2 RG&TC-Code

2e

In[54]:=
```
g = {{Exp[2 β[r]], 0, 0},
    {0, r^2, 0},
    {0, 0, r^2 Sin[θ]^2}};
g // MatrixForm
```

Out[55]//MatrixForm=

$$\begin{pmatrix} e^{2\,\beta[r]} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

In[56]:=
```
xcoord = {r, θ, ϕ}
```

Out[56]= $\{r, \theta, \phi\}$

In[57]:=
```
RGtensors[g, xcoord]
```

$$\mathrm{gdd} = \begin{pmatrix} e^{2\,\beta[r]} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

$$\mathrm{LineElement} = e^{2\,\beta[r]}\,\mathrm{d}[r]^2 + r^2\,\mathrm{d}[\theta]^2 + r^2\,\mathrm{d}[\phi]^2\,\mathrm{Sin}[\theta]^2$$

$$\mathrm{gUU} = \begin{pmatrix} e^{-2\,\beta[r]} & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

gUU computed in 0.004038 sec

Gamma computed in 0.001647 sec

Riemann(dddd) computed in 0.001665 sec

Riemann(Uddd) computed in 0.001432 sec

Ricci computed in 0.003464 sec

Weyl computed in 0.000025 sec

Testing for 3-dim conformal flatness...

••• Outer : Heads Times and List at positions 3 and 2 are expected to be the same .

Einstein computed in 0.001583 sec

Out[57]= All tasks completed in 0.02007

In[58]:=
```
Rdd // MatrixForm
```

Out[58]//MatrixForm=

$$\begin{pmatrix} \frac{2\,\beta'[r]}{r} & 0 & 0 \\ 0 & e^{-2\,\beta[r]}\left(-1+e^{2\,\beta[r]}+r\,\beta'[r]\right) & 0 \\ 0 & 0 & e^{-2\,\beta[r]}\,\text{Sin}[\theta]^2\left(-1+e^{2\,\beta[r]}+r\,\beta'[r]\right) \end{pmatrix}$$

In[59]:=
```
DSolve[Rdd == 2 k g, β'[r], r]
```

Out[59]=
$$\text{DSolve}\left[\left\{\left\{\frac{2\,\beta'[r]}{r},\,0,\,0\right\},\,\{0,\,e^{-2\,\beta[r]}\left(-1+e^{2\,\beta[r]}+r\,\beta'[r]\right),\,0\},\,\{0,\,0,\,e^{-2\,\beta[r]}\,\text{Sin}[\theta]^2\left(-1+e^{2\,\beta[r]}+r\,\beta'[r]\right)\}\right\} == \{\{2\,e^{2\,\beta[r]}\,k,\,0,\,0\},\,\{0,\,2\,k\,r^2,\,0\},\,\{0,\,0,\,2\,k\,r^2\,\text{Sin}[\theta]^2\}\},\,\beta'[r],\,r\right]$$

3a

In[60]:=
```
Integrate[1/Sqrt[1 - k r^2], r]
```

Out[60]=
$$\frac{\text{ArcSin}\left[\sqrt{k}\,r\right]}{\sqrt{k}}$$

k=-1

In[61]:=
```
Solve[χ == Limit[ArcSin[Sqrt[k] r]/Sqrt[k], k → -1], r, Reals]
```

Out[61]=
$$\{\{r \to \text{Sinh}[\chi]\}\}$$

k=0

In[62]:=
```
Solve[χ == Limit[ArcSin[Sqrt[k] r]/Sqrt[k], k → 0], r]
```

Out[62]=
$$\{\{r \to \chi\}\}$$

k=1

In[63]:=
```
Solve[χ == Limit[ArcSin[Sqrt[k] r]/Sqrt[k], k → 1], r, Reals]
```

Out[63]=
$$\left\{\left\{r \to \boxed{\text{Sin}[\chi]\ \ \text{if}\ \ -\frac{\pi}{2} \le \chi \le \frac{\pi}{2}}\right\}\right\}$$

