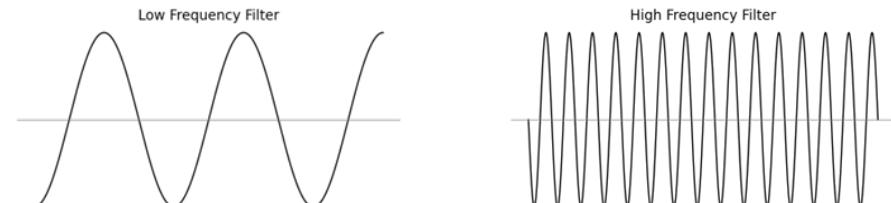
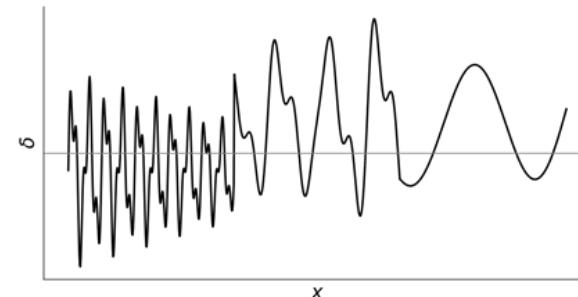


# Traditional Statistics using Fourier Transform

- **Traditional statistics**

- Measuring 2- or 3-point correlation function
- Computing Fourier transform of these n-point correlation functions (power spectrum, bispectrum) might be computationally expensive
- Gaining information in frequency space but no spatial information

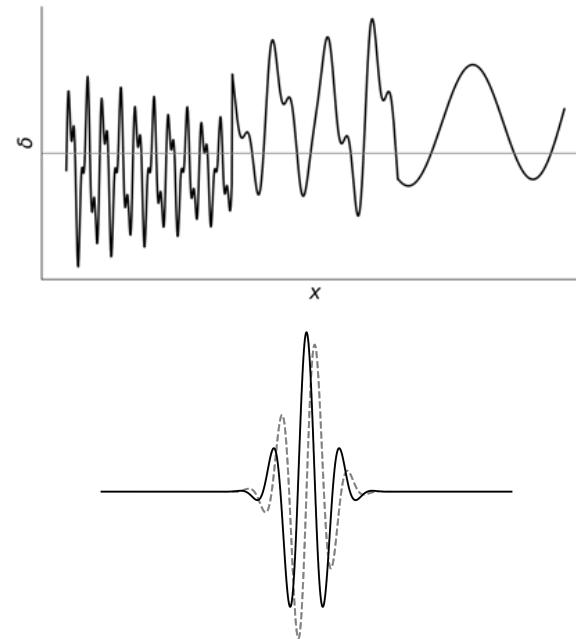


# Traditional Statistics using Fourier Transform

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- **Traditional statistics**

- Measuring 2- or 3-point correlation function
- Computing Fourier transform of these n-point correlation functions (power spectrum, bispectrum) might be computationally expensive
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# Scattering Transform

---

**Operation:**

$$S = \langle |I \star \psi^{j,l}| \rangle$$

I is cosmological field, such as convergence field (function of  $\Omega_m, \Omega_{DE}, \sigma_8 \dots$ )  
 S is scattering coefficient

**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**

(S. Cheng et al. 2020, A new approach to observational cosmology using the scattering transform)

# Scattering Transform

---

**Operation:**

$$S = \langle |I \star \psi^{j,l}| \rangle$$

**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**

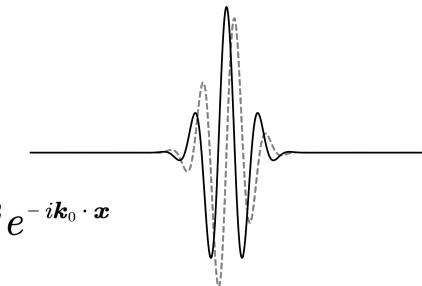
Wavelet: localized oscillating wave

Morlet wavelets:

In real space:

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\mathbf{x}^T \Sigma^{-1} \mathbf{x}/2} e^{-i\mathbf{k}_0 \cdot \mathbf{x}}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$



$\Sigma$  : the covariance of matrix describing the size and shape of the Gaussian envelope  
 $k_0$  : the frequency of the modulated oscillation

