

# CH10009 Workshop Questions

Fiona Dickinson

2020-11-02



# Contents

<b>Welcome</b>	<b>5</b>
Workshops for CH10009 . . . . .	5
Version history . . . . .	5
<b>1 Week 1</b>	<b>7</b>
1.1 Preliminary infomation . . . . .	7
1.2 Questions . . . . .	9
1.3 Answers . . . . .	11
<b>2 Week 2</b>	<b>13</b>
2.1 Preliminary infomation . . . . .	13
2.2 Questions . . . . .	14
2.3 Answers . . . . .	16
<b>3 Week 3</b>	<b>17</b>
3.1 Preliminary infomation . . . . .	17
3.2 Questions . . . . .	19
3.3 Answers . . . . .	21
<b>4 Week 4</b>	<b>23</b>
4.1 Preliminary infomation . . . . .	23
4.2 Questions . . . . .	23
4.3 Answers . . . . .	25
<b>5 Week 5</b>	<b>27</b>
5.1 Preliminary infomation . . . . .	27
5.2 Partial differentiation . . . . .	28
5.3 Questions . . . . .	28
5.4 Answers . . . . .	29



# Welcome

The notes have been prepared in a package called BookDown for RStudio so that the equations are accessible to screen readers. However, by providing the notes as a .html webpage I can also embed short videos to further describe some of the topics. Further you can download the questions (and later the answers, top left of the screen) in a format that suits you (either pdf or epub) to view offline, or change the way this document appears for ease of reading.

If you spot any typos or think there are any errors please let me know and I will do my best to fix them.

## Workshops for CH10009

The topics for LOILS each week are as follows:

- Week 1: General Q&A
- Week 2: Rearranging equations, units and standard form
- Week 3: Logarithms and exponentials
- Week 4: Tables and graphs
- Week 5: Calculus - differentiation - the basics and the chain rule
- Week 6: Calculus - differentiation - the product rule and partial differentiation
- Week 7: Calculus - integration - the basics and definite integrals
- Week 8: Some more examples of integration and revision

This ‘book’ will be updated weekly with workshop questions, answers will be provided and some answers will include ‘process’ as well as answer. Please contact me if you need help.

I am using this format as it is an accessible format.

## Version history

Week 3 workshop released 19th October 2020

Week 2 workshop released 12th October 2020.

Some more video answers for workshop 1 embedded 11th October 2020.

Video answers for workshop 1 embedded 09th October 2020.

The initial commit of this book is dated 2nd October 2020.

# Chapter 1

## Week 1

### 1.1 Preliminary information

#### 1.1.1 SI base units

The SI system of base units has seven fundamental units from which the others are derived.

Table 1.1: The seven base units from which all others are derived.

SI base unit	symbol	quantity symbol (dimension)	quantity
kilogram	kg	M	mass
metre	m	L	length
second	s	T	time
ampere	A	I	electric current
kelvin	K	$\Theta$	temperature
mole	mol	N	amount of substance
candela	cd	J	luminous intensity

#### 1.1.2 SI Derived units

Table 1.2: Some common SI derived units used in chemistry.

symbol	SI derived unit	quantity	SI base units	other SI units
Hz	hertz	frequency	$s^{-1}$	
N	newton	force	$kg\ m\ s^{-2}$	
Pa	pascal	pressure	$kg\ m^{-1}\ s^{-2}$	$N\ m^{-2}$
J	joule	energy	$kg\ m^2\ s^{-2}$	$N\ m$
W	watt	power	$kg\ m^2\ s^{-3}$	$J\ s^{-1}$

symbol	SI derived unit	quantity	SI base units	other SI units
C	coulomb	electrical charge	A s	
V	volt	electrical potential	kg m <sup>2</sup> s <sup>-3</sup> A <sup>-1</sup>	J C <sup>-1</sup>
F	farad	electrical capacitance	kg <sup>-1</sup> m <sup>-2</sup> s <sup>4</sup> A <sup>2</sup>	C V <sup>-1</sup>
$\Omega$	ohm	electrical resistance	kg m <sup>2</sup> s <sup>-3</sup> A <sup>-2</sup>	V A <sup>-1</sup>
S	siemens	electrical conductance	kg <sup>-1</sup> m <sup>-2</sup> s <sup>3</sup> A <sup>2</sup>	A V <sup>-1</sup> or 1/ $\Omega$

### 1.1.3 Other units and conversion factors

There are a number of non-SI base or derived units which are in common usage which are useful to know and be able to convert between. Table 1.3 contains a number of useful unit conversions.

