Logic — Boolean Expressions

Exercise 1. Consider the formulae $\phi_1 = (r \Rightarrow p)$ and $\phi_2 = (p \Rightarrow (q \vee \neg r))$. Transform the formula $\phi = (\neg q \Rightarrow (\phi_1 \wedge \phi_2))$ into **DNF**. Simplify the result as much as possible.

Exercise 2. Digital circuits are often built only from **nand**-gates with two inputs. A **nand** B is defined as $\overline{A \cdot B}$, that is, $\neg(A \wedge B)$. Prove that **nand**-gates are sufficient to encode any Boolean function.

*∗**Exercise 3.***

(a) Give all elements of BOOL(1), that is, all functions over a single boolean variable.

(b) Prove that there are 2^{2^n} elements in BOOL(n) for $n \in \mathbb{P}$.

Exercise 4. In Mew (cf. Week 2, Exercise 4), the Nelsons, who are truars, were leaving their four children with the new babysitter, Nancy, for the evening. Before they left, they told Nancy that three of their children were consistent liars but that one of them was a truar. While she was preparing dinner, one of the children broke a vase in the next room. When she asked who broke the vase the children's answers were:

Betty: Steve broke the vase,

Steve: John broke it,

Laura: I didn't break it,

John: Steve lied when he said I broke it.

Who did it?