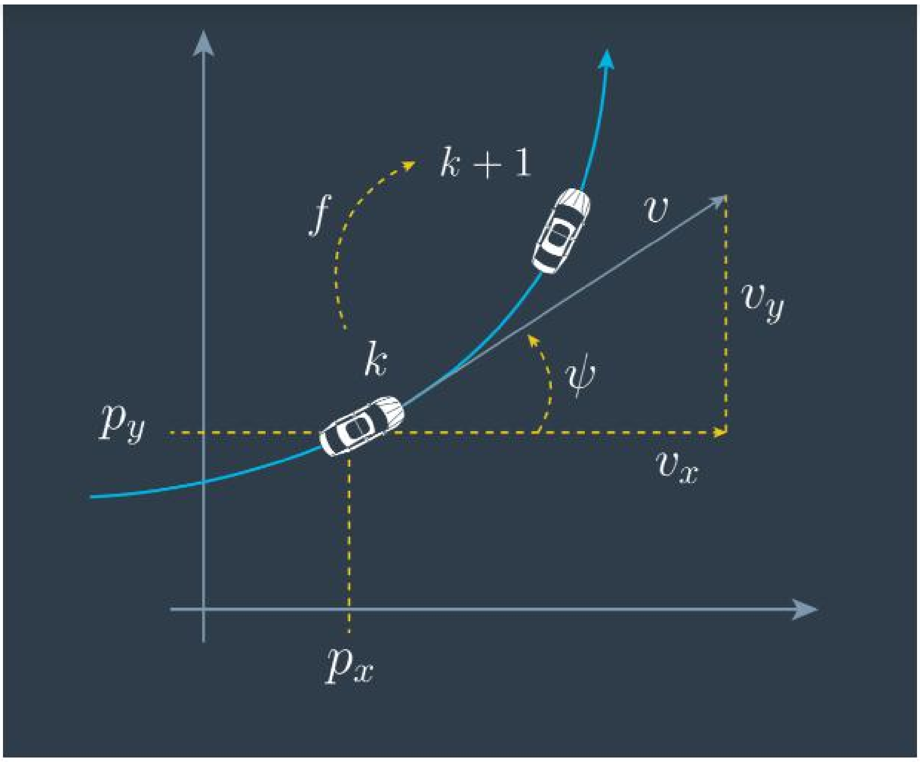

A diagram of the CTRV model.

In the following quizzes, you'll be using state vectors to draw qualitative observations about the motion of turning objects.

General State Vector

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

$$X_{k|k} = [x_{k|k} \qquad x_{k|k} + \sqrt{(\lambda + n_x)P_{k|k}} \qquad x_{k|k} - \sqrt{(\lambda + n_x)P_{k|k}}]$$

remember that $x_{k|k}$ is the first column of the Sigma matrix.

$x_{k|k} + \sqrt{(\lambda + n_x)P_{k|k}}$ is the second through $n_x + 1$ column.

$x_{k|k} - \sqrt{(\lambda + n_x)P_{k|k}}$ is the $n_x + 2$ column through $2n_x + 1$ column.

Augmented State = $x_{a,k} = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \\ \nu_a \\ \nu\ddot{\psi} \end{bmatrix}$

Note: The mean of the process noise is zero.

Augmented Covariance Matrix = $Pa,k|k = \begin{bmatrix} Pk|k & 0 \\ 0 & Q \end{bmatrix}$

$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$

If $\dot{\psi}_k$ is not zero

State = $x_{k+1} = x_k + \begin{bmatrix} \frac{v_k}{\dot{\psi}_k}(sin(\psi_k + \dot{\psi}_k\Delta t) - sin(\psi_k)) \\ \frac{v_k}{\dot{\psi}_k}(-cos(\psi_k + \dot{\psi}_k\Delta t) + cos(\psi_k)) \\ 0 \\ \dot{\psi}_k\Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta t)^2 cos(\psi_k)\nu_a, k \\ \frac{1}{2}(\Delta t)^2 sin(\psi_k)\nu_a, k \\ \Delta t\nu_a, k \\ \frac{1}{2}(\Delta t)^2 \nu\ddot{\psi}, k \\ \Delta t\nu\ddot{\psi}, k \end{bmatrix}$

If $\dot{\psi}_k$ is zero

State = $x_{k+1} = x_k + \begin{bmatrix} v_k cos(\psi_k)\Delta t \\ v_k sin(\psi_k)\Delta t \\ 0 \\ \dot{\psi}_k\Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta t)^2 cos(\psi_k)\nu_a, k \\ \frac{1}{2}(\Delta t)^2 sin(\psi_k)\nu_a, k \\ \Delta t\nu_a, k \\ \frac{1}{2}(\Delta t)^2 \nu\ddot{\psi}, k \\ \Delta t\nu\ddot{\psi}, k \end{bmatrix}$

Notice that when $\dot{\psi}_k = 0$,

the term $\dot{\psi}_k\Delta t$ would also equal zero.

Weights

$w_i = \frac{\lambda}{\lambda + n_a}, i = 1$

$w_i = \frac{1}{2(\lambda + n_a)}, i = 2 \ldots n_\sigma$

Predicted Mean

$x_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i X_{k+1|k,i}$

Predicted Covariance

$P_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i (X_{k+1|k,i} - x_{k+1|k})(X_{k+1|k,i} - x_{k+1|k})^T$

State Vector

$$x_{k+1|k} = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

Measurement Vector

$$z_{k+1|k} = \begin{bmatrix} \rho \\ \varphi \\ \dot{\rho} \end{bmatrix}$$

Measurement Model

$$z_{k+1|k} = h(x_{k+1}) + w_{k+1}$$

$$\rho = \sqrt{p_x^2 + p_y^2}$$

$$\varphi = arctan(\frac{p_y}{p_x})$$

$$\dot{\rho} = \frac{p_x cos(\psi) v + p_y sin(\psi) v}{\sqrt{p_x^2 + p_y^2}}$$

Predicted Measurement Mean

$$z_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i Z_{k+1|k,i}$$

Predicted Covariance

$$S_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i (Z_{k+1|k,i} - z_{k+1|k})(Z_{k+1|k,i} - z_{k+1|k})^T + R$$

$$R = E(w_k \cdot w_k^T) = \begin{bmatrix} \sigma\rho^2 & 0 & 0 \\ 0 & \sigma\varphi^2 & 0 \\ 0 & 0 & \sigma_{\dot{\rho}}^2 \end{bmatrix}$$

Cross-correlation Matrix

$T_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i (X_{k+1|k,i} - x_{k+1|k}) \, (Z_{k+1|k,i} - z_{k+1|k})^T$

Kalman gain K

$K_{k+1|k} = T_{k+1|k} S_{k+1|k}^{-1}$

Update State

$x_{k+1|k+1} = x_{k+1|k} + K_{k+1|k}(z_{k+1} - z_{k+1|k})$

Covariance Matrix Update

$P_{k+1|k+1} = P_{k+1|k} - K_{k+1|k} S_{k+1|k} K_{k+1|k}^T$