

Convex Optimization Homework #2
 Posted Friday April 29, 2022, [Revised May 3](#)
 Due: Saturday May 21, 2022, 11am.

1. (50%) Consider the QCQP optimization problem

$$\begin{aligned} & \underset{x \in \mathbf{R}^n}{\text{minimize}} && \frac{1}{2}x^T P_0 x + q_0^T x + r_0 \\ & \text{subject to} && \frac{1}{2}x^T P_i x + q_i^T x + r_i \leq 0, i = 1, \dots, m, \\ & && Ax = b \end{aligned} \tag{1}$$

where $P_0 \in \mathbf{S}_{++}^n$, $P_i \in \mathbf{S}_+^n$ for $i = 1, \dots, m$, $q_i \in \mathbf{R}^n$, $r_i \in \mathbf{R}$ for $i = 0, 1, \dots, m$, $A \in \mathbf{R}^{m \times n}$, $b \in \mathbf{R}^m$.

- (a) (3%) Determine if Problem (1) is a convex problem. Justify your answer.
- (b) (5%) Write the Lagrangian of the problem.
- (c) (5%) Find the Lagrange dual function $g(\lambda, \nu)$ and its domain **dom** g .
- (d) (5%) Formulate the Lagrange dual problem. Determine if it is a convex problem.
- (e) (7%) If Problem (1) is convex, please take the parameters defined in Table 1 and use **CVX** to solve the problem. Write down the **optimal value** p^* and an **optimal point** you obtained from **CVX**.
- (f) (5%) Determine whether each of the inequality constraints is active or inactive.
- (g) (5%) If the dual problem is convex, use **CVX** to solve the problem. Find the dual optimal value d^* , dual optimal point λ^* , and the duality gap $p^* - d^*$.
- (h) (5%) Does Problem (1) with parameters in Table 1 satisfy the Slater's condition? Justify your answer.
- (i) (5%) Find the KKT conditions for Problem (1). Verify that the solution you get from **CVX** is consistent with the solution of the KKT condition equations.
- (j) (5%) Submit your source code as **problem1.m** or **problem1.py**.

Parameters	(n, m)	P_0	P_1	P_2	(r_0, r_1, r_2)
	$(3, 2)$	$\begin{bmatrix} 8.2 & -3.8 & 8.6 \\ -3.8 & 1.8 & -4.3 \\ 8.6 & -4.3 & 11.9 \end{bmatrix}$	$\begin{bmatrix} 1.2 & 0.3 & -0.6 \\ 0.3 & 1.7 & 2.4 \\ -0.6 & 2.4 & 5.0 \end{bmatrix}$	$\begin{bmatrix} 1.5 & 1.3 & -1.9 \\ 1.3 & 10.1 & -3.2 \\ -1.9 & -3.2 & 3.8 \end{bmatrix}$	$(1, 2, 3)$
Parameters	A	b	q_0	q_1	q_2
	$\begin{bmatrix} 1 & 2 & -3 \\ -7 & 8 & -9 \end{bmatrix}$	$\begin{bmatrix} 1.5 \\ 8 \end{bmatrix}$	$\begin{bmatrix} 7 & 8 & 9 \end{bmatrix}^T$	$\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}^T$	$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$

Table 1: Parameters for the QCQP Problem

2. (50%) Consider the two-way partitioning optimization problem

$$\begin{aligned} & \underset{x \in \mathbf{R}^n}{\text{minimize}} && x^T W x \\ & \text{subject to} && x_i^2 = 1, \quad i = 1, \dots, n \end{aligned} \tag{2}$$

where $W \in \mathbf{S}^n$.

- (a) (5%) Write the Lagrangian of the problem.
- (b) (5%) Find the Lagrange dual function $g(\nu)$ and its domain **dom** g .

