Quiz 6: Mathematical Statistics (MATH-UA 234)

In-class 11/21 (15min). Print your name and NetID, write in the box, and | circle your final answer |.

Name: _____ NetID: | | | | | | | | | | | | |

Problem 1. *Let $X_1, \ldots, X_n \sim \mathrm{Ber}(\theta)$ (iid), for some fixed unknown $\theta \in [0, 1]$. Thus, each X_i is 1 with probability θ and 0 with probability $1 - \theta$.*

Suppose we wish to test the null hypothesis that $\theta \leq 2/3$.

(a) *Write the set of parameter Θ_0 so that the null hypothesis is true if and only if $\theta \in \Theta_0$.* (1 pt)

(b) *Let $T(x_1, \ldots, x_n) = x_1 + \cdots + x_n$ and suppose our test has rejection region $R = \{(x_1, \ldots, x_n) : T(x_1, \ldots, x_n) > n - 1/2\}$. That is, given data $X_1, \ldots, X_n$, we reject if and only if $(X_1, \ldots, X_n) \in R$. Compute the power function*

$$\beta(\theta) := \mathbb{P}[(X_1', \ldots, X_n') \in R | X_1', \ldots, X_n' \sim \mathrm{Ber}(\theta)].$$ (6 pts)

Hint: think about what has to happen to reject.

(c) *Find the size of the test*

$$\alpha := \sup_{\theta \in \Theta_0} \beta(\theta).$$ (5 pts)

(d) *Recall that a Type I error means rejecting the null hypothesis even though it is true. If $n = 10$, what is the maximum probability of a Type I error?* (3 pts)