$$\int \sqrt{1+x^2}\,\mathrm{d}x \ (\text{let } x=\tan t)$$

$$= I$$

$$= \int \sec t\,\mathrm{d}\tan t$$

$$= \sec t\tan t - \int \tan t\,\mathrm{d}\sec t$$

$$= \sec t\tan t - \int \tan^2 t\sec t\,\mathrm{d}t$$

$$= \sec t\tan t - \int (\sec^2 t - 1)\sec t\,\mathrm{d}t$$

$$= \sec t\tan t - \int \sec^3 t\,\mathrm{d}t + \int \sec t\,\mathrm{d}t$$

$$= \sec t\tan t - I + \ln|\sec t + \tan t| + C$$

$$\Rightarrow I = \frac{1}{2}(\sec t\tan t + \ln|\sec t + \tan t|) + C$$

$$= \frac{1}{2}\left(x\sqrt{1+x^2} + \ln\left(x + \sqrt{1+x^2}\right)\right) + C$$