Probe Weak Lensing Cosmology with Scattering Transform

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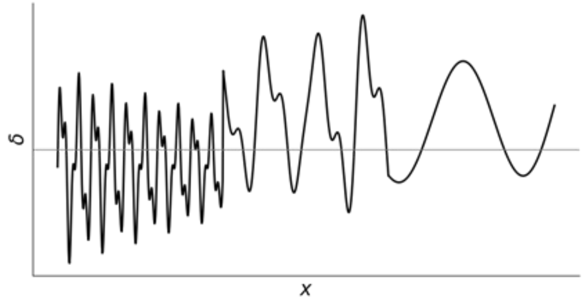
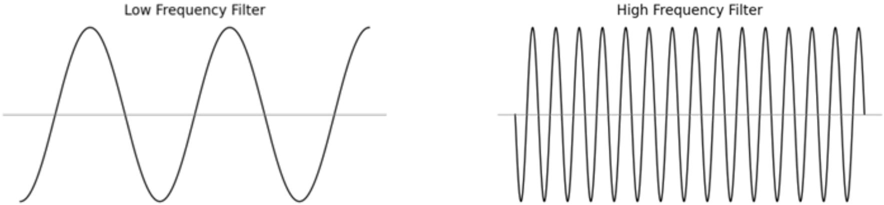
Traditional Statistics using Fourier Transform

- **Traditional statistics**
    - Measuring 2- or 3-point correlation function
    - Computing Fourier transform of these n-point correlation functions (power spectrum, bispectrum) might be computationally expensive
    - Gaining information in frequency space but no spatial information





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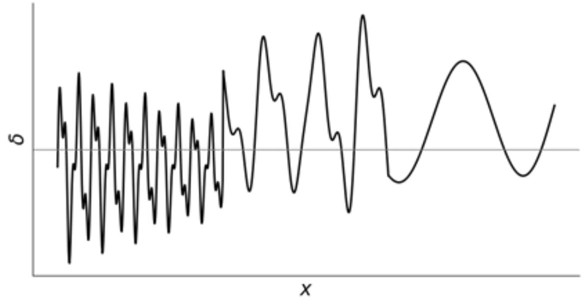
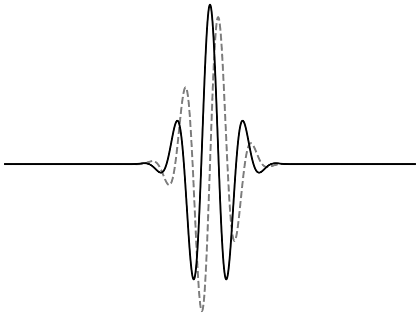




Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

I is cosmological field, such as convergence field (function of Ω_m, Ω_{DE}, σ_8…)

S is scattering coefficient

Scattering transform=

wavelet convolution

+ modulus

+ mean

(S. Cheng et al. 2020, A new approach to observational cosmology using the scattering transform)

Scattering Transform

Operation:

$$S = \left\langle \left| I \star \psi^{j,l} \right| \right\rangle$$

Scattering transform=
wavelet convolution
+ **modulus**
+ **mean**

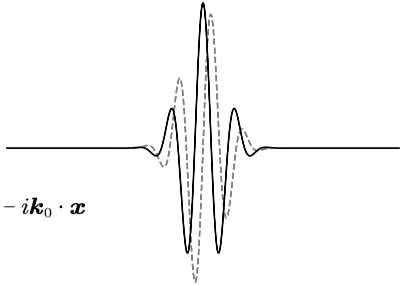
Wavelet: localized oscillating wave

Morlet wavelets:

In real space:

$$\psi(\boldsymbol{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}/2} e^{-i \boldsymbol{k}_0 \cdot \boldsymbol{x}}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$



Σ : the covariance of matrix describing the size and shape of the Gaussian envelope
k_0 : the frequency of the modulated oscillation

Scattering Transform

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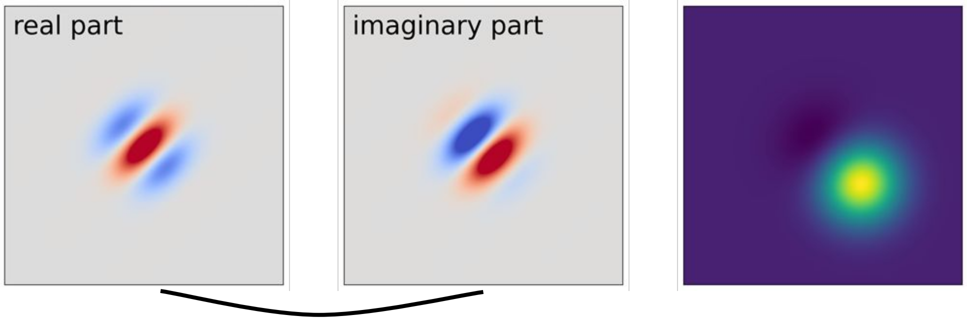
Morlet Wavelets

In real space:

$$\psi(\boldsymbol{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}/2} e^{-i\boldsymbol{k}_0 \cdot \boldsymbol{x}}$$

In Fourier space

$$\tilde{\psi}(\boldsymbol{k}) = \frac{1}{\sqrt{\Sigma}} e^{-(\boldsymbol{k}-\boldsymbol{k}_0)^T \Sigma (\boldsymbol{k}-\boldsymbol{k}_0)/2}$$



Real Space

Fourier Space

Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=

wavelet convolution

+ modulus

+ mean

Morlet Wavelets

$$L = 4,\ \ell = 0,\ 1,\ 2,\ 3$$

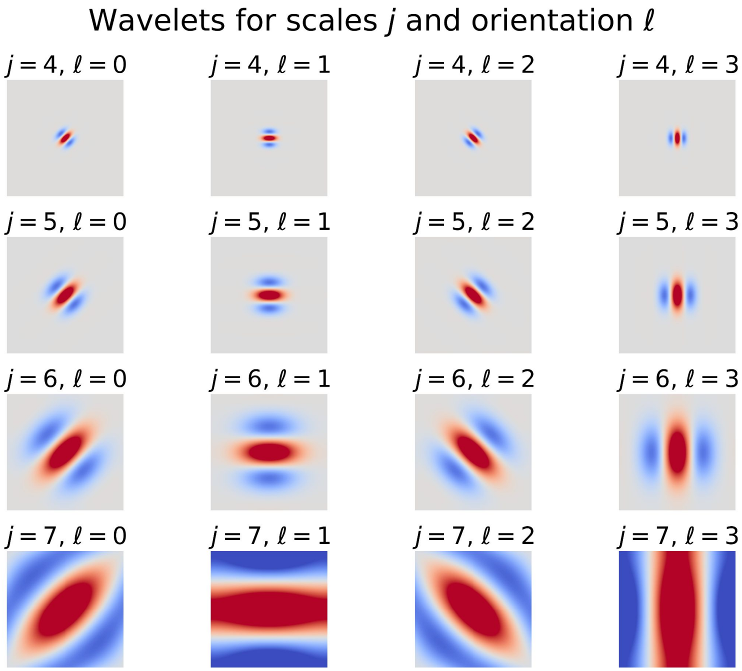
- **ℓ : orientation**

$$J = 8,\ j = 0,\ 1,\ \dots,\ 7$$

$$\tilde{\psi}(\boldsymbol{k}) = \frac{1}{\sqrt{\Sigma}} e^{-(\boldsymbol{k}-\boldsymbol{k}_0)^T \Sigma (\boldsymbol{k}-\boldsymbol{k}_0)/2}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$

