

Homework 1: Mathematical Statistics (MATH-UA 234)

Due 09/08 at the beginning of class on Gradescope

Problem 1 (Wasserman 1.19 (Bayes theorem)). Suppose that 30 percent of computer owners use a Macintosh, 50 percent use Windows, and 20 percent use Linux. Suppose that 65 percent of the Mac users have succumbed to a computer virus, 82 percent of the Windows users get the virus, and 50 percent of the Linux users get the virus. We select a person at random and learn that her system was infected with the virus. What is the probability that they are a Windows user?

Problem 2. Suppose $\mathbb{P}[A] = 2/3$ and $\mathbb{P}[B^c] = 1/4$. Can A and B be disjoint (only one can occur)? Why or why not?

Problem 3. Use the axioms of a probability distribution to show or answer the following:

- (a) Show $\mathbb{P}[\emptyset] = 0$.
- (b) Show $A \subseteq B \implies \mathbb{P}[A] \leq \mathbb{P}[B]$.
- (c) Does $A \subsetneq B$ imply $\mathbb{P}[A] < \mathbb{P}[B]$?
- (d) Show $0 \leq \mathbb{P}[A] \leq 1$ for all A .

Problem 4. Use the axioms of a probability distribution to answer the following:

- (a) Describe a situation where we might have a sample space Ω such that $\mathbb{P}[\{\omega\}] = 0$ for all $\omega \in \Omega$.
- (b) Explain what is wrong with the following proof that $1 = 0$.

Proof. By definition $\cup_{\omega \in \Omega} \{\omega\} = \{\omega : \exists \omega' \in \Omega \text{ with } \omega \in \{\omega'\}\} = \Omega$. Thus,

$$\begin{aligned} 1 &= \mathbb{P}[\Omega] && \text{(Axiom 2)} \\ &= \mathbb{P}\left[\bigcup_{\omega \in \Omega} \{\omega\}\right] && \text{(definition of union)} \\ &= \sum_{\omega \in \Omega} \mathbb{P}[\{\omega\}] && \text{(Axiom 3)} \\ &= \sum_{\omega \in \Omega} 0 && \text{(assumption)} \\ &= 0 && \square \end{aligned}$$

Problem 5 (Wasserman 1.22 (simulate coin flipping)). Suppose we flip a coin n times and let p denote the probability of heads. Let X be the number of heads. We call X a binomial random variable, which is discussed in the next chapter. Intuition suggests that X will be close to np . To see if this is true, we can repeat this experiment many times and average the X values. Carry out a simulation and compare the average of the X 's to np .

Try this for $p = .3$ and $n = 10$, $n = 100$, and $n = 1000$ repeating each experiment 100 and then 1000 times. Make a histogram of your results, or report the sample mean and variance.

Problem 6 (Wasserman 2.2 (computing probabilities from distributions)). Let X be such that $\mathbb{P}[X = 2] = \mathbb{P}[X = 3] = 1/10$ and $\mathbb{P}[X = 5] = 8/10$.

Plot the CDF F for X and use the CDF to find $\mathbb{P}[2 < X < 4.8]$ and $\mathbb{P}[2 \leq X \leq 4.8]$.

problems with a textbook reference are based on, but not identical to, the given reference

Problem 7 (Wasserman 2.17 (conditional probabilities from distributions)). *Let*

$$f_{X,Y}(x,y) = \begin{cases} c(x+y^2) & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find $\mathbb{P}[X < 1/2 | Y = 1/2]$. Here c is a normalizing constant so that $f_{X,Y}$ is a probability density function.

Problem 8 (Wasserman 2.21 (important trick about iid maxes)). *Let $X_1, \dots, X_n \sim \text{Exp}(\eta)$ be iid. Let $Y = \max\{X_1, \dots, X_n\}$. Find the PDF of Y . Hint: $Y \leq y$ if and only if $X_i \leq y$ for $i = 1, \dots, n$.*

Problem 9. *Look for an instance of one of the probability concepts we've seen in the course which you noticed in a different part of your life (e.g. in other classes, on the subway, at the park, on TV, etc.).*

In a few sentences, explain this instance and how it could be described mathematically.