## 2 RG&TC-Code

# Appendix

## 0.1

In[54]:=
```
xCoord = {t, χ, θ, φ};
g = {
    {-x y, 0, 0, 0},
    {0, x y t, 0, 0},
    {0, 0, z, 0},
    {0, 0, 0, x t}
  };
RGtensors[g, xCoord]
```

$$\text{gdd} = \begin{pmatrix} -x\,y & 0 & 0 & 0 \\ 0 & t\,x\,y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & t\,x \end{pmatrix}$$

$$\text{LineElement} = -x\,y\,d[t]^2 + z\,d[\theta]^2 + t\,x\,d[\varphi]^2 + t\,x\,y\,d[\chi]^2$$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{x\,y} & 0 & 0 & 0 \\ 0 & \frac{1}{t\,x\,y} & 0 & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & \frac{1}{t\,x} \end{pmatrix}$$

```
gUU computed in 0.00122 sec
Gamma computed in 0.002656 sec
Riemann(dddd) computed in 0.00472 sec
Riemann(Uddd) computed in 0.002312 sec
Ricci computed in 0.000301 sec
Weyl computed in 0.003843 sec
Einstein computed in 0.00016 sec
```

Out[56]=
```
All tasks completed in 0.020573
```

In[57]:=
```
(* Ricci Scalar *)
```

In[58]:=
```
R
```

Out[58]=

$$-\frac{1}{2\,t^2\,x\,y}$$

In[59]:=
```
(* Einstein Tensor *)
```

In[60]:=
```
EUd
```

Out[60]=

$$\left\{\left\{-\frac{1}{4\,t^2\,x\,y},\,0,\,0,\,0\right\},\,\left\{0,\,\frac{1}{4\,t^2\,x\,y},\,0,\,0\right\},\,\left\{0,\,0,\,\frac{1}{4\,t^2\,x\,y},\,0\right\},\,\left\{0,\,0,\,0,\,\frac{1}{4\,t^2\,x\,y}\right\}\right\}$$

In[61]:=
```
(* Christoffel Symbol *)
```

In[62]:=
```
GUdd // MatrixForm
```

Out[62]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2\,y} \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{1}{2\,t} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2\,t} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2\,t} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{2\,t} \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[63]:=
```
Part[GUdd, 1, 2, 2]
Part[GUdd, 2, 2, 1]
```

Out[63]=

$$\frac{1}{2}$$

Out[64]=

$$\frac{1}{2\,t}$$

In[65]:=
```
(* Riemann tensor *)
```

In[66]:= RUddd

Out[66]=
$$\left\{\left\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\right\},\left\{\left\{0,-\frac{1}{4\,t},0,0\right\},\left\{\frac{1}{4\,t},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\left\{\left\{0,0,0,-\frac{1}{4\,t\,y}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4\,t\,y},0,0,0\right\}\right\}\right\},$$
$$\left\{\left\{\left\{0,-\frac{1}{4\,t^2},0,0\right\},\left\{\frac{1}{4\,t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\left\{\{0,0,0,0\},\left\{0,0,0,\frac{1}{4\,t\,y}\right\},\{0,0,0,0\},\left\{0,-\frac{1}{4\,t\,y},0,0\right\}\right\}\right\},$$
$$\{\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\},$$
$$\left\{\left\{\left\{0,0,0,-\frac{1}{4\,t^2}\right\},\{0,0,0,0\},\{0,0,0,0\},\left\{\frac{1}{4\,t^2},0,0,0\right\}\right\},\left\{\{0,0,0,0\},\left\{0,0,0,-\frac{1}{4\,t}\right\},\{0,0,0,0\},\left\{0,\frac{1}{4\,t},0,0\right\}\right\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\},\{\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\}\right\}\right\}$$

In[67]:= (* Ricci Tensor *)

In[68]:= Rdd

Out[68]=
$$\left\{\left\{\frac{1}{2\,t^2},0,0,0\right\},\{0,0,0,0\},\{0,0,0,0\},\{0,0,0,0\}\right\}$$

In[69]:= Part[Rdd, 1, 1]

