$$\lim_{x\to\infty}\left(\frac{ax+b}{ax+c}\right)^{hx+k} \quad (a\neq 0)$$

$$= \lim_{x\to\infty} e^{(hx+k)\ln\left(\frac{ax+b}{ax+c}\right)}$$

$$= \lim_{x\to\infty} e^{(hx+k)\ln\left(\frac{ax+b+c-c}{ax+c}\right)}$$

$$= \lim_{x\to\infty} e^{(hx+k)\ln\left(1+\frac{b-c}{ax+c}\right)}$$

$$= \lim_{x\to\infty} e^{(hx+k)\frac{b-c}{ax+c}}$$

$$= \lim_{x\to\infty} e^{\frac{(b-c)hx+(b-c)k}{ax+c}}$$

$$= e^{\frac{(b-c)h}{a}}$$