

# Homework 1: Mathematical Statistics (MATH-UA 234)

Due 09/08 at the beginning of class on Gradescope

**Problem 1** (Wasserman 1.19 (Bayes theorem)). Suppose that 30 percent of computer owners use a Macintosh, 50 percent use Windows, and 20 percent use Linux. Suppose that 65 percent of the Mac users have succumbed to a computer virus, 82 percent of the Windows users get the virus, and 50 percent of the Linux users get the virus. We select a person at random and learn that her system was infected with the virus. What is the probability that they are a Windows user?

**Problem 2.** Suppose  $\mathbb{P}[A] = 2/3$  and  $\mathbb{P}[B^c] = 1/4$ . Can  $A$  and  $B$  be disjoint (only one can occur)? Why or why not?

**Problem 3.** Use the axioms of a probability distribution to show or answer the following:

- (a) Show  $\mathbb{P}[\emptyset] = 0$ .
- (b) Show  $A \subseteq B \implies \mathbb{P}[A] \leq \mathbb{P}[B]$ .
- (c) Does  $A \subseteq B$  imply  $\mathbb{P}[A] < \mathbb{P}[B]$ ?
- (d) Show  $0 \leq \mathbb{P}[A] \leq 1$  for all  $A$ .

**Problem 4.** Use the axioms of a probability distribution to answer the following:

- (a) Describe a situation where we might have a sample space  $\Omega$  such that  $\mathbb{P}[\{\omega\}] = 0$  for all  $\omega \in \Omega$ .
- (b) Explain what is wrong with the following proof that  $1 = 0$ .

**Proof.** By definition  $\cup_{\omega \in \Omega} \{\omega\} = \{\omega : \exists \omega' \in \Omega \text{ with } \omega \in \{\omega'\}\} = \Omega$ . Thus,

$$\begin{aligned} 1 &= \mathbb{P}[\Omega] && \text{(Axiom 2)} \\ &= \mathbb{P}\left[\bigcup_{\omega \in \Omega} \{\omega\}\right] && \text{(definition of union)} \\ &= \sum_{\omega \in \Omega} \mathbb{P}[\{\omega\}] && \text{(Axiom 3)} \\ &= \sum_{\omega \in \Omega} 0 && \text{(assumption)} \\ &= 0 && \square \end{aligned}$$

**Problem 5** (Wasserman 1.22 (simulate coin flipping)). Suppose we flip a coin  $n$  times and let  $p$  denote the probability of heads. Let  $X$  be the number of heads. We call  $X$  a binomial random variable, which is discussed in the next chapter. Intuition suggests that  $X$  will be close to  $np$ . To see if this is true, we can repeat this experiment many times and average the  $X$  values. Carry out a simulation and compare the average of the  $X$ 's to  $np$ .

Try this for  $p = .3$  and  $n = 10$ ,  $n = 100$ , and  $n = 1000$  repeating each experiment 100 and then 1000 times.

**Problem 6** (Wasserman 2.2 (computing probabilities from distributions)). Let  $X$  be such that  $\mathbb{P}[X = 2] = \mathbb{P}[X = 3] = 1/10$  and  $\mathbb{P}[X = 5] = 8/10$ .

Plot two different possible CDFs  $F$  for  $X$  and use these CDFs to find  $\mathbb{P}[2 < X < 4.8]$  and  $\mathbb{P}[2 \leq X \leq 4.8]$ .

problems with a textbook reference are based on, but not identical to, the given reference

**Problem 7** (Wasserman 2.17 (conditional probabilities from distributions)). *Let*

$$f_{X,Y}(x,y) = \begin{cases} c(x+y^2) & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

*Find  $\mathbb{P}[X < 1/2 | Y = 1/2]$ .*

**Problem 8** (Wasserman 2.21 (important trick about iid maxes)). *Let  $X_1, \dots, X_n \sim \text{Exp}(\eta)$  be iid. Let  $Y = \max\{X_1, \dots, X_n\}$ . Find the PDF of  $Y$ . Hint:  $Y \leq y$  if and only if  $X_i \leq y$  for  $i = 1, \dots, n$ .*

**Problem 9.** *Look for an instance of one of the probability concepts we've seen in the course which you noticed in a different part of your life (e.g. in other classes, on the subway, at the park, on TV, etc.).*

*In a few sentences, explain this instance and how it could be described mathematically.*