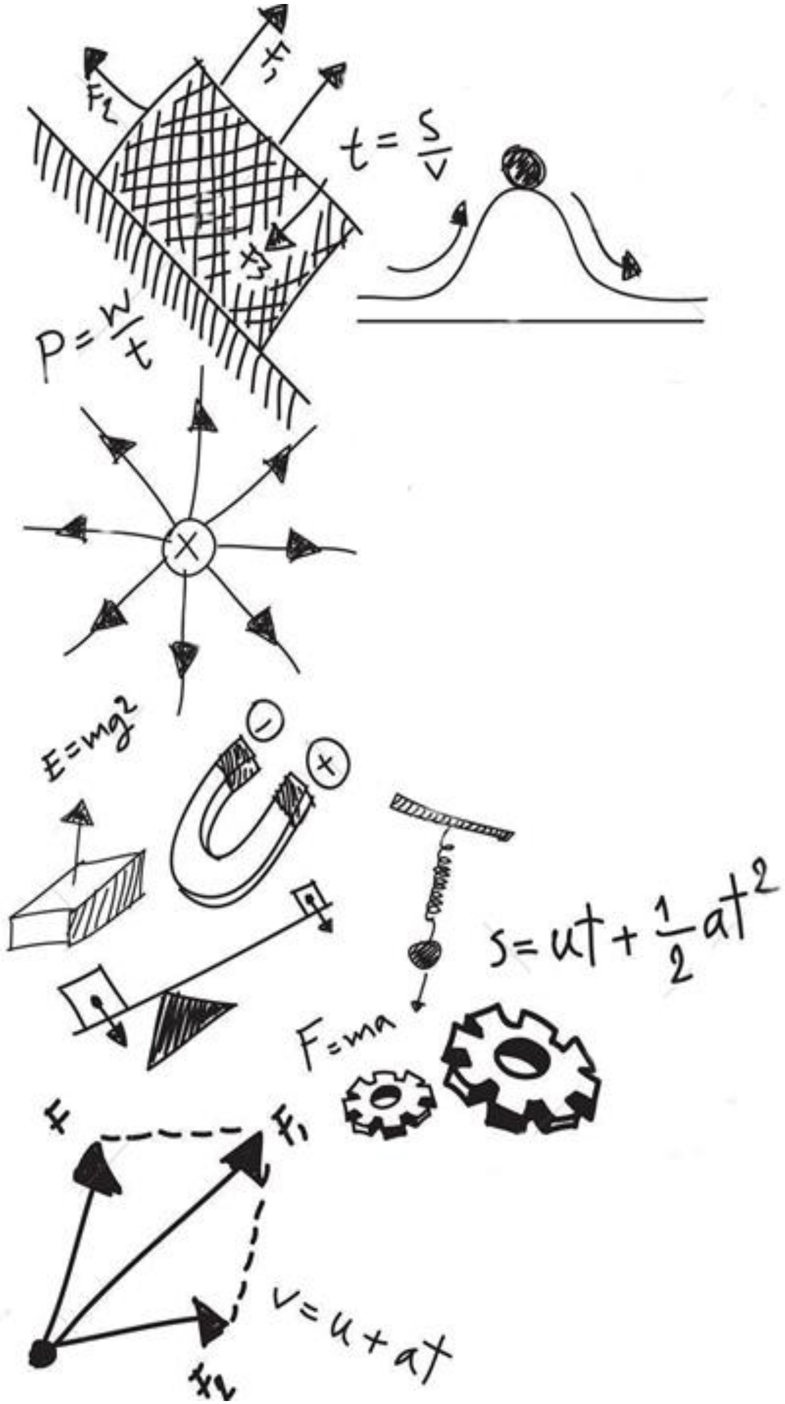


简谐振动的能量
和合成

一、简谐振动的能量

◆（1）以弹簧振子为例

$$F=-kx \quad \begin{cases} x=A\cos(\omega t+\varphi) \\ v=-A\omega\sin(\omega t+\varphi) \end{cases}$$

