Probe Weak Lensing Cosmology with Scattering Transform

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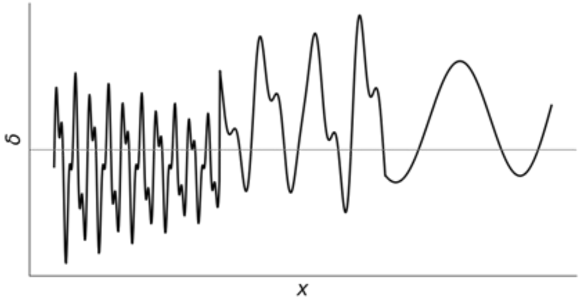
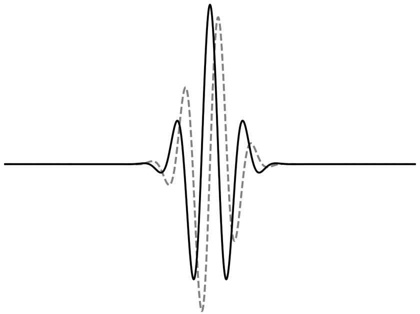
MPE

Traditional Statistics using Fourier Transform

- **Traditional statistics**
  - Measuring 2- or 3-point correlation function
  - Computing Fourier transform of these n-point correlation functions (power spectrum, bispectrum) might be computationally expensive
  - Gaining information in frequency space but no spatial information





Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

I is cosmological field, such as convergence field (function of Ω_m, Ω_{DE}, σ_8...)

S is scattering coefficient

Scattering transform=

wavelet convolution

+ modulus

+ mean

(S. Cheng et al. 2020, A new approach to observational cosmology using the scattering transform)

Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

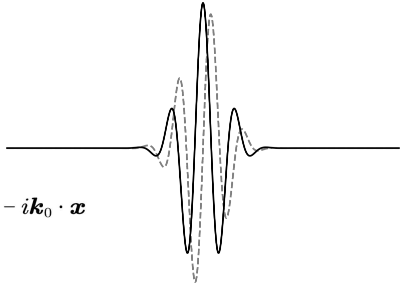
Scattering transform=
wavelet convolution
+ modulus
+ mean

Wavelet: localized oscillating wave

Morlet wavelets:

In real space:

$$\psi(\boldsymbol{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\boldsymbol{x}^T \Sigma^{-1} \boldsymbol{x}/2} e^{-i\boldsymbol{k}_0 \cdot \boldsymbol{x}}$$



$$|k_0| = \frac{3\pi}{4 \times 2^j}$$

Σ : the covariance of matrix describing the size and shape of the Gaussian envelope
k_0 : the frequency of the modulated oscillation

Scattering Transform

Operation:

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Scattering transform=

wavelet convolution

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Morlet Wavelets

$$L = 4,\ \ell = 0,\ 1,\ 2,\ 3$$

- **ℓ : orientation**

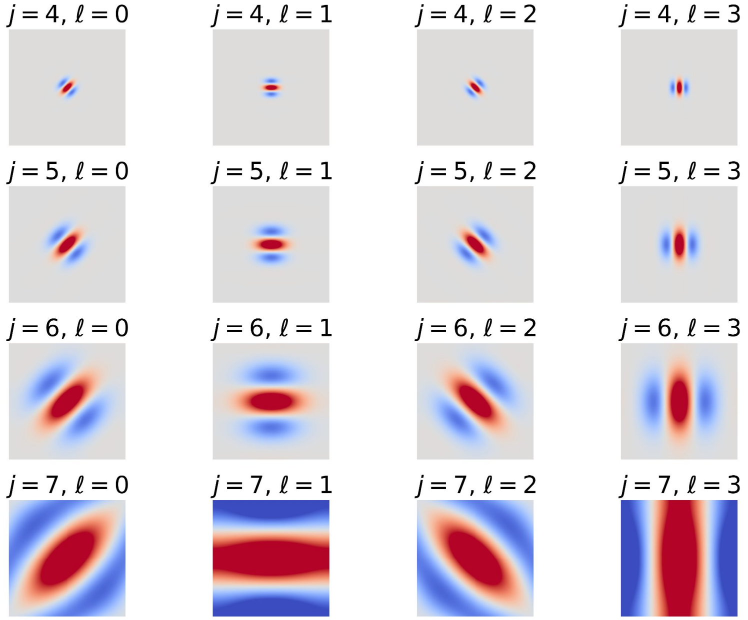
$$J = 8,\ j = 0,\ 1,\ \ldots,\ 7$$

$$\tilde{\psi}(\boldsymbol{k}) = \frac{1}{\sqrt{\Sigma}} e^{-(\boldsymbol{k}-\boldsymbol{k}_0)^T \Sigma (\boldsymbol{k}-\boldsymbol{k}_0)/2}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$

