COMP9020 17s1 • Problem Set 2 • 10 March 2017

Solutions

Exercise 1. We are given

1. $H \wedge \neg R$
2. $(H \wedge N) \Rightarrow R$

From 1 we can conclude H and $\neg R$. If N were true, then from H and N we could conclude R by 2, which contradicts $\neg R$. Hence, N cannot be true, which proves $\neg N$.

Hint: You can also use a truth table to show that $\neg N$ is true in every row in which formulae 1 and 2 are true.

Exercise 2.

- First question: Yes. In fact, the conclusion follows directly from just the first requirement.
- Second question: No. The third requirement states that the alarm should sound whenever there is a fire. On the other hand, the first requirement does not require the alarm to sound at all (it only states a requirement about when the alarm should *not* sound); and the second requirement mentions nothing about fire at all.

Exercise 3. If n is odd, then $n-1$ and $n+1$ are both even and one of them must be divisible by 4. It follows that $n^2 - 1 = (n+1)(n-1) = 2k \cdot 4\ell = 8k\ell$, for some $k, \ell \in \mathbb{N}$. Therefore, $8 \mid (n^2 - 1)$.

Hint: Other proofs are possible.

Exercise 4. Model the "character" of each of the three persons (Joan, Shane and Peter) with a proposition J,S,P. These are true if and only if that person is a truar. Then, we write their statements as follows:

$J \Leftrightarrow \neg S \wedge \neg P$

$S \Leftrightarrow \neg\neg S$

$P \Leftrightarrow \neg S$

Using a truth table we can see that the only assignments consistent with the above are: J=F, S=T, P=F or J=F, S=F, P=T. In both cases there are two liars and one truar.