LaTeX Note Template

1 表格展示

1.1 表格内脚注

表 1: FSTSP 模型符号及含义

符号	含义
0	起点仓库
$c+1$	终点仓库（和起点仓库相同，只是为了建模方便的另一个记号）
$C=\{1,2,\cdots,c\}$	全部客户集合
$C'\subseteq C$	无人机可访问的客户集合
$N_0=\{0,1,2,\cdots,c\}$	流出节点集合
$N_+=\{1,2,\cdots,c+1\}$	流入节点集合
$N=\{0,1,2,\cdots,c,c+1\}$	全部节点集合
$\langle i,j,k\rangle\in P, i\in N_0, j\in\{C':j\neq i\}$, $k\in\{N_+:k\neq i,k\neq j,\tau'_{ij}+\tau'_{jk}\leq e\}$	无人机飞行路径集合
$\tau'_{ij}/\tau_{ij}, i\in N_0, j\in N_+, i\neq j, \tau_{0,c+1}\equiv 0$[a]	弧 $\langle i,j\rangle$ 的飞行/行驶时间成本
s_L/s_R	无人机发射/回收耗时
e	无人机续航时长，以单位时间来衡量
$x_{ij}\in\{0,1\}, i\in N_0, j\in N_+, i\neq j$	卡车路由决策变量
$y_{ijk}\in\{0,1\}, i\in N_0, j\in C, k\in\{N_+:\langle i,j,k\rangle\in P\}$	无人机路由决策变量
$t'_i/t_i\geq 0, i\in N_+, t'_0=t_0=0$	无人机/卡车有效到达时间戳辅助变量
$p_{ij}\in\{0,1\}$[b], $p_{0j}=1,\forall j\in C$	卡车访问次序先后辅助变量（为了确保无人机连续的 sortie 和卡车访问的顺序一致[c]）
$1\leq u_i\leq c+2, i\in N_+$	卡车破子圈辅助变量（和标准 TSP 的 MTZ 形式破子圈辅助变量类似[d]）

[a] 出于完备性的考虑，当只有一个顾客节点的时候，这个顾客将由无人机从仓库直接起飞进行服务。
[b] 当卡车访问顾客节点 $j\in\{C:j\neq i\}$ 时，顾客节点 $i\in C$ 已经在之前的某个时间点被卡车访问过了，则 $p_{ij}=1$。
[c] 当顾客节点 i 或者 j 仅被无人机服务时，p_{ij} 的取值就不重要。
[d] u_i 表示顾客点 i 在卡车访问的路径中的次序，比如 $u_5=1$ 表示顾客点 $i=5$ 是卡车访问路径中的第 1 个节点，但是不同于 TSP，在 FSTSP 中需要通过约束将无人机服务的顾客点 i 排除在外。

1.2 跨页表格

表 3: Symmetric Traveling Salesman Problem Examples

数据名称	城市数量	距离计算方式	最优值
a280	280	EUC_2D	2579
berlin52	52	EUC_2D	7542
bier127	127	EUC_2D	118282

表 3(续)

数据名称	城市数量	距离计算方式	最优值
ch130	130	EUC_2D	6110
ch150	150	EUC_2D	6528
d198	198	EUC_2D	15780
d493	493	EUC_2D	35002
d657	657	EUC_2D	48912
eil51	51	EUC_2D	426
eil76	76	EUC_2D	538
eil101	101	EUC_2D	629
fl417	417	EUC_2D	11861
gil262	262	EUC_2D	2378
kroA100	100	EUC_2D	21282
kroB100	100	EUC_2D	22141
kroC100	100	EUC_2D	20749
kroD100	100	EUC_2D	21294
kroE100	100	EUC_2D	22068
kroA150	150	EUC_2D	26524
kroB150	150	EUC_2D	26130
kroA200	200	EUC_2D	29368
kroB200	200	EUC_2D	29437
lin105	105	EUC_2D	14379
lin318	318	EUC_2D	42029
linhp318	318	EUC_2D	41345
p654	654	EUC_2D	34643
pcb442	442	EUC_2D	50778
pr76	76	EUC_2D	108159
pr107	107	EUC_2D	44303
pr124	124	EUC_2D	59030
pr136	136	EUC_2D	96772
pr144	144	EUC_2D	58537
pr152	152	EUC_2D	73682
pr226	226	EUC_2D	80369
pr264	264	EUC_2D	49135
pr299	299	EUC_2D	48191
pr439	439	EUC_2D	107217
rat99	99	EUC_2D	1211
rat195	195	EUC_2D	2323
rat575	575	EUC_2D	6773
rat783	783	EUC_2D	8806
rd100	100	EUC_2D	7910
rd400	400	EUC_2D	15281
st70	70	EUC_2D	675

