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EPR-B correlations: non-locality or geometry?

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Abstract

A photoelectron-by-photoelectron classical simulation of EPR-B correlations is described. It is shown that this model can be made compatible with Bell's renowned "no-go" theorem by restricting the source to that which produces only what is known as paired photons.

1 The problem

John Bell tells us:

> In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the settings of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant.[3]

This quotation may be taken as a statement of what has been labeled "Bell's Theorem."[4] It is, of course, not the statement of a theorem as the term is used in Mathematics and Logic. As a statement about a physics theory, it could never be a theorem, that is: a hypothesis proved by syllogism, because there is no chain of predecessor syllogisms ultimately based on an axiom set to call on. Physics, as a discipline is not built up that way; rather the other way around. Physics is the enterprise of trying to divine those basic theories that could be the axioms of a mathematical structure describing the material world. These basic theories, the *axioms,* as it were, are largely unknown and may in fact ever remain so, as least in part. Thus, there can be no theorems in Physics, *per se*, i.e., as distinct from mathematics used for physics.

The truth of Bell's statement depends on a more precise delineation of the elements of its 'should-have-been' hypothesis. Herein I will show, that strictly as it is, this statement is not true. A counter example can be displayed. Of course, my counter example may not put all disputes pertinent to the relevant physics to rest. One may argue, although not yet so brought to my attention, that it does not faithfully represent the situation envisioned by Bell, or that experiments done to test Bell's statement are somehow fundamentally different; or whatever. All in all, however, what the counter example arguably does show, is what does in fact need to be added to make Bell's statement valid.

2 A local and realistic EPR-B simulation

Elsewhere I have shown how to calculate the correlations seen in all experiments which are considered to demonstrate Bell's result.[6] These calculations use the classical definition of correlation among electromagnetic fields; they do not in any way exploit structure from quantum mechanics. They are, therefore, fully 'local' and 'realistic,' and as such serve already as counter examples to Bell's statement. Nevertheless, these calculations are somewhat like a "black-box": what goes in and comes out is clear, but what happens inside is mysterious. Apparently for this reason they have not been widely acclaimed.

Below I will describe a photoelectron-by-photoelectron simulation, ultimately based on the above calculation, of an EPR experiment. While this simulation adds nothing to the rigor of the analysis, it does foster much deeper insight into exactly how EPR correlations arise and why Bell made his statement. We shall see that Bell was operating under an implicit assumption, whose validity is a separate issue.

For the simulation, of a simple Clauser-Aspect type setup (see [2] for a current survey of such experiments) with two arms, five elements are considered: the signal source, $\boldsymbol{S}$, two "experimenter inputs," i.e., the polarizer settings, $\boldsymbol{A}$ and $\boldsymbol{B}$, and finally two analyzer stations, one on each arm, $\boldsymbol{X}$ and $\boldsymbol{Y}$.

$\boldsymbol{S}$ is implemented as an equal, flat, random selection of one of two possible signal pairs, one comprised of a vertically polarized pulse to the left, say, and a horizontally polarized pulse to the right; the second signal exchanges the polarizations.

$\boldsymbol{A}$ and $\boldsymbol{B}$ are implemented as simple random number generators with a flat distribution between 1 and 0, such that if a random number is ≤ 0.5, one polarizer setting is used, otherwise the other, etc. This is done to explicitly cater to the notion seen oft in the literature that the "free will" of the experimenter is a vital element of the experiment. Whether true of not, this slight elaboration is easily included in the simulation.

$\boldsymbol{X}$ and $\boldsymbol{Y}$ are implemented as models of polarizing beam splitters (PBS) for which the axis of the left (right) one is at an angle $\theta_{l\,(r)}$, each PBS feeds two photo detectors. These photo detectors are taken to adhere to Malus' Law, that is, they produce photoelectron streams for which the intensity is proportional to the incoming field intensity and the arrival times of the photoelectrons is a random variable described by a Poisson process.[1] In so far as these photo detectors are all independent, each Poisson process is event-wise uncorrelated with respect to the others. As a matter of detail, the photoelectron generation process is modeled by comparing a random number with the field such that if the random number is less that the field intensity, it is taken that a photoelectron was generated; if greater than the incoming intensity, none was generated. In other words, N_{hl} (i.e., the number of electrons in the horizontal channel on the left) is increased by 1, when the random number $\leq \cos^2(\theta_l)$; and N_{vl} is increased by 1 when the random number $\leq \sin^2(\theta_l)$, etc.

The final step of the simulation is simply to register the 'creation' of photoelectrons.