Table 1.3: The relationship between some other common units and the SI system.

unit	quantity	SI equivalent
torr (or mm Hg)	pressure	$\frac{101325}{760}$ Pa
atm	pressure	101325 Pa
bar	pressure	100000 Pa
eV	energy	$1.602176634 \times 10^{-19}$ J
cal	energy	4.184 J
Å	length	$1 \times 10^{-10}$ m

There are myriad other units in use, either with historical or geographic preference, or just for niche purposes (where would we be without olympic swimming pools or London buses). Examples such as the mile, furlong or beard-second are all units of length.

Further, the unit °C is formally an SI derived unit. The temperature in Kelvin is:

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$$

### 1.1.4 SI prefixes and standard form

In general lower case prefixes are used for negative powers and upper case prefixes are used for positive powers, however k (kilo) is an obvious exception to this rule. (Other exceptions are da (deca, 10<sup>1</sup>) and h (hecto, 10<sup>2</sup>)).



Table 1.4: The more common SI prefixes used in chemistry.

SI prefix	SI prefix 'name'	standard form multiplier
y	yocto	$10^{-24}$
z	zepto	$10^{-21}$
a	atto	$10^{-18}$
f	femto	$10^{-15}$
p	pico	$10^{-12}$
n	nano	$10^{-9}$
	micro	$10^{-6}$
m	milli	$10^{-3}$
c	centi	$10^{-2}$
d	deci	$10^{-1}$
k	kilo	$10^3$

## 1.2 Questions

### 1.2.1 Rearranging equations

Answers for these questions are in Section 1.3.1.

For each of the following rearrange to make the specified variable the subject of the equation.

1.  $[A] = [A]_0 - kt$ ,  $t$
2.  $E = \frac{1}{2}mv^2$ ,  $v$
3.  $F = \frac{q_1q_2}{4\pi\epsilon_0r^2}$ ,  $r$
4.  $\frac{1}{[A]} = \frac{1}{[A]_0} + kt$ ,  $[A]_0$
5.  $\ln(x_A) = -\frac{\Delta H}{R}(\frac{1}{T_1} - \frac{1}{T_2})$ ,  $T_1$
6.  $K_a = \frac{\alpha^2c}{1-\alpha}$ ,  $\alpha$

### 1.2.2 Unit conversion questions

Answers for these questions are in Section 1.3.2.

1. Convert the following:
    - a.  $3.4 \mu\text{m}$  to mm and m
    - b.  $270.4 \text{ g mol}^{-1}$  to  $\text{kg mol}^{-1}$  and  $\text{yg (molecule}^{-1})$
    - c.  $23.4 \text{ g dm}^{-3}$  to  $\text{mg dm}^{-3}$ ,  $\text{g m}^{-3}$ , and  $\text{kg m}^{-3}$
    - d.  $17.5 \mu\text{Hz}$  to Hz
    - e.  $5796 \text{ cm}^{-1}$  to  $\mu\text{m}^{-1}$  and  $\text{m}^{-1}$
-

2. If a box has dimensions 0.234 m x 34.5 cm x 26.8 mm. What is the volume of the box in:
- cm<sup>3</sup>?
  - dm<sup>3</sup>?
  - m<sup>3</sup>?
  - Å<sup>3</sup>?

3. The Gibbs free energy of a reaction,  $\Delta G$  is given by equation (1.1).

$$\Delta G = \Delta H - T\Delta S \quad (1.1)$$

Determine the value of  $\Delta G$  at 40 °C when the enthalpy of reaction,  $\Delta H = -10.235 \text{ kJ mol}^{-1}$  and the molar entropy,  $\Delta S = +34 \text{ J K}^{-1} \text{ mol}^{-1}$

### 1.2.3 Determining units of variables in equations

Answers for these questions are in Section 1.3.3.

1. The ideal gas equation is given in equation (1.2).

$$pV = nRT \quad (1.2)$$

The units of the variables are:  $p$  (pressure), Pa (pascals)  $V$  (volume), m<sup>3</sup>  $n$  (number of moles), mol  $T$  (absolute temperature), K

- Determine the SI base units of the gas constant,  $R$ .
- Determine the pressure in mmHg of 1.00 mmol of an ideal gas that occupies 1.65 dm<sup>3</sup> at 25 °C.

2. The famous Einstein equation  $E = mc^2$  is more properly written as:

$$E^2 = p^2c^2 + m_0^2c^4$$

Determine the units of the variable  $p$ .

3. At low temperatures the molar heat capacity of a material  $C_{p,m}$  (J K<sup>-1</sup> mol<sup>-1</sup>) is given by equation (1.3).

$$C_{p,m} = aT^3 \quad (1.3)$$

Determine the units of the constant,  $a$ .

