The Problem

1. You have one box of Chocolate Cookies and one box of Vanilla Cookies.

2. You will deliver one of the boxes to Alice and one of the boxes to Bob. Neither has a preference of flavor.

3. You flip a coin to decide who gets which box.[1]

4. Alice has a favorite angle, α. Bob also has a favorite angle, β. You write down the numbers $100 \times \cos^2 \alpha$, $100 \times \sin^2 \alpha$, $100 \times \cos^2 \beta$, and $100 \times \sin^2 \beta$. Also, you have a pair of ten-sided dice. You can roll these dice to get a number between zero and ninety-nine.

5. Suppose Alice is getting the box of Chocolate Cookies. You roll the dice. If the number you roll is less than $100 \times \cos^2 \alpha$, then you write $+1$ on the box. Otherwise you write -1.[2]

6. Suppose, on the other hand, that Alice is getting the box of Vanilla Cookies. You roll the dice. If the number you roll is less than $100 \times \sin^2 \alpha$, then you write $+1$ on the box. Otherwise you write -1.

7. Suppose Bob is getting the box of Chocolate Cookies. You roll the dice. If the number you roll is less than $100 \times \cos^2 \beta$, then you write $+1$ on the box. Otherwise you write -1.

8. Suppose, on the other hand, that Bob is getting the box of Vanilla Cookies. You roll the dice. If the number you roll is less than $100 \times \sin^2 \beta$, then you write $+1$ on the box. Otherwise you write -1.

[1] This step is equivalent to a light source generating a pair of perpendicularly polarized photons.
[2] This is equivalent to a photon being detected in one of the channels of a polarizing beam splitter.

9. Calculate the correlation coefficient of the numbers written on the cookie boxes, as a function of $\alpha - \beta$.[3]

The Solution

First let us solve the problem for $\alpha = \alpha_0$ and $\beta = \beta_0 = 0$. We will use β_0 as an *angular origin*. Then

$$\cos^2 \alpha = \cos^2 \alpha_0$$

$$\sin^2 \alpha = \sin^2 \alpha_0$$

$$\cos^2 \beta = 1$$

$$\sin^2 \beta = 0$$

The choice of markings as $\{+1, -1\}$ makes the correlation coefficient simple to compute. It is

$$\rho_0 = \rho_0^{++} + \rho_0^{+-} + \rho_0^{-+} + \rho_0^{--}$$

where

[3] The answer from quantum mechanics is $-\cos\{2(\alpha - \beta)\}$, and physicists assure us only quantum mechanical systems can achieve this correlation. According to physicists, not even logically equivalent systems can do so!

$$\rho_0^{++} = \frac{1}{2}\,(+1)\,(+1)\,\sin^2\alpha_0$$

$$\rho_0^{+-} = \frac{1}{2}\,(+1)\,(-1)\,\cos^2\alpha_0$$

$$\rho_0^{-+} = \frac{1}{2}\,(-1)\,(+1)\,\cos^2\alpha_0$$

$$\rho_0^{--} = \frac{1}{2}\,(-1)\,(-1)\,\sin^2\alpha_0$$

Thus (by a double-angle identity found in reference books)

$$\rho_0 = -(\cos^2\alpha_0 - \sin^2\alpha_0) = -\cos(2\alpha_0)$$

Let $\alpha = \alpha_0 + \beta$. Then ρ_0 is the desired solution, so rename it ρ without the subscript:

$$\rho = -\cos\{2(\alpha - \beta)\}$$

Although the particular arrangement of $+1$ and -1 marks depends on the specific values of α and β, the correlation coefficient depends only on their difference.[4] Of course the corresponding fact was always true of Bell test experiments.

[4] This solution is the same as the one from quantum mechanics. Presumably some physicists would Gish Gallop a lot of objections to my solution. Such persons are allowed to teach classes?