- **j : size (logarithmic spacing)**



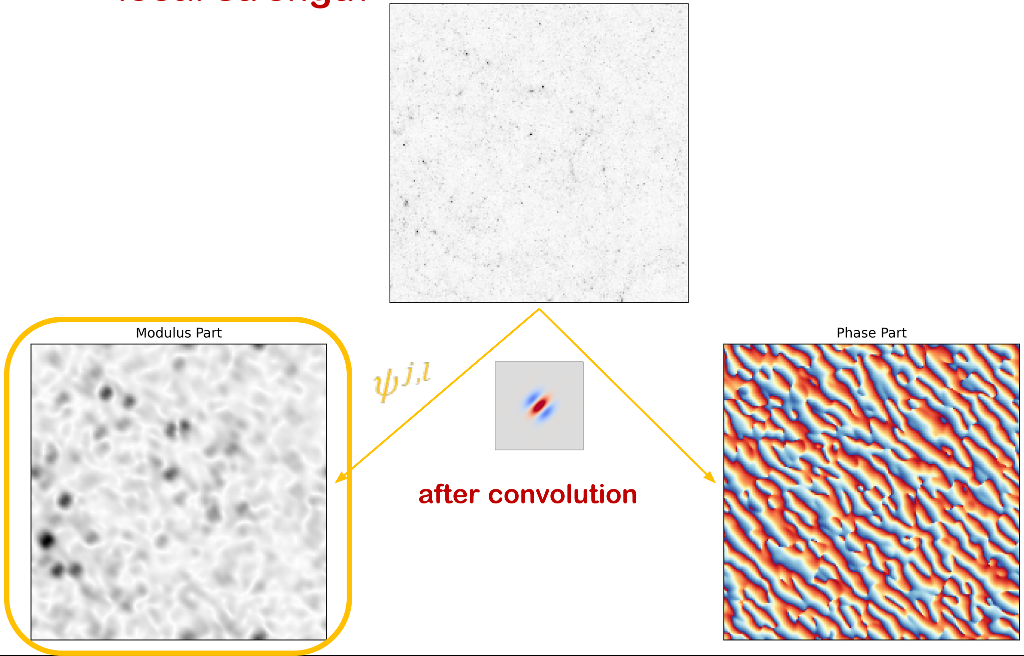
Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=
wavelet convolution
+ modulus
+ mean

Modulus: convert selected fluctuations into their local strength



Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=
wavelet convolution
+ modulus
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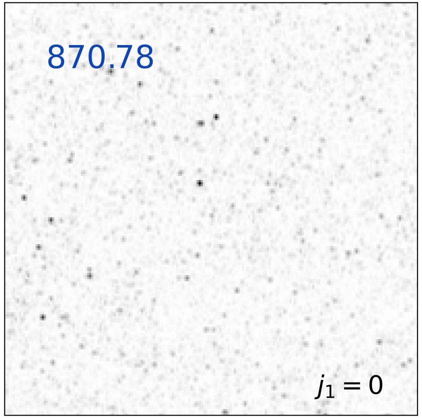
Mean: spatial average of the field



$$S_0 \equiv \langle I_0 \rangle$$

$$S_1^{j_1,l_1} \equiv \langle I_1^{j_1,l_1} \rangle = \langle | I_0 \star \psi^{j_1,l_1} | \rangle$$

$$S_2^{j_1,l_1,j_2,l_2} \equiv \langle I_2^{j_1,l_1,j_2,l_2} \rangle = \langle || I_0 \star \psi^{j_1,l_1} | \star \psi^{j_2,l_2} | \rangle$$

Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=

wavelet convolution

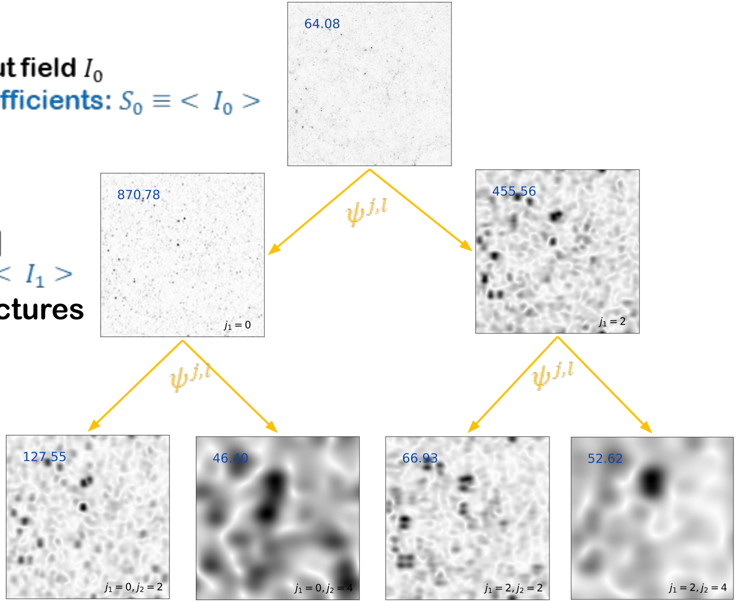
\+ **modulus**

\+ **mean**

Input field I_0
Coefficients: $S_0 \equiv < I_0 >$

Fields: $I_1 \equiv |I_0 \star \psi_1|$
Coefficients: $S_1 \equiv < I_1 >$
clustering of structures

Fields $I_2 \equiv ||I_0 \star \psi_1| \star \psi_2|$
Coefficients: $S_2 \equiv < I_2 >$
clustering of clustering



(all the convolved fields here has orientation index $l_1 = 1, l_2 = 1$)

(All the numbers here are 10^6 times the real coefficients

Scattering Transform

Operation:

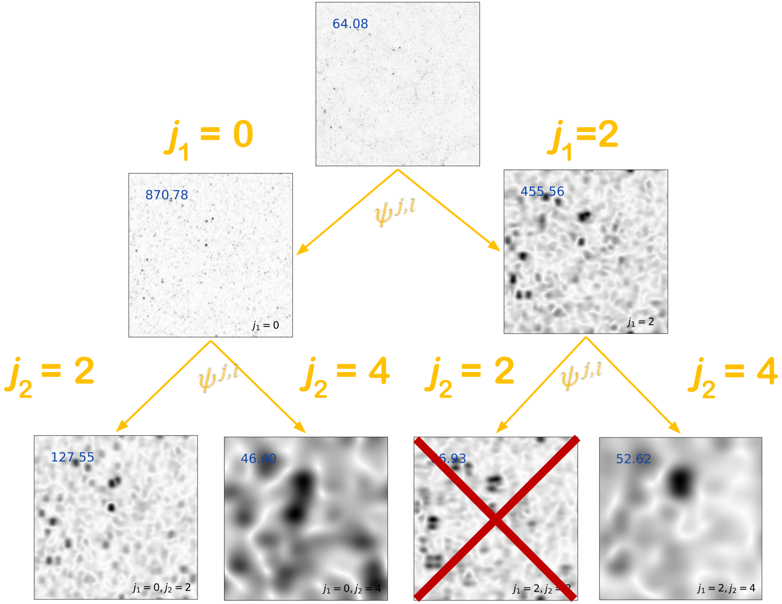
$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=

