Graphs

Exercise 1. True or false?

(a) The complete bipartite graph $K_{5,5}$ has no cycle of length five.

(b) If T is a tree with at least four edges, then $\chi(T) = 3$.

(c) Let C_n denote a cycle on n vertices. For all $n \geq 5$ it holds $\chi(C_n) \neq \chi(C_{n-1})$.

(d) It is possible to remove two edges from K_6 so that the resulting graph has a clique number of 4.

Exercise 2. Let G be an undirected graph on 20 vertices with exactly two connected components. What is the minimum and the maximum possible number of edges in G?

Exercise 3. For what pairs of integers (i, j), $i \geq j \geq 1$, are the graphs $K_{i,j}$ planar?

Exercise 4. Consider the complete 3-partite graphs $K_{4,1,1}$, $K_{3,2,1}$, $K_{2,2,2}$.

(a) What is the chromatic number of each of these graph?

(b) Which of these graphs are planar?

∗**Exercise 5.** What is the minimum number of edges that need to be removed from K_5 so that the resulting graph has a chromatic number of

(a) 3?

(b) 2?

(c) 1?