# Scattering Transform

**Operation:**

$$S = \langle |I \star \psi^{j,l}| \rangle$$

**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**

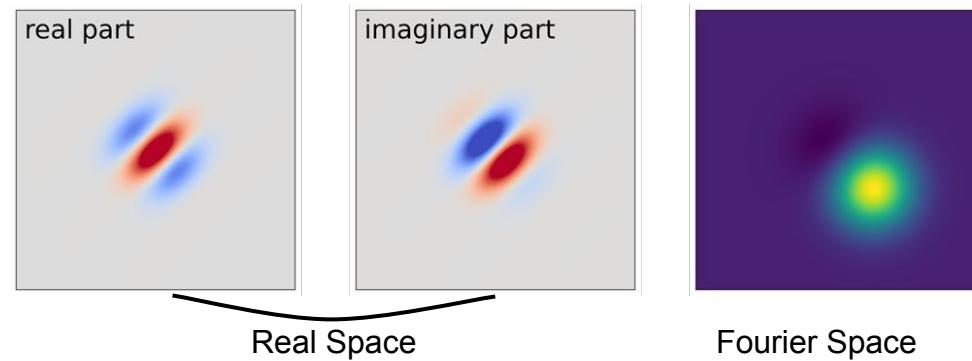
**Morlet Wavelets**

**In real space:**

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\mathbf{x}^T \Sigma^{-1} \mathbf{x}/2} e^{-i\mathbf{k}_0 \cdot \mathbf{x}}$$

**In Fourier space**

$$\tilde{\psi}(\mathbf{k}) = \frac{1}{\sqrt{\Sigma}} e^{-(\mathbf{k} - \mathbf{k}_0)^T \Sigma (\mathbf{k} - \mathbf{k}_0)/2}$$



# Scattering Transform

## Operation:

$$S = \langle |I \star \psi^{j,l}| \rangle$$

$L = 4, \ell = 0, 1, 2, 3$   
 •  $\ell$ : orientation

**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**

$$J = 8, j = 0, 1, \dots, 7$$

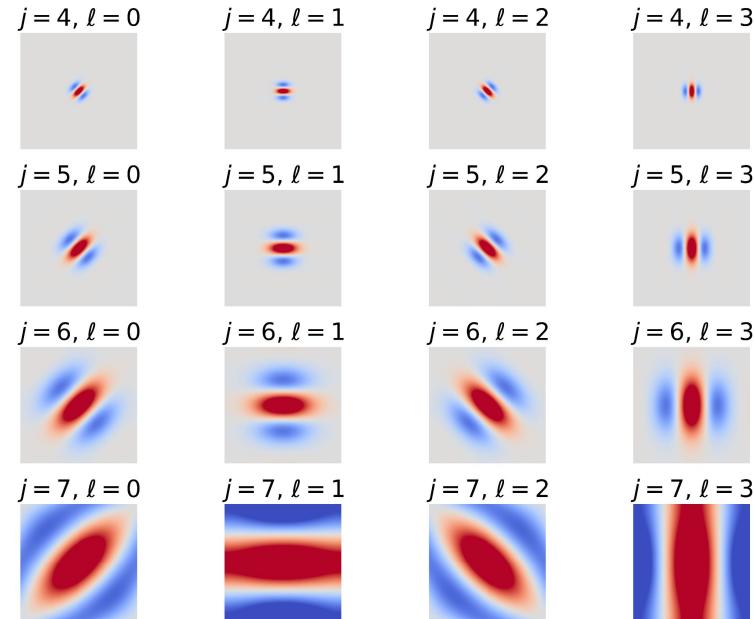
$$\tilde{\psi}(\mathbf{k}) = \frac{1}{\sqrt{\Sigma}} e^{-(\mathbf{k} - \mathbf{k}_0)^T \Sigma (\mathbf{k} - \mathbf{k}_0) / 2}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$

- $j$ : size (logarithmic spacing)

## Morlet Wavelets

Wavelets for scales  $j$  and orientation  $\ell$



# Scattering Transform

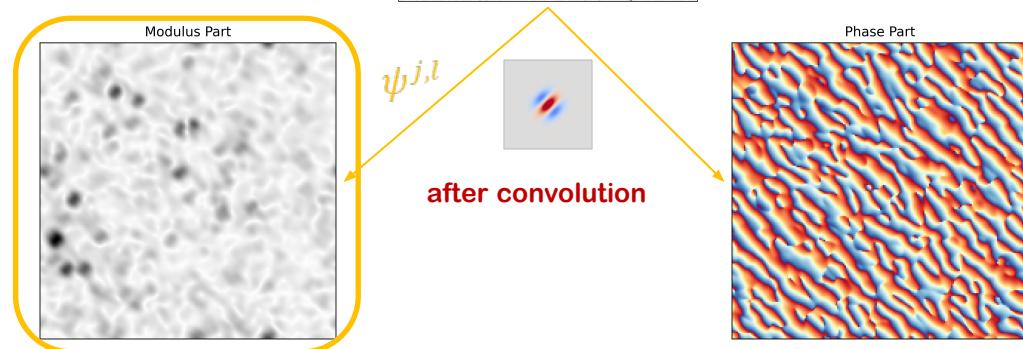
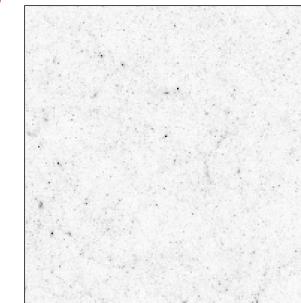
**Operation:**

