Quiz 7: Mathematical Statistics (MATH-UA 234)

In-class 12/06 (15min). Print your name and NetID, write in the box, and circle your final answer .

Name: _____   NetID: [　][　][　][　][　][　][　][　][　][　][　][　]

Problem 1. *Suppose Y is a random variable with density*

$$f_Y(y) \propto \begin{cases} y & y \in [0, 1] \\ 0 & otherwise \end{cases}.$$

Write the exact formula for $f_Y(y)$. I.e. with equality rather than "$\propto$".   *(5 pts)*

Problem 2. *For some fixed but unknown parameter $\theta > 0$, suppose we get data iid $X_1, X_2, X_3 \sim \text{Pareto}(1, \Theta)$. Recall that if $\Theta = \theta$, the PDF for a Pareto random variable with parameters $(1, \Theta)$ is*

$$f_{X|\Theta=\theta}(x) = \begin{cases} \frac{\theta}{x^{\theta+1}} & x \geq 1 \\ 0 & \text{otherwise} \end{cases}.$$

Suppose we use a prior distribution $\Theta \sim \text{Exp}(1)$. Recall the PDF for an exponential random variable with parameter 1 is

$$f_\Theta(\theta) = \begin{cases} \exp(-\theta) & \theta \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

(a) *Write down the conditional PDF*

$$f_{X_1, X_1, X_3|\Theta=\theta}(x_1, x_2, x_3).$$

(5 pts)

(b) *Given that $X_1 = 3, X_2 = 2, X_3 = 5$, write down a function proportional to the posterior PDF*

$$f_{\Theta|X_1=3, X_2=2, X_3=5}(\theta).$$

(5 pts)

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