Parity-Odd Power Spectra: Concise Statistics for Cosmological Parity Violation

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Parity

Parity transform:

$$\mathbb{P} : \boldsymbol{x} \to -\boldsymbol{x}$$

$$\mathbb{P} : (x, y, z) \to (-x, -y, -z)$$

Parity transformation is equal to **mirror** + **rotation** in 3D space. For example, if we want to parity transform a position vector, (x, y, z), we could decompose this transformation into two steps.

1. Mirror the position vector on $x - y$ plane and then we get $(x, y, -z)$.
2. Rotate the position vector around z axis by π and finally get $(-x, -y, -z)$

Vector and Pseudovector

- **Vector**: Parity odd under parity. $\mathbb{P} : \boldsymbol{A} \to -\boldsymbol{A}$
- **Pseudovector**: Parity even under parity. $\boldsymbol{A} = \boldsymbol{B} \times \boldsymbol{C}$ (imagine angular momentum $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$).

$$\mathbb{P}\boldsymbol{A} = (-\boldsymbol{B}) \times (-\boldsymbol{C}) = \boldsymbol{B} \times \boldsymbol{C}$$

Scalar and Pseudoscalar

- **Scalar**: Parity even under parity. $\mathbb{P}a \to a$
- **Pseudoscalar**: Parity odd under parity. $s = \boldsymbol{A} \cdot (\boldsymbol{B} \times \boldsymbol{C})$.

$$\mathbb{P}s = -\boldsymbol{A} \cdot ((-\boldsymbol{B}) \times (-\boldsymbol{C})) = -s$$

Scalar field

- Under parity, a scalar field $\delta(\boldsymbol{x})$ transforms as:

$$\mathbb{P} : \delta(\boldsymbol{x}) \to \delta(-\boldsymbol{x})$$

- the modes of the field transform according to

$$\mathbb{P} : \delta(\boldsymbol{k}) \to \delta(-\boldsymbol{k})$$

- If $\delta(\boldsymbol{x})$ is a real-valued field, its modes satisfy $\delta(-k) = \delta^*(k)$. Parity replaces the modes of a field with the complex conjugate of the modes.

$$\begin{aligned}\mathbb{P} : \langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\ldots\delta(\boldsymbol{k}_N)\rangle &\mapsto \langle \delta(-\boldsymbol{k}_1)\delta(-\boldsymbol{k}_2)\ldots\delta(-\boldsymbol{k}_N)\rangle \\ &= \langle \delta(\boldsymbol{k}_1)^*\delta(\boldsymbol{k}_2)^*\ldots\delta(\boldsymbol{k}_N)^*\rangle\end{aligned}$$

  For parity even quantities, the value does not change under parity, while for parity odd quantities, the value changes the sign under parity, which means that parity-violating part of any scalar field's N-point spectrum is the imaginary part of that spectrum.

Power Spectrum

Power spectrum:

$$\left\langle \delta\left(\boldsymbol{k}\right)\delta\left(\boldsymbol{k'}\right)\right\rangle = \left(2\pi\right)^3 \delta_D^{(3)}\left(\boldsymbol{k}+\boldsymbol{k'}\right) P\left(k\right)$$

The power spectrum is an average over the squared amplitudes of the field's modes: $\delta\left(\boldsymbol{k}\right)\delta\left(-\boldsymbol{k}\right) = \delta\left(\boldsymbol{k}\right)\delta^*\left(\boldsymbol{k}\right) = \left|\delta\left(\boldsymbol{k}\right)\right|^2$, which is purely real. So, power spectrum is **NOT** sensitive to parity.

Bispectrum

$$\langle \delta(\boldsymbol{k}_1)\,\delta(\boldsymbol{k}_2)\,\delta(\boldsymbol{k}_3)\rangle = (2\pi)^3\,\delta_D^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,B(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$

The parity even property is less clear here.

- One way to think of this is to construct a pseudoscalar, which is parity odd. To achieve this, we take three vectors to form a triple product, $\boldsymbol{k}_1 \cdot (\boldsymbol{k}_2 \times \boldsymbol{k}_3)$. However, the Dirac delta function enforces $\boldsymbol{k}_1 = -(\boldsymbol{k}_2 + \boldsymbol{k}_3)$, so these three vectors are on the same plane and the triple product vanish. Therefore, there is no way to form a parity-odd three-point function.

