Homework 1 Linear Algbera I

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Instructions:

- Due TBD at 11:59pm on Gradescope.
- You must follow the submission policy in the syllabus

Problem 1 (Complex Numbers)**.**

(a) Show that $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in \mathbb{C}$.

(b) Show that $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.

Problem 2 (Vector Spaces)**.**

(a) Let V be a vector space. Prove that $-(-v) = v$ for all $v \in V$.

(b) Let $V = \mathbf{R}^n$. Suppose $a \in \mathbb{R}$ and $v \in V$. Prove that if $a\mathbf{v} = \mathbf{0}$, then $a = 0$ or $\mathbf{v} = \mathbf{0}$.

(c) The empty set $\{\}$ is not a vector space. Which property of vector spaces is not satisfied by the empty set?

Problem 3 (Subspaces)**.**

(a) Prove that if X and Y are linear susbpaces of V, then so is $X + Y$.

(b) Prove that the set $\{0\}$ consisting of the zero element of V is a subspace of V.

(c) Suppose $b \in \mathbb{R}$. Show that the set of continuous real-valued functions f on the interval $[0, 1]$ such that $\int_0^1 f(x)\mathrm{d}x = b$ is a subspace of the vector space of all continuous real-valued functions on $[0, 1]$ if and only if $b = 0$.

Problem 4 (Span/Independence)**.**

(a) Let $\mathbf{v}_1, \dots, \mathbf{v}_k \in V$. Prove $\operatorname{span}(\mathbf{v}_1, \dots, \mathbf{v}_k)$ is a subspace of V.

(b) Show that if the set of vectors $\mathbf{v}_1, \dots, \mathbf{v}_k$ are linearly independent, then none of the $\mathbf{v}_i$ are the zero vector.