3b

In[64]:=
```
g = {{-1, 0, 0, 0},
  {0, bigR[t]^2, 0, 0},
  {0, 0, Sk[χ]^2, 0},
  {0, 0, 0, Sk[χ]^2 Sin[θ]^2}}
```

Out[64]= $\{\{-1, 0, 0, 0\}, \{0, \text{bigR}[t]^2, 0, 0\}, \{0, 0, \text{Sk}[\chi]^2, 0\}, \{0, 0, 0, \text{Sin}[\theta]^2\ \text{Sk}[\chi]^2\}\}$

In[65]:=
```
xcoord = {t, r, θ, ϕ}
```

Out[65]= $\{t, r, \theta, \phi\}$

In[66]:=
```
g = {{-1, 0, 0, 0},
  {0, 1^2/(1 - k r^2), 0, 0},
  {0, 0, r^2, 0},
  {0, 0, 0, r^2 Sin[θ]^2}}
```

Out[66]= $\left\{\{-1, 0, 0, 0\}, \left\{0, \frac{1}{1 - k\ r^2}, 0, 0\right\}, \{0, 0, r^2, 0\}, \{0, 0, 0, r^2\ \text{Sin}[\theta]^2\}\right\}$

In[67]:=
```
RGtensors[g, xcoord]
```

$$\text{gdd} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{1-k\,r^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

$$\text{LineElement} = -\frac{\text{d}[r]^2}{-1+k\,r^2} - \text{d}[t]^2 + r^2\,\text{d}[\theta]^2 + r^2\,\text{d}[\phi]^2\,\text{Sin}[\theta]^2$$

$$\text{gUU} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1-k\,r^2 & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\text{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

gUU computed in 0.002933 sec

Gamma computed in 0.00534 sec

Riemann(dddd) computed in 0.006296 sec

Riemann(Uddd) computed in 0.004903 sec

Ricci computed in 0.001194 sec

Weyl computed in 0.010067 sec

Conformally Flat

Einstein computed in 0.000458 sec

Out[67]= All tasks completed in 0.039249

In[68]:= **GUdd // MatrixForm**

Out[68]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -\frac{k\,r}{-1+k\,r^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ r\left(-1+k\,r^2\right) \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ r\left(-1+k\,r^2\right)\text{Sin}[\theta]^2 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{1}{r} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{r} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\text{Cos}[\theta]\,\text{Sin}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{r} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \text{Cot}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{r} \\ \text{Cot}[\theta] \\ 0 \end{pmatrix} \end{pmatrix}$$

In[69]:=
$$\text{eqn1} = r''[\lambda] - \frac{k\, r[\lambda]}{-1 + k\, r[\lambda]^2} (r'[\lambda])^2 + r[\lambda] \left(-1 + k\, r[\lambda]^2\right) \text{Sin}\left[\frac{\pi}{2}\right]^2 (\phi'[\lambda])^2;$$

$$\text{eqn2} = \phi''[\lambda] + 2 \frac{1}{r[\lambda]} (r'[\lambda] \times \phi'[\lambda]);$$

k = 1:

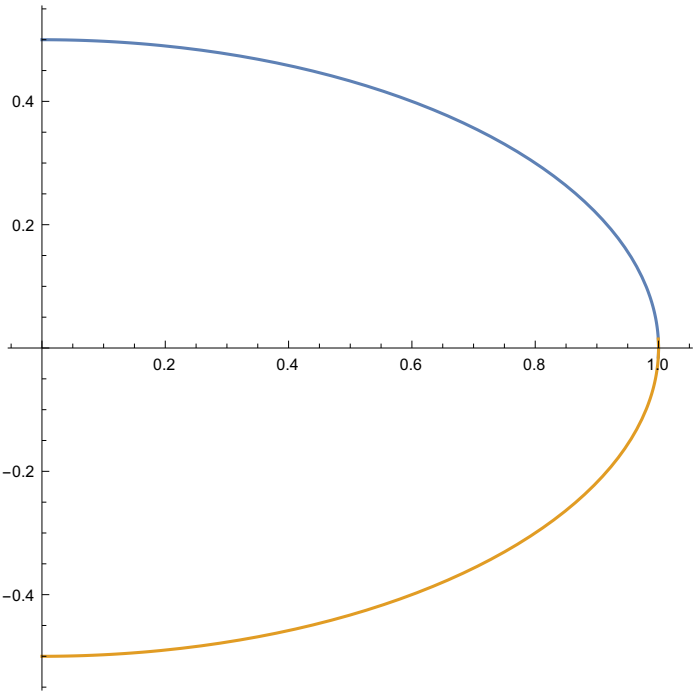
In[71]:=
$$s1 = \text{NDSolve}\left[\left\{\text{eqn1} == 0,\ \text{eqn2} == 0,\ \phi'[0] == -1,\ r'[0] == 0,\ r[0] == 0.5,\ \phi[0] == \frac{\pi}{2}\right\} /. \{k \to 1\},\ \{r[\lambda],\ \phi[\lambda]\},\ \{\lambda,\ 0,\ 5\}\right][[1]];$$

$$s2 = \text{NDSolve}\left[\left\{\text{eqn1} == 0,\ \text{eqn2} == 0,\ \phi'[0] == 1,\ r'[0] == 0,\ r[0] == 0.5,\ \phi[0] == \frac{-\pi}{2}\right\} /. \{k \to 1\},\ \{r[\lambda],\ \phi[\lambda]\},\ \{\lambda,\ 0,\ 5\}\right][[1]];$$

In[73]:=

In[74]:=
```
ParametricPlot[{{Evaluate[r[λ] Cos[ϕ[λ]] /. s1], Evaluate[r[λ] Sin[ϕ[λ]] /. s1]},
  {Evaluate[r[λ] Cos[ϕ[λ]] /. s2], Evaluate[r[λ] Sin[ϕ[λ]] /. s2]}}, {λ, 0, 3.2}]
```

Out[74]=

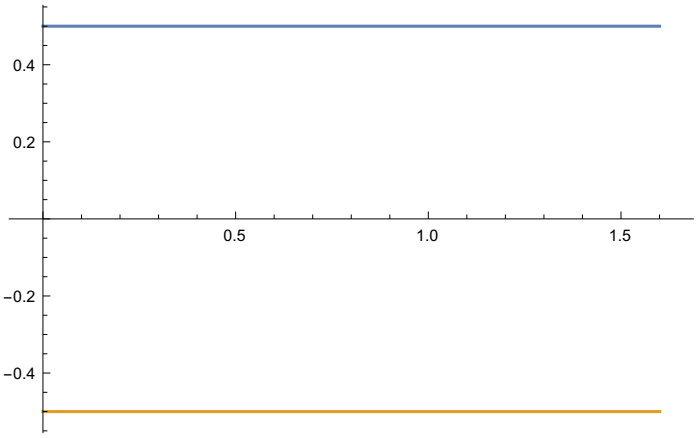


k = 0:

In[75]:=
```
s1 =
  NDSolve[{eqn1 == 0, eqn2 == 0, ϕ'[0] == -1, r'[0] == 0, r[0] == 0.5, ϕ[0] == π/2} /. {k → 0},
    {r[λ], ϕ[λ]}, {λ, 0, 5}][[1]];
s2 = NDSolve[{eqn1 == 0, eqn2 == 0, ϕ'[0] == 1, r'[0] == 0, r[0] == 0.5, ϕ[0] == -π/2} /.
    {k → 0}, {r[λ], ϕ[λ]}, {λ, 0, 5}][[1]];
```