Bispectrum

- Another way to understand the parity even is that the three wave vectors of the bispectrum form a closed triangle. Then the parity transform of a triangle in three dimensions is equivalent to a rotation. For example. if the triangle lies in the $k_z = 0$ plane, its parity transformation is the same as a rotation around the k_z-axis by π. Thus the bispectrum cannot carry information about parity due to isotropy.

Figure: Adapted from Jamieson et.al 2024

Trispectrum

Trispectrum is the lowest-order correlation function that is sensitive to parity for a scalar-like field since we could have three linearly independent wave vectors to construct triple product $\boldsymbol{k}_1 \cdot (\boldsymbol{k}_2 \times \boldsymbol{k}_3)$.

$$\langle \delta(\boldsymbol{k}_1)\,\delta(\boldsymbol{k}_2)\,\delta(\boldsymbol{k}_3)\,\delta(\boldsymbol{k}_4)\rangle = (2\pi)^3\,\delta_D^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4) \times T\left(k_1, k_2, k_3, k_4, |\boldsymbol{k}_1+\boldsymbol{k}_2|, |\boldsymbol{k}_3+\boldsymbol{k}_4|\right)$$

Figure: Adapted from Jamieson et.al 2024

Tetrahedron

- Among the set of 24 tetrahedron, we could split them into two groups of 12 by the sign of $\boldsymbol{k}_1 \cdot (\boldsymbol{k}_2 \times \boldsymbol{k}_3)$, and this is the *handedness* or helicity of the tetrahedron.
- Define the tetrahedrons to be right-handed configurations if $\boldsymbol{k}_1 (\cdot \boldsymbol{k}_2 \times \boldsymbol{k}_3) > 0$, and left-handed configurations for $\boldsymbol{k}_1 (\cdot \boldsymbol{k}_2 \times \boldsymbol{k}_3) < 0$.
- isolate the trispectrum for only right-handed and obtain the right-handed trispectrum $T_R (\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ and similarly for left-handed trispectrum $T_L (\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$. Let $\pi_{123} \equiv \mathrm{sgn} (\boldsymbol{k}_1 \cdot (\boldsymbol{k}_2 \times \boldsymbol{k}_3))$, then

$$T_{R/L} (\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \frac{1 + \pi_{123}}{2} T_R (\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \frac{1 - \pi_{123}}{2} T_L (\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

Tetrahedron

- If left- and right-handed parts of the trispectrum are not necessarily equal, this indicates that the parity trispectrum has a parity-odd component. For a real field, a parity transformation is complex conjugate in Fourier space:

$$\Phi\left(-\boldsymbol{k}\right) = \Phi^{*}\left(\boldsymbol{k}\right)$$

- The parity even component of the trispectrum is the real part and is proportional to the sum of $T_R + T_L$, the parity-odd component is the imaginary part and is proportional to the difference $T_R - T_L$:

$$\begin{aligned} T\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right) = & \frac{1}{2}\left(T_R\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right) + T_L\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right)\right) + \\ & \frac{\pi_{123}}{2}\left(T_R\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right) - T_L\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right)\right) \\ & = T_{+}\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right) + iT_{-}\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\right). \end{aligned}$$

Tetrahedron

The parity-odd part (iT_-) must be proportional to the triple product, so we can further parameterize its shape,

$$T_-(k_1, k_2, k_3, k_4, |\boldsymbol{k}_1+\mathbf{k}_2|, |\boldsymbol{k}_1 + \boldsymbol{k}_4|) = \boldsymbol{k}_1 \cdot (\boldsymbol{k}_2 \times \boldsymbol{k}_3) \times \tau_-(k_1, k_2, k_3, k_4, |\boldsymbol{k}_1 + \boldsymbol{k}_2|, |\boldsymbol{k}_1 + \boldsymbol{k}_4|)$$

Since $\langle \delta(\boldsymbol{k}_1)\, \delta(\boldsymbol{k}_2)\, \delta(\boldsymbol{k}_3)\, \delta(\boldsymbol{k}_4) \rangle$ is symmetric under interchange of any two wave vectors, τ_- must be antisymmetric.

