The Impact of the Screening Subsidy on the Screening and Advertising Decisions made in a Movie Supply Chain

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Abstract

The competition for limited screens has now intensified because an increasing number of new movies are released at the same times. It is well acknowledged that the number of screenings of a certain movie in theaters is based on the movie's popularity and screening costs, which often affect the profits of theaters and distributors. Thus, the performance of the movie supply chain is influenced. In this paper, we probe into how screening subsidies can be used to influence the optimal decisions of theaters and distributors. Assuming that movie popularity is advertising dependent, we construct a two-echelon supply chain in which a distributor provides a movie to a theater, and we regard the equilibrium situation in the movie supply chain as a benchmark. With the theater acting as the supply chain leader, we consider two cases, one in which the distributor pays a subsidy to the theater and another in which there is no such subsidy. Finally, through numerical investigation, we conclude that a proper screening subsidy can improve supply chain efficiency, increasing the total profit of the supply chain.

Keywords: Movie Scheduling; Optimization Model; Movie Supply Chain Efficiency; Screening Subsidy

1. Introduction

Screen scheduling in multiplex movie theaters has received much attention in recent years because the number of mass market movies is sharply increasing while the number of theaters has remained relatively stable. Most modern theaters have multiple screens, so theaters must make everyday decisions regarding how to schedule movies. While movie distributors look for ways to get their movies on as many screens as possible and for as long as possible, theaters carefully schedule movies due to their limited numbers of screens. In this paper, we propose a supply chain model that deals with the decision problems of movie distributors and theaters and investigate the impact of screen-subsidy contracts on the movie supply chain.

So far, Eliashberg et al. (2009) has proposed a decision support model-based column generation technique to help exhibitors make optimal screening schedules with specifications about the days of the week, time, and on which screens different movies ought to be shown. Related research was conducted by Drobouchevitch (2015), who finds that this optimization problem is an NP-hard, thus decreasing the complexity of the problem.

It is widely known that the movie industry is dynamic due to the decay of movies. Decay refers to the inherent weekly decline in box-office attraction and the gross revenues of a movie that is being played in a theater, as defined by Lehmann and Weinberg (2000). Therefore, distributors of movies spend large amounts of money publicizing them through the internet, television, newspapers and so on to promote high box-office revenues. Besides, the demand for specific movies can be promoted by advertising investments from distributors, and Liu (2006) analyzes the impacts of such advertising. The number of screenings of a movie in a cinema is like shelf space for products in a supermarket, a large number of screenings of a movie can motivate audiences to watch this movie, and Moon and Sangkil (2010) have probed into the consumption of movies at individual level. Furthermore, Jun et al. (2011) focus on forecasting the consumption of movies at the aggregate level. However, this motivational effect on the larger watching demand of audience has a decreasing rate as increasing numbers of screenings are scheduled for a movie, and theater attendance rates are used as indicators of the motivational effect in this paper, which has been studied by Marshall et al. (2013) using a specific parametric solution.

When a distributor decides to offer a proper screening subsidy to a theater to get additional screenings for their movie, the net profits of both the distributor and the theater rise. Wang and Gerchak (2001) provided a similar coordinating contract. In particular, Raut et al. (2008) look into the effects of such contracts on movie-screening scheduling. The main difference between our study and their researches is that we focus on improving the whole movie supply chain's profits and efficiency through the use of screening subsidies, which is a new application field of the shelf-space-allocation theory. We optimize the total number of screenings during the obligatory time rather than the hourly screening schedules, but our research can be an interchangeable reference to previous research studies focused on detailed hourly schedules. Furthermore, according to the method that Stackelberg developed to analyze the competition game with leaders and followers, we establish a basic model without a screening subsidy and an improved model with a screening subsidy. In both, we regard the theater as the leader according to the practical experience.

Similar to retail management, our model, which helps both theaters and distributors, was established to profitably and efficiently improve their decisions. In our model, the theaters act as retailers and the distributors are regarded as manufacturers. Therefore, the SHARP model of Bultez and Naert (1988), which helps retailers determine shelf space allocations, is applicable to the short-lived movie industry.

This paper is organized as follows. First, we demonstrate the process of problem formulation and scheduling model construction in Section 2. Then, in Section 3 we analyze theater decisions about screening numbers for particular movies during their run times when the theater is the leader in the supply chain and distributors' advertising strategies after a theater sets a screening schedule. Besides, we present the integrated movie supply chain in which the total profit of the theater and distributor is the greatest, so it is regarded as the benchmark. We then analyze the situation in which the distributor offers a subsidy to the theater to partly compensate for theater's screening costs, and the decisions of theaters and distributors in this situation are compared to the ones without a subsidy in Section 4. In Section 5 we conduct numerical analysis to demonstrate the robustness of our model and further explains the improvements of the supply chain. Finally, we draw conclusions and outline directions for future research.

