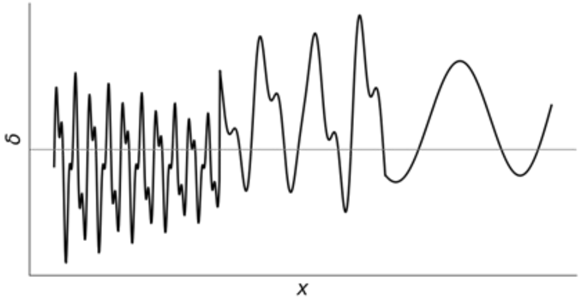
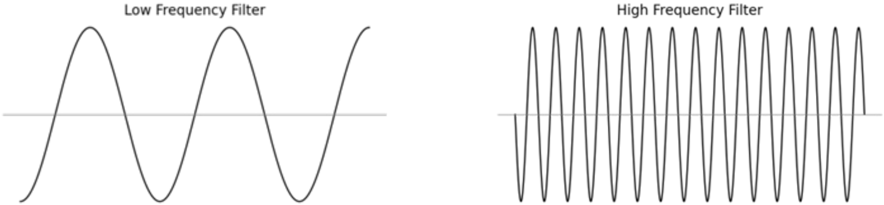
Traditional Statistics using Fourier Transform

- **Traditional statistics**
  - Measuring 2- or 3-point correlation function
  - Computing Fourier transform of these n-point correlation functions (power spectrum, bispectrum) might be computationally expensive
  - Gaining information in frequency space but no spatial information





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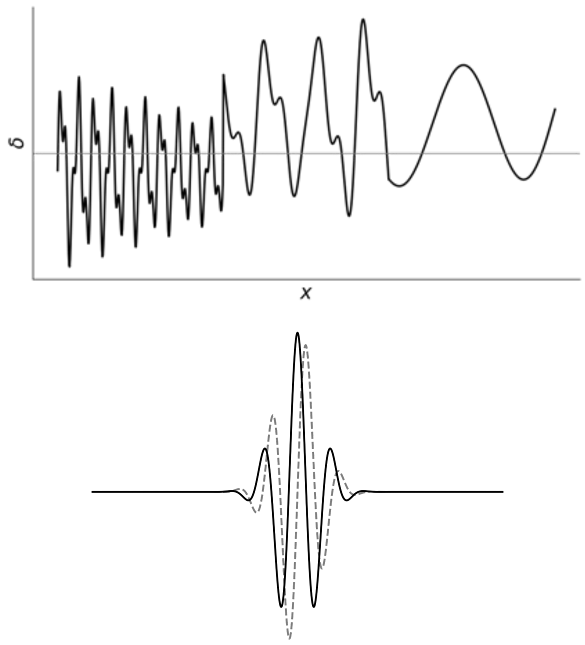


Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

I is cosmological field, such as convergence field (function of Ω_m, Ω_{DE}, σ_8...)

S is scattering coefficient

Scattering transform=

wavelet convolution

\+ **modulus**

\+ **mean**

(S. Cheng et al. 2020, A new approach to observational cosmology using the scattering transform)

Scattering Transform

Operation:

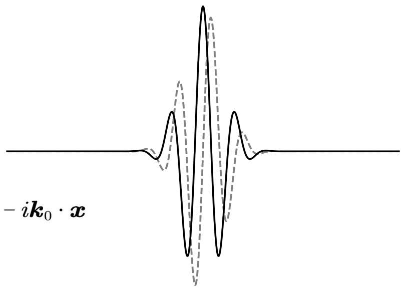
$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=
wavelet convolution
+ modulus
+ mean

Wavelet: localized oscillating wave

Morlet wavelets:

In real space:



$$\psi(\boldsymbol{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}/2} e^{-i\boldsymbol{k}_0 \cdot \boldsymbol{x}}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$

Σ : the covariance of matrix describing the size and shape of the Gaussian envelope
k_0 : the frequency of the modulated oscillation

Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

$J = 8, j = 0, 1 \ldots 7$

$L = 4, l = 0, 1, 2, 3$

Scattering transform=

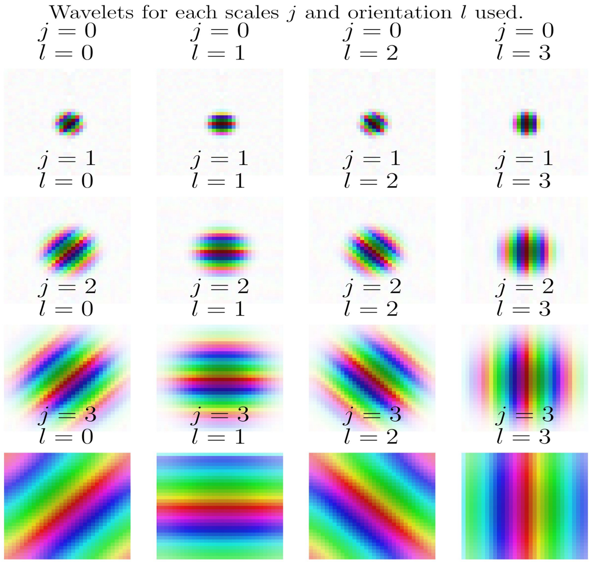
wavelet convolution

+ modulus

+ mean

- j : size (logarithmic spacing)
- ℓ : orientation

Morlet Wavelets



Kymatio python package to carry out the wavelet transform

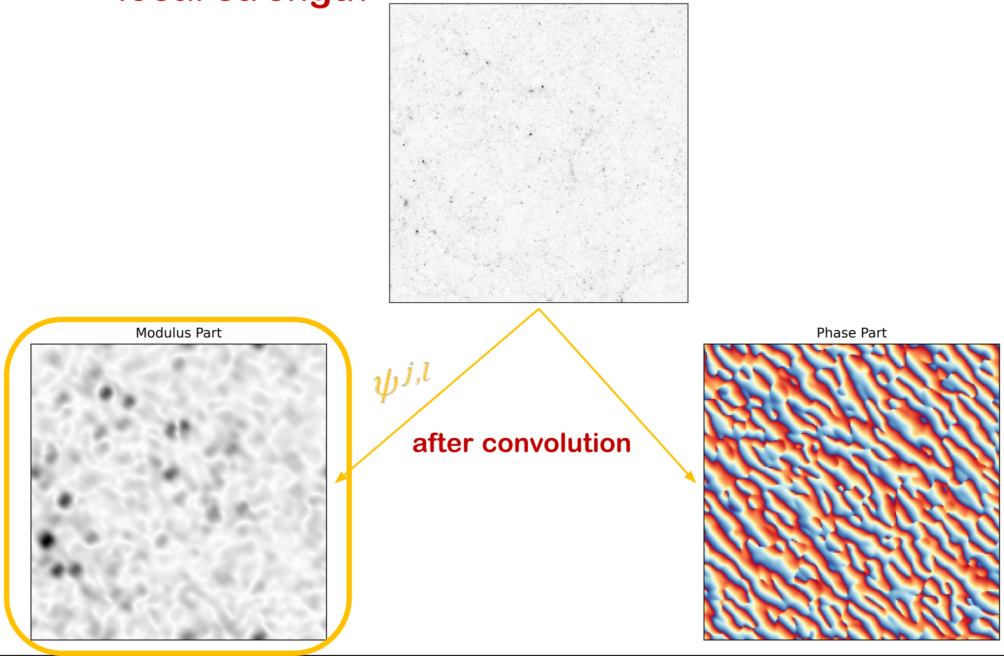
Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=
wavelet convolution
+ modulus
+ mean

Modulus: convert selected fluctuations into their local strength



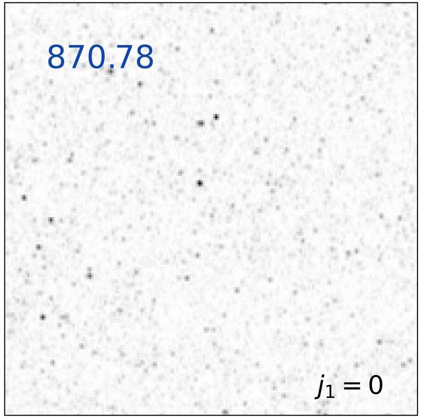
Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=
wavelet convolution
+ modulus
+ mean

Mean: spatial average of the field



$$S_0 \equiv \langle I_0 \rangle$$

$$S_1^{j_1,l_1} \equiv \langle I_1^{j_1,l_1} \rangle = \langle | I_0 \star \psi^{j_1,l_1} | \rangle$$

$$S_2^{j_1,l_1,j_2,l_2} \equiv \langle I_2^{j_1,l_1,j_2,l_2} \rangle = \langle | | I_0 \star \psi^{j_1,l_1} | \star \psi^{j_2,l_2} | \rangle$$

Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=

wavelet convolution

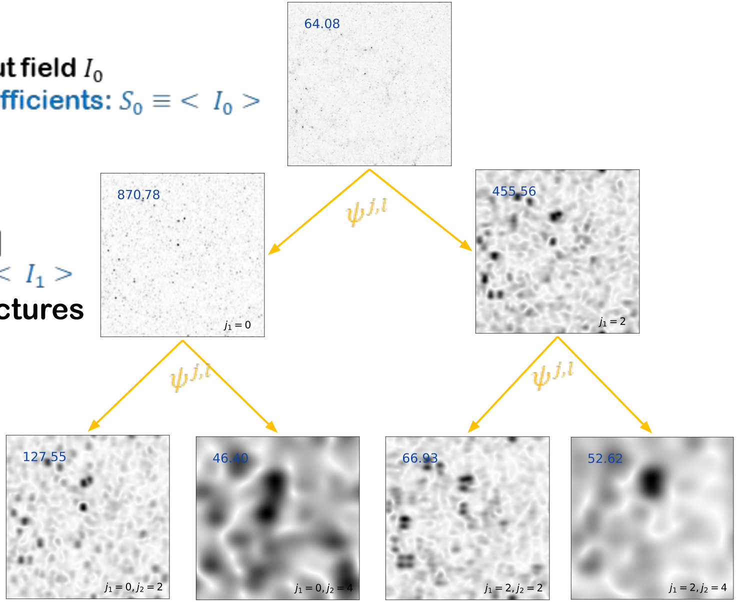
+ modulus

+ mean

Input field I_0
Coefficients: $S_0 \equiv < I_0 >$

Fields: $I_1 \equiv |I_0 \star \psi_1|$
Coefficients: $S_1 \equiv < I_1 >$
clustering of structures

Fields $I_2 \equiv ||I_0 \star \psi_1| \star \psi_2|$
Coefficients: $S_2 \equiv < I_2 >$
clustering of clustering



(all the convolved fields here has orientation index $l_1 = 1, l_2 = 1$)

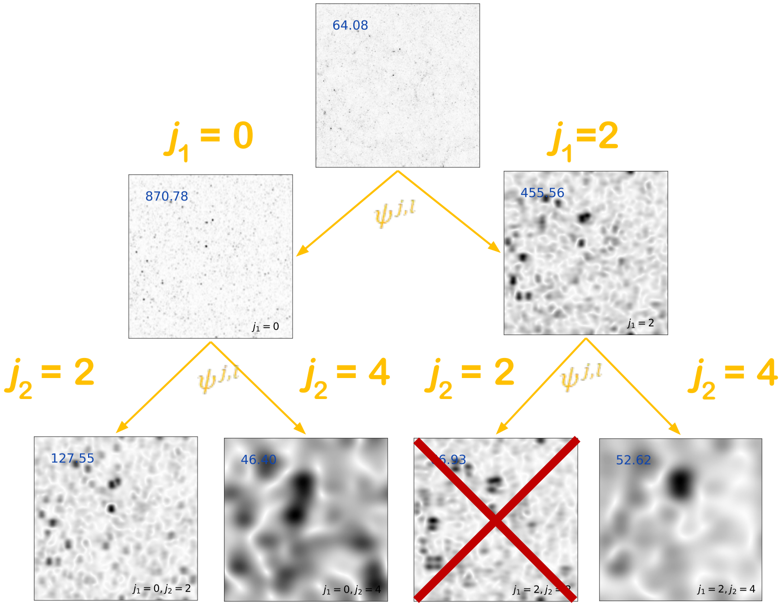
(All the numbers here are 10^6 times the real coefficients

Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=
wavelet convolution
+ modulus
+ mean

