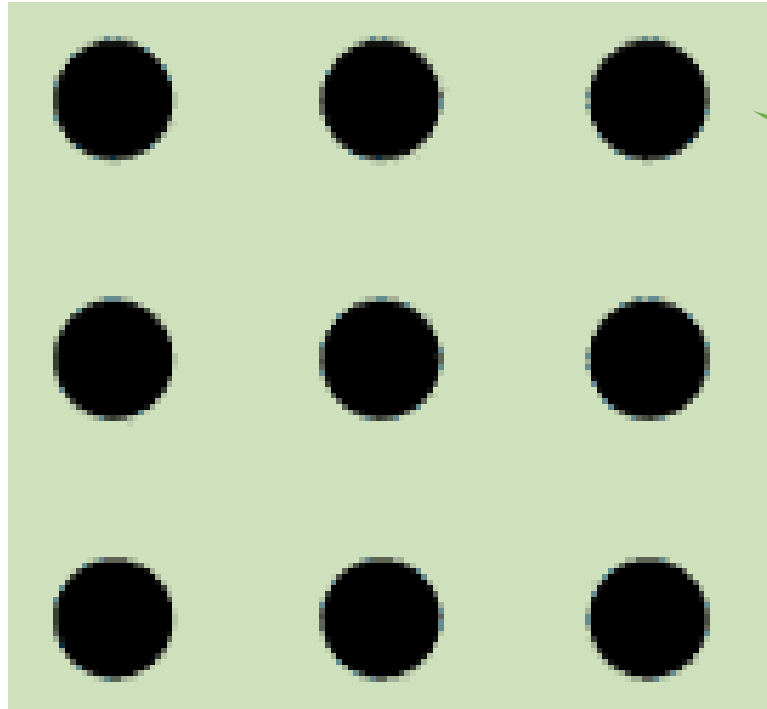
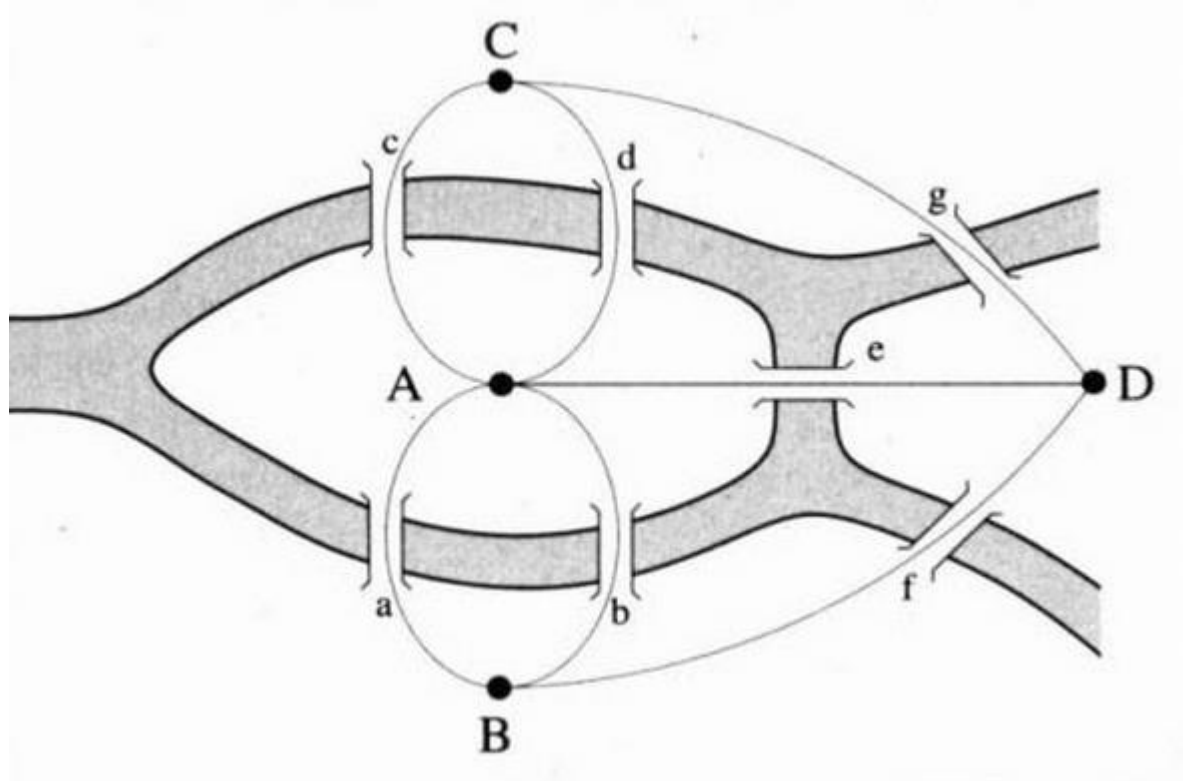


# Network Analysis

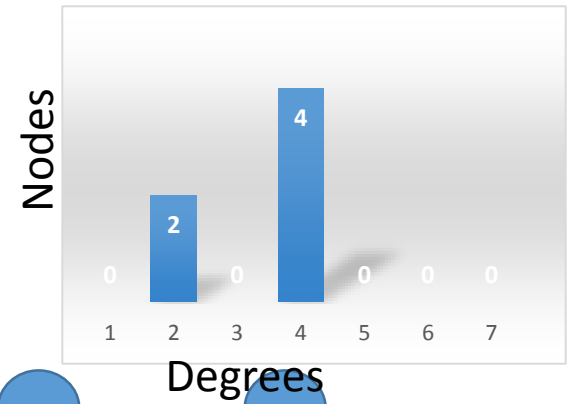
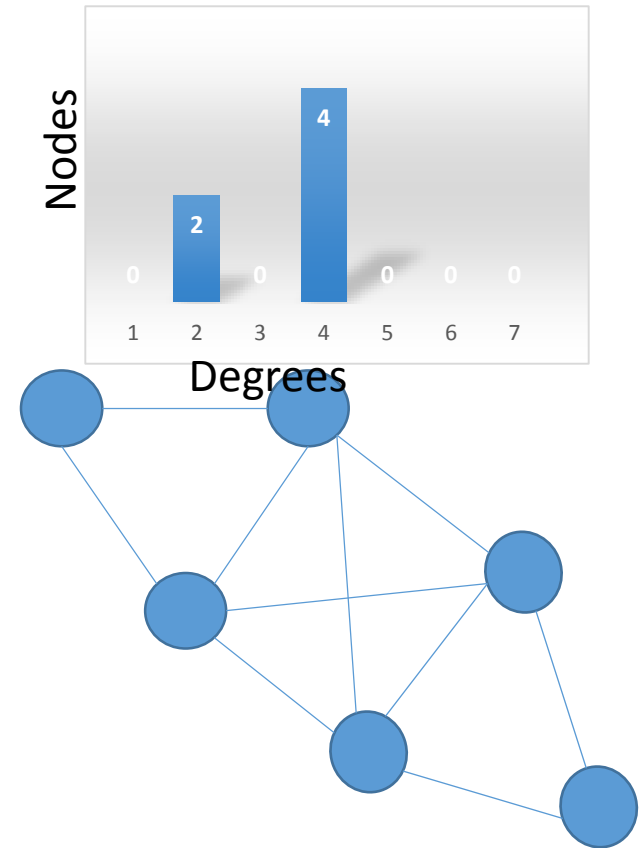
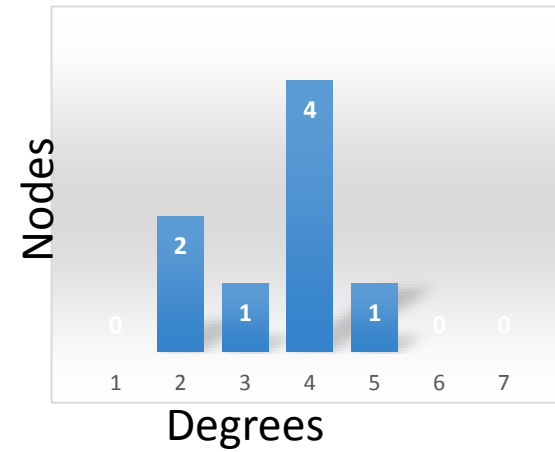
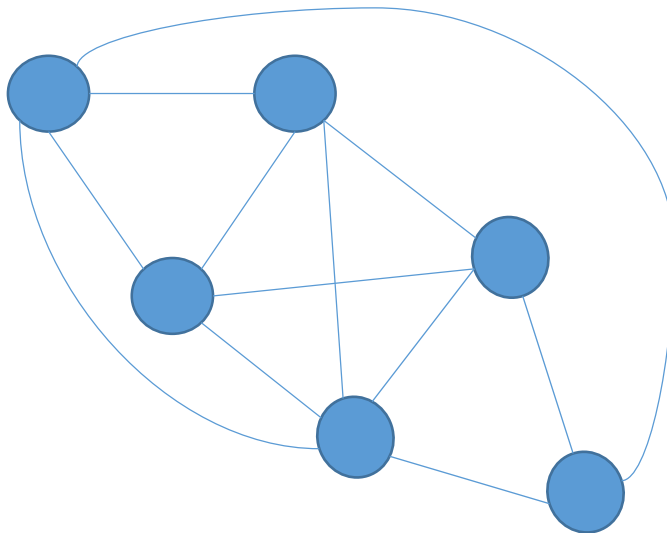
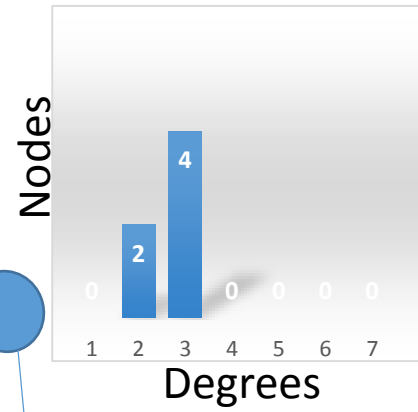
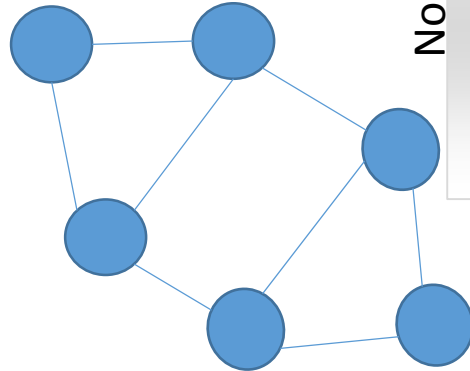
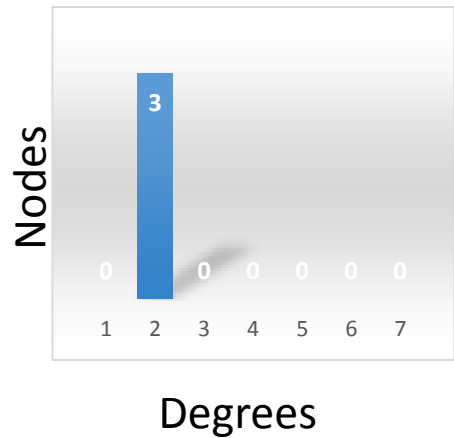
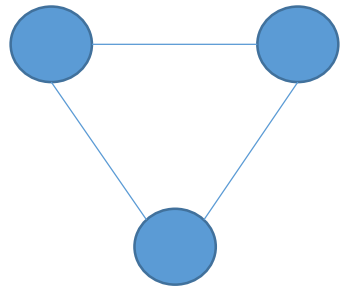


Connect all 9 points with 4 interconnected lines. Each point must be touched once. Last point must be touched twice. Lines are allowed to intersect. Keep the pencil point always on the paper.

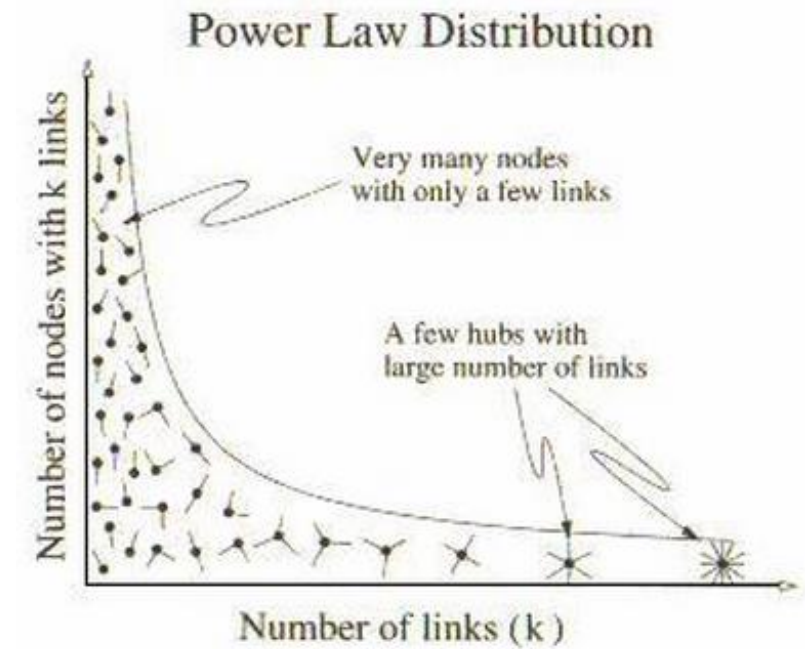
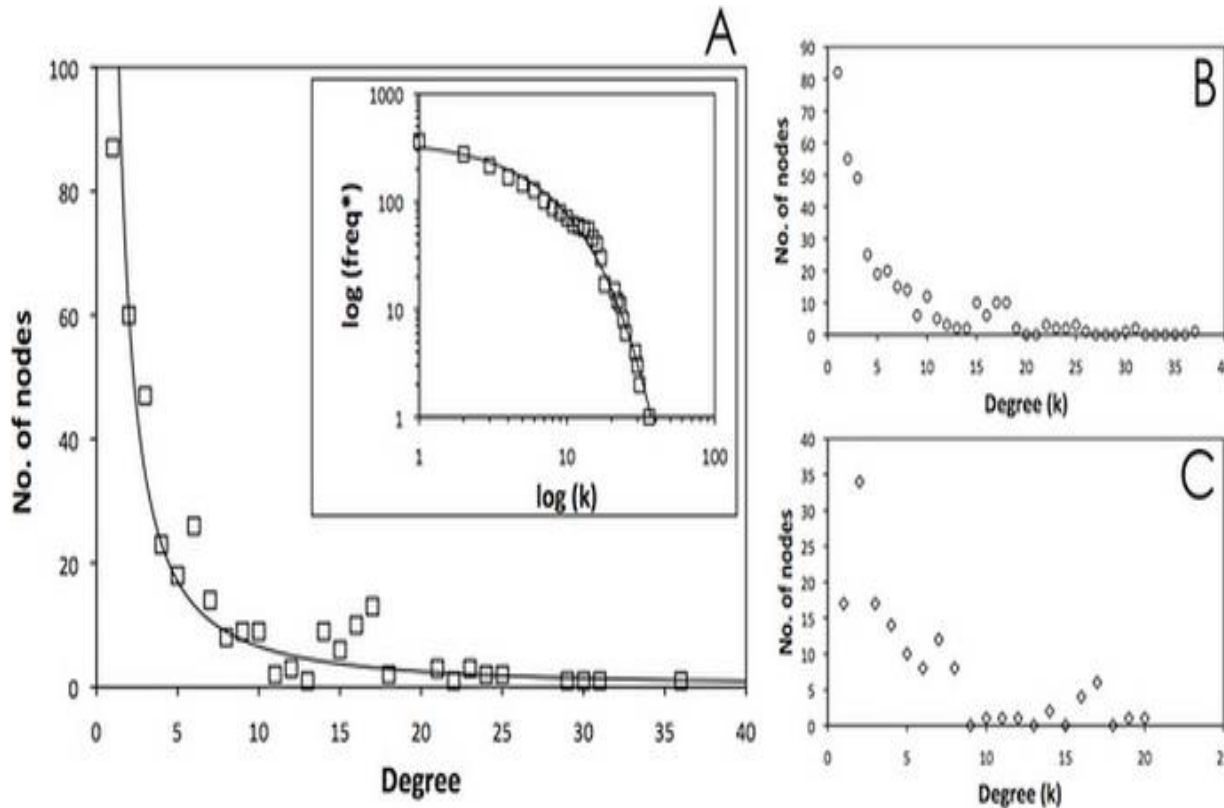
# The Bridges of Königsberg (Euler: Graph Theory)



