#### YAWTB: Yet Another Wavelet ToolBox



http://www.fyma.ucl.ac.be/projects/yawtb/

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P. Vandergheynst (Sig. Proc. Inst., EPFL, Switzerland)

A. Rivoldini (Royal Observatory of Belgium)

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- 4. TODO list

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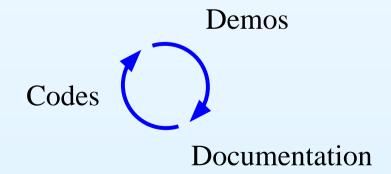
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- Various useful tools: (universal yashow, special functions, thresholdings, (P)SNR, frequency grid, ..),

#### Less stable

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- Spherical DOG frame (half continuous and discrete)
- Multiselectivity analysis of images

**CWT 1D** Given a signal s, and a wavelet  $\psi$ :

$$W_f(b,a) = \int_{\mathbb{R}} dt \ s(t) \frac{1}{\sqrt{a}} \psi^*(\frac{t-b}{a}).$$

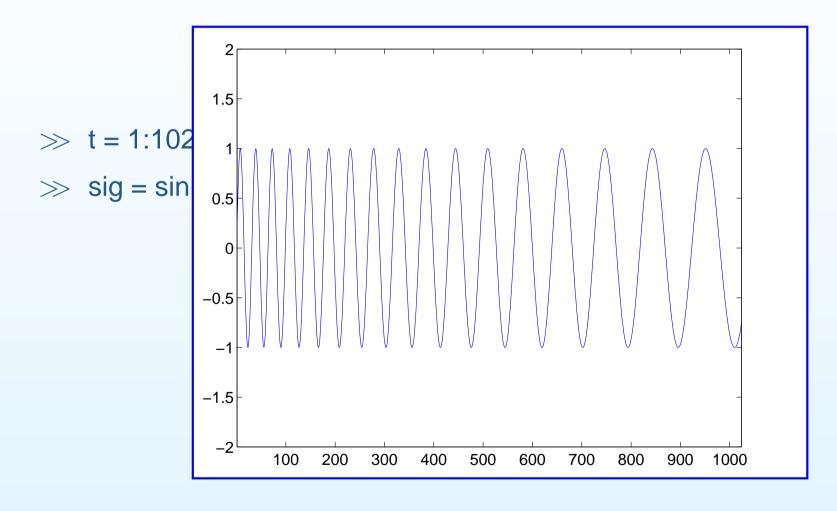
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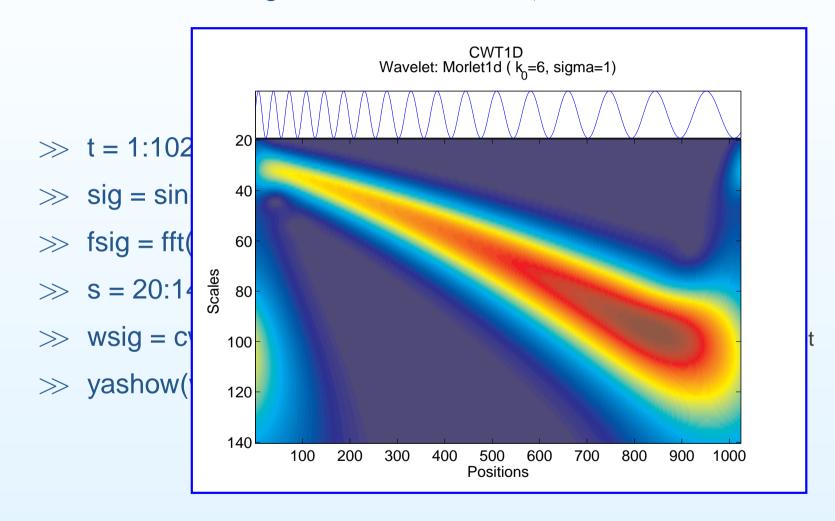
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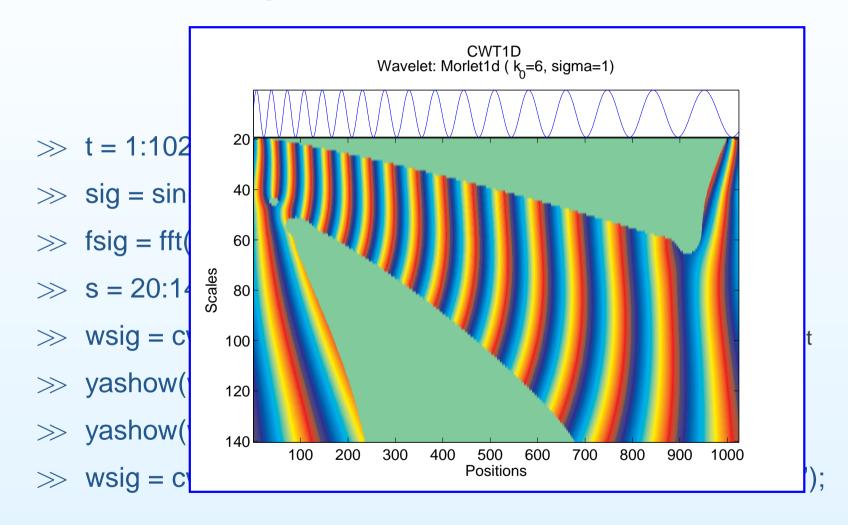
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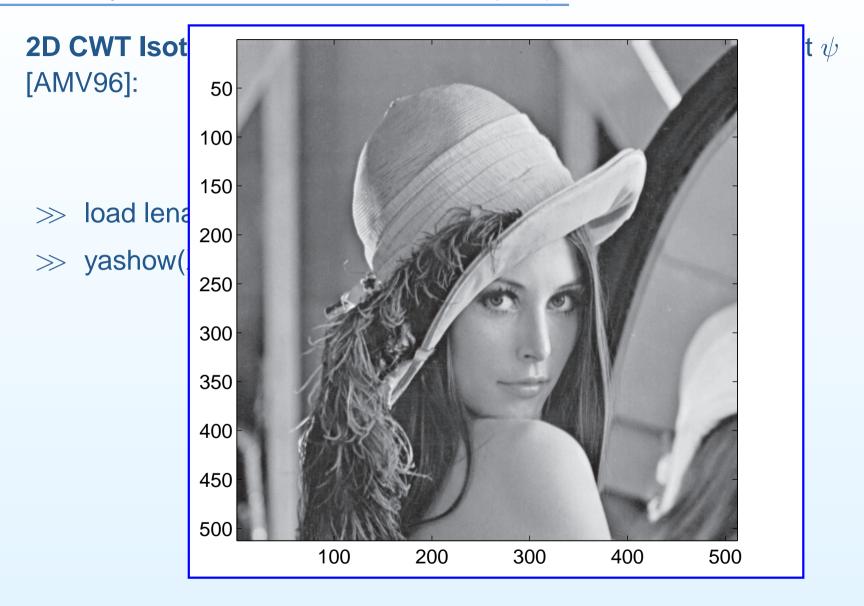
**2D CWT Isotropic** Given an image I, and a 2D isotropic wavelet  $\psi$  [AMV96]:

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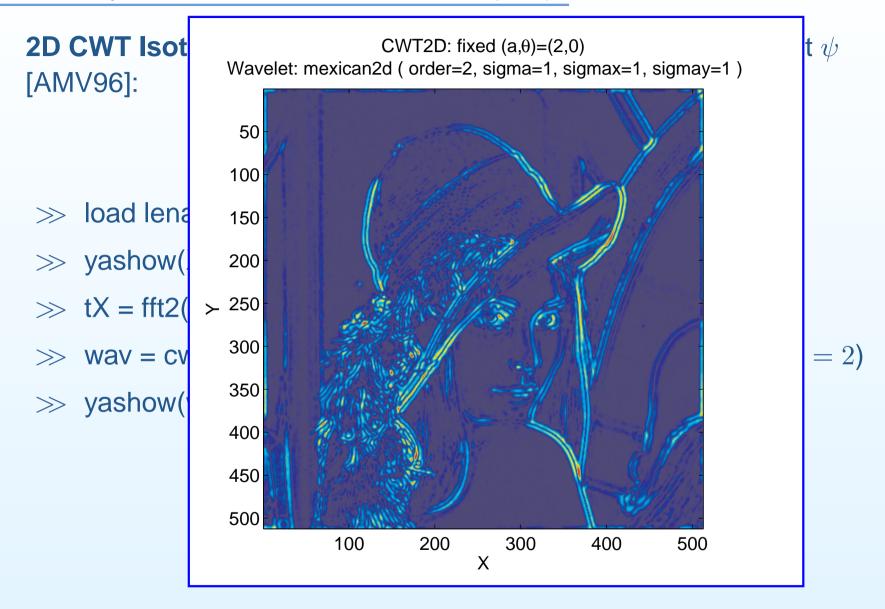
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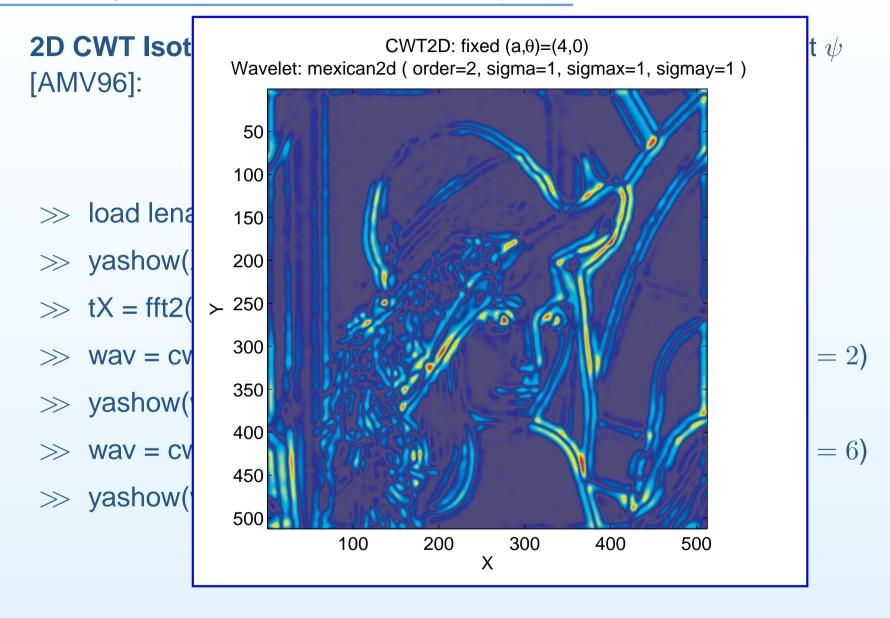


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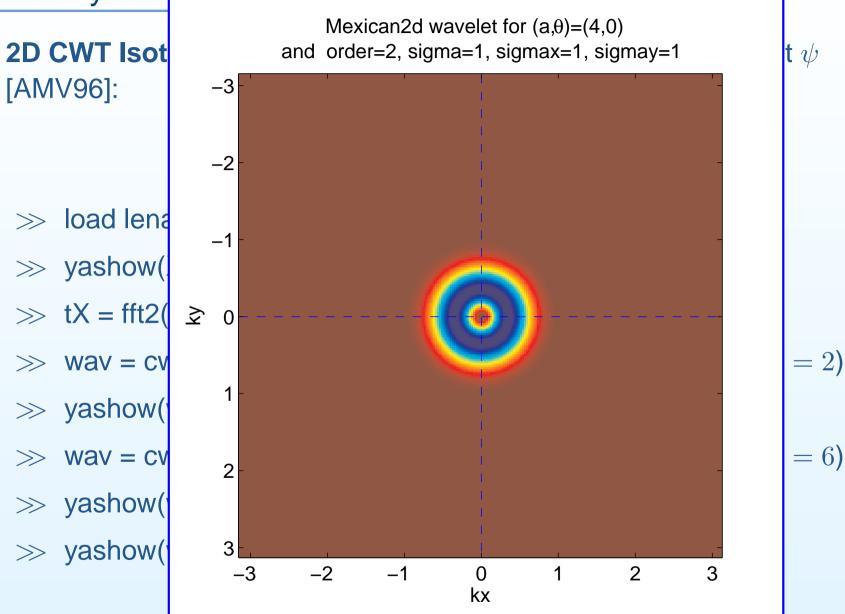
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**2D CWT Directional** Given an image  $I(\vec{x})$  and a directional wavelet  $\psi$  [AMV99]:

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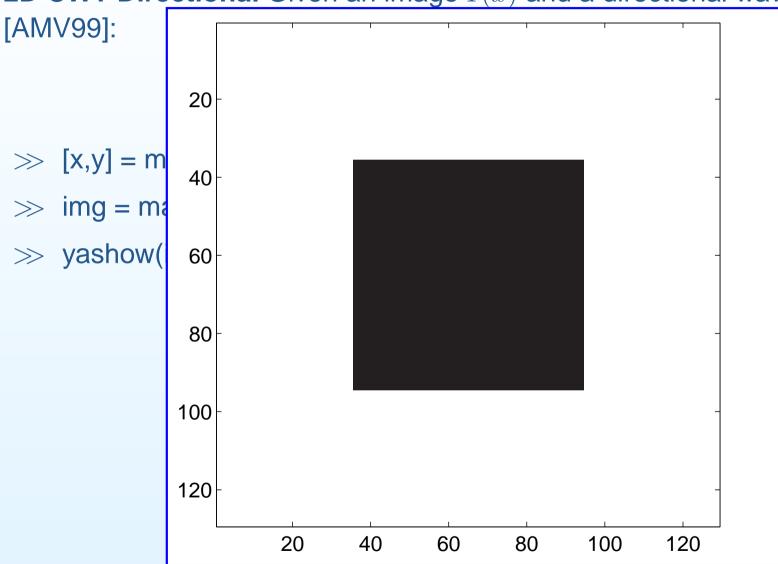
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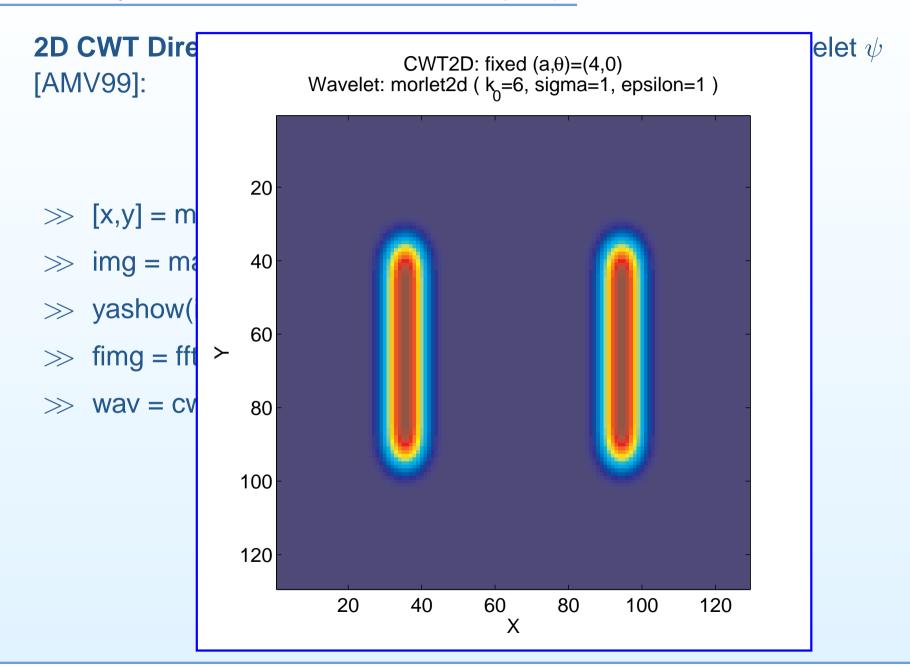
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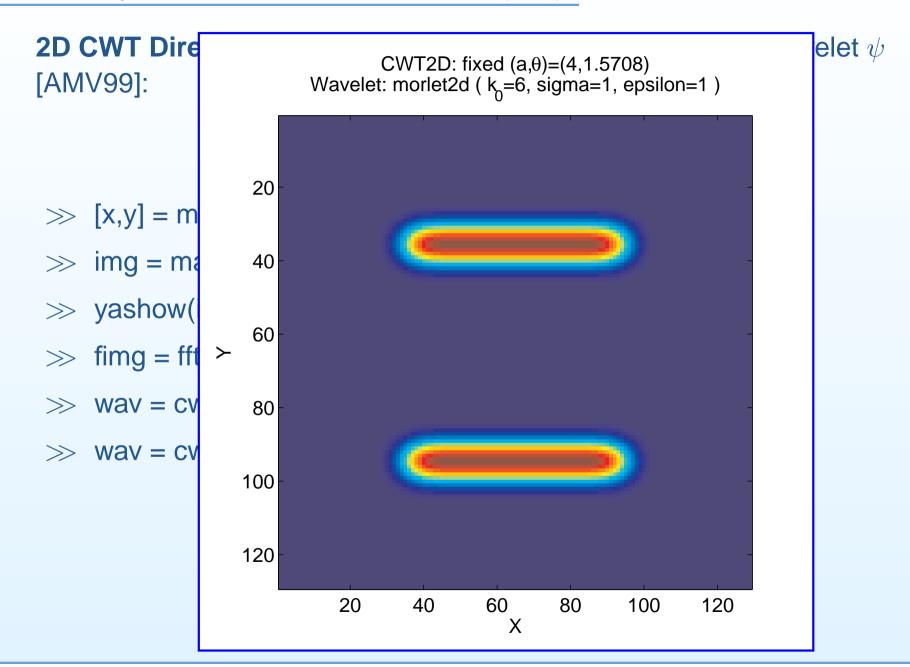
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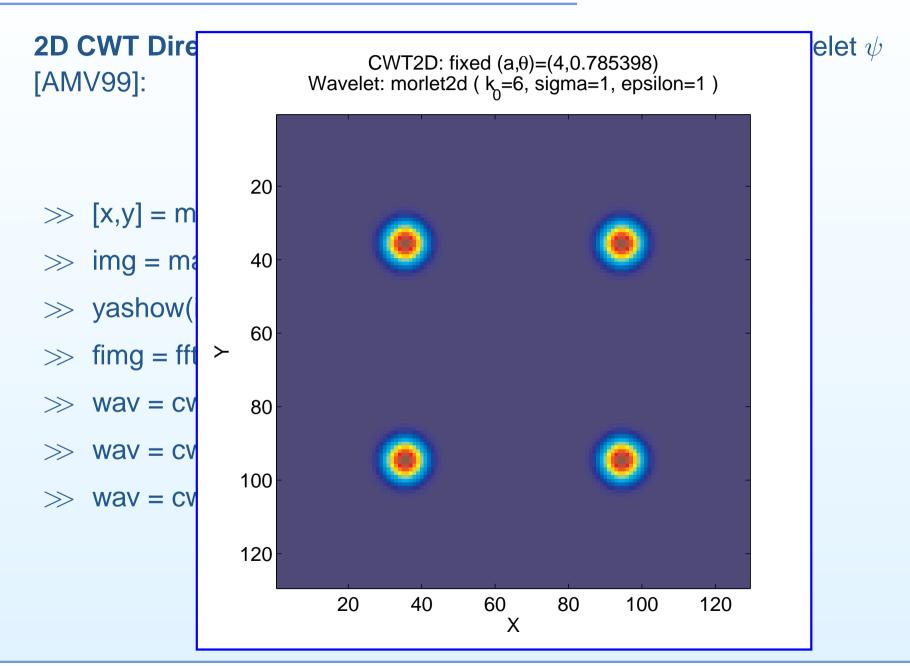
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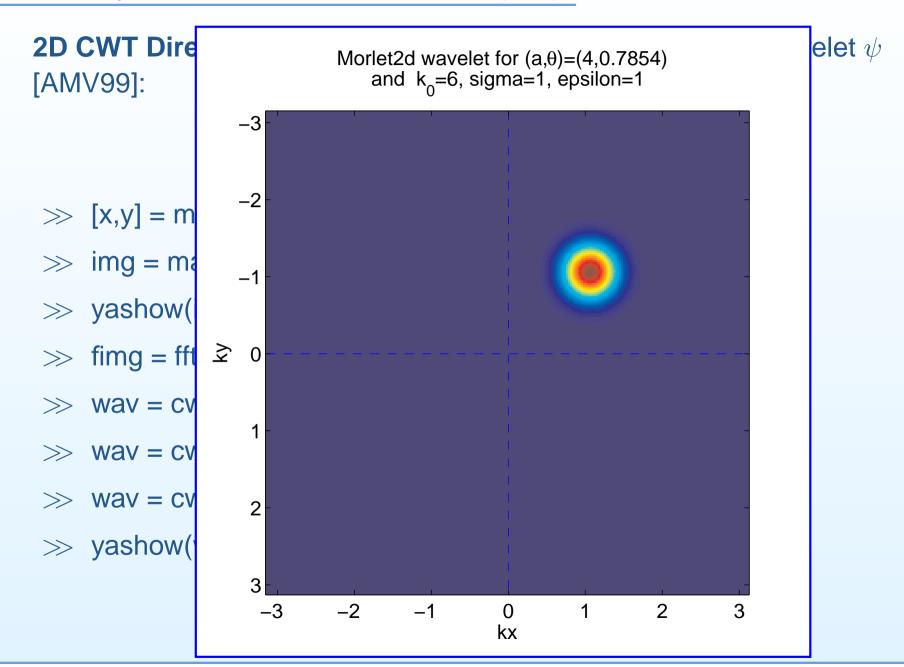
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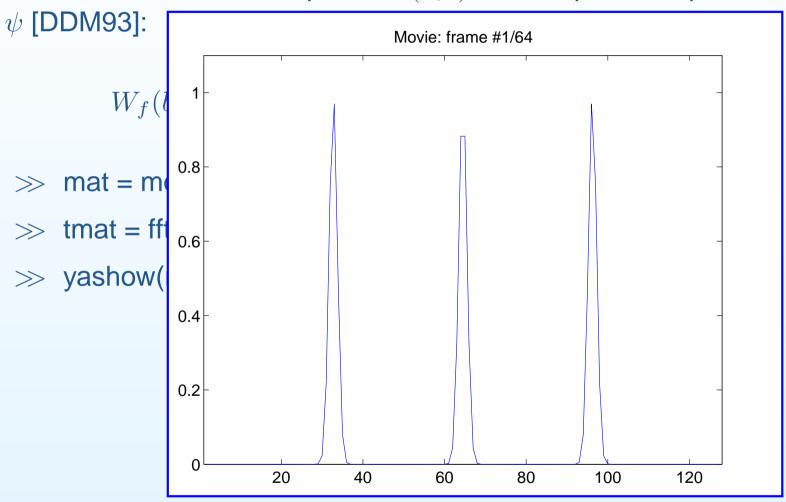
>>> mat = movgauss; %% Three moving Gaussians (64 frames)