wavelet convolution

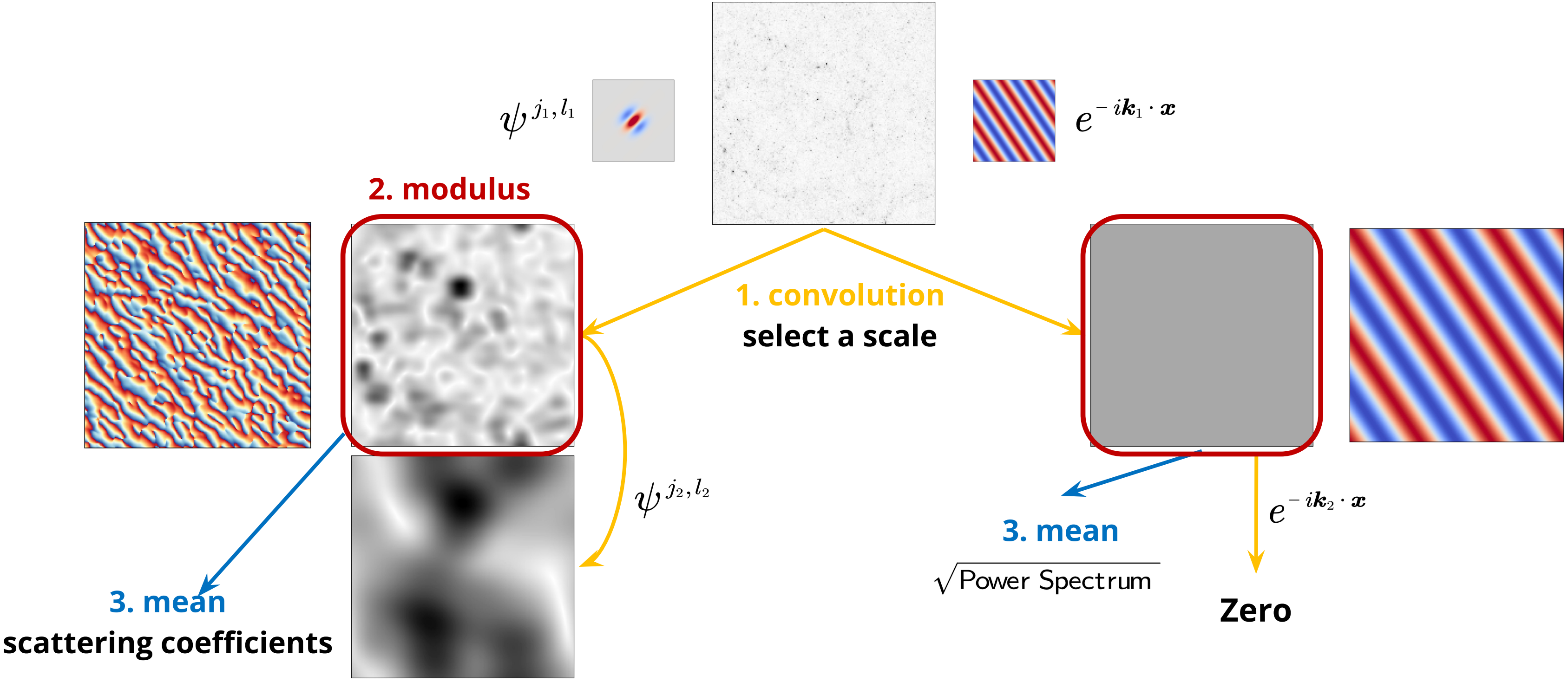
+ modulus

+ mean



When doing second order convolution, choose filter ψ^{j_2,l_2} with $j_2 > j_1$

Scattering vs. Power Spectrum



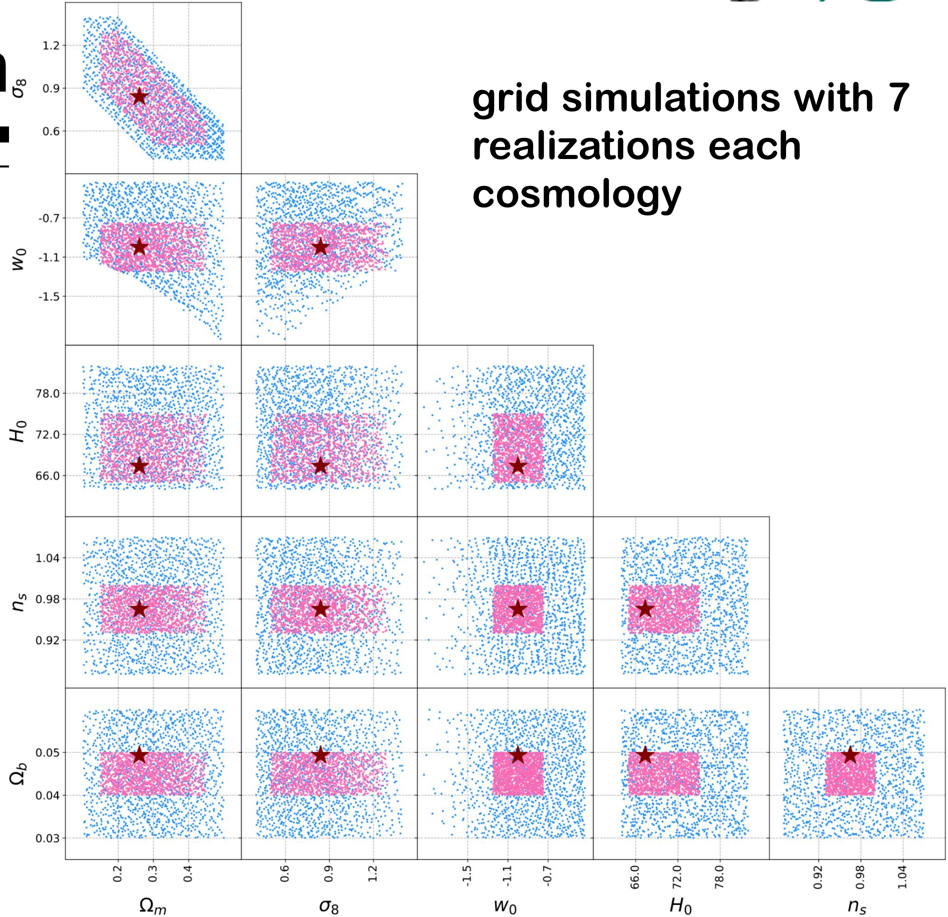
CosmoGridV1 Simulation

	fiducial	Δ fid.	wide grid prior	narrow grid prior
Ω_{m}	0.26	$\pm$ 0.01	$\in$ [0.10, 0.50]	$\in$ [0.15, 0.45]
σ_8	0.84	$\pm$ 0.015	$\in$ [0.40, 1.40]	$\in$ [0.50, 1.30]
w_0	-1	$\pm$ 0.05	$\in$ [-2.00, -0.33]	$\in$ [-1.25, -0.75]
n_s	0.9649	$\pm$ 0.02	$\in$ [0.87, 1.07]	$\in$ [0.93, 1.00]
Ω_b	0.0493	$\pm$ 0.001	$\in$ [0.03, 0.06]	$\in$ [0.04, 0.05]
H_0	67.3	$\pm$ 2.0	$\in$ [64.0, 82.0]	$\in$ [65.0, 75.0]