动能：$E_k=\frac{1}{2}mv^2=\boxed{\frac{1}{2}m\omega^2A^2}\sin^2(\omega t+\varphi)$

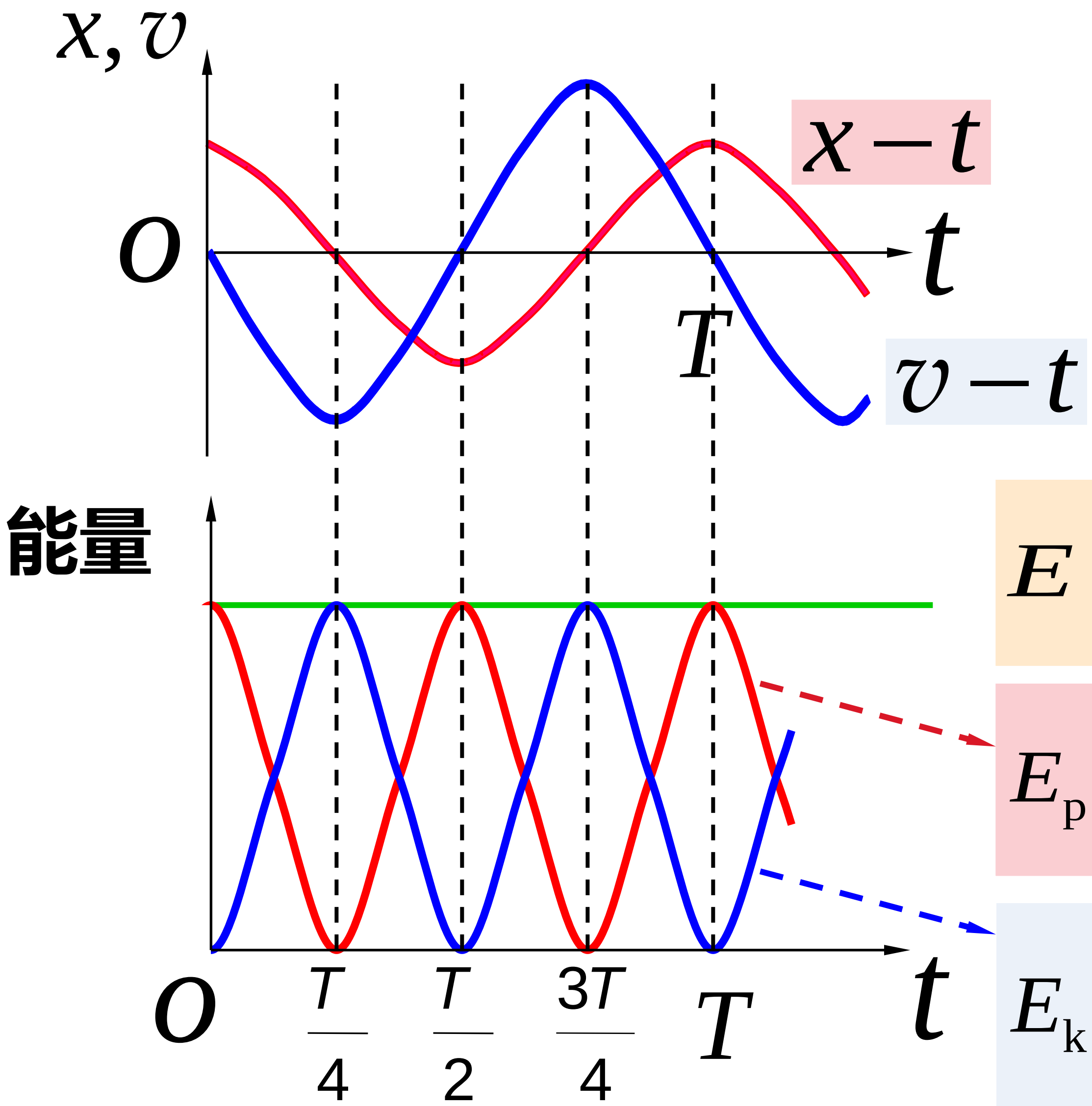
势能：$E_p=\frac{1}{2}kx^2=\boxed{\frac{1}{2}kA^2}\cos^2(\omega t+\varphi)$

