
Probe Weak Lensing Cosmology with Scattering Transform

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Gong

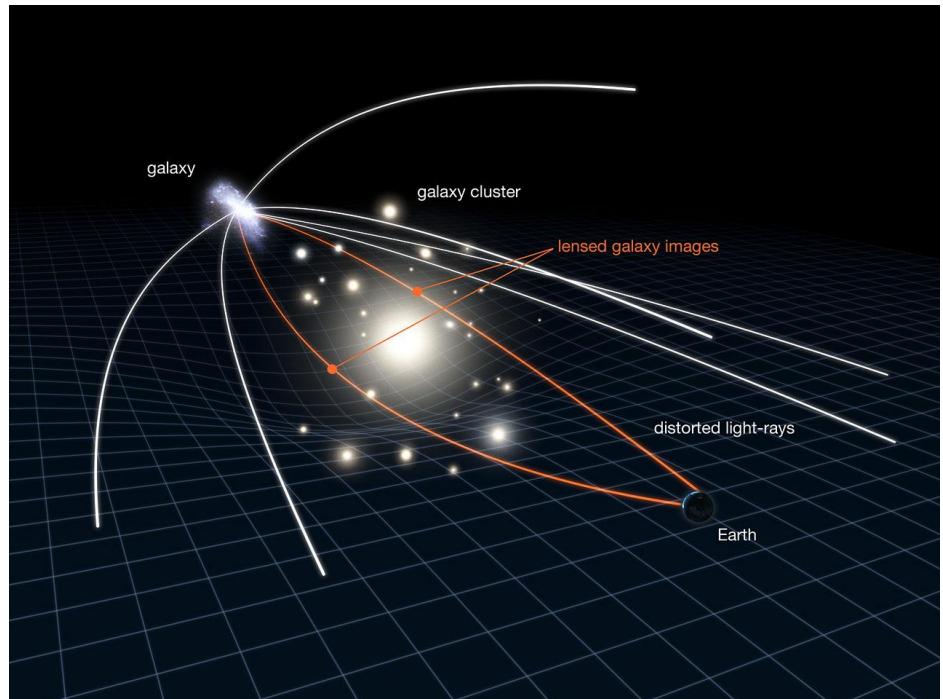


ITP Heidelberg, Feb 26, 2025



Weak Lensing

Light from distant galaxies is slightly bent by matter, subtly distorting their shapes



Weak Lensing

Lens equation:

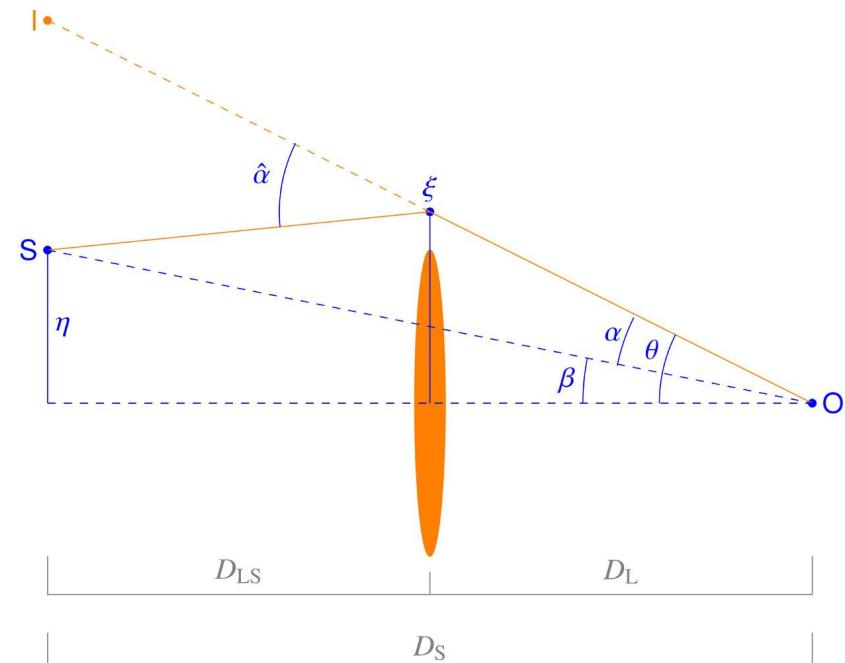
$$\beta = \theta - \alpha$$

Distortion matrix:

$$A = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

$$\kappa = \frac{1}{2} (\partial_1 \partial_1 + \partial_2 \partial_2) \psi, \quad \gamma_1 = \frac{1}{2} (\partial_1 \partial_1 - \partial_2 \partial_2) \psi, \quad \gamma_2 = \partial_1 \partial_2 \psi$$

ψ is the lensing potential (projected 3D gravitational potential)

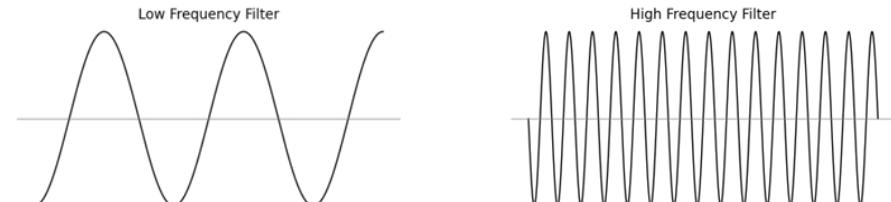
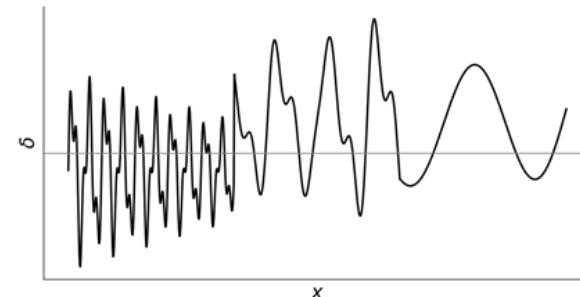


Bartelmann et al. 1996

Traditional Statistics using Fourier Transform

- **Traditional statistics**

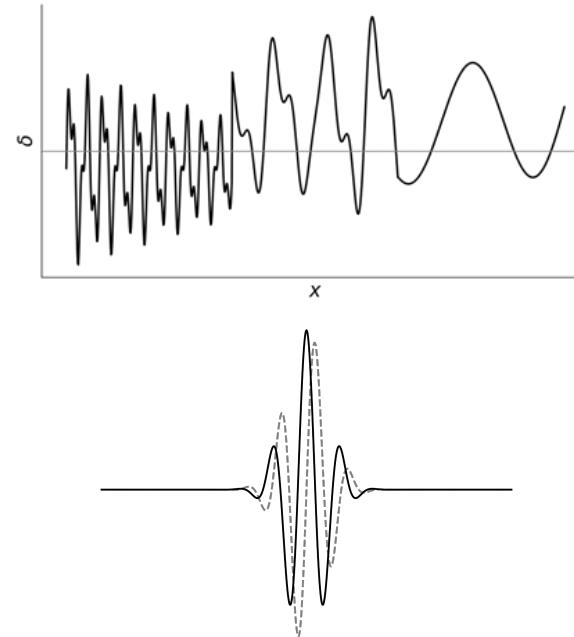
- Measuring 2- or 3-point correlation function
- Computing Fourier transform of these n-point correlation functions (power spectrum, bispectrum) might be computationally expensive
- Gaining information in frequency space but no spatial information



Traditional Statistics using Fourier Transform

- **Traditional statistics**

- Measuring 2- or 3-point correlation function
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Scattering Transform

Operation:

$$S = \langle |I \star \psi^{j,l}| \rangle$$

I is cosmological field, such as convergence field (function of $\Omega_m, \Omega_{DE}, \sigma_8 \dots$)
 S is scattering coefficient

Scattering transform=
wavelet convolution

- + **modulus**
- + **mean**

(S. Cheng et al. 2020, A new approach to observational cosmology using the scattering transform)

Scattering Transform

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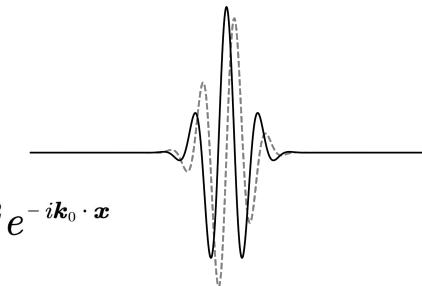
Wavelet: localized oscillating wave

Morlet wavelets:

In real space:

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\mathbf{x}^T \Sigma^{-1} \mathbf{x}/2} e^{-i\mathbf{k}_0 \cdot \mathbf{x}}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$



Σ : the covariance of matrix describing the size and shape of the Gaussian envelope
 k_0 : the frequency of the modulated oscillation

