

## Functions and Relations

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**Exercise 1.** Consider the relation  $\mathcal{R} \subseteq \mathbb{R} \times \mathbb{R}$  defined by  $a\mathcal{R}b$  if, and only if,  $b + 0.5 \geq a \geq b - 0.5$ . Is  $\mathcal{R}$

- (a) reflexive?
- (b) antireflexive?
- (c) symmetric?
- (d) antisymmetric?
- (e) transitive?

**Exercise 2.** Prove each of the following statements.

- (a)  $(\mathbf{A}^T)^T = \mathbf{A}$  for any matrix  $\mathbf{A}$ .
- (b) If two matrices  $\mathbf{A}$  and  $\mathbf{B}$  are of the same size, then  $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$ .
- (c)  $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$  for any matrix  $\mathbf{A}$  of size  $m \times n$  and matrices  $\mathbf{B}, \mathbf{C}$  of size  $n \times p$ .

**\*Exercise 3.** Consider a relation  $\mathcal{R}$  on  $\text{Pow}(U)$  for some set  $U$  defined by  $A\mathcal{R}B$  iff  $|A \cap B| \geq 1$ . Prove that  $\mathcal{R}$  is transitive iff  $|U| \leq 1$ .