2. Problem Formulation and Scheduling Model of Movies

2.1 The Problem of Movie Supply Chain

Consider a movie supply chain with a theater and a movie producer. There are multiple screens in the theater. The revenue of the theater is determined by the ticket price, seating capacity, screening number, and attendance rate. The ticket price and seat capacity are assumed to be exogenous. The attendance rate is partly determined by the attractiveness of a movie, which can be influenced by advertising expenditures. The box-office revenues are distributed between the theater and the movie producer at 40% and 60%, respectively, which is estimated based on the data given by Radovilsky and Vereen (2003).

This problem of making decisions on advertising expenditure and movie schedules needs to be further explored as theaters and distributors make contracts to share the box-office revenues. Consequently, the percentages of total revenue of theaters and distributors vary depending on the contract. Given the complicated variability of the movie industry and the sharing scheme between theaters and distributors, we construct a movie-scheduling model to help the distributors decide how much money to spend on advertising.

Definition of Variables

$\mathbf{A}$ expenditures on advertising
$\mathbf{P}$ price of movie ticket
$\mathbf{S}$ number of seats in all screening rooms scheduled to show a movie
$\mathbf{N}$ total screening number in a theater during run time
$\boldsymbol{\alpha}$ proportion of total box-office revenues the theater receives
$\mathbf{a_0}$ upper limit of average attendance rate stimulated by advertising
$\mathbf{r}$ coefficient of the advertising impacts on average attendance rate
$\mathbf{c}$ scale parameter of theater's screening cost
$\mathbf{f}$ ratio coefficient of screening subsidy

Attendance Rate Function We use a decreasing exponential model to estimate the attendance rate, as influenced by advertising. The demand for a movie increases with advertising expenditures, but the increase has a marginal diminishing impact. Consistent with the empirical results in the research of Krider and Weinberg (1998), and based on our analysis above, we assume that the attendance rate function follows $a_0 - e^{-rA}$, where $a_0 > 0, r > 0,$ where a_0 denotes the potential upper limit of the attendance rate of a given movie, which varies depending on movie and theater, r is the scale parameter, and A is the advertising expenditures of the movie producer.

Theater's Cost Function The cost of scheduling movies increases with the number of movies that are planned to be shown at the theater, which includes lighting, rent, the salaries of the theater staff, and so on. The more screenings that are arranged for this movie during its run time, the more difficult it is to schedule these screenings, and the higher the involved costs are, which are assumed to go up in an increasing rate, so the cost function can be modeled as cN^2, where $c > 0$ and c is a scale parameter aimed to adjust the theater cost for a certain movie.

Problem Statement Based on the two functions discussed above, we can write the net profit functions of both the theater and the distributor.
The theater seeks to maximize its net profit by determining the number of screenings of a movie (N), and the net profit πT of a theater is calculated as follows:

$$\pi T = \alpha PNS(a_0 - e^{-rA}) - cN^2 \quad (1)$$

The distributor determines the advertising expenditures (A) and the number of screenings of a movie(N) to maximize its net profit (πP) using the following formula:

$$\pi P = (1 - \alpha)PNS(a_0 - e^{-rA}) - A \quad (2)$$

The total net profit of the supply chain is $\pi C = \pi T + \pi P$. That is,

$$\pi C = PNS(\alpha - e^{-rA}) - cN^2 - A \ (3)$$

3. The Basic Scheduling Model without Screening Subsidy

Firstly, the distributor determines the advertising expenditures that are used to attract people to watch the movie, with the ultimate goal of generating the largest net profit for itself. Then the theater that is regarded as the supply chain leader will determine the total number of screenings to display during the run time to maximize its net profit, which is based on Stackelberg model. Finally, the distributor adjusts its advertising expenditures according to the theater's decision.

Proposition 1 shows how the theater makes decisions about screening number to get maximum net profit and the distributor's subsequent decision about advertising expenditures.

Proposition 1. The optimal screening number for theater is $\mathbf{N_{T1}^{*}} = \frac{\mathbf{a_0 SP\alpha}}{\mathbf{2c}}$ and the distributor accordingly invests $\mathbf{A_{P1}^{*}} = -\frac{\mathbf{1}}{\mathbf{r}}\mathbf{ln}\frac{\mathbf{2c}}{\mathbf{ra_0\alpha(1-\alpha)(SP)^2}}$ in advertising. (See proof in Appendix A)

In proposition 1, it can be directly seen that N_{T1}^{*} is proportional to α while N_{T1}^{*} and A_{P1}^{*} are inversely proportional to c, which is mainly due to the fact that when the theater can get more a larger proportion of the movie's total box-office revenues, then it will certainly schedule additional screenings for this movie to earn extra profit. However, if the theater incurs high costs to project this movie, then it will accordingly arrange fewer screenings of this movie to reduce costs, and with fewer screenings available for the distributor, the distributor will be less passionate in advertising the movie than is offered a large number of screenings by theaters.

