Dynamic Pricing for Sports Event Ticket

Ziyun Huang
Purdue University, Department of Management, 403 W. State St
West Lafayette, IN 47907
huang747@purdue.edu

Wenying Huang
Purdue University, Department of Management, 403 W. State St
West Lafayette, IN 47907
huang814@purdue.edu

Wei-Cheng Chen
*Purdue University, Department of Management, 403 W. State St *
West Lafayette, IN 47907
chen1614@purdue.edu

Matthew A. Lanham
Purdue University, Department of Management, 403 W. State St
West Lafayette, IN 47907
lanahamm@purdue.edu

***Abstract*— This paper discusses a research that studies the market demand for the tickets of a major NFL team and develops a dynamic pricing model for the price of the tickets based on the understanding of the market demand. The team utilized R together with packages like h2o and ggplot2 to build a predictive model that could reflect future demand of tickets and developed an optimization strategy based on this model for the use of dynamic pricing. A Tableau dashboard was also created using simulation data from one of the previous games to demonstrate the revenue increase potential of this model.**

Keywords*— *Dynamic Pricing, Demand Forecast, Predictive Analytics, R, h2o

I. Introduction

Nowadays, in the world that is flooded by digital data, one price for all strategy is no longer optimized for products like sports tickets. With continuously increasing costs in operation, maintenance for the venue, and contracts for players, sports team management are in need of a modern strategy for pricing to optimize their revenue and profits. Dynamic pricing strategies are widely adopted in the hotel industry in which the demand for products is very flexible. According to Hoisington [1], the hotel industry in the US reports the highest revenue in the budget hotel market and revenue for budget hotels increased 3.5 percent over year thanks to dynamic pricing strategy adapted in the industry. The demand for sports tickets could also change dynamically with time, participants in the events, and other factors. With its similarity to the hotel industry, it is believed that the sports industry could also be greatly benefited from a dynamic pricing strategy. In fact, some sports teams have already employed data analytics in their pricing strategy to maximize their ticket revenue. According to SAS [2], the Orlando Magic from the NBA league has accomplished a game ticket revenue grown of 91% since the 2013-2014 season with its data-driven strategy in pricing. 91% is a huge number, and not to mention that the baseline revenue from an NBA team is tremendous by itself. According to Khan [3], the benefits of dynamic pricing include 1) greater control over pricing strategy 2) brand value with flexibility 3) cost efficiency in the long run 4) efficiency in management.

With the benefits and potential growth in profits that could be brought by dynamic pricing, this paper aims at discussing how to apply dynamic pricing strategy to individual sports teams in practice. A deep understanding of the methodologies of dynamic pricing in the sports industry is necessary to maximize the benefits of dynamic pricing in the industry. The scale of the sports industry is one of a kind in the U.S economy and the growth that could be brought to the sports industry by dynamic pricing would be a strong boost to the overall economy.

It is impossible to develop an effective price optimization model without an accurate prediction of the demand under different circumstances. Therefore, after data cleaning, the first and the most important part of constructing the optimization model would be to develop a predictive model for the demand. In the process, multiple machine learning methods would be applied to the cleaned data, and the model that performs the best among the candidate models would be chosen as the foundation to build the dynamic pricing model. With predictable demands, economic rules could be utilized to develop a dynamic pricing model. With the developed model, the optimized price could be generated based on historical data to obtain a potential optimized revenue compared to the real historical revenue. The dynamic pricing model which builds upon this process could provide sport teams insights and methods to capture the lost revenue due to ineffective pricing. The potential of this model could be quantified by the historical data to prove its effectiveness.

In the remainder of this paper, literature reviews would be discussed at first to explore the current studies in the field of dynamic pricing. Followed by the data section that gives the sources and context of the data used. The third section would be detailed explanations of methodologies to develop the predictive model and the optimization model. The evaluation and performance demonstration of the developed model would be the topic of the fourth section. The paper would then be concluded with the consideration of future application and study direction of this research.