Vector Field

Under parity, a vector field $\boldsymbol{V}(\boldsymbol{x})$ transforms,

$$\mathbb{P} : \boldsymbol{V}(\boldsymbol{x}) \to -\boldsymbol{V}(-\boldsymbol{x})$$

For the modes of the fields, we can choose an orthonormal basis $\boldsymbol{e}_i(\boldsymbol{k})$ such that,

$$\boldsymbol{V}(\boldsymbol{k}) = \boldsymbol{e}_i(\boldsymbol{k}) V^i(\boldsymbol{k})$$

Vector Field

- Choose the longitudinal unit basis vector,

$$\boldsymbol{e}_{\parallel}(\boldsymbol{k}) = i\frac{\boldsymbol{k}}{k}$$

- Pick any unit vector $\boldsymbol{n}(\boldsymbol{k})$ that is not parallel to $\boldsymbol{k}$, construct transverse basis:

$$\mathbf{e}_{+}(\mathbf{k}) = -\frac{\mathbf{k}\times\mathbf{n}(\mathbf{k})}{\sqrt{k^2-(\mathbf{k}\cdot\mathbf{n})^2}} \quad \mathbf{e}_{-}(\mathbf{k}) = -\frac{i\mathbf{k}\times(\mathbf{k}\times\mathbf{n}(\mathbf{k}))}{k\sqrt{k^2-(\mathbf{k}\cdot\mathbf{n})^2}}$$

- Decompose the vector field:

$$\mathbf{V}(\mathbf{k}) = \mathbf{e}_{\parallel}(\mathbf{k})V^{\parallel}(\mathbf{k}) + \mathbf{e}_{+}(\mathbf{k})V^{-}(\mathbf{k}) + \mathbf{e}_{-}(\mathbf{k})V^{+}(\mathbf{k})$$

Vector Field

- Decomposition:

$$\mathbf{V}(\mathbf{k}) = \mathbf{e}_{\parallel}(\mathbf{k})V^{\parallel}(\mathbf{k}) + \mathbf{e}_{+}(\mathbf{k})V^{-}(\mathbf{k}) + \mathbf{e}_{-}(\mathbf{k})V^{+}(\mathbf{k})$$

- Parity property of components and basis:

$$\mathbb{P} : \{\mathbf{e}_{\parallel}(\mathbf{k}), \mathbf{e}_{+}(\mathbf{k}), \mathbf{e}_{-}(\mathbf{k})\} \mapsto \{-\mathbf{e}_{\parallel}(-\mathbf{k}), \mathbf{e}_{+}(-\mathbf{k}), -\mathbf{e}_{-}(-\mathbf{k})\}$$
$$\mathbb{P} : \{V^{\parallel}(\mathbf{k}), V^{-}(\mathbf{k}), V^{+}(\mathbf{k})\} \mapsto \{V^{\parallel}(-\mathbf{k}), -V^{-}(-\mathbf{k}), V^{+}(-\mathbf{k})\}$$

- $\langle V^{+}(\mathbf{k})V^{-}(\mathbf{k})\rangle$ is parity odd, so for a vector field we can access parity information at the level of the power spectrum.

Vector Field

- We can also use this to form new basis vectors,

$$\mathbf{e}_{\mathrm{R/L}}(\mathbf{k}) = \frac{1}{\sqrt{2}}\left(\mathbf{e}_{+}(\mathbf{k}) \mp i\mathbf{e}_{-}(\mathbf{k})\right)$$

- Parity swaps the right-handed and left-handed modes

$$\mathbb{P} : \mathbf{e}_{\mathrm{R/L}}(\mathbf{k}) \mapsto \mathbf{e}_{\mathrm{L/R}}(-\mathbf{k})$$

- Parity-odd cross power spectrum:

$$\langle V^{+}(\mathbf{k})V^{-}(\mathbf{k}')\rangle = \frac{1}{2}\langle V^{\mathrm{R}}(\mathbf{k})V^{\mathrm{R}}(\mathbf{k}')\rangle - \frac{1}{2}\langle V^{\mathrm{L}}(\mathbf{k})V^{\mathrm{L}}(\mathbf{k}')\rangle$$

parity-odd cross-power spectrum from the transverse components of the vector field measures the difference between power in right-handed modes and left-handed modes.