Consider an integrated movie supply chain in which A and N are determined using a central planner.

We use the A and N decisions of the integrated movie supply chain as the benchmark, which is the most profitable and ideal situation for the theater and the distributor in the movie supply chain.
The parties involved in the movie industry aim to earn maximum net profit. Through mathematical analysis, which is shown in Appendix B, we get the integrated decisions described in Proposition 2.

Proposition 2. The integrated decisions for the supply chain are the following

$$\mathbf{N_C^{*}} = \frac{\mathbf{a_0SP}}{\mathbf{4c}} + \frac{\sqrt{\mathbf{a_0{}^2(SP)^2 - \frac{8c}{r}}}}{\mathbf{4c}} \quad (4)$$

$$A_C^* = -\frac{1}{r}\ln\frac{4c}{SPr(a_0SP+\sqrt{a_0{}^2(SP)^2-\frac{8c}{r}}} \quad (5)$$

Based on Equation 4 and Equation 5, we can observe that with a central control, the theater it is supposed to schedule to screen the movie N_C^* time during the run time, and the distributor should spend A_C^* on advertising to improve the performance of the movie supply chain and improve profitably.

It is straightforward to show that N_C^* increases with r because more people will be encouraged to see the movie by advertising when r increases. However, N_C^* and A_C^* vary inversely with c, for the same reason mentioned above in the explanation of Proposition 1.

In the next section, we further analyze the decisions of the decentralized supply chain in which a subsidy is offered by the distributor to the theater, and we compare these decisions to the supply chain without a subsidy discussed in Proposition 1.

4. A Screening Subsidy Contract Based on the Stackelberg Model

To promote the theater to give more projections for the distributor's movie, the distributor will choose to compensate for the theater's costs. Taking account of this, we rewrite the model after adding the subsidy into the supply chain on the basis of the Stackelberg model.

Suppose that there is a movie supply chain with a theater as the leader in the Stackelberg model and a distributor as the follower. The theater first determines the screening number for the movie, then the distributor sets is advertising expenditures and the modified ratio coefficient of screening subsidy (f) for this movie accordingly. The purpose of the subsidy offered by the distributor to the theater is to compensate for some of the theater's costs, so we establish a function in the same form as the theater's cost function: fN^2, where $0 < f < 1$, f is a ratio coefficient of screening subsidy with the objective of adjusting the subsidy for the movie. It can be explicitly seen that $\frac{f}{c}$ denotes the percentage of the theater's cost that the distributor pays.

Therefore, the theater's net profit is rewritten as Equation (6):

$$\pi T = \alpha SPN(a_0 - e^{-rA}) - (c - f)N^2 \quad (6)$$

which is similar to Equation (1), except c has been changed into $c - f$.

The net profit of the distributor becomes Equation (7),

$$\pi P = (1 - \alpha)SPN(a_0 - e^{-rA}) - A - fN^2 \quad (7)$$

The total net profit of the supply chain is calculated using Equation 3 because the subsidy offered by the distributor is offset by the one received by the theater

$$\pi C = PNS(\alpha - e^{-rA}) - cN^2 - A$$

Provided with a subsidy by the distributor, the theater's optimal strategy is shown in proposition 3 and the distributor's sequent decisions are also described in it.

Proposition 3. When the theater projects a movie $\mathbf{N_{T2}^{*}=\frac{SP(1+\alpha)a_0}{4c}}$ times during the run time, the distributor of this movie can maximize its profit by spending $\mathbf{A_{T2}^{*}=-\frac{1}{r}\ln\frac{4c}{rSP^2(1-\alpha^2)a_0}}$ on advertising and offering a screening subsidy equal to $\mathbf{f^{*}=\frac{2c(1-\alpha)}{1+\alpha}-\frac{8c^2}{r(SP(1+\alpha)a_0)^2}}$. (See the proof in the Appendix C)

In proposition 3, it is obvious that N_{T2}^{*} goes up with α but N_{T2}^{*} and A_{T2}^{*} vary inversely with c, for the same reason mentioned above in the explanation of Proposition 1. Besides, f^{*} is in inverse proposition to r, which is mainly because when r increases, more people will be encouraged to see the movie by advertising, so with a bigger profit, the theater will ask for a smaller subsidy from the distributor.

Furthermore, we consider the supply chain's total net profit. It is proved to increase with the screening number N (see proof in Appendix E), so we can prove that $\pi C_{C}^{*} > \pi C_{T1}^{*} > \pi C_{T2}^{*}$ based on $N_{C}^{*} > N_{T2}^{*} > N_{T1}^{*}$, which is proved in the following mathematical reasoning. Besides, the relationship between πC_{T1}^{*} and πC_{T2}^{*} indicates that the movie supply chain is improved profitably when a subsidy is offered. Proposition 3 implies that the total screenings for the movie in the theater is boosted compared to the case without subsidies in Section 3. Because $0<\alpha<1$, it can be easily deduced that $N_{T2}^{*}=\frac{SP(1+\alpha)a_0}{4c}>\frac{SP\alpha a_0}{2c}=N_{T1}^{*}$.