Out[69]=
$$\frac{1}{2\,t^2}$$

# 2.1

In[70]:= xCoord = {ρ, ϕ};
g = {{1, 0}, {0, ρ^2}}

Out[71]=
$$\{\{1,0\},\{0,\rho^2\}\}$$

In[72]:= RGtensors[g, xCoord]

$$gdd = \begin{pmatrix} 1 & 0 \\ 0 & \rho^2 \end{pmatrix}$$

LineElement $= d[\rho]^2 + \rho^2\, d[\phi]^2$

$$gUU = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\rho^2} \end{pmatrix}$$

gUU computed in 0.000357 sec

Gamma computed in 0.000264 sec

Riemann (dddd) computed in 0.000135 sec

**Flat Space!**

Out[72]= Aborted after 0.006891

In[73]:= **GUdd // MatrixForm**

Out[73]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -\rho \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{1}{\rho} \end{pmatrix} & \begin{pmatrix} \frac{1}{\rho} \\ 0 \end{pmatrix} \end{pmatrix}$$

In[74]:= **Part[gUU, 2, 2]**

Out[74]= $\frac{1}{\rho^2}$

Since we’re in flat space, Mathematica doesn’t calculate the other tensors

# 2.2

In[75]:= **xCoord = {θ, ϕ};**
**g = {{r^2, 0}, {0, r^2 Sin[θ]^2}}**

Out[76]= $\{\{r^2, 0\}, \{0, r^2 \operatorname{Sin}[\theta]^2\}\}$

In[77]:= **RGtensors[g, xCoord]**

$$gdd = \begin{pmatrix} r^2 & 0 \\ 0 & r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

LineElement = $r^2\,d[\theta]^2 + r^2\,d[\phi]^2\,\mathrm{Sin}[\theta]^2$

$$gUU = \begin{pmatrix} \frac{1}{r^2} & 0 \\ 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

gUU computed in 0.002156 sec

Gamma computed in 0.000793 sec

Riemann (dddd) computed in 0.000404 sec

Riemann (Uddd) computed in 0.000229 sec

Ricci computed in 0.000155 sec

Weyl computed in 0.000019 sec

**Conformally Flat**

Einstein computed in 0.000057 sec

**Einstein Space**

Out[77]= All tasks completed in 0.006174

We have einstein tensor:

In[78]:= **EUd**

Out[78]= {{0, 0}, {0, 0}}

And Ricci Scalar

In[79]:= **R**

Out[79]= $\frac{2}{r^2}$

Can we find a non-vanishing Einstein tensor?

In[80]:= **RGtensors[{{f1[x, y], f2[x, y]}, {f2[x, y], f3[x, y]}}, {x, y}];**

$$gdd = \begin{pmatrix} f1[x, y] & f2[x, y] \\ f2[x, y] & f3[x, y] \end{pmatrix}$$

$$LineElement = d[x]^2 f1[x, y] + 2\, d[x] \times d[y] \times f2[x, y] + d[y]^2 f3[x, y]$$

$$gUU = \begin{pmatrix} \frac{f3[x,y]}{-f2[x,y]^2+f1[x,y]\times f3[x,y]} & \frac{f2[x,y]}{f2[x,y]^2-f1[x,y]\times f3[x,y]} \\ \frac{f2[x,y]}{f2[x,y]^2-f1[x,y]\times f3[x,y]} & \frac{f1[x,y]}{-f2[x,y]^2+f1[x,y]\times f3[x,y]} \end{pmatrix}$$

gUU computed in 0.002126 sec

Gamma computed in 0.004489 sec

Riemann (dddd) computed in 0.004371 sec

Riemann (Uddd) computed in 0.008025 sec

Ricci computed in 0.010105 sec

Weyl computed in 0.000024 sec

**Conformally Flat**

Einstein computed in 0.004559 sec

**Einstein Space**

In[81]:= **EUd**

Out[81]= {{0, 0}, {0, 0}}

So no, any 2d metric has a vanishing Einstein tensor

### non constant Ricci Scalar:

In[82]:=
```
xCoord = {θ, ϕ};
g = {{-Cos[ϕ]^2 r^2, 0}, {0, -r^2 Sin[θ]^2}};
RGtensors[g, xCoord]
```

$$\text{gdd} = \begin{pmatrix} -r^2 \,\text{Cos}[\phi]^2 & 0 \\ 0 & -r^2 \,\text{Sin}[\theta]^2 \end{pmatrix}$$

