Convex Optimization Homework #1

Posted Friday April 1, 2021, Revised April 3
Due: Monday April 18, 2022, 11am.

1. (40%) Consider the optimization problem

$$\begin{aligned} \underset{x\in\mathbf{R}_{++},y\in\mathbf{R}_{++}}{\text{minimize}} \quad & \frac{x^2}{y} \\ \text{subject to} \quad & x+y \geq 1 \\ & \frac{1}{x}+\frac{1}{y} \leq 1 \\ & x^{1.5}+y^{1.5} \leq 5^{1.5} \end{aligned} \tag{1}$$

   (a) (10%) Determine if the problem if (1) is a convex problem. Justify your answer.

   (b) (10%) If Problem (1) is convex, use `CVX` to solve the problem. Write down the **optimal value** p^* and an **optimal point** you obtained from `CVX`.

   (c) (10%) Verify that the reported optimal point (x^*, y^*) satisfies the constraint inequality and yields the optimal value.

   (d) (10%) Submit your source code as `problem1.m` or `problem1.py`.

2. (60%) Consider the optimization problem

$$\begin{aligned} \underset{x\in\mathbf{R}^n}{\text{minimize}} \quad & f_0(x) \\ \text{subject to} \quad & ||Ax-b||_1 \leq \epsilon \end{aligned} \tag{2}$$

   where $A \in \mathbf{R}^{m\times n}$, $b \in \mathbf{R}^m$, and $\epsilon > 0$, and $f_0(x) = \text{length}(x)$.

   (a) (10%) Determine if Problem (2) is a convex problem. Justify your answer.

   (b) (10%) Show that Problem (2) is a quasiconvex problem. (*Hint: You need to show that $f_0(x)$ is a quasiconvex function in x.*)

   (c) (10%) Construct a family of functions $\phi_t : \mathbf{R}^n \to \mathbf{R}$ such that

$$f(x) \leq t \Longleftrightarrow \phi_t(x) \leq 0$$

   and ϕ_t is convex for all t.

   (d) (10%) Let $n = 3$ and $m = 3$. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $\epsilon = 0.1$. Use `CVX` and the bisection method your learned in class to solve the problem. Write down the **optimal value** p^* and an **optimal point** x^* you obtained from your algorithm. Make sure that the reported optimal point x^* satisfies the constraint inequality and yields the optimal value.

   (e) (10%) Repeat (2d) for $n = 4$, $m = 5$, $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 16 & 15 \\ 17 & 18 & 20 & 19 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \\ 5 \end{bmatrix}$, and $\epsilon = 0.1$.

   (f) (10%) Submit your source code as `problem2.m` or `problem2.py` (along with its subroutines if any).

Guidelines of Homework Submission:

- You are allowed to discuss with other students, ask for hints from the TAs. But you have to write your answers and argument solely on your own, without looking at any part of anyone else's answers. Sharing your written (or typed) answers with others is *strongly prohibited*. Both parties will get a zero-score penalty for this mis-conduct.
- Submit your answer online as a document file (in *.pdf only) that contains all answers in this problem set.
- Submit your files online onto the NTU Cool system. No paper shall be handed in. If needed, you can choose to write (sketch) your answers on a sheet first and convert the image(s) to a single pdf file.
- Late submissions will be processed according to the following rules.

  (1) Homework received by 11am, April 18 (t_1) will be counted in full.

  (2) Homework received after 2pm, April 18 (t_2) will not be counted.

  (3) Homework received between t_1 and t_2 will be counted with a discount rate

  $$\frac{t_2 - t}{t_2 - t_1}$$

  where t is the submission time. Note that $t_2 - t_1$ is three hours.