Correlation Coefficient of a Two-Channel Bell Test by Probability Theory

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***Abstract*—It is widely believed that only quantum mechanics can characterize the correlations of a Bell test. In fact, quantum mechanics is superfluous to the problem, and a correlation coefficient may be derived by probability theory.**

I. Introduction

John S. Bell famously proposed a test **FILL IN REFERENCE HERE** supposed to prove that quantum mechanics was, in essence, magical. I put the matter this way because the proposed test is soluble by probability theory, and yet Bell seems to have claimed quantum mechanics alone could provide the One True Answer that surpassed all other solutions. At the time of this writing, Bell's test is considered passed and the matter resolved in favor of the One True Answer.

Of course, what actually happened is that Bell *did not know* the test could be analyzed by means other than quantum mechanics, and also surely *already entertained the notion* that quantum mechanics might indeed be magical. Quantum mechanics has widely been regarded as some kind of "magic" for a very long time. However, some of us do not—at least when supposedly engaged in the scientific endeavor—entertain the notion that magic—or, indeed, anything remotely resembling it—exists. Furthermore, the Bell test problem is not, in fact, one of quantum mechanics at all. It is more properly a problem in random process analysis. It merely happens that quantum mechanics "knows" how to solve such a problem, without the user having to understand the subject of random process analysis.

We, however, wish to understand—and so shall recast the problem as one of random signal analysis. We will state a problem in random signal analysis, and the reader can confirm for themself that it is equivalent to a Bell test. Then we shall derive a correlation coefficient.

II. The Problem

There is a transmitter that sends a *signal* randomly from the set

$$S = \{\curvearrowleft\,,\ \curvearrowright\} \tag{1}$$

The transmission goes into both of two channels. Each channel attaches a *tag* to the signal, according to an algorithm to be specified below, and re-transmits the tagged signal. The tag comes from the set

$$T = \{\oplus\,,\ \ominus\} \tag{2}$$

The tagging algorithm works as follows. The channel is "tuned" by an angular setting $\zeta \in [0, 2\pi]$. Let r represent a number chosen uniformly from $[0, 1]$. Now suppose the signal σ is $\curvearrowleft$. In that case, if $r < \cos^2\zeta$ then re-transmit $(\oplus\,,\ \curvearrowleft)$. Otherwise re-transmit $(\ominus\,,\ \curvearrowleft)$. On the other hand, suppose the signal is σ is $\curvearrowright$. Then, if $r < \sin^2\zeta$ re-transmit $(\oplus\,,\ \curvearrowright)$, else re-transmit $(\ominus\,,\ \curvearrowright)$.

At the end of both channels is a receiver-recorder, which makes a record of received pairs of tagged signals, for some pair of "tunings" (ζ_1, ζ_2) for the two channels. For example, one hundred thousand or one million pairs of tagged signals might be recorded.

Now suppose we map tags to numbers, $T \to T' = \{-1, +1\}$, thus:

$$\oplus \mapsto +1 \tag{3}$$
$$\ominus \mapsto -1 \tag{4}$$

The problem is to use these numbers to calculate a correlation coefficient ρ, characterizing the correlation between tags in the received signal pairs.

III. Solution

We will use subscripts to refer to channel numbers. Thus, for example, ζ_2 may refer to a ζ parameter for channel 2, τ_1 to a tag value for channel 1, and so on. An unsubscripted letter may refer to either channel. Thus, for instance, τ may stand in for either τ_1 or τ_2. And so on like that.

We will use more or less conventional probability notation, though also always adding the letter "λ" as a condition, meaning something such as "any relevant information we may so far have neglected". It pays to be cautious.

Note also that all angles will be in some relation relative to each other, and positioned as a whole with respect to some "landmark." We will call the landmark ϕ_1 and associate it, purely for convenience, with the "calibration" of channel 1. The landmark may be any angle, and thus "$\phi_1 \in [0, 2\pi]$" will often appear as a condition.