表 3(续)

数据名称	城市数量	距离计算方式	最优值
ts225	225	EUC_2D	126643
tsp225	225	EUC_2D	3919
u159	159	EUC_2D	42080
u574	574	EUC_2D	36905
u724	724	EUC_2D	41910

1.3 单元格内换行

<table>
<tr><td>双行
表头</td><td>双行
表头</td></tr>
<tr><td rowspan="2">简单
粗暴</td><td>ABCD
EF</td></tr>
<tr><td>更大的竖直空距</td></tr>
</table>

1.4 跨行跨列表格

<table>
<tr><td rowspan="2">A Text!</td><td>ABC</td><td>DEF</td></tr>
<tr><td>abc</td><td>def</td></tr>
<tr><td colspan="2">Nothing</td><td>XYZ
xyz</td></tr>
</table>

1.5 嵌套表格

<table>
<tr><td>a</td><td colspan="2">bbb</td><td>c</td></tr>
<tr><td rowspan="2">a</td><td>a</td><td>b</td><td rowspan="2">c</td></tr>
<tr><td>aa</td><td>bb</td></tr>
<tr><td>a</td><td colspan="2">b</td><td>c</td></tr>
</table>

2 参考文献、数学公式

2.1 Traveling Salesman Problem

旅行商问题（Traveling Salesman Problem, TSP）是组合优化领域的经典问题之一，其核心目标是给定城市列表和每对城市之间的距离，求恰好访问每个城市一次并返回起始城市的最短可能路线。该问题于 1930 年正式提出，是优化中研究最深入的问题之一，被用作许多优化方法的基准。自从该问题被正式提出以来，一直是运筹学、计算机科学和物流管理等领域的研究热点，尽管该问题在计算上很困难，但许多启发式方法和精确算法是已知的[1-2]。

TSP 可以表述为整数线性规划模型[3]：假设共有 N 个城市，每个城市的编号为 $1, \cdots, N$，从城市 i 到城市 j 的旅行成本（距离）为 $c_{ij} > 0$。旅行商的目标是从任意一个城市出发访问完所有的城市，每个城市只能访问一次，最后回到最初的城市，目标是找到一条依次访问所有城市且访问城市不重

