COMP9020 17s1 • Problem Set 7 • 5 May 2017

Induction and Recursion

Exercise 1.

(a) Prove by induction that

$$1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1 \quad \text{for } n \geq 1$$

(b) Given the recursive definition,

$$\begin{array}{ll} \text{(B)} & s_1 = 1 \\ \text{(R)} & s_{n+1} = \frac{1}{1+s_n} \end{array}$$

prove by induction that

$$s_n = \frac{\text{FIB}(n)}{\text{FIB}(n+1)} \quad \text{for } n \geq 1$$

Exercise 2. Prove that in any rooted tree, the number of leaves is one more than the number of nodes with a right sibling.

Hint: This assumes a given order among the children of every node from left to right; see page 38 of the lecture slides week 7 for an example.

$*$**Exercise 3.** Prove by induction that every *connected* graph $G = (V, E)$ must satisfy $e(G) \geq v(G) - 1$.

Hint: You can use the fact from the previous lecture that $\sum_{v \in V} deg(v) = 2 \cdot e(G)$.

Exercise 4. Let $T(n)$ be defined by the recurrence

$$T(n) = T(n-1) + g(n) \quad \text{for } n > 1$$

Prove by induction on i that if $1 \leq i < n$, then

$$T(n) = T(n-i) + \sum_{j=0}^{i-1} g(n-j)$$