

## Advanced Counting and Probability

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**Exercise 1.** How many 5-letter words over the alphabet  $\Sigma = \{a, c, e, n, s\}$

- (a) include the substring *ace*?
- (b) include all letters from  $\Sigma$  with *a* before *e* (for example, *canes*)?
- (c) have all their letters in alphabetical order (for example, *aceen*)?

**Exercise 2.** Prove that

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | A_1 \cap A_2) \cdots P(A_n | A_1 \cap A_2 \cap \dots \cap A_{n-1})$$

**Exercise 3.** Consider three urns: Urn 1 contains one red and two black marbles, Urn 2 contains three red and four black marbles, Urn 3 contains two red and two black marbles. One urn is selected at random and then two marbles are randomly drawn from that urn without replacement. Given that these two marbles are red, what is the probability that Urn 2 was chosen?

**\*Exercise 4.** Alice and Bob repeatedly toss a coin (outcome *H* – head, or *T* – tails) until either Alice's or Bob's winning sequence is observed. What is the probability for Alice to win if

- (a) Alice's winning sequence is *HTH* and Bob's is *HHH*?
- (b) Alice's winning sequence is *HTH* and Bob's is *THT*?