From Data to Insights - Exercise sheet 12

discussed July 11 and 12

July 5, 2024

1 Gibbs sampling a Bayesian Hierarchical Model

The mean value μ of a Gaussian is drawn from a wide, uniform distribution. From the Gaussian defined by this μ and a standard deviation $\sigma = 1$, the true value t of a parameter is drawn. A measurement x is equal to t with a Gaussian random error with mean 0 and standard deviation of 1.

1. Draw a PGM that describes this process.
2. Write a Python code that enables Gibbs sampling of the joint posterior of μ and t given an observed $x = 2$.
3. Plot the resulting samples as a 2d and a 1d histogram and give a verbal interpretation of what the posterior looks like.
4. Describe qualitatively how the algorithmic and computational difficulty of this sampling compares to what you would have to do if you wanted to sample the posterior of μ and t with the Metropolis-Hastings algorithm.

Hint: This can be done with 20 lines of Python code or less. If you would like a starting point, this how the core of your algorithm could look.

```
x = 2 # the data are fixed

nsamples=10000

# random starting point - let's pick mu=t=0
mu = np.zeros(nsamples)
t = np.zeros(nsamples)

for i in np.arange(1,nsamples):
    t[i] = sample_p_t_given_mu_x(mu[i-1],x)
    mu[i] = sample_p_mu_given_t_x(t[i],x)
    # need to implement these functions
```

2 Probabilistic Graphical Models

Draw a PGM for the following experiments:

A) An object whose flux F_g is drawn from a Gamma distribution with fixed shape and rate parameters α, β is observed with a CCD to produce N analog-digital units of count. The conversion of flux to expected count is $\mu = \langle N \rangle = C \cdot F_g$ in the instrument setup you have used.

B) An object whose flux is drawn from a Gamma distribution with fixed parameters α_g, β_g is observed with a photon counter, with an additive background signal that is drawn from a Gamma distribution with different fixed parameters α_b, β_b. The count is N_s. In addition, a flux measurement with equal setup is made in an empty region of sky, leading to a count of purely background signal of N_b.

C) Any other experiment.

3 Variational inference

A parameter t has its value drawn from an Exponential distribution with $\lambda = 1$. x is a measurement of t with a Gaussian random error with mean 0 and standard deviation of 1.

Using variational inference to analytically derive an approximation to the posterior of t in this setting. For this, assume as the family of posteriors the Exponential distributions with free parameters λ.

Hint: You could use the lecture notes of lecture 11 or (probably even better) this Youtube channel by TUM graduate studente Felix Koehler https://www.youtube.com/@MachineLearningSimulation.