说明：
$\omega^2=k/m$

机械能：$E=E_k+E_p=\frac{1}{2}kA^2\propto A^2$　（振幅的动力学意义）

线性回复力是保守力，作简谐运动的系统机械能守恒。

（2）简谐振动的能量图



$\varphi = 0$

$x = A\cos\omega t$

$v = -A\omega\sin\omega t$

$E = \frac{1}{2}kA^2$ 机械能

$E_p = \frac{1}{2}kA^2\cos^2\omega t$ 势能

$E_k = \frac{1}{2}m\omega^2 A^2\sin^2\omega t$ 动能

（3）讨论

能量守恒 $\xrightarrow{\text{推导}}$ **简谐运动方程**

$$E=\frac{1}{2}mv^2+\frac{1}{2}kx^2=\text{常量}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv^2+\frac{1}{2}kx^2\right)=0$$

$$m\cancel{v}\frac{\mathrm{d}v}{\mathrm{d}t}+kx\cancel{\frac{\mathrm{d}x}{\mathrm{d}t}}=0$$

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\frac{k}{m}x=0$$

二、简谐振动的合成

（1）同方向同频率的简谐振动的合成

① **分振动**：$\begin{cases} x_1 = A_1\cos(\omega t+\varphi_1) \\ x_2 = A_2\cos(\omega t+\varphi_2) \end{cases}$

② **合振动**：$x = x_1 + x_2 = A_1\cos(\omega t+\varphi_1) + A_2\cos(\omega t+\varphi_2)$

$$= \underbrace{(A_1\cos\varphi_1 + A_2\cos\varphi_2)}_{\substack{\shortparallel \\ A\cos\varphi}}\cos\omega t - \underbrace{(A_1\sin\varphi_1 + A_2\sin\varphi_2)}_{\substack{\shortparallel \\ A\sin\varphi}}\sin\omega t$$

$$= A\cos\varphi\cos\omega t - A\sin\varphi\sin\omega t$$

$$\boxed{= A\cos(\omega t+\varphi)}$$

结论：合振动 x 仍是简谐振动

② 合振动:

$$\boxed{x = A\cos(\omega t + \varphi)}$$

根据分振动的振幅和初相位确定

$$A\cos\varphi = A_1\cos\varphi_1 + A_2\cos\varphi_2$$

$$A\sin\varphi = A_1\sin\varphi_1 + A_2\sin\varphi_2$$

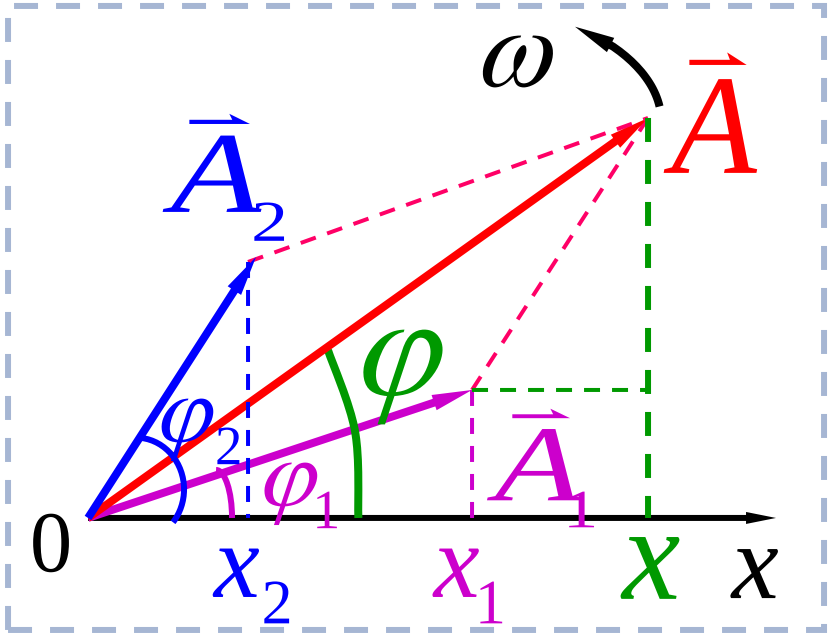
$$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)}}$$

$$\boxed{\tan\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}}$$

③ 旋转矢量法合成:

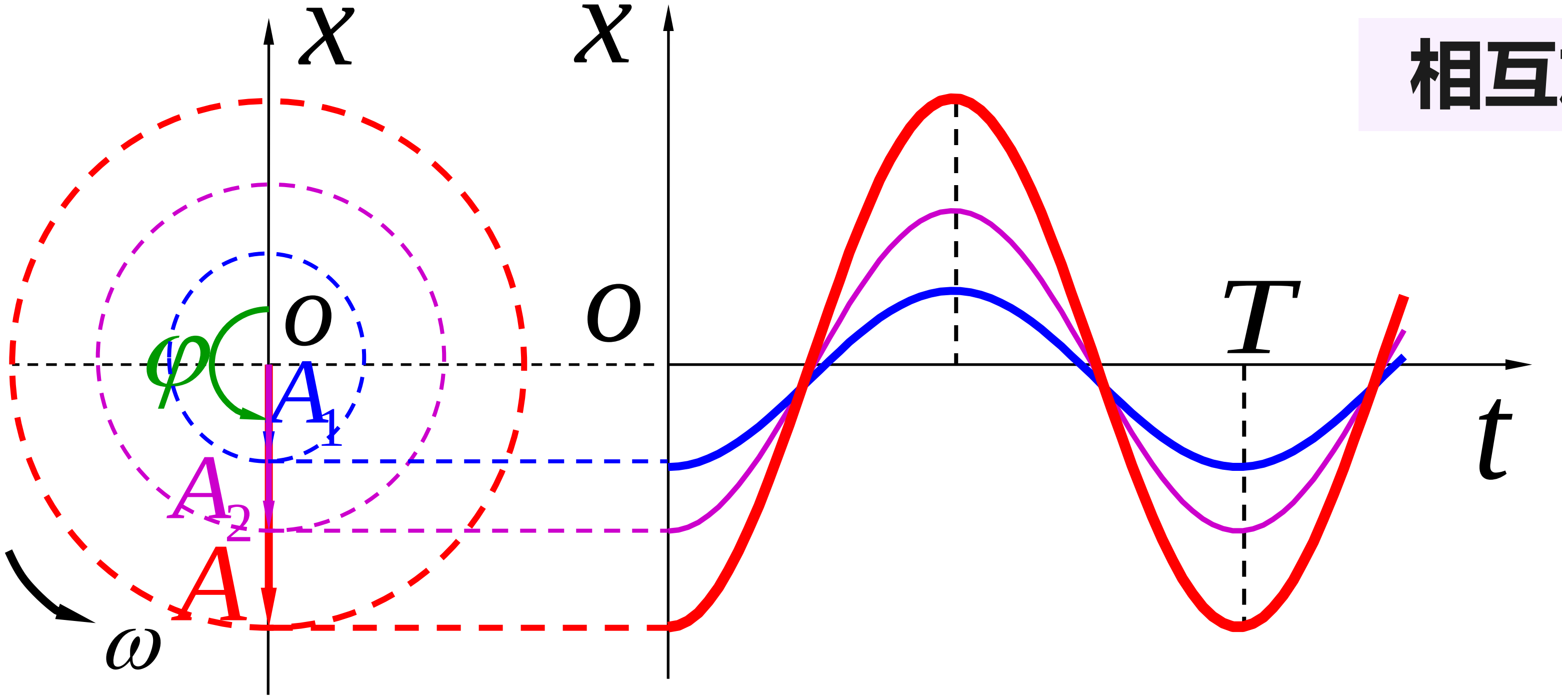
$$\begin{cases} x_1 = A_1\cos(\omega t + \varphi_1) \\ x_2 = A_2\cos(\omega t + \varphi_2) \end{cases}$$