- 
4. Determine the units of the coulomb constant,  $k_e$ , in equation (1.4), given that  $r$  is the separation of two charges,  $F$  the force of attraction between the two charges, and  $q_x$  is the charge (in coulombs, C) on each of the particles.

$$F = k_e \frac{q_1 q_2}{r^2} \quad (1.4)$$

## 1.3 Answers

### 1.3.1 Rearranging equations answers

1.  $t = \frac{[A]_0 - [A]}{k}$
2.  $v = \sqrt{\frac{2E}{m}}$
3.  $r = \sqrt{\frac{q_1 q_2}{4\pi\epsilon_0 F}}$
4.  $[A]_0 = \frac{[A]}{1 - [A]kt}$
5.  $\frac{\Delta H T_2}{\Delta H - RT \ln x_A}$
1.  $\alpha = \frac{-K_a \pm \sqrt{K_a^2 + 4cK_a}}{2c}$

### 1.3.2 Unit conversion answers

1.
  - a.  $3.4 \times 10^{-3}$  mm;  $3.4 \times 10^{-6}$  m
  - b.  $0.2704$  kg mol $^{-1}$ ;  $449.0$  yg
  - c.  $23.4 \times 10^{-3}$  mg dm $^{-3}$ ;  $23.4 \times 10^{-3}$  g m $^{-3}$ ; and  $23.4 \times 10^{-6}$  kg m $^{-3}$
  - d.  $17.5 \times 10^{-6}$  Hz
  - e.  $0.5796$   $\mu$ m $^{-1}$  and  $5.796 \times 10^5$  m $^{-1}$

- 
2.
    - a.  $2.16 \times 10^3$  cm $^3$
    - b.  $2.16$  dm $^3$
    - c.  $2.16 \times 10^{-3}$  m $^3$
    - d.  $2.16 \times 10^{27}$  Å $^3$
- 

3.  $-21$  kJ mol $^{-1}$  - please note this value is correct to the appropriate sig figs

### 1.3.3 Determining units of variables in equations answers

1.
  - a. • kg m $^2$  s $^{-2}$  K $^{-1}$  mol $^{-1}$  (this is more usually written as J K $^{-1}$  mol $^{-1}$ )
  - b. •  $11.3$  mm Hg ( $1.50$  kPa)

2.  $\text{kg m s}^{-1}$

3.  $\text{J K}^{-4} \text{ mol}^{-1}$

4.  $\text{kg m}^3 \text{ s}^{-4} \text{ A}^{-2}$  or  $\text{N m}^2 \text{ C}^{-2}$

# Chapter 2

## Week 2

### 2.1 Preliminary information

#### 2.1.1 Rules of powers and exponents

$$m^a \times m^b = m^{a+b} \quad (2.1)$$

$$\frac{p^a}{p^b} = p^{a-b} \quad (2.2)$$

$$(q^a)^b = q^{a \times b} \quad (2.3)$$

*Anything* raised to the power 0 is equal to 1.

$$x^0 = 1$$

Roots may be expressed as fractional powers:

$$\sqrt[n]{x} = x^{\frac{1}{n}} \quad (2.4)$$

When we see negative powers it is the same as the inverser of the positive power.

$$x^{-n} = x^{\frac{1}{-n}} \quad (2.5)$$

### 2.1.2 Rules of logs

Logs are the inverse function of exponents, and can have many bases:

When we use ‘natural logs’ we use the terminology  $\ln$ , a natural log is the inverse of ‘ $e$ ’.

$$x = \ln e^x \quad (2.6)$$

Other logs are usually marked with the *base*, however if no base is indicated it should be considered that this is  $\log_{10}$ .

$$x = \log_{10} 10^x \quad (2.7)$$

When combining logs (these rules are the same regardless of base):

$$\log_x A + \log_x B = \log_x (AB) \quad (2.8)$$

$$\log_x A - \log_x B = \log_x \left( \frac{A}{B} \right) \quad (2.9)$$

$$\log_x (A^n) = n \log_x A \quad (2.10)$$

If we want to change the bases of logs (such as in the Beer-Lambert law):

$$\log_b A = \frac{\log_x A}{\log_x b} \quad (2.11)$$

## 2.2 Questions

### 2.2.1 Simple log practice

Answers for these questions are in Section 2.3.1.

Evaluate the following expressions:

1.  $\log_{10} 10^6$
2.  $\log_{10} 10^{-5}$
3.  $\log_{10} (5^4 \times 3^{-2})$
4.  $\ln \pi 6^2$
5.  $e^{\log_e x} = \ln y$

### 2.2.2 Rearranging equations

Answers for these questions are in Section 2.3.2

1. Rearrange the following to make the highlighted term the subject:

- $\Delta S = k_B \ln W$ ,  $W$
- $\Delta S = nR \ln \frac{V_f}{V_i}$ ,  $V_f$
- $\nu = \frac{1}{2\pi} \left( \frac{k}{\mu} \right)^{\frac{1}{2}}$ ,  $\mu$
- $\ln K = \frac{nFE}{RT}$ ,  $E$
- $\ln K' - \ln K = \frac{\Delta H}{R} \left( \frac{1}{T} - \frac{1}{T'} \right)$ ,  $\Delta H$

2. The integrated rate equation for a first order reaction is  $[A] = [A]_0 e^{-kt}$ .
- Rearrange this equation in order to make  $k$  the subject.
  - What units must  $k$  have?