POP

- Parity-Odd Power (POP) spectra, a novel set of observables for probing parity violation in cosmological N -point statistics. POP spectra are derived from composite fields obtained by applying nonlinear transformations, involving also gradients, curls, and filtering functions, to a scalar field. This compresses the parity-odd trispectrum into a power spectrum. These new statistics offer several advantages: they are computationally fast to construct, estimating their covariance is less demanding compared to estimating that of the full parity-odd trispectrum, and they are simple to model theoretically.
- Vector-pseudovector power spectrum: correlate a vector and a pseudovector
- Scalar-pseudoscalar power spectrum: correlate a scalar and a pseudoscalar

Vector POP Spectrum

$$\Phi_a(\boldsymbol{x}) = \int \mathrm{d}^3 y f_a(\boldsymbol{x}-\boldsymbol{y})\,\Phi(\boldsymbol{y}) = \int_{\boldsymbol{k}} f_a(\boldsymbol{k})\,\Phi(\boldsymbol{k})\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

- Construct composite vector and pseudovector fields out of a scalar field such that the crosspower spectrum between them is a compressed estimator of the parity-odd trispectrum. By taking the gradient of a scalar, $\Phi_a(\mathbf{x})$, we get a vector $\nabla_a\Phi(\mathbf{x})$.
- For two *distinct* filtering functions, $f_a(\mathbf{x})$ and $f_b(\mathbf{x})$, the composite vector field $\mathbf{V}_{ab}(\mathbf{x}) \equiv \Phi_a(\boldsymbol{x})\,\nabla_b\Phi(\boldsymbol{x})$ is not the gradient of a scalar, so it has a curl component. Decompose it,

$$\mathbf{V}_{ab}(\mathbf{x}) = \nabla\phi_{ab}(\boldsymbol{x}) - \nabla\times\boldsymbol{A}_{ab}(\boldsymbol{x})$$

where $\phi_{ab}(\boldsymbol{x}) \equiv \frac{1}{2}\Phi_a(\boldsymbol{x})\,\Phi_b(\boldsymbol{x})$

Vector POP Spectrum

$$\mathbf{V}_{ab}(\mathbf{x}) = \nabla\phi_{ab}(\boldsymbol{x}) - \nabla \times \boldsymbol{A}_{ab}(\boldsymbol{x})$$

- Since $\mathbf{V}_{ab}$ and spatial derivatives are parity odd, $\mathbf{A}_{ab}$ is a pseudovector. (If the two fields, Φ_a and Φ_b, were identical, $\mathbf{A}_{ab}$ would vanish). By taking the curl of $\mathbf{V}_{ab}(\mathbf{x})$, we could get

$$\boldsymbol{A}_{ab}(\boldsymbol{k}) = -\frac{1}{k^2} i\boldsymbol{k} \times \boldsymbol{V}_{ab}(\boldsymbol{k})$$

- We are free to choose four distinct filter functions $f_a(\boldsymbol{x})$, $f_b(\boldsymbol{x})$, $f_c(\boldsymbol{x})$ and $f_d(\boldsymbol{x})$. From the first two, we define the vector field $\mathbf{V}_{ab}(\mathbf{x})$. From the second two, we define the pseudovector field $\mathbf{A}_{cd}(\mathbf{x})$. The dot product $\boldsymbol{V}_{ab}(\boldsymbol{x}) \cdot \boldsymbol{A}_{cd}(\boldsymbol{x})$ is parity odd. Then we define POP spectrum,

$$\langle \boldsymbol{V}_{ab}(\boldsymbol{k}) \cdot \boldsymbol{A}_{cd}(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}+\boldsymbol{k}') P_{vector}(\boldsymbol{k})$$

Vector POP Spectrum

- Explicitly, we can express the power spectrum as

$$P_{\text{vector}}(K) = \frac{1}{\mathcal{N}_{\text{vector}}(k)} \int_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4} \Big[(2\pi)^6 \delta_D^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{K}) \delta_D^{(3)}(\boldsymbol{k}_3 + \boldsymbol{k}_4 + \boldsymbol{K})\, f_a(\boldsymbol{k}_1) f_b(\boldsymbol{k}_2) f_c(\boldsymbol{k}_3) f_d(\boldsymbol{k}_4) \left[\boldsymbol{K} \cdot (\boldsymbol{k}_1 \times \boldsymbol{k}_3)\right]^2 \times \tau_-(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4, \boldsymbol{K}, |\boldsymbol{k}_1 + \boldsymbol{k}_4|) \Big]$$

