2 RG&TC-Code

Part 4

In[65]:=
```
g = {{- Exp[2 α[r]], 0, 0, 0},
  {0, Exp[2 β[r]], 0, 0},
  {0, 0, r^2, 0},
  {0, 0, 0, r^2 Sin[θ]^2}}
```

Out[65]= $\{\{-e^{2\,\alpha[r]}, 0, 0, 0\}, \{0, e^{2\,\beta[r]}, 0, 0\}, \{0, 0, r^2, 0\}, \{0, 0, 0, r^2\,\text{Sin}[\theta]^2\}\}$

In[66]:=
```
g // MatrixForm
```

Out[66]//MatrixForm=

$$\begin{pmatrix} -e^{2\,\alpha[r]} & 0 & 0 & 0 \\ 0 & e^{2\,\beta[r]} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

In[67]:=
```
xcoord = {t, r, θ, ϕ}
```

Out[67]= $\{t, r, \theta, \phi\}$

In[68]:=
```
RGtensors[g, xcoord]
```

$$\text{gdd} = \begin{pmatrix} -e^{2\,\alpha[r]} & 0 & 0 & 0 \\ 0 & e^{2\,\beta[r]} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\text{Sin}[\theta]^2 \end{pmatrix}$$

$$\text{LineElement} = e^{2\,\beta[r]}\,d[r]^2 - e^{2\,\alpha[r]}\,d[t]^2 + r^2\,d[\theta]^2 + r^2\,d[\phi]^2\,\text{Sin}[\theta]^2$$

$$\text{gUU} = \begin{pmatrix} -e^{-2\,\alpha[r]} & 0 & 0 & 0 \\ 0 & e^{-2\,\beta[r]} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\text{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

```
gUU computed in 0.001197 sec

Gamma computed in 0.001942 sec

Riemann(dddd) computed in 0.003173 sec

Riemann(Uddd) computed in 0.003476 sec

Ricci computed in 0.003509 sec

Weyl computed in 0.01012 sec

Einstein computed in 0.002856 sec
```

Out[68]=
```
All tasks completed in 0.029595
```

In[70]:= `GUdd // MatrixForm`

Out[70]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix} 0 \\ \alpha'[r] \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \alpha'[r] \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} e^{2\,\alpha[r]-2\,\beta[r]}\,\alpha'[r] \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \beta'[r] \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ -e^{-2\,\beta[r]}\,r \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -e^{-2\,\beta[r]}\,r\,\mathrm{Sin}[\theta]^2 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{1}{r} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{r} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{r} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Cot}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{r} \\ \mathrm{Cot}[\theta] \\ 0 \end{pmatrix}
\end{pmatrix}$$

In[71]:= `RUddd // MatrixForm`

Out[71]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \alpha'[r]^2 - \alpha'[r]\,\beta'[r] + \alpha''[r] \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 & -e^{2\,\alpha[r]-2\,\beta[r]}\left(\alpha'[r]^2 - \alpha'[r]\,\beta'[r] + \alpha''[r]\right) & 0 & 0 \\ e^{2\,\alpha[r]-2\,\beta[r]}\left(\alpha'[r]^2 - \alpha'[r]\,\beta'[r] + \alpha''[r]\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & -\frac{e^{2\,\alpha[r]-2\,\beta[r]}\,\alpha'[r]}{r} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{e^{2\,\alpha[r]-2\,\beta[r]}\,\alpha'[r]}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & 0 & -\frac{e^{2\,\alpha[r]-2\,\beta[r]}\,\alpha'[r]}{r} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{e^{2\,\alpha[r]-2\,\beta[r]}\,\alpha'[r]}{r} & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
\end{pmatrix}$$

In[59]:= `Rdd // Simplify // MatrixForm`

Out[59]//MatrixForm=

$$\begin{pmatrix}
\frac{e^{2\,\alpha[r]-2\,\beta[r]}\left(r\,\alpha'[r]^2+\alpha'[r]\,(2-r\,\beta'[r])+r\,\alpha''[r]\right)}{r} & 0 & 0 & \\
0 & -\alpha'[r]^2 + \frac{2\,\beta'[r]}{r} + \alpha'[r]\,\beta'[r] - \alpha''[r] & 0 & \\
0 & 0 & e^{-2\,\beta[r]}\left(-1 + e^{2\,\beta[r]} - r\,\alpha'[r] + r\,\beta'[r]\right) & \\
0 & 0 & 0 & e^{-2\,\beta[r]}\,\mathrm{S}
\end{pmatrix}$$

In[60]:=
```
Part[Rdd, 3, 3]
```

Out[60]= $e^{-2\beta[r]}\left(-1+e^{2\beta[r]}-r\,\alpha'[r]+r\,\beta'[r]\right)$

Part 5

In[61]:=
```
g = {{-(1 - Rs / r), 0, 0, 0},
  {0, (1 - Rs / r)^-1, 0, 0},
  {0, 0, r^2, 0},
  {0, 0, 0, r^2 Sin[θ]^2}}
xcoord = {t, r, θ, ϕ}
```

Out[61]= $\left\{\left\{-1+\frac{Rs}{r},0,0,0\right\},\left\{0,\frac{1}{1-\frac{Rs}{r}},0,0\right\},\left\{0,0,r^2,0\right\},\left\{0,0,0,r^2\,\mathrm{Sin}[\theta]^2\right\}\right\}$

Out[62]= $\{t, r, \theta, \phi\}$

In[63]:=
```
RGtensors[g, xcoord]
```

$$gdd = \begin{pmatrix} -1+\frac{Rs}{r} & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{Rs}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

$$LineElement = \frac{r\,d[r]^2}{r-Rs}-\frac{(r-Rs)\,d[t]^2}{r}+r^2\,d[\theta]^2+r^2\,d[\phi]^2\,\mathrm{Sin}[\theta]^2$$

$$gUU = \begin{pmatrix} -\frac{r}{r-Rs} & 0 & 0 & 0 \\ 0 & \frac{r-Rs}{r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

gUU computed in 0.006343 sec

Gamma computed in 0.00172 sec

Riemann (dddd) computed in 0.002396 sec

Riemann (Uddd) computed in 0.00189 sec

Ricci computed in 0.000136 sec

Weyl computed in 0.000013 sec

Ricci Flat

Out[63]= All tasks completed in 0.019952

In[64]:= **GUdd // MatrixForm**

Out[64]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ \frac{Rs}{2\,r\,(r-Rs)} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{Rs}{2\,r\,(r-Rs)} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} \frac{(r-Rs)\,Rs}{2\,r^3} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -\frac{Rs}{2\,r\,(r-Rs)} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ -r+Rs \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\left((r-Rs)\,\mathrm{Sin}[\theta]^2\right) \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{1}{r} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{r} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{r} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Cot}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{r} \\ \mathrm{Cot}[\theta] \\ 0 \end{pmatrix} \end{pmatrix}$$