# Degree Distribution



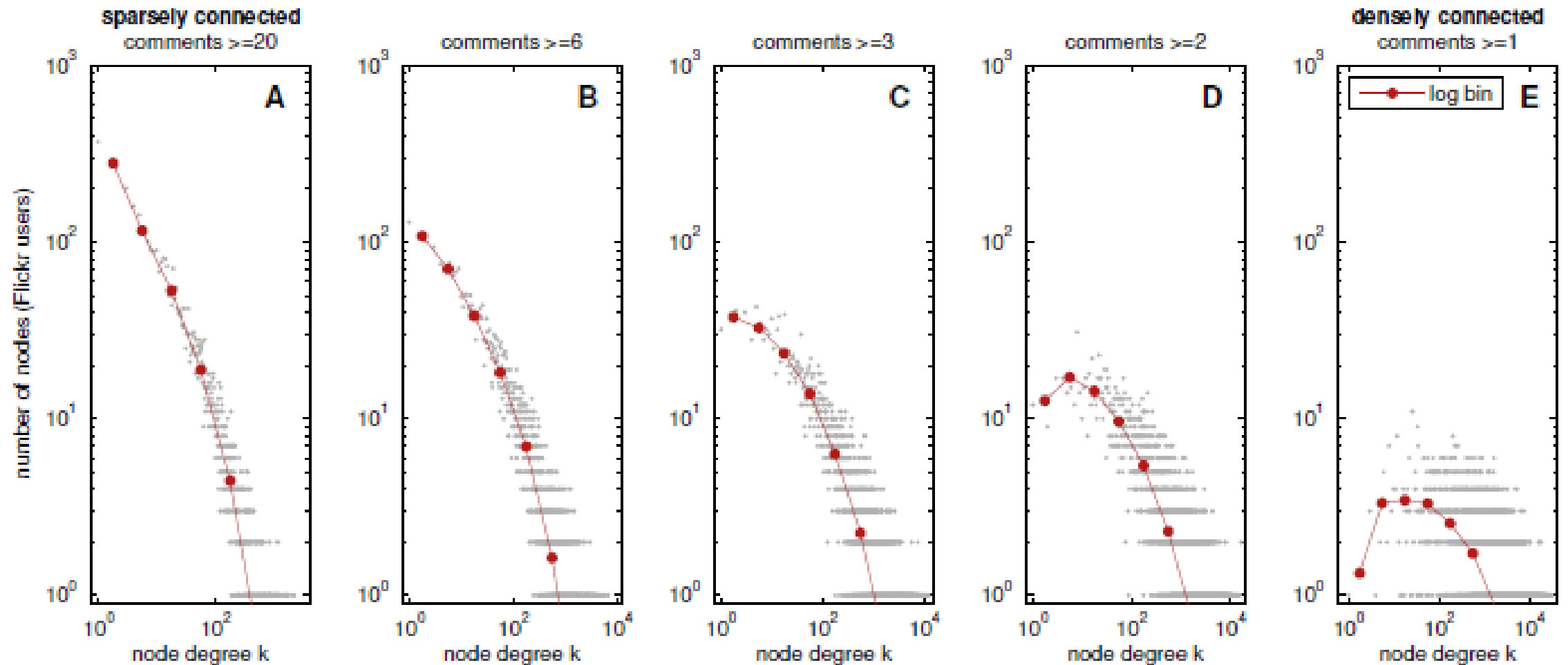
# Degree Distribution: Tens of degrees



~~T-test  
ANOVA  
Least-Squares Regression  
K-means  
GMM  
BernoulliNB  
PCA / CCA / SEM~~

(Partially reproduced from here: <http://www.nature.com/srep/2011/110811/srep00061/full/srep00061.html#close>)

# Degree Distribution: Higher Connectivity

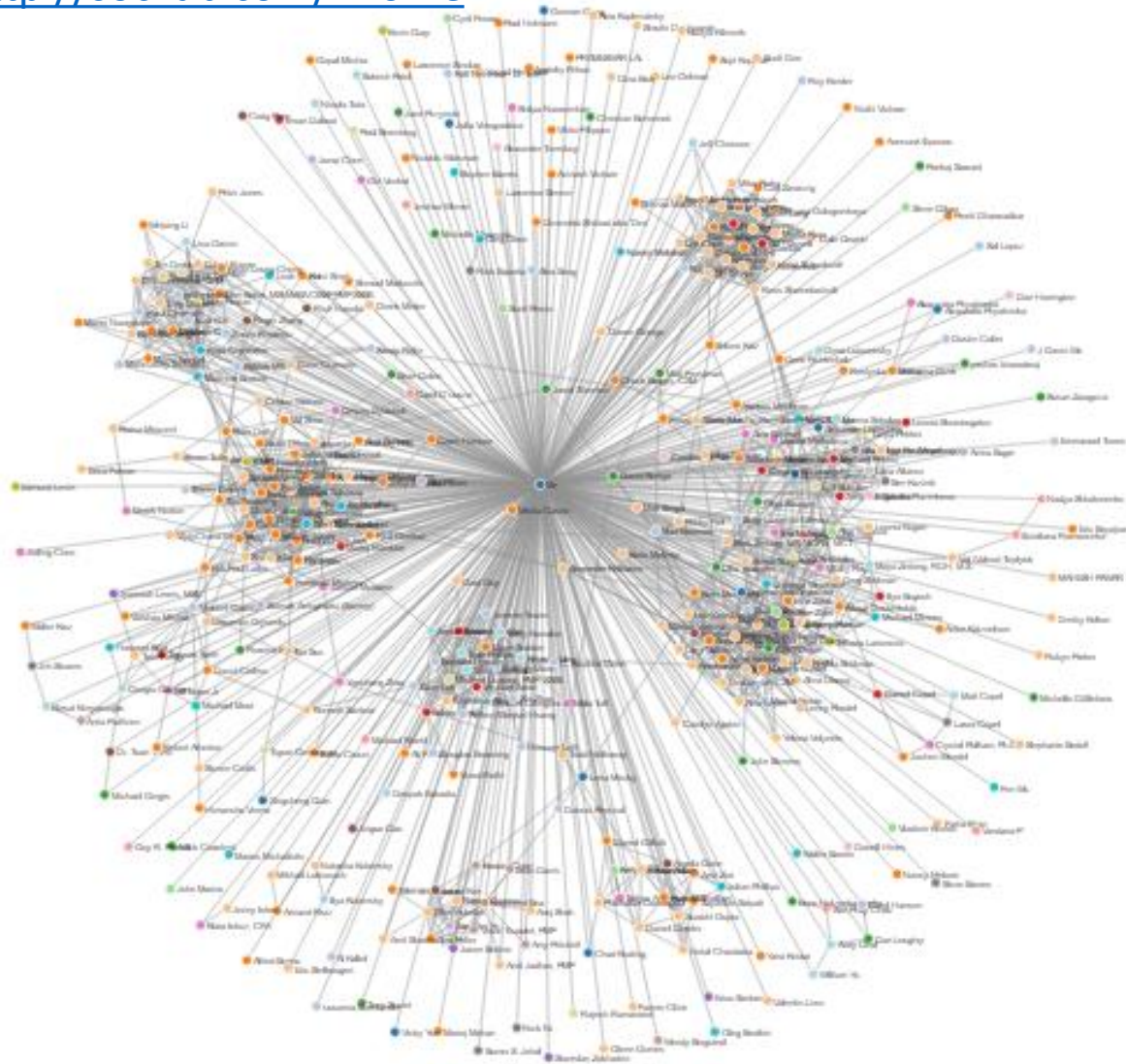


Flickr comment counts: the denser the network, the fewer comments per edge

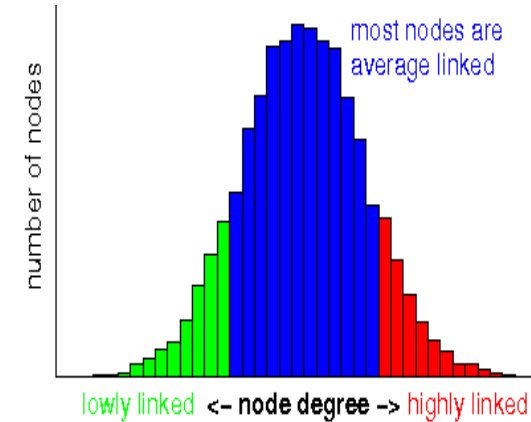
(Study conducted by Matthias Scholz: <http://jdmdh.episciences.org/77>)

# Degree Distribution: Higher Connectivity: Less Order

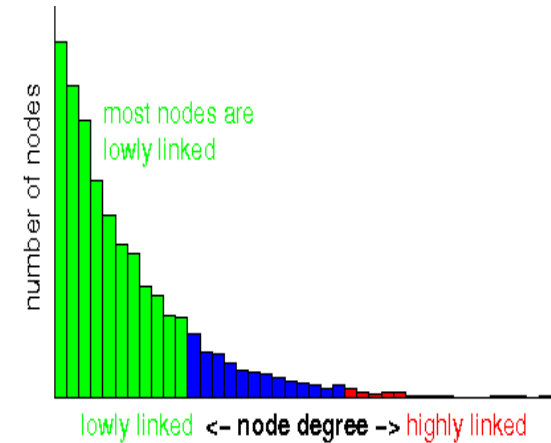
<http://socilab.com/#home>



— Random networks:  
Poisson distribution



— Organized networks:  
power-law / scale-free



www.network-science.org

1. Degree Distribution points to network's randomness
2. Be careful comparing networks.
3. Be careful analyzing networks: what works for one, may fail for another.
4. Stay on top of research.

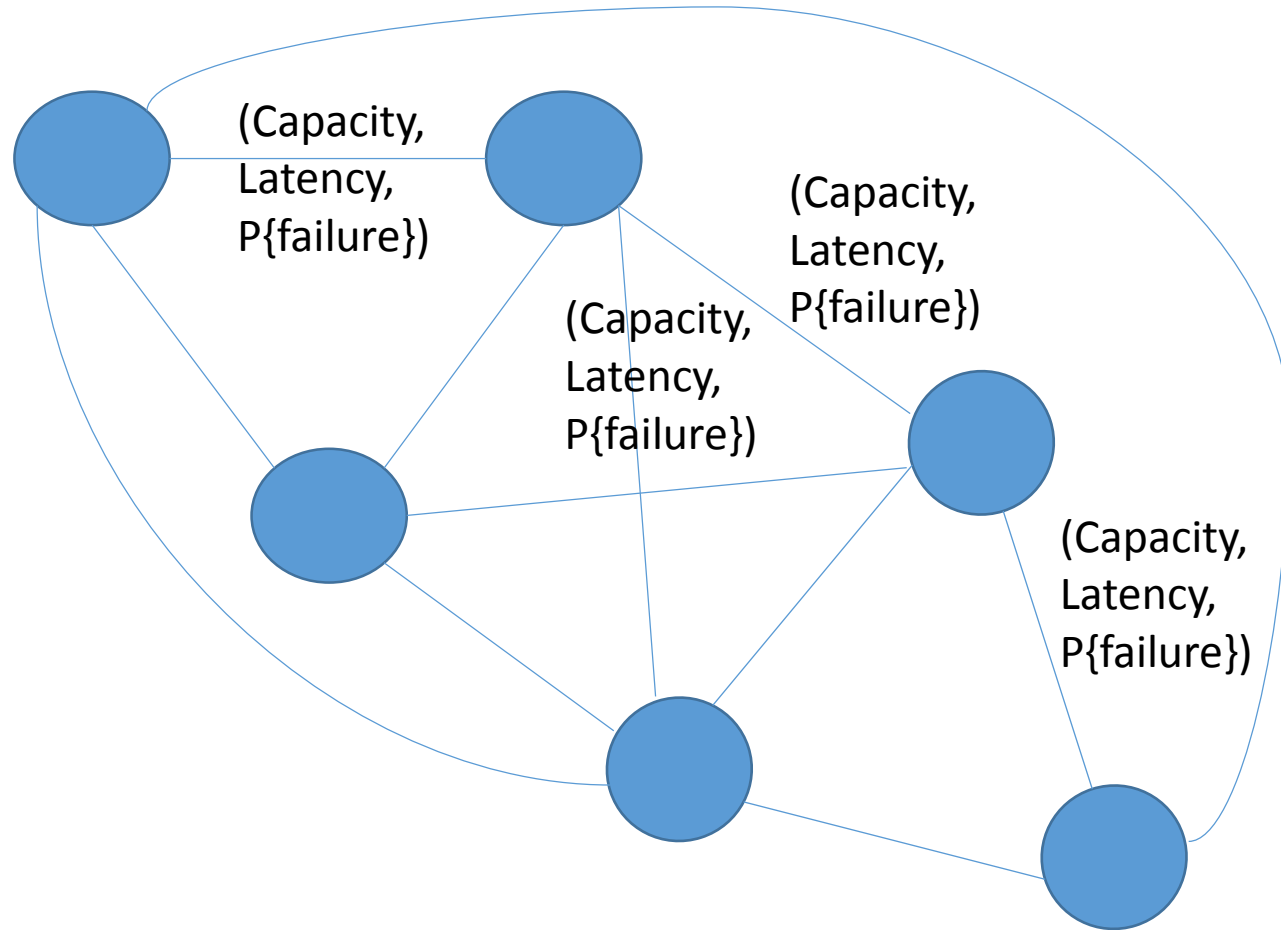
1. Use Non-Parametric Analysis.
2. Use Monte-Carlo Analysis.
3. Avoid Assumptions!