$$S = \langle |I \star \psi^{j,l}| \rangle$$

**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**

**Modulus: convert selected fluctuations into their local strength**



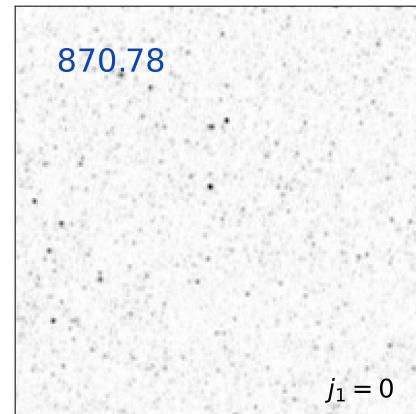
# Scattering Transform

---

**Operation:**

$$S = \langle |I \star \psi^{j,l}| \rangle$$

Mean: spatial average of the field



**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**

$$S_0 \equiv \langle I_0 \rangle$$

$$S_1^{j_1, l_1} \equiv \langle I_1^{j_1, l_1} \rangle = \langle |I_0 \star \psi^{j_1, l_1}| \rangle$$

$$S_2^{j_1, l_1, j_2, l_2} \equiv \langle I_2^{j_1, l_1, j_2, l_2} \rangle = \langle ||I_0 \star \psi^{j_1, l_1}| \star \psi^{j_2, l_2}| \rangle$$

# Scattering Transform

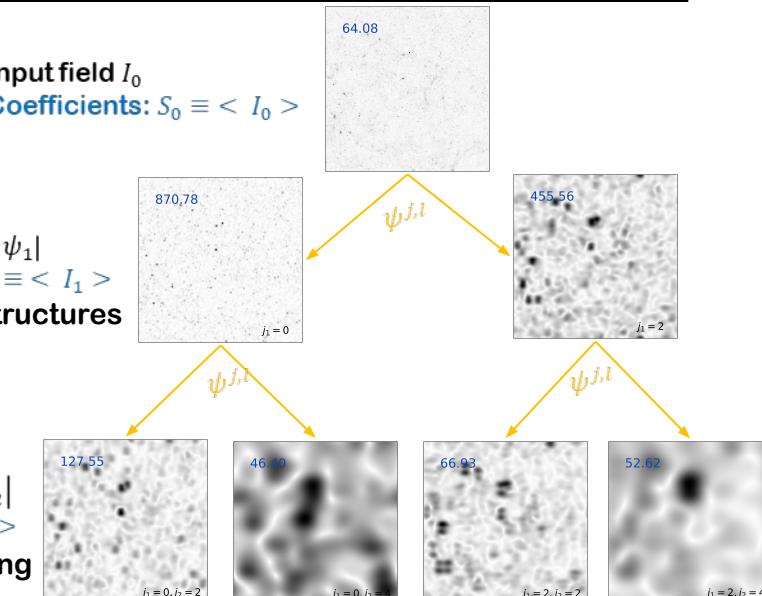
**Operation:**

$$S = \langle |I \star \psi^{j,l}| \rangle$$

**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**

Fields  $I_1 \equiv |I_0 \star \psi_1|$   
Coefficients:  $S_1 \equiv \langle I_1 \rangle$   
clustering of structures



(all the convolved fields here has orientation index  $l_1 = 1, l_2 = 1$ )

(All the numbers here are  $10^6$  times the real coefficients)