In[77]:=
```
ParametricPlot [{{Evaluate[r[λ] Cos[ϕ[λ]] /. s1], Evaluate[r[λ] Sin[ϕ[λ]] /. s1]},
  {Evaluate[r[λ] Cos[ϕ[λ]] /. s2], Evaluate[r[λ] Sin[ϕ[λ]] /. s2]}}, {λ, 0, 3.2}]
```

Out[77]=

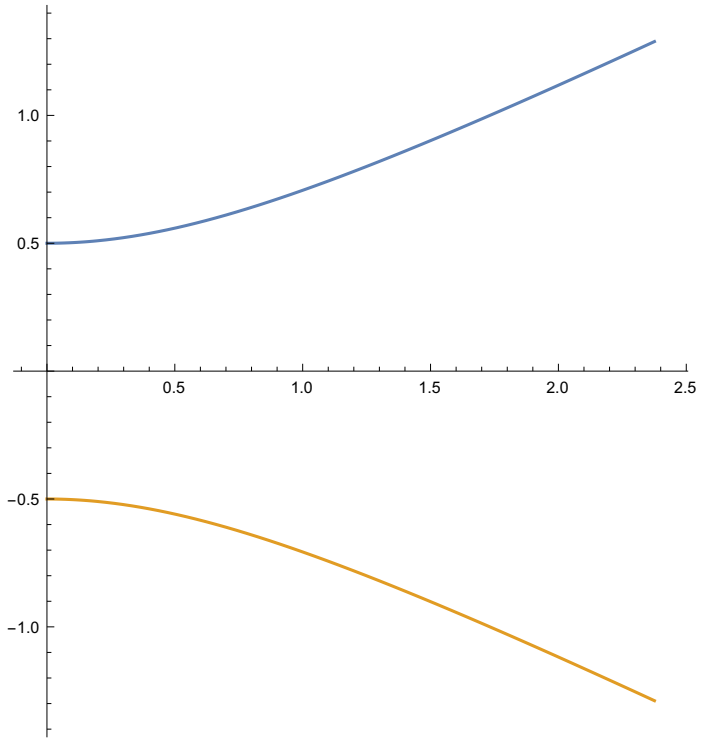


k = -1:

In[78]:=
```
s1 = NDSolve[{eqn1 == 0, eqn2 == 0, ϕ'[0] == -1, r'[0] == 0, r[0] == 0.5, ϕ[0] == π/2} /.
    {k → -1}, {r[λ], ϕ[λ]}, {λ, 0, 5}][[1]];
s2 = NDSolve[{eqn1 == 0, eqn2 == 0, ϕ'[0] == 1, r'[0] == 0, r[0] == 0.5, ϕ[0] == -π/2} /.
    {k → -1}, {r[λ], ϕ[λ]}, {λ, 0, 5}][[1]];
```

In[80]:=
```
ParametricPlot [{{Evaluate [r[λ] Cos[ϕ[λ]] /. s1], Evaluate [r[λ] Sin[ϕ[λ]] /. s1]},
  {Evaluate [r[λ] Cos[ϕ[λ]] /. s2], Evaluate [r[λ] Sin[ϕ[λ]] /. s2]}}, {λ, 0, 3.2}]
```

Out[80]=



4a

In[96]:=
```
g = {{-1, 0, 0, 0},
   {0, a[t]^2/(1 - κ r^2), 0, 0},
   {0, 0, a[t]^2 r^2, 0},
   {0, 0, 0, a[t]^2 r^2 Sin[θ]^2}};
g // MatrixForm
```

Out[97]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a[t]^2}{1-r^2\,\kappa} & 0 & 0 \\ 0 & 0 & r^2\,a[t]^2 & 0 \\ 0 & 0 & 0 & r^2\,a[t]^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

```
xcoord = {t, r, θ, ϕ}
```

