School of Electronics and Computer Science
University of Southampton

COMP6212 Computational Finance 2017/18, Assignment (Part I (1 of 3 from MN): 20%)

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1. Consider two assets whose expected returns and covariances on returns are given by $\mathbf{m} = \begin{bmatrix} 0.10 & 0.10 \end{bmatrix}^{\mathbf{t}}$ and $\begin{bmatrix} 0.005 & 0.0 \\ 0.0 & 0.005 \end{bmatrix}$ respectively. Derive the Markowitz efficient frontier for this data, deriving your solution from basic algebra and not using a computer.

2. Consider three securities whose expected returns are

$$\mathbf{m} = \begin{bmatrix} 0.10 & 0.20 & 0.15 \end{bmatrix}^{t},$$

and their corresponding covariances are

$$C = \begin{bmatrix} 0.005 & -0.010 & 0.004 \\ -0.010 & 0.040 & -0.002 \\ 0.004 & -0.002 & 0.023 \end{bmatrix}$$

Generate 100 random portfolios and plot a scatter diagram in the $E - V$ space as shown in Fig. 1 of [1].

Use `MATLAB`'s financial toolbox to draw the efficient portfolio frontier for this three-asset model, and for the three two-asset portfolios, taking the assets pair-wise. Draw the graphs and scatter of points using the same scale and briefly comment on what you observe.

3. Set up the `CVX` Convex Programming toolbox in MATLAB and familiarize yourself with it. Work through one or two simple examples given in the documentation.

Replace the two optimization steps in the `NaiveMV` function (*i.e.* calls to `linprog` and `quadprog` by `CVX` and show that similar results (or identical results?) are produced.

4. Obtain daily `FTSE 100` data for the past three years and data for the prices of 30 companies in the `FTSE` index. `Yahoo Finance` may be a convenient source.

Select three stocks at random and estimate the expected returns and covariances from the first half of the time series. Select assets for which approximately equal lengths of data is available and use a simple method to impute any missing values in them. Using the above estimates design an efficient portfolio.

Would the portfolio have performed better than the simple $\frac{1}{N}$ portfolio during the remainder of the period for which you have data (see [2])?

5. Implement one of the enhancements reviewed in [2], selecting the method to implement according to the last digit of your student number as shown in Table 1.

Last digit of student number	Method to implement
$0, 1, 2$	Bayes-Stein shrinkage portfolio (Eqn. (4))
$3, 4, 5$	MacKinlay & Pastor model (Eqn. 7)
$6, 7, 8\&9$	Shortsale constrained portfolio (Eqn. 8)

Table 1: Choice of method to implement

6. Implement and compare two strategies for *Index Tracking* using the returns on the FTSE index and 30 constituent considered in the earlier section : **(a)** a greedy forward selection algorithm that selects about a fifth of the available stocks; and **(b)** sparse index tracking portfolio using l_1 regularization as discussed in [3]. Tune the regularization parameter so that the number of stocks selected by the regularization scheme is similar to the number selected in method (a). Comment on the subsets selected by the two methods.

7. Lobo *et al.* [4] discuss how transaction costs may be included in optimizing adjustments to a portfolio. Study how they impose various constraints, and explain "in your own words" aspects of the conclusions explained in Fig. 2, and the optimization problem formulated in the example of Section 1.6 (what the objective function is, what the constraints are and how you might extend the work you have done so far to implement this). What is needed is an insightful discussion, you need not actually implement this.

Marking Scheme

For each part, 12/20 marks will be awarded for correctly doing what you are instructed to do. Additional marks are gained by "going the extra mile", *i.e.* for showing initiative of reading slightly outside what was taught, demonstrating clear understanding, presenting the work to high standards etc.

Report

Write a report of no more than ten pages describing the work you have done, answering any questions above.

References

[1] H. Markowitz, "Portfolio selection," *The Journal of Finance*, vol. 7, no. 1, pp. 77 – 91, 1952.

[2] V. DeMiguel, L. Garlappi, and R. Uppal, "Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy?" *The Review of Financial Studies*, vol. 22, no. 5, pp. 1915 – 1953, 2009.

[3] J. Brodie, I. Daubechies, C. De Mol, D. Giannone, and I. Loris, "Sparse and stable Markowitz portfolios," *PNAS*, vol. 106, no. 30, pp. 12 267 – 12 272, 2009.

[4] M. Lobo, M. Fazel, and S. Boyd, "Portfolio optimization with linear and fixed transaction costs," *Annals of Operations Research*, vol. 152, no. 1, pp. 341–365, 2007.

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