These three types of links are explained below and represented in Figure 1. They all follow the same form $P\left(y \prec = C_q\right) = \Phi\left(b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}\right)$ for a continuous cdf Φ .

Logit. Logit link function is the most widely used function for Proportional Odds Models. The logit link is shown in [1].

$$logit\left[P\left(y \prec= C_q\right)\right] = log\frac{P\left(y \prec= C_q\right)}{1- P\left(y \prec= C_q\right)} = b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}, \quad \text{q=1, ...,} Q - 1$$

or what is the same

$$P\left(y \prec= C_q\right) = \frac{1}{1+e^{-(b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x})}} \tag{1}$$

Probit. Probit link function is the inverse of the standard normal cummulative distribution function (cdf) Φ. Its expression is shown in 2.

$$\Phi^{-1}\left[P\left(y \prec = C_q\right)\right] = b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}, \quad \text{q= 1, ...,} Q - 1$$

$$P\left(y \prec = C_q\right) = \Phi(b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}), \quad \text{q= 1, ...,} Q - 1 \tag{2}$$

or what is the same

$$P\left(y \prec= C_q\right) = \int_{-\infty}^{b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx \tag{2}$$

Complementary log-log. Like the logit and the probit transformation, the complementary log-log transformation takes a response that is restricted to the (0,1) interval and converts it into something in (−infinito,+infinito) interval. Complementary log-log expression is shown in 3.

$$log\left[-log\left[1 - P(y \prec= C_q)\right]\right] = b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}, \quad \text{q= 1, ...,} Q - 1$$

$$P(y \prec= C_q) = 1 - \exp\left[-\exp(b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x})\right]$$

Logit and probit links are symmetric, that is

$$link[P(y \prec= C_q)] = -link\left[1 - P(y \prec= C_q)\right]$$

This means that the response curve for $P(y \prec= C_q)$ has a symmetric appearance about the point $P(y \prec= C_q) = 0.5$ and so $P(y \prec= C_q)$ has the same rate for approaching 0 as well as for approaching 1.

i) If we define $P(y \prec= C_q) \equiv \text{p}$, we have to

$$link(p) = \log it(p) = \log\left(\frac{p}{1-p}\right) = \log(p) - \log(1-p)$$

while

$$-link(1-p) = -\log it(1-p) = -\log\left(\frac{1-p}{p}\right) = -\log(1-p) + \log(p)$$

q.e.d.

ii) If we define

$$p \equiv P\left(y \prec= C_q\right) = \Phi(b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}) = \int_{-\infty}^{b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

then

$$probit\left(p\right) = \Phi^{-1}(p) = b_q - \mathbf{w}^{\mathbf{T}}\mathbf{x}$$

and

$$-probit\ \left(1-p\right) = \Phi^{-1}(1-p) = -b_q + \mathbf{w}^{\mathbf{T}}\mathbf{x}$$

de donde

$$1 - p = \int_{-\infty}^{-b_q + \mathbf{w}^{\mathbf{T}}\mathbf{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

from where

$$p = 1 - \int_{-\infty}^{-b_q + \mathbf{w}^{\mathbf{T}}\mathbf{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

q.e.d.

Unlike logit and probit the complementary log-log model is asymmetrical, it is frequently used when the probability of an event is very small or very large. When the data given is not symmetric in the [0,1] interval and increase slowly at small to moderate value but increases sharply near 1. The logit and probit models are inappropriate. However, in this situation, the complementary log-log model might give a satisfied answer.