$$
\begin{aligned}
\Gamma(s) &= \int_0^{+\infty} e^{-x}x^{s-1}\ \mathrm{d}x \quad (s>0) \\
&= \int_0^{+\infty} -x^{s-1}\ \mathrm{d}(e^{-x}) \\
&= 0 - \int_0^{+\infty} e^{-x}\ \mathrm{d}(-x^{s-1}) \\
&= \int_0^{+\infty} e^{-x}\ \mathrm{d}(x^{s-1}) \\
&= \int_0^{+\infty} e^{-x}(s-1)x^{s-2}\ \mathrm{d}x \\
&= (s-1)\int_0^{+\infty} e^{-x}x^{s-2}\ \mathrm{d}x \\
&= (s-1)\Gamma(s-1) \\
&= (s-1)(s-2)\Gamma(s-2) \\
&= (s-1)(s-2)\cdots\Gamma(1) \\
&= (s-1)(s-2)\cdots 1 \\
&= (s-1)!
\end{aligned}
$$