- If apply isotropic filters, P_{vector} does not vanish if parity-odd part is non-zero.

$$\tau_-(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4, K, \tilde{K}) = \mathcal{N}_{\tau_-}^{-1} \int_{\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4} -i\, \mathbf{q}_1 \cdot (\mathbf{q}_2 \times \mathbf{q}_3) \langle \Phi(\mathbf{q}_1)\Phi(\mathbf{q}_2)\Phi(\mathbf{q}_3)\Phi(\mathbf{q}_4) \rangle$$

- The normalization factor $\mathcal{N}_{vector}(\boldsymbol{k})$ is chosen to be

$$\mathcal{N}_{\text{vector}}(K) = \int_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4} (2\pi)^6 \delta_D^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{K}) \delta_D^{(3)}(\boldsymbol{k}_3 + \boldsymbol{k}_4 + \boldsymbol{K}) f_a(\boldsymbol{k}_1) f_b(\boldsymbol{k}_2) f_c(\boldsymbol{k}_3) f_d(\boldsymbol{k}_4) \left[\boldsymbol{K} \cdot (\boldsymbol{k}_1 \times \boldsymbol{k}_3)\right]^2$$

Vector POP Spectrum

- The vector-pseudovector construction compresses the trispectrum down to a power spectrum by correlating two fields of order Φ^2.
- The vector POP spectrum is a weighted average over all trispectrum configurations while holding fixed the diagonal side length k, which is the argument of this POP spectrum. For this reason, the vector POP carries crucial information about the soft limit where the **diagonal** approaches zero.

Scalar POP Spectrum

- Construct another POP spectrum by cross-correlating a scalar field with a pseudoscalar field. First is to construct a pseudovector,

$$\boldsymbol{B}_{bc}(\boldsymbol{x}) = \nabla\Phi_b(\boldsymbol{x}) \times \nabla\Phi_c(\boldsymbol{x})$$

$$\boldsymbol{B}_{bcd}(\boldsymbol{K}) = -f_d(\boldsymbol{K}) \int_{\boldsymbol{k}_3,\boldsymbol{k}_4} (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}_3 + \boldsymbol{k}_4 - \boldsymbol{K}) \\ f_b(\boldsymbol{k}_3) f_c(\boldsymbol{k}_4)(\boldsymbol{k}_3 \times \boldsymbol{k}_4)\Phi(\boldsymbol{k}_3)\Phi(\boldsymbol{k}_4)$$

- Then construct the triple product field,

$$\Psi_{abcd}(\boldsymbol{x}) = \nabla\Phi_a(\boldsymbol{x}) \cdot \boldsymbol{B}_{bcd}(\boldsymbol{x})$$

- Its cross-power spectrum with $\Phi(\mathbf{k})$ is a POP spectrum,

$$\langle \Phi(\boldsymbol{k})\Psi(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k} + \boldsymbol{k}') P_{scalar}(k)$$

Scalar POP Spectrum

- The scalar POP spectrum is given by

$$P_{\text{scalar}}(k) = \frac{1}{\mathcal{N}_{\text{scalar}}(k)} \int_{\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{K}} \Big[(2\pi)^6 \delta_D^{(3)}(\boldsymbol{k}+\boldsymbol{k}_2+\boldsymbol{K}) \\ \delta_D^{(3)}(\boldsymbol{k}_3+\boldsymbol{k}_4-\boldsymbol{K}) \times f_a(\boldsymbol{k}_2) f_b(\boldsymbol{k}_3) f_c(\boldsymbol{k}_4) f_d(\boldsymbol{K}) \\ [\boldsymbol{K}\cdot(\boldsymbol{k}_2\times\boldsymbol{k}_4)]^2 \, \tau_-(k,k_2,k_3,k_4,K,|\boldsymbol{k}_2+\boldsymbol{k}_3|) \Big]$$

