

$$\begin{aligned}
\Gamma(s) &= \int_0^{+\infty} e^{-x} x^{s-1} dx \quad (s > 0) \\
&= \int_0^{+\infty} -x^{s-1} d(e^{-x}) \\
&= 0 - \int_0^{+\infty} e^{-x} d(-x^{s-1}) \\
&= \int_0^{+\infty} e^{-x} d(x^{s-1}) \\
&= \int_0^{+\infty} e^{-x} (s-1) x^{s-2} dx \\
&= (s-1) \int_0^{+\infty} e^{-x} x^{s-2} dx \\
&= (s-1) \Gamma(s-1) \\
&= (s-1)(s-2) \Gamma(s-2) \\
&= (s-1)(s-2) \cdots \Gamma(1) \\
&= (s-1)(s-2) \cdots 1 \\
&= (s-1)!
\end{aligned}$$