COMP9020 17s1 • Problem Set 9 • 19 May 2017

Solutions

Exercise 1.

(a) One member from each profession (out of 5) must be selected; therefore $5^3 = 125$ panels.

(b) $4^3 = 64$ panels possible, out of which we need to subtract the 4 panels including Brent and David $\Rightarrow$ 60 panels.

(c) A 5-member panel can either consist of 3 members of one profession and 1 member from each of the other two, so $3 \cdot \binom{5}{1}\binom{5}{1}\binom{5}{3} = 750$ panels; or of 2 members each from two professions and 1 member of the remaining profession, so $3 \cdot \binom{5}{2}\binom{5}{2}\binom{5}{1} = 1500$ panels. The total is 2250 panels.

(d) As above, we find there are a total of $3 \cdot \binom{4}{1}\binom{4}{1}\binom{4}{3} + 3 \cdot \binom{4}{2}\binom{4}{2}\binom{4}{1} = 624$ possible panels. Out of these we need to subtract the panels that include Brent and David; one way to count them is that there are $\binom{10}{3} = 120$ ways to select the remaining 3 members, minus $\binom{6}{3} = 20$ that do not include any accountant, therefore $120 - 20 = 100$ panels that include Brent and David $\Rightarrow$ total of $624 - 100 = 524$ possible panels.

Exercise 2.

(a) For each $x \in S$ there are $|T|$ choices for $f(x)$; hence, the number of functions is $|T|^{|S|} = 3^4 = 81$.

(b) Each pair $(x, y) \in S \times T$ is either related or not; hence, there are $2^{|S|\cdot|T|} = 2^{12} = 4096$ relations.

(c) Among the 81 functions from (a), *not* onto are all the functions $S \longrightarrow \{e, f\}$ ($2^4 = 16$ functions), $S \longrightarrow \{e, g\}$ ($2^4 = 16$ functions), $S \longrightarrow \{f, g\}$ ($2^4 = 16$ functions). This, however, counts twice the functions $S \longrightarrow \{e\}$, $S \longrightarrow \{f\}$ and $S \longrightarrow \{g\}$. Hence, there are $81 - 3 \cdot 16 + 3 = 36$ onto functions $S \longrightarrow T$.

(d) If $\mathcal{R}$ is an antireflexive relation on $S \times S$, then $(x, x) \notin \mathcal{R}$ for all $x \in S$. Each other pair, that is $(x, y) \in S \times S$ with $x \neq y$, is either related or not. There are $|S| \cdot (|S| - 1) = 12$ such pairs; hence, there are $2^{12} = 4096$ such relations.

Exercise 3.

(a) Possible outcomes are $(i, j, k) \in \{1, \ldots, 6\} \times \{1, \ldots, 6\} \times \{1, \ldots, 6\}$. Possible prime sums are 3 (1 outcome), 5 (6 outcomes), 7 (15 outcomes), 11 (27 outcomes), 13 (21 outcomes) and 17 (3 outcomes). Hence, the overall probability is $\frac{1+6+15+27+21+3}{216} = \frac{73}{216} \approx 0.338$.

(b) Select 2 out of 3 dice to have the same value, which can be any of $1 \ldots 6$, while the third number is different, hence one of the 5 remaining values. Thus the overall probability is $\frac{\binom{3}{2}\cdot 6\cdot 5}{216} = \frac{90}{216} = \frac{5}{12}$.

Exercise 4. By definition,

$$P(E_1 \setminus E_2) = \sum_{\omega \in E_1 \setminus E_2} P(\omega) = \sum_{\omega \in E_1} P(\omega) - \sum_{\omega \in E_1 \cap E_2} P(\omega) = P(E_1) - \sum_{\omega \in E_1 \cap E_2} P(\omega)$$

Hence, if $P(E_1 \setminus E_2) = P(E_1) - P(E_2)$ then $\sum_{\omega \in E_1 \cap E_2} P(\omega) = \sum_{\omega \in E_2} P(\omega)$. Therefore, $P(\omega) = 0$ for all $\omega \in E_2 \setminus E_1$, hence $\sum_{\omega \in E_2 \setminus E_1} P(\omega) = 0$.