- Normalization factor:

$$\mathcal{N}_{\text{scalar}}(k) = \int_{\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{K}} (2\pi)^6 \delta_D^{(3)}(\boldsymbol{k}+\boldsymbol{k}_2+\boldsymbol{K}) \delta_D^{(3)}(\boldsymbol{k}_3+\boldsymbol{k}_4-\boldsymbol{K}) \\ \times f_a(\boldsymbol{k}_2) f_b(\boldsymbol{k}_3) f_c(\boldsymbol{k}_4) f_d(\boldsymbol{K}) \, [\boldsymbol{K}\cdot(\boldsymbol{k}_2\times\boldsymbol{k}_4)]^2$$

Scalar POP Spectrum

- The scalar-pseudoscalar construction compresses the trispectrum down to a power spectrum by correlating two fields of order Φ and Φ^3.
- The scalar POP spectrum is a weighted average over all trispectrum configurations while holding fixed the magnitude of one of the wave vector sides, which is the argument of this POP spectrum. For this reason, the vector POP carries crucial information about the soft limit where the **side** approaches zero.

Random Parity-Violating Realizations

1. Generate grid Gaussian random fields (primordial curvature fields):

$$\zeta_G(\boldsymbol{k}) = \sqrt{\frac{V_{Box}\pi^2 A_s}{k^3}\left(\frac{k}{k_p}\right)^{n_s-1}}\left[N_1(\boldsymbol{k}) + iN_2(\boldsymbol{k})\right]$$

   where N_1 and N_2 are drawn from a standard normal distribution, A_s is the amplitude of the primordial power spectrum, n_s is the spectral tilt and $k_p = 0.05\ \mathrm{Mpc}^{-1}$ is the pivot scale. There are 64 simulations.

2. Non-Gaussian field

$$\zeta(\boldsymbol{x}) = \zeta_{\mathrm{G}}(\boldsymbol{x}) + g_- \nabla\zeta_{\mathrm{G}}^{[\alpha]}(\boldsymbol{x}) \cdot \left[\nabla\zeta_{\mathrm{G}}^{[\beta]}(\boldsymbol{x}) \times \nabla\zeta_{\mathrm{G}}^{[\gamma]}(\boldsymbol{x})\right]$$

   where the coefficient g_- controls the amplitude of the primordial parity-odd trispectrum, and the modes of the fields in the triple product are

$$\zeta_G^{[\alpha]}(\boldsymbol{k}) = k^\alpha \zeta_G(\boldsymbol{k})$$

Random Parity-Violating Realizations

The leading order imaginary part of the trispectrum is given by:

$$T_-(k_1, k_2, k_3) = g_- \, \boldsymbol{k}_1 \cdot (\boldsymbol{k}_2 \times \boldsymbol{k}_3) \left(2\pi^2 A_s\right)^3 \left(k_1^{\alpha-4+n_s} k_2^{\beta-4+n_s} k_3^{\gamma-4+n_s} k_4^0 \mp 23 \text{ signed permutations}\right)$$

The trispectrum is proportional to P^3 and $P(k) \propto k^{-3}$ from prediction of inflation. The RHS of above equation requires $\propto k^{-9}$, meaning $\alpha + \beta + \gamma = -3$. We choose $\alpha = -2, \beta = -1, \gamma = 0$. The filters are follows

$$f_a(k) = k^2 \Theta(k - k_{\text{min}}) \Theta(k_{\text{max}} - k)$$
$$f_c(k) = k^{-2} \Theta(k - k_{\text{min}}) \Theta(k_{\text{max}} - k)$$
$$f_b(k) = f_d(k) = \Theta(k - k_{\text{min}}) \Theta(k_{\text{max}} - k)$$

where $k_{min} = 5 \times 10^{-3} \ \mathrm{Mpc}^{-1} \ h$ and $k_{max} = 2 \times 10^{-1} \ \mathrm{Mpc}^{-1} \ h$.

Vector-Pseudovector Parity-odd Power Spectrum

Figure: Jamieson et al. (2024)

Scalar-Pseudoscalar Parity-odd Power Spectrum

Figure: Jamieson et al. (2024)

Summary

Summary:

- POP is a novel way to extract parity-odd information from cosmological spin-0 field by compressing parity-odd trispectrum information into simple power spectra that is fast to compute

Future work:

- Apply POP from 3D space to weak lensing 2D space by constructing tetrahedron on different tomographic redshift bins