Advanced Mathematics Notes

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Contents

1 Common Taylor Series

$$e^x = 1 + x + \frac{1}{2}x^2 + o(x^2) = \sum_{n=0}^{\infty} \frac{1}{n!}x^n \tag{1}$$

$$\sin x = x - \frac{1}{6}x^3 + o(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} \tag{2}$$

$$\xRightarrow{Derivation} \cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n} \tag{3}$$

$$\arcsin x = x + \frac{1}{6}x^3 + o(x^3) = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n(n!)^2(2n+1)}x^{2n+1} \tag{4}$$

$$\tan x = x + \frac{1}{3}x^3 + o(x^3) = \sum_{n=1}^{\infty} \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!}x^{2n-1} \tag{5}$$

$$\arctan x = x - \frac{1}{3}x^3 + o(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}x^{2n+1} \tag{6}$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + o(x^2) = \sum_{n=0}^{\infty} C_\alpha^n x^n \tag{7}$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + o(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}x^{n+1} \tag{8}$$

$$\xRightarrow{Derivation} \frac{1}{1+x} = 1 - x + x^2 + o(x^2) = \sum_{n=0}^{\infty} (-1)^n x^n \tag{9}$$

$$\xRightarrow{x \to (-x)} \frac{1}{1-x} = 1 + x + x^2 + o(x^2) = \sum_{n=0}^{\infty} x^n \tag{10}$$