Assume that if the theater projects the movie during run time at number $N_{C}^{*}=\frac{a_0 SP}{4c}+\frac{\sqrt{a_0{}^2(SP)^2-\frac{8c}{r}}}{4c}$ as the theater in integrated supply chain does, then the net profit of the theater (πT) will go down due to the fact that πT decreases as N is increases when $N<\frac{(3\alpha-1)a_0 SP}{8c}$ (see the proof in Appendix D). Therefore, even with a screening subsidy, the theater still cannot project as many movies as the theater in integrated supply chain, which means a subsidy for the theater can improve the performance of the supply chain but cannot coordinate it due to the fact that $N_{C}^{*} > N_{T2}^{*} > N_{T1}^{*}$. Because $N_{T2}^{*} > N_{T1}^{*}$, it can also be seen that the performance of the supply chain is improved because additional screenings will induce the emerging demand of potential customers to watch this movie. That is, the improvement of the supply chain is associated with the rising power of the distributor. And the distributor's desired number of screening during the time is satisfied in this way, which is similarly suggested by Cairns (1962).

5. Numerical Studies

In this section, we probe into the impact of the scale parameter of theater's cost (c) and the occupancy rate elasticity (r). Although the price of a cinema ticket (P), the total number of seats (S) in a theater available for the movie, the limited occupancy rate (a_0), and the ratio of total box-office revenue that the theater can share, α, vary in different situations, in real life, these variables can be estimated before scheduling movies via empirical experience.

According to the data collected from an authoritative website that contains movie statistics, including ticket price, theater size, and so on, we select a set of representative data and assume that the screening room is of medium size and the movie is a hit. Then we can set the following variables of our model: $\mathrm{S} = 100, \mathrm{P} = 35, \mathrm{a}_0 = 0.4, \alpha = 0.57$.

Then we conduct the sensitivity analysis to demonstrate that our conclusions in section 4 are robust to the variations of our parameters $(\mathrm{c},\ \mathrm{f}$ and $\alpha)$. We further discuss the reasons for the trends shown in the following figures and the insights gained from the movie supply chain.

5.1 Impact of r on Advertising Expenditures (A)

We set $\mathrm{c} = 2$ and vary r from 0.0001 to 0.001 to investigate the impact of the occupancy rate elasticity (r) on advertising expenditures (A). Figure 1 demonstrates the result.

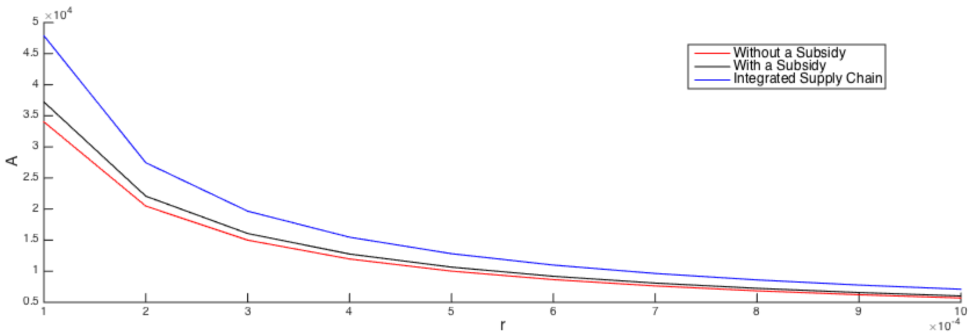

Figure 1 Impact of r on the advertising expenditures (A)

Figure 1 shows that advertising expenditures decreases as the occupancy rate elasticity increases. This trend demonstrates that when advertising has a large impact on occupancy rate, the distributor can spend less money on advertising. There are two turning points when occupancy rate elasticity is around 2 and 3. This indicates that advertising has a decreasing influence on occupancy rate as occupancy rate elasticity increases, the impact will be smaller after the turning points. It can also be seen that when the supply chain is integrated, the distributor has to spend the largest amount of money on advertising compared to the other two situations.

5.2 Impact of c on Screening Number (N)

We set r to 0.001 and vary c from 1 to 20 to investigate the impact of the scale parameter of the theater's cost (c) on the screening number (N). The effect is depicted in Figure 2.