### 2.2.3 pH question.

Answers for these questions are in Section 2.3.3

HCl fully dissociates in water. If 5 cm<sup>3</sup> (measured using a glass pipette) of 38% w/w HCl solution ( $\rho = 1.189 \text{ kg dm}^{-3}$ ) is ‘added to 20 cm<sup>3</sup> water’ made up in a 25 cm<sup>3</sup> standard flask.

- What is the pH of the resulting solution?
- What mass of NaOH is required to neutralise the resulting solution?

*Hint: w/w means weight-weight, i.e. the number of g in 100 g. In this case 38 g of HCl in 100 g total of mixture.*

*Hint: you will need to think about units and rearranging equations from Week 1.*

### 2.2.4 pK<sub>a</sub> question.

Answers for these questions are in Section 2.3.4

The degree of dissociation of an acid,  $\alpha$  is related to the acid dissociation constant,  $K_a$  and the concentration of the acid,  $c$ , as shown in equation (2.12)

$$K_a = \frac{\alpha^2 c}{1 - \alpha} \quad (2.12)$$

Determine the pH of hydrofluoric acid solutions ( $\text{p}K_a = 3.18$ ) when the concentration of acid is:

- 1.00 M
- 2.50 mM

*Hint: We rearranged this equation last week for  $\alpha$*

## 2.3 Answers

### 2.3.1 Simple log practice answers

1. 6
2. -5
3. 1.841...
4. 4.728...
5.  $x = \ln y$

### 2.3.2 Rearranging equations answers

1. Rearrange the following to make the highlighted term the subject:

- a.  $W = e^{\frac{\Delta S}{k_B}}$

- b.  $V_f = V_i e^{\frac{\Delta S}{nR}}$

- c.  $\mu = \frac{1}{4\pi^2} \frac{k}{\nu^2}$

- d.  $E = \frac{RT}{nF} \ln K$

- e.  $\Delta H = \left( \frac{TT'}{T'-T} \right) R \ln \frac{K'}{K}$

---

2.
  - a.  $k = \frac{\ln[A]_0 - \ln[A]}{t}$
  - b.  $s^{-1}$

### 2.3.3 pH answer

- a. pH -0.394
  - b.  $m_{\text{NaOH}} = 2.5 \text{ g}$

### 2.3.4 pK<sub>a</sub> answer

- a. Two roots: 0.025 and -0.026 and we can't have a negative degree of dissociation. pH 1.6
  - b. Two roots: 0.40 and -0.66 and we can't have a negative degree of dissociation. pH 3.0



# Chapter 3

## Week 3

### 3.1 Preliminary information

#### 3.1.1 Presentation of units on tables and graphs

Your tables should have units expressed in the column headers where appropriate, and uncertainties should be expressed in the header if they are constant across the whole range, or in parentheses for each measurement if different. Units should be expressed as “Quantity / unit” to allow you to express them as dimensionless quantities in the table or on the axis of your graph (please note brackets are absolutely incorrect).

This is because this is how our governing body (IUPAC) tell us how to do it. See the IUPAC Green Book (2012 edition, page 3).

You can think of the horizontal line below the header as an equals sign, so:

Table 3.1: The effect of potassium iodide concentration on emission intensity and fluorescence lifetime of acridone in aqueous solution.

[KI] / M	[KNO <sub>2</sub> ] / M	Emission intensity / 10 <sup>3</sup> cps	/ ns
0	1.100	16.580	17.60
0.040	1.060	3.753	3.90
0.100	1.000	1.566	1.80
0.200	0.900	0.721	0.95
0.300	0.800	0.446	0.64
0.500	0.600	0.242	0.39
0.800	0.300	0.121	0.25

in the table above the third value down in the first column can be expressed as:

$$[\text{KI}] / \text{M} = 0.100$$

which of course rearranges to:

$$[\text{KI}] = 0.100 \text{ M.}$$

If we look at the top value in the third column:

$$\text{Emission intensity} / 10^3 \text{ cps} = 16.580$$

which of course rearranges to:

$$\text{Emission intensity} = 16.580 \times 10^3 \text{ cps}$$

### 3.1.2 Plotting of graphs

When ever we plot a graph we have to determine what we plot on each of the two axes.