$$\text{LineElement} = -r^2 \,\text{Cos}[\phi]^2 \,\text{d}[\theta]^2 - r^2 \,\text{d}[\phi]^2 \,\text{Sin}[\theta]^2$$

$$\text{gUU} = \begin{pmatrix} -\frac{\text{Sec}[\phi]^2}{r^2} & 0 \\ 0 & -\frac{\text{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

gUU computed in 0.00301 sec

Gamma computed in 0.000838 sec

Riemann (dddd) computed in 0.000354 sec

Riemann (Uddd) computed in 0.000628 sec

Ricci computed in 0.000705 sec

Weyl computed in 0.000014 sec

## Conformally Flat

Einstein computed in 0.000333 sec

## Einstein Space

Out[84]= All tasks completed in 0.00851

In[85]:= R // Simplify

Out[85]=
$$-\frac{2\left(\text{Csc}[\theta]^2 + \text{Sec}[\phi]^2\right)}{r^2}$$

Can we make a 2d manifold with constant negative curvature?

In[86]:=
```
xCoord = {θ, ϕ};
g = {{-r^2, 0}, {0, -r^2 Sin[θ]^2}};
RGtensors[g, xCoord]
```

gdd = $\begin{pmatrix} -r^2 & 0 \\ 0 & -r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$

LineElement = $-r^2\, d[\theta]^2 - r^2\, d[\phi]^2\, \mathrm{Sin}[\theta]^2$

gUU = $\begin{pmatrix} -\frac{1}{r^2} & 0 \\ 0 & -\frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$

gUU computed in 0.000492 sec

Gamma computed in 0.000451 sec

Riemann (dddd) computed in 0.000194 sec

Riemann (Uddd) computed in 0.000157 sec

Ricci computed in 0.000116 sec

Weyl computed in 0.000013 sec

**Conformally Flat**

Einstein computed in 0.000051 sec

**Einstein Space**

Out[88]= All tasks completed in 0.003855

In[89]:= **R // Simplify**

Out[89]= $-\frac{2}{r^2}$

so yes, we can.

In[90]:=

## 3.1

In[91]:=
```
g = {{-a[t]^2 , 0 , 0, 0},
  {0, 1, 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```

Out[91]= $\{\{-a[t]^2, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

In[92]:= **coord = {t, x, y, z}**

Out[92]= {t, x, y, z}

In[93]:= **RGtensors[g, coord]**

$$gdd = \begin{pmatrix} -a[t]^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LineElement = $-a[t]^2\, d[t]^2 + d[x]^2 + d[y]^2 + d[z]^2$

$$gUU = \begin{pmatrix} -\frac{1}{a[t]^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0.000483 sec

Gamma computed in 0.006737 sec

Riemann (dddd) computed in 0.00187 sec

## Flat Space!

Out[93]= Aborted after 0.011206

let’s instead try:

```
In[94]:= g = {{-1 , 0 , 0, 0},
   {0, a[t]^2, 0, 0},
   {0, 0, 1, 0},
   {0, 0, 0, 1}}
```

Out[94]= $\{\{-1, 0, 0, 0\}, \{0, a[t]^2, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

```
In[95]:= coord = {t, x, y, z}
```

Out[95]= {t, x, y, z}

```
In[96]:= RGtensors[g, coord]
```

$$gdd = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a[t]^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$LineElement = -d[t]^2 + a[t]^2\, d[x]^2 + d[y]^2 + d[z]^2$$

$$gUU = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{a[t]^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gUU computed in 0.000444 sec

Gamma computed in 0.001094 sec

Riemann (dddd) computed in 0.001814 sec

Riemann (Uddd) computed in 0.001374 sec

Ricci computed in 0.000218 sec

Weyl computed in 0.002341 sec

Einstein computed in 0.00009 sec
```

Out[96]=

```
All tasks completed in 0.009561
```

In[97]:=

```
EUd // MatrixForm
```

Out[97]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{a''[t]}{a[t]} & 0 \\ 0 & 0 & 0 & -\frac{a''[t]}{a[t]} \end{pmatrix}$$

