

Quiz 3: Mathematical Statistics (MATH-UA 234)

In-class 10/13 (15min). Print your name and NetID, write in the box, and circle your final answer.

Name: _____

NetID:

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Problem 1. Suppose $X \sim \text{Exp}(\beta)$ for some $\beta > 0$ (here we are using the book's parametrization). That is, suppose X has probability density function

$$f_X(t) = \begin{cases} \beta^{-1} \exp(-\beta^{-1}t) & t \geq 0 \\ 0 & t < 0 \end{cases}.$$

It is a well-known fact that $\mathbb{E}[X] = \beta$ and $\mathbb{V}[X^2] = \beta^2$.

Suppose X_1, \dots, X_n are iid copies of X . That is, X_1, \dots, X_n are all independent Exponential random variables with parameter β . Define, the estimator $\hat{\beta}_n$ for β by

$$\hat{\beta}_n = \frac{1}{n+1}(X_1 + \dots + X_n).$$

(a) Compute $\text{Bias}_n = \mathbb{E}[\hat{\beta}_n] - \beta$. (5pts)

(b) Compute $\text{se}_n = \sqrt{\mathbb{V}[\hat{\beta}_n]}$. (5pts)

(c) Compute $\text{MSE}_n = \mathbb{E}[(\hat{\beta}_n - \beta)^2]$. (5pts)

Justify all steps!