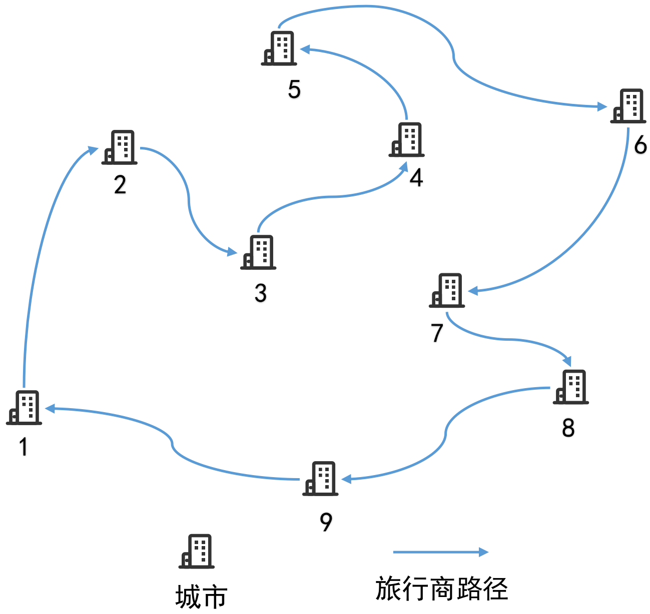

图 1: TSP 示意图

复的最短路线。TSP 中的决策变量为 $x_{ij} = \begin{cases} 1, & \text{存在从城市 } i \text{ 到城市 } j \text{ 的路径} \\ 0, & \text{其他} \end{cases}$，城市节点集合表示为 $V(|V| = N)$。由于可能存在子回路，所以在构建 TSP 模型时需要消除会产生子回路的情况，这里采用 Miller-Tucker-Zemlin (MTZ) 约束进行子回路的消除[4]，引入连续变量 $u_i(\forall i \in V, u_i \geq 0)$，其取值可以为任何非负实数（实数集合表示为 R）。这里用 u_i 表示编号为 i 的城市的访问次序，比如当 $u_i = 5$ 时表示编号为 1 的城市是从出发点开始，第 5 个被访问到的点。因此，TSP 的数学模型可以表示为 (1)-(6)。

$$
\begin{aligned}
\min \quad & \sum_{i \in V} \sum_{j \in V, i \neq j} c_{ij} x_{ij} && && (1) \\
\text{s.t.} \quad & \sum_{i \in V} x_{ij} = 1, && \forall j \in V, i \neq j && (2) \\
& \sum_{j \in V} x_{ij} = 1, && \forall i \in V, i \neq j && (3) \\
& u_i - u_j + N x_{ij} \leq N - 1, && \forall i, j \in V; i \neq j && (4) \\
& u_i \geq 0, && u_i \in R && (5) \\
& x_{ij} \in \{0, 1\}, && i, j \in V; i \neq j && (6)
\end{aligned}
$$

目标函数(1)表示最小化访问所有城市的成本（距离），约束(2)和(3)保证每个城市节点的入度和出度为 1，即每个城市只进入一次和出去一次，保证了每个城市只访问一次，不会被重复访问，约束(4)消除子回路，约束(5)和(6)表示变量的取值范围。

3 颜色

3.1 颜色名称测试（dvipsnames, svgnates, x11names）

- Red (dvipsname)
- DeepSkyBlue2 (x11names)
- ForestGreen (dvipsname)
- DarkOrange1 (x11names)

3.2 自定义颜色示例

这是自定义的蓝色文本。

这是自定义的黄色文本。

3.3 颜色混合示例

混合 50% 蓝色和 50% 红色

混合 60% 绿色和 40% 白色

混合 30% 橙色、70% 黄色

3.4 背景色示例

这是 SkyBlue 背景色的文本。

图 2: 图片示例

(a) 子图 A

(b) 子图 B

(c) 子图 C

(d) 子图 D

图 3: 多张子图排列示例 1

(a) 子图 1

(b) 子图 2

(c) 子图 3

图 4: 多张子图排列示例 2

4 插入图片及子图

4.1 插入图片

4.2 子图排列

5 伪代码

Algorithm 1: what

Input: This is some input
Output: This is some output
/ This is a comment */*

some code here
$x \leftarrow 0$
$y \leftarrow 0$
if $x > 5$ **then**
    x is greater than 5 *// This is also a comment*
else
    x is less than or equal to 5
end if
foreach y *in 0..5* **do**
    $y \leftarrow y + 1$
end foreach
for y *in* 0..5 **do**
    $y \leftarrow y - 1$
end for
while $x > 5$ **do**
    $x \leftarrow x - 1$
end while
return Return something here

6 代码环境

6.1 多行代码

Python 代码：

```python
def hello_en():
    print("Hello, world!")

def hello_cn():
    print(" 你好，世界！ ")

hello_en()
hello_cn()
```