All information flow is from $\boldsymbol{S}$, $\boldsymbol{A}$, and $\boldsymbol{B}$ to $\boldsymbol{X}$ and $\boldsymbol{Y}$. There is no information flow between $\boldsymbol{X}$ and $\boldsymbol{Y}$, so that there is no non-local interaction. At this point please note, however, that the usual analysis of EPR experiments is incomplete; it neglects to consider

[1] I use time units of "windows;" i.e., a photoelectron is lifted into a conduction band in the first or second or ... n-th window. What happens exactly within a window is considered unknowable for practical reasons. Thus,these random variables exhibit Poisson statistics, not exponential.

the data acquisition procedures and subsequent factual (as opposed to *supposed*) analysis procedures. Specifically, too often one does not consider the "coincidence circuitry" and how its output feeds the subsequent data analysis.

At this point conventional analysis proceeds to delineate how the data is to be analyzed. What it calls for is the total of the number of times the outputs are equal, i.e., $N^{=}_{ab}$, for the polarizer settings a and b, as well as the number of times they are unequal, $N^{\neq}_{ab}$, and the total number of trials, N_{ab}. With these numbers, the correlation, κ, for each setting pair, ab, is then:

$$\kappa_{ab} = \frac{N^{=}_{ab} - N^{\neq}_{ab}}{N_{ab}}. \tag{2.1}$$

These correlations, in turn, are used to compute the CHSH contrast:

$$S = k_{12} + k_{11} + k_{21} - k_{22}, \tag{2.2}$$

which is to be tested for violation of Bell's limit, $|S| \leq 2$, as is well known.[1, 2]

In my simulation, however, the data analysis proceeds differently. To model the "coincidence circuitry" I take it that, rather than register single photoelectrons in each output channel, coincidence pairs between the channels are counted. Here again, relative intensity between channels is given purely by geometrical considerations according to Malus' Law; i.e., according to $\cos^2(\theta_r - \theta_l)$ for like channels, and $\sin^2(\theta_r - \theta_l)$ for unlike channels. This is essentially equivalent to requiring internal self consistency. That is, if the axis of one PBS is parallel to the axis of the source, $\theta_r = 0$, say, then it is quite obvious that the relative intensity as measured by photo detectors on the outputs of the other PBS must follow Malus' Law by virtue of the photocurrent generation mechanism. Further, since this geometric fact must be independent of the choice of coordinate system, it follows that a transformation of coordinates can be effected using

$$\begin{aligned} \cos(\theta_r - \theta_l) &= \cos(\theta_r)\cos(\theta_l) + \sin(\theta_r)\sin(\theta_l), \\ \sin(\theta_r - \theta_l) &= \sin(\theta_r)\cos(\theta_l) - \cos(\theta_r)\sin(\theta_l); \end{aligned} \tag{2.3}$$

which are just trigonometric identities. Note that although (2.3) are not in general factorable (sometimes said to be a criteria for 'non-locality'), all the information required is available after-the-fact from both sides independently, i.e., no violation of locality occurs. Thus, in modeling the "coincidence circuitry" I use the fact that the individual terms on the right side of (2.3) are, by virtue of photo detector physics, proportional to the square root of the number of counts in the channel, i.e.,

$$\begin{aligned} \cos(\theta_l) &= \lim_{N\to\infty} \sqrt{N_{hl}/N}, \\ \sin(\theta_l) &= \lim_{N\to\infty} \sqrt{N_{vl}/N}; \end{aligned} \tag{2.4}$$

where N is the total number of signals intercepted by the considered photo detector in the experiment. In other words, the relative frequency of coincidences is determined as a function of the intensity of the photocurrent after-the-fact. No communication between right and left sides is involved; what correlation there is, is there on account of the equality of the amplitudes (and therefore *intensities*) at the source of the signals making up the singlet state. Of course, in doing a simulation, provision must be made to resolve the sign ambiguity; doing so, however, also does not violate locality as the required information is all available after-the-fact. Nature herself (in the form of a PBS setting), as it were, "knows" which sign is valid without super luminal communication. In sum, instead of Eq. (2.1), I use:

$$\kappa^* = \cos^2(\theta_r - \theta_l) - \sin^2(\theta_r - \theta_l), \tag{2.5}$$

as required by Malus' Law, and then break down the terms using (2.3) and (2.4) to get the result for each individual 'photoelectron.'

An example of the results from the simulation are presented on Fig. 1. The top curve