(From [http://www.network-science.org/powerlaw\\_scalefree\\_node\\_degree\\_distribution.html](http://www.network-science.org/powerlaw_scalefree_node_degree_distribution.html))

# Growing Directions in Network Research

- Hematology (angionetworks)
- Physics & Chemistry
- Network Resilience Analysis
  - Centrality & Reliability
  - Centrality & Phase Transition (in physics)
- Network Dynamics
  - Sociology
  - Law Enforcement
  - Computer Science:
    - GitHub / Perforce / SVN / CVS / ...
    - Internet
  - Transport and Communication Networks:
    - Aviation / Automotive / Railroad / Waterways
    - Television / Mobile / Fixed-Line Telephone
  - Water Supply Networks:
    - Macro-level: water distribution
    - Micro-level: water permeation through the soil

# Applications In the IT world



Link_ID	Capacity	Latency	P{failure}
a12	100	100	1.00E-05
a13	50	65	1.00E-05
a14	500	128	5.00E-06
a23	700	17	1.00E-04
a24	640	250	7.00E-05
a25	480	78	8.00E-06
...	...	...	...
a34	300	360	8.00E-03
a37	80	175	9.00E-07

**Goal:** Configure and operate the network to maximize QoS for all users

The following 24 slides are from Alex Gilgur, Brian Eck. “Sources of Traffic Demand Variability and Use of Monte Carlo for Network Capacity Planning” – Performance and Capacity by CMG International Conference (CMG’14) – Atlanta, GA 2014.



# QoS = what's important to user

## 1. High Probability of:

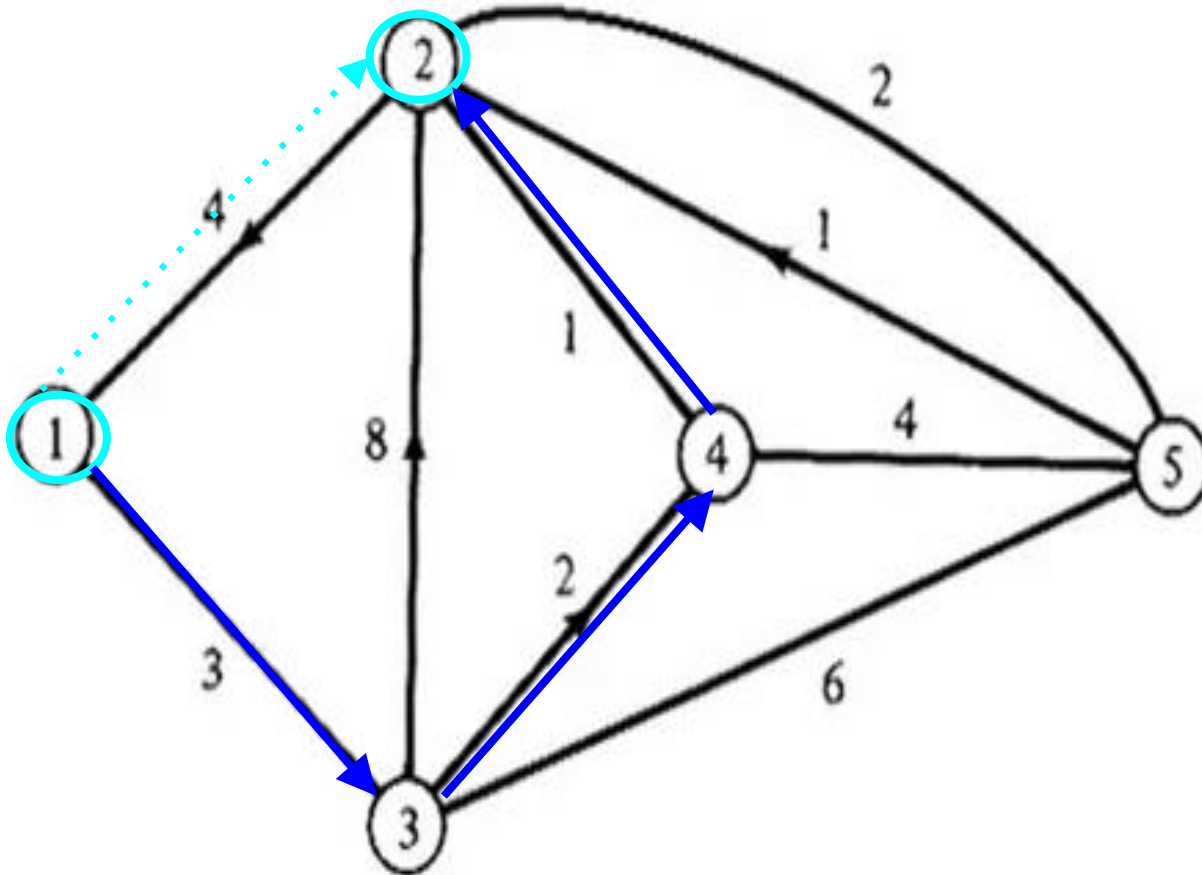
- a. Delivery
- b. Accuracy

## 2. Low Latency

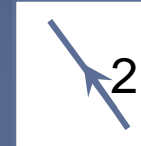
1.  $\text{QoS} = \text{“Goodput”} = \text{Throughput} * \text{Pr}\{\text{delivery}\}$

2.  $\text{QoS} = 1 / \text{Latency}$

# Routing for Low Latency: SPF: “Travelling Salesman”



= Node 4



= “Latency of this link = 2 units”

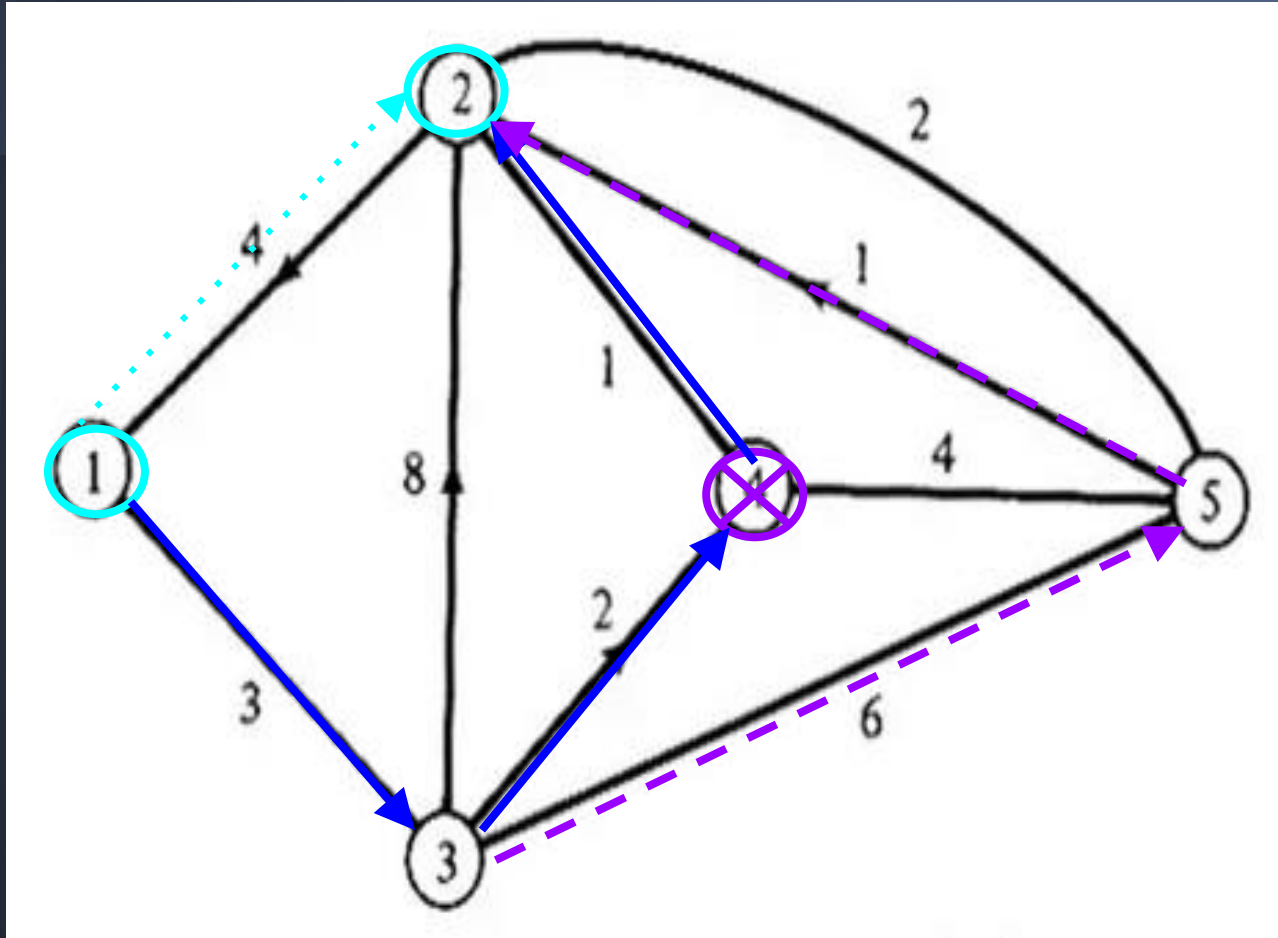
$$Cost_{Path} = \sum_{Link_1}^{Link_n} Cost_{Link} = \sum_{Link_1}^{Link_n} Latency_{Link}$$

Find shortest path from Node 1 to Node 2

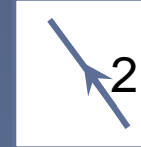
Cost = Latency

QoS =  $1/\text{Cost} = 1/\text{Latency}$

# Routing for Low Latency: SPF: “Travelling Salesman”



= Node 4



= “Latency of this link = 2 units”

$$Cost_{Path} = \sum_{Link_1}^{Link_n} Cost_{Link} = \sum_{Link_1}^{Link_n} Latency_{Link}$$

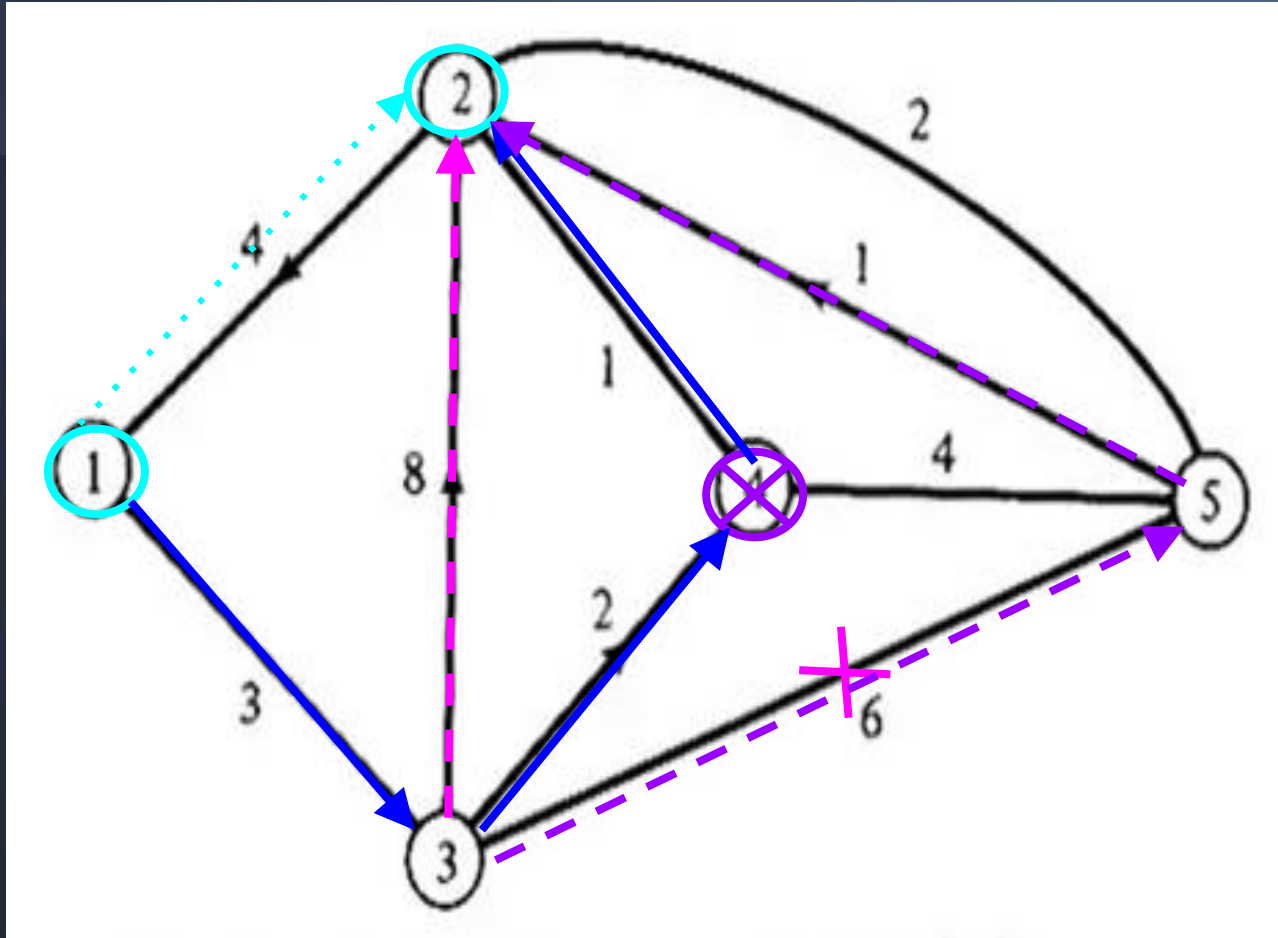
Find shortest path from Node 1 to Node 2

Find shortest path from Node 1 to Node 2 **IF** Node 4 is down

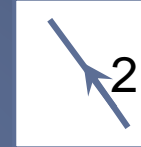
Cost = Latency

QoS =  $1/\text{Cost} = 1/\text{Latency}$

# Routing for Low Latency: SPF: “Travelling Salesman”



= Node 4



= “Latency of this link = 2 units”

$$Cost_{Path} = \sum_{Link_1}^{Link_n} Cost_{Link} = \sum_{Link_1}^{Link_n} Latency_{Link}$$

Find shortest path from Node 1 to Node 2

Find shortest path from Node 1 to Node 2 **IF** Node 4 is down ...