C++ 代码：

```cpp
#include <iostream>
#include <vector>
#include <string>

using namespace std;

void getNext(int *next, const string &p) {
    int m = p.size(), j = 0; // j is the length of the previous longest prefix suffix
    next[0] = 0; // The first character has no proper prefix or suffix
    for (int i = 1; i < m; ++i) { // Start from the second character
        // Check if the j > 0 because we need to index to the previous next value
        while (j > 0 && p[i] != p[j]) { // Backtrack if there is a mismatch
            j = next[j - 1]; // j - 1 because next is 0-indexed
        }
        if (p[i] == p[j]) { // If characters match, increment j, namely the length of the current prefix
            ++j;
        }
        next[i] = j; // Set the next value for the current character
    }
}

int kmp(const string &s, const string &p) {
    int n = s.size(), m = p.size();
    if (n < m) { // Check if the pattern is longer than the text
        cout << "Pattern is longer than text." << endl;
        return -1;
    }
    if (m == 0) { // Check if the pattern is empty
        cout << "Empty pattern." << endl;
        return -1;
    }
    vector<int> next(m);
    getNext(&next[0], p); // Initialize the next array for the pattern p
    for (int i = 0, j = 0; i < n; ++i) { // i is the index in the main string s and j is the index in the pattern p
        while (j > 0 && s[i] != p[j]) { // Mismatch after j matches
            j = next[j - 1]; // Use the next array to skip unnecessary comparisons, minus 1 because next is 0-indexed
        }
        if (s[i] == p[j]) { // Match found and increment j to check next character
            ++j;
        }
        if (j == m) { // If j equals the length of the pattern, the first occurrence is found
            cout << "Pattern found at index: " << i + 1 - m << endl;
            return 0; // Return after finding the first occurrence
        }
    }
    return -1; // If no match is found, return -1
}

int main() {
    string s, p;
    cin >> s >> p;
    cout << "The main string is: " << s << endl;
    cout << "The pattern string is: " << p << endl;
    int result = kmp(s, p);
    if (result == -1) {
        cout << "Pattern not found." << endl;
    } else {
        cout << "Pattern found successfully." << endl;
    }
```

```
    return 0;
}
```

6.2 行内代码

This is an example of minted in the same line `print("Hello world!")`.

7 编辑体验

7.1 超链接

The url (outer link) color is just like this. The inner link color is just like this one → 7.1. The citation is like this[1].

7.2 脚注

This is a footnote example[1].

7.3 边注

Such a marginpar is like that. This is after the marginpar. A marginpar. Hello world!

7.4 智能交叉引用

这是一个数学公式引用：公式 3，这是一个图片引用：图 3a，这是一个表格引用：表 1。

8 Section example

8.1 Subsection example

This is a subsection example.

8.1.1 Subsubsection example

This is a subsubsection example.

Unnumbered section example

Unnumbered subsection example

This is a unnumbered subsection example.

Unnumbered subsubsection example

This is a unnumbered subsubsection example.

[1] Related footnote is here.

9 中文字体测试

9.1 中文主字体（LXGWWenKaiScreen）

这是默认的中文字体（加粗和*斜体*分别为方正黑体和方正楷体）。

加粗文本示例：这是一段加粗的中文文本。

斜体文本示例：这是一段斜体的中文文本。

9.2 中文无衬线字体（FZHTJW）

这是无衬线字体（**加粗**效果由 AutoFake 生成，*斜体*为方正仿宋）。

加粗文本示例：这是一段加粗的无衬线中文文本。

斜体文本示例：这是一段斜体的无衬线中文文本。

9.3 等宽字体（LXGWWenKaiMonoScreen）

这是等宽字体（**加粗**效果由 `AutoFake` 生成，*斜体*为方正书宋）。

加粗文本示例：这是一段加粗的等宽中文文本。

斜体文本示例：这是一段斜体的等宽中文文本。

9.4 混合字体测试

主字体（**LXGWWenKaiScreen**）

无衬线字体（FZHTJW）

等宽字体（`LXGWWenKaiMonoScreen`）

主字体和**无衬线**的切换效果。

等宽字体与**普通字体**的对比。

10 English Font Test

10.1 English Main Font (Times New Roman)

This is the default English font (serif).