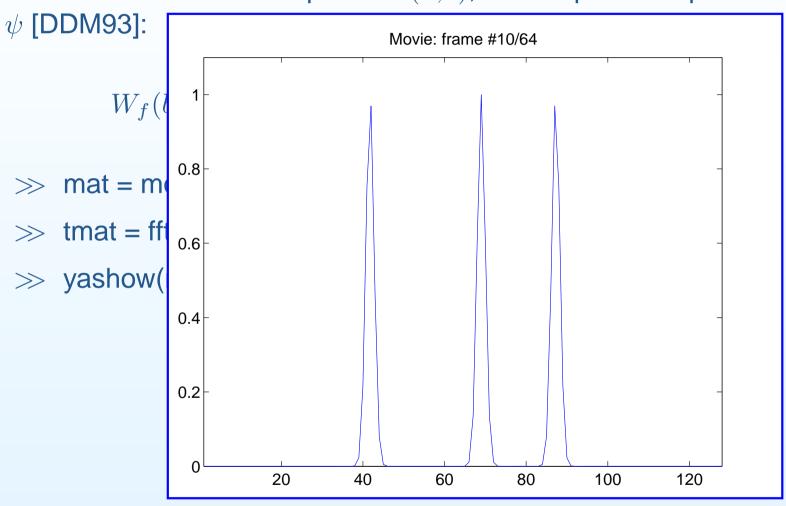
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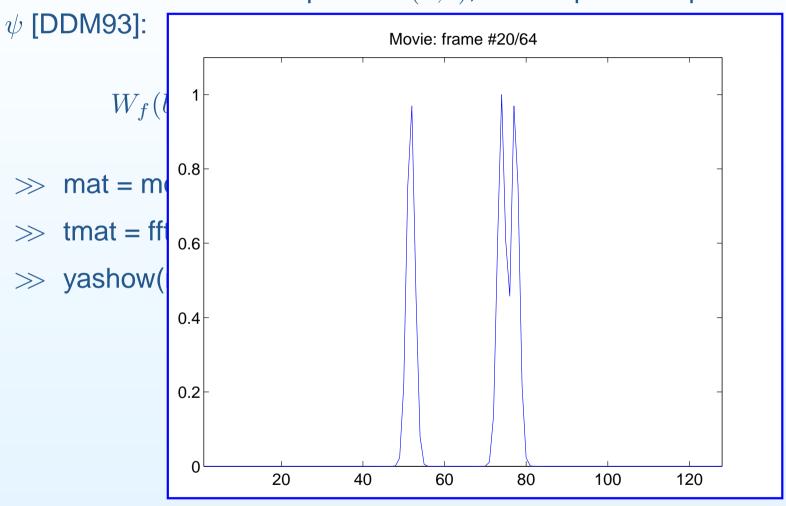
- >>> mat = movgauss; %% Three moving Gaussians (64 frames)
- $\gg$  tmat = fft2(mat);

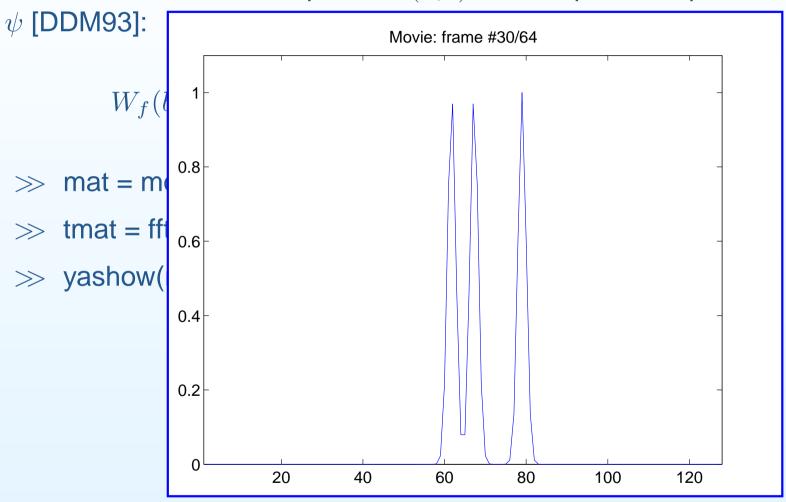
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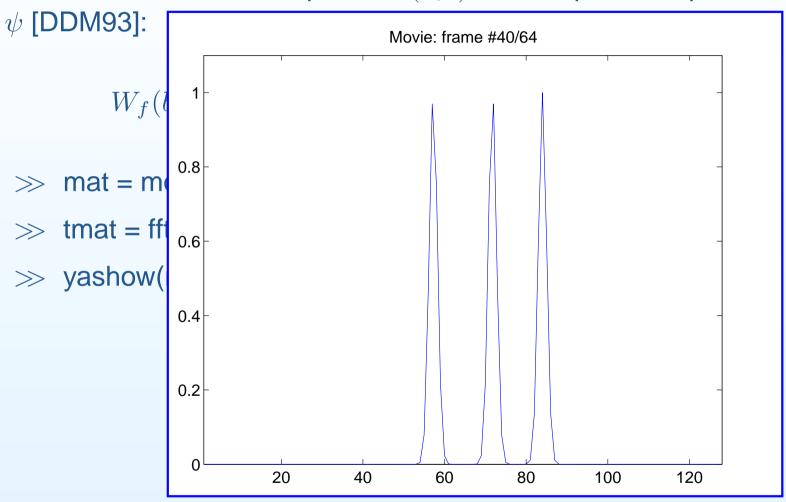
- >>> mat = movgauss; %% Three moving Gaussians (64 frames)
- $\gg$  tmat = fft2(mat);
- >> yashow(mat, 'timeseq');