Scattering Transform

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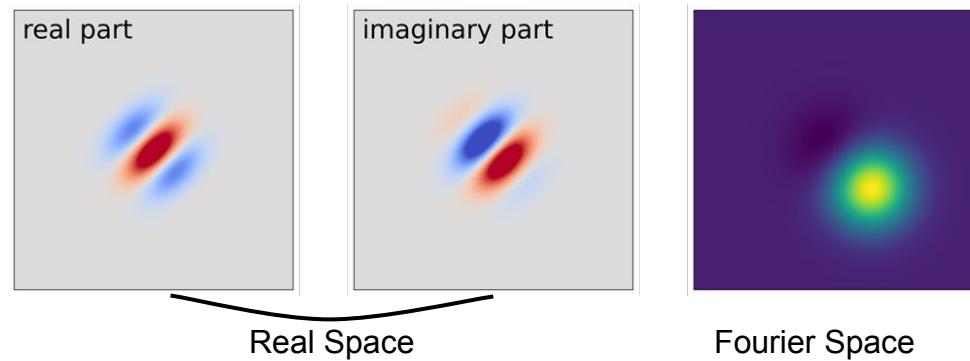
Morlet Wavelets

In real space:

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{|\Sigma|}} e^{-\mathbf{x}^T \Sigma^{-1} \mathbf{x}/2} e^{-i\mathbf{k}_0 \cdot \mathbf{x}}$$

In Fourier space

$$\tilde{\psi}(\mathbf{k}) = \frac{1}{\sqrt{\Sigma}} e^{-(\mathbf{k} - \mathbf{k}_0)^T \Sigma (\mathbf{k} - \mathbf{k}_0)/2}$$



Scattering Transform

Operation:

$$S = \langle |I \star \psi^{j,l}| \rangle$$

$$L = 4, \ell = 0, 1, 2, 3$$

- ℓ : orientation

Scattering transform=
wavelet convolution

- + **modulus**
- + **mean**

$$J = 8, j = 0, 1, \dots, 7$$

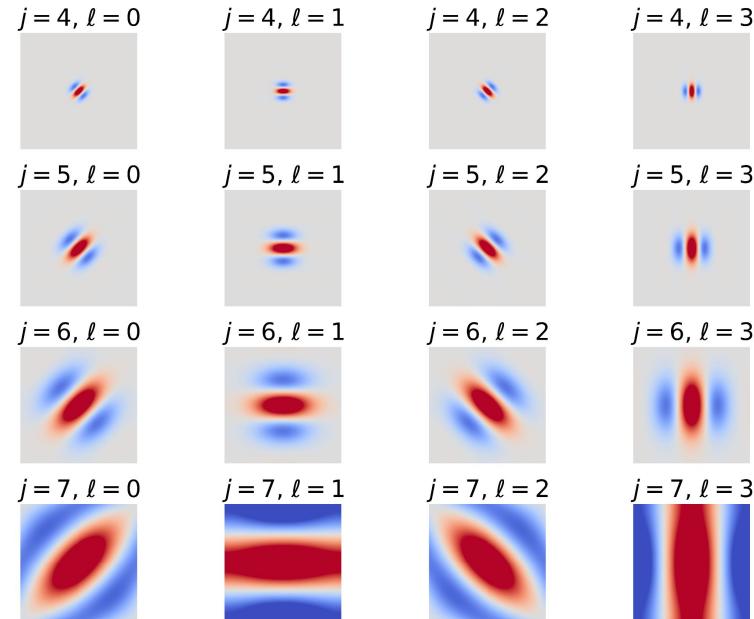
$$\tilde{\psi}(\mathbf{k}) = \frac{1}{\sqrt{\Sigma}} e^{-(\mathbf{k} - \mathbf{k}_0)^T \Sigma (\mathbf{k} - \mathbf{k}_0) / 2}$$

$$|k_0| = \frac{3\pi}{4 \times 2^j}$$

- j : size (logarithmic spacing)

Morlet Wavelets

Wavelets for scales j and orientation ℓ



Scattering Transform

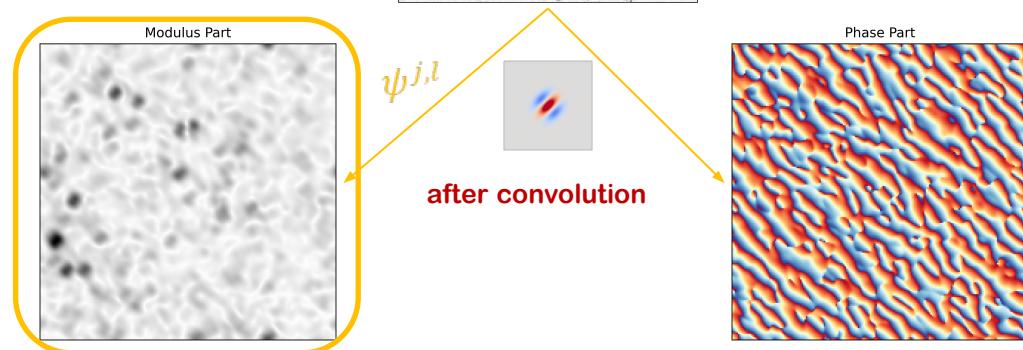
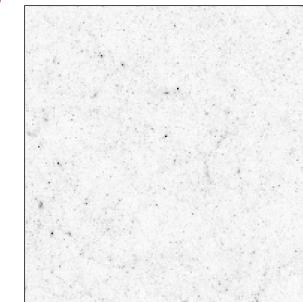
Operation:

$$S = \langle |I \star \psi^{j,l}| \rangle$$

Scattering transform=
wavelet convolution

- + **modulus**
- + **mean**

Modulus: convert selected fluctuations into their local strength

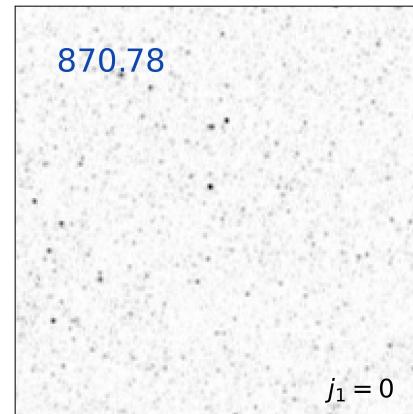


Scattering Transform

Operation:

$$S = \langle |I \star \psi^{j,l}| \rangle$$

Mean: spatial average of the field



Scattering transform=
wavelet convolution

- + **modulus**
- + **mean**

$$S_0 \equiv \langle I_0 \rangle$$

$$S_1^{j_1, l_1} \equiv \langle I_1^{j_1, l_1} \rangle = \langle |I_0 \star \psi^{j_1, l_1}| \rangle$$

$$S_2^{j_1, l_1, j_2, l_2} \equiv \langle I_2^{j_1, l_1, j_2, l_2} \rangle = \langle ||I_0 \star \psi^{j_1, l_1} \star \psi^{j_2, l_2}| \rangle$$

Scattering Transform

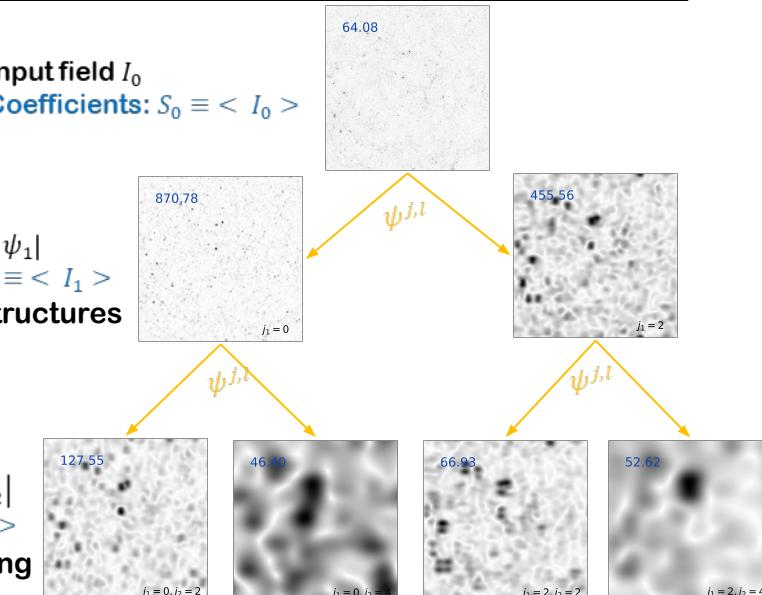
Operation:

$$S = \langle |I \star \psi^{j,l}| \rangle$$

Scattering transform=
wavelet convolution

- + **modulus**
- + **mean**

Fields $I_1 \equiv |I_0 \star \psi_1|$
Coefficients: $S_1 \equiv \langle I_1 \rangle$
clustering of structures



(all the convolved fields here has orientation index $l_1 = 1, l_2 = 1$)