Bold and *Italic* effects are generated automatically.

10.2 English Sans-serif Font (Arial)

This is the English sans-serif font.

Bold and *Italic* effects are generated automatically.

10.3 English Monospace Font (Consolas)

`This is the English monospace font.`

Bold and *Italic* effects are generated automatically.

10.4 中英文混合字体测试

Main Font: Times New Roman + **中文主字体:** LXGWWenKaiScreen.

Sans-serif Font: Arial + **中文无衬线字体**: FZHTJW.

Monospace Font: Consolas + **中文等宽字体:** LXGWWenKaiMonoScreen.

11 滕王阁序

豫章故郡，洪都新府。星分翼轸 (zhěn)，地接衡庐。襟三江而带五湖，控蛮荆而引瓯（ōu）越。物华天宝，龙光射牛斗之墟；人杰地灵，徐孺下陈蕃 (fān) 之榻。雄州雾列，俊采星驰，台隍 (huáng) 枕夷夏之交，宾主尽东南之美。都督阎公之雅望，棨 (qǐ) 戟遥临；宇文新州之懿 (yì) 范，襜 (chān) 帷 (wéi) 暂驻。十旬休假，胜友如云；千里逢迎，高朋满座。腾蛟起凤，孟学士之词宗；紫电清霜，王将军之武库。家君作宰，路出名区；童子何知，躬逢胜饯。

时维九月，序属三秋。潦（lǎo）水尽而寒潭清，烟光凝而暮山紫。俨 (yǎn) 骖騑 (cān fēi) 于上路，访风景于崇阿 (ē)。临帝子之长洲，得天人之旧馆。层峦耸翠，上出重霄；飞阁流（一作翔）丹，下临无地。鹤汀（tīng）凫 (fú) 渚（zhǔ），穷岛屿之萦 (yíng) 回；桂殿兰宫，即（一作列）冈峦之体势。

披绣闼（tà），俯雕甍 (méng)。山原旷其盈视，川泽纡 (yū) 其骇瞩。闾 (lǘ) 阎 (yán) 扑地，钟鸣鼎食之家；舸（gě）舰弥津，青雀黄龙之舳（zhú）。云销雨霁 (jì)，彩彻区明（或作虹销雨霁，彩彻云衢 qú）。落霞与孤鹜 (wù) 齐飞，秋水共长天一色。渔舟唱晚，响穷彭蠡（l ǐ）之滨；雁阵惊寒，声断衡阳之浦。

遥襟甫畅，逸兴遄 (chuán) 飞。爽籁发而清风生，纤歌凝而白云遏 (è)。睢 (suī) 园绿竹，气凌彭泽之樽；邺 (yè) 水朱华，光照临川之笔。四美具，二难并。穷睇眄 (dì miǎn) 于中天，极娱游于暇日。天高地迥 (jiǒng)，觉宇宙之无穷；兴尽悲来，识盈虚之有数。望长安于日下，目吴会（kuài）于云间。地势极而南溟 (míng) 深，天柱高而北辰远。关山难越，谁悲失路之人；萍水相逢，尽是他乡之客。怀帝阍 (hūn) 而不见，奉宣室以何年。

嗟 (jiē) 乎！时运不齐，命途多舛 (chuǎn)；冯唐易老，李广难封。屈贾谊（yì）于长沙，非无圣主；窜梁鸿于海曲，岂乏明时？所赖君子见机，达人知命。老当益壮，宁移白首之心？穷且益坚，不坠青云之志。酌贪泉而觉爽，处涸辙（hé zhé）以犹欢。北海虽赊（shē），扶摇可接；东隅 (yú) 已逝，桑榆非晚。孟尝高洁，空余报国之情；阮籍猖狂，岂效穷途之哭！