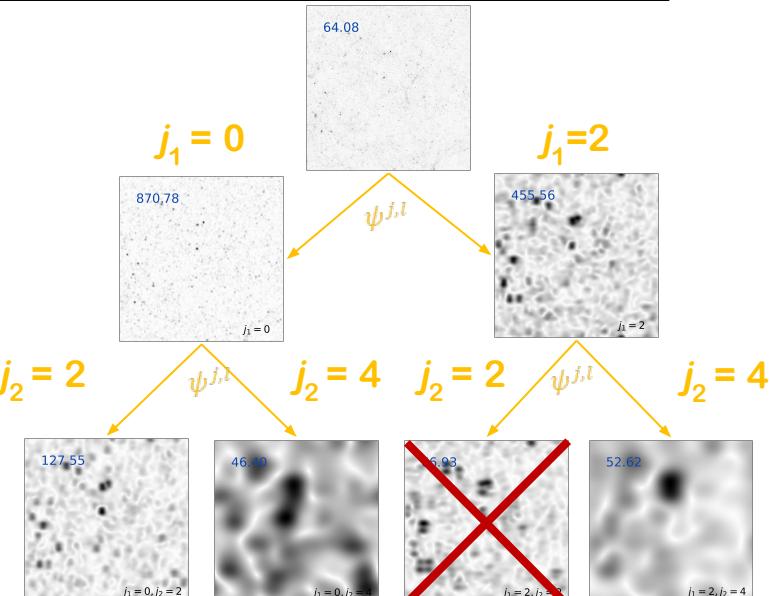
# Scattering Transform

**Operation:**

$$S = \langle |I \star \psi^{j,l}| \rangle$$

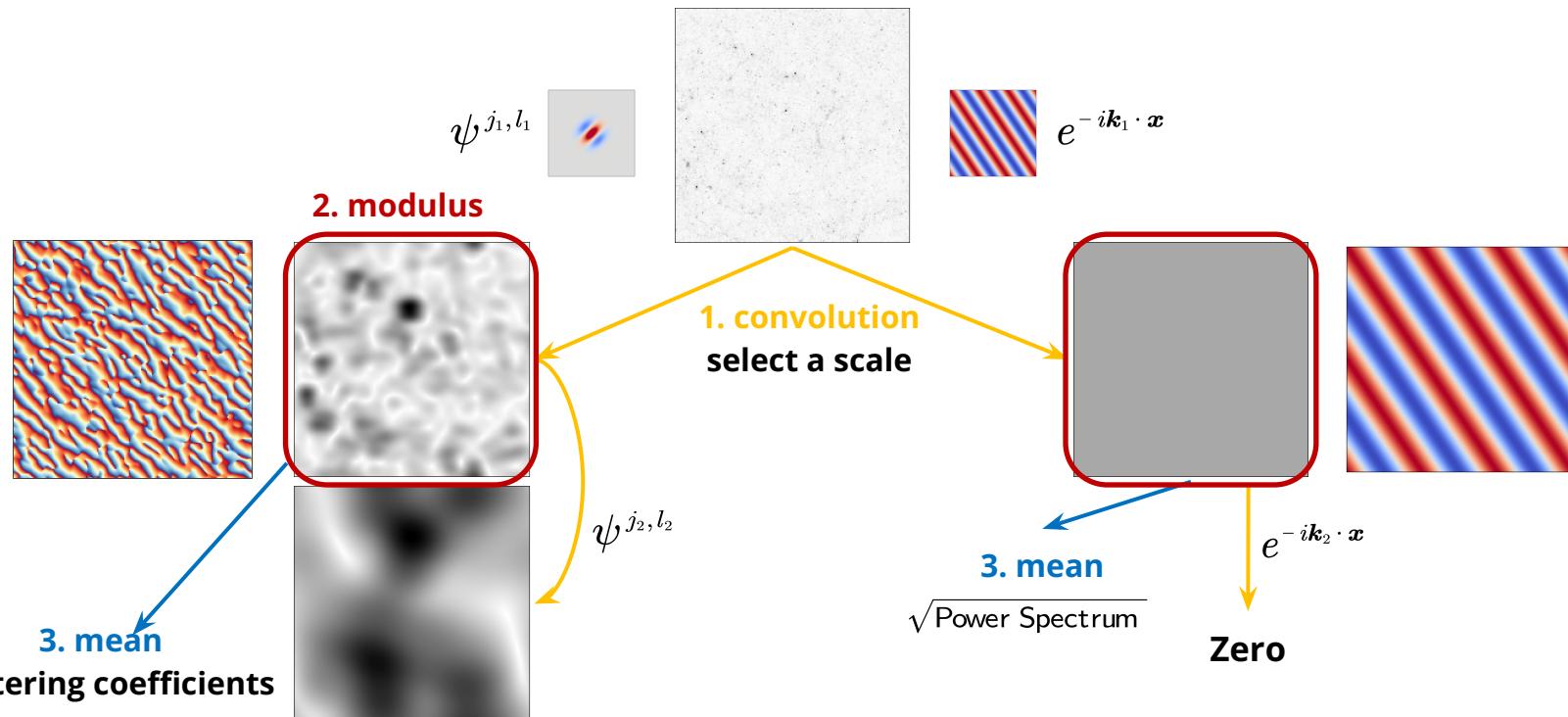
**Scattering transform=**  
**wavelet convolution**

- + **modulus**
- + **mean**



When doing second order convolution,  
choose filter  $\psi^{j_2, l_2}$  with  $j_2 > j_1$