**Cost = Latency**

**QoS = 1/Cost = 1/Latency**

... **and** Link 3-5 is losing packets

# QoS = what's important to user

## 1. High Probability of:

- a. Delivery
- b. Accuracy

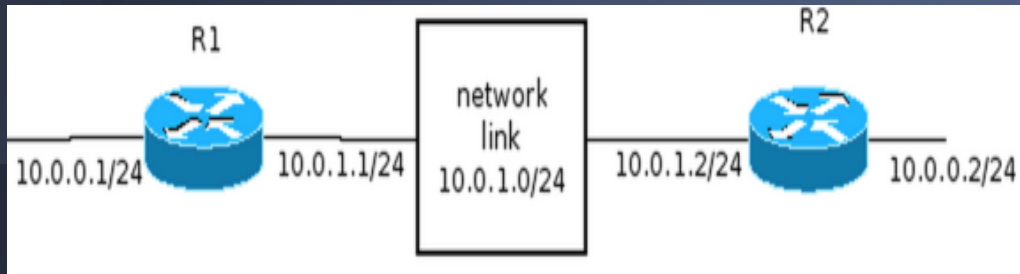
## 2. Low Latency

1.  $\text{QoS} = \text{“Goodput”} = \text{Throughput} * \text{Pr}\{\text{delivery}\}$

2.  $\text{QoS} = 1 / \text{Latency}$

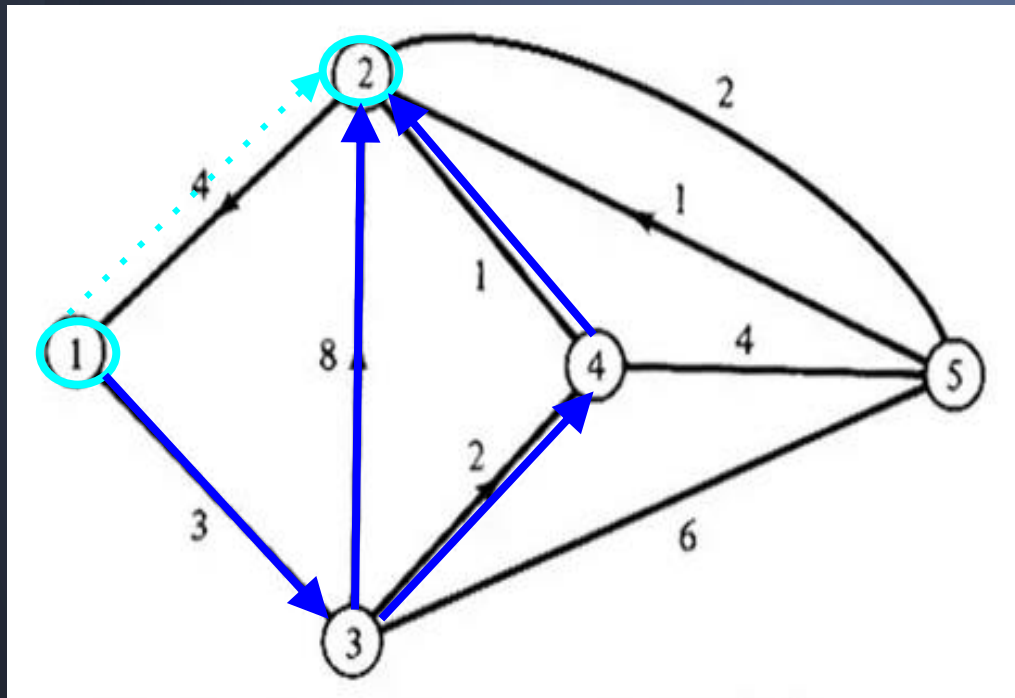


# Routing for “Goodput”: Nonlinear optimization



$$Pr\{delivery_{Link}\} = Pr\{NOT\ blocking \ \& \ NOT\ failure\}$$

$$P_D^{Link} = (1 - P_B^{Link}) * (1 - P_F^{Link})$$



$$P_D^{Path} = \prod_{Link_1}^{Link_n} P_D^{Link}$$

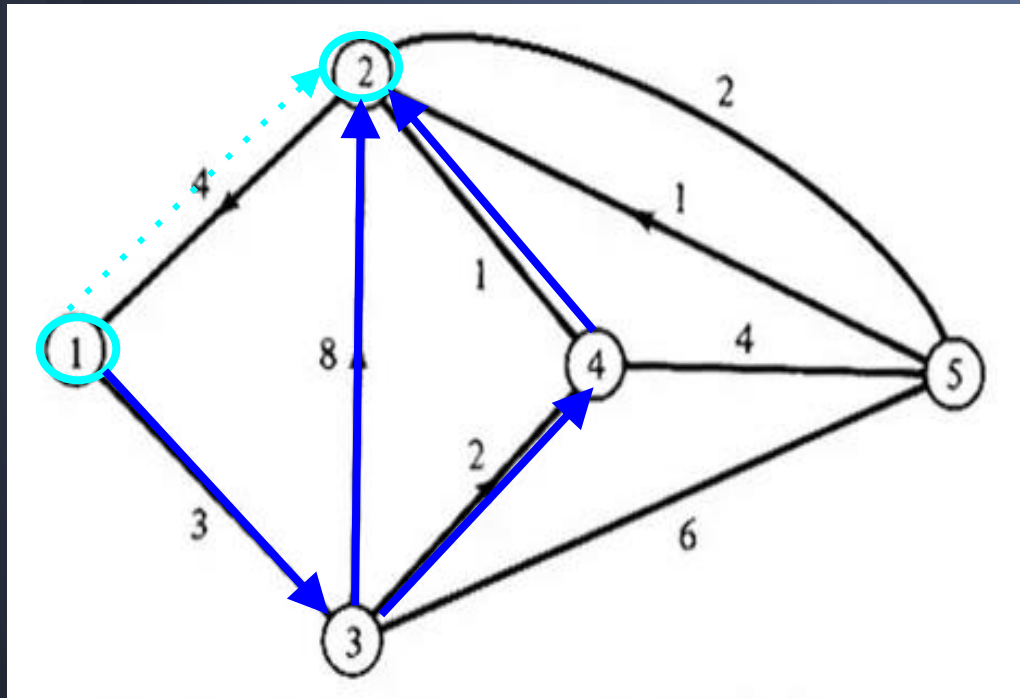
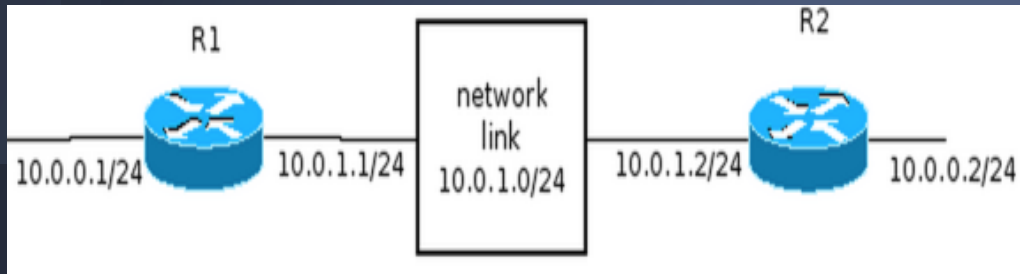
$$Cost_{Path} = \frac{1}{X} * \{1 - \prod_{Link_1}^{Link_n} P_D^{Link}\}$$

$$Cost_{Path} = \left(\frac{1}{X}\right) * \{1 - \prod_{Link_1}^{Link_n} [(1 - P_B^{Link}) * (1 - P_F^{Link})]\}$$

“Travelling Salesman”

