Computer Project 1, Reservoir Simulation I, Fall 2011

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Simulator

Develop a 3D two-phase (oil-water) flow reservoir simulator using the using the purely implicit method based on an $N_x \times N_y \times N_z$ block-centered grid. The code should be able to handle multiple water injection wells bottomhole pressure or injection rate specified. We should be able change the rate at specified times, including setting the rate to zero. The code should be able to do $x - y - z$ area simulation for a horizontal reservoir.

All producing wells will be produced at a specified the flowing bottomhole pressure or the total fluid withdrawal rate in RB/D. More information on the well operating conditions is provided later. I will provide the location of the wells by specifying the (i, j) indices together with the k indices of the completed gridblocks. The wellbore radius and skin factor for each well will also be specified. You should be able to read in all this data. We should have the option of shutting in a producing well when the WOR reaches a prescribed input value. Material balance checks should be done on each component (oil and water) both over each time step and on a cumulative basis.

The primary variables will be oil pressure and water saturation. We will want to investigate the relative effects of capillary pressure, gravity and viscous forces and to investigate the effect of the mobility ratio on results. We also wish to investigate the effect of time step size and gridblock size.

I will give you the data files that you should try to run. The first one is for a small problem on a $20 \times 30 \times 3$. In this case, there will be six wells specified at the locations specified at the (i, j) locations in the data file, well_location.dat The wells are given in increasing order, that is the first set of coordinates in this file pertains to well 1 and the last set to well 6. The wells indices are the (i, j, k) of well locations with the k index giving the location in which the well is completed. Note that the two injectors wells (wells 1 and 2) are completed only in layer 3, i.e., $k = 3$ and the four

producers are completed in only layer 1.

You would like to develop the simulator for two scenarios:

(a) Specified flowing bottomhole pressures or water injection rate STB/D specified at injectors but the pressures or rates can be different at each well. This is the option you should develop first. **If you do all the parts of the project using only this option, you may still receive up to 80% of full credit.**

(b) Specified oil rate in STB/D, specified total liquid rate in RB/D or flowing bottomhole pressure specified at each producer.

The following comments are important. (c) The easiest specification to program is to specify the flowing bottomhole pressure at each injector and each producer and I suggest you put that option in first and get it running. In fact, it is must better to do this option for the cases discussed later, before trying to implement other options. When rates are specified, the bottomhole pressure at datum becomes an unknown and we must add to the set of equations and the Jacobian, the equations that couple the rates from each well, the rates from each segment and the wellbore pressure. (d) Normally, if a production rate is specified a minimum bottomhole pressure would also be specified and if during the simulator run, p_{wf} decreases below this value, then the well control switches to production at this minimum bottomhole pressure. Similarly at an injection well, a maximum injection pressure should be specified. You do not need to include these options.

For simplicity, we will assume oil and water viscosity are constant In transmissibilities, upsteam weight all pressure and saturation dependent terms, i.e., upstream weight both the relative permeabilities viscosity and formation volume factor terms. Generally, we will have a table for $B_0 = B_o(p_o)$ and $B_w = B_w(p_w)$ and $\phi(p_o)$ but for simplicity if you wish you can just use the equations given below. Specifically, above bubble-point, the formation volume factors can be approximated by

$$B_o = B_{ob}(1 - c_o(p_o - p_b)), \tag{1}$$

and

$$B_w = B_{wb}(1 - c_w(p_w - p_b)), \tag{2}$$

where for simplicity, we assume that the bubble point pressure, p_b, is the same for both oil and

water although this is not true in general. Here b_{wb} denotes the inverse of the water formation volume evaluated at bubblepoint pressure and b_{wo} denotes the inverse of the oil formation volume at bubblepoint pressure. We assume that oil compressibility (c_o) and water compressibility c_w are constants that will be read in via an imput file. For simplicity here, we will simply use $p_b = 14.7$ psi, i.e., we have no dissolved gas. We use $B_{ob} = B_{wb} = 1.0$. Since the equations for formation volume factors are linear functions of pressure, you only need two points in the table to do correct interpolation at pressure above bubblepoint pressure, a value at p_b psi and a value at the maximum pressure which in the maximum injection wellbore pressure plus any increase due to gravity. Again, it may be worthwhile longterm for you to prepare for reading in tables of FVF's but I do not require it for the project.

Similar to formation volume factors, we will model porosity as

$$\phi(p_0) = \phi_b(1 + c_r(p_o - p_b)), \tag{3}$$

where c_r is rock compressibility and ϕ_b is a the specified value of porosity at the base pressure p_b. This one will be bit tricky because porosity is a function of position. Thus, each gridblock will have to be treated separately and the gridblock value I give you will be treated as ϕ_b for that gridblock.