II. Literature review

Dynamic pricing is becoming a prevailing method in the sporting event industry; hence, numerous researches have been developed to discuss the implementation of this approach. Dittmer and Carbaugh[4] have mentioned the frequency of game and size of the stadium in the sports industry provided an ideal condition for the adoption of dynamic pricing. The discussion mainly focused on applying microeconomics theory to the model. A dynamic model reduced the cost of re-pricing the ticket every time there is a gap in the price and consumer's perceived value because it will automatically match the expected value based on selected factors. Sweeting[5]'s paper discussed dynamic pricing behavior in the secondary markets for Major League Baseball tickets and concluded that sellers cut prices by more than 40% as event day approaches and adopting dynamic pricing raises the average seller's expected payoff by 16%. Our work will be similar to that of Sweeting[5]'s paper, but we also take into account data from the primary market in addition to the secondary market.

Before getting in touch with dynamic pricing, assessing consumer demand is a critical element before building a price optimization model. Strnad and Nerrat[6]'s paper examined the accuracy of using different neural network models to capture soccer match attendance. The paper suggested that all neural networks outperform linear predictor by a large margin and can capture attendance patterns. A more recent paper by Sahin and Erol[7] also evaluated the performance of neural network models on predicting soccer match demand. The paper used data of three sports teams and found that the ANN model with Elman network using 1 hidden layer and 20 neurons has the most accurate result in demand prediction. Even though neural network models are difficult to interpret the impact of each feature brings to the result, our paper will design NN models for comparison with other models.

Xu, Fader, and Veeraraghavan[8][9] evaluated the effect of dynamic pricing policy by developing a demand model for single-game ticket sales that were used to predict the revenue associated with a pricing strategy over the course of an MLB franchise in a season. The original dynamic pricing policies of this franchise resulted in a revenue decrease of 0.79% compared to static pricing. Hence, the authors proposed alternative pricing policies that helped the franchise find an optimized dynamic pricing policy that improved revenue by 14.3% compared to fixed pricing policy. A similar approach was proposed by Kemper and Breuer[10] where they applied mathematical theory to estimate a demand function before building a price optimization model. Their paper discovered that the consumers' willingness to pay is significantly higher than the original ticket price. The average willingness to pay ranged between € 70 and € 178, depending on the seat and price category. Our paper will also first develop a demand function for a sport event to obtain the probability that a ticket would be sold, then build a dynamic pricing model that reoptimizes prices on a daily or weekly basis.

Diehl and Maxcy[11] investigated elasticity of demand in the secondary market for NFL tickets and how elasticity varies across different seat types. The authors used the standard inverse elasticity rule to maximize profit when elasticity exceeds the unit elastic point. Price elasticity of demand is estimated for the entire venue and then location-specifically. Their research indicated that demand in the secondary market is price elastic and that the demand for higher quality seats is more price elastic than the demand for lower quality seats. An important factor that their research did not incorporate is that their data lack the timing of the sales relative to the game. Shapiro and Drayer[12] assessed factors that influenced the ticket price for a sports team in both the primary and secondary markets. Correlation designs were first used to observe the relationships between other factors and ticket prices. Their paper is different from our paper in that they devised two regression models which include a 2-stage least square regression (2SLS) using season ticket prices and secondary market price as the independent variable and an OLS model excluding these variables. The main finding of their paper suggested that season ticket price is an important feature that affects secondary market price and that the secondary market affects dynamic ticket pricing. Our paper only includes single-game ticket data which could potentially reduce the effect of our dynamic price optimization model.

III. DATA

To estimate the ticket demand for sports events and train all the necessary machine learning models, data is required to include all ticket transactions from both the primary and secondary markets. The datasets used in this study were retrieved from the internal database of a primary NFL team and used with the permission of the team.