$$\boldsymbol{x = x_1 + x_2 = A\cos(\omega t + \varphi)}$$



◆（2）讨论 $A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$

① **相位差** $\Delta\varphi=\varphi_2-\varphi_1=2k\pi \quad (k=0,\pm1,\pm2,\cdots)$



相互加强

$$\begin{cases} A=A_1+A_2 \\ \varphi=\varphi_2=\varphi_1+2k\pi \end{cases}$$

$$x=(A_1+A_2)\cos(\omega t+\varphi)$$

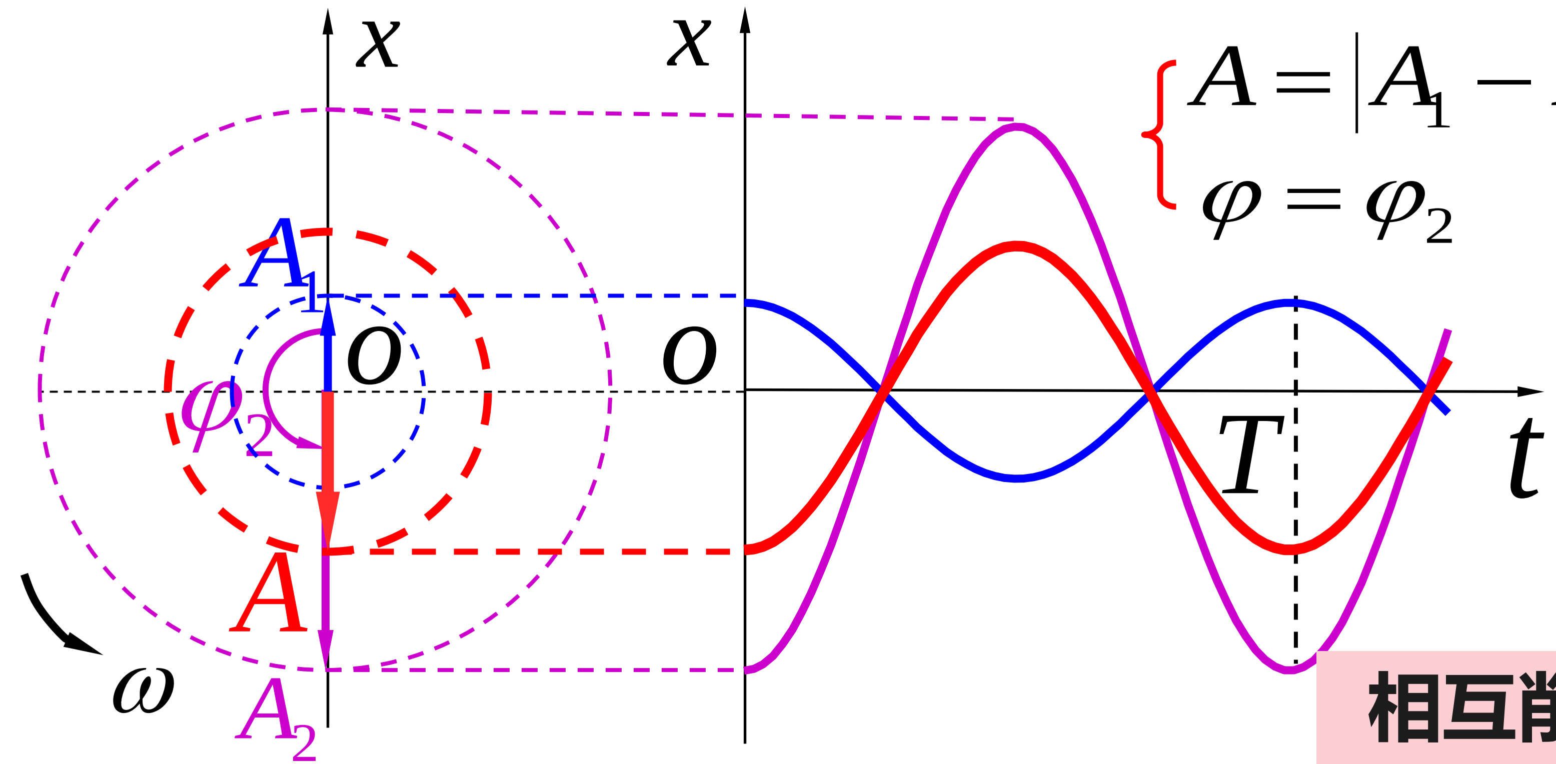
（2）讨论

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

② **相位差** $\Delta\varphi=\varphi_2-\varphi_1=(2k+1)\pi \quad (k=0,\pm1,\cdots)$