Non-linear optimization

# Routing for “Goodput”: Can it be simplified?



Non-linear optimization

$$P_D^{Link} = (1 - P_B^{Link}) * (1 - P_F^{Link})$$

$$P_B^{Link} = 0$$

**Assume:**

- No Queueing
  - No Blocking

$$P_D^{Link} = 1 - P_F^{Link}$$

$$Cost_{Link} = \frac{1}{X} * \{1 - P_D^{Link}\}$$

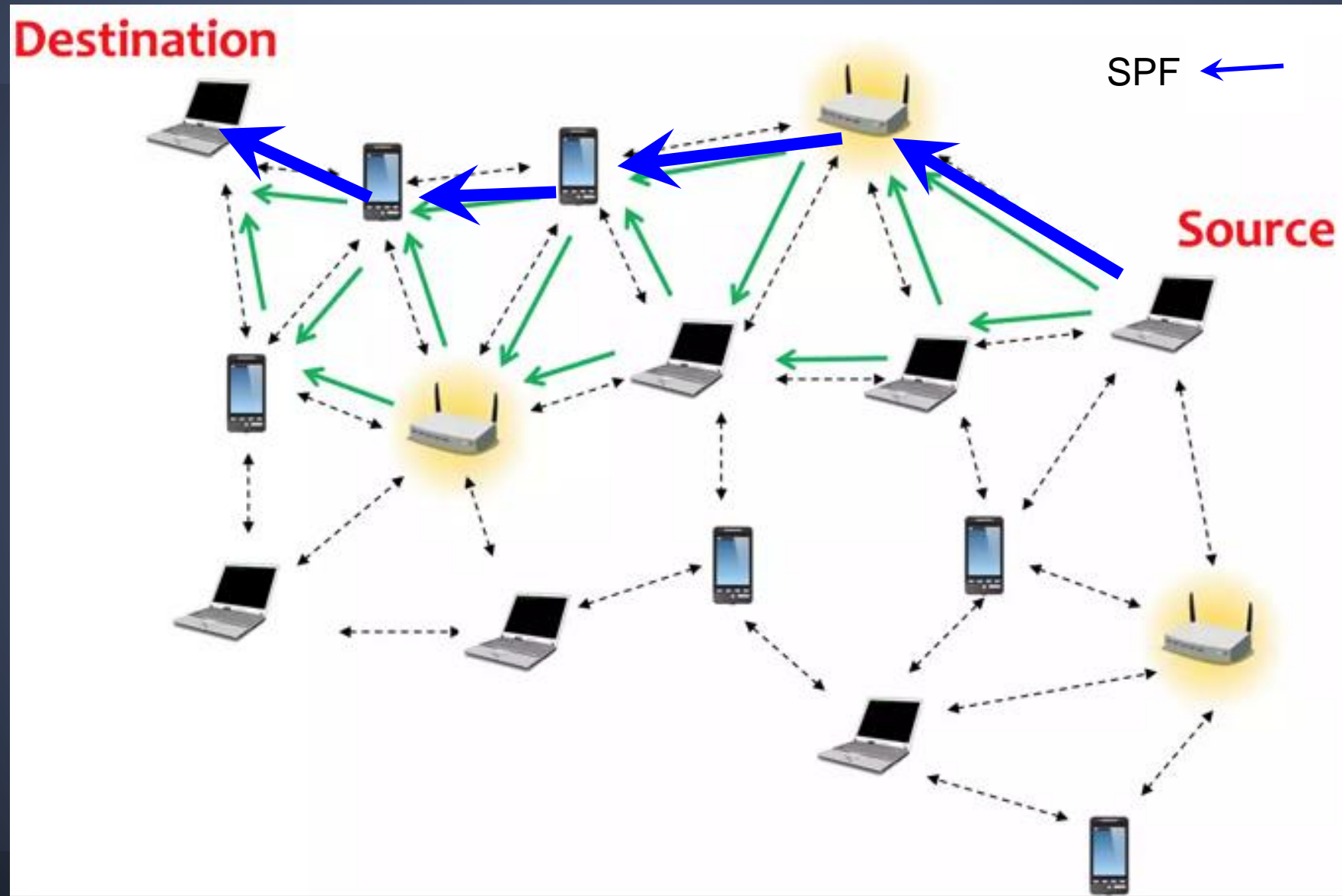
$$= \frac{1}{X} * P_F^{Link}$$

**Redefine:**

$$\ln(Cost) = \ln(P_F) - \ln(X)$$

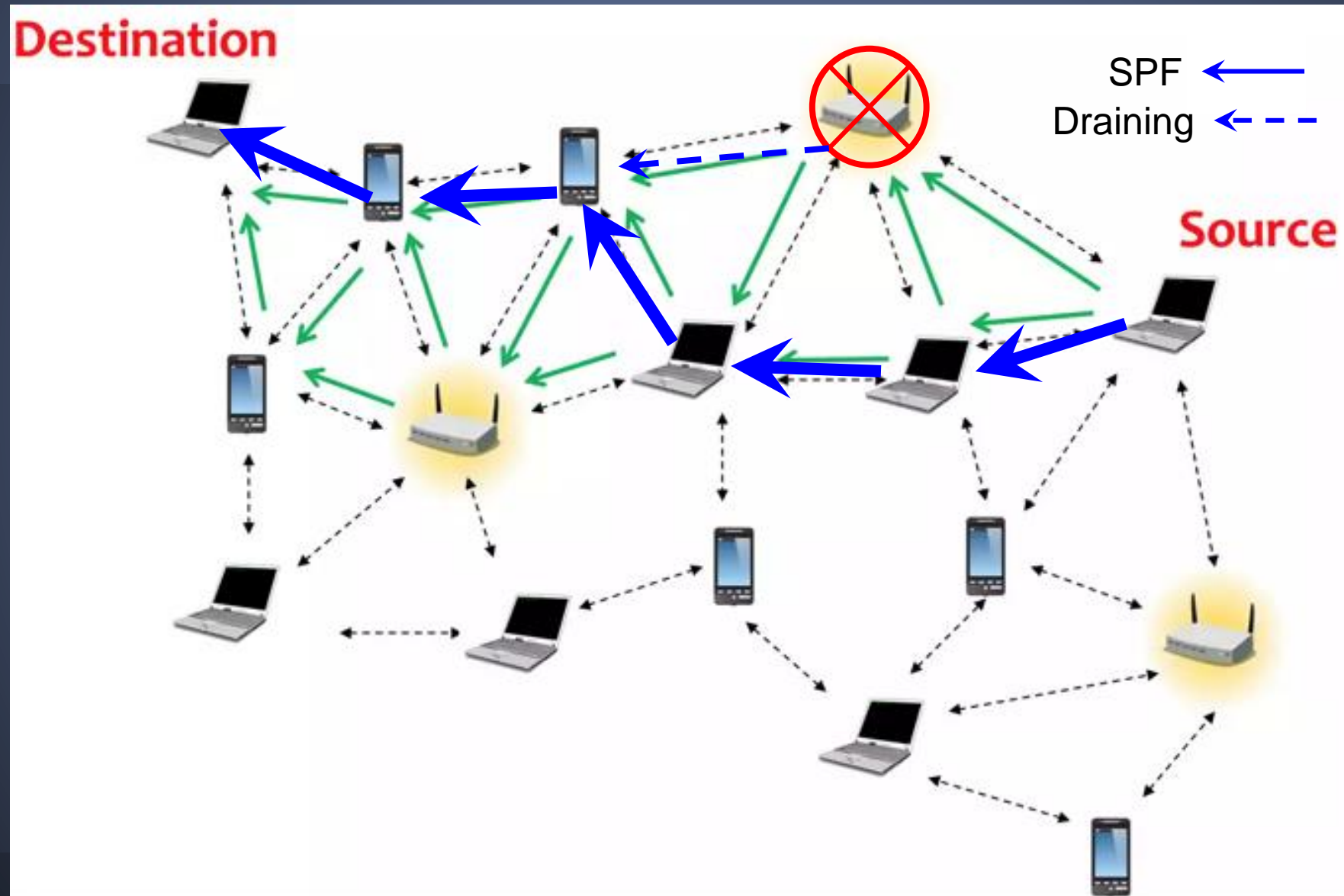
Can be pseudo-linearized

# Routing As a Process

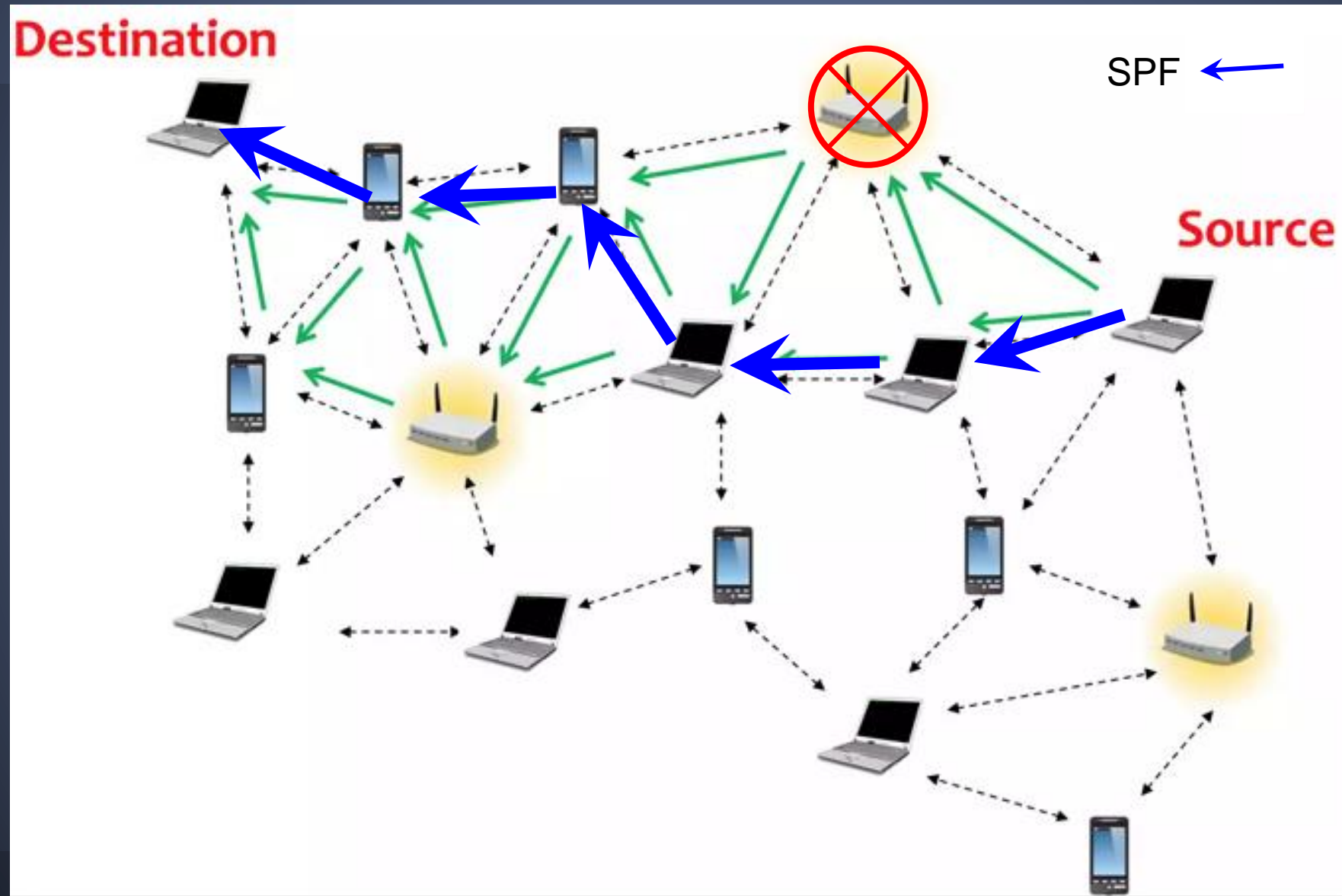




