

$$\int \sqrt{1+x^2} \, dx$$

$$\text{let } x = \tan t$$

$$= I$$

$$= \int \sec t \, d \tan t$$

$$= \sec t \tan t - \int \tan t \, d \sec t$$

$$= \sec t \tan t - \int \tan^2 t \sec t \, dt$$

$$= \sec t \tan t - \int (\sec^2 t - 1) \sec t \, dt$$

$$= \sec t \tan t - \int \sec^3 t \, dt + \int \sec t \, dt$$

$$= \sec t \tan t - I + \ln |\sec t + \tan t| + C$$

$$\Rightarrow I = \frac{1}{2}(\sec t \tan t + \ln |\sec t + \tan t|) + C$$

$$= \frac{1}{2}(x\sqrt{1+x^2} + \ln(x + \sqrt{1+x^2})) + C$$