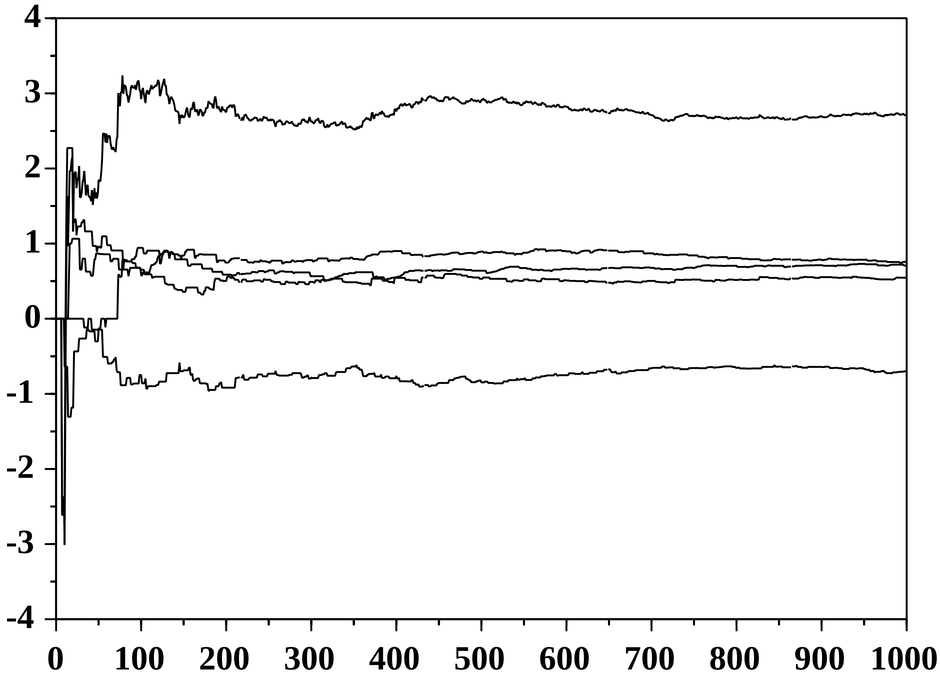

Figure 1. An example of a simulation of an EPR-B experiment. The top curve is the CHSH contrast; the others are the individual correlation coefficients, all plotted as a function of the number of trials. (Scilab code available on request.)

is the CHSH contrast; the lower four curves are the individual correlations for the four combinations of PBS settings. The statistics stabilize after circa 700 trials and exhibit clear violation of the Bell limit of 2, and then by virtually exactly the amount, $2\sqrt{2}$, calculated for the singlet state and observing angles $\theta_l = 0,\ \pi/4$ and $\theta_r = \pm\pi/8$ using Bell inequalities. That these statistics can be found in a sequence of individual events, each of which is calculated without recourse to non-locality, constitutes a direct, unambiguous counterexample to Bell-type "theorems." This simulation is not just an hypothetical, academic exercise, laboratory confirmation can be found in.[5]

3 Consequences

In addition to a direct simulation of a standard EPR experiment, the numerical machinery developed above can be used to explore other setups. Two such alternatives are of special interest. One results when instead of using for the source, $\boldsymbol{S}$, what is a model of essentially classical electromagnetic pulses, a model of what can be taken as pairs of "photons" is used. In this case, instead of independent Poisson processes at each photo detector, one would have a single process at the source. The results do *not* violate Bell's inequality. It appears that this is the case that most proponents of Bell's analysis have in mind, and therefore can be taken to indicate what the essential change needed is, in order to invest validity in Bell's statement. That is: if radiation is ontologically comprised of "photons," then it entails non-locality with respect to EPR-B correlations. (Actually, this still does not fully rectify Bell's analysis—which I hold to be fundamentally defective for other reasons discussed elsewhere[7]—but rather calls on a consequence of dichotomic arithmetic as involved in Kochen-Specker type theorems. This matter is both too involved to be covered here and not fully worked out in detail; the interested reader is referred to:[8].)

(Fortuitously, this difference may be observable. If radiation is digitized as photons, then at very low intensity (i.e., when the average interval between emission of photon pairs is long in comparison to the time tolerance between electrons of the same pair as permitted by Heisenberg uncertainty), then the coincidence current as a function of the window-width used to define a coincidence, should increase stepwise as additional pairs are captured. On the other hand, if radiation is continuous, then the coincidence current as a function of window-width should increase monotonically as the independent processes at separate photo detectors register additional candidates to form pairs. Since the window-width in many experiments is set by software, reanalysis of existing data may be able to address this issue.)

Another especially interesting case is that involving counter factual reasoning, which can not be tested empirically, as it is impossible to exactly recreate all the circumstances of a particular experimental run. Simulations, however, can be rerun with identical 'random' inputs; it is only necessary to retain whatever random values were used for one run, for the execution of another run. A careful study by Adenier of counter factual executions of EPR experiments shows that the results *should* violate Bell inequalities.[1] Here I report that simulations of such experiments confirm Adenier's results.

Finally, the results of these simulations, together with Occam's principle, would seem to argue that, at least with respect to EPR-B correlations, the photon conception brings with it non-locality, while a purely classical interpretation of these phenomena does not. Whether this conclusion can be extended to cover Compton scattering, anti-coincidence detection at the single photoelectron level at a half silvered mirror, etc., so as to further elucidate the photon concept, remains to be studied. In any case, it is easily seen that a simple geometric effect, projection according to Malus' Law, *can* fully account for EPR-B correlations. The case for non-locality is not without caveats.

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