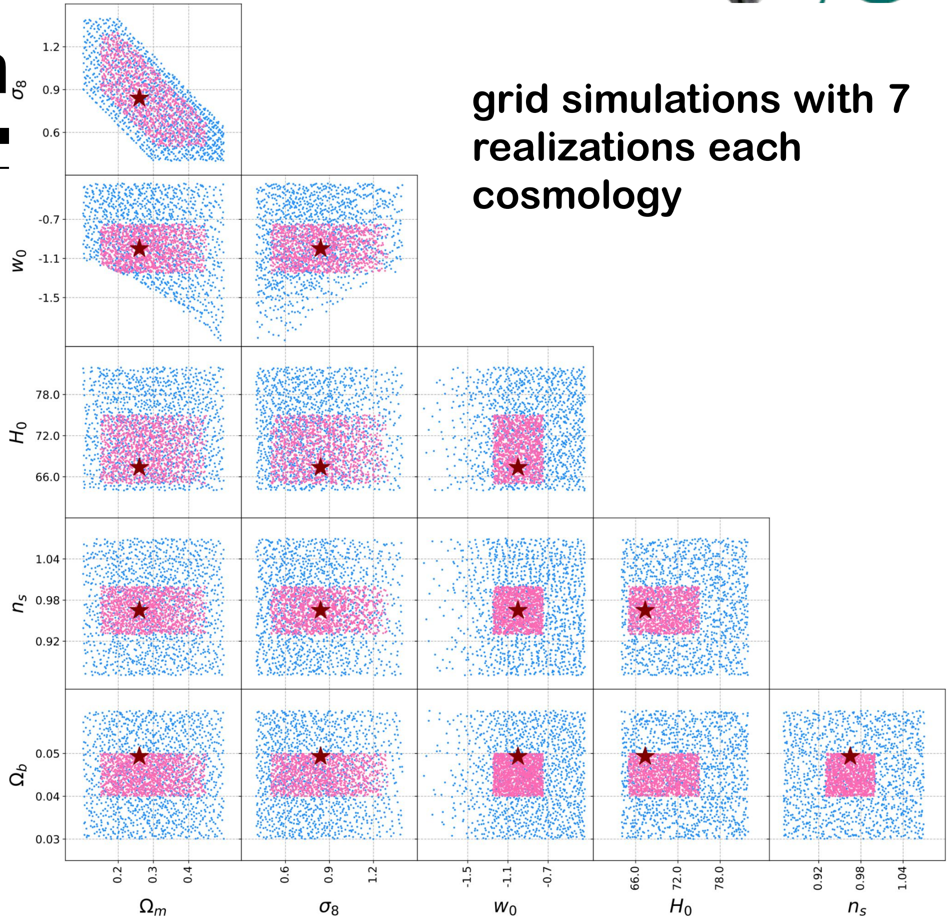


When doing second order convolution, choose filter ψ^{j_2,l_2} **with** $j_2 > j_1$

CosmoGridV1 Simulation

	fiducial	Δ fid.	wide grid prior	narrow grid prior
Ω_{m}	0.26	$\pm$ 0.01	$\in$ [0.10, 0.50]	$\in$ [0.15, 0.45]
σ_8	0.84	$\pm$ 0.015	$\in$ [0.40, 1.40]	$\in$ [0.50, 1.30]
w_0	-1	$\pm$ 0.05	$\in$ [-2.00, -0.33]	$\in$ [-1.25, -0.75]
n_s	0.9649	$\pm$ 0.02	$\in$ [0.87, 1.07]	$\in$ [0.93, 1.00]
Ω_b	0.0493	$\pm$ 0.001	$\in$ [0.03, 0.06]	$\in$ [0.04, 0.05]
H_0	67.3	$\pm$ 2.0	$\in$ [64.0, 82.0]	$\in$ [65.0, 75.0]

