$$\int \frac{mx+n}{ax^2+bx+c}\,\mathrm{d}x \quad (a\neq 0)$$

$$= \int \frac{\frac{m}{a}x+\frac{n}{a}}{x^2+\frac{b}{a}x+\frac{c}{a}}\,\mathrm{d}x$$

$$= \int \frac{\frac{m}{a}x+\frac{n}{a}}{(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$= \frac{m}{a}\int \frac{x}{(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x + \frac{n}{a}\int \frac{1}{(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$= \frac{m}{2a}\int \frac{2x+\frac{b}{a}-\frac{b}{a}}{(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x + \frac{n}{a}\int \frac{1}{(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$= \frac{m}{2a}\int \frac{2x+\frac{b}{a}}{(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x + \frac{2na-mb}{2a^2}\int \frac{1}{(x+\frac{b}{2a})^2+\frac{c}{a}-\frac{b^2}{4a^2}}\,\mathrm{d}x$$

$$= \frac{m}{2a}\int \frac{1}{x^2+\frac{b}{a}x+\frac{c}{a}}\,\mathrm{d}(x^2+\frac{b}{a}x+\frac{c}{a}) + \frac{2na-mb}{2a^2}\int \frac{1}{(x+\frac{b}{2a})^2+(\sqrt{\frac{4ac-b^2}{4a^2}})^2}\,\mathrm{d}(x+\frac{b}{2a})$$

$$= \frac{m}{2a}\ln|x^2+\frac{b}{a}x+\frac{c}{a}| + \frac{2na-mb}{2a^2}\frac{1}{\sqrt{\frac{4ac-b^2}{4a^2}}}\arctan\frac{x+\frac{b}{2a}}{\sqrt{\frac{4ac-b^2}{4a^2}}} + C$$

$$= \frac{m}{2a}\ln|ax^2+bx+c| + \frac{2na-mb}{a\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} + C$$