(All the numbers here are 10^6 times the real coefficients)

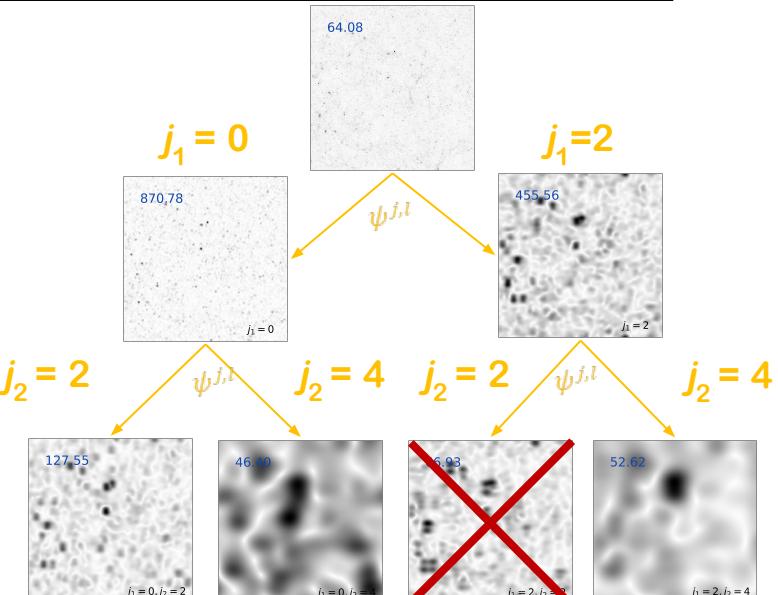
Scattering Transform

Operation:

$$S = \langle |I \star \psi^{j,l}| \rangle$$

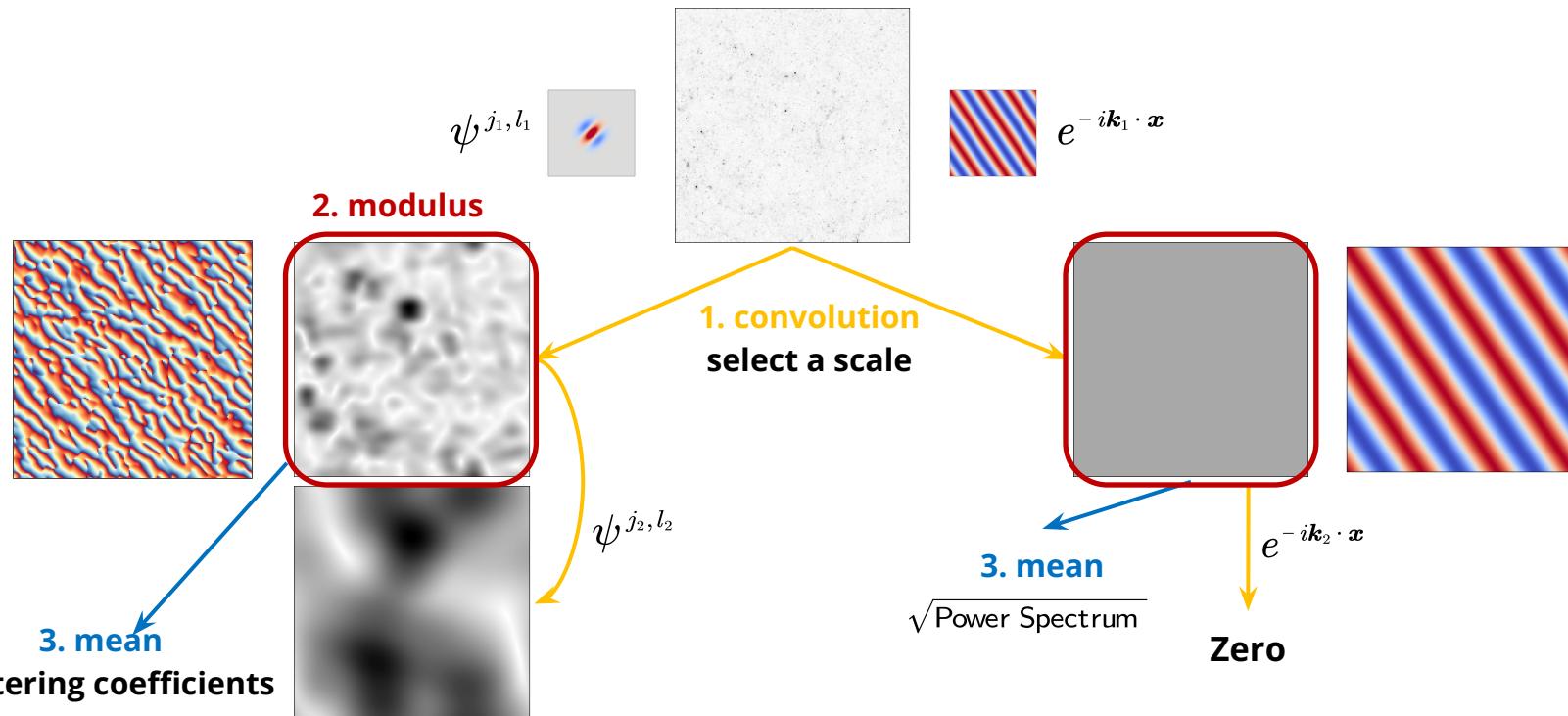
Scattering transform=
wavelet convolution

- + **modulus**
- + **mean**



When doing second order convolution,
choose filter ψ^{j_2, l_2} with $j_2 > j_1$

Scattering vs. Power Spectrum



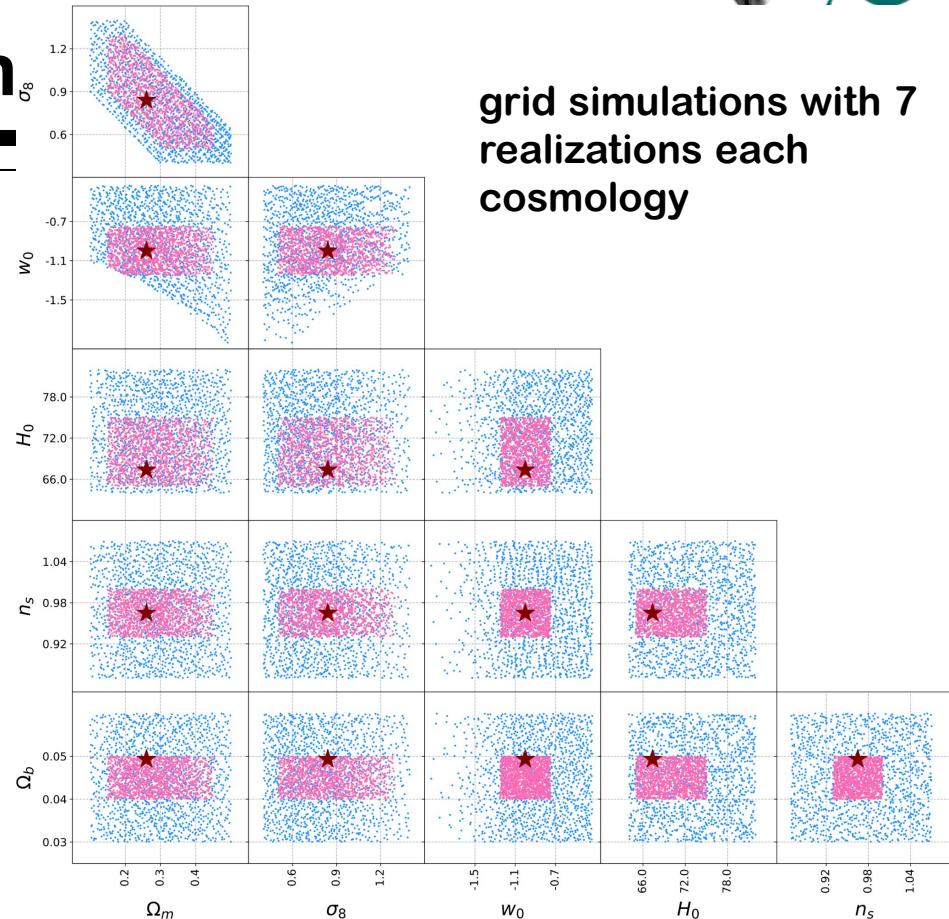
CosmoGridV1 Simulation

	fiducial	Δ fid.	wide grid prior	narrow grid prior
Ω_m	0.26	± 0.01	$\in [0.10, 0.50]$	$\in [0.15, 0.45]$
σ_8	0.84	± 0.015	$\in [0.40, 1.40]$	$\in [0.50, 1.30]$
w_0	-1	± 0.05	$\in [-2.00, -0.33]$	$\in [-1.25, -0.75]$
n_s	0.9649	± 0.02	$\in [0.87, 1.07]$	$\in [0.93, 1.00]$
Ω_b	0.0493	± 0.001	$\in [0.03, 0.06]$	$\in [0.04, 0.05]$
H_0	67.3	± 2.0	$\in [64.0, 82.0]$	$\in [65.0, 75.0]$