Kacprzak et al. 2022



grid simulations with 7 realizations each cosmology

fiducial simulations with 200 realizations each cosmology

N-body simulations with baryon feedback effect, 6.8′ x 6.8′ each pixel

Square Maps Extraction

40°×40°, 15 square maps from each full sky map, 8′ x 8′ each pixel

Scattering Coefficient

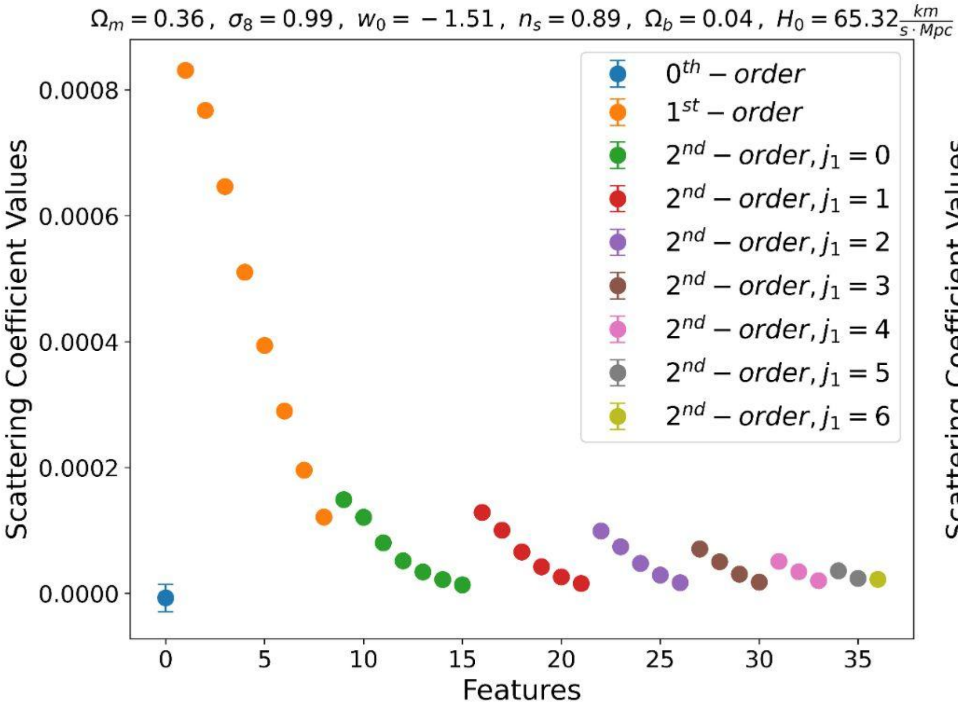
- **When using isotropic fields, the scattering coefficients S_n could be further reduced by taking the average over all orientation indices of same scale**