# Scattering vs. Power Spectrum



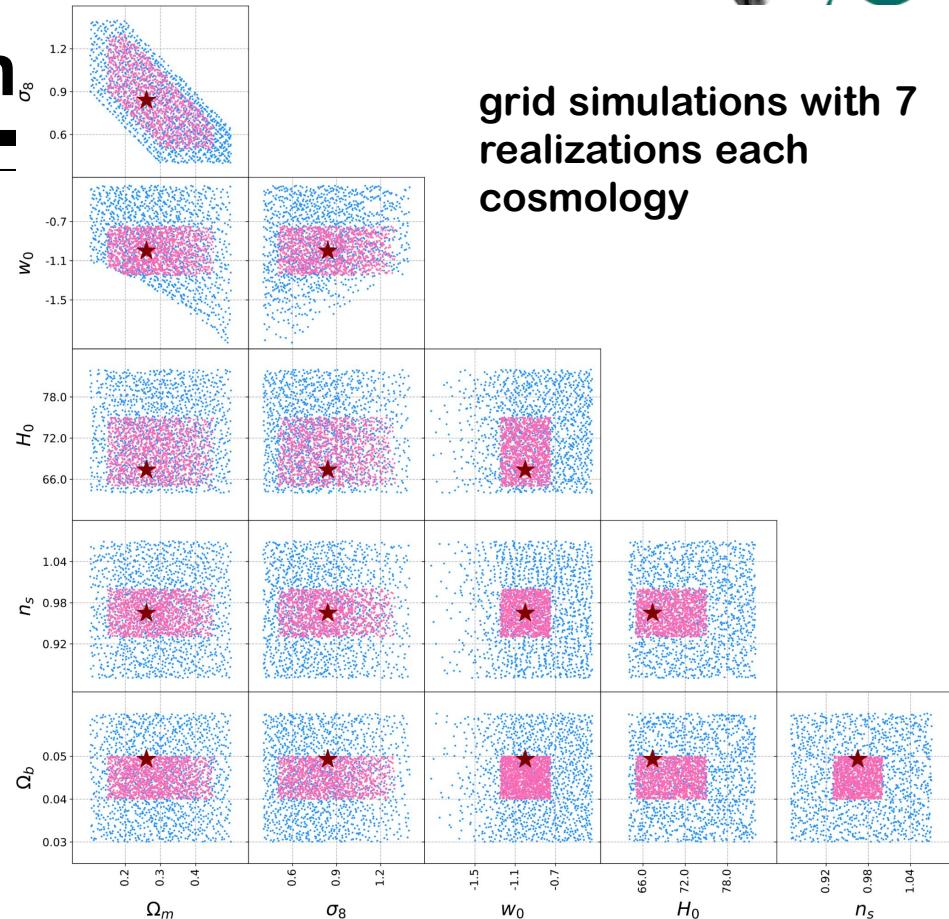
# CosmoGridV1 Simulation

	fiducial	$\Delta$ fid.	wide grid prior	narrow grid prior
$\Omega_m$	0.26	$\pm 0.01$	$\in [0.10, 0.50]$	$\in [0.15, 0.45]$
$\sigma_8$	0.84	$\pm 0.015$	$\in [0.40, 1.40]$	$\in [0.50, 1.30]$
$w_0$	-1	$\pm 0.05$	$\in [-2.00, -0.33]$	$\in [-1.25, -0.75]$
$n_s$	0.9649	$\pm 0.02$	$\in [0.87, 1.07]$	$\in [0.93, 1.00]$
$\Omega_b$	0.0493	$\pm 0.001$	$\in [0.03, 0.06]$	$\in [0.04, 0.05]$
$H_0$	67.3	$\pm 2.0$	$\in [64.0, 82.0]$	$\in [65.0, 75.0]$

Kacprzak et al. 2022

**fiducial simulations with 200 realizations each cosmology**

**N-body simulations with baryon feedback effect, 6.8' x 6.8' each pixel**



**grid simulations with 7 realizations each cosmology**

# Square Maps Extraction

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$40^\circ \times 40^\circ$ , 15 square maps from each full sky map,  $8' \times 8'$  each pixel

# Scattering Coefficient

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- When using isotropic fields, the scattering coefficients  $S_n$  could be further reduced by taking the average over all orientation indices of same scale