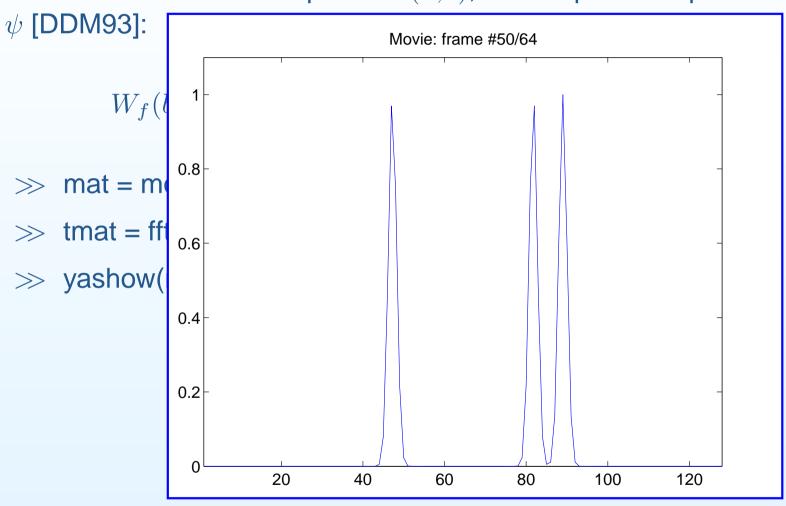


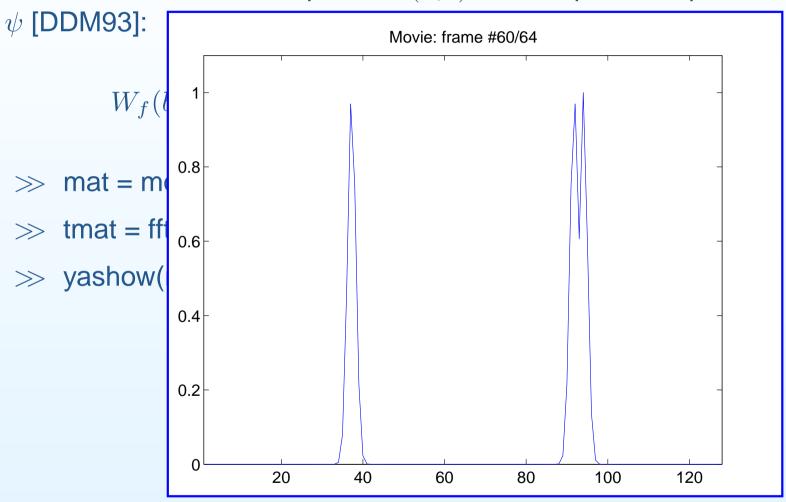












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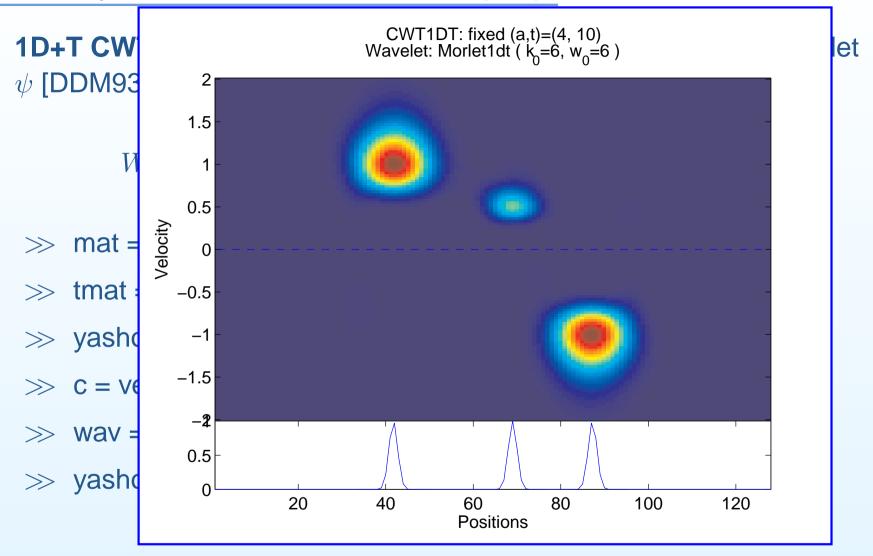
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- $\gg$  c = vect(-2, 2, 128); %% Defining velocities

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**3D CWT Isotropic** Given a volume of data  $I(\vec{x})$ , and a 3D isotropic wavelet  $\psi$ :

$$W_f(\vec{b}, a) = \int_{\mathbb{R}^3} d^3 \vec{x} I(\vec{x}) \frac{1}{a^{\frac{3}{2}}} \psi^*(\frac{\vec{x} - \vec{b}}{a}).$$

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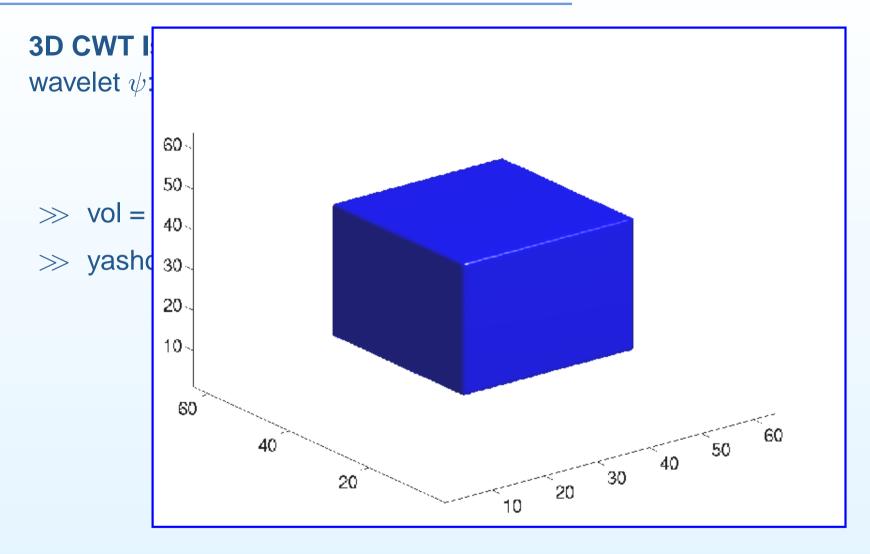
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 $\gg$  vol = cube(64);

