Quiz 4: Mathematical Statistics (MATH-UA 234)

In-class 10/25 (15min). Print your name and NetID, write in the box, and circle your final answer.

Name: _____ NetID: | | | | | | | | | | | |

Problem 1. *For some fixed but unknown parameter $p \in (0, 1/2)$, define the cumulative distribution function*

$$F_p(x) = \begin{cases} 0 & x < -1 \\ p & -1 \le x < 0 \\ 1-p & 0 \le x < 1 \\ 1 & x \ge 1. \end{cases}$$

Suppose $X_1, \dots, X_n \sim F_p$ are all independent and define $\bar{X}_n = \frac{1}{n}(X_1 + \cdots + X_n)$.

(a) Suppose $X \sim F_p$. What is $\mathbb{E}[X] = \int x \mathrm{d}F_p(x)$ and $\mathbb{V}[X] = \int (x - \mathbb{E}[X])^2 \mathrm{d}F_p(x)$? *(5pts)*

(b) What is $\mathbb{E}[\bar{X}_n]$ and $\mathbb{V}[\bar{X}_n]$? *(2 pts)*

(c) Find an interval (a_n, b_n) depending on $X_1, \dots, X_n$ and $\alpha \in (0, 1)$ such that

$$\mathbb{P}[p \in (a_n, b_n)] \ge 1 - \alpha.$$

Your interval should not depend on p and should get smaller as n gets larger. *(8 pts)*

Hint: it may help to use Chebyshev's inequality:

$$\mathbb{P}[|Z - \mathbb{E}[Z]| \ge \epsilon] \le \frac{\mathbb{V}[Z]}{\epsilon^2}, \qquad \forall Z.$$