Question:

$$\int_0^{+\infty} e^{-x^2}\ \mathrm{d}x$$

Answer:

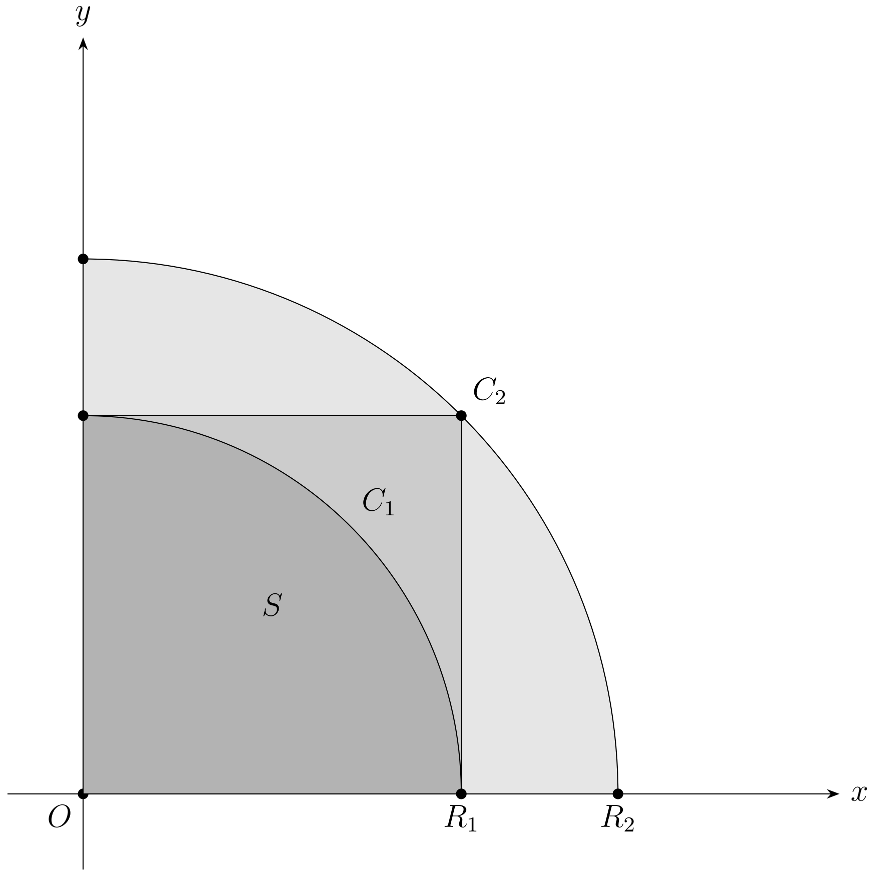

Figure 1: C_1: small circle. C_2: big circle. S: Square.

let

$$f(x,y) = e^{-x^2-y^2}$$

$because$

$$R_2 = \sqrt{2}R_1$$

$therefore$

$$\begin{aligned}
&\iint_{C_1} f(x,y)\ \mathrm{d}x\ \mathrm{d}y && \leq \iint_{S} f(x,y)\ \mathrm{d}x\ \mathrm{d}y && \leq \iint_{C_2} f(x,y)\ \mathrm{d}x\ \mathrm{d}y \\
&\iint_{C_1} e^{-x^2-y^2}\ \mathrm{d}x\ \mathrm{d}y && \leq \int_0^{R_1} e^{-x^2}\ \mathrm{d}x \cdot \int_0^{R_1} e^{-y^2}\ \mathrm{d}y && \leq \iint_{C_2} e^{-x^2-y^2}\ \mathrm{d}x\ \mathrm{d}y \\
&\frac{\pi}{4}(1-e^{-R_1^2}) && \leq \left(\int_0^{R_1} e^{-x^2}\ \mathrm{d}x\right)^2 && \leq \frac{\pi}{4}(1-e^{-2R_1^2}) \\
&\lim_{R_1\to+\infty}\frac{\pi}{4}(1-e^{-R_1^2}) && \leq \lim_{R_1\to+\infty}\left(\int_0^{R_1} e^{-x^2}\ \mathrm{d}x\right)^2 && \leq \lim_{R_1\to+\infty}\frac{\pi}{4}(1-e^{-2R_1^2}) \\
&\frac{\pi}{4} && \leq \left(\int_0^{+\infty} e^{-x^2}\ \mathrm{d}x\right)^2 && \leq \frac{\pi}{4}
\end{aligned}$$

$therefore$

$$\int_0^{+\infty} e^{-x^2}\ \mathrm{d}x = \frac{\sqrt{\pi}}{2}$$