Quiz 1 (bonus)   Numerical Analysis Fall 2023

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Do not begin until instructed. Circle your final answer.

Problem 1. A matrix $\mathbf{T}$ is said to be *tridiagonal* if $[\mathbf{T}]_{i,j} = 0$ for all i, j where $|i - j| > 1$.

Let $\mathbf{A}$ be a $n \times n$ tridiagonal matrix and $\mathbf{D}$ a $n \times n$ diagonal matrix. Prove that $\mathbf{AD}$ is tridiagonal.

Hint: for $n \times n$ matrices $\mathbf{X}, \mathbf{Y}, [\mathbf{XY}]_{i,j} = \sum_{k=1}^{n}[\mathbf{X}]_{i,k}[\mathbf{Y}]_{k,j}$.

Problem 2. Suppose that

$$\begin{bmatrix} | & | \\ \mathbf{x}_1 & \mathbf{x}_2 \\ | & | \end{bmatrix}\begin{bmatrix} \pi & e \\ \pi^2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \begin{bmatrix} | & | & | \\ \mathbf{y}_1 & \mathbf{y}_2 & \mathbf{y}_3 \\ | & | & | \end{bmatrix}\begin{bmatrix} 8 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Compute the following:

a) $\quad \begin{bmatrix} | & | \\ \mathbf{x}_1 & \mathbf{x}_2 \\ | & | \end{bmatrix}\begin{bmatrix} \pi - e \\ \pi^2 + 3 \end{bmatrix},$
b) $\quad \begin{bmatrix} | & | & | & | & | \\ \mathbf{x}_1 & \mathbf{y}_1 & \mathbf{y}_2 & \mathbf{y}_3 & \mathbf{x}_2 \\ | & | & | & | & | \end{bmatrix}\begin{bmatrix} \pi & 2e \\ 8 & 0 \\ 2 & 0 \\ -3 & 0 \\ -\pi^2 & -6 \end{bmatrix}$

a)

b)