# Homework 1: Mathematical Statistics (MATH-UA 234)

Due 09/08 at the beginning of class on Gradescope

**Problem 1** (Wasserman 1.19 (Bayes theorem)). *Suppose that 30 percent of computer owners use a Macintosh, 50 percent use Windows, and 20 percent use Linux. Suppose that 65 percent of the Mac users have succumbed to a computer virus, 82 percent of the Windows users get the virus, and 50 percent of the Linux users get the virus. We select a person at random and learn that her system was infected with the virus. What is the probability that they are a Windows user?*

**Problem 2.** *Suppose $\mathbb{P}[A] = 2/3$ and $\mathbb{P}[B^c] = 1/4$. Can A and B be disjoint (only one can occur)? Why or why not?*

**Problem 3.** *Use the axioms of a probability distribution to show or answer the following:*

(a) *Show $\mathbb{P}[\emptyset] = 0$.*

(b) *Show $A \subseteq B \implies \mathbb{P}[A] \leq \mathbb{P}[B]$.*

(c) *Does $A \subsetneq B$ imply $\mathbb{P}[A] < \mathbb{P}[B]$?*

(d) *Show $0 \leq \mathbb{P}[A] \leq 1$ for all A.*

**Problem 4.** *Use the axioms of a probability distribution to answer the following:*

(a) *Describe a situation where we might have a sample space $\Omega$ such that $\mathbb{P}[\{\omega\}] = 0$ for all $\omega \in \Omega$.*

(b) *Explain what is wrong with the following proof that $1 = 0$.*

***Proof.*** *By definition $\cup_{\omega\in\Omega}\{\omega\} = \{\omega : \exists\omega' \in \Omega \text{ with } \omega \in \{\omega'\}\} = \Omega$. Thus,*

$$\begin{aligned} 1 &= \mathbb{P}[\Omega] && \text{(Axiom 2)} \\ &= \mathbb{P}\Big[\bigcup_{\omega\in\Omega}\{\omega\}\Big] && \text{(definition of union)} \\ &= \sum_{\omega\in\Omega}\mathbb{P}[\{\omega\}] && \text{(Axiom 3)} \\ &= \sum_{\omega\in\Omega} 0 && \text{(assumption)} \\ &= 0 && \square \end{aligned}$$

**Problem 5** (Wasserman 1.22 (simulate coin fipping)). *Suppose we flip a coin n times and let p denote the probability of heads. Let X be the number of heads. We call X a binomial random variable, which is discussed in the next chapter. Intuition suggests that X will be close to np. To see if this is true, we can repeat this experiment many times and average the X values. Carry out a simulation and compare the average of the X's to np.*

*Try this for $p = .3$ and $n = 10$, $n = 100$, and $n = 1000$ repeating each experiment 100 and then 1000 times.*

**Problem 6** (Wasserman 2.2 (computing probabilities from distributions)). *Let X be such that $\mathbb{P}[X = 2] = \mathbb{P}[X = 3] = 1/10$ and $\mathbb{P}[X = 5] = 8/10$.*

*Plot two different possible CDFs F for X and use these CDFs to find $\mathbb{P}[2 < X < 4.8]$ and $\mathbb{P}[2 \leq X \leq 4.8]$.*

problems with a textbook reference are based on, but not identical to, the given reference

**Problem 7** (Wasserman 2.17 (conditional probabilities from distributions)). *Let*

$$f_{X,Y}(x,y) = \begin{cases} c(x+y^2) & 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

*Find* $\mathbb{P}[X < 1/2 | Y = 1/2]$. *Here* $c$ *is a normalizing constant so that* $f_{X,Y}$ *is a probability density function.*

**Problem 8** (Wasserman 2.21 (important trick about iid maxes)). *Let* $X_1, \ldots, X_n \sim \mathrm{Exp}(\eta)$ *be iid. Let* $Y = \max\{X_1, \ldots, X_n\}$. *Find the PDF of* $Y$. *Hint:* $Y \le y$ *if and only if* $X_i \le y$ *for* $i = 1, \ldots, n$.

**Problem 9.** *Look for an instance of one of the probability concepts we've seen in the course which you noticed in a different part of your life (e.g. in other classes, on the subway, at the park, on TV, etc.).*

*In a few sentences, explain this instance and how it could be described mathematically.*