Quiz 2: Mathematical Statistics (MATH-UA 234)

In-class 09/27 (15min). Print your name and NetID and leave space at the edge of the page.

Name: _____ NetID: | | | | | | | | | | | |

Problem 1. *Suppose $X \sim \text{Pareto}(\alpha)$ for some $\alpha > 0$. That is, suppose X has cumulative distribution function*

$$F_X(t) = \mathbb{P}[X \leq t] = \begin{cases} 1 - t^{-\alpha} & t \geq 1 \\ 0 & t < 1 \end{cases}.$$

(a) Compute the probability density function $f_X(t)$. *(5pts)*

(b) Suppose $\alpha = 3$. Compute $\mathbb{E}[X^2]$. *(5pts)*

Problem 2. *Suppose X and Y are random variables with joint probability mass function,*

$$f_{X,Y}(x,y) = \mathbb{P}[X = x, Y = y] = \begin{cases} .1 & X = -2, Y = 1 \\ .3 & X = -1, Y = 1 \\ .2 & X = 1, Y = 1 \\ .1 & X = 3, Y = 2 \\ .3 & X = 1, Y = -1 \end{cases}.$$

Compute $\mathbb{E}[X|Y = 1]$. (hint: recall $\mathbb{P}[A|B] = \mathbb{P}[A \cap B]/\mathbb{P}[B]$) *(5pts)*

Solution.

(a) For $t \geq 1$, $f_X(t) = F_X'(t) = (1 - t^{-\alpha})' = \alpha t^{-\alpha-1} = \alpha t^{-(\alpha+1)}$. For $t \leq 1$, $f_X(t) = F_X'(t) = 0$.

(b) $\mathbb{E}[X^2] = \int_1^\infty t^2 f_X(t)\mathrm{d}t = \int_1^\infty t^2 3t^{-4}\mathrm{d}t = \int_1^\infty 3t^{-2}\mathrm{d}t = [-3t^{-1}]_{t=1}^\infty = 0 - (-3) = 3.$

Solution. We have

$$\mathbb{E}[X|Y = 1] = \sum_x x\mathbb{P}[X = x|Y = 1] = \sum_x x\frac{\mathbb{P}[X = x, Y = 1]}{\mathbb{P}[Y = 1]}.$$

We can compute

$$\mathbb{P}[Y = 1] = \sum_x \mathbb{P}[X = x, Y = 1] = 0.1 + 0.2 + 0.3 = 0.6.$$

Thus,

$$\mathbb{E}[X|Y = 1] = (-2)\frac{0.1}{0.6} + (-1)\frac{0.3}{0.6} + (1)\frac{0.2}{0.6} = -2/6 + -3/6 + 2/6 = -1/2.$$