- **j : size (logarithmic spacing)**

Wavelets for scales j and orientation ℓ



Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

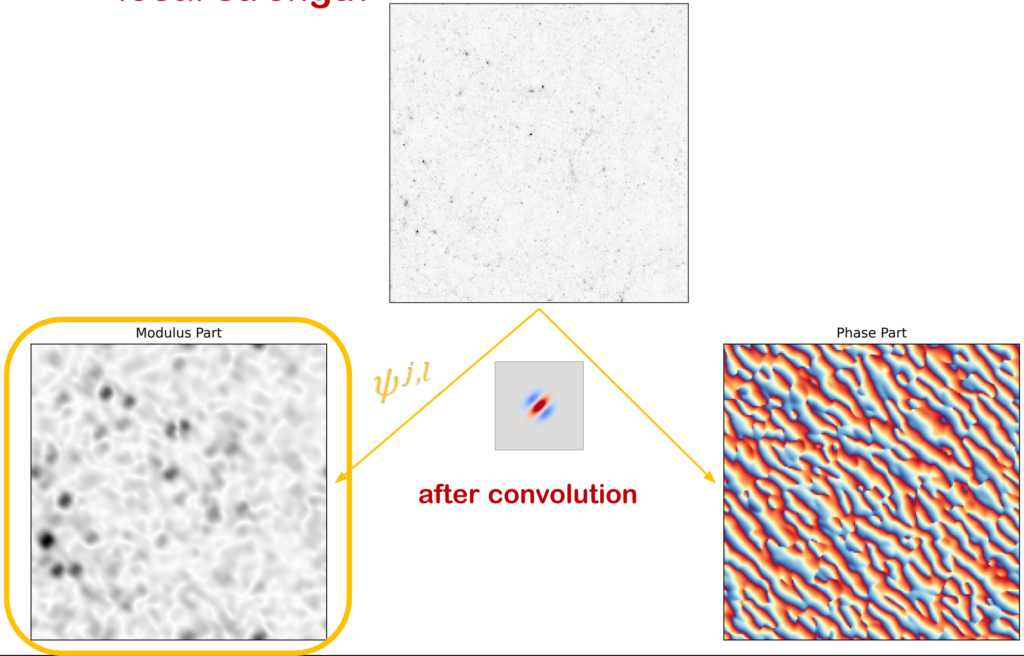
Scattering transform=

wavelet convolution

+ modulus

+ mean

Modulus: convert selected fluctuations into their local strength



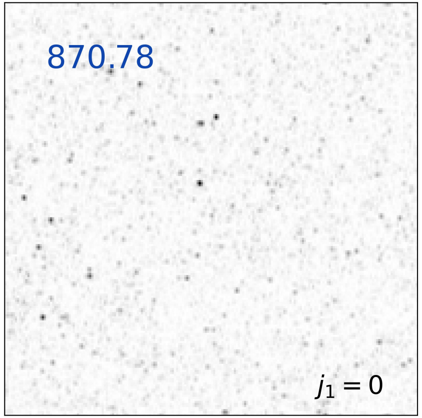
Scattering Transform

Operation:

$$S = \langle | I \star \psi^{j,l} | \rangle$$

Scattering transform=
wavelet convolution
+ modulus
+ mean

Mean: spatial average of the field



$$S_0 \equiv \langle I_0 \rangle \qquad s_0 \equiv S_0$$

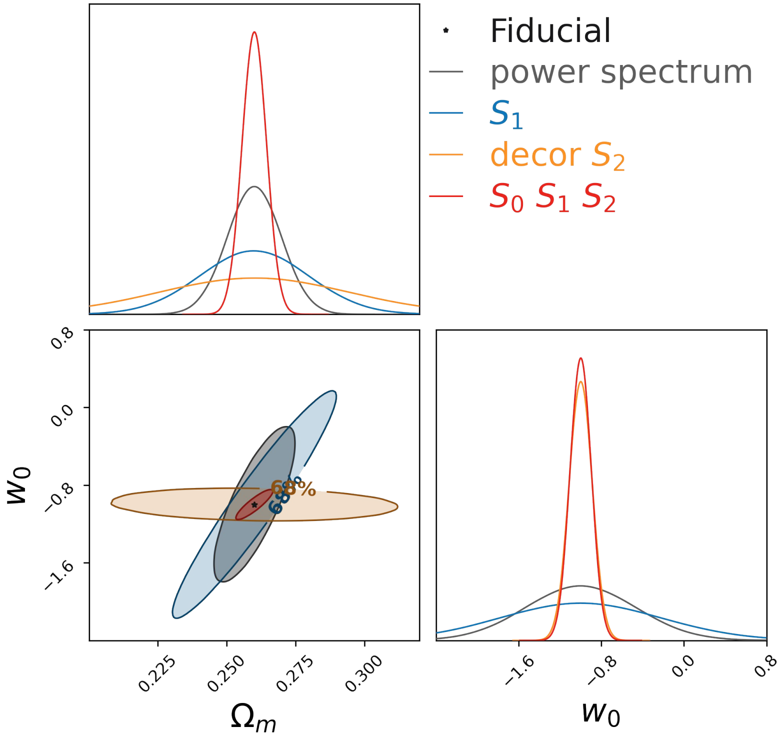
$$S_1^{j_1,l_1} \equiv \langle I_1^{j_1,l_1} \rangle = \langle | I_0 \star \psi^{j_1,l_1} | \rangle \qquad s_1(j_1) \equiv \langle S_1^{j_1,l_1} \rangle_{l_1}$$