The independent variable is the variable we have control over, we plot it (or something related to it) on the x-axis (the horizontal axis).

The dependent variable is the variable we measure, we plot it (or something related to it) on the y-axis (the vertical axis).

These axis should under most circumstances not contain constants unless those constants are specific to the experiment.

There is no need to start a graph axis at zero - you do not need a 'squiggle' on the axis to show that it is not starting at zero (that is what the numbers on the axis tell you), this squiggle (or more formally an axis break) does have a job when plotting graphs, but usually the use by first year undergraduates is incorrect.

When plotting a graph, we want to have broken down the equation we are using to fit the linear relationship:

$$y = mx + c \tag{3.1}$$

We should never force a graph through a particular intercept (c).

It is important when we ever have performed a calculation to determine values to plot on either x or y axis that we have taken account for any multipliers in the header, and that units are SI (such as temperature in K!).

A gradient is calculated by determining the change in y divided by the change in x:

$$m = \frac{\Delta y}{\Delta x}$$

Don't forget units on gradients! The units of a gradient are the units on y divided by the units on x.

### 3.1.3 Example of plotting graphs

The Stern-Volmer (Equation (3.4)) shows how the lifetime  $\tau$  of a dye changes with concentration of a quencher,  $[Q]$ . The data in Table 3.1 can be modeled using the Stern-Volmer equation, you will note that this equation uses terms with a subscript 0 ( $\tau_0$ ), when values like this are frequently used in chemistry (and physics).

$$\frac{\tau_0}{\tau} = 1 + k[Q] \quad (3.2)$$

where  $k = k_q\tau_0$ .

These ‘subscript 0’ values are referencing values when the independent variable is zero, so  $\tau_0 = 17.60$  ns.

You will notice that Equation (3.4) is already arranged into  $y = mx + c$ , where the x variable is  $[Q]$  and the y variable is  $\frac{\tau_0}{\tau}$ . This means that the gradient is  $k$ , and the intercept 1.

## 3.2 Questions

When plotting graphs feel free to plot either by hand, in python, excel or numbers. The choice is entirely up to you.

### 3.2.1 Sketch graphs

Answers for these questions are in Section 3.3.1.

For the following equations sketch suitable linear plots indicating values of the intercept and gradient on each sketch.

1. Show how absorption,  $A$ , changes with concentration,  $c$ :  $A = \epsilon cl$
2. Show how pressure,  $p$ , changes with temperature,  $T$ :  $pV = nRT$
3. Show how pressure,  $p$ , changes with volume,  $V$ :  $pV = nRT$

### 3.2.2 Second order kinetics

Answers for these questions are in Section 3.3.2.

Use the following table of data to plot a graph of  $\frac{1}{[A]}$  against  $t$ . Find the gradient and intercept.

(Just to note here you are plotting the equation  $\frac{1}{[A]} = \frac{1}{[A]_0} + kt$ ).

t / s	$10^4 [A] / \text{mol dm}^{-3}$
100	21.0

t / s	$10^4$ [A] / mol dm <sup>-3</sup>
200	12.0
300	8.4
400	7.1
500	5.6

### 3.2.3 Clausius-Clapeyron equation

Answers for these questions are in Section 3.3.4.

The relationship between the vapour pressure and temperature of diethyl ether (Table 3.3) can be modeled using the Clausius-Clapeyron equation (Equation (3.3)) to determine the enthalpy of vapourisation  $\Delta_{vap}H^\circ$ .

$$\ln p = -\frac{\Delta_{vap}H^\circ}{RT} + C \quad (3.3)$$

1. Determine  $\Delta_{vap}H^\circ$ .
2. Determine the temperature diethyl ether boils at, at 1 atmosphere (760 mm Hg). *Hint: determine C*

Table 3.3: The measured vapor pressure of ether (in mm Hg) at sub-zero temperatures.

p / mm Hg	T / °C
17	-38
28	-30
40	-25
55	-20
75	-15
97	-10
125	-5
157	0

### 3.2.4 Arrhenius equation

Answers for these questions are in Section 3.3.4.

The Arrhenius equation (Equation (3.4)) shows how the rate of reaction,  $k$ , depends upon the temperature of the reaction,  $T$ .

$$k = Ae^{-\frac{E_a}{RT}} \quad (3.4)$$

For the data shown in Table 3.4 plot an appropriate linear plot in order to

determine the activation energy,  $E_a$ , and the pre-exponential factor,  $A$ .

Table 3.4: The variation of measured rate constant with temperature for an undergraduate's experiment.

$T / ^\circ\text{C}$	$k / 10^5 \text{ s}^{-1}$
283	0.000352
356	0.0302
393	0.219
427	1.16
508	39.5

### 3.3 Answers

#### 3.3.1 Sketch graphs

1.