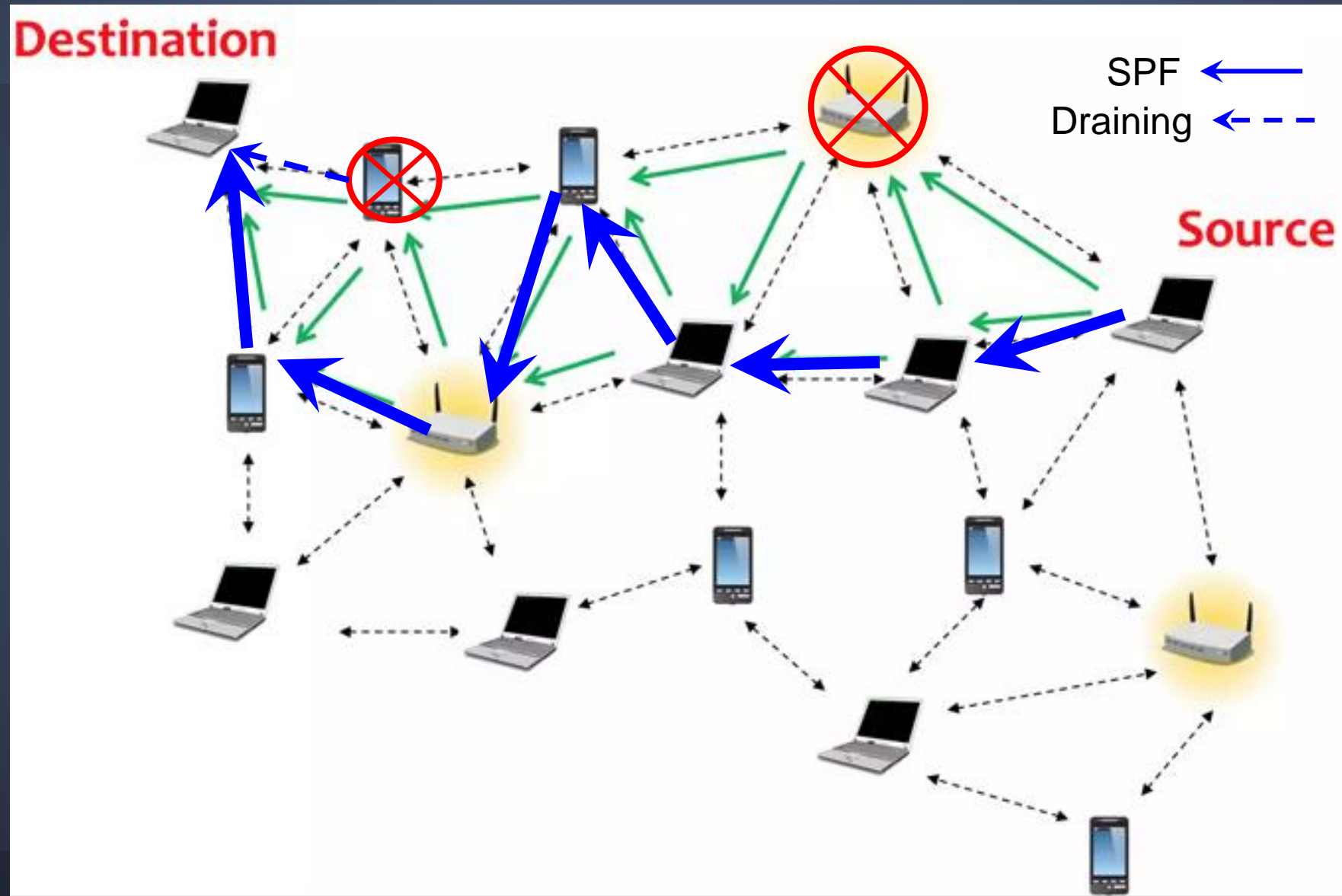
# Routing As a Process



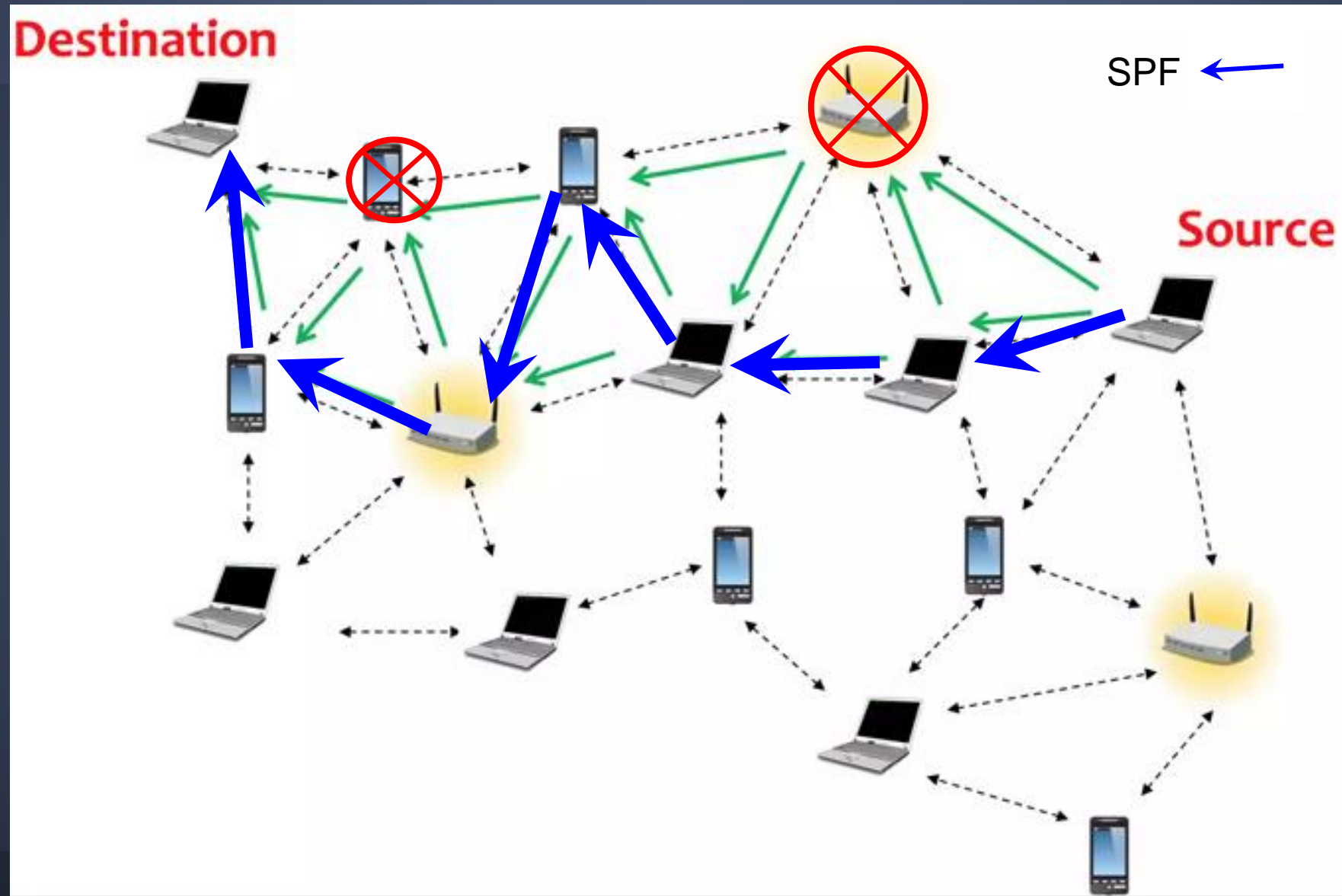
# Routing As a Process



# Routing As a Process

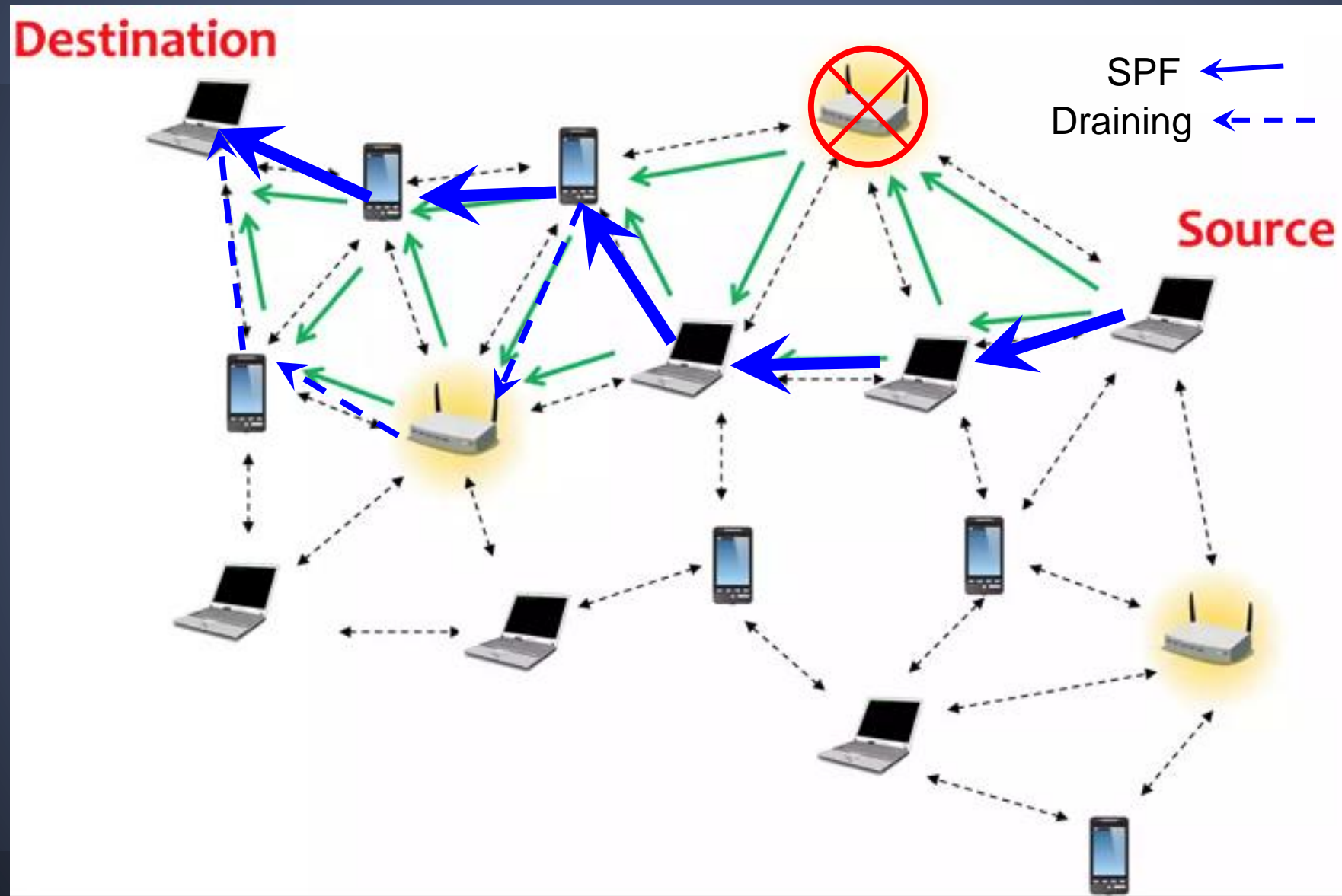


# Routing As a Process

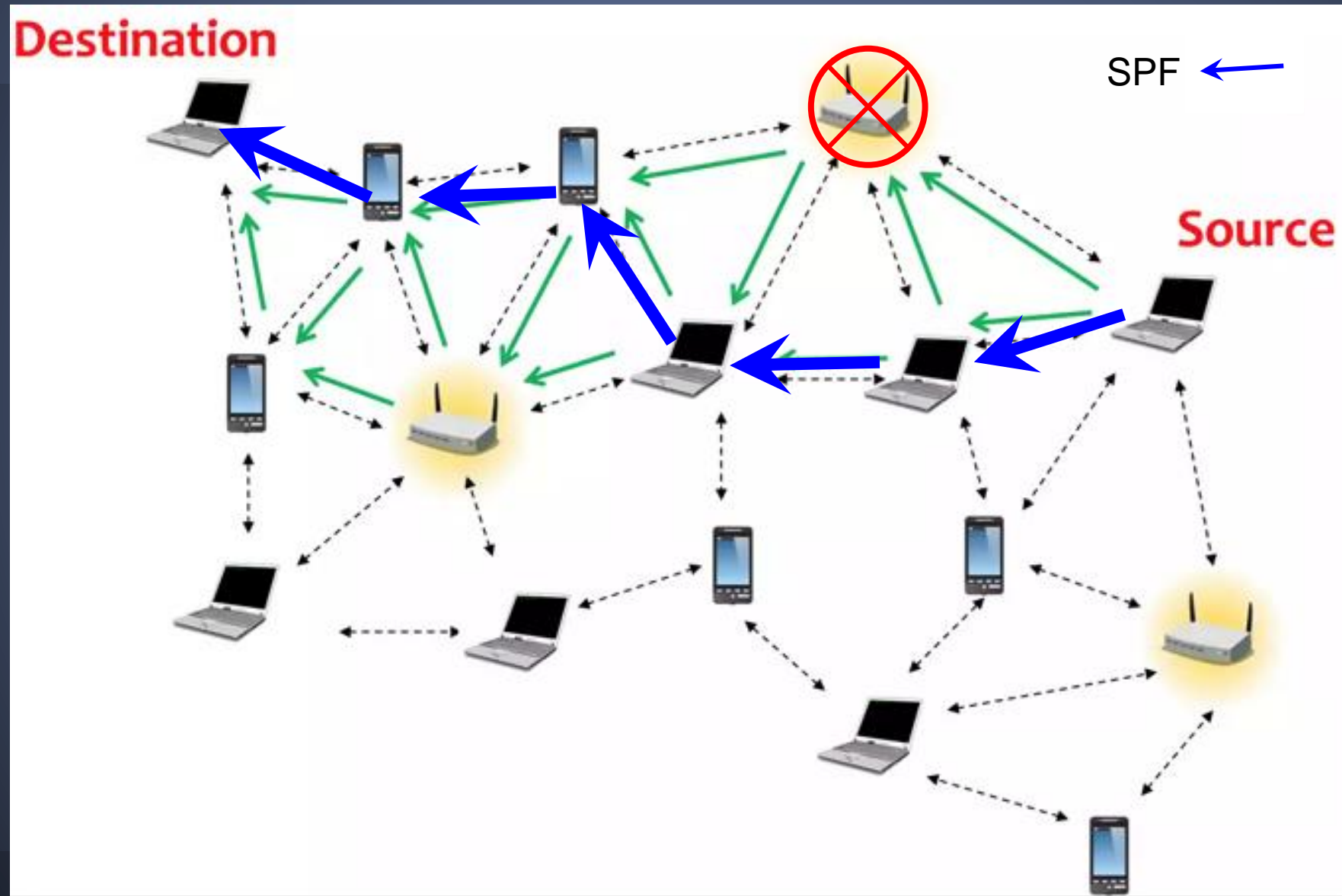




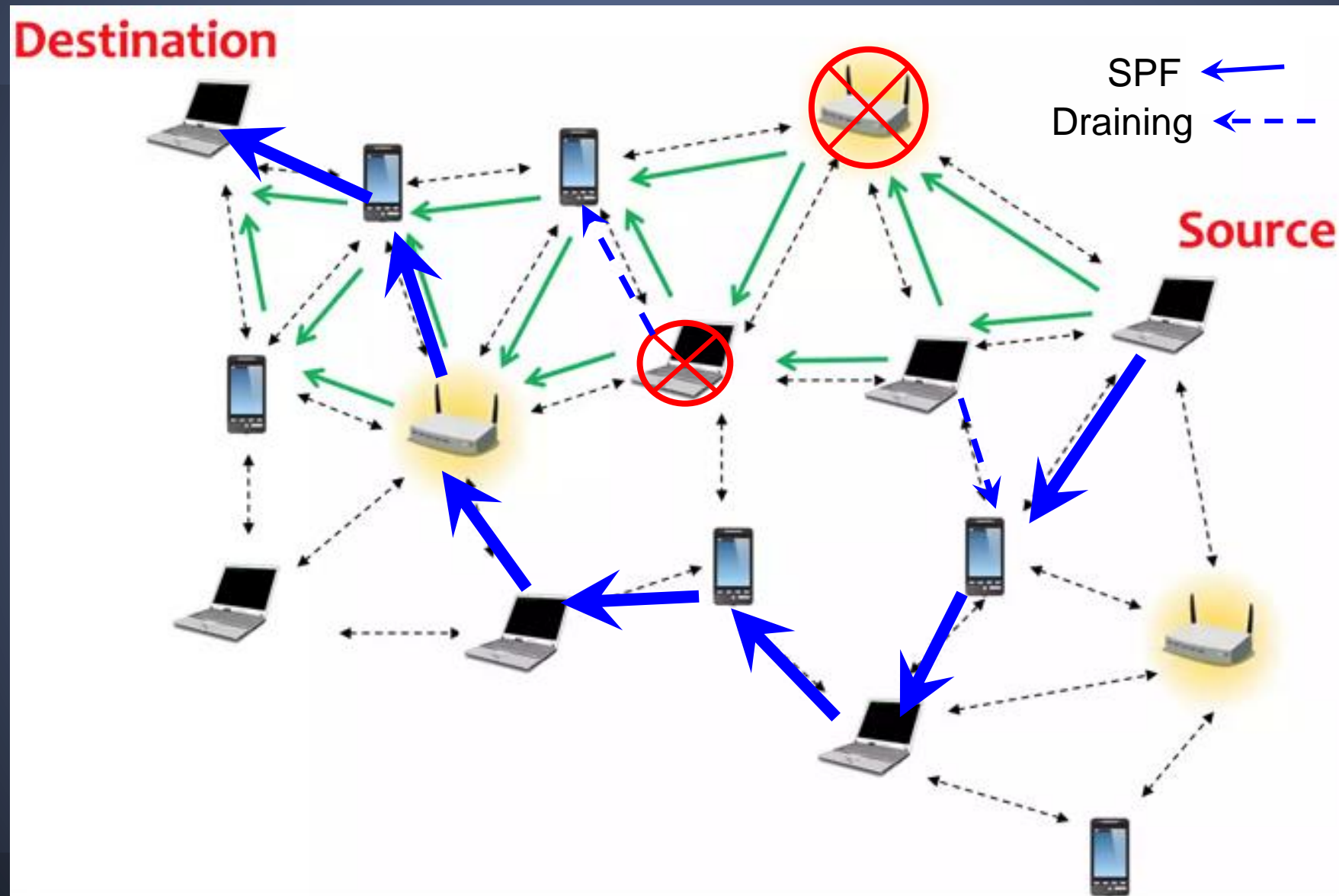
# Routing As a Process



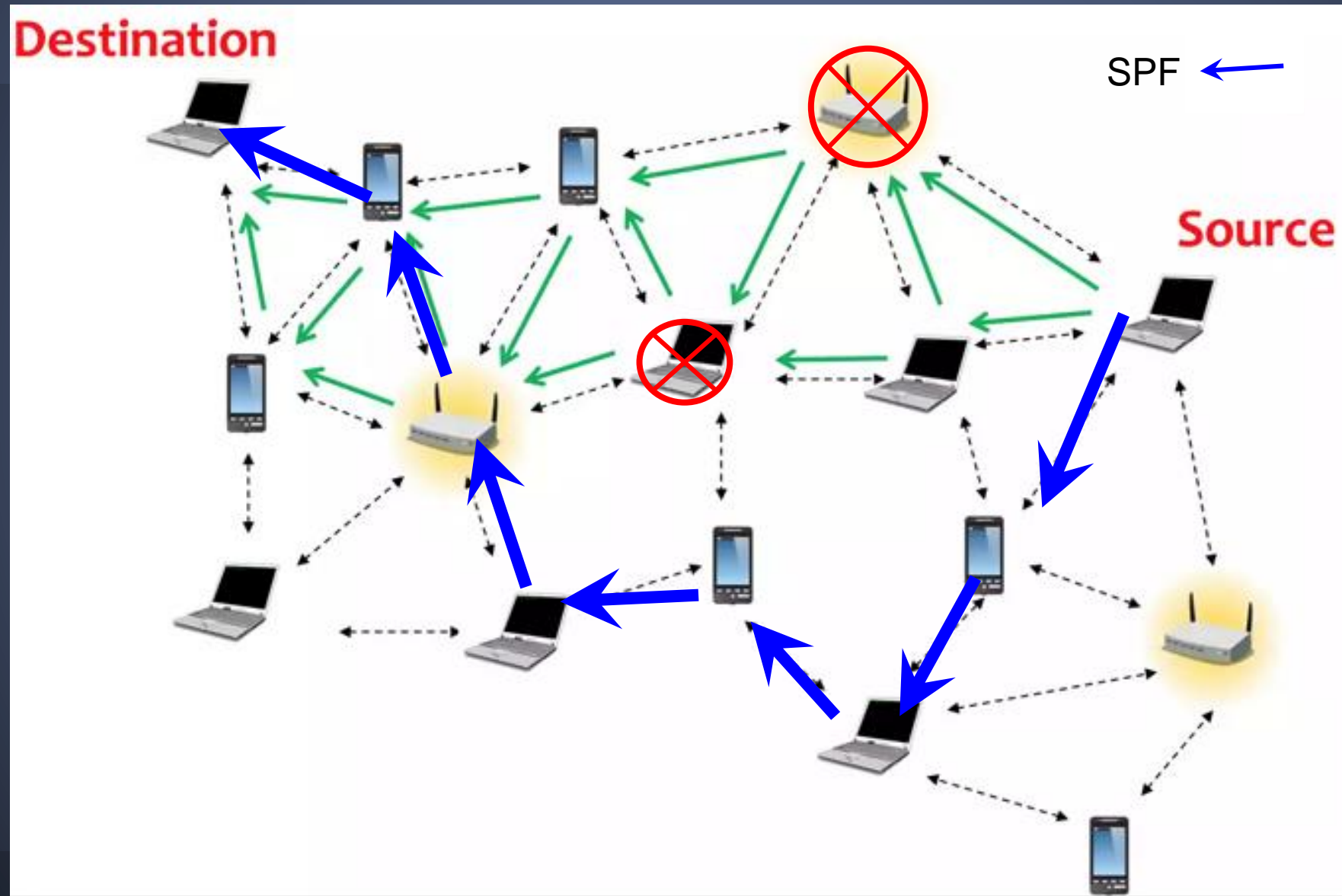
# Routing As a Process



# Routing As a Process

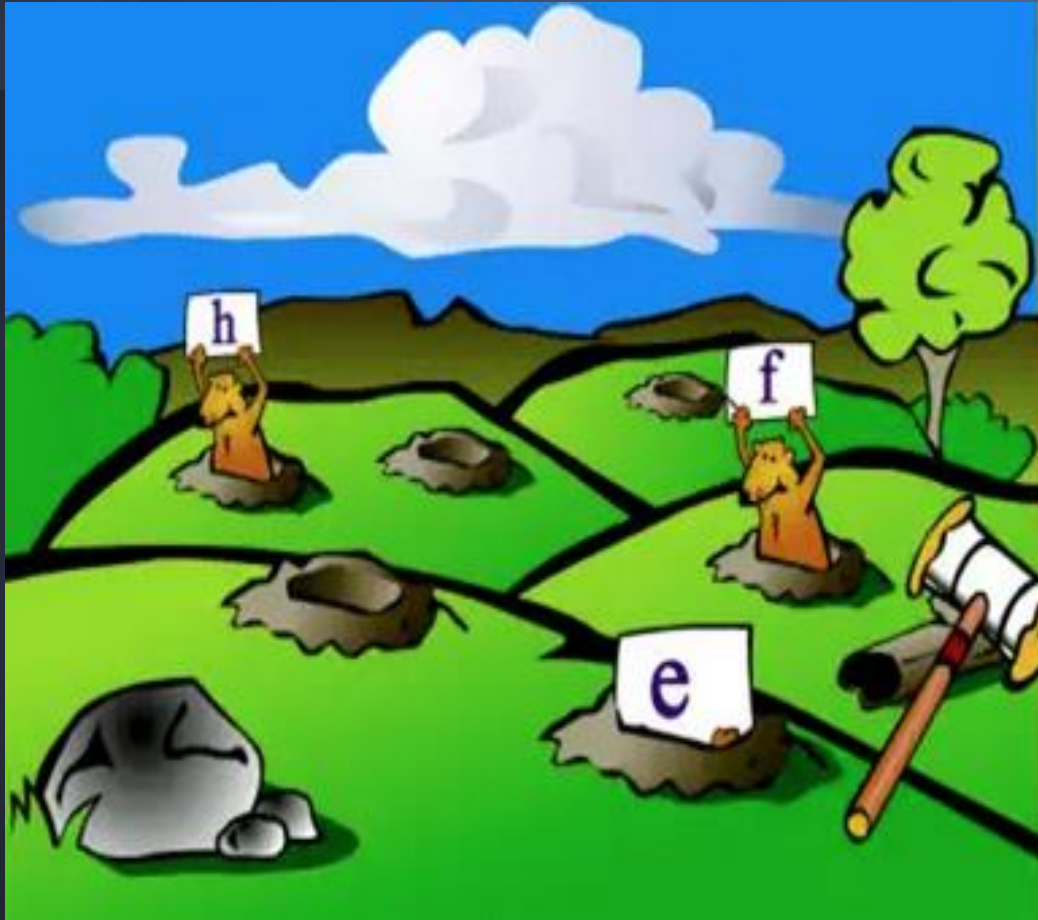


# Routing As a Process





# “Whack-a-Mole!”



Routing is updated all the time via:

- Protocol (e.g., TCP)
- SDN Control

We need to accommodate each Flow's:

- Primary Paths
- Alternate Paths

# We can forecast Demand

## Demand:

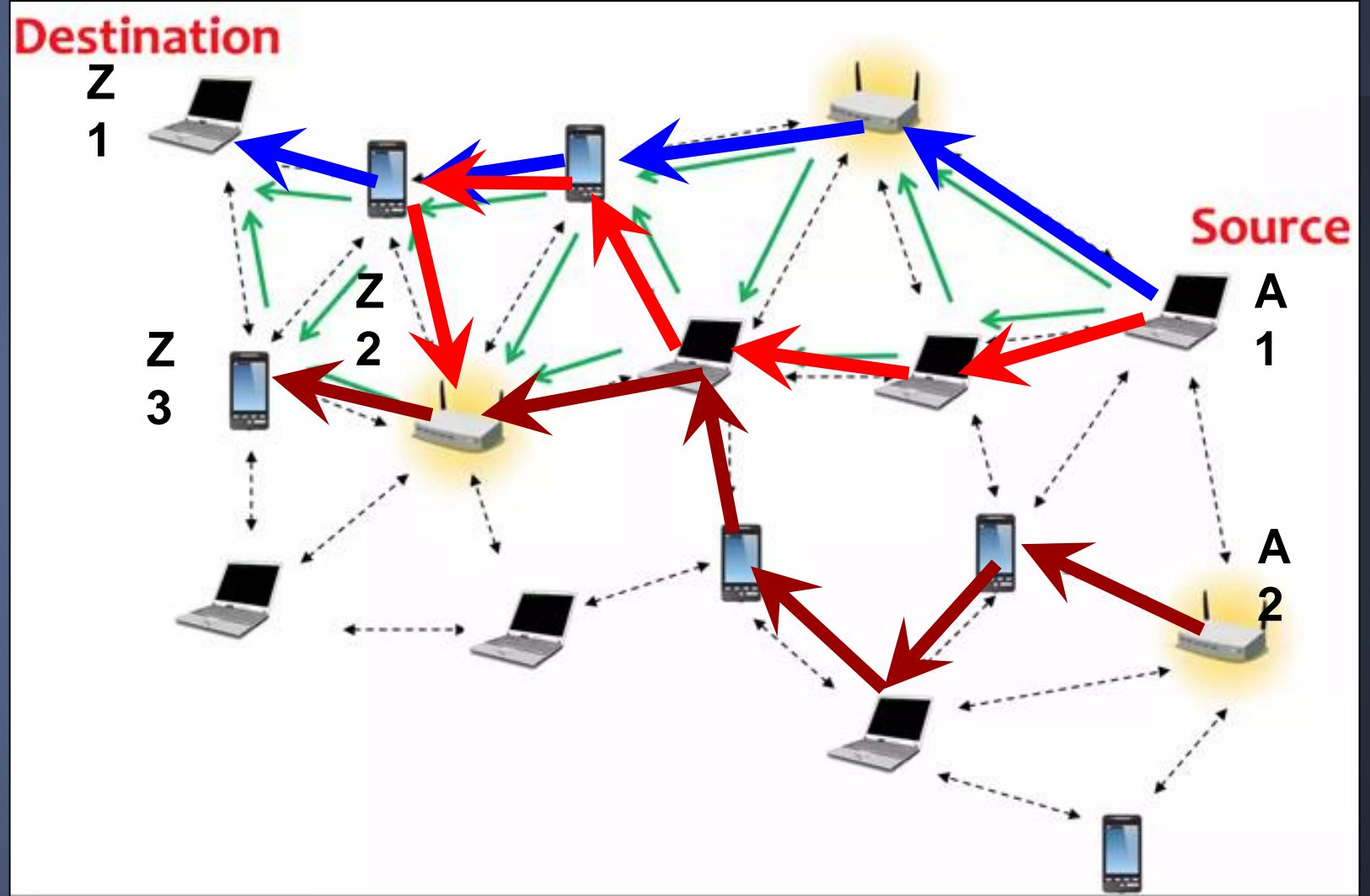
- A1 -> Z1 : X11 Gbps
- A1 -> Z2 : X12 Gbps
- A2 -> Z3 : X23 Gbps



Throughput  
on each Link



Capacity  
for each Link



# We can forecast Demand

Demand:

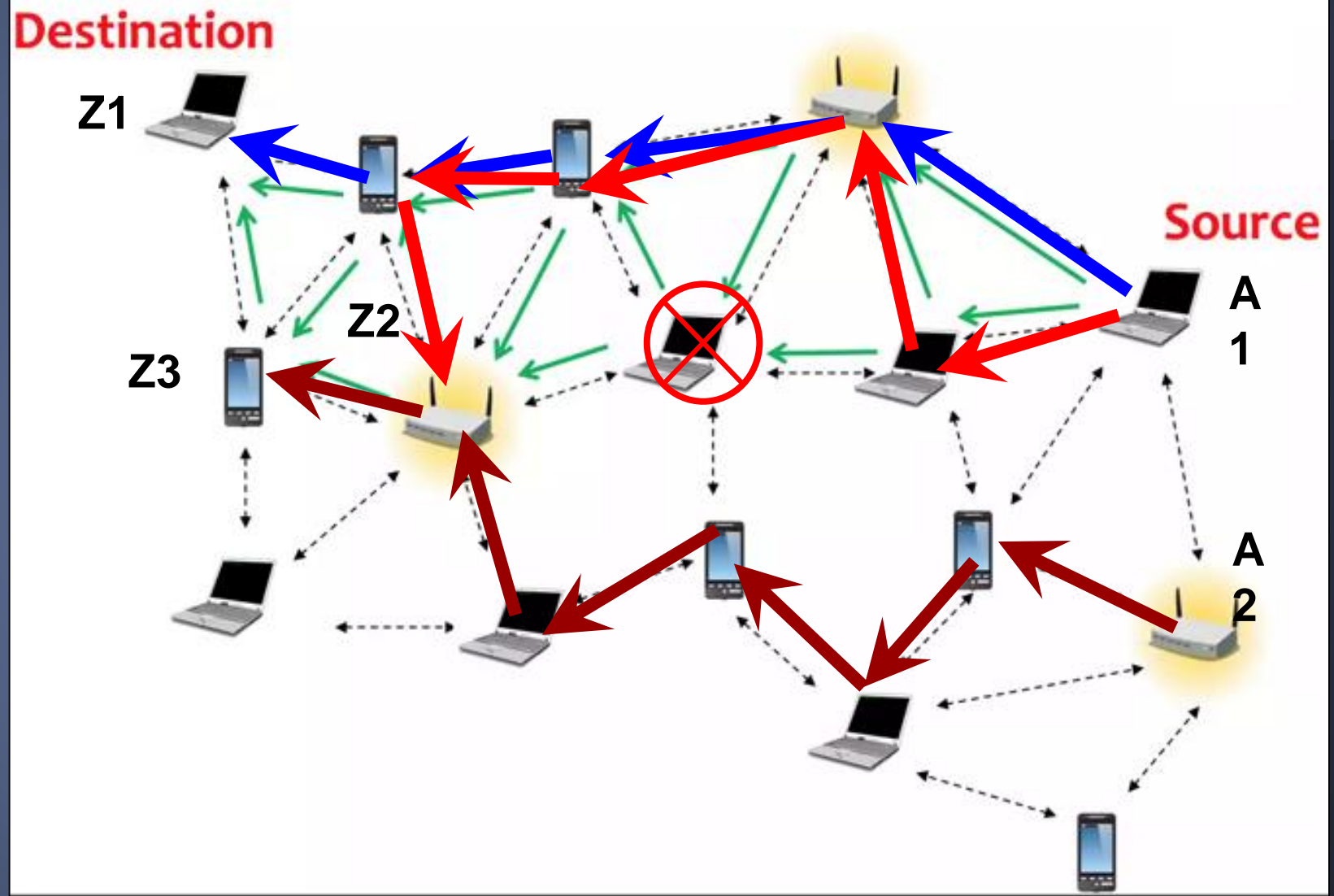
- A1 -> Z1 : X11 Gbps
- A1 -> Z2 : X12 Gbps
- A2 -> Z3 : X23 Gbps



Throughput  
on each Link



Capacity  
for each Link



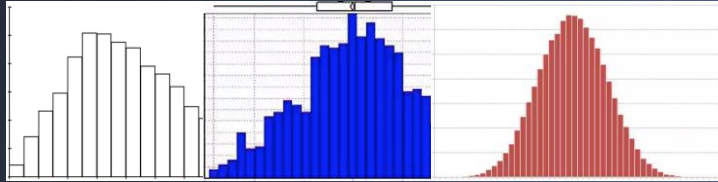
Throughput is combinatorial



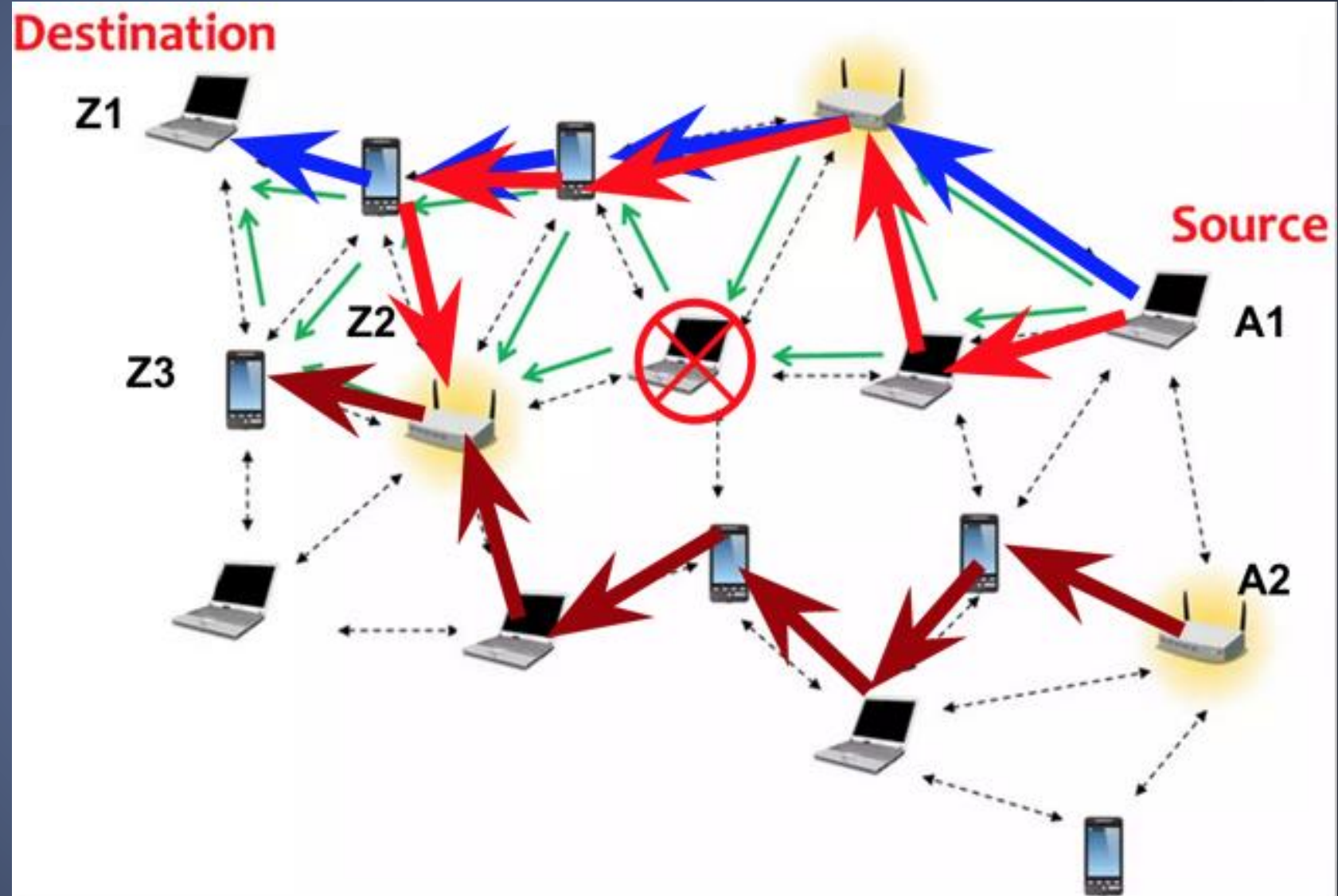
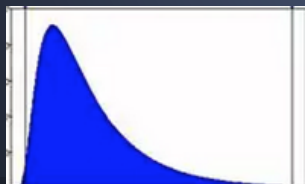
# Demand is NOT Deterministic

Demand:

- A1 -> Z1 : X11 Gbps
- A1 -> Z2 : X12 Gbps
- A2 -> Z3 : X23 Gbps



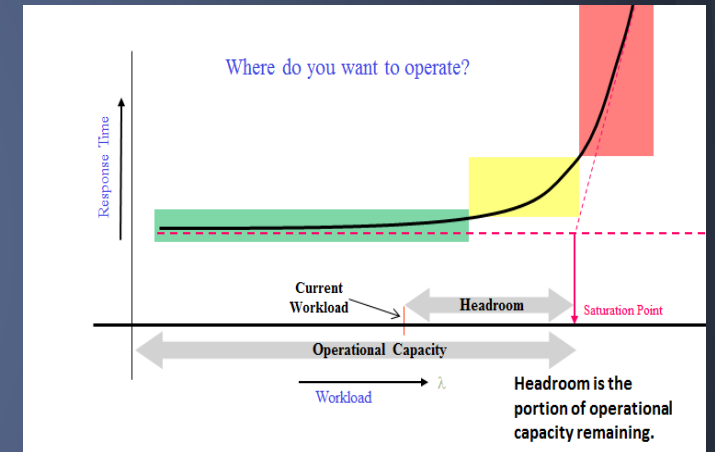
Throughput  
on each Link



Neither is Throughput

# Demand Predictability

- Not all forecasting tools were created equal:
  - Non-Gaussian distributions
  - Non-stationarity
  - Congestion Control
- TSA is not the only way to forecast Demand:
  - Explanatory variables:
    - Timestamp
    - Power
    - CPU
    - Business Metrics



