Cookies for Alice and Bob

— The Problem —

1. You have one box of Chocolate Cookies and one box of Vanilla Cookies.

2. You will deliver one of the boxes to Alice and one of the boxes to Bob. Neither has a preference of flavor.

3. You flip a coin to decide who gets which box.[1]

4. Alice has a favorite angle, α. Bob also has a favorite angle, β. You write down the numbers $\cos^2\alpha$, $\sin^2\alpha$, $\cos^2\beta$, and $\sin^2\beta$. Also, you have a pair of ten-sided dice. You can roll these dice to get a number between 0.00 and 0.99, inclusive.

5. Suppose Alice is getting the box of Chocolate Cookies. You roll the dice. If the number you roll is less than $\cos^2\alpha$, then you write $+1$ on the box. Otherwise you write -1.[2]

6. Suppose, on the other hand, that Alice is getting the box of Vanilla Cookies. You roll the dice. If the number you roll is less than $\sin^2\alpha$, then you write $+1$ on the box. Otherwise you write -1.

7. Suppose Bob is getting the box of Chocolate Cookies. You roll the dice. If the number you roll is less than $\cos^2\beta$, then you write $+1$ on the box. Otherwise you write -1.

8. Suppose, on the other hand, that Bob is getting the box of Vanilla Cookies. You roll the dice. If the number you roll is less than $\sin^2\beta$, then you write $+1$ on the box. Otherwise you write -1.

9. Calculate the correlation coefficient of the numbers written on the cookie boxes, as a function of $\alpha - \beta$.[3]

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[1] This step is equivalent to a light source generating a pair of perpendicularly polarized photons.

[2] This is equivalent to a photon being detected in one of the channels of a polarizing beam splitter.

[3] The answer from quantum mechanics is $-\cos 2(\alpha-\beta)$, and physicists assure us only quantum mechanical systems can achieve this solution. According to physicists, even logically equivalent systems cannot do so, because they have no entangled superposition states nor action at a distance!

— The Solution —

First let us solve the problem for $\alpha = \alpha_0$ and $\beta = \beta_0 = 0$. For this particular case we have

$$\begin{aligned}
\cos^2 \alpha &= \cos^2 \alpha_0 \\
\sin^2 \alpha &= \sin^2 \alpha_0 \\
\cos^2 \beta &= 1 \\
\sin^2 \beta &= 0
\end{aligned}$$

The choice of markings as $\{+1, -1\}$ makes the correlation coefficient simple to compute. It is

$$\rho_0 = \rho_0^{++} + \rho_0^{+-} + \rho_0^{-+} + \rho_0^{--}$$

where

$$\begin{aligned}
\rho_0^{++} &= \frac{1}{2}\,(+1)\,(+1)\,\sin^2 \alpha_0 \\
\rho_0^{+-} &= \frac{1}{2}\,(+1)\,(-1)\,\cos^2 \alpha_0 \\
\rho_0^{-+} &= \frac{1}{2}\,(-1)\,(+1)\,\cos^2 \alpha_0 \\
\rho_0^{--} &= \frac{1}{2}\,(-1)\,(-1)\,\sin^2 \alpha_0
\end{aligned}$$

Thus (by a double-angle identity found in reference books)

$$\rho_0 = -(\cos^2 \alpha_0 - \sin^2 \alpha_0) = -\cos 2\alpha_0$$

Let $\alpha = \alpha_0 + \beta$. Then $\beta = \beta_0 + \beta$, so β is a change of angular coordinates. Now ρ_0 is the desired solution, so rename it ρ without the subscript. The solution is

$$\rho = -\cos 2(\alpha - \beta)$$

Although the particular 'scattering' of $+1$ and -1 marks (were a person to do the operation many times) depends on the specific values of α and β, the correlation coefficient depends only on the difference $\alpha - \beta$. There is a relationship between $+1$ and -1 marks that is invariant under in-unison rotation of the angles.[4] □

[4] This solution is the same as the one from quantum mechanics. Of course, the same rotational invariance was always true of Bell test experiments, but little remarked on. But be forewarned: there are many 'doctors of philosophy' who would object to my solution yet are permitted to teach university classes.