The original datasets describe ticket pricings, online transactions, and event-related information in four separate tables: primary, secondary, unsold inventory, and opponent. The data covers all events in the team's main stadium from 2012 to 2019 seasons, and the total number of games played during the period was 79 after removing pre-season and post-season events. These four tables were cleaned, merged, and standardized before predictive modeling to acquire only significant measures for better forecasting results. Variables directly related to ticket pricing (e.g. seat location in the stadium, sale time, final sale price) were the primary internal measures for demand prediction and revenue management study. In addition to the internal factors, external factors that affect customer purchasing behaviors such as weather measures (e.g. temperature, snow/rain) or competitor measures (e.g. ranking, win probability) were also considered in the modeling. The data dictionary of the pre-processed input variables is given in Table 1.

TABLE I. DATA DICTIONARY OF THE PRE-PROCESSED INPUT VARIABLES

Variable	Type	Description
Target	Factor	The seat sold status, 1 = yes, 0 = no
Event Name	Factor	The event code of game
Section Name	Factor	The name of section
Row Name	Factor	The row number
Seat Num	Factor	The seat number on ticket
Team	Factor	The opponent team state and name
Season	Factor	The year of game
Event Date	Date	The date of game
Sale Date	Date	Ticket purchase's final transaction date (used the date time from the final transaction if resold in the secondary market)
Final Price	Numeric	Ticket's final price (used the final transaction price if resold)
Original Purchase Price	Numeric	Ticket's original purchase price from the primary market
Sales Channel	Factor	Ticket's sales channel
Opp Win	Numeric	The winning probability of opponent team
Opp LS Win	Numeric	Last season's win probability of opponent team
Team Win	Numeric	The winning probability of the team
Team LS Win	Numeric	Last season's win probability of the team
Road Attendance	Numeric	Attendance Percentage for opponent team. This is an indicator if a team has a big draw attendance wise or not.
FB Fans	Numeric	The Facebook fans number of opponent team
Distance	Numeric	Distance between the team's city and opponent city (in miles)
Home Opener	Factor	Whether the team was home opener of the season, 1 = yes, 0 = no
Temp at Kick	Numeric	Temperature at the first kick of the game
Rain/Snow	Factor	Weather condition during the game, 1 = rain/snow, 0 = no
Team Contention	Factor	Whether the team was out of contention, 1 = yes, 0 = no
Last Visit Years	Numeric	Number of years since opponent's last visit
Opp Scored LY	Numeric	Opponents points scored last year (previous year due to at the time we pull this data it is February for the next year and wanted to remain consistent). Regular Season only.
Opp Def GivenLY	Numeric	Opponents points given up last year (previous year due to at the time we pull this data it is February for the next year and wanted to remain consistent). Regular Season only.
Opp Playoff	Factor	Whether opponent had playoff game, 1 = yes, 0 = no
Off MVP	Numeric	Offensive NFL MVP votes the year before
Def MVP	Numeric	Defensive NFL MVP votes the year before
Odds f	Numeric	Super Bowl odds from February of that year
GA_indy_L10	Numeric	Google Trends Index for opponent team in Indianapolis over the past 10 years

IV. METHODOLOGY

According to fundamental economic theories, the demand for a product increases with the lowering of price and decreases with increasing price. In many economic studies, demand is modeled in a simple linear function between quantity and price. However, a more sophisticated model is in need to learn the demand of each ticket for this problem. The data that is available for this study contains information for both sold and unsold tickets. With this data, a predictive model for the binomial categorization problem of whether a certain ticket could be sold at a given price point and other factors could be built. With information about time also available in the data, this predictive model could give the probability of a ticket being sold at any given price point and point of time.

$$E(Revenue) = P(Demand \mid Price \,\&\, other\ factors) \times Price$$

Because the demand change with the price, reaching the goal of maximizing the revenue is not just about increasing price or increasing demand. It is necessary to find the balance point between price and the effects the price has on demand. With the predictive model, it is possible to use a recurring method to develop an optimization method to find the price point at which expected revenue is maximized for each ticket available at any given point of time.

