# CS 111 (S22): Homework 2

**Due Monday, April 11th by 11:59 PM**

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**Homework Buddy:** ( remove entire line if not applicable)

**Additional Instructions**: Please enter the LaTeX command *\newpage* at the end of each of your answers in your LaTeX source code. This will put each answer on at least one page and make it easier to grade on Gradescope.

1. *(30 pts)* The following three statements are all **false**. For each one, give a counterexample consisting of a 3-by-3 matrix or matrices (to show that they are indeed false), and show the Python computation that proves that the statement fails (i.e. snapshot or text copy of the actual code and also the aftermath of its execution).

   A good way to solve this problem (other than thinking about Linear Algebra theories) can start with trial-and-error computations on Python: you should get comfortable with this language and environment, which in turn will help with your intuitive thinking about these sorts of problems.

   - If $P$ is a permutation matrix and $A$ is any matrix, then $PA = AP$.
   - If matrix $A$ is nonsingular, then it has a factorization $A = LU$ where $L$ is lower triangular and $U$ is upper triangular.
   - The product of two symmetric matrices is a symmetric matrix.

2. *(20 pts)* Write the following matrix in the form $A = LU$, where $L$ is a unit lower triangular matrix (that is, a lower triangular matrix with ones on the diagonal) and $U$ is an upper triangular matrix. You can check your answer using Python, but for this exercise, you need to also show the steps you took to get to your answer with some explanation to go with them. In particular, if you have to use pivoting, then I need you to also tell me what the permutation matrix you used was.

$$A = \begin{pmatrix} 5 & 3 & 3 \\ 3 & 5 & 3 \\ 3 & 3 & 5 \end{pmatrix}$$

3. *(20 pts)* Similar to the above exercise, write this matrix in the form $A = LU$. Again, you can check your answer using Python, but for this exercise, you need to also show the steps you took to get to your answer with some explanation to go with them. In particular, if you have to use pivoting, then I need you to also tell me what the permutation matrix you used was.

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

4. *(30 pts)* Write `Usolve()`, analogous to `Lsolve()` in the class lecture file `lect04_A=LU.ipynb` (this is the demo file used for **Lecture 4**), to solve an upper triangular system $Ux = y$. Your submission can just be the code for the *function definition* of `Usolve()`.

   Obviously, you should also check your work beforehand - this is best done in the `lect04_A=LU.ipynb` file (provided for you in the same place you got these instructions). To best utilize this, you should run the .ipynb file on Jupyter Notebook.

   Warning: Notice that, unlike in `Lsolve()`, the diagonal elements of $U$ don't have to be equal to one. Test your answer, both by itself and with `LUsolve()`, and turn in the result.

   Hint: *(You don't have to use this hint, but if you do, here it is...)* Loops can be run backward in Python, say from $n-1$ down to 0, by writing

   ```
   for i in reversed(range(n)):
   ```