- (c) (5%) Formulate the Lagrange dual problem by writing the domain of g as an explicit dual constraint. Determine if it is a convex problem.
- (d) (5%) Let $n = 5$ and $W = W_1$ as in Table 2. Solve the primal problem (Problem (2)) using an exhaustive search method to evaluate the objective function at all 2^n feasible points. Find the optimal value p^* and optimal point x^* .
- (e) (5%) If the dual problem you formulated in (c) is convex, use **CVX** to solve the dual problem. Find the optimal value d^* , the dual optimal point ν^* , and the duality gap $p^* - d^*$. (*Hint: You may need to use `cvx_begin sdp` to indicate that is a semidefinite program.*)
- (f) (5%) Let $\nu = -\lambda_m \mathbf{1}$ where λ_m is the smallest eigenvalue (likely a negative value). Calculate $g(\nu)$ and find the difference $d^* - g(\nu)$.
- (g) (15%) Let $n = 10$ and $W = W_2$ as in Table 2. Repeat (d)-(f).
- (h) (5%) Submit your source code as **problem2.m** or **problem2.py**.

Parameters	W_1	W_2
	$\begin{bmatrix} -0.8 & 0.2 & 0.3 & 1.0 & -0.3 \\ 0.2 & -0.1 & -0.8 & 1.3 & 1.1 \\ 0.3 & -0.8 & -1.0 & -0.2 & 0.4 \\ 1.0 & 1.3 & -0.2 & 0.9 & 0.8 \\ -0.3 & 1.1 & 0.4 & 0.8 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.7 & -0.3 & 0.3 & 0.3 & 0.1 & 0.6 & -0.6 & -1.3 & 1.0 & 0.1 \\ -0.3 & 0.4 & 0.4 & 0 & -0.6 & -0.1 & 0 & 1.2 & -0.3 & 0.4 \\ 0.3 & 0.4 & 0.1 & 0 & -0.6 & -0.3 & 0.5 & 0.2 & 1.2 & 0.3 \\ 0.3 & 0 & 0 & 0.2 & 0.3 & 0.1 & 0.6 & 1.9 & 0.5 & 0.5 \\ 0.1 & -0.6 & -0.6 & 0.3 & -1.6 & -1.2 & -0.2 & 0.3 & 0.7 & -0.9 \\ 0.6 & -0.1 & -0.3 & 0.1 & -1.2 & 0.9 & 0.7 & -0.8 & -0.8 & -0.4 \\ -0.6 & 0 & 0.5 & 0.6 & -0.2 & 0.7 & -0.9 & -0.5 & 0 & -0.3 \\ -1.3 & 1.2 & 0.2 & 1.9 & 0.3 & -0.8 & -0.5 & -1.7 & -0.1 & 0.3 \\ 1.0 & -0.3 & 1.2 & 0.5 & 0.7 & -0.8 & 0 & -0.1 & 1.9 & 1.2 \\ 0.1 & 0.4 & 0.3 & 0.5 & -0.9 & -0.4 & -0.3 & 0.3 & 1.2 & -0.3 \end{bmatrix}$

Table 2: Parameters for the two-way partitioning problem

Guidelines of Homework Submission:

- You are allowed to discuss with other students, ask for hints from the TAs. But you have to write your answers and argument solely on your own, without looking at any part of anyone else's answers. Sharing your written (or typed) answers with others is *strongly prohibited*. Both parties will get a zero-score penalty for this mis-conduct.
- Submit your answer online as a document file (in *.pdf only) that contains all answers in this problem set.
- Submit your files online onto the NTU Cool system. No paper shall be handed in. If needed, you can choose to write (sketch) your answers on a sheet first and convert the image(s) to a single pdf file.
- Late submissions will be processed according to the following rules.
 - Homework received by 11am, May 21 (t_1) will be counted in full.
 - Homework received after 2pm, May 21 (t_2) will not be counted.
 - Homework received between t_1 and t_2 will be counted with a discount rate

$$\frac{t_2 - t}{t_2 - t_1}$$

where t is the submission time. Note that $t_2 - t_1$ is three hours.