勃，三尺微命，一介书生。无路请缨，等终军之弱冠（guàn）；有怀投笔，慕宗悫（què）之长风。舍簪（zān）笏（hù）于百龄，奉晨昏于万里。非谢家之宝树，接孟氏之芳邻。他日趋庭，叨 (tāo) 陪鲤对；今兹捧袂 (mèi)，喜托龙门。杨意不逢，抚凌云而自惜；钟期既遇，奏流水以何惭？

呜呼！胜地不常，盛筵 (yán) 难再；兰亭已矣，梓 (zǐ) 泽丘墟。临别赠言，幸承恩于伟饯；登高作赋，是所望于群公。敢竭鄙怀，恭疏短引；一言均赋，四韵俱成。请洒潘江，各倾陆海云尔。

滕王高阁临江渚，佩玉鸣鸾罢歌舞。

画栋朝飞南浦云，珠帘暮卷西山雨。

闲云潭影日悠悠，物换星移几度秋。

阁中帝子今何在？槛外长江空自流。

12 Do Not Go Gentle into That Good Night

Do not go gentle into that good night,

Old age should burn and rave at close of day;

Rage, rage against the dying of the light.

Though wise men at their end know dark is right,

Because their words had forked no lightning they

Do not go gentle into that good night.

Good men, the last wave by, crying how bright

Their frail deeds might have danced in a green bay,

Rage, rage against the dying of the light.

Wild men who caught and sang the sun in flight,

And learn, too late, they grieved it on its way,

Do not go gentle into that good night.

Grave men, near death, who see with blinding sight

Blind eyes could blaze like meteors and be gay,

Rage, rage against the dying of the light.

And you, my father, there on the sad height,

Curse, bless, me now with your fierce tears, I pray.

Do not go gentle into that good night.

Rage, rage against the dying of the light.

13 Theorem, Proposition, Proof

Theorem 13.1. *If $1 < p < \infty$ and $m > n/p$, or $p = 1$ and $m \geq n$, there exist a constant $C = C(m, n, \gamma, p)$, such that*

$$\|R^m u\|_{L^\infty(\Omega)} \leq C d^{m-n/p} |u|_{W_p^m(\Omega)}$$

for all $u \in W_p^m(\Omega)$.

Proof. First, we assume that $u \in C^m(\Omega) \cap W_p^m(\Omega)$. We can use the pointwise representation of $R^m u(x)$.

$$\begin{aligned}|R^m u(x)| &= m\left|\sum_{|\alpha|=m}\int_{C_x} k_\alpha(x,z)D^\alpha u(z)\,dz\right| \\ &\le C\sum_{|\alpha|=m}\int_\Omega |x-z|^{-n+m}|D^\alpha u(z)|\,dz \\ &\le C' d^{m-n/p}|u|_{W_p^m(\Omega)}.\end{aligned}$$

The proof can be completed via a density argument. □

Theorem 13.2 (xxx). *If $1 < p < \infty$ and $m > n/p$, or $p = 1$ and $m \ge n$, there exist a constant $C = C(m,n,\gamma,p)$, such that*

$$\|R^m u\|_{L^\infty(\Omega)} \le C d^{m-n/p}|u|_{W_p^m(\Omega)}$$

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The proof can be completed via a density argument. □

Proposition 13.3.

$$Q^m u(x) = \sum_{|\lambda|<m}\left(\int_B \psi_\lambda(y)u(y)\,dy\right)x^\lambda$$

where $\psi_\lambda \in C_0^\infty(\mathbb{R}^n)$ *and* $\operatorname{supp}(\phi_\lambda) \in \overline{B}$.