Kacprzak et al. 2022

fiducial simulations with 200 realizations each cosmology

N-body simulations with baryon feedback effect, 6.8' x 6.8' each pixel



grid simulations with 7 realizations each cosmology

Square Maps Extraction



$40^\circ \times 40^\circ$, 15 square maps from each full sky map, $8' \times 8'$ each pixel

Scattering Coefficient

- When using isotropic fields, the scattering coefficients S_n could be further reduced by taking the average over all orientation indices of same scale

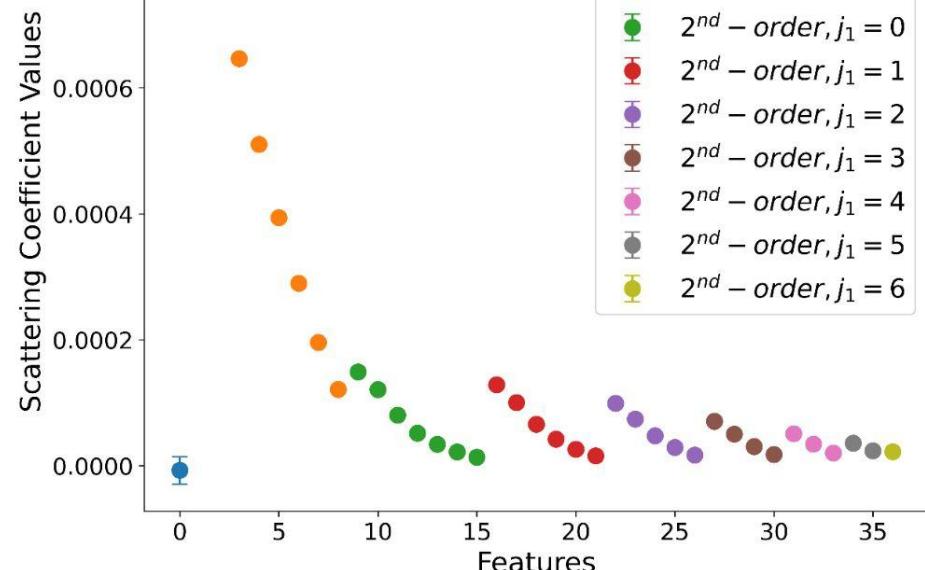
$$s_0 \equiv S_0$$

$$s_1(j_1) \equiv \langle S_1^{j_1, l_1} \rangle_{l_1}$$

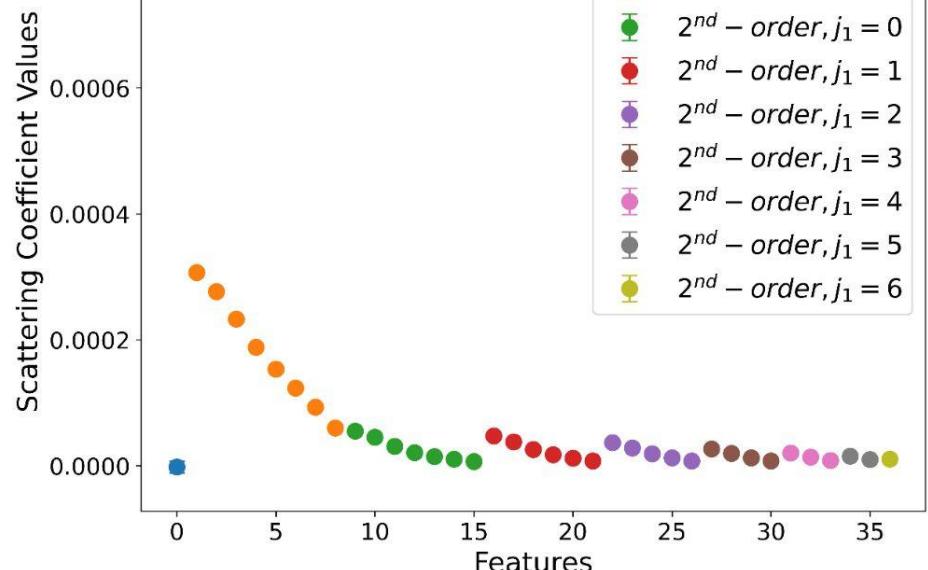
$$s_2(j_1, j_2) \equiv \langle S_2^{j_1, l_1, j_2, l_2} \rangle_{l_1, l_2}$$

Scattering Coefficient

$$\Omega_m = 0.36, \sigma_8 = 0.99, w_0 = -1.51, n_s = 0.89, \Omega_b = 0.04, H_0 = 65.32 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$



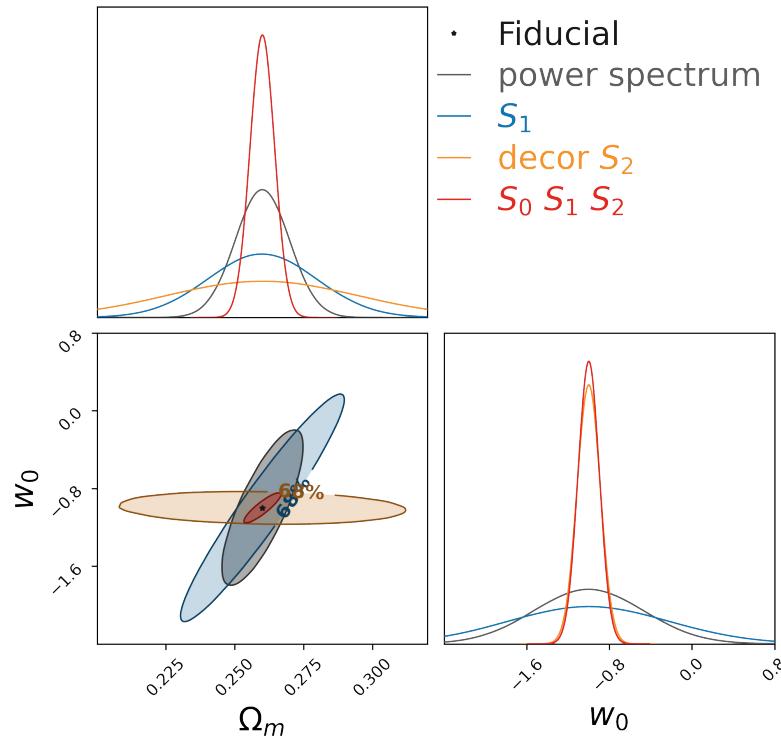
$$\Omega_m = 0.18, \sigma_8 = 0.79, w_0 = -0.52, n_s = 0.97, \Omega_b = 0.04, H_0 = 80.22 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$



Fisher Forecast

De-correlated 2nd order coefficients:

$$\frac{S_2}{S_1}$$



Emulator

Input parameters:

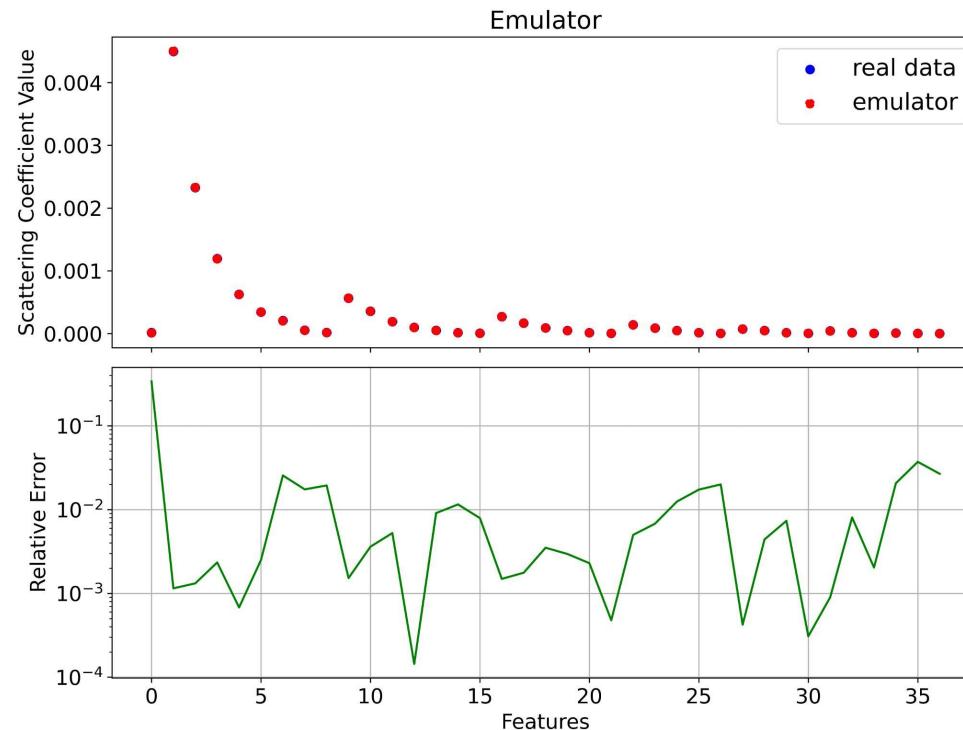
$$\Omega_m, \sigma_8, w_0, n_s, \Omega_b, H_0$$