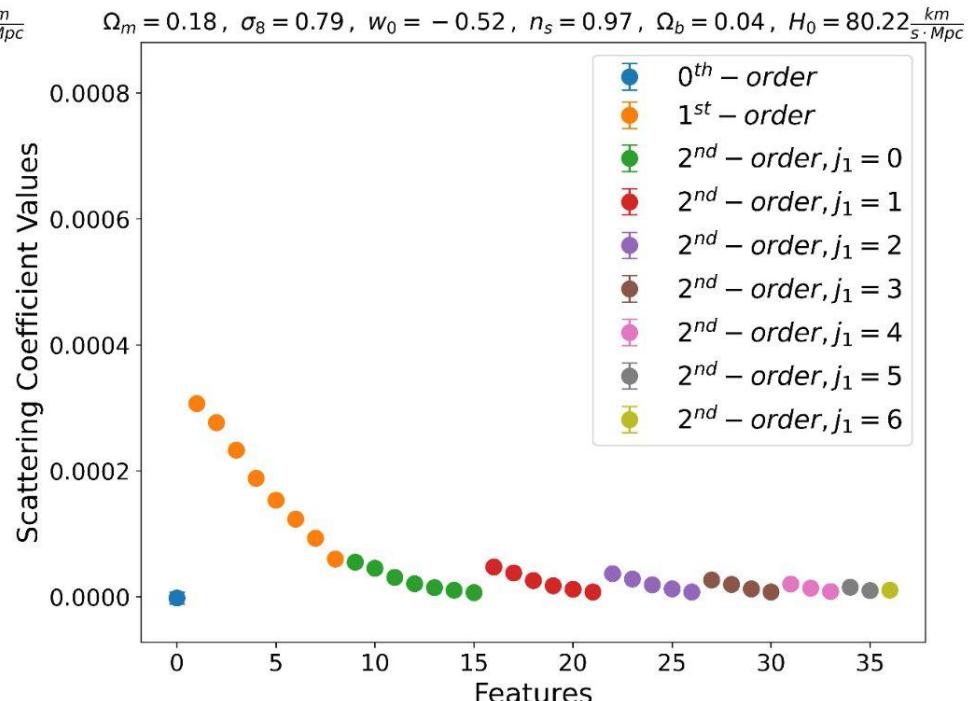
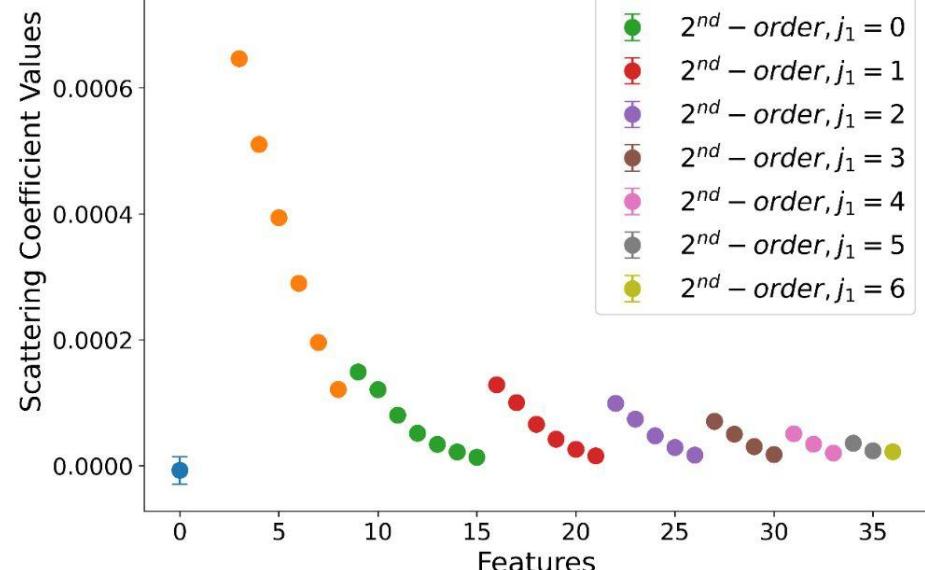
$$s_0 \equiv S_0$$

$$s_1(j_1) \equiv \langle S_1^{j_1, l_1} \rangle_{l_1}$$

$$s_2(j_1, j_2) \equiv \langle S_2^{j_1, l_1, j_2, l_2} \rangle_{l_1, l_2}$$

# Scattering Coefficient

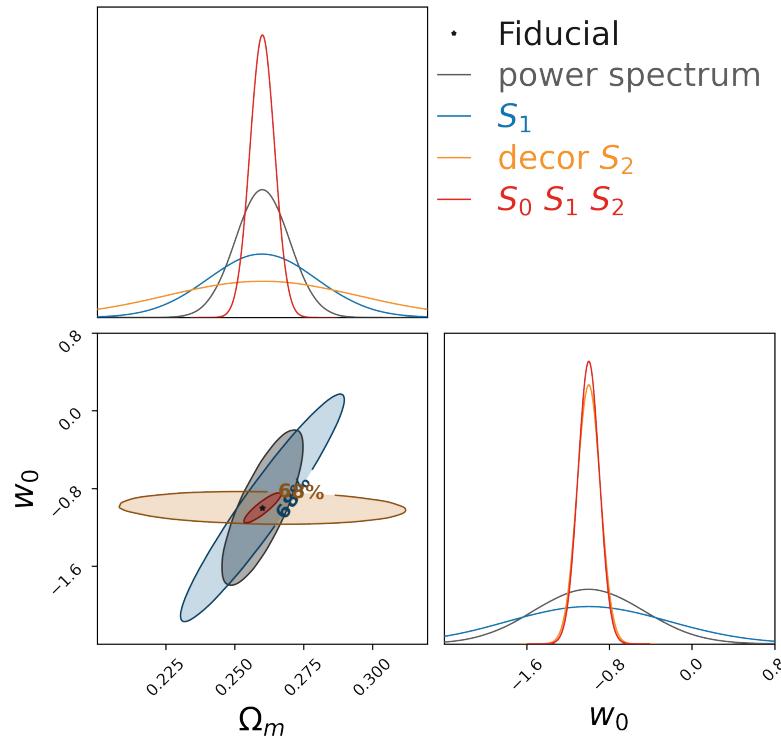
$$\Omega_m = 0.36, \sigma_8 = 0.99, w_0 = -1.51, n_s = 0.89, \Omega_b = 0.04, H_0 = 65.32 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$