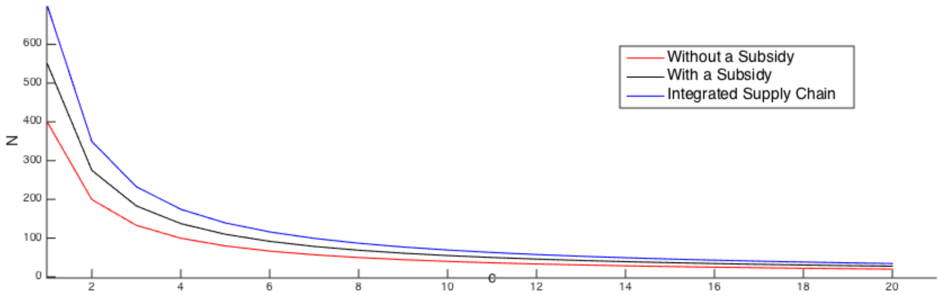

Figure 2 Impact of c on the screening number (N)

The screening number (N) decreases when c increases in the three situations shown in Figure 2. The reason for such trend is that when the theater has to assume more costs to project the movie, it will accordingly arrange fewer screenings for this movie to reduce these costs, regardless of whether the distributor offers it a subsidy to share part of the cost.

In Figure 2, it is also implied that the theater raises the total screening number during run time when it is offered a subsidy. That is, because the distributor shares part of the theater's cost, the theater can have more budget to arrange additional screenings for this movie to stimulate and satisfy people's watching demands. Interestingly, the supply chain still cannot be coordinated when the distributor offers a subsidy to the theater, even though the performance of supply chain is improved in this way.

5.3 Impact of c on Supply Chain Efficiency

We set $r = 0.001$ and vary c from 1 to 20 to look into the the scale parameter of the theater's cost (c) on supply chain efficiency. The definition of movie supply chain efficiency is how the segregated movie supply chain performs compared to the integrated movie supply chain. Hence, the movie supply chain efficiency equals the ratio of the total net profit of the segregated supply chain and the integrated supply chain's net profit. The result is shown in Figure 2.

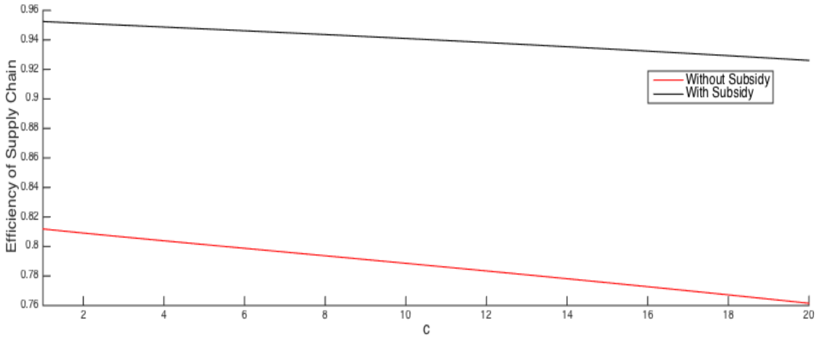

Figure 3 Impact of c on Supply Chain Efficiency

From Figure 3, it is obvious to see that the efficiency of the supply chain is significantly improved by means of offering a subsidy to the theater. Furthermore, the efficiency decrease is relatively small in a supply chain with a subsidy compared to one without a subsidy because cost scale parameter (c) rises. Given the definition of the efficiency of the movie supply chain, the increase in efficiency is identical to the increase of the total profits of the supply chain. Hence, it can be concluded that subsidies can majorly improve the profitability and efficiency of the movie supply chain.

5.4 Impact of α on f and movie Supply Chain Efficiency

We set r to 0.001, c to 20, and the proportion of theater's box-office revenues (α) to 0.70 to investigate how f and supply chain efficiency change with α, which is shown in figures 4 and 5.

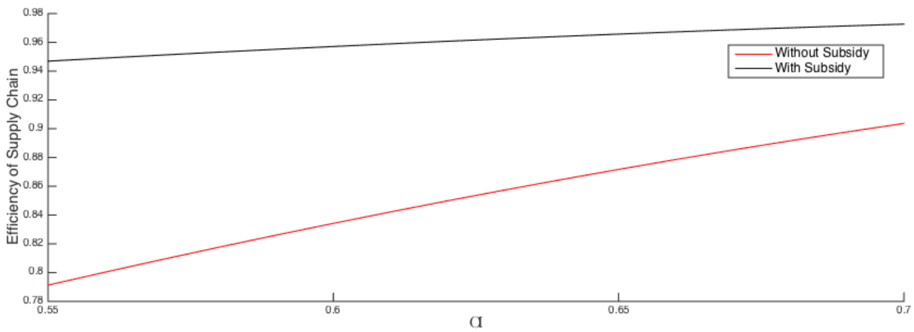

Figure 4 Impact of α on Supply Chain Efficiency

Figure 4 depicts that movie supply chain efficiency rises with α, which is more obvious in the supply chain without a subsidy than that with one. It shows that when the theater shares more box-office revenues with distributors, the supply chain efficiency can be boosted, especially in the case that the distributor does not offer a subsidy to the theater. Therefore, a theater that is not offered a subsidy by the distributor can ask for a larger proportion of box-office revenues to improve the supply chain's performance.