Grid

You should be able to input N_x, N_y, N_z and the Δx_i, $i = 1, 2, \cdots N_x$, Δy_j, $j = 1, 2, \cdots N_y$ and Δz_k, $k = 1, 2, \cdots N_z$, however, for the problems I require in this project, there will be a uniform grid in each direction so we need only read in one value of Δx, Δy and Δz.

For the **base case**, we will use $\Delta x = \Delta y = 200$ feet and $\Delta z = 25$ ft. The grid will be $20 \times 30 \times 3$, i.e., the reservoir will have three layers.

Permeability

The program should be able to (i) read in k_x and k_y and k_z on a gridblock by gridblock basis, or (ii) assume a homogeneous reservoir and read in a single k_x value and a single k_y value and a single k_z value. For the base $20 \times 30 \times 3$ case, I will provide a file that has four entries per row,

where each row pertains to a gridblock and has the value of ϕ, k_x, k_y and k_z for that gridblock. For each column, the values pertain the ordering where the i index (x-direction) varies the fastest and the y the slowest. Note for each layer k if fixed equal to the layer index where the first layer is the top layer. You should write a subroutine so that you can order the 3-D array of gridblocks and property values and way you wish, for example in the z-direction, then the x-direction then the y direction which will be the optimal ordering for the problems I will give you. **You may use an linear solver you wish. As the problems I ask you to do are quite small, it is appropriate to use a sparse direct solver such as PARDISO solver which is a routine of Intel MKL (Math Kernel Library). I will put relevant information in a shares folder called Paridiso including the file 24.rb file contains the compressed row format sparse matrix, which is solved suing the PARDISO solver. Prior to running the code, you need to activate the Intel MKL using the following steps: (1) Go to Project menu ¿project properties; (2) From the configuration properties on the left expand Fortran and select libraries and (3) Activate the use of MKL.**

Viscosities/Compressibilities

Oil and water viscosity will be assumed to be constants. Fluid and rock compressibilities will assumed to be constant. These will all be input via a data file you prepare.

Transmissibilities and grids

You should compute the constant part of gridblock transmissibilities only once. This will require computing the gridpoints, i.e., the gridblock centers $((x_i, y_j, z_k))$ as transmissibilities involve terms like $(1.127\times10^{-3}\Delta y_j\Delta x_j)/(x_i - x_{i-1})$. Similarly, each storativity term involves $(\Delta y_j\Delta x_j\Delta z_k)/5.615$

Technical Report

You will need to write a technical report on your results. The report must be written in acceptable English, must be concise and complete so a reasonably knowledgeable reader can understand exactly what you have done. Figures should be of good quality

with appropriate labels and detailed captions so that a knowledgeable reader can understand what the results are by simply reading the captions.The quality of the writing will constitute 15% of the grade.

This report should include whatever verification you have done to ensure that your simulator is correct. This could include comparison with Buckley Leverett solutions for a linear case and comparison with single-phase flow analytical solutions, and should definitely include phase material balance computations.

The report should also include a discussion on the physics of the problem. The intent of the specific exercises is intended to reinforce your understanding of how physical properties effect your results. No plot should be included without a discussion of the results shown and a physical explanation of those results.

Specific Plots: You should plots of layer properties with well locations, the cumulative material balances of each component as a function of time, water oil ratio at producers, phase rates at producers, water injection rate at injectors, field oil production as a function of time, field water production as a function of time, field water oil ratio cut as a function of time, oil saturation distribution in each layer, after 2 years, 5 years and at the end of the simulation, cumulative oil produced as a function of time versus pore volume of water injected and recovery factor as a function of time.

I may not return your report so your should keep a copy.

Relative Permeabilities - Capillary Pressure

Relative permeabilities will be read in as tables. The tables should contain up to 100 uniformly spaced points between S_{iw} and $1 - S_{or}$. One can either read them in directly this way or read them in using non-uniformly spaced tables containing fewer points and then interpolate to obtain a table of 100 points. Capillary pressure will be treated similarly. In addition each table should have an entry at $S_w = 0$ and $S_w = 1$. We let $p_{c,dr}$ and $p_{c,im}$ respectively denote the drainage and imbibition capillary pressure curve. For water flooding, the imbibition curve is used but the drainage curve is needed to compute the initial saturation distribution.

