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Problem 1

First we use equation

$$G_{0,N} \equiv \sum_{i=0}^{N-1} A^{N-i-1} B R^{-1} B^T (A^T)^{N-i-1} \quad (1)$$

get $G_{0,N} = 5.3281$.

Then we use following equation to get optimal control

$$u_k^* = -R^{-1} B^T \lambda_{k+1} = R^{-1} B^T (A^T)^{N-k-1} G_{0,N}^{-1} (r_N - A^N x_0) \quad (2)$$

The control value I have is

$$u_0 = 0.1173$$

$$u_1 = 0.2346$$

$$u_2 = 0.4692$$

$$u_3 = 0.9384$$

$$u_4 = 1.8768$$

Problem 2

Use `c2d` to discretized the system. Then we have following parameters for discrete system.

$$\begin{aligned} A &= \begin{bmatrix} x1 & x2 \\ x1 & 1 & 0.5 \\ x2 & 0 & 1 \end{bmatrix} \\ B &= \begin{bmatrix} u1 \\ x1 & 0.125 \\ x2 & 0.5 \end{bmatrix} \\ C &= \begin{bmatrix} x1 & x2 \\ 1 & 0 \end{bmatrix} \\ D &= \begin{bmatrix} u1 \\ y1 & 0 \end{bmatrix} \end{aligned} \quad (3)$$

Then solve Riccati difference equation

$$S_k = A_k^T \left[S_{k+1} - S_{k+1} B_k (B_k^T S_{k+1} B_k + R_k)^{-1} B_k^T S_{k+1} \right] A_k + Q_k s \quad (4)$$

we get

s11:

4.0368 4.0435 4.0678 4.1417 4.3448 4.8755 6.2840 10.4679 26.4338 95.3311
100.0000

s22:

4.1499 4.1632 4.1976 4.2775 4.4464 4.7800 5.4147 6.6293 9.0036 11.2041
100.0000

b) use equation

$$K_k = (B_k^T S_{k+1} B_k + R_k)^{-1} B_k^T S_{k+1} A_k \quad (5)$$

We get kalman gain

K:

```
0.6527  0.6566  0.6680  0.6972  0.7644  0.9085  1.2052  1.8094  2.8929  0.4535
1.3169  1.3223  1.3359  1.3663  1.4276  1.5418  1.7409  2.0664  2.4838  2.0408
```

c) we use state equation and $u_k = -K_k * x_k$, we get x and u

x1:

```
10.0000 12.5380 11.5599 9.2410 6.7202 4.5113 2.7813 1.5272 0.6841 0.1878
0.0126
```

x2:

```
10.0000 12.5380 11.5599 9.2410 6.7202 4.5113 2.7813 1.5272 0.6841 0.1878
0.0126
```

u_k:

```
-19.6958 -8.4336 -2.2920 0.6764 1.8188 2.0119 1.7964 1.4914 1.2829 1.2860 0
```