By the problem definition, one immediately gets

$$P(\sigma = \curvearrowleft\ \mid \lambda) = 1/2 \tag{5}$$
$$P(\sigma = \curvearrowright\ \mid \lambda) = 1/2 \tag{6}$$

and

$$P(\tau = \oplus \,|\, \sigma = \curvearrowleft,\ \zeta = \phi,\ \phi_1 \in [0, 2\pi],\ \lambda) = \cos^2\phi \tag{7}$$
$$P(\tau = \ominus \,|\, \sigma = \curvearrowleft,\ \zeta = \phi,\ \phi_1 \in [0, 2\pi],\ \lambda) = \sin^2\phi \tag{8}$$
$$P(\tau = \oplus \,|\, \sigma = \curvearrowright,\ \zeta = \phi,\ \phi_1 \in [0, 2\pi],\ \lambda) = \sin^2\phi \tag{9}$$
$$P(\tau = \ominus \,|\, \sigma = \curvearrowright,\ \zeta = \phi,\ \phi_1 \in [0, 2\pi],\ \lambda) = \cos^2\phi \tag{10}$$

We want to construct a table of probabilities of tagged signal pairs received by the receiver-recorder, so let us start by finding an expression for the following.

$$P_1 = P(\sigma = \curvearrowleft,\ \tau_1 = \oplus,\ \tau_2 = \oplus \,|\, \zeta_1 = \phi_1,\ \zeta_2 = \phi_2 \quad \phi_1 \in [0, 2\pi],\ \lambda) \tag{11}$$

However, because that does not fit well into a line of text, let us first introduce a shorthand, by writing something like

$$P_1 = P(\curvearrowleft\ \oplus_1\ \oplus_2 \mid \phi_1,\ \phi_2,\ [0, 2\pi],\ \lambda) \tag{12}$$

to mean the same thing. Then, by probability theory, and taking into account that ϕ_1 and ϕ_2 respectively pertain exclusively to channel 1 or channel 2 (so conditionality on the opposite channel's ϕ may be dropped),

$$P_1 = P(\curvearrowleft\ \oplus_1\ \oplus_2 \mid \phi_1,\ \phi_2,\ [0, 2\pi],\ \lambda) \tag{13}$$
$$= P(\curvearrowleft\ \mid \lambda) P(\oplus_1\ \oplus_2 \mid\ \curvearrowleft,\ \phi_1,\ \phi_2,\ [0, 2\pi],\ \lambda) \tag{14}$$
$$= P(\curvearrowleft\ \mid \lambda) P_{11} P_{12} = \frac{1}{2} P_{11} P_{12} \tag{15}$$

where

$$P_{11} = P(\oplus_1 \mid\ \curvearrowleft,\ \phi_1,\ [0, 2\pi],\ \lambda) = \cos^2\phi_1 \tag{16}$$
$$P_{12} = P(\oplus_2 \mid\ \curvearrowleft,\ \phi_2,\ [0, 2\pi],\ \lambda) = \cos^2\phi_2 \tag{17}$$

By that and similar calculations the following table may be constructed.

$$P(\curvearrowleft\ \oplus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\cos^2\phi_1\ \cos^2\phi_2 \tag{18}$$
$$P(\curvearrowleft\ \oplus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\cos^2\phi_1\ \sin^2\phi_2 \tag{19}$$
$$P(\curvearrowleft\ \ominus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\sin^2\phi_1\ \cos^2\phi_2 \tag{20}$$
$$P(\curvearrowleft\ \ominus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\sin^2\phi_1\ \sin^2\phi_2 \tag{21}$$
$$P(\curvearrowright\ \oplus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\sin^2\phi_1\ \sin^2\phi_2 \tag{22}$$
$$P(\curvearrowright\ \oplus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\sin^2\phi_1\ \cos^2\phi_2 \tag{23}$$
$$P(\curvearrowright\ \ominus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\cos^2\phi_1\ \sin^2\phi_2 \tag{24}$$
$$P(\curvearrowright\ \ominus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)=\frac{1}{2}\cos^2\phi_1\ \cos^2\phi_2 \tag{25}$$

Disregarding the signal values gives (by addition of probabilities)

$$\begin{aligned}P(\oplus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\\ =P(\ominus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\\ =\frac{1}{2}\cos^2\phi_1\ \cos^2\phi_2+\frac{1}{2}\sin^2\phi_1\ \sin^2\phi_2&\end{aligned} \tag{26}$$

and

$$\begin{aligned}P(\oplus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\\ =P(\ominus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\\ =\frac{1}{2}\cos^2\phi_1\ \sin^2\phi_2+\frac{1}{2}\sin^2\phi_1\ \cos^2\phi_2&\end{aligned} \tag{27}$$

We want to calculate the correlation coefficient

$$\rho=\frac{E(\tau_1'\tau_2')}{\sqrt{E({\tau_1'}^2)}\sqrt{E({\tau_2'}^2)}} \tag{28}$$

where $\tau_1',\tau_2'\in T'$ and the expectations E are calculated with respect to the conditional probabilities derived above. The numerator is the covariance and the denominator is the product of the standard deviations.