The process of the study could be referred as the following figure:

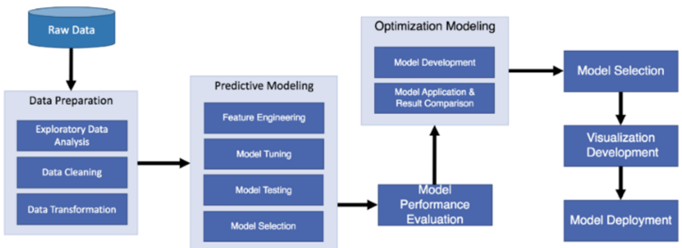

Fig. 1. Process Flow Chart

In the predictive model development process, one specific game from the 2019 season was taken out from the training process and was used as the testing set. 2:8 random partition was conducted to ensure random performance testing. Selected seat sections from this testing game were then used as a demonstration for the optimization model. A game from the 2019 season, which is more recent, could be in more proximity to the current and future situations. Leaving this game out of the training set could guarantee the precision testing statistics and business performance analysis. The Area Under Curve (AUC) would be the criteria used to determine the performance of the predictive model

For the game that was be used for demonstration, the expected revenue of the selected sections using the optimization method was calculated and compared to the actual historical revenue to demonstrate the performance of the model. The recommended price of each section at different point of time would be the range from the lower 1.5 IQR bound to the maximum of the optimized ticket price in the section at that time.

*A. D*ata Cleaning

a) Primary Data: This table includes the information (e.g. opponent, event date, sale date, section, row) of single-game tickets bought by customers directly from the team's website from 2012 to 2019. An individual with a ticket account ID could purchase multiple tickets at once; hence, a column named "num_seats" would indicate the number of tickets bought by the customer. We decomposed the rows with more than one ticket possession into separate rows. We also added a target column with all 1s to this table indicating every ticket in the table is sold.

b) Secondary Data: This table consists of tickets being sold on the secondary market. The table has over 600,000 rows of data because it includes different activities (e.g. update price, cancel posting, expired, successful resold) of the same ticket by the same sellers. To simplify the data, we subset only the resold, expired records. Then we assigned 1s to those whose activity name is resold, and 0s to the records that are expired. In this table, there were a lot of extreme posting price; therefore, we set a condition to only include prices that are below $1000. Event dates for this table also deviate from the actual event date by 1 day, we added 1 day to every event date in this table.

c) Unsold Inventory Data: This table includes all the unsold single game tickets of the team's website from 2012 to 2019. A target column of 0s was added indicating every ticket in the table is unsold. All the records in this table were combined with the Primary table.

d) Opponent Data: Opponent information including last season' win rate, weather conditions, Facebook fans, etc. Attributed to this table will be merged to combine ticket information and opponent information.

*B. Feature Engine*ering

Time is an important factor in a dynamic pricing model; Ticket buyers usually purchase tickets days or weeks before the event occurs. Hence, we inserted a new column that took the difference between the sales and game dates which shows how many days in advance people purchase the tickets. In addition, we extracted the day from the event date column to observe whether a particular day of the week could affect the demand. Categorical encoding was also necessary before moving into model building. Ticket information contains the section names, which generally have hundreds within a sports stadium. Therefore, performing one-hot encoding would impede the efficiency of the models. We used target encoding for the sections, which label the variable by the ratio of the target value occurrences. The remaining categorical features that have low numbers of levels were one-hot encoded.

Numerical features such as Facebook Fans and distance were standardized using min-max scaling to ensure the model does not bias to the feature that has larger numbers. This method rescales every numeric value to lie in the range between 0 and 1 and puts equal weight among all of them.