Output features:

$$\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2$$

37 features in total

$$1 \mathbf{s}_0, 8 \mathbf{s}_1, 28 \mathbf{s}_2$$



MCMC

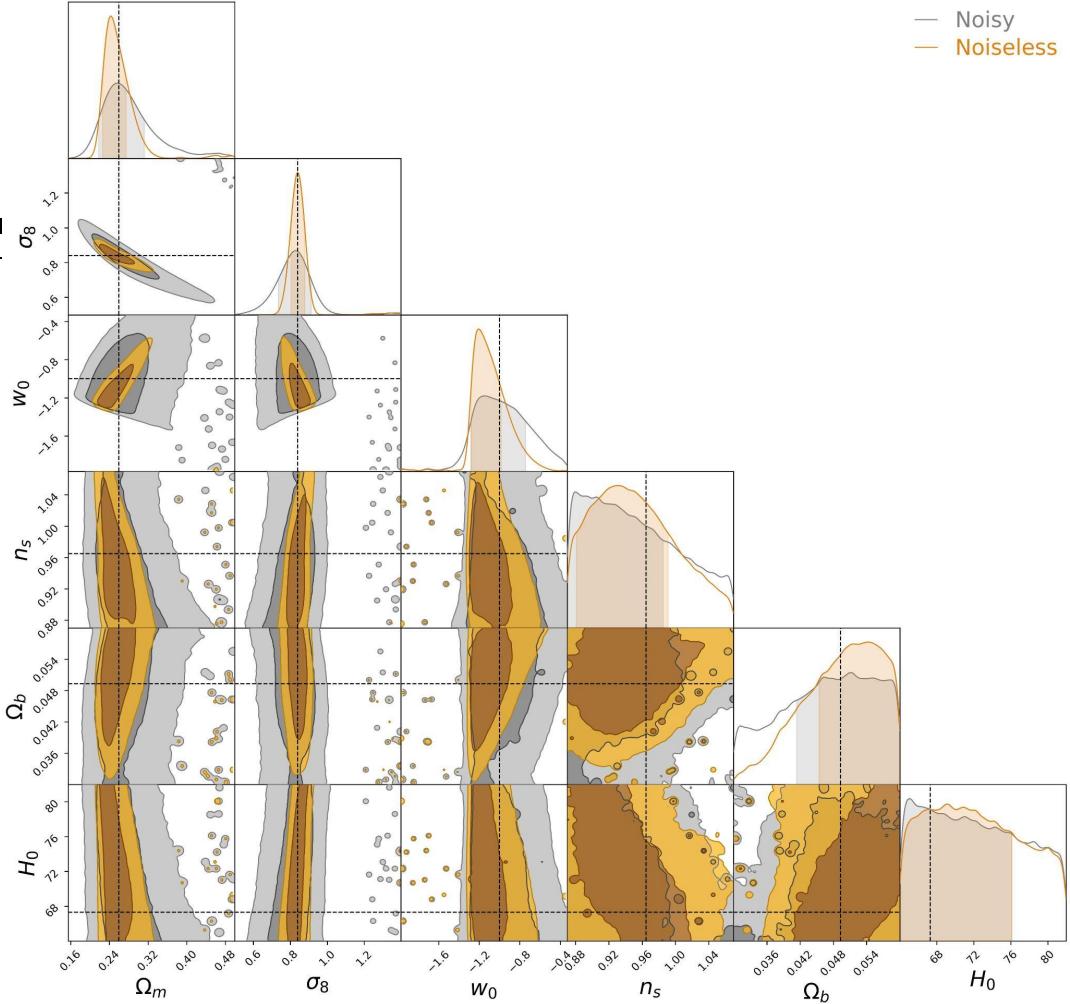
$$\sigma^2 = \frac{\varepsilon^2}{An_g}$$

$\varepsilon = 0.26$: shape noise

A : area of each pixel

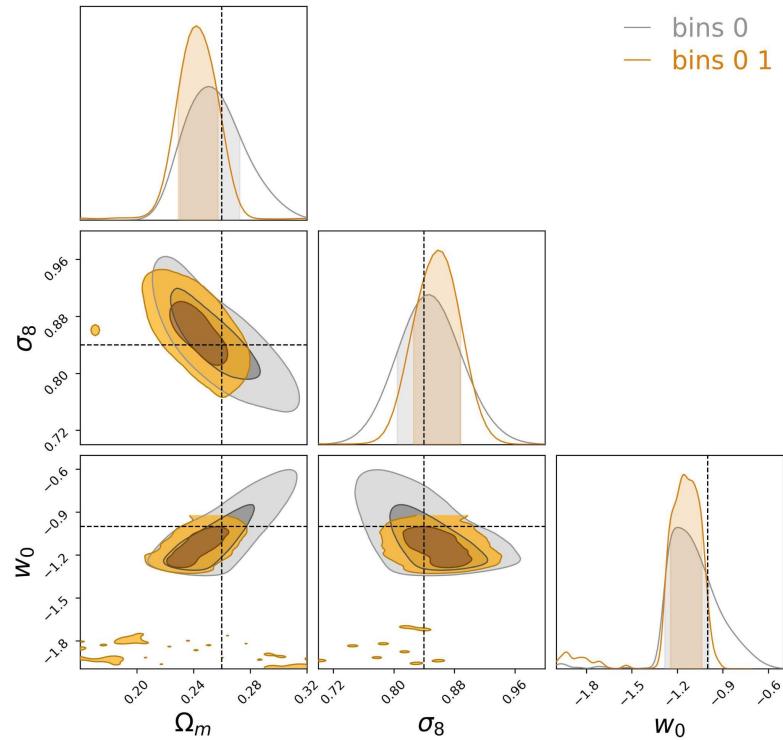
$n_g = 5.59$: galaxy number density per arcmin²

(DES Collaboration et al. 2022)