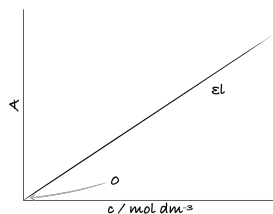


Figure 3.1: The Beer-Lambert relationship to show how absorbance, the dependent variable, changes with concentration, the independent variable.

2.

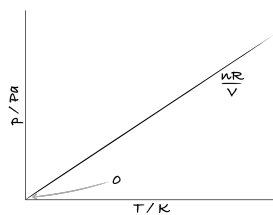


Figure 3.2: A sketch to show how the pressure of an ideal gas, the dependent variable, changes with temperature, the independent variable.

3.



Figure 3.3: A sketch to show how the pressure of an ideal gas, the dependent variable, changes with volume, the independent variable - note for a linear relationship we have  $1/V$  on the x-axis.

### 3.3.2 Second order kinetics

$$m = 3.19 \text{ M}^{-1} \text{ s}^{-1}, c = 180 \text{ M}^{-1} ([A]_0 = 5.56 \text{ mM})$$

### 3.3.3 Clausius-Clapeyron equation

$$\Delta_{vap}H^\circ = 31.6 \text{ kJ mol}^{-1}, 39^\circ\text{C}.$$

### 3.3.4 Arrhenius equation

$$m = -22.4 \times 10^3 \text{ K}, c = 43.7, E_a = 186 \text{ kJ mol}^{-1}, A = 9.7 \times 10^{18} \text{ s}^{-1}$$

# Chapter 4

## Week 4

### 4.1 Preliminary information

$f(x) \equiv y$	$f'(x) \equiv \frac{dy}{dx}$
$A$	$0$
$Ax^n$	$nAx^{n-1}$
$\frac{A}{x^n} \equiv Ax^{-n}$	$-nAx^{-n-1}$
$e^{Ax}$	$Ae^{Ax}$
$A \ln Bx$	$\frac{A}{x}$
$\sin Ax$	$A \cos Ax$
$\cos Ax$	$-A \sin Ax$
$\tan Ax$	$-A \sec^2 Ax$
$Au + Bv$	$A \frac{du}{dx} + B \frac{dv}{dx}$

Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

### 4.2 Questions

#### 4.2.1 'Simple' differentiation

1. Find  $\frac{dy}{dx}$  of the following functions:
  - a.  $y = 6x + 4$
  - b.  $y = 5x^2 + 3x - 7$

- c.  $y = x^{-\frac{1}{2}} + 3x - 4$
- d.  $6x^{\frac{2}{3}} - x + 9$

2. Find  $\frac{dy}{dx}$  of the following functions:

- a.  $y = 3x^2 + \cos x$
- b.  $y = 6 \ln 2x$
- c.  $y = \frac{2}{3}e^{\frac{3x}{2}}$
- d.  $y = -e^{-5x} - e^{5x}$

### 4.2.2 Turning points

Find the turning points  $(x, y)$  for the following functions:

- 1.  $y = 2x^3 - 9x^2 + 12x$
- 2.  $y = 12x^5 - 45x^4 + 40x^3$

### 4.2.3 Chain rule

Find the turning points  $(x, y)$  for the following functions:

- 1.  $y = \sin^2(x)$
- 2.  $y = \sin(e^{4x})$
- 3.  $y = e^{(7x^2+2x)}$
- 4.  $y = (x - 1)^9$

### 4.2.4 Applied questions

- 1. If two electric dipoles are aligned in an adjacent parallel manner such that they are separated by distance  $r$ , their potential energy of interaction,  $V$ , is given by:

$$V = \frac{\mu_1 \mu_2}{4\pi \epsilon_0 r^3}$$

determine  $\frac{dV}{dr}$

- 2. Light is scattered as it passes through solutions containing small particles with an intensity  $I$ , given by the following expression:

$$I = I_0 \frac{\pi \alpha^2}{\epsilon_r^2 \lambda^4 r^2} (\sin \phi)^2$$

determine expressions for how the intensity varies with:

- a. wavelength,
- b. angle,



- c. distance from scatterer,  $r$

*formally these are partial derivatives and we will learn about them next week*

3. The radius of a spherical balloon is increasing at a rate of  $0.2 \text{ m s}^{-1}$ . Find the rate of increase of (a) the volume and (b) the surface area of the balloon at the instant when the radius is  $1.6 \text{ m}$ .