$$S_2^{j_1,l_1,j_2,l_2} \equiv \langle I_2^{j_1,l_1,j_2,l_2} \rangle = \langle || I_0 \star \psi^{j_1,l_1} | \star \psi^{j_2,l_2} | \rangle \qquad s_2(j_1,j_2) \equiv \langle S_2^{j_1,l_1,j_2,l_2} \rangle_{l_1,l_2}$$

Fisher Forecast

De-correlated 2nd order coefficients:

$$\frac{s_2}{s_1}$$



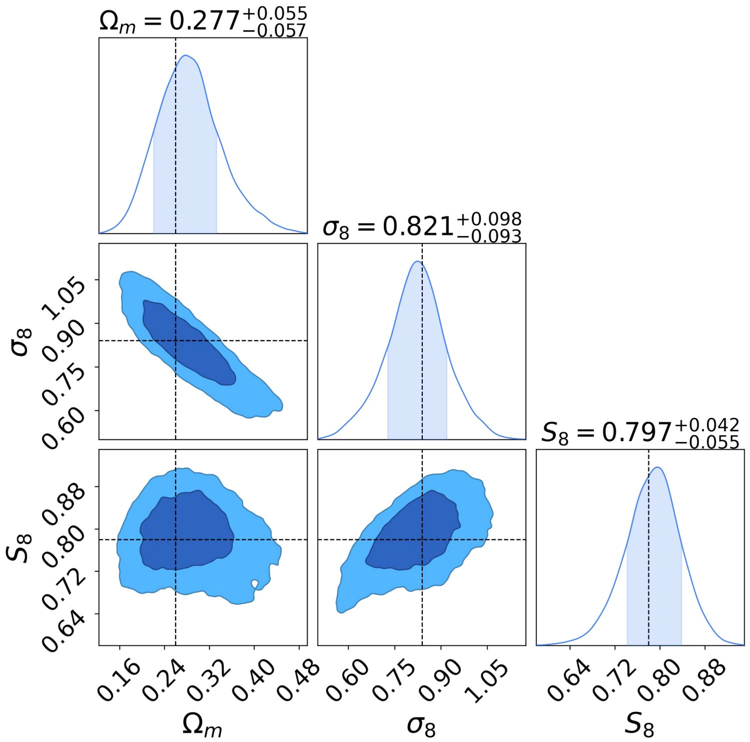
Preliminary result of constraints

$$\kappa_{mock} = (1+m)\,(\kappa_{sim} + A_{IA} \cdot \kappa_{IA}) + shape\ noise$$

- The prior of MCMC

Parameter	Fiducial Value	Prior Distribution
Ω_m	0.26	$\mathcal{U}(0.10, 0.50)$
σ_8	0.84	$\mathcal{U}(0.40, 1.40)$
w_0	-1.00	$\mathcal{U}(-2.00, -0.33)$
n_s	0.9649	$\mathcal{U}(0.87, 1.07)$
Ω_b	0.0493	$\mathcal{U}(0.03, 0.06)$
H_0	67.30	$\mathcal{U}(64, 82)$
$A_{\rm IA}$	0	$\mathcal{U}(-5, 5)$
Δz^1	0.0	$\mathcal{N}(0.0, 0.018)$
Δz^2	0.0	$\mathcal{N}(0.0, 0.015)$
Δz^3	0.0	$\mathcal{N}(0.0, 0.011)$
Δz^4	0.0	$\mathcal{N}(0.0, 0.017)$
m_1	-0.006	$\mathcal{N}(-0.006, 0.009)$
m_2	-0.020	$\mathcal{N}(-0.020, 0.008)$
m_3	-0.024	$\mathcal{N}(-0.024, 0.008)$
m_4	-0.037	$\mathcal{N}(-0.037, 0.008)$

Gatti et al. 2023



Future Work

- Tune hyperparameters and improve the accuracy of emulators

- Apply ST on DES data
  - operate on masked maps
  - try ST on DES Y3 shear maps