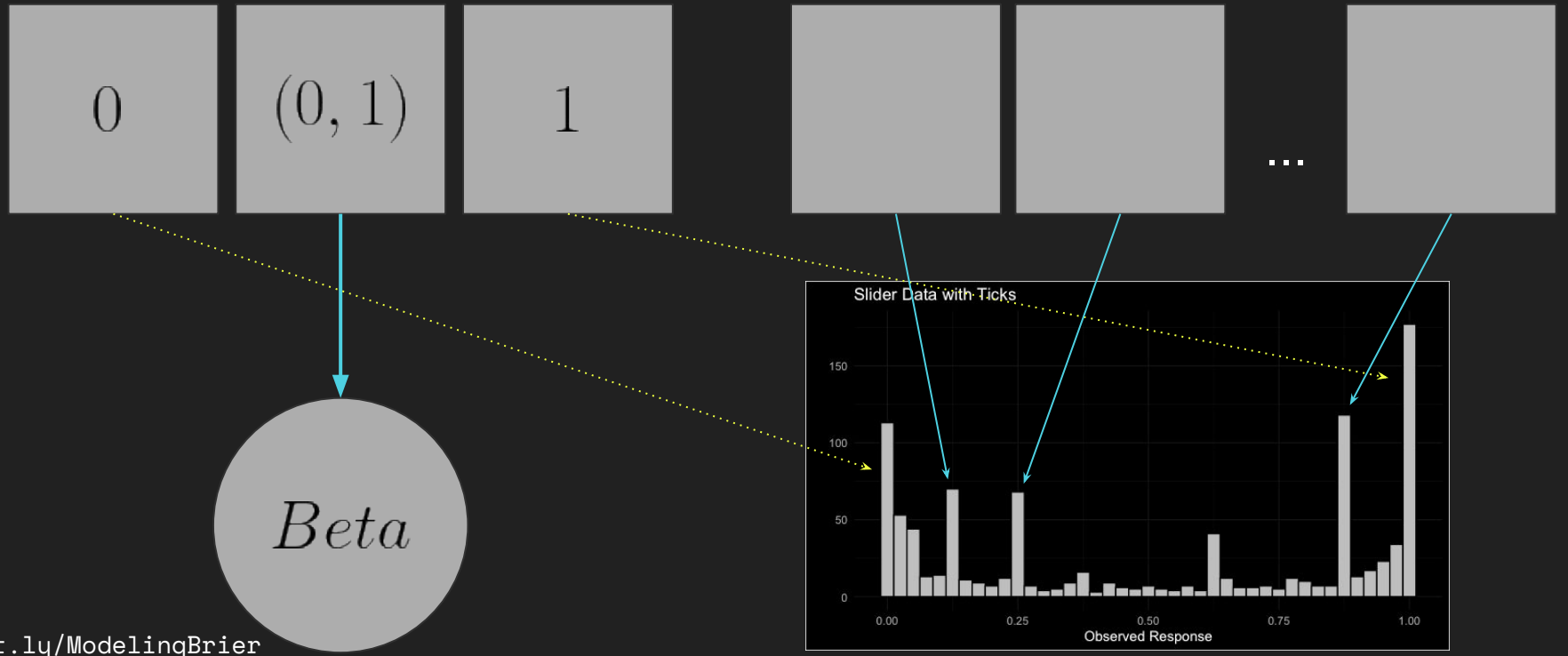
An IB Analysis of Sliding Scale Data

Probabilities:
[P1 P2 P3 ... Pn]



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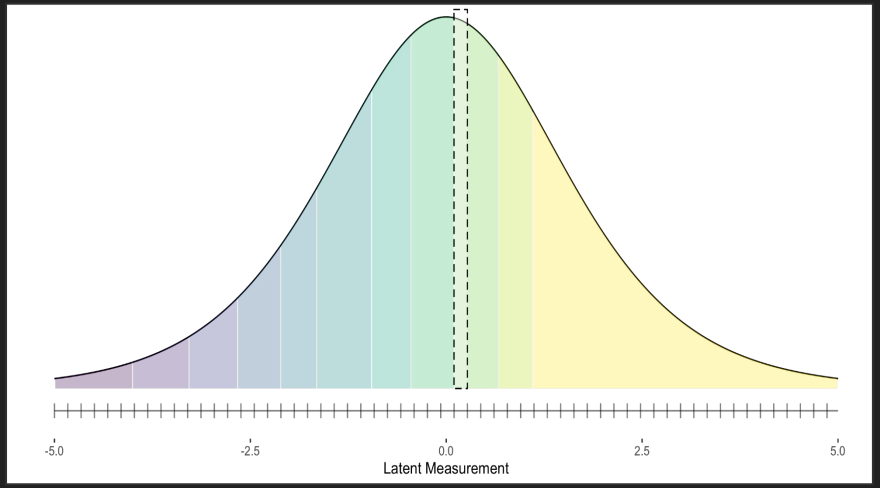
Other Options: Ordered Beta



Ordered Beta Regression: A Parsimonious, Well-Fitting Model for Continuous Data with Lower and Upper Bounds

Robert Kubinec
New York University Abu Dhabi

June 24th, 2021

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6. Alternatives to IB

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Thank You