Homework 5: Mathematical Statistics (MATH-UA 234)

Due 11/03 at the beginning of class on Gradescope

Problem 1. *Suppose $X_1, \dots, X_n$ are real numbers. Then the likelihood function is*

$$L_n(\theta) = \prod_{i=1}^{n} f_\theta(X_i).$$

In your own words, describe the meaning of $L_n(\theta)$ for a specific value of θ when

(a) f_θ is a probability mass function

(b) f_θ is a probability density function

Problem 2 (Wasserman 9.1)**.** *In this problem, you will fill in some pieces we skipped in lecture. If you take the approach of searching for critical points, you should justify why your response is a global maximum.*

(a) Let $X_1, \dots, X_n \sim \mathrm{Ber}(p)$ where p is an unknown parameter. Show $\ell_n(p) = S\log(p) + (n-S)\log(1-p)$, where $S = X_1 + \cdots + X_n$, and then find the maximizer of $\ell_n(p)$.

(b) Let $X_1, \dots, X_n \sim N(\mu, \sigma^2)$ where μ and σ are unknown parameters. Show $\ell(\mu, \sigma) = c - n\log(\sigma) - nS^2/(2\sigma^2) - n(\bar{X}-\mu)^2/(2\sigma^2)$, where $\bar{X} = n^{-1}(X_1 + \cdots + X_n)$, $S = n^{-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$, and c is some constant, and then find the maximizer of $\ell_n(\mu, \sigma)$.

Problem 3 (Wasserman 9.1)**.** *Let $X_1, \dots, X_n \sim \mathrm{Gamma}(\alpha, \beta)$ where α and β are unknown parameters. Find the method of moments estimator for α and β.*

Problem 4 (Wasserman 9.2)**.** *Let $X_1, \dots, X_n \sim \mathrm{Unif}(a, b)$ where a and b are unknown parameters with $a < b$.*

(a) Find the method of moments estimator for a and b.

(b) Find the maximum likelihood estimators for a and b.

Problem 5 (Wasserman 9.5)**.** *Let $X_1, \dots, X_n \sim \mathrm{Poisson}(\lambda)$ where λ is an unknown parameter.*

(a) Find the method of moments estimator for λ.

(b) Find the maximum likelihood estimators for λ.

Problem 6 (Wasserman 9.6)**.** *Let $X_1, \dots, X_n \sim N(\theta, 1)$ where θ is an unknown parameter. Define*

$$Y_i = \begin{cases} 1 & X_i > 0 \\ 0 & X_i \leq 0. \end{cases}$$

Let $\psi = \mathbb{P}[Y_1 = 1]$.

Find the maximum likelihood estimator for ψ.

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problems with a textbook reference are based on, but not identical to, the given reference