Kacprzak et al. 2022

fiducial simulations with 200 realizations each cosmology

N-body simulations with baryon feedback effect, 6.8′ x 6.8′ each pixel



grid simulations with 7 realizations each cosmology

Square Maps Extraction

40°×40°, 15 square maps from each full sky map, 8′ x 8′ each pixel

Scattering Coefficient

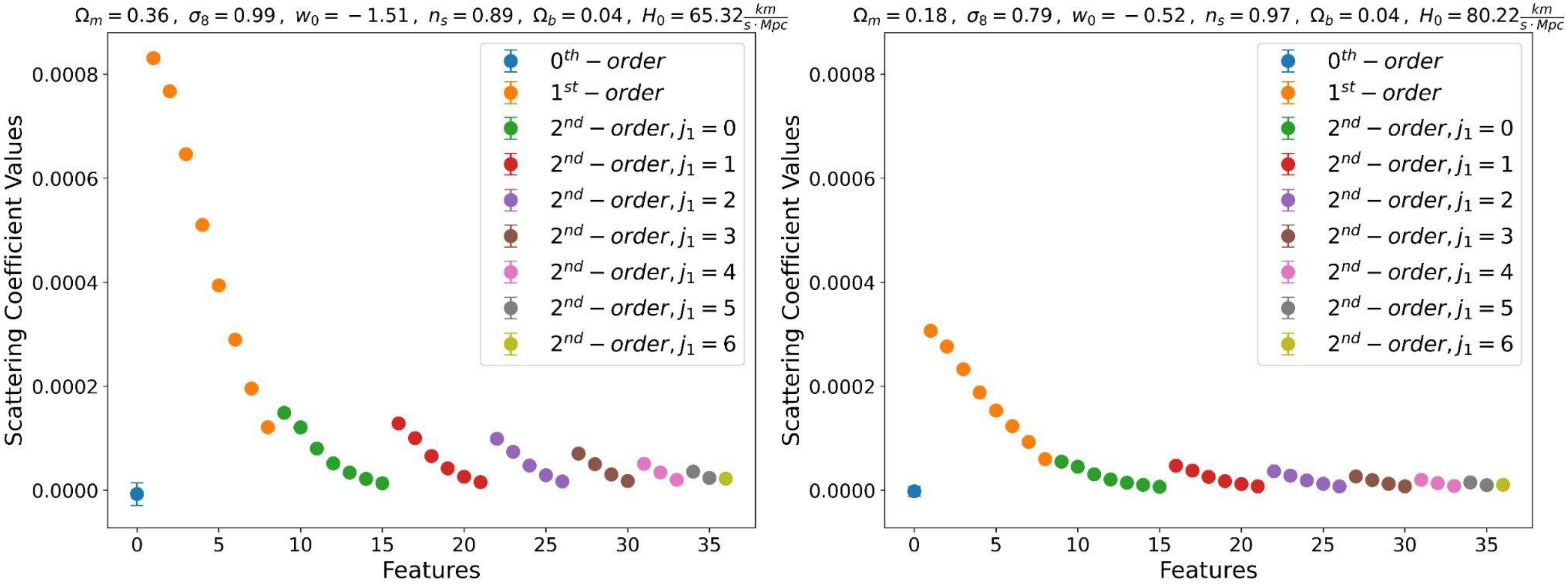
- **When using isotropic fields, the scattering coefficients S_n could be further reduced by taking the average over all orientation indices of same scale**

$$s_0 \equiv S_0$$

$$s_1(j_1) \equiv \langle S_1^{j_1, l_1} \rangle_{l_1}$$

$$s_2(j_1, j_2) \equiv \langle S_2^{j_1, l_1, j_2, l_2} \rangle_{l_1, l_2}$$

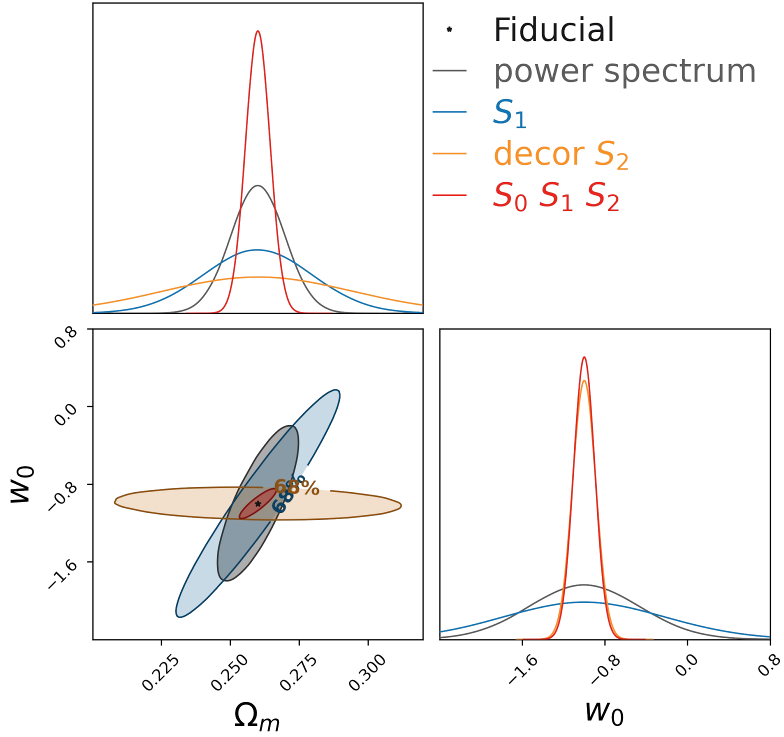
Scattering Coefficient



Fisher Forecast

De-correlated 2nd order coefficients:

$$\frac{s_2}{s_1}$$



Emulator

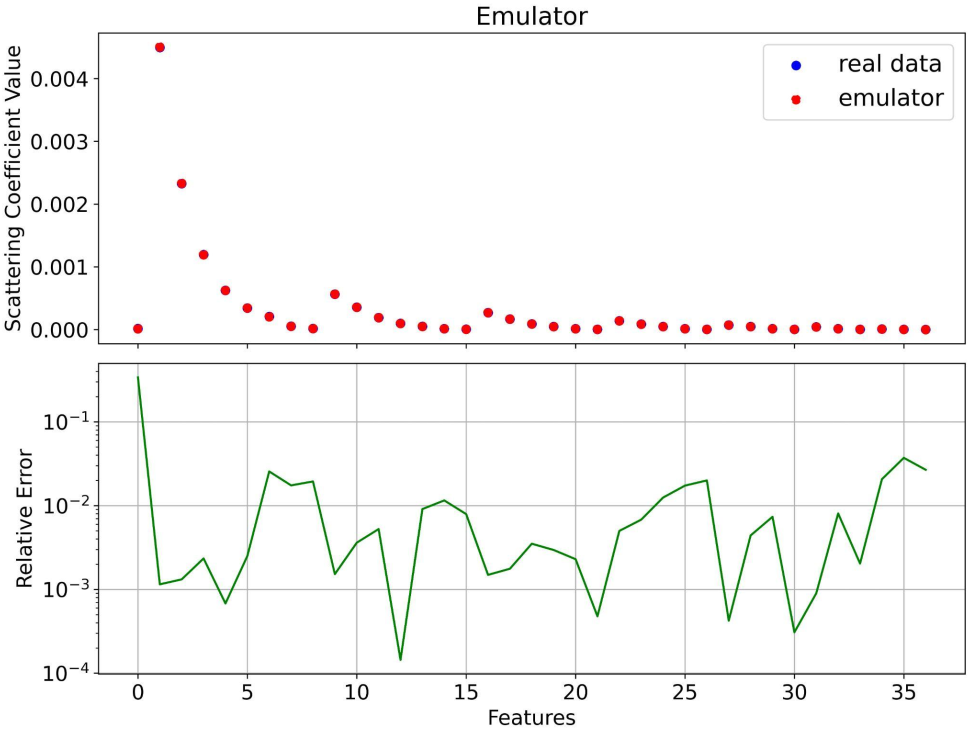
Input parameters:

$$\Omega_m,\ \sigma_8,\ w_0,\ n_s,\ \Omega_b,\ H_0$$

Output features:

$$\mathbf{s_0},\ \mathbf{s_1},\ \mathbf{s_2}$$

37 features in total
1 $\mathbf{s_0}$, 8 $\mathbf{s_1}$, 28 $\mathbf{s_2}$

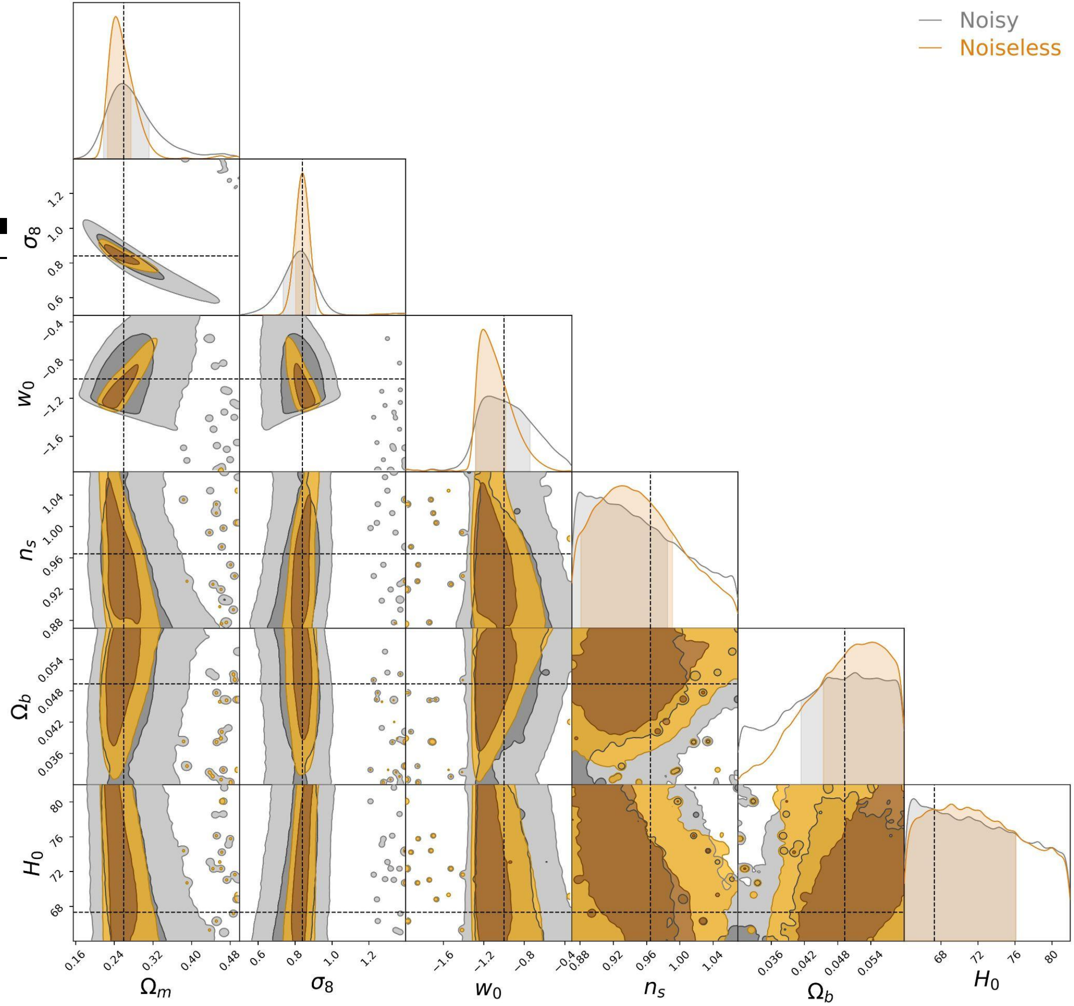


MCMC

$$\sigma^2 = \frac{\varepsilon^2}{A n_g}$$

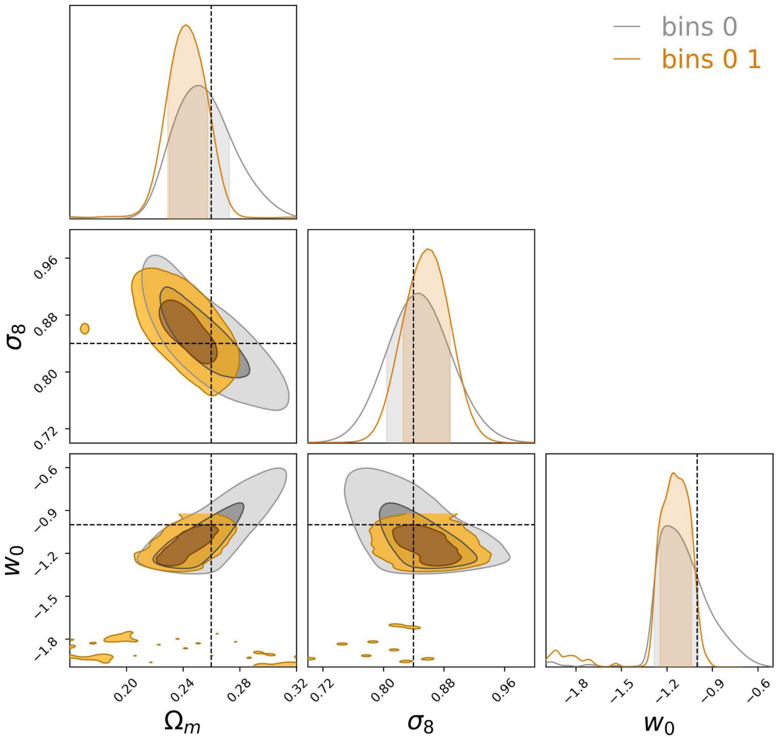
ε = 0.26 : shape noise
A : area of each pixel
n_g = 5.59 : galaxy number density per arcmin^2

(DES Collaboration et al. 2022)



Tomographic Analysis

- Build emulators on redshift bins 0 and 1
- Tomographic analysis



Future Work

- **Apply ST on DES data**
  - try ST on shear fields
  - operate on masked maps
  - use real DES data

Power Spectrum

$$F(k_n) = \sum_{n=0}^{2N} f(x) e^{-ik_n x}$$

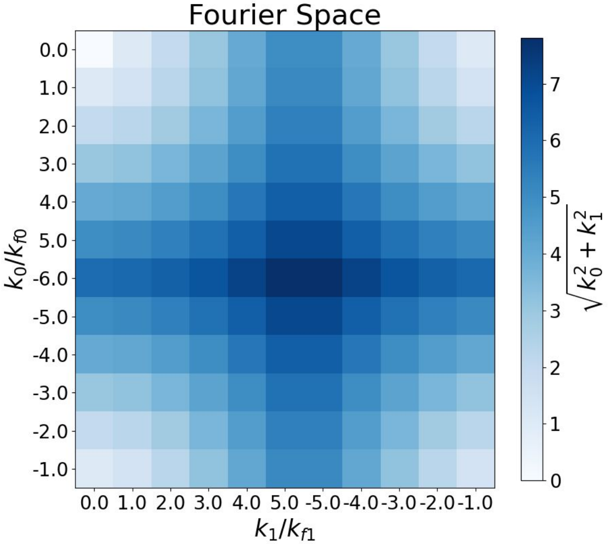
$$k_n = \frac{2\pi}{L}[0, 1, 2, \ldots, N, -N, -(N-1), \ldots, -2, -1]$$