**Not** a vacuum solution, einstein tensor is not 0.

In[98]:=

```
GUdd // MatrixForm
```

Out[98]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ a[t]\, a'[t] \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ \frac{a'[t]}{a[t]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{a'[t]}{a[t]} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[99]:=
```
Part[GUdd, 1, 2, 2]
```
Out[99]= $a[t]\, a'[t]$

In[100]:=
```
Part[GUdd, 2, 1, 2]
```
Out[100]= $\frac{a'[t]}{a[t]}$

In[101]:=
```
Part[GUdd, 2, 2, 1]
```
Out[101]= $\frac{a'[t]}{a[t]}$

---

# 3.2

In[102]:=
```
g = {{-a[x]^2 , 0 , 0, 0},
  {0, 1, 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```
Out[102]= $\{\{-a[x]^2, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

In[103]:=
```
coord = {t, x, y, z}
```
Out[103]= {t, x, y, z}

In[104]:=
```
RGtensors[g, coord]
```

$$\text{gdd} = \begin{pmatrix} -a[x]^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = -a[x]^2\, d[t]^2 + d[x]^2 + d[y]^2 + d[z]^2$$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{a[x]^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0.00047 sec

Gamma computed in 0.000952 sec

Riemann (dddd) computed in 0.001817 sec

Riemann (Uddd) computed in 0.001241 sec

Ricci computed in 0.000185 sec

Weyl computed in 0.002388 sec

Einstein computed in 0.000078 sec

Out[104]= All tasks completed in 0.009187

In[105]:= **GUdd // MatrixForm**

Out[105]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ \frac{a'[x]}{a[x]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{a'[x]}{a[x]} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} a[x]\,a'[x] \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[106]:=
```
g = {{-X^2 , 0 , 0, 0},
  {0, 1, 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```

Out[106]= $\{\{-X^2, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

In[107]:= **coord = {t, x, y, z}**

Out[107]= {t, x, y, z}

In[108]:= **RGtensors[g, coord]**

$$\text{gdd} = \begin{pmatrix} -X^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LineElement $= -X^2\, d[t]^2 + d[x]^2 + d[y]^2 + d[z]^2$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{X^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0.000668 sec

Gamma computed in 0.001673 sec

Riemann (dddd) computed in 0.002794 sec

**Flat Space!**

Out[108]= Aborted after 0.007828

In[109]:=
```
Rdddd
```

Out[109]=
```
{{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}},
 {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

In[110]:=

In[111]:=
```
g = {{-x^4 , 0 , 0, 0},
  {0, 1, 0, 0},
  {0, 0, 1, 0},
  {0, 0, 0, 1}}
```

Out[111]= $\{\{-x^4, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$

In[112]:=
```
coord = {t, x, y, z}
```

Out[112]= {t, x, y, z}

In[113]:=
```
RGtensors[g, coord]
```

$$\text{gdd} = \begin{pmatrix} -x^4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = -x^4\, d[t]^2 + d[x]^2 + d[y]^2 + d[z]^2$$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{x^4} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gUU computed in 0.000577 sec

Gamma computed in 0.00096 sec

Riemann(dddd) computed in 0.001984 sec

Riemann(Uddd) computed in 0.001498 sec

Ricci computed in 0.000183 sec

Weyl computed in 0.002395 sec

Einstein computed in 0.000082 sec
```

Out[113]= All tasks completed in 0.00965

In[114]:=

In[115]:= **RUddd // MatrixForm**

Out[115]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -\frac{2}{x^2} & 0 & 0 \\ \frac{2}{x^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & -2x^2 & 0 & 0 \\ 2x^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{pmatrix}$$

In[116]:= **Part[RUddd, 1, 2, 1, 2]**

Out[116]= $-\frac{2}{x^2}$

In[117]:=
```
Part[RUddd, 1, 2, 2, 1]
```

Out[117]= $\frac{2}{x^2}$

In[118]:=
```
R
```

Out[118]= $-\frac{4}{x^2}$

In[119]:=
```
g = {{-1 , 0 , 0, 0},
  {0, Exp[-(x^2 + y^2 + z^2)], 0, 0},
  {0, 0, Exp[-(x^2 + y^2 + z^2)], 0},
  {0, 0, 0, Exp[-(x^2 + y^2 + z^2)]}}
```

