Hyperbolic Trig Identities

Version 1

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1 Introduction

A compiled list of the most commonly used hyperbolic trig identities. Enjoy.

All credits goes to math2.org. If you have comments, corrections, or clarifications, please submit an issue or pull request on GitHub.

2 Hyperbolic Definitions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

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$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x - e^{-x}}$$
$$\coth x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\cosh^2 x - \sinh^2 x = 1$$
$$\tanh^2 x + \operatorname{sech}^2 x$$
$$\coth^2 x - \operatorname{csch}^2 x$$

3 Inverse Hyperbolic Definitions

$$\operatorname{arcsinh} z = \ln\left(z + \sqrt{z^2+1}\right)$$
$$\operatorname{arccosh} z = \ln\left(z \pm \sqrt{z^2-1}\right)$$
$$\operatorname{arctanh} z = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$$

$$\operatorname{arccsch} z = \ln\left(\frac{1+\sqrt{1+z^2}}{z}\right)$$
$$\operatorname{arcsech} z = \ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$$
$$\operatorname{arccoth} z = \frac{1}{2}\ln\left(\frac{z+1}{z-1}\right)$$

4 Relations to Trigonometric Functions

$$\sinh z = -i\sin(iz)$$
$$\operatorname{csch} z = i\csc(iz)$$

$$\cos z = \cos(iz)$$
$$\operatorname{sech} z = \sec(iz)$$

$$\tanh z = -i\tan(iz)$$
$$\coth z = i\cot(iz)$$