CS 292C Winter 2020 - Assignment 1

1 Syntax

$$a \in AE \to n \in \mathbb{N} \mid \textbf{true} \mid \textbf{false} \mid a_1 + a_2 \mid a_1 * a_2$$
$$\mid a_1 \leq a_2 \mid \textbf{if } a_1 \textbf{ then } a_2 \textbf{ else } a_3$$

2 Semantics

$$\frac{a_1 \to a_1'}{a_1 + a_2 \to a_1' + a_2} \text{ ADD}1 \qquad \frac{a_1 \to a_1'}{a_1 * a_2 \to a_1' * a_2} \text{ MUL}1 \qquad \frac{a_1 \to a_1'}{a_1 \leq a_2 \to a_1' \leq a_2} \text{ REL}1$$

$$\frac{a_2 \to a_2'}{n + a_2 \to n + a_2'} \text{ ADD}2 \qquad \frac{a_2 \to a_2'}{n * a_2 \to n * a_2'} \text{ MUL}2 \qquad \frac{a_2 \to a_2'}{n \leq a_2 \to n \leq a_2'} \text{ REL}2$$

$$\frac{n_1 + n_2 = n_3}{n_1 + n_2 \to n_3} \text{ ADD}3 \qquad \frac{n_1 \times n_2 = n_3}{n_1 * n_2 \to n_3} \text{ MUL}3 \qquad \frac{n_1 \leq n_2}{n_1 \leq n_2 \to \textbf{true}} \text{ REL}3$$

$$\frac{n_1 > n_2}{n_1 \leq n_2 \to \textbf{false}} \text{ REL}4$$

$$\frac{a_1 \to a_1'}{\textbf{if } a_1 \textbf{ then } a_2 \textbf{ else } a_3 \to \textbf{if } a_1' \textbf{ then } a_2 \textbf{ else } a_3} \text{ IF}1$$

$$\frac{}{\textbf{if true then } a_2 \textbf{ else } a_3 \to a_2} \text{ IF}2 \qquad \frac{}{\textbf{if false then } a_2 \textbf{ else } a_3 \to a_3} \text{ IF}3$$

3 Type System

$$\frac{}{n : \texttt{nat}} \text{ NUM} \qquad \frac{}{\textbf{true} : \texttt{bool}} \text{ BOOL}1 \qquad \frac{}{\textbf{false} : \texttt{bool}} \text{ BOOL}2$$

$$\frac{a_1 : \texttt{nat} \quad a_2 : \texttt{nat}}{a_1 + a_2 : \texttt{nat}} \text{ ADD} \qquad \frac{a_1 : \texttt{nat} \quad a_2 : \texttt{nat}}{a_1 * a_2 : \texttt{nat}} \text{ MUL} \qquad \frac{a_1 : \texttt{nat} \quad a_2 : \texttt{nat}}{a_1 \leq a_2 : \texttt{bool}} \text{ REL}$$

$$\frac{a_1 : \texttt{bool} \quad a_2 : T \quad a_3 : T}{\textbf{if } a_1 \textbf{ then } a_2 \textbf{ else } a_3 : T} \text{ IF}$$