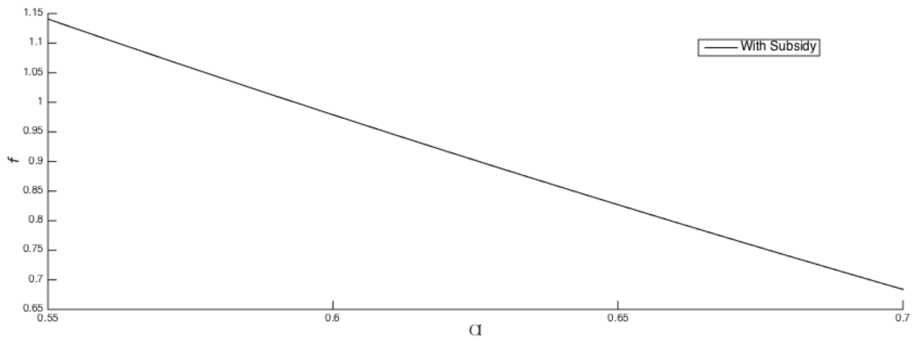

Figure 5 Impact of α on f

Figure 5 shows that f reduces as α increases, which means that when the distributor gains less from total box-office revenues, it will accordingly compensate less for the theater's costs.

5.5 Impact of Subsidies on the Movie Supply Chain

We firstly set S to 100, P to 30, a_0 to 0.4, and α to 0.65 and vary $\frac{c}{r}$ from 1 to $180{,}000$, which is the upper limit of $\frac{c}{r}$, to make the radical expression in N_C^* valid and compare the screening number in the integrated supply chain (N_C^*) to the screening number in the supply chain with a subsidy (N_{T2}^*), and then we choose different sets of S, P, a_0, and α to see if the relationship between N_C^* and N_{T2}^* is always consistent. The results are shown in Figure 6 and Table 1. In our research, c is the scale parameter of theater's movie-screening costs and r is the coefficient of the advertising impacts on average attendance rate.

In Figure 6, it can be seen $N_C^* > N_{T2}^*$ is always established between the interval where $\frac{c}{r} < 180{,}000$. That is, under such situation, theaters in an integrated movie supply chain will project more movies during the run time than those in the movie supply chain, even when a subsidy is not offered by the distributor.

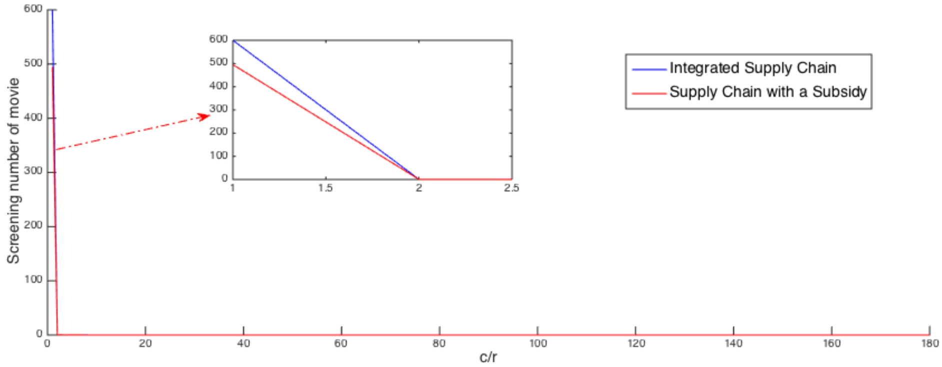

Figure 6 relationship of N_C^* and N_{T2}^*

We then change the values of S, P, a_0, and α to focus on whether $N_C^* > N_{T2}^*$ is always established. Some of the representative sets of S, P, a_0, and α are demonstrated in Table 1. Via Matlab plotting, we can always get the similar trends and relationship of and between N_C^* and N_{T2}^* to those shown in Figure 6 where $N_C^* > N_{T2}^*$.

$\mathbf{S}$	$\mathbf{P}$	$\mathbf{a_0}$	$\boldsymbol{\alpha}$	**Relationship between N_C^* *and* N_{T2}^***
100	30	0.4	0.65	$N_C^* > N_{T2}^*$
100	30	0.4	0.55	$N_C^* > N_{T2}^*$
100	30	0.3	0.65	$N_C^* > N_{T2}^*$
100	40	0.4	0.65	$N_C^* > N_{T2}^*$
200	30	0.4	0.65	$N_C^* > N_{T2}^*$

Table 1 relationship between N_C^* and N_{T2}^* under different sets of S, P, a_0, α

By comparing the five groups in Table 1, we can conclude that when S, P, a_0, and α vary according to the situations of different theaters, $N_C^* > N_{T2}^*$ will always hold true.

