Calculate $u = a + bi$ to the $v = c + di$ power: $z = u^v$.

$$
\begin{aligned}
z &= u^v \\
&= (a + bi)^{c+di} \\
&\quad \text{let } m = \sqrt{a^2 + b^2} = |u|,\ w = \arctan\left(\frac{b}{a}\right) = \arg(u) \\
&= \left(me^{iw}\right)^{c+di} \\
&= \left(e^{\ln m} e^{iw}\right)^{c+di} \\
&= e^{(\ln m + iw)(c+di)} \\
&= e^{(c \ln m - dw) + i(cw + d \ln m)} \\
&\quad \text{let } \theta = cw + d \ln m \\
&= e^{(c \ln m - dw) + i\theta} \\
&= e^{c \ln m - dw} e^{i\theta} \\
&= e^{c \ln m - dw} (\cos\theta + i \sin\theta) \\
&= \left(m^c e^{-dw}\right)(\cos\theta + i \sin\theta) \\
&\quad \text{let } r = m^c e^{-dw} \\
&= r(\cos\theta + i \sin\theta) \\
&= \underbrace{r\cos\theta}_{\text{real part}} + \underbrace{r\sin\theta}_{\text{imag part}}\ i \\
&\quad r = |u|^c e^{-d \arg(u)},\ \theta = c \arg(u) + d \ln|u|
\end{aligned}
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