$$k_n = \frac{2\pi}{L}[0, 1, 2, \ldots, N, N+1, N+2, \ldots, 2N]$$

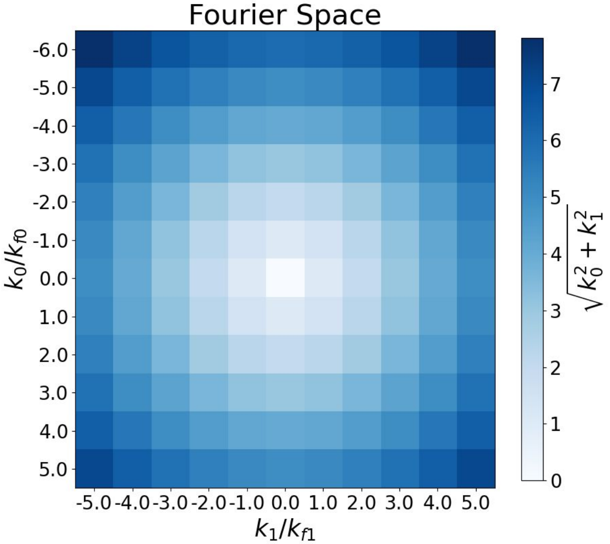
Power Spectrum

$$
\left[\begin{array}{cccc|ccccc}
(0,0) & (0,1) & \cdots & \left(0,\frac{N-1}{2}\right) & \left(0,-\frac{N-1}{2}\right) & \left(0,-\left(\frac{N-1}{2}-1\right)\right) & \cdots & (0,-2) & (0,-1) \\
(1,0) & (1,1) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\left(\frac{M}{2}-1,0\right) & \ddots & \ddots & \left(\frac{M}{2}-1,\frac{N-1}{2}\right) & \left(\frac{M}{2}-1,-\frac{N-1}{2}\right) & \ddots & \ddots & \ddots & \left(\frac{M}{2}-1,-1\right) \\
\hline
\left(-\frac{M}{2},0\right) & \ddots & \ddots & \left(-\frac{M}{2},\frac{N-1}{2}\right) & \left(-\frac{M}{2},-\frac{N-1}{2}\right) & \ddots & \ddots & \ddots & \left(-\frac{M}{2},-1\right) \\
\left(-\left(\frac{M}{2}-1\right),0\right) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \left(-\left(\frac{M}{2}-1\right),-1\right) \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
(-2,0) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & (-2,-2) & (-2,-1) \\
(-1,0) & \cdots & \cdots & \left(-1,\frac{N-1}{2}\right) & \left(-1,-\frac{N-1}{2}\right) & \left(-1,-\left(\frac{N-1}{2}-1\right)\right) & \cdots & (-1,-2) & (-1,-1)
\end{array}\right]
$$

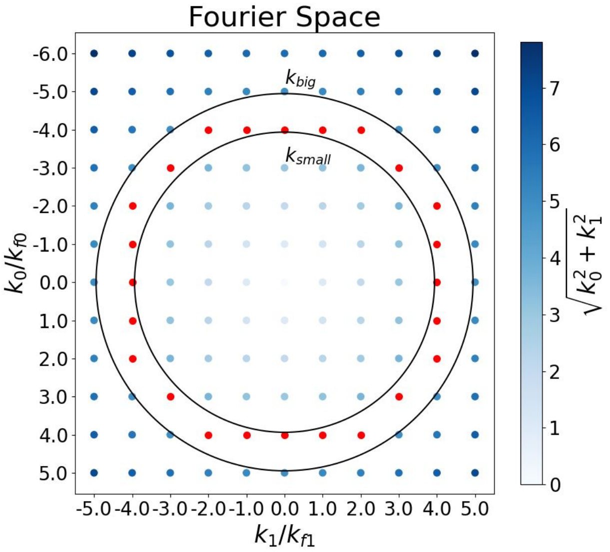
Power Spectrum



Before shift min frequency to center



After shift min frequency to center



Average value of the elements in the same logarithmic bins

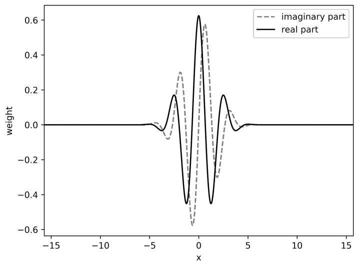
Morlet Wavelet

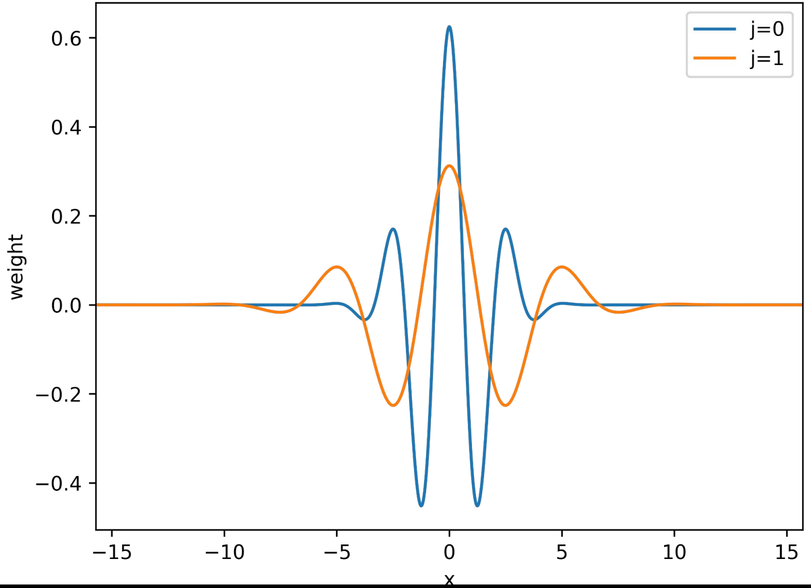
$\boldsymbol{\Sigma}$ is selected to have 1 eigenvalue different from others, and k_0 to be along that eigen-direction. Thus we denote the eigen-value along k_0 by σ^2 and other eigen-value by σ^2/s^2