6. Conclusion

Due to the features of the movie industry that many movies are released on similar dates, theaters must deal with challenges involved in scheduling these movies. Movie schedules influence the revenues of not only the theater but also the distributor. Therefore, in this paper, we consider how to optimize the movie supply chain, which means increasing the total profit of the supply chain composed of a theater and a distributor through a subsidy offered by the distributor to the theater. We propose a model in which the advertising expenditures, the screening number, and the subsidy offered to the theater are the variables. With the integrated supply chain as the benchmark, we have proved that a subsidy offered by the distributor can stimulate the theater to arrange more screenings for a particular movie, so more watching demand for the movie will be spurred and satisfied and then the segregated supply chain is improved. Finally, we conducted numerical studies to examine the robustness of our model.

Future work will be devoted to the investigation of how to improve the performance of the movie supply chain when there is obvious internal competition among distributors themselves and theaters themselves. Furthermore, it is also worth looking at the impact, on the movie supply chain, of the situation that some distributors decide to offer a subsidy to the theater while others do not.

Appendix A

Using equation (2), the first derivative of A is,

$$\frac{d\pi P}{dA} = rSPNe^{-rA}(1-\alpha) - 1$$

and the second derivative of A is,

$$\frac{d^2\pi P}{d^2A} = -r^2SPNe^{-rA} < 0$$

Hence, by solving $\frac{d\pi P}{dA} = 0$, we can get $A_P = -\frac{1}{r}\ln\frac{1}{r\alpha SPN}$, which is set at first by the distributor. Substituting A_P into the equation of πT, then πT is rewritten as

$$\pi T = \alpha SP\left(\alpha N - \frac{1}{r(1-\alpha)SP}\right) - cN^2.$$

The first derivative of N is

$$\frac{d\pi T}{dA} = SPa_0 - 2cN,$$

and the second derivative of A,

$$\frac{d^2\pi T}{d^2N} = -2c < 0$$

With the solution of $\frac{d\pi T}{dN} = 0$, the optimal screening for theater is $N^*_{T1} = \frac{a_0 SP\alpha}{2c}$, then according to the equation of A_P, we can calculate that $A^*_{P1} = -\frac{1}{r}\ln\frac{2c}{ra_0\alpha(1-\alpha)(SP)^2}$.

Appendix B

The first derivative of πC concerning N and A are as follows:

$$\frac{d\pi C}{dN} = \frac{d(PNS(\alpha - e^{-rA}) - cN^2 - A)}{dN}$$
$$= SP(a_0 - e^{-rA}) - 2cN$$

$$\frac{d\pi C}{dA} = SPNre^{-rA} - 1$$

Solving $\frac{d\pi C}{dN} = 0$ and $\frac{d\pi C}{dA} = 0$ simultaneously, we get the result $N_C^* = \frac{a_0 SP}{4c} + \frac{\sqrt{a_0{}^2(SP)^2 - \frac{8c}{r}}}{4c}$ and $A_C^* = -\frac{1}{r}\ln\frac{4c}{SPr(a_0SP+\sqrt{a_0{}^2(SP)^2-\frac{8c}{r}}}$ as equation (4) and (5) in proposition 2.

We then examine the second derivative of πC,

$$\frac{d^2\pi C}{d^2N} = -2c < 0$$

$$\frac{d^2\pi C}{d^2A} = -r^2 SPNe^{-rA} < 0$$

$$\frac{d^2\pi C}{dN \cdot dA} = 0$$

It can be easily seen that the Hessian matrix related with function of πC is always negative. Therefore, the conditions of getting integrated decisions are sufficient.

Appendix C

Solving $\frac{d\pi P}{dN} = 0$ and $\frac{d\pi P}{dA} = 0$ separately, it can be got that $A_P = -\frac{1}{r}\ln\frac{4c}{SP(1-\alpha)Nr}$ and $N_P = \frac{SP(1-\alpha)\,(a_0 - e^{-rA})}{2f}$. Then we combine A_P and N_P together to calculate the value of f_P which is $f_P = \frac{SP(1-\alpha)a_0}{2N} - \frac{1}{2rN^2}$. Substituting A_P and f_P into πT next, then πT changes into

$$\pi T = SPN \cdot \left(a_0 - \frac{1}{rSP(1-\alpha)N}\right) - cN^2 + \frac{SPN(1-\alpha)a_0}{2} - \frac{1}{2r}$$

