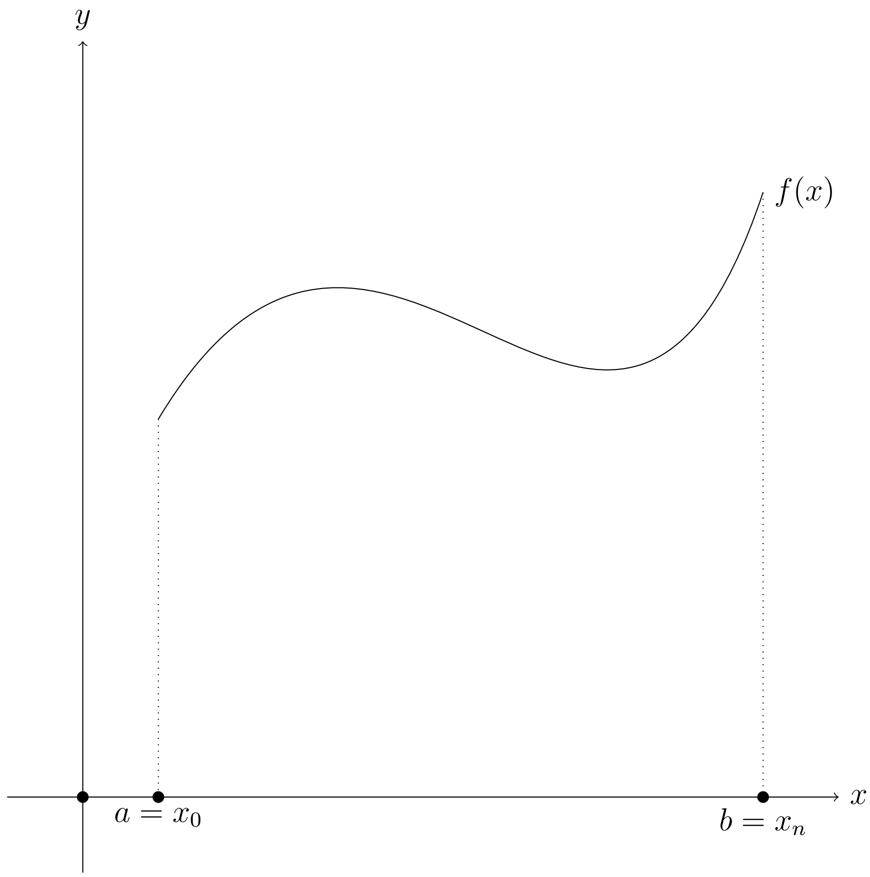

For each small rectangle, the Width is:

$$W_i = \frac{b-a}{n} \qquad \text{(Divided evenly into n parts)}$$

And Height is:

1. Take the right endpoint $x_i \quad (i = 1, 2, 3, \cdots, n)$

$$H_i = f(x_i) = f(a + i \cdot W) = f(a + i \cdot \frac{b-a}{n})$$

2. Take the left endpoint $x_{i-1} \quad (i = 1, 2, 3, \cdots, n)$

$$H_i = f(x_{i-1}) = f(a + (i-1) \cdot W) = f(a + (i-1) \cdot \frac{b-a}{n})$$

3. Take the middle point $c_i \quad (i = 1, 2, 3, \cdots, n)$

$$c_i = \frac{x_{i-1} + x_i}{2} = \frac{a + (i-1) \cdot \frac{b-a}{n} + a + i \cdot \frac{b-a}{n}}{2} = a + \frac{2i-1}{2} \cdot \frac{b-a}{n}$$

$$H_i = f(c_i) = f(a + \frac{2i-1}{2} \cdot \frac{b-a}{n})$$

So, the Area of the small rectangle:

$$\begin{aligned} A_i &= W_i \cdot H_i \quad (i = 1, 2, 3, \cdots, n) \\ &= \begin{cases} \frac{b-a}{n} \cdot f(a + i \cdot \frac{b-a}{n}) & \text{Take the right endpoint } x_i \\ \frac{b-a}{n} \cdot f(a + (i-1) \cdot \frac{b-a}{n}) & \text{Take the left endpoint } x_{i-1} \\ \frac{b-a}{n} \cdot f(a + \frac{2i-1}{2} \cdot \frac{b-a}{n}) & \text{Take the middle point } c_i \end{cases} \end{aligned}$$

So, the Definite Integral:

$$\int_a^b f(x)\ \mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

$$= \begin{cases} \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{b-a}{n} \cdot f(a + i \cdot \dfrac{b-a}{n}) & \text{Take the right endpoint } x_i \\ \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{b-a}{n} \cdot f(a + \dfrac{(i-1)(b-a)}{n}) & \text{Take the left endpoint } x_{i-1} \\ \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{b-a}{n} \cdot f(a + \dfrac{(2i-1)(b-a)}{2n}) & \text{Take the middle point } c_i \end{cases}$$

Specifically, when a = 0 and b = 1:

$$\int_0^1 f(x)\ \mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

$$= \begin{cases} \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1}{n} \cdot f(\dfrac{i}{n}) & \text{Take the right endpoint } x_i \\ \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1}{n} \cdot f(\dfrac{i-1}{n}) & \text{Take the left endpoint } x_{i-1} \\ \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1}{n} \cdot f(\dfrac{2i-1}{2n}) & \text{Take the middle point } c_i \end{cases}$$