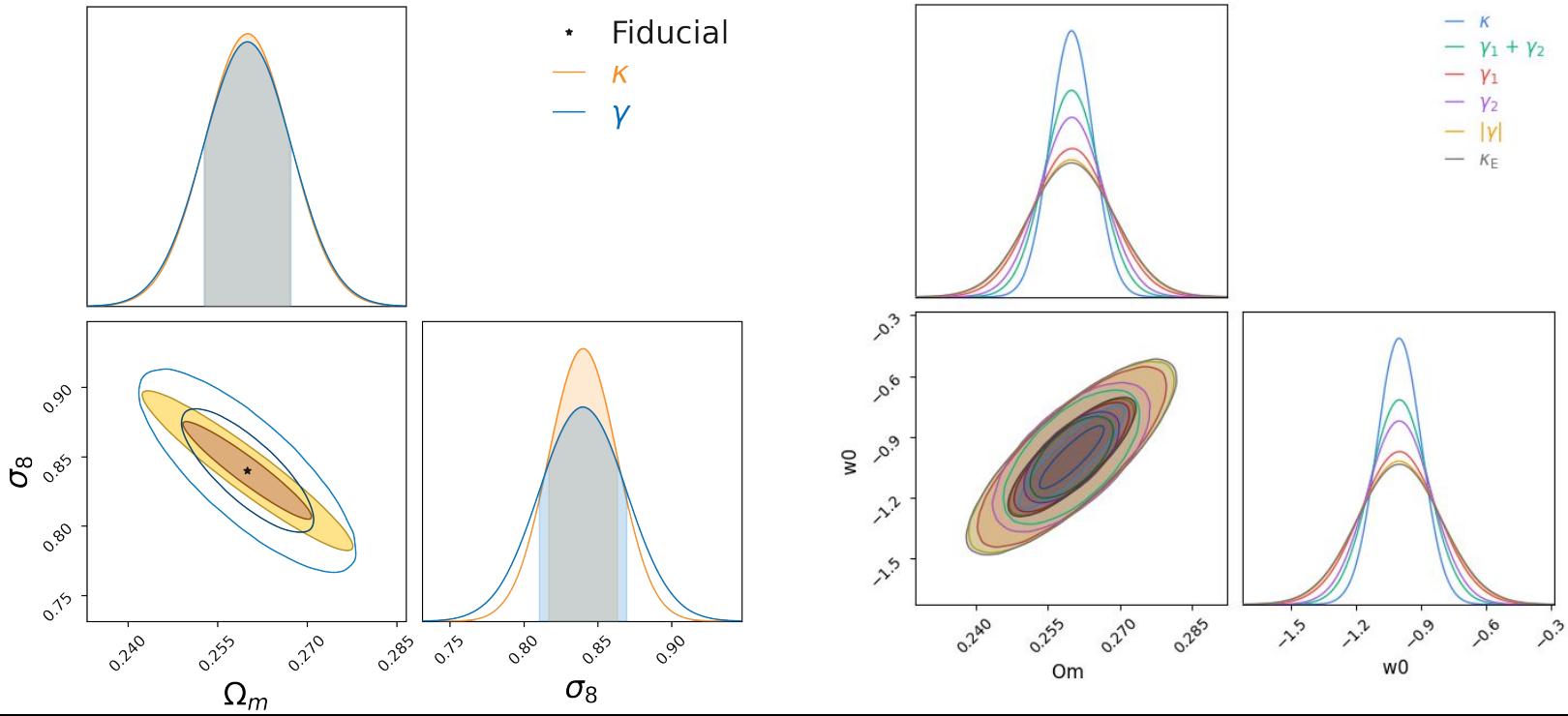


Tomographic Analysis

- Build emulators on redshift bins 0 and 1
- Tomographic analysis



Shear maps



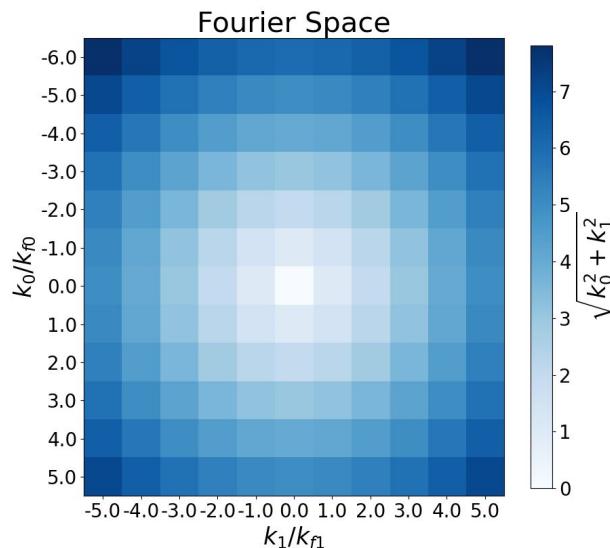
Power Spectrum

Discrete Fourier Transform

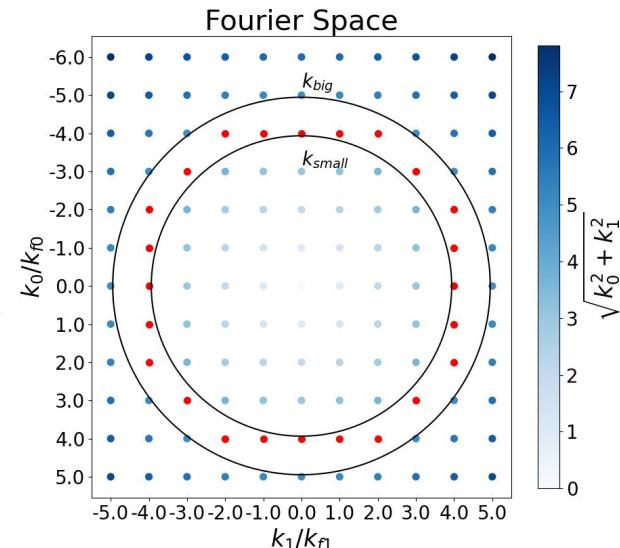
$$F(k_n) = \sum_{n=0}^{2N} f(x) e^{-ik_n x}$$

$$k_n = \frac{2\pi}{L} [0, 1, 2, \dots, N, -N, -(N-1), \dots, -2, -1]$$

Here is the length of the signal



shift min frequency to center



Average value of the elements in the same logarithmic bins

Future Work (doing now)

- **Apply ST on DES data**
 - operate on masked maps
 - use real DES data



Power Spectrum

$$F(k_n) = \sum_{n=0}^{2N} f(x) e^{-ik_n x}$$

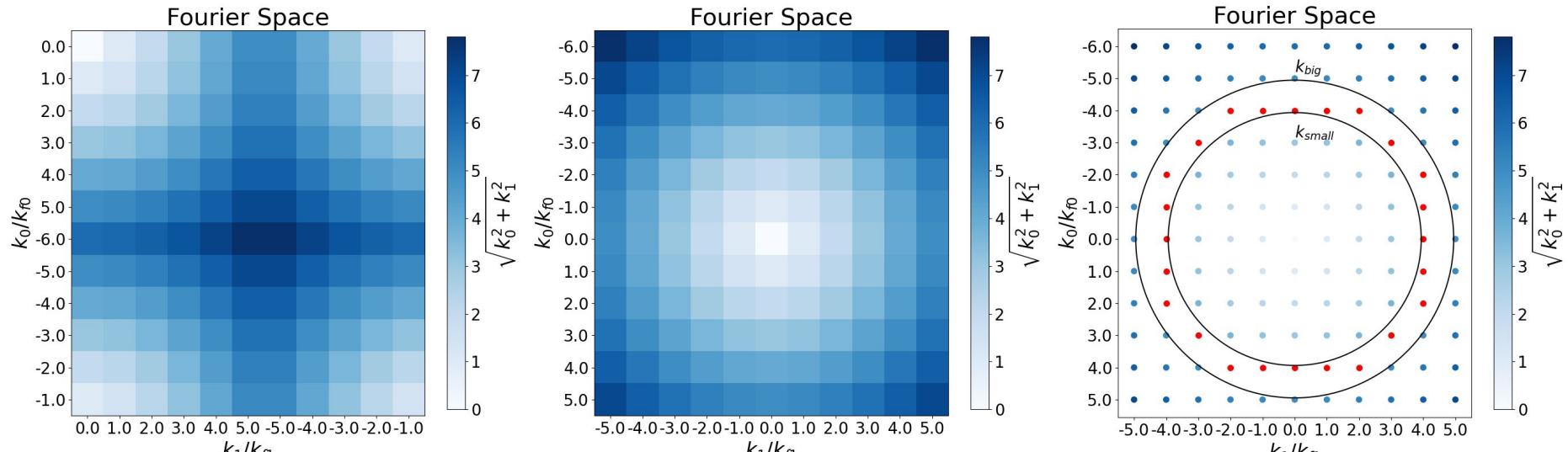
$$k_n = \frac{2\pi}{L} [0, 1, 2, \dots, N, -N, -(N-1), \dots, -2, -1]$$

$$k_n = \frac{2\pi}{L} [0, 1, 2, \dots, N, N+1, N+2, \dots, 2N]$$

Power Spectrum

$$\left[\begin{array}{cccc|cccc|c} (0, 0) & (0, 1) & \cdots & \left(0, \frac{N-1}{2}\right) & \left(0, -\frac{N-1}{2}\right) & \left(0, -\left(\frac{N-1}{2}-1\right)\right) & \cdots & (0, -2) & (0, -1) \\ (1, 0) & (1, 1) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \left(\frac{M}{2}-1, 0\right) & \ddots & \ddots & \left(\frac{M}{2}-1, \frac{N-1}{2}\right) & \left(\frac{M}{2}-1, -\frac{N-1}{2}\right) & \ddots & \ddots & \ddots & \left(\frac{M}{2}-1, -1\right) \\ \hline \left(-\frac{M}{2}, 0\right) & \ddots & \ddots & \left(-\frac{M}{2}, \frac{N-1}{2}\right) & \left(-\frac{M}{2}, -\frac{N-1}{2}\right) & \ddots & \ddots & \ddots & \left(-\frac{M}{2}, -1\right) \\ \left(-\left(\frac{M}{2}-1\right), 0\right) & \ddots & \left(-\left(\frac{M}{2}-1\right), -1\right) \\ \vdots & \ddots & \vdots \\ (-2, 0) & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & (-2, -2) & (-2, -1) \\ (-1, 0) & \cdots & \cdots & \left(-1, \frac{N-1}{2}\right) & \left(-1, -\frac{N-1}{2}\right) & \left(-1, -\left(\frac{N-1}{2}-1\right)\right) & \cdots & (-1, -2) & (-1, -1) \end{array} \right]$$