C. Data Imputation

After combining the value of the day in advance column, it would appear as null for those tickets that were unsold. We used the mice package to impute the missing values by using the cart (classification/regression tree) method to replace missing values. However, it is crucial that we state the assumption that the distribution of the missing values follows the same distribution of the existing values. A density plot of the imputed and existing values was created to compare the distributions.

V. MODELS

To obtain the most accurate demand forecasting models, four different supervised learning algorithms were compared.

A. Logistic Regression

Logistic regression is a generalized linear regression for binary classification problems. The probability of both positive and negative event outcomes could be given by logistics. This suits our need for the predictive model. Nevertheless, a regression method could provide specific insights on how each factor affects the change of the probability of outcomes. However, as will discuss in the following section, the performance of the logistic regression on this data set is not ideal. Therefore, more robust algorithms were tested.

B. Gradient Boosting / XGBoosting

Gradient Boosting algorithm is suitable for both regression and classification problems. The algorithm is based upon the classical decision tree algorithm in an iterative stage-wise method. Gradient boosting seeks to optimize the mean squared error (MSE) of the model by performing gradient descent.
XGBoosting is the abbreviate for eXtreme Gradient Boosting. Indicating that XGBoosting is a derivative of Gradient Boosting. According to Tianqi Chen[13] who is the author of XGBoosting, the engineering goal for the model was to push the limit of computations resources of boosted tree algorithm. He also stated that 'XGBoost used a more regularized model formalization to control over-fitting, which gives it better performance'.

The performance of these two algorithms was challenged by the random forest algorithm which is finally adopted for the development of the predictive model.

C. Random Forest

Similar to the gradient boosting algorithm, random forest operates by building a multitude of the decision tree. The algorithm is closely related to another algorithm called bootstrap aggregating (also known as bagging). The bootstrap aggregating algorithm repeatedly chooses random samples from the data with replacement to train decision trees. This method helps to reduce overfitting that is commonly seen in simple decision trees algorithm. The random forest takes one step further from bagging by limiting the candidate features that could be chosen at each split of a tree. This further reduces the risk of overfitting by decreasing the correlations between individual trees generated in the model training process.

D. Optimization

Having used the random forest algorithm to develop a predictive model, a function that optimizes the price of individual tickets at different point of time was then developed using an iterative method. The process was summarized in figure 2 below. Given any other features equal of a ticket, the price of the ticket was set at 0 initially and is increased by 1 dollar at each step of the iteration. The expected revenue of the ticket was calculated by

$$E(Revenue) = P(Demand \mid Price \,\&\, other\ factors) \times Price$$

at each step of the iteration. The demand at each step is

$$P(Demand \mid Price \,\&\, \ldots)$$

which was obtained from the predictive model. The expected revenue at each step is $E(Revene)t$ which was compared to the expected revenue at the last step $E(Revenue)t-1$. The iteration would stop when the increase of revenue between two steps is less than 0.4.

Fig. 2. Optimization Process

VI. RESULTS

The results are illustrated in two sections: the results of demand forecasting from the predictive models and the results of dynamic pricing from the optimization models as explained above.

A. Demand Forecasting Results from Predictive Models

In the predictive modeling phase, all the season tickets were excluded while all the final ticket transaction records, including listed "resold" and "expired" tickets in the secondary market and all tickets from the NFL team's internal ticket transaction records of the primary market and unsold inventory. The time measure to predict demand used in the models was the number of days difference between the event date and ticket sales date. To demonstrate the prediction accuracy for future use, one event in the 2019 season was selected as the testing dataset while all other events from 2012 to 2019 seasons were used as the training set to train models. Here, a game with event code 'CLT19DEN' was used to exemplify the prediction and optimization results. Since the testing and training data partition is very specific on the individual event, sophisticated machine learning models such as Random Forest and Gradient Boosting were used to improve prediction results.

In the binary classification problem of this study, the model performance evaluation metric is the Area Under Curve (AUC).