The first two rows of entries should be

S_w	k_{rw}	k_{ro}	$p_{c,im}$	$p_{c,dr}$
0.0	0.0	$k_{ro,max}$	$p_{c,max}$	$p_{c,max}$
S_{iw}	0.0	$k_{ro,max}$	$p_{c,max}$	$p_{c,max}$

Letting $p_{c,th}$ denote the threshold pressure for the drainage capillary pressure curve, the last two entries of the tables should be

S_w	k_{rw}	k_{ro}	$p_{c,im}$	$p_{c,dr}$
$1 - S_{or}$	$k_{rw,max}$	0.0	0.0	x
1.0	1	0.0	0.0	$p_{c,th}$

We will make relative permeability tables from analytical expressions for relative permeabilities. This will make it easier to check value of the Jacobian matrix as then its entries can be computed analytically if there is no capillary pressure. To define power law relative permeabilities, we first define dimensionless water saturation by

$$S_{wD} = \frac{S_w - S_{iw}}{1 - S_{or} - S_{iw}} \tag{4}$$

Set 1 Relative Perms.

$$k_{rw} = a_1 S_{wD}^{m_1}, \tag{5}$$

and

$$k_{ro} = a_2 \big(1 - S_{wD}\big)^{m_2}, \tag{6}$$

where $S_{iw} = 0.3$ and $S_{or} = 0.2$, $a_1 = 0.2$, $a_2 = 1.0$, $m_1 = m_2 = 2.5$.

Set 2, relative permeabilities

$$k_{rw} = 0.5 S_{wD}^2, \tag{7}$$

and

$$k_{ro} = 1.0 \big(1 - S_{wD}\big)^{3.0}, \tag{8}$$

where $S_{iw} = 0.25$ and $S_{or} = 0.2$. Note Set 2 rel perms have the same form at those of set 1 except the a_i's and m_i's are different. We use the preceding sets of relative permeabilities to generate table entries from $S_w = S_{iw}$ to $S_w = 1 - S_{or}$ and the first and last two table entries as discussed previously. Alternatively, you should be able to read in tables of relative permeabilities directly.

For fast table lookup, it is always best to use uniform saturation (or pressure increments) in the tables. For the relative permeability and capillary pressure tables, one can obtain a uniform ΔS_w increment (for entries between S_{iw} and $1 - S_{or}$ by dividing $1 - S_{or} - S_{iw}$ by the desired number of increments. Make your tables for on the order of 103 total entries.

Common Data

Capillary Pressure

The table for imbibition and capillary pressure data are given below:

Table 1: Drainage and imbibition capillary pressure tables.

S_w	p_c, psi, drainage	p_c, psi, imbibition
0	6.45	6.45
0.3	6.45	6.45
0.32	4.2	4.2
0.35	2.78	2.74
0.4	1.83	1.81
0.45	1.35	1.28
0.5	1.03	0.89
0.6	0.64	0.49
0.7	0.38	0.34
0.8	0.25	0
0.9	0.1	0
1.0	0.04	0

In some cases, this data will be modified to investigate the effect of capillary pressure. Alternatively, we can use capillary pressure defined by ....

Initial Conditions

The drainage capillary pressure curve should be used to calculate the initial fluid distribution that satisfies gravity-capillary equilibrium. **Here, the initial oil water contact coincides with the bottom boundary of the reservoir and we will read in the initial oil phase pressure equal to 3500 psi at the bottom of the reservoir which we assumes coincides with the**

depth of the initial water oil contact. This is of course unrealistic as if the WOC is the bottom of the reservoir, we would need an aquifer model to simulate water influx but you do not have time to do more. We will calculate the phase pressures at centers of the shallower gridblocks using the standard procedure discussed in class. We assume a water-wet system with $p_c - p_o - p_w$. For the simple problem consider here, the oil and water phase densities can be written, respectively, as

$$\rho_o = \frac{\rho_{o,sc}}{B_o},$$

and

$$\rho_w = \frac{\rho_{w,sc}}{B_w},$$

We will assume that $\rho_{w,sc} = 65$ (lb-mass)/(ft^3) and that $\rho_{0,sc} = 53$ (lb-mass)/(ft^3).

Viscosities

We will assume constant fluid viscosities. Although viscosities should be input data, we will use $\mu_w = 0.8$ cp in all cases. Oil viscosity will vary from case to case.

Compressibilities

Water, oil and rock compressibility, respectively, are given by $c_w = 3 \times 10^{-6}$ psi^{-1} and $c_o = 10^{-5}$ psi^{-1} $c_r = 0$ psi unless specifically specified otherwise.