# Fisher Forecast

De-correlated 2nd order coefficients:

$$\frac{S_2}{S_1}$$



# Emulator

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Input parameters:

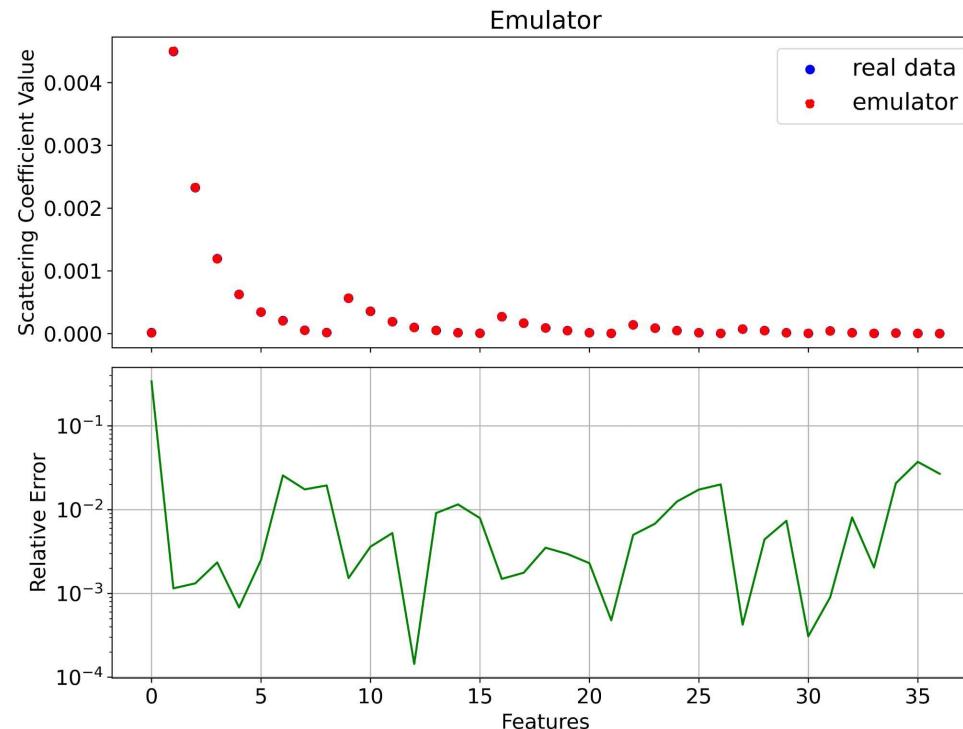
$$\Omega_m, \sigma_8, w_0, n_s, \Omega_b, H_0$$

Output features:

