NAVARCH 568: Mobile Robotics
Problem Set 2

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Task 1

Part A: EKF

Propagation Step

$$
\begin{aligned}
\bar{\mu}_{k+1} &= \mu_k + \begin{bmatrix} -\frac{\hat{v}}{\hat{\omega}} sin\theta_k + \frac{\hat{v}}{\hat{\omega}} sin(\theta_k + \hat{\omega}\Delta t) \\ \frac{\hat{v}}{\hat{\omega}} cos\theta_k - \frac{\hat{v}}{\hat{\omega}} cos(\theta_k + \hat{\omega}\Delta t) \\ \hat{\omega}\Delta t \end{bmatrix} \\
G &= \begin{bmatrix} 1 & 0 & -\frac{\hat{v}}{\hat{\omega}} cos\theta_k + \frac{\hat{v}}{\hat{\omega}} cos(\theta_k + \hat{\omega}\Delta t) \\ 0 & 1 & -\frac{\hat{v}}{\hat{\omega}} sin\theta_k + \frac{\hat{v}}{\hat{\omega}} sin(\theta_k + \hat{\omega}\Delta t) \\ 0 & 0 & 1 \end{bmatrix} \\
V &= \begin{bmatrix} \frac{-sin\theta + sin(\theta_k + \hat{\omega}\Delta t)}{\hat{\omega}} & \frac{v(sin\theta - sin(\theta_k + \hat{\omega}\Delta t))}{\hat{\omega}^2} + \frac{vcos(\theta_k + \hat{\omega}\Delta t))}{\hat{\omega}} \\ \frac{cos\theta - cos(\theta_k + \hat{\omega}\Delta t)}{\hat{\omega}} & -\frac{v(cos\theta - cos(\theta_k + \hat{\omega}\Delta t))}{\hat{\omega}^2} + \frac{vsin(\theta_k + \hat{\omega}\Delta t))}{\hat{\omega}} \end{bmatrix} \\
M &= \begin{bmatrix} \alpha_1 \hat{v}^2 + \alpha_2 \hat{\omega}^2 & 0 & 0 \\ 0 & \alpha_3 \hat{v}^2 + \alpha_4 \hat{\omega}^2 & 0 \\ 0 & 0 & \alpha_5 \hat{v}^2 + \alpha_6 \hat{\omega}^2 \end{bmatrix} \\
\bar{\Sigma}_{k+1} &= G \Sigma_k G^T + VMV^T
\end{aligned}
$$

Correction Step

NOTE: Q (measurement noise) is inherent and not calculated.

$$
\begin{aligned}
\hat{z} &= \begin{bmatrix} atan2(m_y - \bar{\mu}_{k+1,y}, m_x - \bar{\mu}_{k+1,x}) - \bar{\mu}_{k+1,\theta} \\ \sqrt{(m_x - \bar{\mu}_{k+1,x})^2 + (m_y - \bar{\mu}_{k+1,y})^2} \end{bmatrix} \\
H &= \begin{bmatrix} \frac{m_y - \bar{\mu}_{k+1,y}}{(m_x - \bar{\mu}_{k+1,x})^2 + (m_y - \bar{\mu}_{k+1,y})^2} & -\frac{m_x - \bar{\mu}_{k+1,x}}{(m_x - \bar{\mu}_{k+1,x})^2 + (m_y - \bar{\mu}_{k+1,y})^2} & -1 \\ -\frac{m_x - \bar{\mu}_{k+1,x}}{\sqrt{(m_x - \bar{\mu}_{k+1,x})^2 + (m_y - \bar{\mu}_{k+1,y})^2}} & -\frac{m_y - \bar{\mu}_{k+1,y}}{\sqrt{(m_x - \bar{\mu}_{k+1,x})^2 + (m_y - \bar{\mu}_{k+1,y})^2}} & 0 \end{bmatrix} \\
S &= H \bar{\Sigma}_{k+1} H^T + Q \\
K &= \bar{\Sigma}_{k+1} H^T S^{-1} \\
\mu_{k+1} &= \bar{\mu}_{k+1,y} + K(z(1:2) - \hat{z}) \\
\Sigma_{k+1} &= (I - KH)\bar{\Sigma}_{k+1}
\end{aligned}
$$

