$$\lim_{x\to\infty}\left(\frac{ax+b}{ax+c}\right)^{hx+k}\quad(a\neq 0)$$

$$=\lim_{x\to\infty}e^{(hx+k)\ln\left(\frac{ax+b}{ax+c}\right)}$$

$$=\lim_{x\to\infty}e^{(hx+k)\ln\left(\frac{ax+b+c-c}{ax+c}\right)}$$

$$=\lim_{x\to\infty}e^{(hx+k)\ln\left(1+\frac{b-c}{ax+c}\right)}$$

$$=\lim_{x\to\infty}e^{(hx+k)\frac{b-c}{ax+c}}$$

$$=\lim_{x\to\infty}e^{\frac{(b-c)hx+(b-c)k}{ax+c}}$$

$$=e^{\frac{(b-c)h}{a}}$$