The choice of values for the elements of T' makes it so the standard deviations in (28) equal one, and thus the correlation coefficient simplifies to the covariance

$$\rho=E(\tau_1'\tau_2') \tag{29}$$

which we now must calculate. To do so, not only must we compute a sum weighted by the probabilities in (26) and (27), but we must also eliminate the $\phi_1\in[0,2\pi]$ condition by integrating with respect to ϕ_1. We have no information ahead of time on what this angular landmark ϕ_1 will be, and so the probability distribution is uniform, which simplifies our integration. We will encounter a complication, but it will be minor: we will have to break the integration into pieces and solve the problem separately for each.

First let us calculate the sum for (26) and (27) (call the sum ρ'), and then we will integrate the result of that sum.

$$\begin{aligned}\rho'=(+1)(+1)P(\oplus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\\ +(+1)(-1)P(\oplus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\\ +(-1)(+1)P(\ominus_1\ \oplus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\\ +(-1)(-1)P(\ominus_1\ \ominus_2\mid\phi_1\,,\ \phi_2\,,\ [0,2\pi]\,,\ \lambda)&\end{aligned} \tag{30}$$

Substituting for the probabilities gives

$$\begin{aligned}\rho'=\cos^2\phi_1\ \cos^2\phi_2-\cos^2\phi_1\ \sin^2\phi_2&\\ -\sin^2\phi_1\ \cos^2\phi_2+\sin^2\phi_1\ \sin^2\phi_2&\end{aligned} \tag{31}$$

and so

$$\rho'=(\cos^2\phi_2-\sin^2\phi_2)(\cos^2\phi_1-\sin^2\phi_1) \tag{32}$$
$$=\cos(2\phi_2)\ \cos(2\phi_1) \tag{33}$$

with the last step by one of the double-angle identities.

This expression must now be integrated with respect to ϕ_1 over all of $[0,2\pi]$, using a uniform probability distribution function (pdf) as the weight. We run into this complication: we have to treat all landmark angles alike—a coordinate system does not care what quadrant its landmark is in—yet the cosine sometimes is reversed in sign, leading to ρ being reversed in sign. However, the sign of a correlation coefficient is arbitrary, so we can simply reverse the sign in those quadrants.

$$\rho=\int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}}\cos(2\phi_2)\cos(2\phi_1)\,d\phi_1\qquad\phi_1\in[-\frac{\pi}{4},+\frac{\pi}{4}] \tag{34}$$
$$-\rho=\int_{+\frac{\pi}{4}}^{+\frac{3\pi}{4}}\cos(2\phi_2)\cos(2\phi_1)\,d\phi_1\qquad\phi_1\in[+\frac{\pi}{4},+\frac{3\pi}{4}] \tag{35}$$
$$\rho=\int_{+\frac{3\pi}{4}}^{+\frac{5\pi}{4}}\cos(2\phi_2)\cos(2\phi_1)\,d\phi_1\qquad\phi_1\in[+\frac{3\pi}{4},+\frac{5\pi}{4}] \tag{36}$$
$$-\rho=\int_{+\frac{5\pi}{4}}^{+\frac{7\pi}{4}}\cos(2\phi_2)\cos(2\phi_1)\,d\phi_1\qquad\phi_1\in[+\frac{5\pi}{4},+\frac{7\pi}{4}] \tag{37}$$

In each case the result is the same. The correlation coefficient is

$$\rho=\cos(2\phi_2)=\cos^2\phi_2-\sin^2\phi_2 \tag{38}$$

regardless of quadrant.

Now suppose we have finished calibrating our transmission channels and wish to mark the dials. Where the landmark is on channel 1 we mark a zero. Where ϕ_2 is on channel 2 we mark the value of $\phi_2-\phi_1$. Then we complete the two circles using a protractor or similar device. We label the dials "θ".