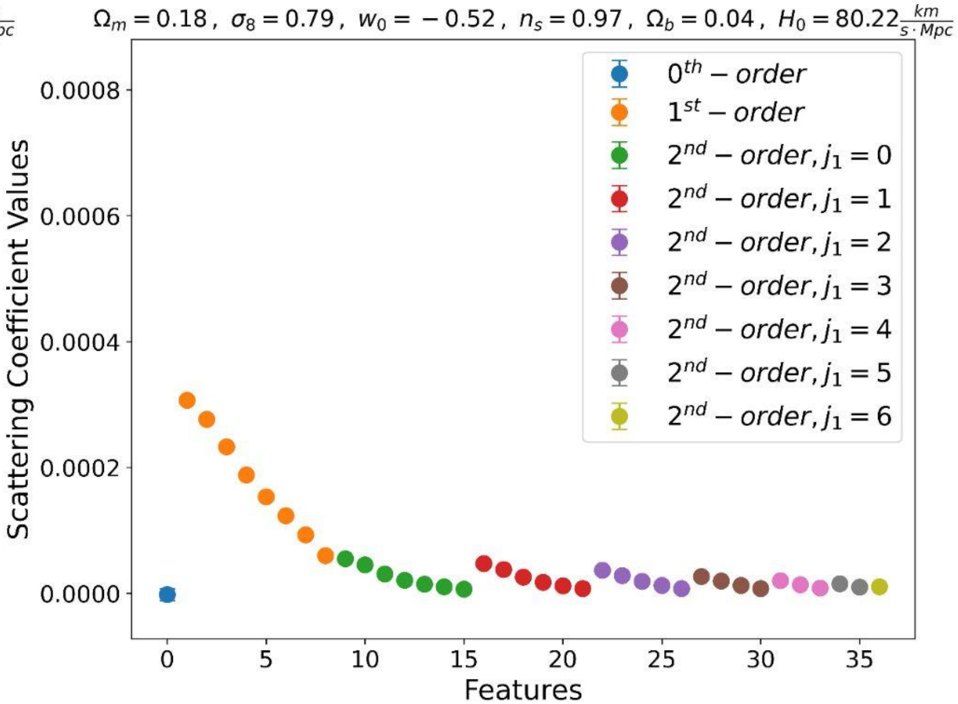
$$s_0 \equiv S_0$$

$$s_1(j_1) \equiv \langle S_1^{j_1, l_1} \rangle_{l_1}$$

$$s_2(j_1, j_2) \equiv \langle S_2^{j_1, l_1, j_2, l_2} \rangle_{l_1, l_2}$$

Scattering Coefficient

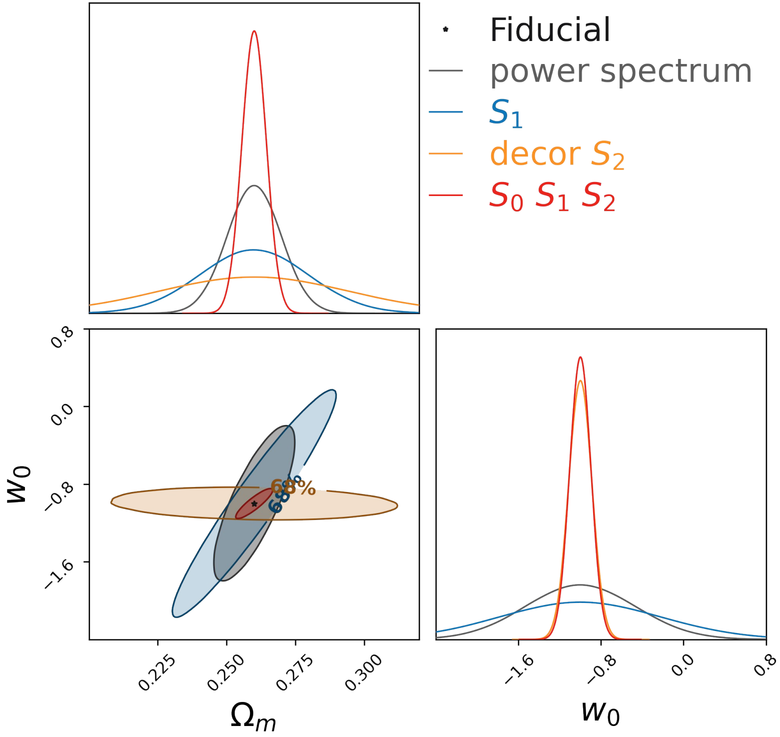




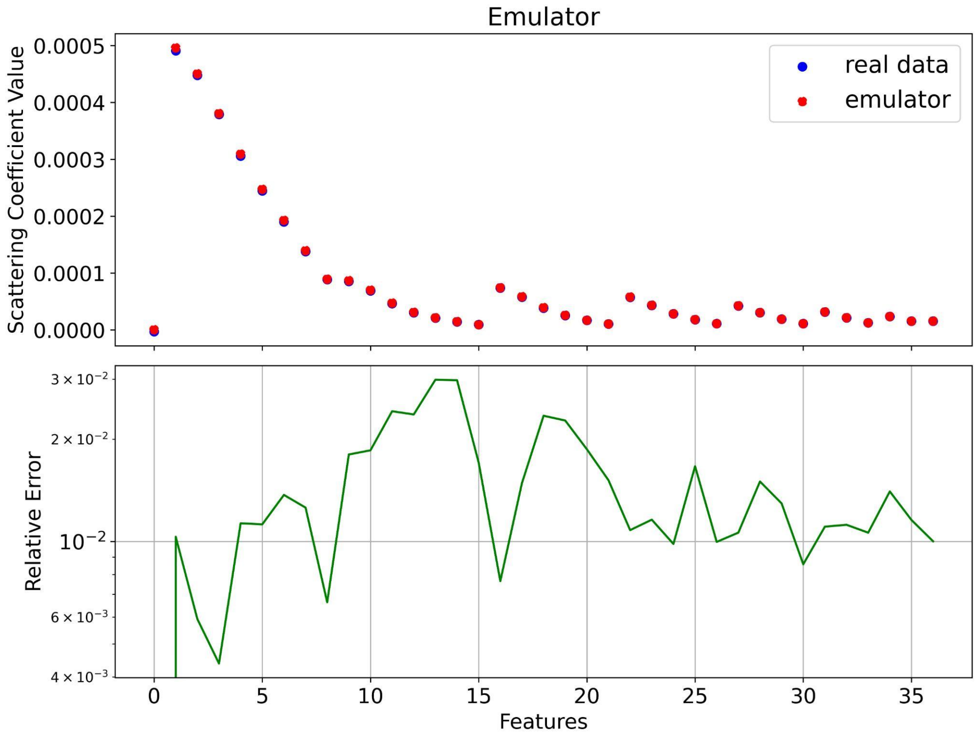
Fisher Forecast

De-correlated 2nd order coefficients:

$$\frac{s_2}{s_1}$$



Emulator



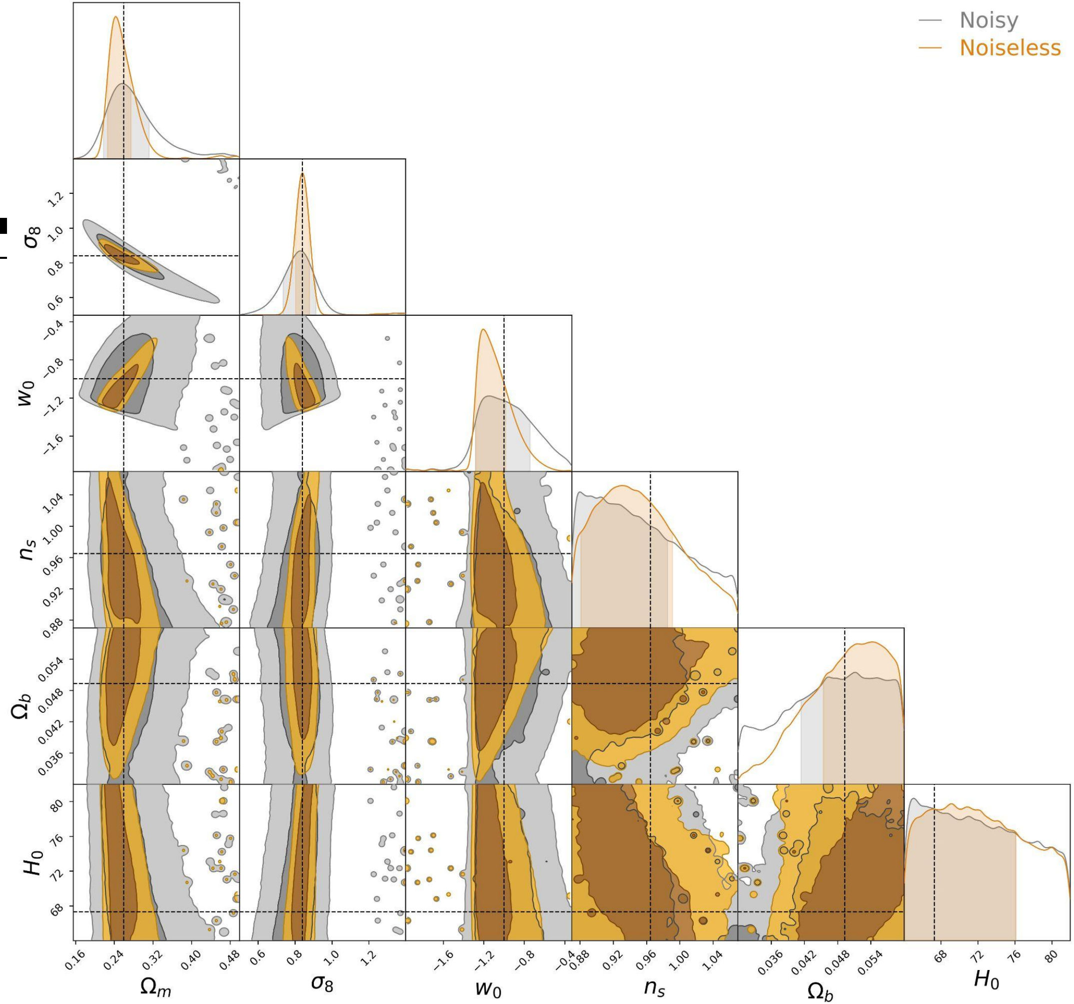
Emulator
Scattering Coefficient Value
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
real data
emulator
Relative Error
3×10^{-2}
2×10^{-2}
10^{-2}
6×10^{-3}
4×10^{-3}
0
5
10
15
20
25
30
35
Features

MCMC

$$\sigma^2 = \frac{\varepsilon^2}{An_g}$$

ε = 0.26 : shape noise
A : area of each pixel
n_g = 5.59 : galaxy number density per arcmin^2

(DES Collaboration et al. 2022)



Future Work

- Shear map test
  - Try scattering transform on γ_1 and γ_2 fields
  - Compare the effect of shear map and convergence map
- Mask test
  - Try ST on masked map so that we could try it on real data
  - Try more tomographic redshift bins