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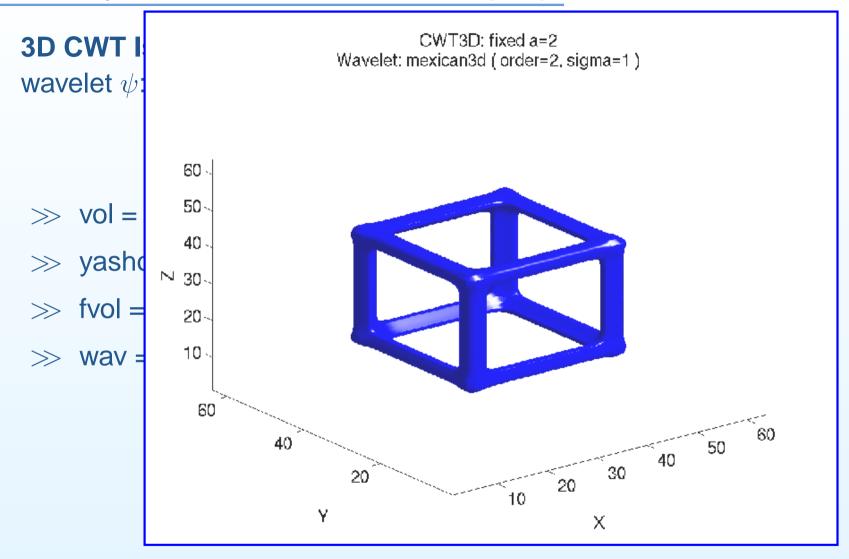
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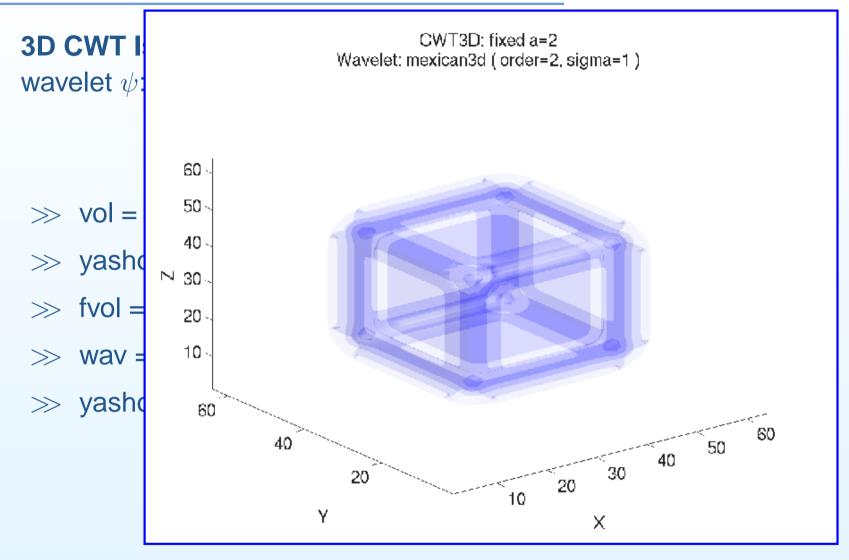
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- $\gg$  wav = cwt3d(fvol, 'mexican', 2, [0 0]); yashow(wav);



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- >>> yashow(wav, 'levels', 3) %% Three level sets

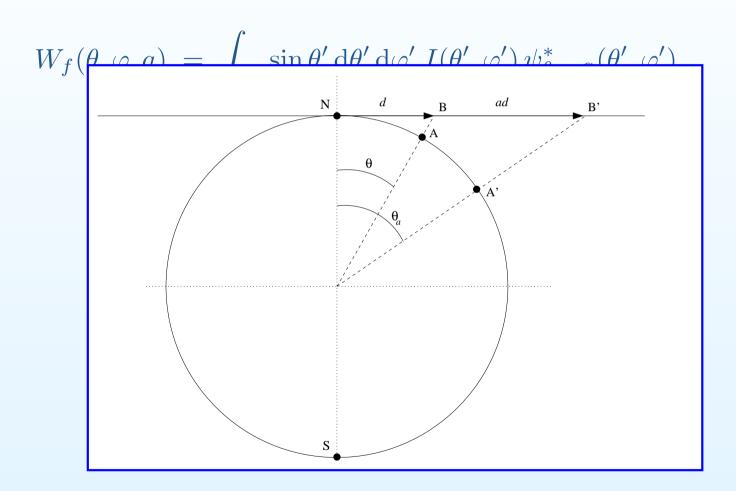


$$W_f(\theta, \varphi, a) = \int_{S_2} \sin \theta' \, d\theta' \, d\varphi' \, I(\theta', \varphi') \, \psi_{\theta, \varphi, a}^*(\theta', \varphi')$$

**Spherical CWT Isotropic** Given a spherical data  $I(\theta, \varphi)$ , and a spheric isotropic wavelet  $\psi$ :

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Stereographical Dilation



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>>> load world2;

$$W_f(\theta, \varphi, a) = \int_{S_2} \sin \theta' \, d\theta' \, d\varphi' \, I(\theta', \varphi') \, \psi_{\theta, \varphi, a}^*(\theta', \varphi')$$

- >> load world2;
- >> yashow(mat, 'spheric', 'relief');

**Spherical CWT Isotropic** Given a spherical data  $I(\theta, \varphi)$ , and a

spheric isotropic wavelet 1/2:

 $W_f$ 

>> load wor

>>> yashow(



$$W_f(\theta, \varphi, a) = \int_{S_2} \sin \theta' \, d\theta' \, d\varphi' \, I(\theta', \varphi') \, \psi_{\theta, \varphi, a}^*(\theta', \varphi')$$

- >>> load world2;
- >> yashow(mat, 'spheric', 'relief');
- >>> fmat = fst(mat); %% Precomputing FFT

$$W_f(\theta, \varphi, a) = \int_{S_2} \sin \theta' \, d\theta' \, d\varphi' \, I(\theta', \varphi') \, \psi_{\theta, \varphi, a}^*(\theta', \varphi')$$

- >>> load world2;
- >> yashow(mat, 'spheric', 'relief');
- >> fmat = fst(mat); %% Precomputing FFT
- $\gg$  wav = fcwtsph(fmat, 'dog', 0.05, 0);

$$W_f(\theta, \varphi, a) = \int_{S_2} \sin \theta' \, d\theta' \, d\varphi' \, I(\theta', \varphi') \, \psi_{\theta, \varphi, a}^*(\theta', \varphi')$$

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- >> yashow(wav, 'relief');