Times Steps

Option 1 Times steps will be specified by defining an initial time step and using time step multipliers to increase the time step (after every x time steps where x may be 1) until one of the following options holds: (i) time step reaches a maximum specified time step size.

You should have an option to specify the total simulation time. You should also have an option to stop the simulation run if the FWOR reaches a given limit or of course if all producing wells become shut-in.

Well Constraints

Minimum bottomhole pressure equal 800 **psi, initial pressure is 3500 psi at the bottom of the reservoir, which, as noted above, is at the depth of the initial water oil contact. Maximum bottomhole pressure at injectors equal to 5500 psi. Assume a wellbore radius of** 0.5 **ft and** $s = 0$ **at all wells when computing the well indices for Peaceman's formula.**

Shut in a well when the water oil ratio exceeds 7.0 STB/STB wells. Terminate the run when the the field water oil ratio exceeds 5 **STB/STB. There are other options, here, e.g., we could simply shut-in the well with the higher WOR everytime the FWOR exceeds 5 STB/STB**

Relative Permeabilities

We use either Set 1 or Set 2 relative permeability curves in all cases; see Eqs. 4–8. We will specify which case to use.

Coding, Jacobian

Spending some time designing your code, e.g., a flow chart, is generally a good investment. I always write a driver or main program which simply reads in input from a set of data files and calls a set of subroutines. It is best to partition the code into several modules and/or subroutines that are relatively simple and have a limited number of functions for each in debugging. I have attached a few pages from Diego Oliver's report from a previous Reservoir Simulation Class so you can see a reasonable code structure and what a good report looks like.

Depending on the linear solver you end up using or if you wish to test solvers, it might be worthwhile to be able to store the Jacobian as a 2D-band matrix, in compressed row format or even as a full matrix and have a subroutine to convert from one format to another. **Note the solver I suggest above (Pardiso) uses compressed row format.**

Base Case

For the base case, water injection wells are completed in only the bottom layer and producing wells are completed in only the top layer. The well locations and completions are specified in a companion data file. Well's 1 and 2 are water injection wells in the base case with the injection pressure set at 5000 psi. All five other wells are produced at a constant bottomhole pressure of 800 psi until the producing WOR exceeds the specified limits. In all subcases, use an initial time step of 10^{-1} days and increase the time step by a factor of 1.05 each time step until $\Delta t = 30$ days. Set $\mu_o = 1.1$, $\mu_w = 0.8$ cp; ϕ, k_x, k_y and k_z for each layer are defined in a set of data files. In all cases, we will specify a well maximum producing WOR of 7 and a maximum FWOR (field producing WOR) of 5. If you wish to run the base case using water-cuts instead of WOR, that is fine as long as you do the same in all cases.

Part a Run the base case with capillary pressure, then with capillary pressure set to zero and compare results. After this all runs will be done with zero capillary pressure.

Part b Rerun (a) with zero capillary pressure and rock compressibility set equal to $c_r = 5 \times 10^{-6}$ psi^{-1} and explain the effect. In all runs from this point on, use $c_r = 5 \times 10^{-6}$ psi^{-1}. From this point on (b) will be referred to as the base case.

Part c, effect of mobility ratio Redo the (b) with zero capillary and set 2 relative permeability curves. Explain why the two results are different.

Part d Redo part (c) but with water viscosity doubled. Explain why the two sets of results are different.

Part (e), field development and optimal well location Assuming injectors well be operated at 5000 psi and producers at 8000 psi and that we could have put the two injectors at any reservoir locations, try to find the optimal location of injection wells that will give you higher oil production at abandonment than obtained for case (c). We still must maintain field and well WOR constraints and well operating conditions.

Part f, field development Assume the seven wells with the operating conditions and WOR constraints of the base case. You have money in your budget to drill one more well at the end of

3 years of production. It can be either an injector or a producer. The cost of drilling the well is $10,000,000$. Should we drill the well and if so what is its location and in which layers should it be completed. How do things change if the cost of the well is $50,000,000? Assume for simplicity that oil is sold at $80/STB and ignore other operational costs.

Part g Divide each gridblock for the $20 \times 30 \times 3$ into 8 uniform gridblocks and assign ϕ, k_x, k_y, k_z and to each of these 8 to be equal to the value of the coarse parent gridblock. With this fine $40 \times 60 \times 6$ grid redo part (c). Does the fine grid make any significant difference, e.g., on cumulative oil and water produced? Do the problem again but now with the maximum time step size reduced to 10 days. Does the time step refinement make any difference.