Out[119]= $\{\{-1, 0, 0, 0\}, \{0, e^{-x^2-y^2-z^2}, 0, 0\}, \{0, 0, e^{-x^2-y^2-z^2}, 0\}, \{0, 0, 0, e^{-x^2-y^2-z^2}\}\}$

In[120]:=
```
coord = {t, x, y, z}
```

Out[120]= {t, x, y, z}

In[121]:=
```
RGtensors[g, coord]
```

$$gdd = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{-x^2-y^2-z^2} & 0 & 0 \\ 0 & 0 & e^{-x^2-y^2-z^2} & 0 \\ 0 & 0 & 0 & e^{-x^2-y^2-z^2} \end{pmatrix}$$

LineElement $= -d[t]^2 + e^{-x^2-y^2-z^2}\, d[x]^2 + e^{-x^2-y^2-z^2}\, d[y]^2 + e^{-x^2-y^2-z^2}\, d[z]^2$

$$gUU = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & e^{x^2+y^2+z^2} & 0 & 0 \\ 0 & 0 & e^{x^2+y^2+z^2} & 0 \\ 0 & 0 & 0 & e^{x^2+y^2+z^2} \end{pmatrix}$$

gUU computed in 0.002237 sec

Gamma computed in 0.003972 sec

Riemann (dddd) computed in 0.004701 sec

Riemann (Uddd) computed in 0.00167 sec

Ricci computed in 0.001064 sec

Weyl computed in 0.006724 sec

Einstein computed in 0.002555 sec

Out[121]= All tasks completed in 0.026086

In[122]:= **RUddd // MatrixForm**

Out[122]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \\
\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&2-z^2&y\,z\\0&-2+z^2&0&-x\,z\\0&-y\,z&x\,z&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&y\,z&2-y^2\\0&-y\,z&0&x\,y\\0&-2+y^2&-x\,y&0\end{pmatrix} \\
\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&-2+z^2&-y\,z\\0&2-z^2&0&x\,z\\0&y\,z&-x\,z&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&-x\,z&x\,y\\0&x\,z&0&2-x^2\\0&-x\,y&-2+x^2&0\end{pmatrix} \\
\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&-y\,z&-2+y^2\\0&y\,z&0&-x\,y\\0&2-y^2&x\,y&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&x\,z&-x\,y\\0&-x\,z&0&-2+x^2\\0&x\,y&2-x^2&0\end{pmatrix} & \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}
\end{pmatrix}$$

^^ And so no singularities

In[123]:= **R**

Out[123]= $-2\,e^{x^2+y^2+z^2}\left(-6+x^2+y^2+z^2\right)$

In[124]:= **EUd // MatrixForm**

Out[124]//MatrixForm=

$$\begin{pmatrix}
e^{x^2+y^2+z^2}\left(-6+x^2+y^2+z^2\right) & 0 & 0 & 0 \\
0 & e^{x^2+y^2+z^2}\left(-2+x^2\right) & e^{x^2+y^2+z^2}\,x\,y & e^{x^2+y^2+z^2}\,x\,z \\
0 & e^{x^2+y^2+z^2}\,x\,y & e^{x^2+y^2+z^2}\left(-2+y^2\right) & e^{x^2+y^2+z^2}\,y\,z \\
0 & e^{x^2+y^2+z^2}\,x\,z & e^{x^2+y^2+z^2}\,y\,z & e^{x^2+y^2+z^2}\left(-2+z^2\right)
\end{pmatrix}$$

Integrating from R=0 to R=∞, we get:

In[125]:= **Integrate[Exp[$\frac{-1}{2}$ r$^2$], {r, 0, ∞}]**

Out[125]= $\sqrt{\frac{\pi}{2}}$

# 3.3

In[126]:=
```
g = {{-1 + f[x - t] , 0 , 0, 0},
    {0, 1 + f[x - t], 0, 0},
    {0, 0, 1, 0},
    {0, 0, 0, 1}};
g // MatrixForm
```

Out[127]//MatrixForm=

$$\begin{pmatrix} -1+f[-t+x] & 0 & 0 & 0 \\ 0 & 1+f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[128]:=
```
coord = {t, x, y, z}
```