The first and second derivative of the rewritten πT in is as follows:

$$\frac{d\pi T}{dN} = SPa_0 - 2cN + \frac{SP(1-\alpha)a_0}{2}$$

$$\frac{d^2\pi T}{d^2N} = -2c < 0$$

Solving $\frac{d\pi T}{dN} = 0$, we get $N_{T2}^* = \frac{SP(1+\alpha)a_0}{4c}$. Finally substitute N_{T2}^* into A_P and f_P, f^* is got as below,

$$f^* = \frac{2cSP(1-\alpha)}{2SP + SP(1-\alpha)} - \frac{8c^2}{r(SP(3-\alpha)a_0)^2}$$

$$= \frac{2c(1-\alpha)}{1+\alpha} - \frac{8c^2}{r(SP(1+\alpha)a_0)^2}$$

$$A_{T2}^* = -\frac{1}{r}\ln\frac{4c}{rSP^2(1-\alpha^2)a_0}$$

Appendix D

In APPENDIX C, we get $A_P = -\frac{1}{r}\ln\frac{4c}{SP(1-\alpha)Nr}$ and $N_P = \frac{SP(1-\alpha)\,(a_0-e^{-rA})}{2f}$ by calculating $\frac{d\pi P}{dN} = 0$ and $\frac{d\pi P}{dA} = 0$. Therefore, $e^{-rA} = \frac{1}{rPS(1-\alpha)N}$ and $f = \frac{a_0PS(1-\alpha)}{2N} + \frac{1}{2rN^2}$ can be worked out from the equation of A_P and N_P.

Recalling the theater's net profit when a subsidy is given to the theater as equation (6), we substitute e^{-rA} and f into πT, then we can rewrite πT as below,

$$\pi T = \alpha PNS(a_0 - e^{-rA}) - (c - f)cN^2$$

$$= \alpha PNS\left(a_0 - \frac{1}{rPS(1-\alpha)N}\right) - \left(c - \frac{a_0PS(1-\alpha)}{2N} + \frac{1}{2rN^2}\right)N^2$$

$$= \alpha PNSa_0 - \frac{\alpha}{r(1-\alpha)} - cN^2 + \frac{a_0PS(1-\alpha)N}{2} - \frac{1}{2r}$$

Then we get the first derivative of πT concerning N as below,

$$\frac{d\pi T}{dN} = \alpha PSa_0 - 2cN + \frac{a_0PS(1-\alpha)}{2}$$

when $N \leq \frac{a_0PS(1+\alpha)}{4c}, \frac{d\pi T}{dN} \geq 0$, there is an inverse relationship between N and πT.

Because $N_C^* > N_{T2}^*$, the theater's net profit goes down when it changes the N from N_{T2}^* to N_C^* that the integrated supply chain sets.

Appendix E

Solving $\frac{d\pi C}{dN} = 0$ and $\frac{d\pi C}{dA} = 0$, we get that $A = \frac{1}{rPSN}$ and $e^{-rA} = \frac{1}{rPNS}$ which next are substituted into πC as below,

$$\pi C = PNS(a_0 - e^{-rA}) - cN^2 - A$$

$$= PNS\left(a_0 - \frac{1}{rPNS}\right) - cN^2 + \frac{1}{r}\ln\left(\frac{1}{rPSN}\right)$$

$$= a_0PNS - \frac{1}{r} - cN^2 + \frac{1}{r}\ln\left(\frac{1}{rPSN}\right)$$

Then we get the first derivative of πC concerning N as below,

$$\frac{d\pi C}{dN} = a_0PS - 2cN - \frac{1}{rN}$$

When $\frac{a_0SP}{4c} - \frac{\sqrt{{a_0}^2(SP)^2 - \frac{8c}{r}}}{4c} \leq N \leq \frac{a_0SP}{4c} + \frac{\sqrt{{a_0}^2(SP)^2 - \frac{8c}{r}}}{4c}$, $\frac{d\pi C}{dN} \geq 0$. Given $N_C^* = \frac{a_0SP}{4c} + \frac{\sqrt{{a_0}^2(SP)^2 - \frac{8c}{r}}}{4c}, N_{T1}^* = \frac{\alpha SPa_0}{2c}, N_{T2}^* = \frac{SP(1+\alpha)a_0}{4c}$ and $\frac{c}{r}$ is in its satisfying interval, it can be deduced that $\frac{a_0SP}{4c} - \frac{\sqrt{a_0{}^2(SP)^2 - \frac{8c}{r}}}{4c} < \frac{(1-\alpha)a_0SP}{4c}$ and it is obvious that $\frac{a_0SP}{4c} + \frac{\sqrt{a_0{}^2(SP)^2 - \frac{8c}{r}}}{4c} = N_C^* > N_{T2}^* > N_{T1}^* > \frac{a_0SP}{4c} - \frac{\sqrt{a_0{}^2(SP)^2 - \frac{8c}{r}}}{4c}$. That is to say that $\pi C_C^* > \pi C_{T2}^* > \pi C_{T1}^*$.

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