 $W_f$  (

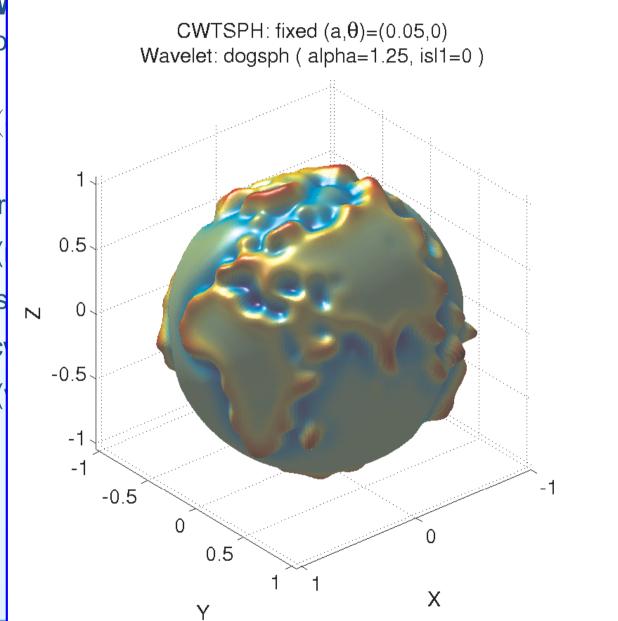
>>> load wor

>>> yashow(

 $\gg$  fmat = fs

 $\gg$  wav = fc

>>> yashow(



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- >> yashow(wav);

Spherical CWT Isotropic Given a spherical data I(A) and a

spheric isotro

 $W_f$ 

>>> load wor

>>> yashow(

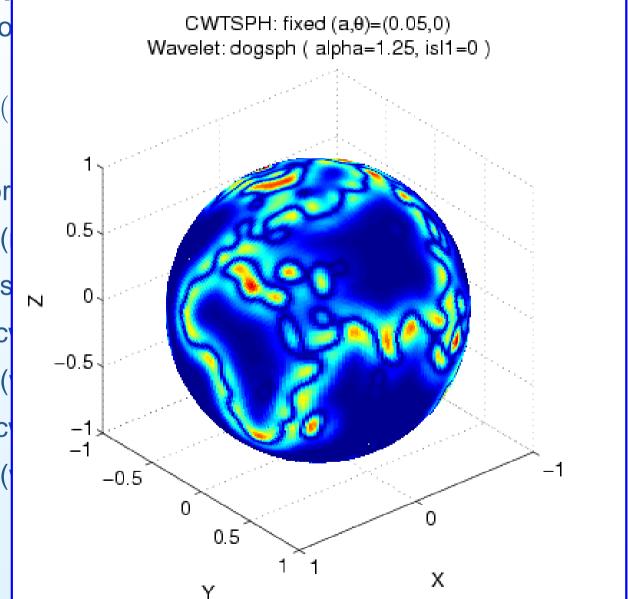
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>>> yashow(





 $W_f$  (

≫ load wor

>>> yashow(

 $\gg$  fmat = fs

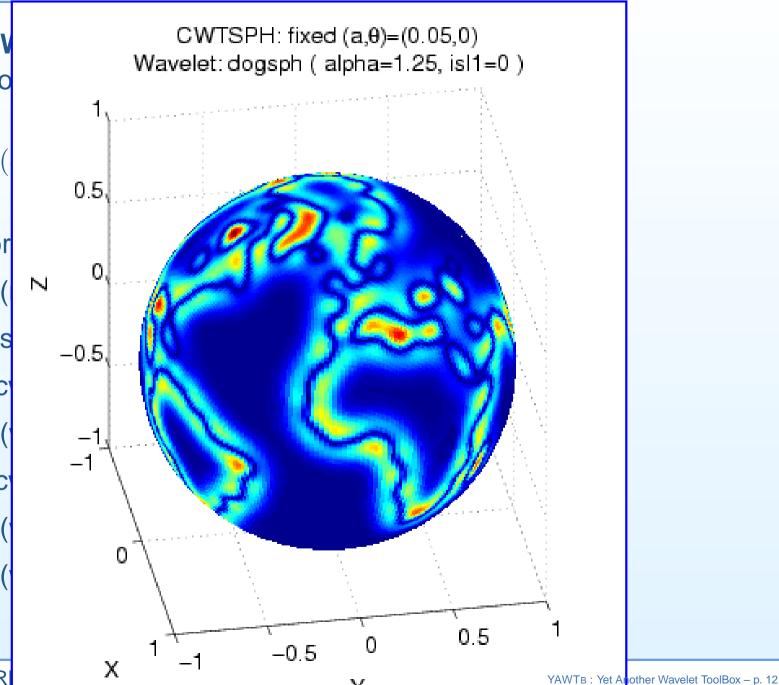
 $\gg$  wav = fc

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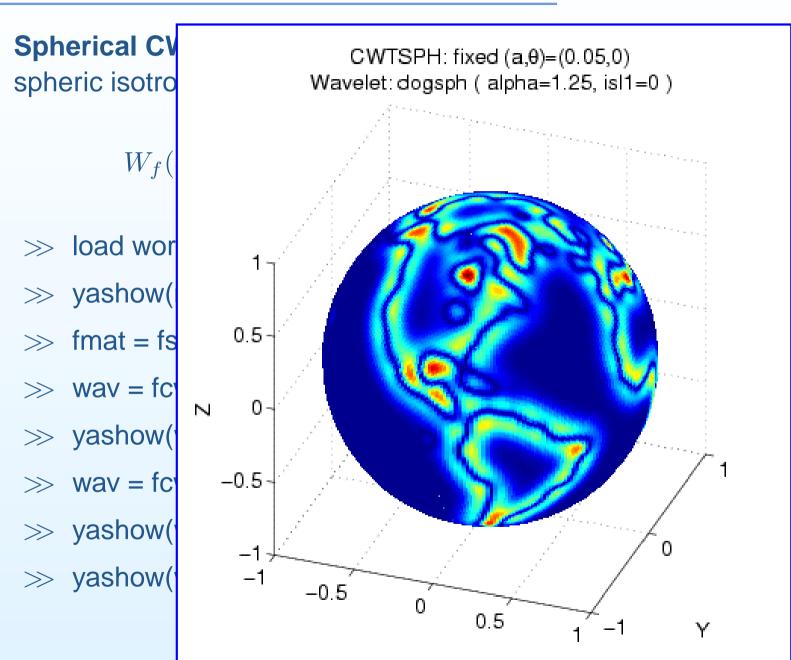
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YAWTB: Yet Another Wavelet ToolBox - p. 12

Spherical CWT Isotropic Given a spherical data I(A) and a

spheric isotro

 $W_f$  (

>> load wor

>>> yashow(

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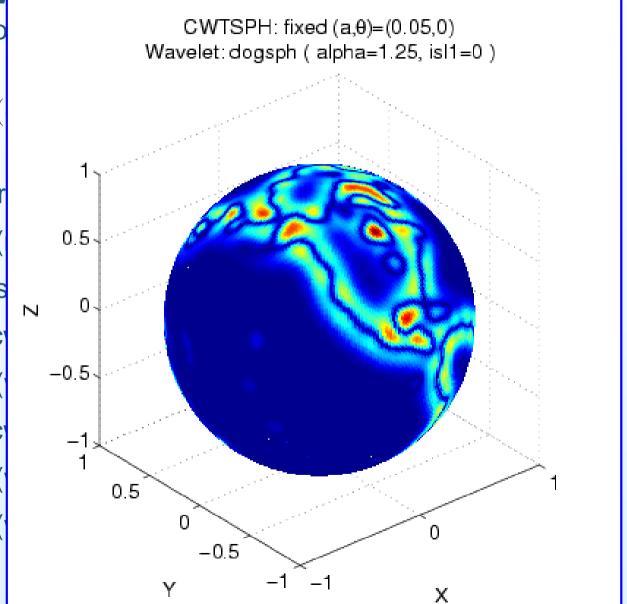
 $\gg$  wav = fc