Power Spectrum



Before shift min frequency to center

After shift min frequency to center

Average value of the elements in the same logarithmic bins

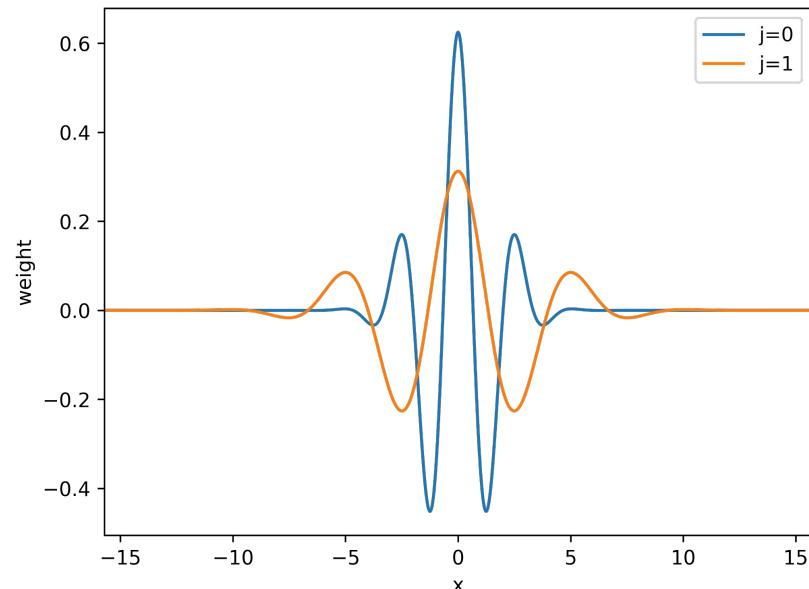
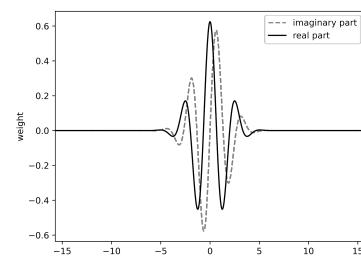
Morlet Wavelet

Σ is selected to have 1 eigenvalue different from others, and k_0 to be along that eigen-direction. Thus we denote the eigen-value along k_0 by σ^2 and other eigen-value by σ^2/s^2

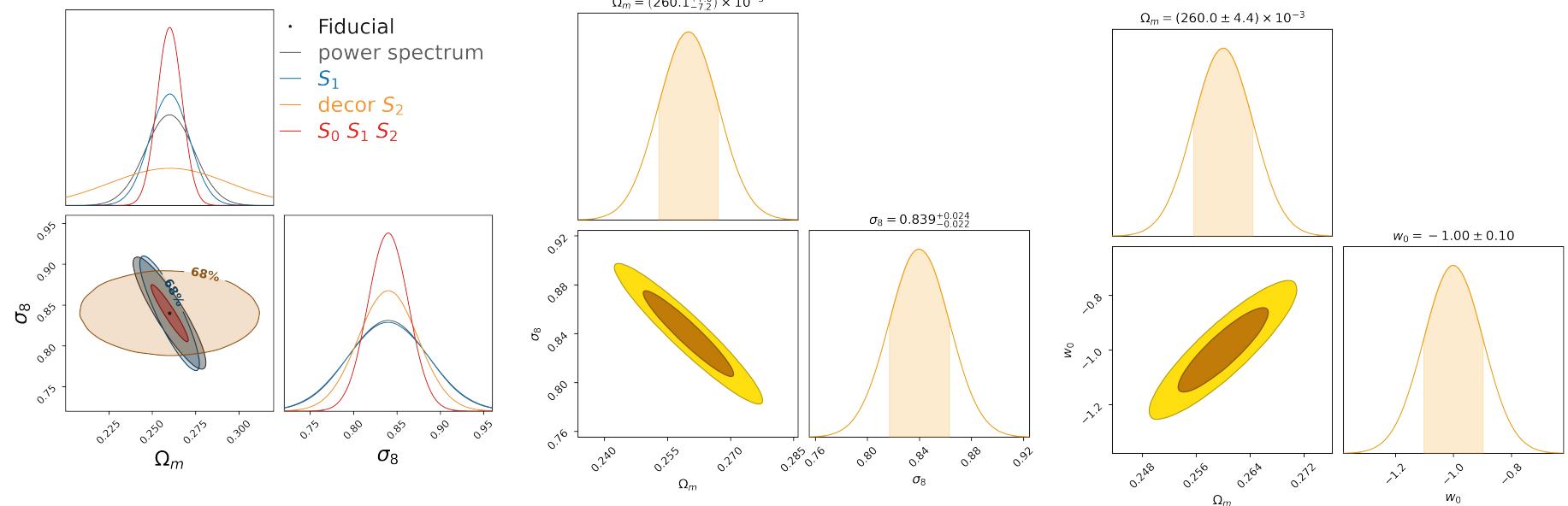
$$\sigma = 0.8 \times 2^j, \text{ where } \sigma \text{ is the unit of pixel}$$