$$\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2$$

37 features in total

$$1 \mathbf{s}_0, 8 \mathbf{s}_1, 28 \mathbf{s}_2$$



# MCMC

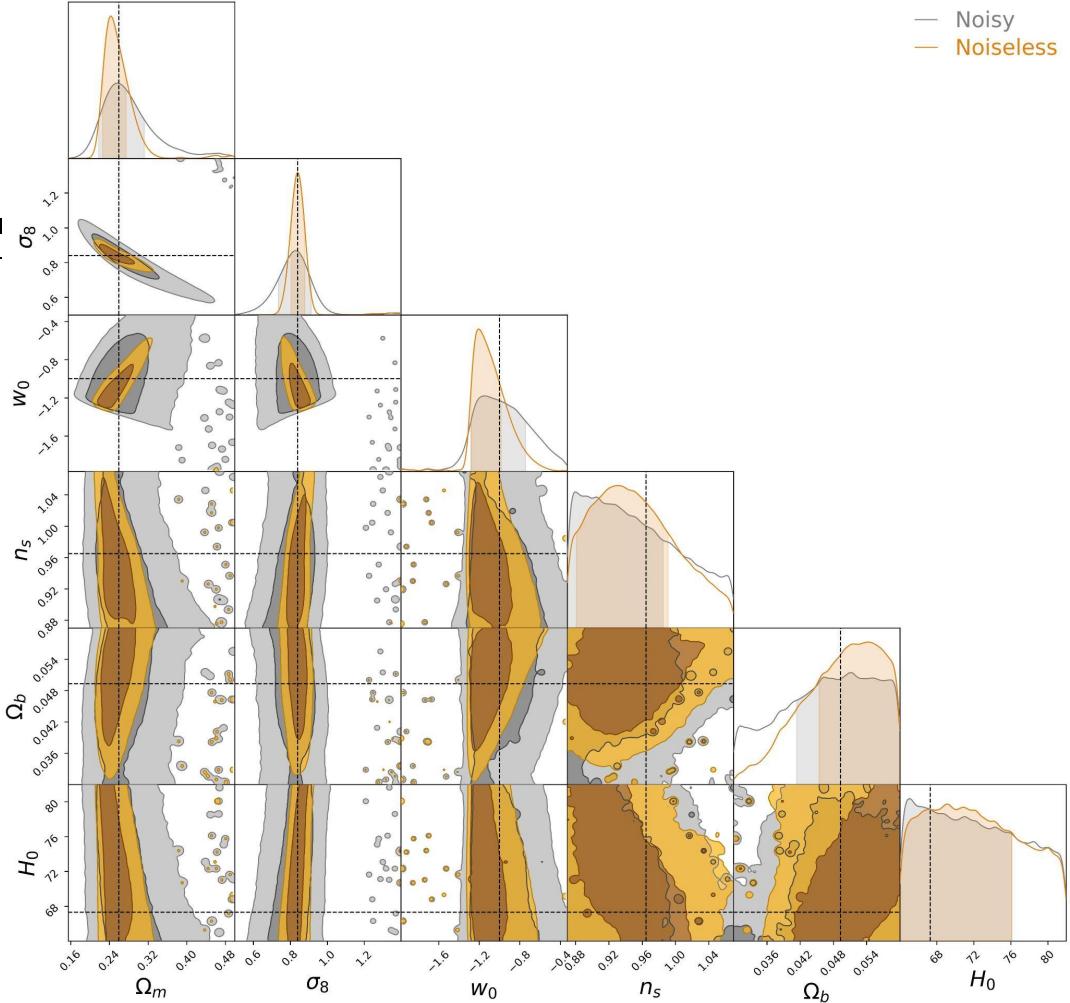
$$\sigma^2 = \frac{\varepsilon^2}{An_g}$$

$\varepsilon = 0.26$  : shape noise

$A$  : area of each pixel

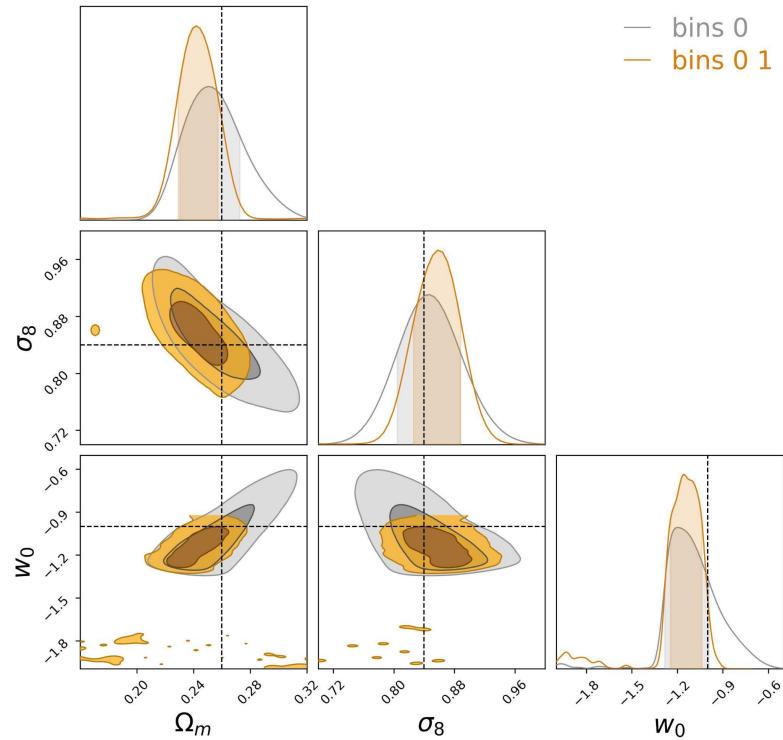
$n_g = 5.59$  : galaxy number density per arcmin<sup>2</sup>

(DES Collaboration et al. 2022)



# Tomographic Analysis

- Build emulators on redshift bins 0 and 1
- Tomographic analysis



# Future Work

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- **Apply ST on DES data**
  - try ST on shear fields
  - operate on masked maps
  - use real DES data