课堂练习

练习1：证明如下等式： (1) $[\hat{A},\hat{B}\hat{C}]=[\hat{A},\hat{B}]\hat{C}+\hat{B}[\hat{A},\hat{C}]$； (2) $[\hat{A}\hat{B},\hat{C}]=\hat{A}[\hat{B},\hat{C}]+[\hat{A},\hat{C}]\hat{B}$

证明：因为：

$$\begin{aligned}[\hat{A},\hat{B}\hat{C}]&=\hat{A}(\hat{B}\hat{C})-(\hat{B}\hat{C})\hat{A}=\hat{A}\hat{B}\hat{C}-\hat{B}\hat{C}\hat{A}+\hat{B}\hat{A}\hat{C}-\hat{B}\hat{A}\hat{C}=(\hat{A}\hat{B}\hat{C}-\hat{B}\hat{A}\hat{C})+(-\hat{B}\hat{C}\hat{A}+\hat{B}\hat{A}\hat{C})\\&=(\hat{A}\hat{B}-\hat{B}\hat{A})\hat{C}+\hat{B}(\hat{A}\hat{C}-\hat{C}\hat{A})=[\hat{A},\hat{B}]\hat{C}+\hat{B}[\hat{A},\hat{C}]\end{aligned}$$

$$\begin{aligned}[\hat{A}\hat{B},\hat{C}]&=(\hat{A}\hat{B})\hat{C}-\hat{C}(\hat{A}\hat{B})=\hat{A}\hat{B}\hat{C}-\hat{C}\hat{A}\hat{B}+\hat{A}\hat{C}\hat{B}-\hat{A}\hat{C}\hat{B}=(\hat{A}\hat{B}\hat{C}-\hat{A}\hat{C}\hat{B})+(-\hat{C}\hat{A}\hat{B}+\hat{A}\hat{C}\hat{B})\\&=\hat{A}(\hat{B}\hat{C}-\hat{C}\hat{B})+(\hat{A}\hat{C}-\hat{C}\hat{A})\hat{B}=\hat{A}[\hat{B},\hat{C}]+[\hat{A},\hat{C}]\hat{B}\end{aligned}$$

故原题得证

练习2：以$|s_x+\rangle$和$|s_x-\rangle$为基矢来表示$|s_z\pm\rangle$和$|s_y\pm\rangle$

解：我们知道$|s_x+\rangle=\frac{1}{\sqrt{2}}(|s_z+\rangle+|s_z-\rangle)$，$|s_x-\rangle=\frac{1}{\sqrt{2}}(|s_z+\rangle-|s_z-\rangle)$，因此两式相加得 $|s_x+\rangle+|s_x-\rangle=\sqrt{2}|s_z+\rangle$，即$|s_z+\rangle=\frac{1}{\sqrt{2}}(|s_x+\rangle+|s_x-\rangle)$；两式相减得 $|s_x+\rangle-|s_x-\rangle=\sqrt{2}|s_z-\rangle$，即$|s_z-\rangle=\frac{1}{\sqrt{2}}(|s_x+\rangle-|s_x-\rangle)$。因此，$|s_z\pm\rangle$的表达式为 $|s_z\pm\rangle=\frac{1}{\sqrt{2}}(|s_x+\rangle\pm|s_x-\rangle)$。

我们再来看看$|s_y\pm\rangle$，由于$|s_y\pm\rangle=\frac{1}{\sqrt{2}}(|s_z+\rangle\pm\mathrm{i}|s_z-\rangle)$，因此将上述的$|s_z\pm\rangle$的表达式代入，得

$$|s_y+\rangle=\frac{1}{\sqrt{2}}(|s_z+\rangle+\mathrm{i}|s_z-\rangle)=\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(|s_x+\rangle+|s_x-\rangle)+\frac{\mathrm{i}}{\sqrt{2}}(|s_x+\rangle-|s_x-\rangle)]=\frac{1+\mathrm{i}}{2}|s_x+\rangle+\frac{1-\mathrm{i}}{2}|s_x-\rangle$$

$$|s_y-\rangle=\frac{1}{\sqrt{2}}(|s_z+\rangle-\mathrm{i}|s_z-\rangle)=\frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(|s_x+\rangle+|s_x-\rangle)-\frac{\mathrm{i}}{\sqrt{2}}(|s_x+\rangle-|s_x-\rangle)]=\frac{1-\mathrm{i}}{2}|s_x+\rangle+\frac{1+\mathrm{i}}{2}|s_x-\rangle$$

从而$|s_y\pm\rangle=\frac{1\pm\mathrm{i}}{2}|s_x+\rangle+\frac{1\mp\mathrm{i}}{2}|s_x-\rangle$