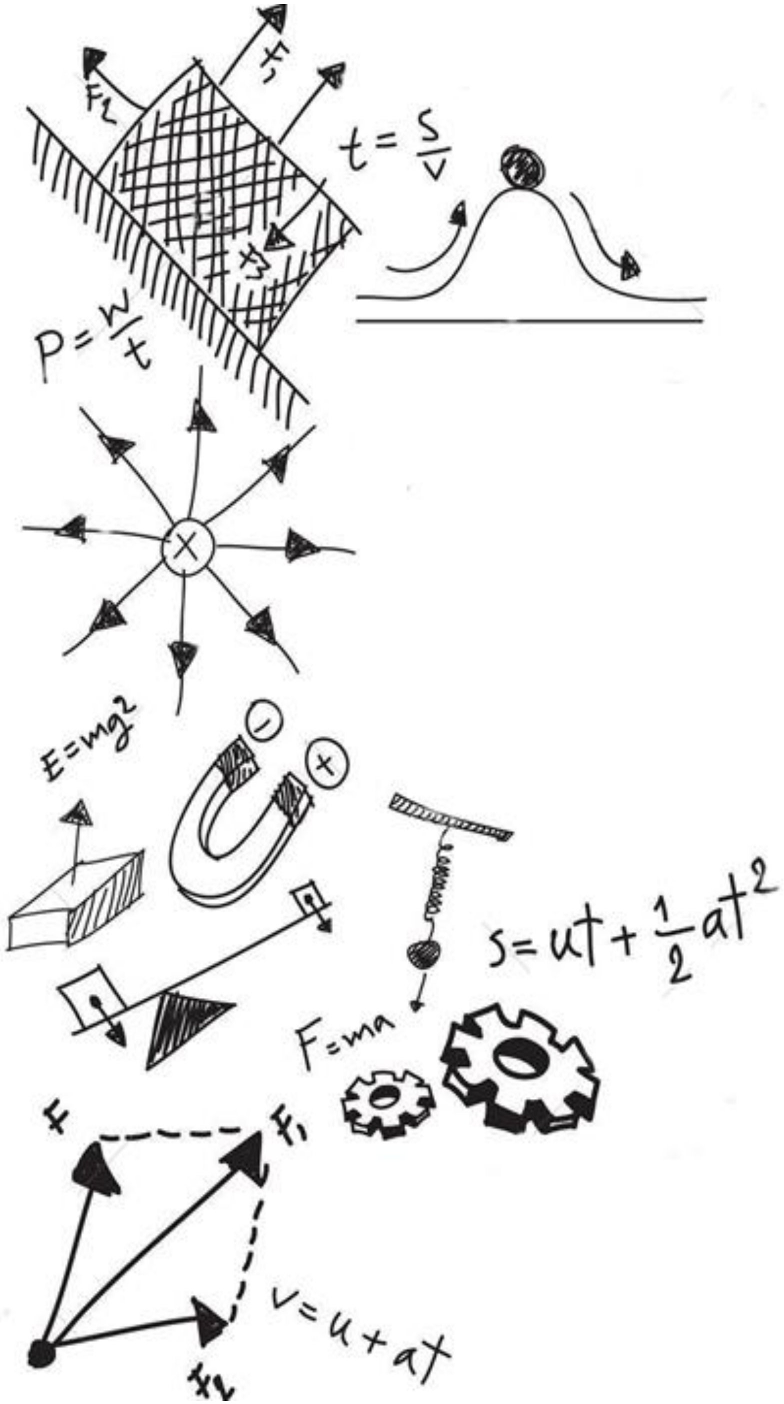
$$\begin{cases} x_1=A_1\cos\omega t \\ x_2=A_2\cos(\omega t+\pi) \end{cases}$$

$$x=(A_2-A_1)\cos(\omega t+\pi)$$



$$\begin{cases} A=|A_1-A_2| \\ \varphi=\varphi_2 \end{cases}$$

相互削弱


$t = \frac{S}{V}$
$P = \frac{W}{t}$
$F = ma$
$s = ut + \frac{1}{2}at^2$
$v = u + at$

Thanks!


$PE = mgh$
$W = F \times S$
$S_x = V_x t$
$S = \left(\frac{u+v}{2}\right)t$
$PE = m \times g \times h$
$t = \frac{S}{V}$