COMP9020 17s1 • Problem Set 1 • 3 March 2017

Numbers, Sets, Words

Exercise 1. How many numbers are there between 100 and 1000 that are

(a) divisible by 3?

(b) divisible by 5?

(c) divisible by 15?

Exercise 2. Prove that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

(a) using Venn diagrams,

$*$(b) without Venn diagrams.

Exercise 3. Let $\Sigma = \{a, b, c\}$ and $\Phi = \{a, c, e\}$.

(a) How many words are in the set Σ^2?

(b) What are the elements of $\Sigma^2 \setminus \Phi^*$?

(c) Is it true that $\Sigma^* \setminus \Phi^* = (\Sigma \setminus \Phi)^*$? Why?

Exercise 4. Recall the algorithm for computing the gcd of two positive numbers[1]:

$$\gcd(m, n) = \begin{cases} m & \text{if } m = n \\ \gcd(m - n, n) & \text{if } m > n \\ \gcd(m, n - m) & \text{if } m < n \end{cases}$$

Recall the correctness proof given in class. What needs to change to adapt it to the potentially faster version below?

$$\gcd(m, n) = \begin{cases} m & \text{if } m = n \\ \gcd(m \bmod n, n) & \text{if } m > n \\ \gcd(m, n \bmod m) & \text{if } m < n \end{cases}$$

[1] The way it's defined here works for positive numbers only.