Part B: UKF

Propagation Step

The method I chose was to create augmented state and covariance, μ_k^a and Σ_k^a, and create my sigma points from there. Consequently, each sigma point has its own corresponding input and measurement noise we use with it. For these sigma points, X^a indicates the full augmented 7x1 vector, X^x is a 3x1 holding the state, X^u is a 3x1 vector of the associated input noise, and X^z is a 2x1 vector of the associated measurement noise.
NOTE: Q is inherent and not calculated, and κ is a tuned parameter.

$$
\begin{aligned}
M &= \begin{bmatrix} \alpha_1\hat{v}^2+\alpha_2\hat{\omega}^2 & 0 & 0 \\ 0 & \alpha_3\hat{v}^2+\alpha_4\hat{\omega}^2 & 0 \\ 0 & 0 & \alpha_5\hat{v}^2+\alpha_6\hat{\omega}^2 \end{bmatrix} \\
\mu_k^a &= \begin{bmatrix} \mu_k \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
\Sigma_k^a &= \begin{bmatrix} \Sigma_k & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & Q \end{bmatrix} \\
n &= dim(\mu_k^a) \\
L &= chol(\Sigma_k^a, 'lower') \\
X_k^{[0]} &= \mu_k^a \\
X_k^{[i]} &= \mu_k^a + \sqrt{n+\kappa}col_i L, \quad i = 1:n \\
X_k^{[i]} &= \mu_k^a - \sqrt{n+\kappa}col_{i-n} L, \quad i = n+1:2n \\
w^{[i]} &= \frac{\kappa}{n+\kappa}, \quad i = 0 \\
w^{[i]} &= \frac{1}{2(n+\kappa)}, \quad i = 1:2n \\
\bar{X}_{k+1} &= g(X_k^x, u + X_k^u)
\end{aligned}
$$

NOTE: $g(X, u)$ is the motion model provided in PS2 and was also used in EKF for $\bar{\mu}_{k+1}$.

$$
\begin{aligned}
\bar{\mu}_{k+1} &= \sum_{i=0}^{2n} w^{[i]} \bar{X}_{k+1}^{[i],x} \\
\bar{\Sigma}_{k+1} &= \sum_{i=0}^{2n} w^{[i]} (\bar{X}_{k+1}^{[i],x} - \bar{\mu}_{k+1})(\bar{X}_{k+1}^{[i],x} - \bar{\mu}_{k+1})^T
\end{aligned}
$$

Correction Step

$$
\bar{Z} = h(m_x, m_y, \bar{X}_{k+1}^x) + \bar{X}_{k+1}^z
$$

NOTE: $h(m_x, m_y, X)$ is the measurement model used in EKF for $\hat{z}$.

$$
\begin{aligned}
\hat{z}_{k+1} &= \sum_{i=0}^{2n} w^{[i]} \bar{Z}^{[i]} \\
S &= \sum_{i=0}^{2n} \omega^{[i]} (\bar{Z}^{[i]} - \hat{z}_{k+1})(\bar{Z}^{[i]} - \hat{z}_{k+1})^T \\
\Sigma_{k+1}^{xz} &= \sum_{i=0}^{2n} w^{[i]} (\bar{X}_{k+1}^{[i],x} - \bar{\mu}_{k+1})(\bar{Z}^{[i]} - \hat{z}_{k+1})^T \\
K &= \Sigma_{k+1}^{xz} S^{-1} \\
\mu_{k+1} &= \bar{\mu}_{k+1} + K(z(1:2) - \hat{z}) \\
\Sigma_{k+1} &= \bar{\Sigma}_{k+1} - KSK^T
\end{aligned}
$$

Part C: PF

Propagation Step

for i = 1:n

$$
\begin{aligned}
x_{k+1}^{[i]} &= g(x_k^{[i]}, u + chol(M(u) randn(3,1))) \\
\hat{z}_{k+1}^{[i]} &= h(m_x, m_y, x_{k+1}^{[i]}) \\
w_{k+1}^{[i]} &= w_k^{[i]} p(\hat{z}_{k+1}^{[i]} | N(z_{k+1}^{[i]}, Q)) \\
\bar{X}_{k+1} &= \bar{X}_{k+1} + < x_{k+1}^{[i]}, w_{k+1}^{[i]} >
\end{aligned}
$$

end for

Correction Step

$$
\begin{aligned}
w_{k+1} &= \frac{w_{k+1}}{\sum_{i=1}^{n} w_{k+1}^{[i]}} \qquad \textit{(normalize weights)} \\
n_{eff} &= \frac{1}{\sum_{i=1}^{n} (w_{k+1}^{[i]})^2}
\end{aligned}
$$

if $n_{eff} < n_t \quad \Rightarrow \quad$ *resample particles according to weight*