Out[128]= {t, x, y, z}

In[129]:=
```
RGtensors[g, coord]
```

$$\text{gdd} = \begin{pmatrix} -1+f[-t+x] & 0 & 0 & 0 \\ 0 & 1+f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = d[y]^2 + d[z]^2 + d[t]^2\,(-1+f[-t+x]) + d[x]^2\,(1+f[-t+x])$$

$$\text{gUU} = \begin{pmatrix} \frac{1}{-1+f[-t+x]} & 0 & 0 & 0 \\ 0 & \frac{1}{1+f[-t+x]} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0.000903 sec

Gamma computed in 0.001293 sec

Riemann (dddd) computed in 0.001973 sec

Riemann (Uddd) computed in 0.001473 sec

Ricci computed in 0.000773 sec

Weyl computed in 0.0032 sec

Einstein computed in 0.000361 sec

Out[129]= All tasks completed in 0.012435

In[130]:=
```
Rdd // MatrixForm
```

Out[130]//MatrixForm=

$$\begin{pmatrix} -\frac{-f[-t+x]\,f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])\,(1+f[-t+x])^2} & 0 & 0 & 0 \\ 0 & -\frac{-f[-t+x]\,f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\,f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[131]:= **EUd // MatrixForm**

Out[131]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])^2} & 0 \\ 0 & 0 & 0 & \frac{-f[-t+x]\, f'[-t+x]^2 - f''[-t+x] + f[-t+x]^2\, f''[-t+x]}{(-1+f[-t+x])^2\,(1+f[-t+x])^2} \end{pmatrix}$$

We now consider the following metric:

In[132]:=
```
g = {{-1 + f[x - t] , - f[x - t], 0, 0},
   {- f[x - t], 1 + f[x - t], 0, 0},
   {0, 0, 1, 0},
   {0, 0, 0, 1}};
g // MatrixForm
```

Out[133]//MatrixForm=

$$\begin{pmatrix} -1+f[-t+x] & -f[-t+x] & 0 & 0 \\ -f[-t+x] & 1+f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[134]:= **coord = {t, x, y, z}**

Out[134]= {t, x, y, z}

In[135]:= **RGtensors[g, coord]**

$$\text{gdd} = \begin{pmatrix} -1+f[-t+x] & -f[-t+x] & 0 & 0 \\ -f[-t+x] & 1+f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = d[y]^2 + d[z]^2 + d[t]^2\,(-1+f[-t+x]) - 2\, d[t] \times d[x] \times f[-t+x] + d[x]^2\,(1+f[-t+x])$$

$$\text{gUU} = \begin{pmatrix} -1-f[-t+x] & -f[-t+x] & 0 & 0 \\ -f[-t+x] & 1-f[-t+x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0.000596 sec

Gamma computed in 0.001291 sec

Riemann (dddd) computed in 0.001803 sec

**Flat Space!**

Out[135]= Aborted after 0.006029

In[136]:=
```
Rdddd // MatrixForm
```
Out[136]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \\ \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \\ \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \\ \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} & \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \end{pmatrix}$$

Generalising, we find:

In[137]:=
```
g = {{-1 + f[x - t, y, z], -f[x - t, y, z], 0, 0},
   {-f[x - t, y, z], 1 + f[x - t, y, z], 0, 0},
   {0, 0, 1, 0},
   {0, 0, 0, 1}};
g // MatrixForm
```
Out[138]//MatrixForm=

$$\begin{pmatrix} -1+f[-t+x,y,z] & -f[-t+x,y,z] & 0 & 0 \\ -f[-t+x,y,z] & 1+f[-t+x,y,z] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[139]:=
```
coord = {t, x, y, z}
```
Out[139]= {t, x, y, z}

In[140]:=
```
RGtensors[g, coord]
```

$$gdd = \begin{pmatrix} -1+f[-t+x,\ y,\ z] & -f[-t+x,\ y,\ z] & 0 & 0 \\ -f[-t+x,\ y,\ z] & 1+f[-t+x,\ y,\ z] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LineElement =

$d[y]^2 + d[z]^2 + d[t]^2 (-1 + f[-t+x,\ y,\ z]) - 2\, d[t] \times d[x] \times f[-t+x,\ y,\ z] + d[x]^2 (1 + f[-t+x,\ y,\ z])$

$$gUU = \begin{pmatrix} -1-f[-t+x,\ y,\ z] & -f[-t+x,\ y,\ z] & 0 & 0 \\ -f[-t+x,\ y,\ z] & 1-f[-t+x,\ y,\ z] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
gUU computed in 0.000624 sec

