$$\int \frac{mx+n}{ax^2+bx+c}\,\mathrm{d}x\ (a\neq 0)$$

$$=\int \frac{\frac{m}{a}x+\frac{n}{a}}{x^2+\frac{b}{a}x+\frac{c}{a}}\,\mathrm{d}x$$

$$=\int \frac{\frac{m}{a}x+\frac{n}{a}}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$=\frac{m}{a}\int \frac{x}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x+\frac{n}{a}\int \frac{1}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$=\frac{m}{2a}\int \frac{2x+\frac{b}{a}-\frac{b}{a}}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x+\frac{n}{a}\int \frac{1}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$=\frac{m}{2a}\int \frac{2x+\frac{b}{a}}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x+\frac{2na-mb}{2a^2}\int \frac{1}{\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$=\frac{m}{2a}\int \frac{1}{x^2+\frac{b}{a}x+\frac{c}{a}}\,\mathrm{d}\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)+\frac{2na-mb}{2a^2}\int \frac{1}{\left(x+\frac{b}{2a}\right)^2+\left(\sqrt{\frac{4ac-b^2}{4a^2}}\right)^2}\,\mathrm{d}\left(x+\frac{b}{2a}\right)$$

$$=\frac{m}{2a}\ln|x^2+\frac{b}{a}x+\frac{c}{a}|+\frac{2na-mb}{2a^2}\frac{1}{\sqrt{\frac{4ac-b^2}{4a^2}}}\arctan\frac{x+\frac{b}{2a}}{\sqrt{\frac{4ac-b^2}{4a^2}}}+C$$

$$=\frac{m}{2a}\ln|ax^2+bx+c|+\frac{2na-mb}{a\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}}+C$$