$$k_0 = 3\pi/(4 \times 2^j)$$

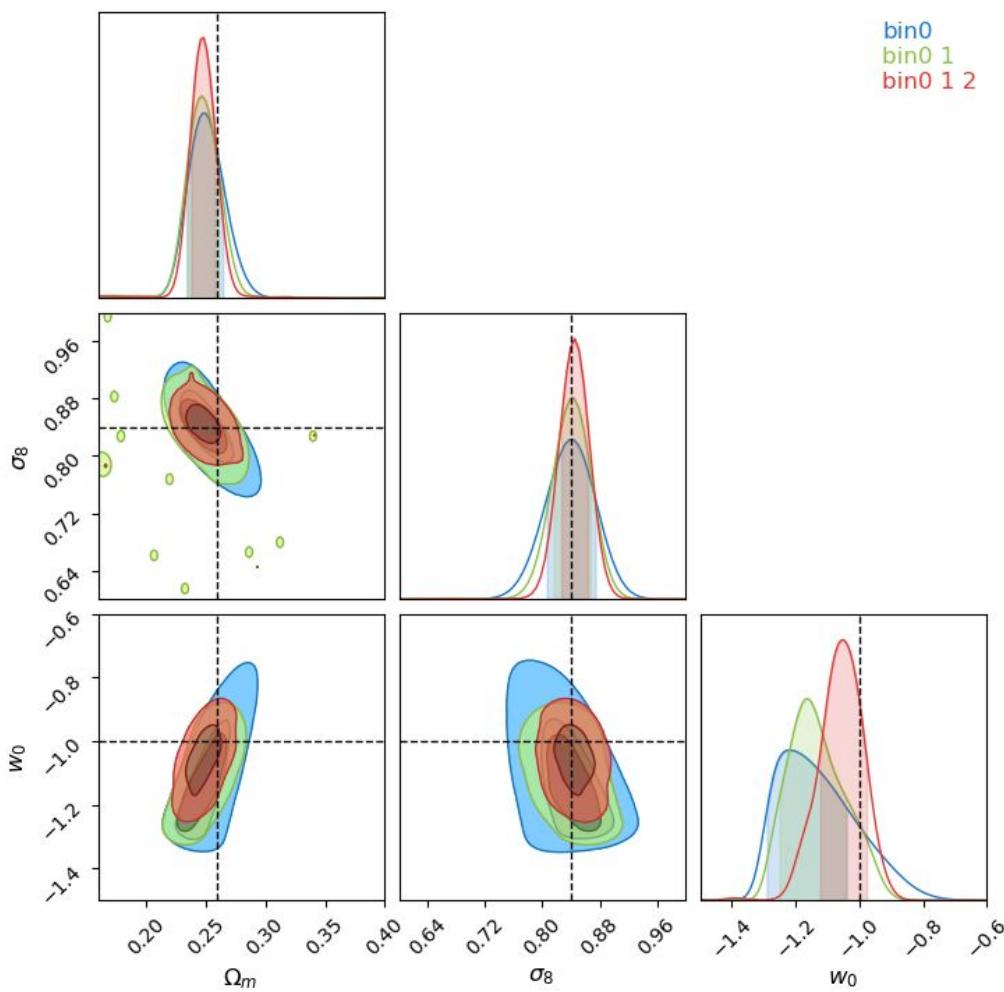
$$s=4/L$$



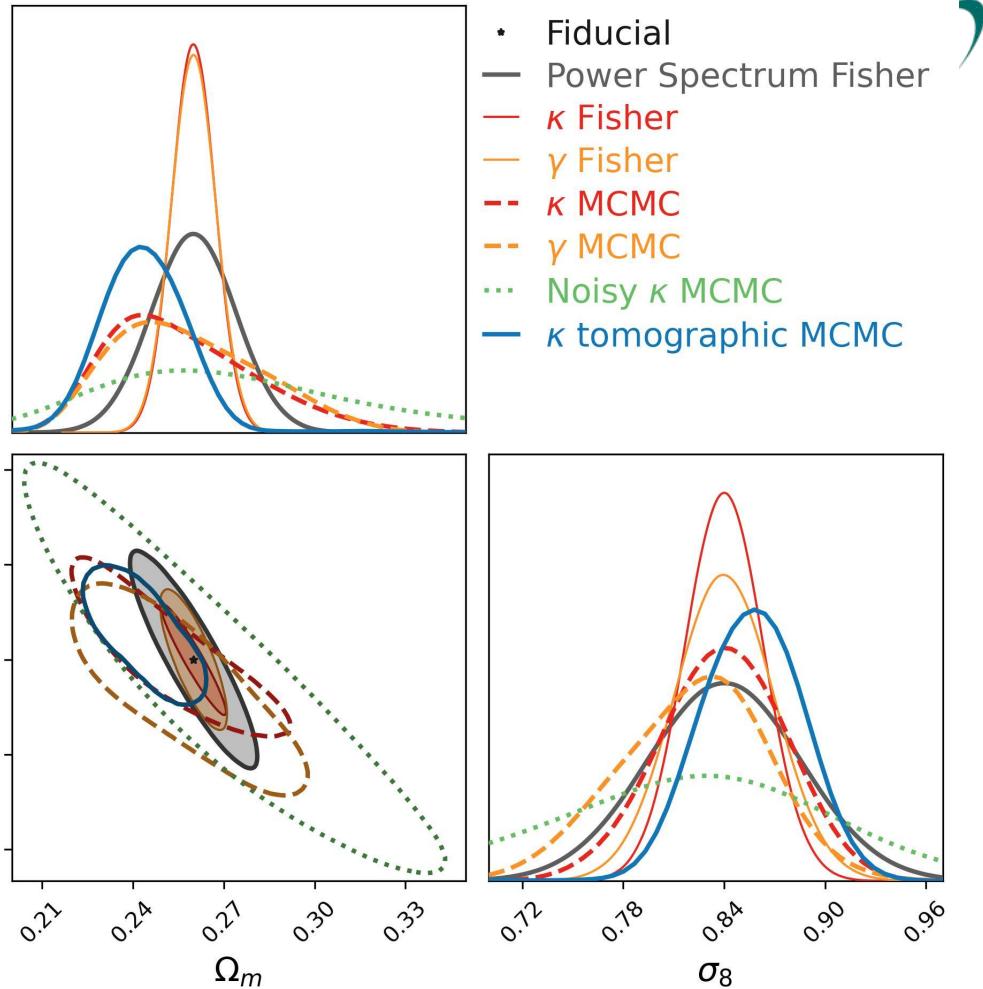
Fisher Forecast



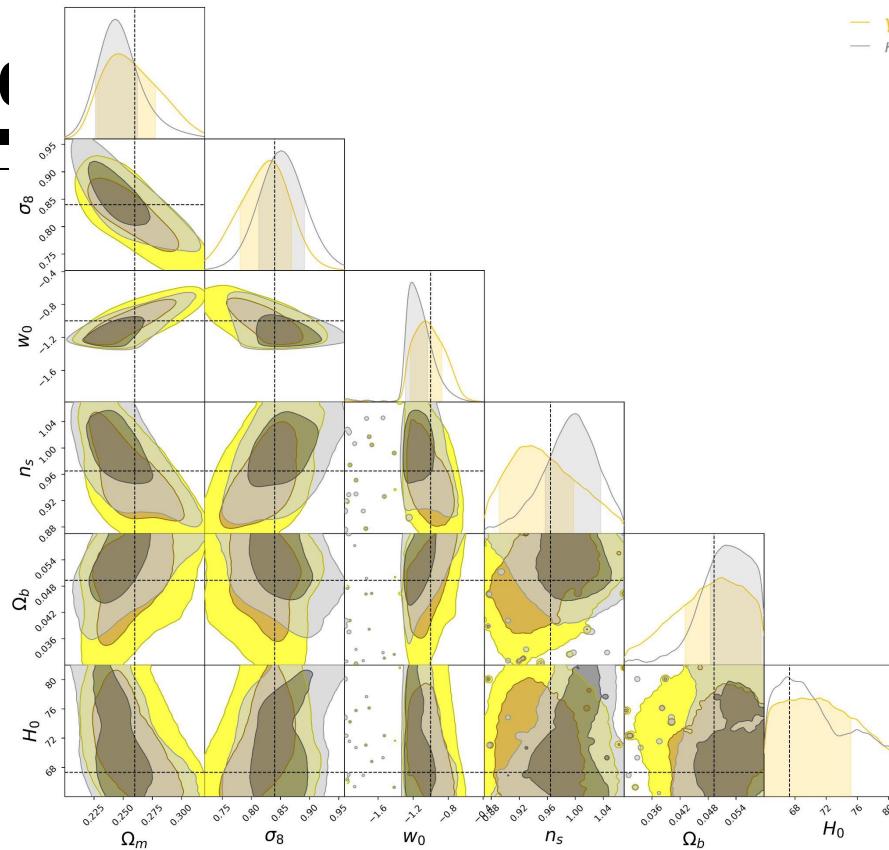
Tomographic Analy



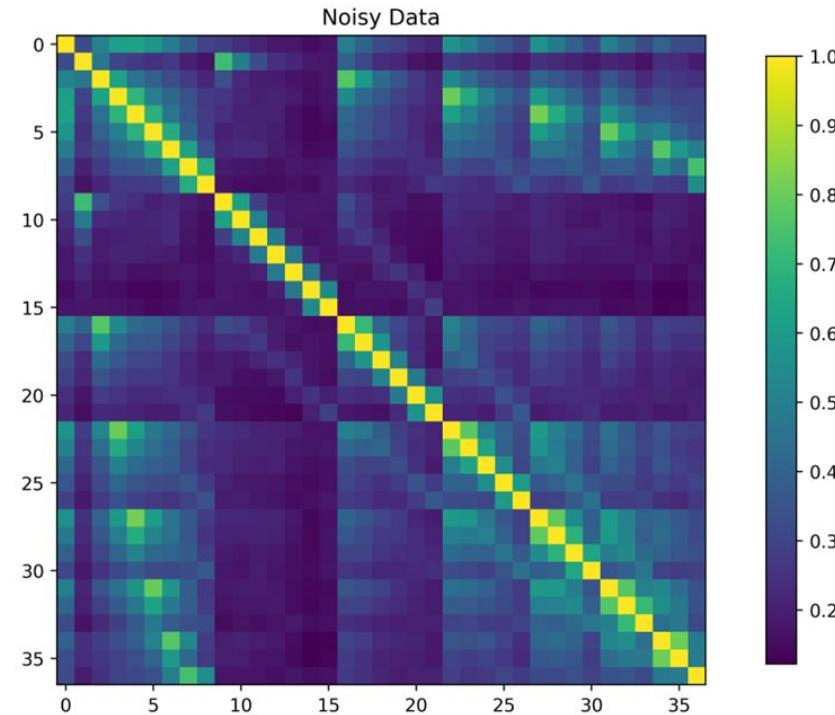
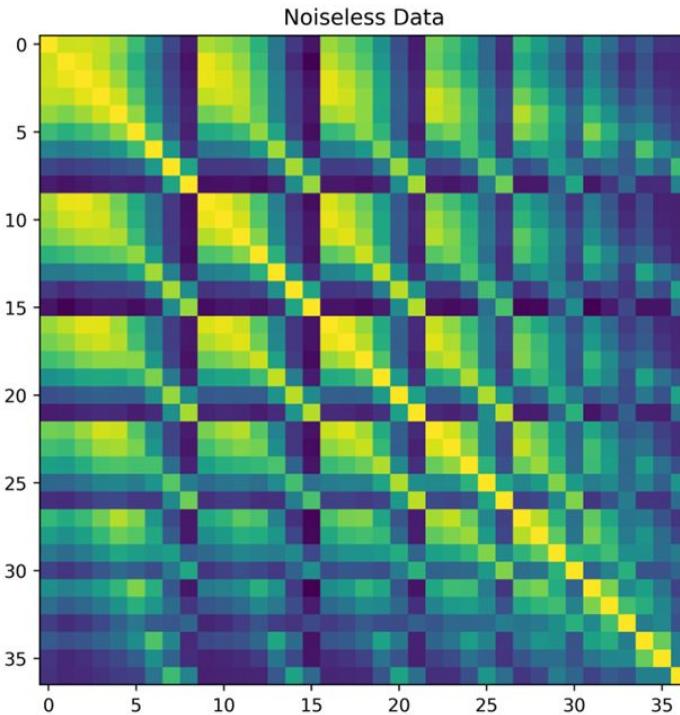
Shear Maps Fisher Forecast



Shear Maps Fisher



Covariance Matrix of Data



Accuracy of Emulator

$$\varepsilon = \frac{\chi^2_{emu}}{\chi^2_{sim}} - 1 = \frac{(\mu_{emu} - \bar{d}_{fid})^T C_{fid}^{-1} (\mu_{emu} - \bar{d}_{fid})}{(\mu_{sim} - \bar{d}_{fid})^T C_{fid}^{-1} (\mu_{sim} - \bar{d}_{fid})} - 1$$