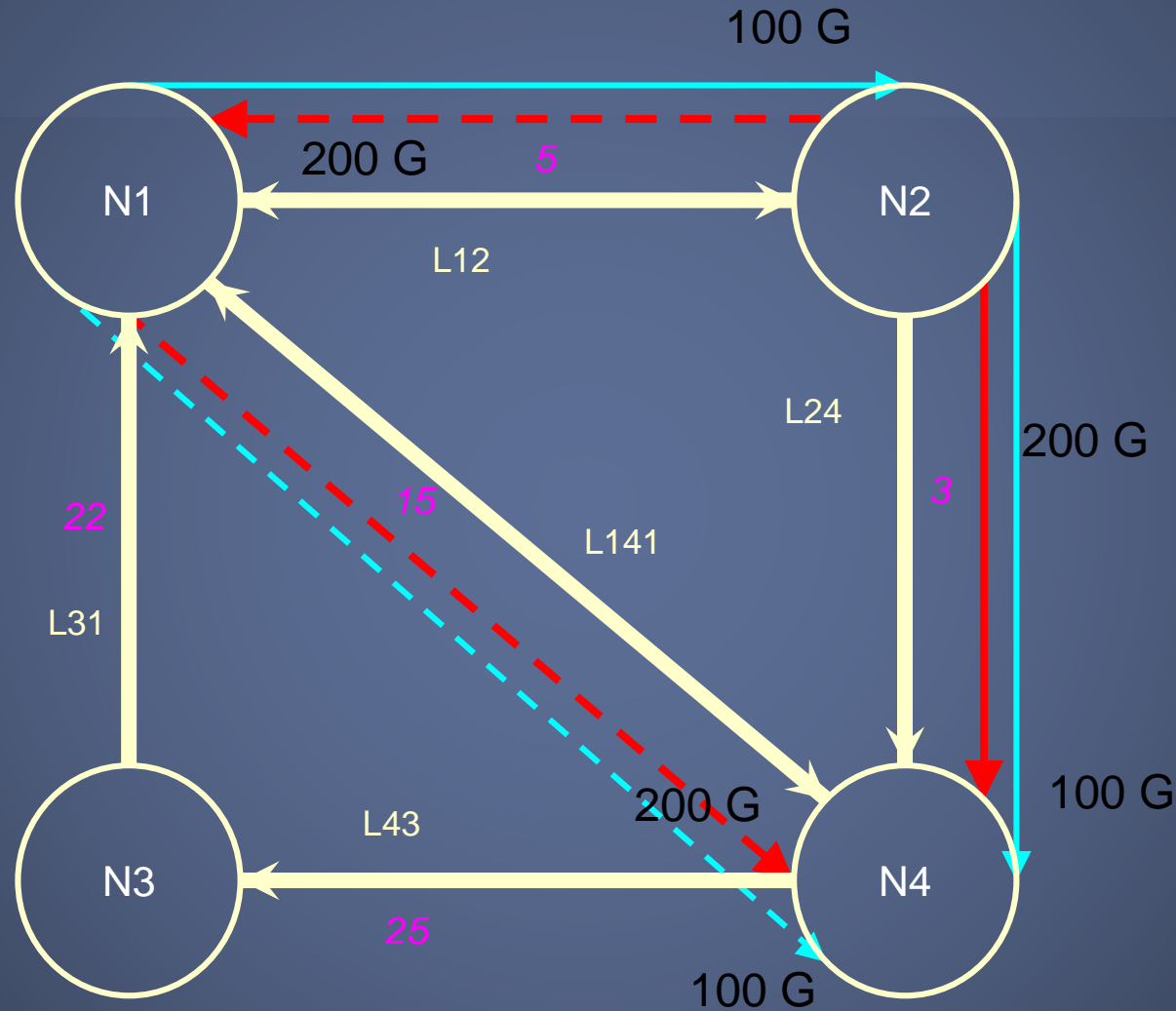
“All models are wrong. Some models are useful” - G.E.P. Box

# From Deterministic Demand to Throughput

## Demand:

N1\_N4: 100 G

N2\_N4: 200 G



## Throughput:

L12 = 100 G

L21 = 200 G

L24 = 300 G

L14 = 300 G

L41 = 0

L43 = 0

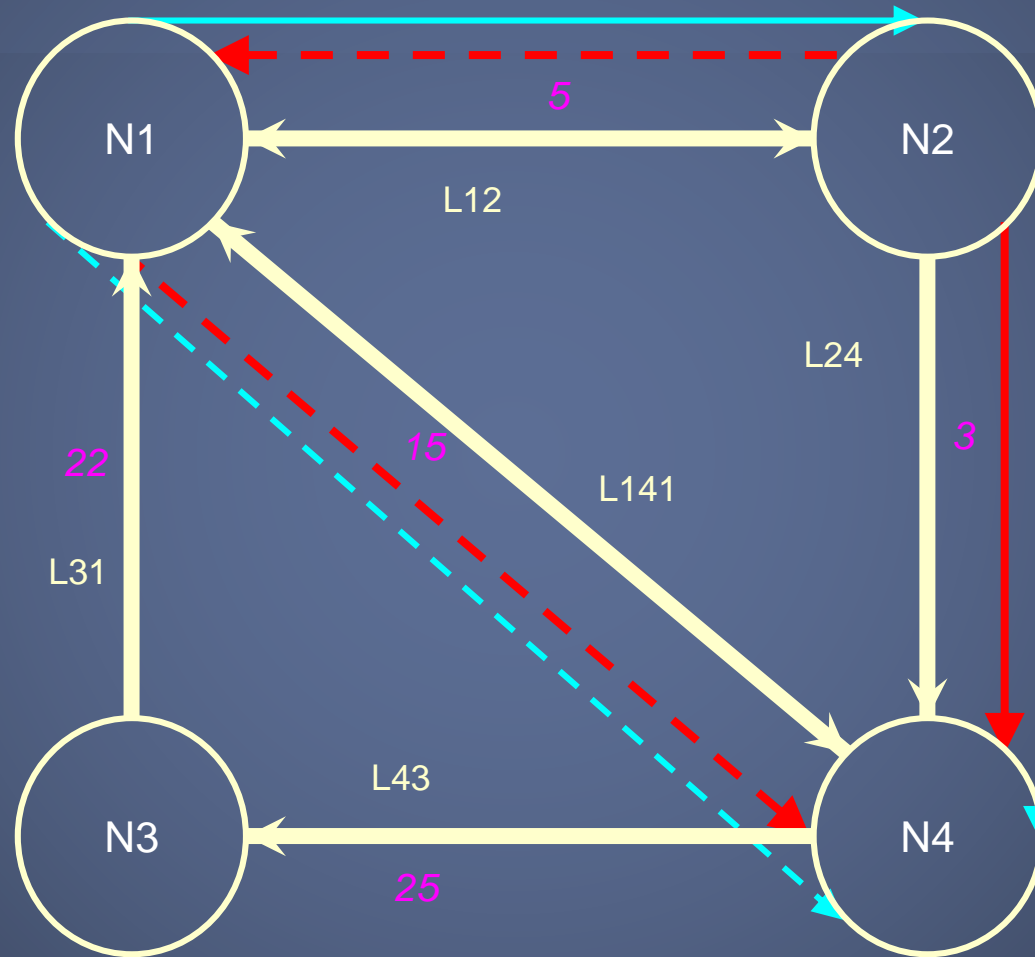
L31 = 0

# From Gaussian Demand to Throughput:

## Demand:

N1\_N4: N (100, 10) G

N2\_N4: N (200, 15) G



## Throughput:

L12 = N (100, 10) G

L21 = N (200, 15) G

L24 = N (300, 18) G

L14 = N (300, 18) G

L41 = 0

L43 = 0

L31 = 0

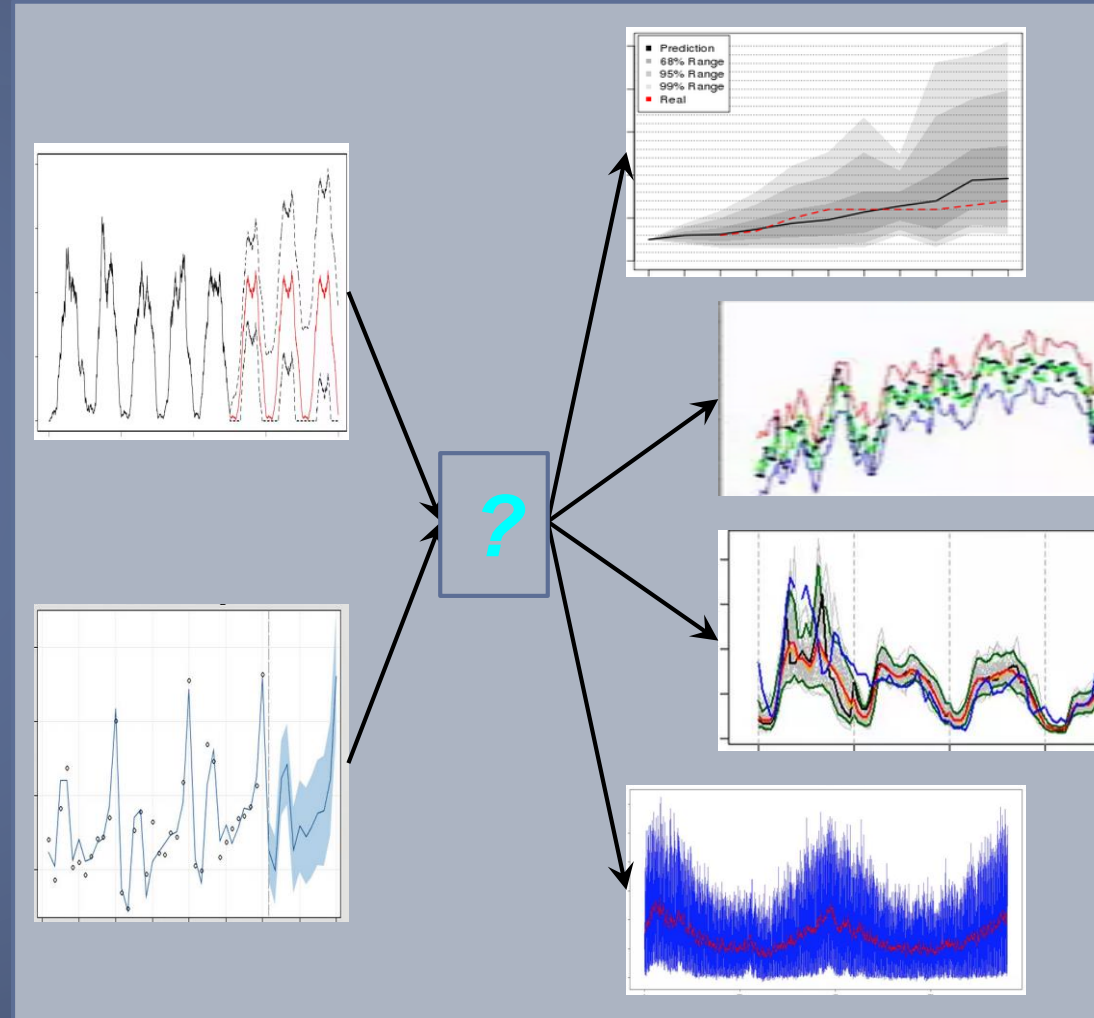
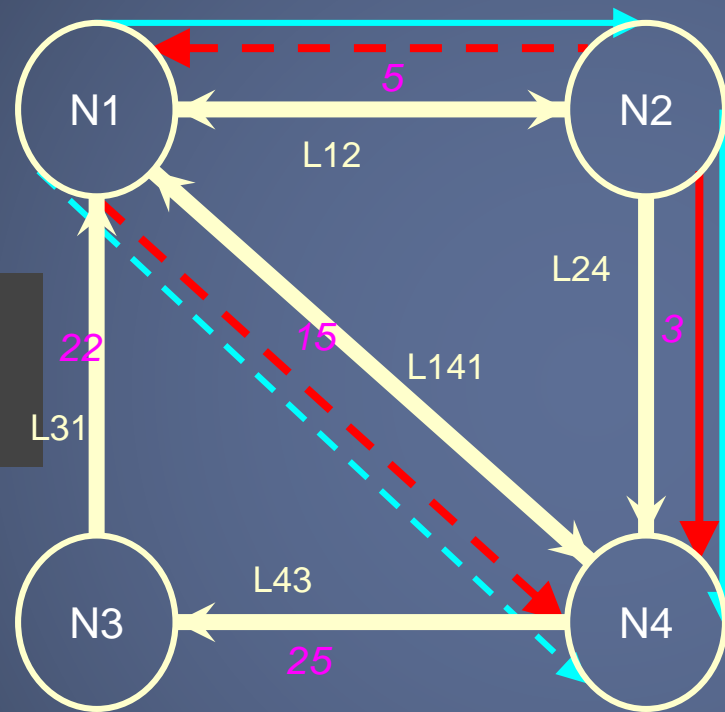


# From Generic Random Demand to Throughput:

## Demand:

N1\_N4: G (100, ...) G

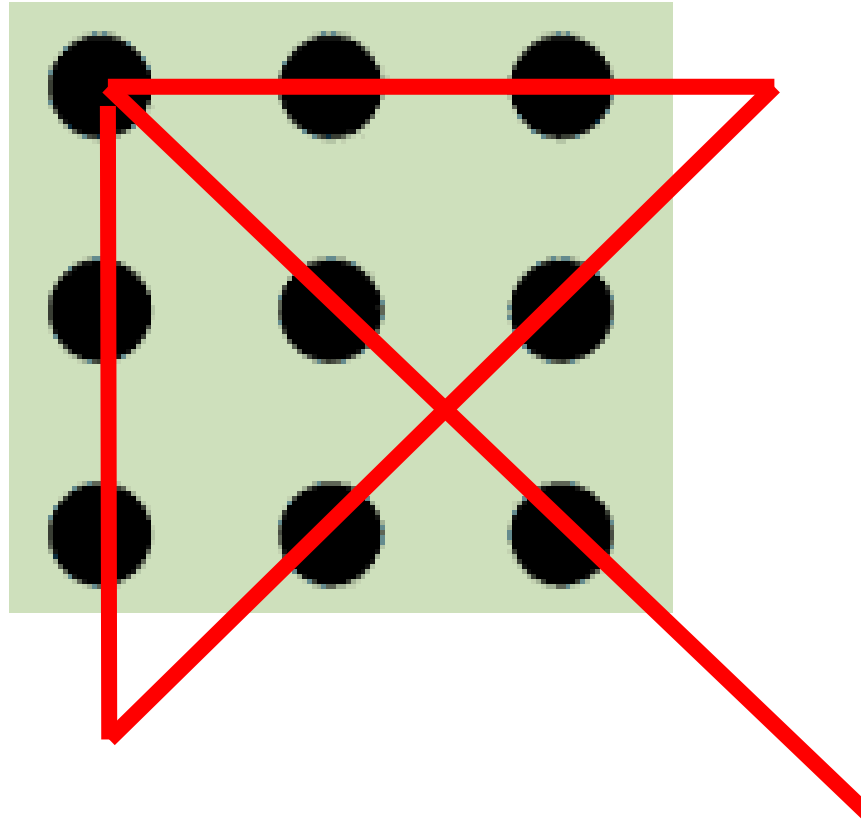
N2\_N4: G (200, ...) G



## Solution: Monte-Carlo



# Solution to the problem on Slide 1:



More to Explore: [https://www.math.washington.edu/~morrow/336\\_11/papers/leo.pdf](https://www.math.washington.edu/~morrow/336_11/papers/leo.pdf)