In[98]:=
```
RGtensors[g, xcoord]
```

$$\text{gdd} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a[t]^2}{1-r^2\,\kappa} & 0 & 0 \\ 0 & 0 & r^2\,a[t]^2 & 0 \\ 0 & 0 & 0 & r^2\,a[t]^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

$$\text{LineElement} = -\frac{a[t]^2\,d[r]^2}{-1+r^2\,\kappa} - d[t]^2 + r^2\,a[t]^2\,d[\theta]^2 + r^2\,a[t]^2\,d[\phi]^2\,\text{Sin}[\theta]^2$$

$$\text{gUU} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -\frac{-1+r^2\,\kappa}{a[t]^2} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2\,a[t]^2} & 0 \\ 0 & 0 & 0 & \frac{\text{Csc}[\theta]^2}{r^2\,a[t]^2} \end{pmatrix}$$

gUU computed in 0.004357 sec

Gamma computed in 0.006784 sec

Riemann(dddd) computed in 0.007511 sec

Riemann(Uddd) computed in 0.004744 sec

Ricci computed in 0.00159 sec

Weyl computed in 0.008458 sec

Conformally Flat

Einstein computed in 0.000941 sec

Out[98]= All tasks completed in 0.042554

In[99]:= **Rdd // MatrixForm**

Out[99]//MatrixForm=

$$\begin{pmatrix} -\frac{3\,a''[t]}{a[t]} & 0 & 0 & 0 \\ 0 & -\frac{2\,\kappa+2\,a'[t]^2+a[t]\,a''[t]}{-1+r^2\,\kappa} & 0 & 0 \\ 0 & 0 & r^2\left(2\,\kappa+2\,a'[t]^2+a[t]\,a''[t]\right) & 0 \\ 0 & 0 & 0 & r^2\,\text{Sin}[\theta]^2\left(2\,\kappa+2\,a'[t]^2+a[t]\,a''[t]\right) \end{pmatrix}$$

In[100]:= **R // Simplify**

Out[100]= $$\frac{6\left(\kappa+a'[t]^2+a[t]\,a''[t]\right)}{a[t]^2}$$

4d

In[101]:= **GUdd // MatrixForm**

Out[101]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -\frac{a[t]\,a'[t]}{-1+r^2\,\kappa} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ r^2\,a[t]\,a'[t] \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ r^2\,a[t]\,\text{Sin}[\theta]^2\,a'[t] \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{a'[t]}{a[t]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{a'[t]}{a[t]} \\ -\frac{r\,\kappa}{-1+r^2\,\kappa} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ r\,(-1+r^2\,\kappa) \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ r\,(-1+r^2\,\kappa)\,\text{Sin}[\theta]^2 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ \frac{a'[t]}{a[t]} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{1}{r} \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{a'[t]}{a[t]} \\ \frac{1}{r} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\text{Cos}[\theta]\,\text{Sin}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{a'[t]}{a[t]} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{r} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \text{Cot}[\theta] \end{pmatrix} & \begin{pmatrix} \frac{a'[t]}{a[t]} \\ \frac{1}{r} \\ \text{Cot}[\theta] \\ 0 \end{pmatrix} \end{pmatrix}$$

5a

In[108]:= **Rdd / g // Simplify // MatrixForm**

Out[108]//MatrixForm=

$$\begin{pmatrix} \frac{3\,a''[t]}{a[t]} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \frac{2\,\kappa+2\,a'[t]^2+a[t]\,a''[t]}{a[t]^2} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \frac{2\,\kappa+2\,a'[t]^2+a[t]\,a''[t]}{a[t]^2} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \frac{2\,\kappa+2\,a'[t]^2+a[t]\,a''[t]}{a[t]^2} \end{pmatrix}$$

each R_{ii}/g_{ii} component is the same:

$$\frac{R_{ii}}{g_{ii}} = \frac{2\,\kappa+2\,a'[t]^2+a[t]\,a''[t]}{a[t]^2}$$