## 4.3 Answers

### 4.3.1 ‘Simple’ differentiation

1.  $\frac{dy}{dx} =$ 
  - a. 6
  - b.  $10x + 3$
  - c.  $-\frac{1}{2}x - \frac{3}{2} + 3$
  - d.  $4x^{-\frac{1}{3}} - 1$
2.  $\frac{dy}{dx} =$ 
  - a.  $6x - \sin x$
  - b.  $\frac{6}{x}$
  - c.  $e^{\frac{3x}{2}}$
  - d.  $5e^{-5x} - 5e^{5x}$

### 4.3.2 Turning points

1. (1,5)max, (2,4)min
2. (0,0) POI(+/+), (1,7)max, (2, -16)min

### 4.3.3 Chain rule

1.  $\frac{dy}{dx} = 2 \sin x \cos x$
2.  $\frac{dy}{dx} = 4e^{4x} \cos(e^{4x})$
3.  $\frac{dy}{dx} = (14x + 2)e^{(7x^2+2x)}$
4.  $\frac{dy}{dx} = 9(x-1)^8$

### 4.3.4 Applied questions

- 1.

$$V = \frac{-3\mu_1\mu_2}{4\pi\epsilon_0 r^4}$$

2. •

a.

$$\frac{dI}{d\lambda} = -4I_0 \frac{\pi\alpha^2}{\varepsilon_r^2 \lambda^5 r^2} (\sin \phi)^2$$

b.

$$\frac{dI}{d\phi} = 2I_0 \frac{\pi\alpha^2}{\varepsilon_r^2 \lambda^4 r^2} (\sin \phi)(\cos \phi)$$

c.

$$\frac{dI}{dr} = -2I_0 \frac{\pi\alpha^2}{\varepsilon_r^2 \lambda^4 r^3} (\sin \phi)^2$$

$$3. \quad \frac{dV}{dt} = 0.8\pi r^2, \quad \frac{dSA}{dt} = 1.6\pi r$$

# Chapter 5

## Week 5

### 5.1 Preliminary information

A recap of standard differentials...

$f(x) \equiv y$	$f'(x) \equiv \frac{dy}{dx}$
$A$	0
$Ax^n$	$nAx^{n-1}$
$\frac{A}{x^n} \equiv Ax^{-n}$	$-nAx^{-n-1}$
$e^{Ax}$	$Ae^{Ax}$
$A \ln Bx$	$\frac{A}{x}$
$\sin Ax$	$A \cos Ax$
$\cos Ax$	$-A \sin Ax$
$\tan Ax$	$-A \sec^2 Ax$
$Au + Bv$	$A \frac{du}{dx} + B \frac{dv}{dx}$

If differentiating a function of a function we use the ‘Chain Rule’:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If differentiate a function times a function we use the ‘Product Rule’, if :

$$y = uv$$

...where  $u$  and  $v$  are different functions of  $x$ . The first derivative of this compound function may be found using the general differential form for products:

$$d(uv) = vdu + u dv \quad (5.1)$$

## 5.2 Partial differentiation

Any good scientist knows that you should only change one variable at a time to understand the effect it has on a system. This is exactly what partial differentiation does, although it recognises that there may be more than one variable in the system...

Partial differentiation uses a different notation to standard differentiation to denote there is more than one variable. Subscripts are used to denote variables which are kept constant.

If  $y$  is a function of both  $x$  and  $z$ , then we can differentiate with respect to either variable, in this case we treat the other variable as a constant and differentiate as we ever have. The notation for differentiating  $y$  with respect to  $z$  as  $x$  is kept constant would be:

$$\left(\frac{\partial y}{\partial z}\right)_x$$

In some circumstances you may wish to differentiate a function first by one variable and then by another, the notation in this case is:

$$\left(\frac{\partial^2 y}{\partial x \partial z}\right)$$

this is implying that the partial derivative of  $y$  with respect to  $x$  was determined first and then the partial derivative of that first derivative was determined with respect to  $z$ . It should not matter in which order you perform these functions, they should both yield the same answer.

## 5.3 Questions

### 5.3.1 Product rule questions

Differentiate the following using the product rule:

1.  $y = xe^{7x} + 7xe^x$
2.  $y = 5x \cos(2x)$

### 5.3.2 More complex differentiation questions

Differentiate the following (you will need to decide if you need the product rule or the chain rule):

1.  $y = x \ln x$
2.  $y = \sin^7(5x) + \cos^5(7x)$

### 5.3.3 Partial differentiation questions

For the following function determine:

$$\left(\frac{\partial y}{\partial z}\right)_x, \left(\frac{\partial y}{\partial x}\right)_z, \left(\frac{\partial^2 y}{\partial z^2}\right)_x, \left(\frac{\partial^2 y}{\partial x^2}\right)_z, \left(\frac{\partial^2 y}{\partial x \partial z}\right) \text{ and } \left(\frac{\partial^2 y}{\partial z \partial x}\right)$$

1.  $y = x^6 z - 3x^2 z^3 + x^2 z + 6z^6 + 5$
2.  $y = e^{3x} + ze^{2x} + x^2 e^{-z} - z^2$
3.  $y = \frac{\sin(3z)}{x} - \frac{1}{e^{3x} z^2}$
4.  $y = \sin(3z^2 + 2xz + x^2)^*$

\*this one is involved - it uses both the chain and the product rule!

