COMP9020 17s1 • Problem Set 4 • 24 March 2017

Solutions

Exercise 1.

(a) Yes, since $a + 0.5 \geq a \geq a - 0.5$ for all $a \in \mathbb{R}$

(b) No; see (a)

(c) Yes, since $(b + 0.5 \geq a) \wedge (a \geq b - 0.5)$ implies $(b \geq a - 0.5) \wedge (a + 0.5 \geq b)$.

(d) No; e.g. $(0, 0.1) \in \mathcal{R}$ and $(0.1, 0) \in \mathcal{R}$.

(e) No; e.g. $(1.1, 1.5) \in \mathcal{R}$ and $(1.5, 1.9) \in \mathcal{R}$ but $(1.1, 1.9) \notin \mathcal{R}$ since $1.9 - 0.5 > 1.1$

Exercise 2.

(a) If $\mathbf{A}$ is of size $m \times n$ then $\mathbf{A}^T$ is of size $n \times m$, hence $(\mathbf{A}^T)^T$ is of size $m \times n$, the same as $\mathbf{A}$. The (i, j)-th entry of $\mathbf{A}^T$ is a_{ji}, hence the (i, j)-th entry of $(\mathbf{A}^T)^T$ is a_{ij}, the same as $\mathbf{A}$.

(b) The (j, i)-th entry of $\mathbf{A} + \mathbf{B}$ is $a_{ji} + b_{ji}$, hence the (i, j)-th entry of $(\mathbf{A} + \mathbf{B})^T$ is $a_{ji} + b_{ji}$. The (i, j)-th entry of $\mathbf{A}^T$ is a_{ji} and the (i, j)-th entry of $\mathbf{B}^T$ is b_{ji}, hence the (i, j)-th entry of $\mathbf{A}^T + \mathbf{B}^T$ is $a_{ji} + b_{ji}$. This proves the claim.

(c) The (j, k)-th entry of $\mathbf{B} + \mathbf{C}$ is $b_{jk} + c_{jk}$, hence the (i, k)-th entry of $\mathbf{A}(\mathbf{B} + \mathbf{C})$ is

$$\sum_{j=1}^{n} a_{ij}(b_{jk} + c_{jk}) \tag{1}$$

The (i, k)-th entry of $\mathbf{AB}$ is $\sum_{j=1}^{n} a_{ij} b_{jk}$ and the (i, k)-th entry of $\mathbf{AC}$ is $\sum_{j=1}^{n} a_{ij} c_{jk}$, hence the (i, k)-th entry of $\mathbf{AB} + \mathbf{AC}$ is

$$\sum_{j=1}^{n} a_{ij} b_{jk} + \sum_{j=1}^{n} a_{ij} c_{jk} \tag{2}$$

This proves the claim since $(1) = \sum_{j=1}^{n}(a_{ij} b_{jk} + a_{ij} c_{jk}) = (2)$

Exercise 3. If $|U| = 0$, there is only one subset (empty) that is not related to itself so there is nothing to violate the transitivity (it holds vacuously). If $|U| = 1$, there is only one case when $\mathcal{R}$ holds, which is $U \mathcal{R} U$ and again nothing violates transitivity. If $|U| \geq 2$, choose any two elements in U (say, a and b) and observe that $\{a\} \mathcal{R} \{a, b\}$ and $\{a, b\} \mathcal{R} \{b\}$, but $\{a\}$ is not in a relation with $\{b\}$, so $\mathcal{R}$ is not transitive.