Counting and Basic Probability

Exercise 1. A management panel at a hospital needs to include at least one member from each of the following three professions: a doctor, a lawyer and an accountant. How many different panels can be formed in each of the following situations?

(a) Each profession offers 5 possible candidates. The panel size is 3.

(b) Each profession offers 4 possible candidates, but A. Brent (doctor) refuses to serve with C. David (lawyer). The panel size is 3.

(c) Each profession offers 5 possible candidates. The panel size is 5.

(d) Each profession offers 4 possible candidates, but A. Brent (doctor) refuses to serve with C. David (lawyer). The panel size is 5.

Exercise 2. Let $S = \{a, b, c, d\}$ and $T = \{e, f, g\}$.

(a) How many different *functions* $f : S \longrightarrow T$ are there?

(b) How many different *relations* on $S \times T$ are there?

∗(c) How many different *onto* functions $f : S \longrightarrow T$ are there?

∗(d) How many different *binary relations* on S are *antireflexive*?

Exercise 3. Three dice are rolled simultaneously.

(a) What is the probability that the sum of the values is a prime number?

(b) What is the probability of a doublet (2 of the 3 values are equal but the third value is different)?

Exercise 4. Let E_1, E_2 be two events. Prove that $P(E_1 \setminus E_2) = P(E_1) - P(E_2)$ implies $P(E_2 \setminus E_1) = 0$.