Gamma computed in 0.001971 sec

Riemann (dddd) computed in 0.00323 sec

Riemann (Uddd) computed in 0.002458 sec

Ricci computed in 0.000348 sec

Weyl computed in 0.003267 sec

Einstein computed in 0.000335 sec
```

Out[140]= All tasks completed in 0.014753

In[141]:= **Rdd // MatrixForm**

Out[141]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2}\left(-f^{(0,0,2)}[-t+x,\ y,\ z] - f^{(0,2,0)}[-t+x,\ y,\ z]\right) & \frac{1}{2}\left(f^{(0,0,2)}[-t+x,\ y,\ z] + f^{(0,2,0)}[-t+x,\ y,\ z]\right) & 0 & 0 \\ \frac{1}{2}\left(f^{(0,0,2)}[-t+x,\ y,\ z] + f^{(0,2,0)}[-t+x,\ y,\ z]\right) & \frac{1}{2}\left(-f^{(0,0,2)}[-t+x,\ y,\ z] - f^{(0,2,0)}[-t+x,\ y,\ z]\right) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[142]:= **f1[xminust_ , y_, z_] = Exp[-y] Sin[z] Sin[xminust]**
**f2[xminust_ , y_, z_] = Log[$y^2$ + $z^2$] Sin[xminust]**

Out[142]= $e^{-y}$ Sin[xminust] Sin[z]

Out[143]= Log[$y^2 + z^2$] Sin[xminust]

In[144]:= **D[D[f1[x - t, y, z], y], y] + D[D[f1[x - t, y, z], z], z] // Simplify**

Out[144]= 0

In[145]:= **D[D[f2[x - t, y, z], y], y] + D[D[f2[x - t, y, z], z], z] // Simplify**

Out[145]= 0

So both f1 and f2 are harmonic functions. Let’s double check they satisfy the vacuum field equaitons:

In[146]:=
```
g = {{-1 + f1[x - t, y, z], -f1[x - t, y, z], 0, 0},
    {-f1[x - t, y, z], 1 + f1[x - t, y, z], 0, 0},
    {0, 0, 1, 0},
    {0, 0, 0, 1}};
g // MatrixForm
```

Out[147]//MatrixForm=

$$\begin{pmatrix} -1 - e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & 0 & 0 \\ e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & 1 - e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[148]:=
```
coord = {t, x, y, z}
```

Out[148]= {t, x, y, z}

In[149]:=
```
RGtensors[g, coord]
```

$$\mathrm{gdd} = \begin{pmatrix} -1 - e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & 0 & 0 \\ e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & 1 - e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LineElement =

$$d[y]^2 + d[z]^2 + 2\,e^{-y}\,d[t] \times d[x]\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] + e^{-y}\,d[x]^2\left(e^{y} - \mathrm{Sin}[t-x]\,\mathrm{Sin}[z]\right) - e^{-y}\,d[t]^2\left(e^{y} + \mathrm{Sin}[t-x]\,\mathrm{Sin}[z]\right)$$

