$$\lim_{x\to 0}\left(\frac{a_1^x+a_2^x+a_3^x+\cdots+a_n^x}{n}\right)^{\frac{1}{x}}$$

$$=\lim_{x\to 0} e^{\left(\frac{1}{x}\ln\frac{a_1^x+a_2^x+a_3^x+\cdots+a_n^x}{n}\right)}$$

$$=\lim_{x\to 0} e^{\left(\frac{1}{x}\ln(1+\frac{a_1^x+a_2^x+a_3^x+\cdots+a_n^x-n}{n})\right)}$$

$$=\lim_{x\to 0} e^{\left(\frac{1}{x}(\frac{a_1^x+a_2^x+a_3^x+\cdots+a_n^x-n}{n})\right)}$$

$$=\lim_{x\to 0} e^{\left(\frac{a_1^x+a_2^x+a_3^x+\cdots+a_n^x-n}{nx}\right)}$$

$$=\lim_{x\to 0} e^{\left(\frac{a_1^x\ln a_1+a_2^x\ln a_2+a_3^x\ln a_3+\cdots+a_n^x\ln a_n}{n}\right)}$$

$$=e^{\left(\frac{\ln a_1+\ln a_2+\ln a_3+\cdots+\ln a_n}{n}\right)}$$

$$=e^{\left(\frac{1}{n}\ln(a_1a_2a_3\cdots a_n)\right)}$$

$$=(a_1a_2a_3\cdots a_n)^{\frac{1}{n}}$$