Then the general formula for the correlation coefficient of the calibrated channels, in terms of any θ_1 and θ_2 in $[0,2\pi]$, is

$$\rho=\cos(2(\theta_2-\theta_1))=\cos^2(\theta_2-\theta_1)-\sin^2(\theta_2-\theta_1) \tag{39}$$

because one could always redo the calibration, using ϕ_1 equal to the new value of θ_1 and ϕ_2 equal to the new $\theta_2-\theta_1$.

This result accords with quantum mechanics **CITATION**.

IV. Discussion

I should not belabor this note with too many details of how Bell went wrong in his arguments. His writings simply are devoid of *correct* methods of logical inference. Instead we are subjected to *imitations* of logical inference.

For example, Bell famously introduces a mathematical contradiction with the goal of producing a physical absurdity **CITATION**. In fact, this is the crux of his argument in its best known form. But this is not a sound method a logical inference. One cannot deduce *anything* from a mathematical contradiction, except that the mathematical assumptions behind it are not sound. In other words, all Bell proved is that he made a mathematical error! Indeed, he made what is probably the most common error made in probability theory, which is to factor a joint probability incorrectly. However, Bell was *imitating* methods of logical inference, not actually employing them. For his naïve audience, this was good enough.

Perhaps, finally, we have reached the limit of what we should tolerate. Perhaps, now that millions and millions of dollars have been wasted, and graduate students' lives increasingly are being wasted, it is time to put our foot down and say it is enough. When it came to physics, Bell was a dunderhead, unable to reason like a scientist.

And now an entire batch of fields related to his work is full of dunderheads and incompetents who are supposed to be our society's greatest "geniuses," yet who fail to visualize that a two-channel Bell test is a totally ordinary, causal, contact-action random process that is, mathematically, shaped like a pair of wheels, which may have imposed upon them any angular coordinate system one wishes.

Among the corollaries of that "shape" is that, if you rotate θ_1 and θ_2 in unison, the frequencies of tag values change (also in unison) *but the correlations remains invariant*. A consequence of this is that it should be experimentally *impossible* to violate the so-called "CHSH inequality" **CITATION**, if the experiment be properly designed. This inequality applies a test that itself is not invariant under rotations of the apparatus. For Bell-test angles, in the case of photon experiments, a rotation by $\pi/8$ should see a drop in magnitude of the CHSH "correlation function" from $\sqrt{1/2}$ to zero, so $|S|$ (in the ideal) adds up to $\sqrt{2}$ instead of $2\sqrt{2}$. Yet experimenters report positive results. Therefore not only is the inequality itself a failure of scientific methods, so must be the experiments.

At this point I shall state unequivocally that there is no such thing as "entanglement." I write not for the journals but directly to the reader and so may speak freely. It is obvious that a "superposition state" or any such stuff must always be simply different notation for some probability expression, wave coherence expression, or the like. The real reason anyone believes there is magic involved seems not to be that the magic exists, but that the Fathers of the early Quantum Church (although perhaps not the great Emissary himself [1]) forbade treatment of quantum mechanics in the classical fashion. Physicists have been true to that stricture since, and simply do not *permit* the heresy of a solution by classical methods. One is permitted *only* to reach some solution different from that of quantum mechanics, which then is used to "prove" that quantum mechanics is "different." Otherwise physicists would have discovered long ago what we discovered above: that there actually *is no* "modern physics"—just "classical physics" that no one has been allowed to investigate.

Quantum physicists obviously do not even teach themselves and their students *how* to reach solutions of random process problems by classical methods. *I* knew how because *I* majored not in physics at all, but in the very closely related subject of *electrical engineering*. Peer review thus serves only to reinforce the orthodoxy. Orthodox papers in quantum physics are reviewed by orthodox quantum physicists for publication in orthodox journals. Peer review is useless, in this instance, to serve the scientific mission. The papers never are subjected to analysis by experts in random process analysis, who would tear the arguments to shreds, but only by ignorant persons who delight in the conclusions. Shining careers are constructed as the peers see each others' papers published in the most elite of journals. Eventually, the Nobel Prize in Physics for 2022 is awarded [2].

The author of *this* paper would tell a Nobel Committee to take their prize and dump it in *Östersjön*, and recommends the same for all Nobels in science and mathematics. Such prizes are unethical and destructive at best [3], [4], but now also shown to be awarded without distinction to both actual science and pseudoscientific claptrap. Is it worth giving up on the scientific enterprise, just to be famous and also insulted with a cash reward?

References

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