$$\mathrm{gUU} = \begin{pmatrix} -e^{-y}\left(e^{y} - \mathrm{Sin}[t-x]\,\mathrm{Sin}[z]\right) & e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & 0 & 0 \\ e^{-y}\,\mathrm{Sin}[t-x]\,\mathrm{Sin}[z] & e^{-y}\left(e^{y} + \mathrm{Sin}[t-x]\,\mathrm{Sin}[z]\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0.002438 sec

Gamma computed in 0.013297 sec

Riemann (dddd) computed in 0.008017 sec

Riemann (Uddd) computed in 0.014566 sec

Ricci computed in 0.00035 sec

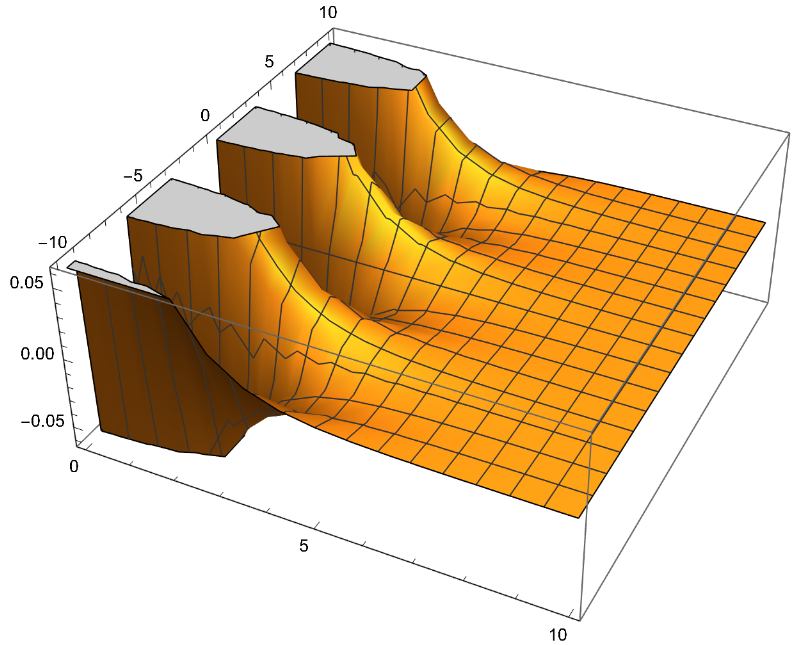
Weyl computed in 0.000031 sec

**Ricci Flat**

Out[149]= All tasks completed in 0.044088

In[150]:=
```
Plot3D[Evaluate[f1[xt, y, z] /. {xt → π / 2}], {y, 0, 10}, {z, -10, 10}]
```

Out[150]=




In[151]:=
```
Rdd // MatrixForm
```

Out[151]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[152]:=
```
g = {{-1 + f2[x - t, y, z] , - f2[x - t, y, z], 0, 0},
   {- f2[x - t, y, z], 1 + f2[x - t, y, z], 0, 0},
   {0, 0, 1, 0},
   {0, 0, 0, 1}};
g // MatrixForm
```

Out[153]//MatrixForm=

$$\begin{pmatrix} -1 - \mathrm{Log}[y^2 + z^2]\,\mathrm{Sin}[t - x] & \mathrm{Log}[y^2 + z^2]\,\mathrm{Sin}[t - x] & 0 & 0 \\ \mathrm{Log}[y^2 + z^2]\,\mathrm{Sin}[t - x] & 1 - \mathrm{Log}[y^2 + z^2]\,\mathrm{Sin}[t - x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[154]:=
```
coord = {t, x, y, z}
```

Out[154]= {t, x, y, z}

In[155]:=
```
RGtensors[g, coord]
```

$$gdd = \begin{pmatrix} -1-\text{Log}[y^2+z^2]\,\text{Sin}[t-x] & \text{Log}[y^2+z^2]\,\text{Sin}[t-x] & 0 & 0 \\ \text{Log}[y^2+z^2]\,\text{Sin}[t-x] & 1-\text{Log}[y^2+z^2]\,\text{Sin}[t-x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{LineElement} = d[y]^2 + d[z]^2 + 2\,d[t] \times d[x]\,\text{Log}[y^2+z^2]\,\text{Sin}[t-x] + d[t]^2\,(-1-\text{Log}[y^2+z^2]\,\text{Sin}[t-x]) + d[x]^2\,(1-\text{Log}[y^2+z^2]\,\text{Sin}[t-x])$$

$$gUU = \begin{pmatrix} -1+\text{Log}[y^2+z^2]\,\text{Sin}[t-x] & \text{Log}[y^2+z^2]\,\text{Sin}[t-x] & 0 & 0 \\ \text{Log}[y^2+z^2]\,\text{Sin}[t-x] & 1+\text{Log}[y^2+z^2]\,\text{Sin}[t-x] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gUU computed in 0.001261 sec

Gamma computed in 0.012206 sec

Riemann (dddd) computed in 0.009286 sec

Riemann (Uddd) computed in 0.01675 sec

Ricci computed in 0.000369 sec

Weyl computed in 0.000063 sec

## Ricci Flat

Out[155]= All tasks completed in 0.044019

In[156]:= **Rdd // MatrixForm**

Out[156]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[157]:= **Plot3D[Evaluate[f2[xt, y, z] /. {xt → π / 2}], {y, -10, 10}, {z, -10, 10}]**

Out[157]=