

Calculate $u = a + bi$ to the $v = c + di$ power: $z = u^v$.

$$z = u^v$$

$$= (a + bi)^{c+di}$$

$$\text{let } m = \sqrt{a^2 + b^2} = |u|, w = \arctan\left(\frac{b}{a}\right) = \arg(u)$$

$$= (me^{iw})^{c+di}$$

$$= (e^{\ln m} e^{iw})^{c+di}$$

$$= e^{(\ln m + iw)(c+di)}$$

$$= e^{(c \ln m - dw) + i(cw + d \ln m)}$$

$$\text{let } \theta = cw + d \ln m$$

$$= e^{(c \ln m - dw) + i\theta}$$

$$= e^{c \ln m - dw} e^{i\theta}$$

$$= e^{c \ln m - dw} (\cos \theta + i \sin \theta)$$

$$= (m^c e^{-dw}) (\cos \theta + i \sin \theta)$$

$$\text{let } r = m^c e^{-dw}$$

$$= r(\cos \theta + i \sin \theta)$$

$$= \underbrace{r \cos \theta}_{\text{real part}} + \underbrace{r \sin \theta}_{\text{imag part}} i$$

$$r = |u|^c e^{-d \arg(u)}, \theta = c \arg(u) + d \ln |u|$$