$\sigma = 0.8\times 2^j$, where σ is the unit of pixel

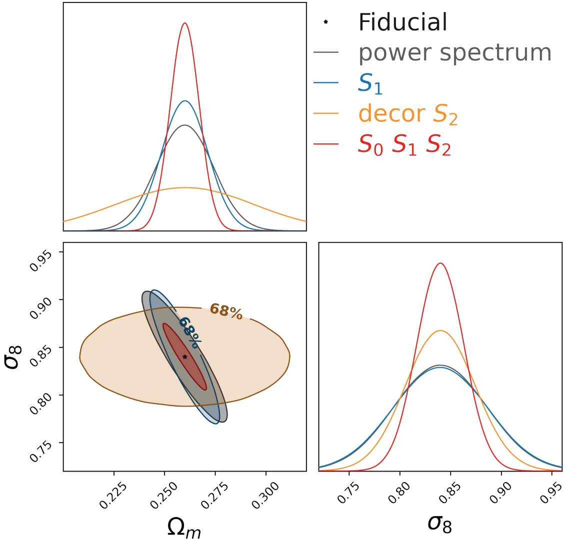
$k_0 = 3\boldsymbol{\pi}/(4\times 2^j)$

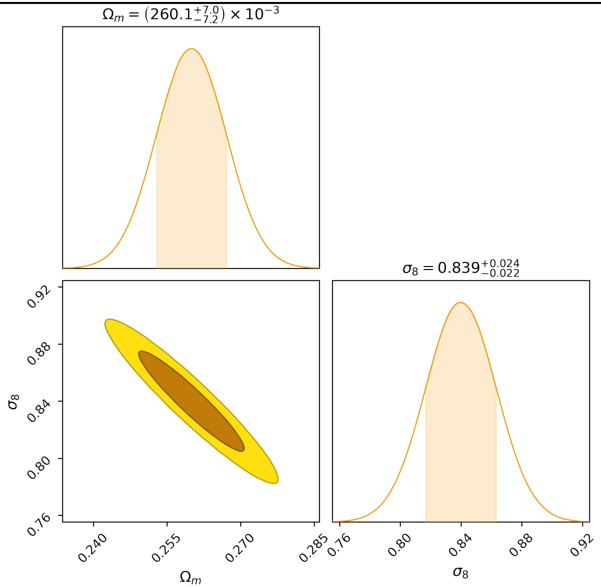
s=4/L

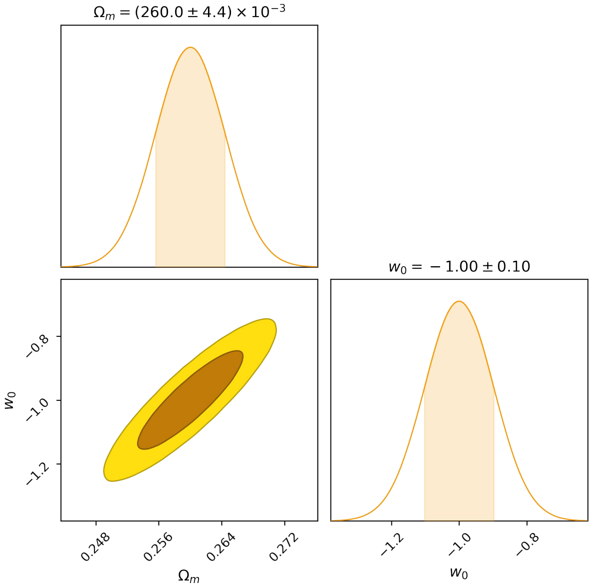




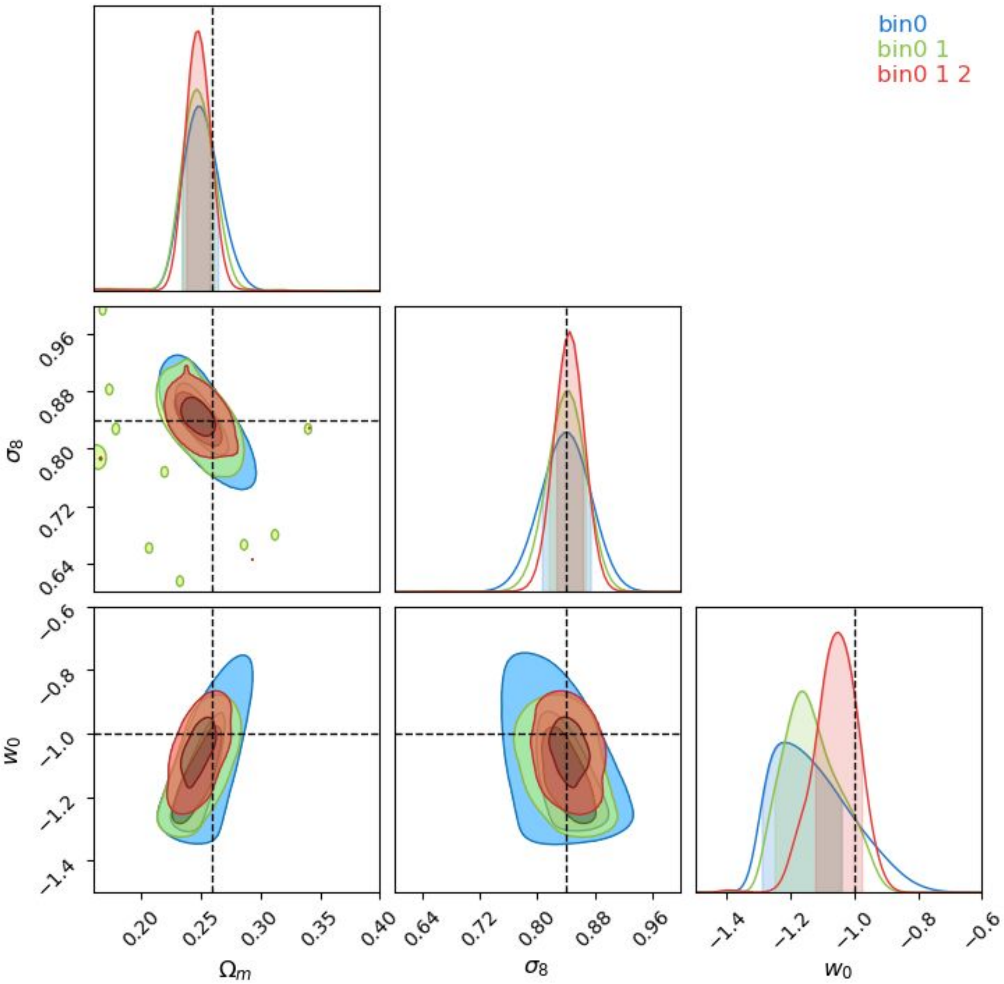
Fisher Forecast



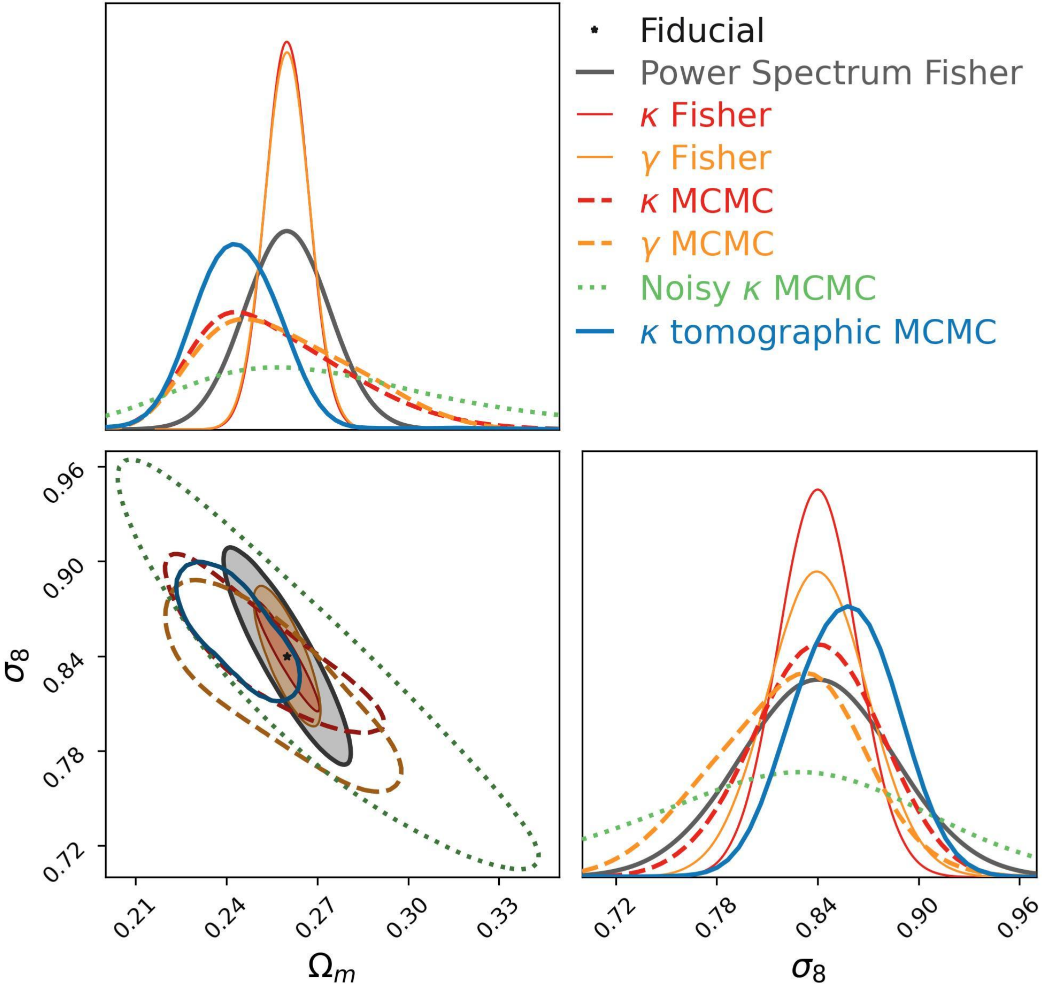




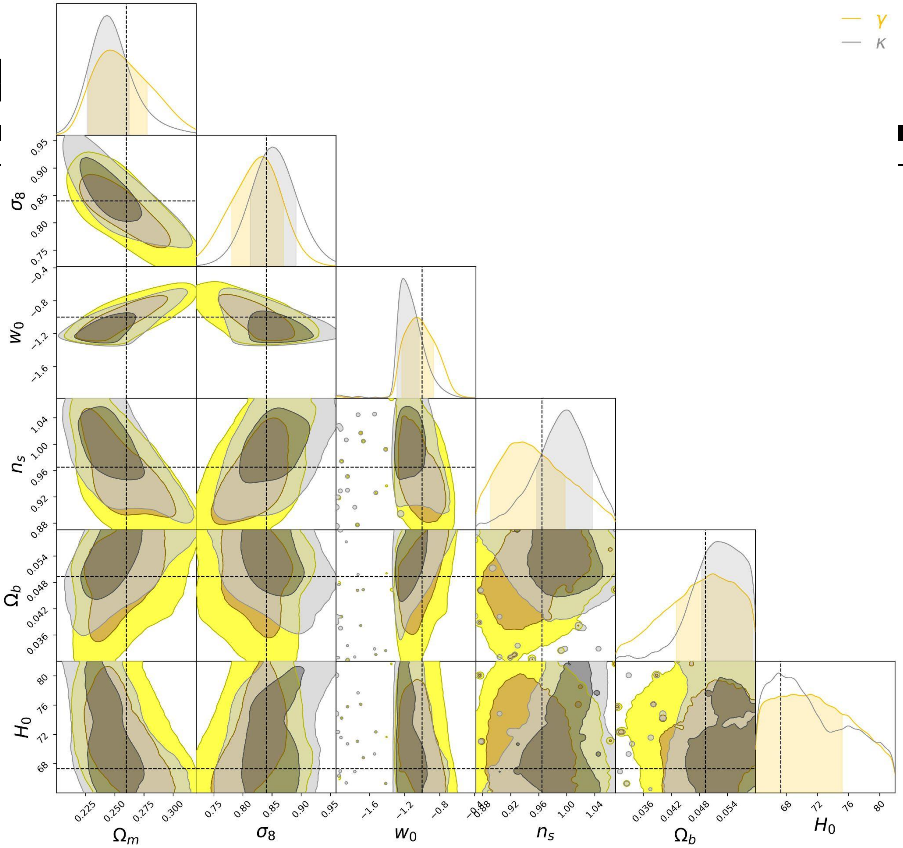
Tomographic Analy