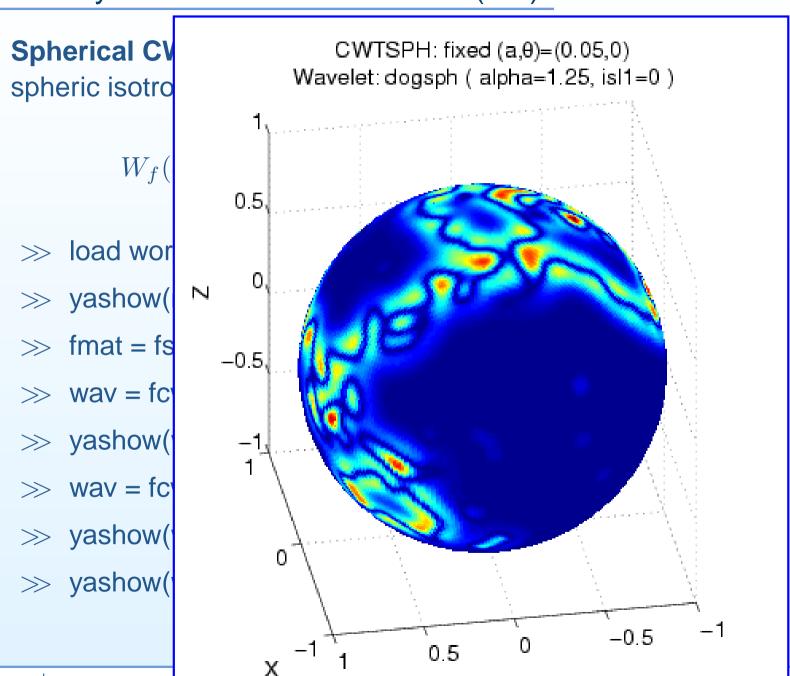
>>> yashow(

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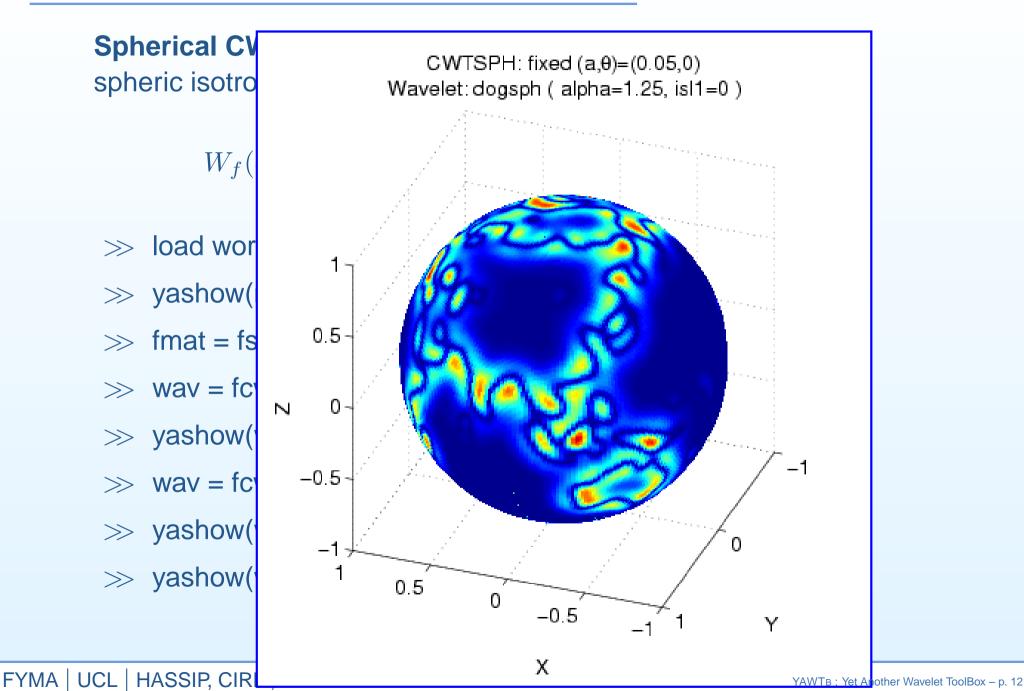
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YAWTB: Yet Another Wavelet ToolBox - p. 12

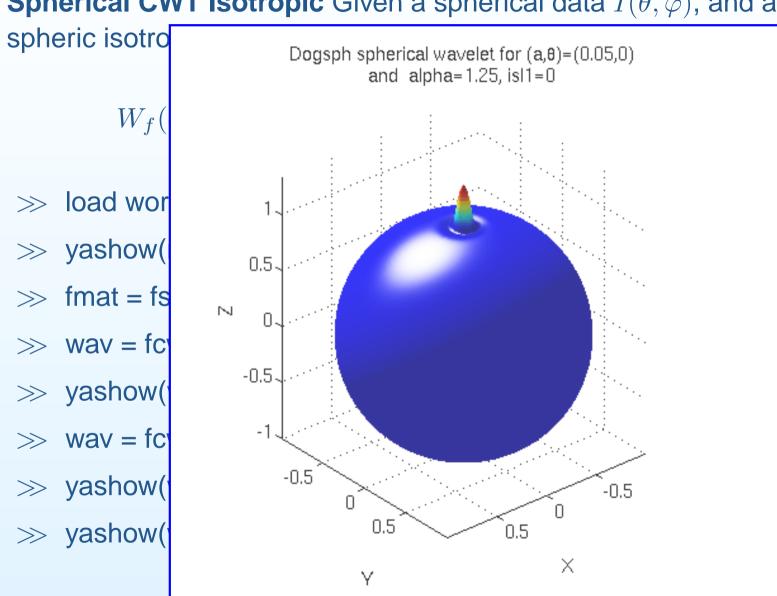


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- $\gg$  wav = fcwtsph(fmat, 'dog', 0.1, 0);
- >> yashow(wav);
- >> yashow(wav,'filter','relief');

### **Spherical CWT Isotropic** Given a spherical data $I(\theta, \varphi)$ , and a



1. Parsing and completing the documentation of the stable code

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- 5. ... (We're looking for interested developers ;-)

#### References

- [AMV96] Jean-Pierre Antoine, Romain Murenzi, and Pierre Vandergheynst.

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- [GPL] GNU General Public License.

  http://www.gnu.org/copyleft/gpl.html.
- [SMK] D. Rockmore S. Moore, D. Healy and P. Kostelec. SpharmonicKit is a freely available collection of C programs for doing Legendre and scalar spherical transforms developed at Dartmouth College. It is available at <a href="http://www.cs.dartmouth.edu/~geelong/sphere/">http://www.cs.dartmouth.edu/~geelong/sphere/</a>.