Part D: InEKF

Propagation Step

NOTE: Q and V (and consequently $Cov(V)$) values are tuned.

$$
\begin{aligned}
X_k &= \mu_k^{\vee} \\
X_{k+1} &= g(X_k, u) \\
\hat{\xi} &= logm(X_k^{-1} X_{k+1}) \\
\bar{\mu}_{k+1} &= \mu_k exp(\hat{\xi} \Delta t)
\end{aligned}
$$

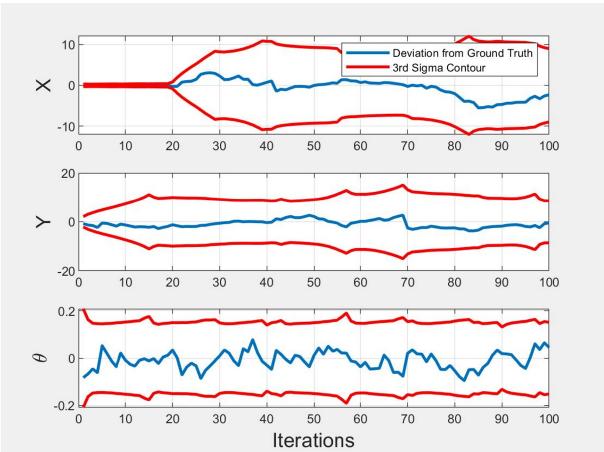
$$\begin{aligned}
Adj_{\mu_k} &= \begin{bmatrix} cos\mu_\theta & -sin\mu_\theta & \mu_y \\ sin\mu_\theta & cos\mu_\theta & -\mu_x \\ 0 & 0 & 0 \end{bmatrix} \\
\bar{\Sigma}_{k+1} &= \Sigma_k + Adj_{\mu_k} Q Adj_{\mu_k}^T
\end{aligned}$$

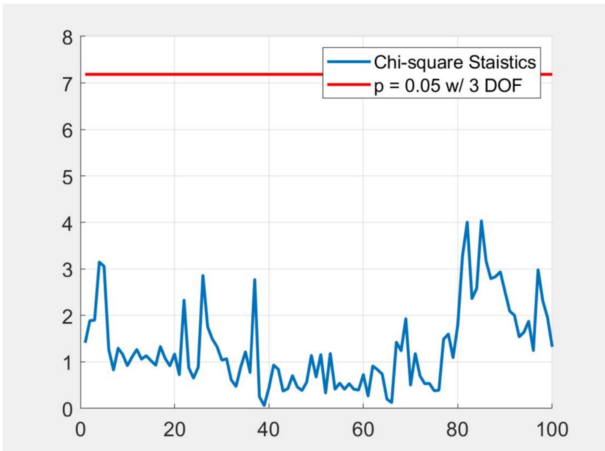
Correction Step

$$\begin{aligned}
H\xi &= -\hat{\xi} b \\
H_1 &= \begin{bmatrix} -1 & 0 & L_{y1} \\ 0 & -1 & -L_{x1} \end{bmatrix} \\
H_2 &= \begin{bmatrix} -1 & 0 & L_{y2} \\ 0 & -1 & -L_{x2} \end{bmatrix} \\
H &= \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \\
\bar{N} &= \bar{\mu}_{k+1} cov(V) \bar{\mu}_{k+1}^T \\
S &= H\bar{\Sigma}_{k+1} H^T + \bar{N} \\
L &= \bar{\Sigma}_{k+1} H^T S^{-1} \\
\eta_1 &= \bar{\mu}_{k+1} Y_1 - b_1 \\
\eta_2 &= \bar{\mu}_{k+1} Y_2 - b_2 \\
\eta &= \begin{bmatrix} \eta_1(1:2) \\ \eta_2(1:2) \end{bmatrix} \\
\mu_{k+1} &= exp((L\eta)^\wedge) \bar{\mu}_{k+1} \\
\Sigma_{k+1} &= (I - LH)\bar{\Sigma}_{k+1}(I - LH)^T + L\bar{N}L^T
\end{aligned}$$