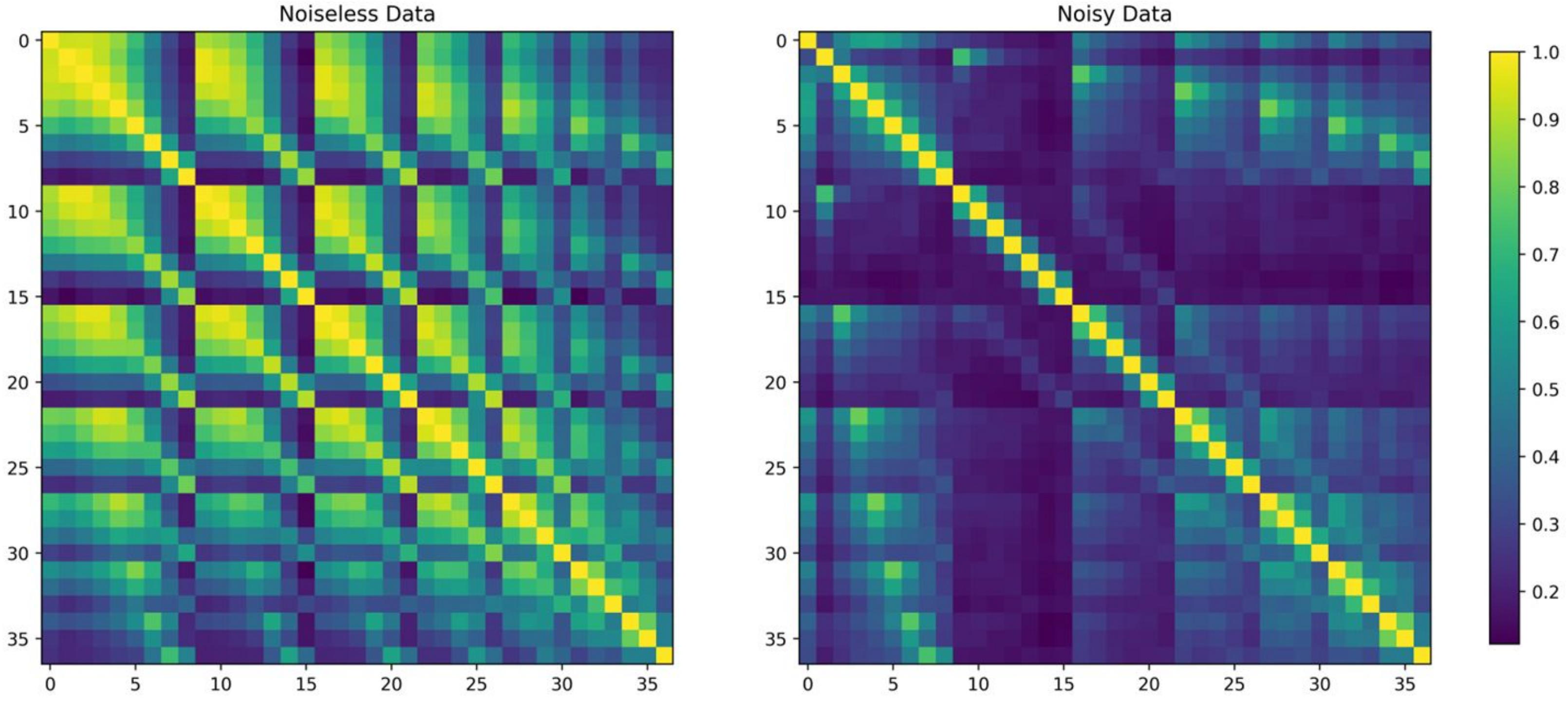
Shear Maps Fisher Foreca



Shear Maps Fish



Covariance Matrix of Data



Accuracy of Emulator

$$\varepsilon = \frac{\chi^2_{emu}}{\chi^2_{sim}} - 1 = \frac{\left(\mu_{emu} - \overline{d}_{fid}\right)^T C^{-1}_{fid}\left(\mu_{emu} - \overline{d}_{fid}\right)}{\left(\mu_{sim} - \overline{d}_{fid}\right)^T C^{-1}_{fid}\left(\mu_{sim} - \overline{d}_{fid}\right)} - 1$$