Proof. This follows from xxx if we define

$$\psi_\lambda(y) = \sum_{\alpha \geq \lambda, |\alpha| < m} \frac{(-1)^{|\alpha|}}{\alpha!} a_{[\lambda, \alpha - \lambda]} D^\alpha (y^{\alpha - \lambda} \phi(y)).$$

□

Proposition 13.4 (xxx).

$$Q^m u(x) = \sum_{|\lambda| < m} \left(\int_B \psi_\lambda(y) u(y)\, dy \right) x^\lambda$$

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□

14 Corollary, Lemma, Claim

Corollary 14.1. *Under the assumption of xxx, the following inequality holds*

$$\inf_{v \in P^{m-1}} \|u - v\|_{W_p^k(\Omega)} \leq C_{m,n,\gamma} d^{m-k} |u|_{W_p^k(\Omega)}, \quad k = 0, 1, \ldots, m,$$

Corollary 14.2 (xxx). *Under the assumption of xxx, the following inequality holds*

$$\inf_{v \in P^{m-1}} \|u - v\|_{W_p^k(\Omega)} \leq C_{m,n,\gamma} d^{m-k} |u|_{W_p^k(\Omega)}, \quad k = 0, 1, \ldots, m,$$

Corollary. *Under the assumption of xxx, the following inequality holds*

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Lemma 14.3. *Let $f \in L^p(\Omega)$ for $p \geq 1$ and $m \geq 1$ and let*

$$g(x) = \int_\Omega |x - z|^{-n+m} |f(z)|\, dz$$

Then

$$\|g\|_{L^p(\Omega)} \leq C_{m,n} d^m \|f\|_{L^p(\Omega)}.$$

Lemma 14.4 (xxx)**.** *Let $f \in L^p(\Omega)$ for $p \geq 1$ and $m \geq 1$ and let*

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Lemma. *Let $f \in L^p(\Omega)$ for $p \geq 1$ and $m \geq 1$ and let*

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Then

$$\|g\|_{L^p(\Omega)} \leq C_{m,n} d^m \|f\|_{L^p(\Omega)}.$$

Claim 14.5. *$Q^m u$ is a polynomial of degree less than m in x.*

Claim 14.6 (xxx)**.** *$Q^m u$ is a polynomial of degree less than m in x.*

Claim. *$Q^m u$ is a polynomial of degree less than m in x.*

15 Definition

Definition 15.1. Ω is star-shaped with respect to the ball B if , for all $x \in \Omega$, the closed convex hull of $\{x\} \cup B$ is a subset of Ω.

Definition 15.2 (xxx)**.** Ω is star-shaped with respect to the ball B if , for all $x \in \Omega$, the closed convex hull of $\{x\} \cup B$ is a subset of Ω.

Definition. Ω is star-shaped with respect to the ball B if , for all $x \in \Omega$, the closed convex hull of $\{x\} \cup B$ is a subset of Ω.

16 Example

Example 16.1. The integral form of the Taylor remainder for $f \in C^m([0,1])$ is given by

$$f(s) = \sum_{k=0}^{m-1} \frac{1}{k!} f^{(k)}(0) + \int_0^s \frac{1}{(m-1)!} f^{(m)}(t)(s-t)^{m-1}\, dt.$$

Example 16.2 (xxx)**.** The integral form of the Taylor remainder for $f \in C^m([0,1])$ is given by

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17 Problem, Solution

Problem 17.1. Calculate the integral of the function $g(x) = 3x^2$ with respect to x.

Solution 17.1. To calculate the integral of $g(x) = 3x^2$, we use the power rule for integration:

$$\int 3x^2\, dx = x^3 + C$$

where C is the constant of integration. □

Problem 17.2 (xxx)**.** Calculate the integral of the function $g(x) = 3x^2$ with respect to x.

Solution 17.2 (xxx)**.** To calculate the integral of $g(x) = 3x^2$, we use the power rule for integration:

$$\int 3x^2\, dx = x^3 + C$$

where C is the constant of integration. □

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Solution. To calculate the integral of $g(x) = 3x^2$, we use the power rule for integration:

$$\int 3x^2\, dx = x^3 + C$$

where C is the constant of integration. □

18 Remark

Remark 18.1. Such a polynomial is not unique, due to the choice od cut-off function ϕ.

Remark 18.2 (xxx). Such a polynomial is not unique, due to the choice od cut-off function ϕ.

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19 Note

Note 19.1. The degree of $Q^m u$ is at most $m - 1$.

Note 19.2 (xxx). The degree of $Q^m u$ is at most $m - 1$.

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