## 5.4 Answers

### 5.4.1 Product rule questions

1.  $\frac{dy}{dx} = e^{7x} + 7xe^{7x} + 7e^x + 7xe^x$

### 5.4.2 More complex differentiation questions

1.  $\frac{dy}{dx} = \ln x + 1$
2.  $\frac{dy}{dx} = 35 \cos(5x) \sin^6(5x) - 35 \sin(7x) \cos^4(7x)$

### 5.4.3 Partial differentiation questions

1.  $\left(\frac{\partial y}{\partial z}\right)_x = x^6 - 9x^2 z^2 + x^2 + 36z^5$ ,  $\left(\frac{\partial y}{\partial x}\right)_z = 6x^5 z - 6xz^3 + 2xz$ ,  $\left(\frac{\partial^2 y}{\partial z^2}\right)_x = -18x^2 z + 180z^4$ ,  $\left(\frac{\partial^2 y}{\partial x^2}\right)_z = 30x^4 z - 6z^3$ ,  $\left(\frac{\partial^2 y}{\partial x \partial z}\right) = 6x^5 - 18xz^2 + 2x$  and  $\left(\frac{\partial^2 y}{\partial z \partial x}\right) = 6x^5 - 18xz^2 + 2x$
2.  $\left(\frac{\partial y}{\partial z}\right)_x = e^{2x} - x^2 e^{-z} - 2z$ ,  $\left(\frac{\partial y}{\partial x}\right)_z = 3e^{3x} + 2e^{2x} z + 2xe^{-z}$ ,  $\left(\frac{\partial^2 y}{\partial z^2}\right)_x = x^2 e^{-z} - 2$ ,  $\left(\frac{\partial^2 y}{\partial x^2}\right)_z = 9e^{3x} + 4e^{2x} z + 2e^{-z}$ ,  $\left(\frac{\partial^2 y}{\partial x \partial z}\right) = 2e^{2x} - 2xe^{-z}$  and  $\left(\frac{\partial^2 y}{\partial z \partial x}\right) = 2e^{2x} - 2xe^{-z}$
3.  $\left(\frac{\partial y}{\partial z}\right)_x = \frac{\cos(3z)}{x} + \frac{2}{e^{3x} z^3}$ ,  $\left(\frac{\partial y}{\partial x}\right)_z = -\frac{\sin(3z)}{x^2} + \frac{3}{e^{3x} z^2}$ ,  $\left(\frac{\partial^2 y}{\partial z^2}\right)_x = -9\frac{\sin(3z)}{x} - \frac{6}{e^{3x} z^4}$ ,  $\left(\frac{\partial^2 y}{\partial x^2}\right)_z = \frac{2\sin(3z)}{x^3} - \frac{9}{e^{3x} z^2}$ ,  $\left(\frac{\partial^2 y}{\partial x \partial z}\right) = \frac{-3\cos(3z)}{x^2} - \frac{6}{e^{3x} z^3}$  and  $\left(\frac{\partial^2 y}{\partial z \partial x}\right) = \frac{-3\cos(3z)}{x^2} - \frac{6}{e^{3x} z^3}$

4.  $\left(\frac{\partial y}{\partial z}\right)_x = (6z + 2x) \cos(3z^2 + 2xz + x^2)$ ,  $\left(\frac{\partial y}{\partial x}\right)_z = (2z + 2x) \cos(3z^2 + 2xz + x^2)$ ,  $\left(\frac{\partial^2 y}{\partial z^2}\right)_x = 2 \cos(3z^2 + 2xz + x^2) - (2z + 2x)^2 \sin(3z^2 + 2xz + x^2)$ ,  $\left(\frac{\partial^2 y}{\partial x^2}\right)_z = 6 \cos(3z^2 + 2xz + x^2) - (6z + 2x)^2 \sin(3z^2 + 2xz + x^2)$ ,  $\left(\frac{\partial^2 y}{\partial x \partial z}\right) = 2 \cos(3z^2 + 2xz + x^2) - (2z + 2x)(6z + 2x) \sin(3z^2 + 2xz + x^2)$  and  $\left(\frac{\partial^2 y}{\partial z \partial x}\right) = 2 \cos(3z^2 + 2xz + x^2) - (2z + 2x)(6z + 2x) \sin(3z^2 + 2xz + x^2)$   
 if you can do this one you can do them all!!!