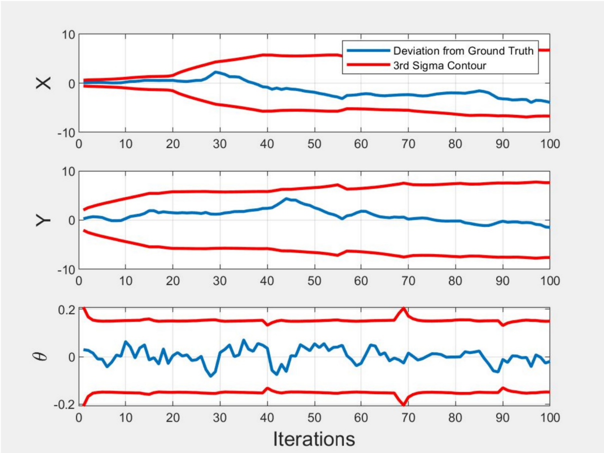
Task 2

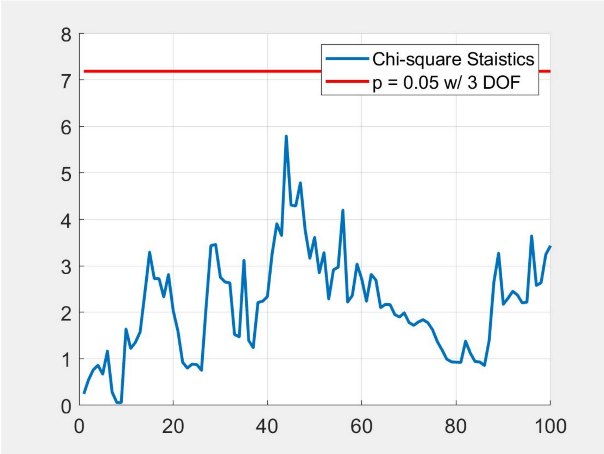
Part A: EKF



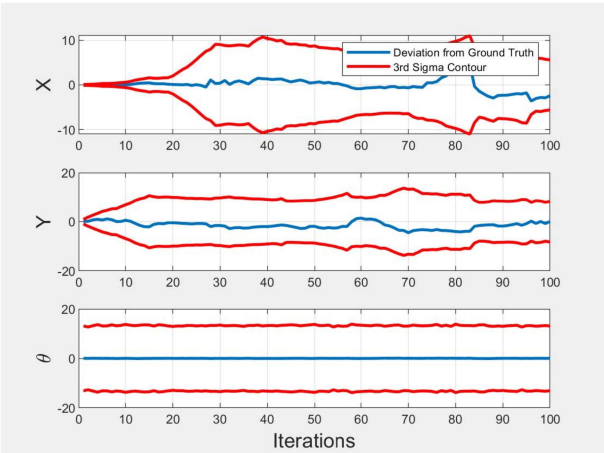


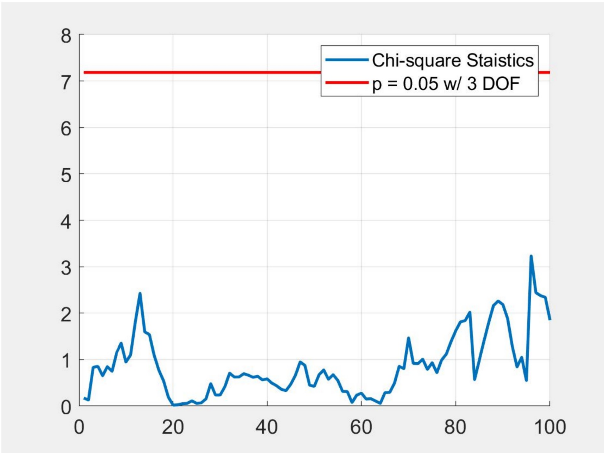
Part B: UKF





Part C: PF





Part D: InEKF

N/A.