Based on the AUC rankings, the top four best performance models are listed in the table and chart below:

TABLE II. MODEL Performance Metric Result

Model	*AUC*	*LogLoss*	*AUCPR*	*Mean Per-Class Error*	*RMSE*	*MSE*	*Gini*	*R^2*
Random Forest	0.7848318	0.5944028	0.6987082	0.2989711	0.4516365	0.2039755	0.5696636	0.1297819
Gradient Boosting	0.7686897	0.5707387	0.6674262	0.3144083	0.4406208	0.1941467	0.5373793	0.1717146
XGBoosting	0.7536417	0.6030912	0.6672684	0.3303482	0.4591871	0.2108528	0.5072833	0.1004412
Logistic Regression	0.6959825	0.6069658	0.5354505	0.3493138	0.4588363	0.2105307	0.3919649	0.1018155

TABLE III. XGBOOSTING Model Confusion Matrix

	0	**1**	**Error**	**Rate**
0	3819	4613	0.547083	4613/ 8432
1	575	4486	0.113614	575/5061
Totals	4394	9099	0.384496	5188/13493

TABLE IV. RANDOM Forest Model Confusion Matrix

	0	**1**	**Error**	**Rate**
0	4753	3679	0.436314	3679/8432
1	818	4243	0.161628	818/5061
Totals	4394	9099	0.384496	5188/13493

TABLE V. Gradient Boosting Model Confusion Matrix

	0	**1**	**Error**	**Rate**
0	4491	3941	0.467386	3941/8432
1	817	4244	0.161431	817/5061
Totals	5571	7922	0.333284	4497/13493

TABLE VI. Logistic Regression Model Confusion Matrix

	0	**1**	**Error**	**Rate**
0	3964	4468	0.529886	4468/8432
1	854	4207	0.168741	854/5061
Totals	4818	8675	0.394427	5322/13493

VII. CONCLUSION

The sports ticket market could have a giant potential increase in profit with the introduction of dynamic pricing to the market. A predictive model that targets whether a ticket would be sold at a given price point and time could portrait the demand for sports tickets. The predictive model could provide the probability of a ticket being sold. With the probability of a ticket being sold, an iterative method could be adopted to find the price at which the expected revenue of the sale of a certain ticket could be found.

A. Limitation

Despite the good performance of the optimization on sections that are lower in price, the optimization result is not ideal for premium tickets like those in section 117. It is possible that the demand for premium tickets is not as elastic as the demand for cheaper tickets. Therefore, the increase in demand given by lower pricing could not compensate for the revenue because of the lowering of price. However, revenue increase by dynamic pricing for cheaper tickets like the upper-deck tickets is still huge. According to Overby[14], San Francisco Giants from the MLB was the first professional sports team to adopt dynamic pricing in their upper-deck seats which increased revenue of $500,000. It is reasonable to believe that optimization for cheaper tickets only could also bring enormous revenue increase to NFL teams.

The iterative method for optimization is time consuming and require strong computing power resources. This is the reason why the research team only used a few sections for performance demonstration instead of using a whole game. With limited computing power, it is difficult to achieve the goal of daily updating for ticket prices.

B. *Future Study Direction*

a) Parallel Computing

Parallel computing could be a solution to the issue of slow computing with the iterative method. Future studies could focus on parallel computing to increase the speed of the optimization process. Other methods to solve the issue of slow computing include but are not limited to using environments other than R which is used in this research.

b) Database Management

The research team would also suggest a better data management system or method to be introduced. Typo and inconsistency are common in the data set and might become the obstacle for future implementation of the optimization method as well as any other potential future studies based on the data set. A potential future research topic could be finding a suitable data management method and/or system for sports teams in the NFL. This would not only benefit individual sports team in terms of their information management but also create a more organized data structure for follow up study in the optimization method discussed in this article.

c) Attendance Rates

When it is essential for the team to optimize ticket prices to maximize potential revenue, it is also important to optimize the attendance rate of games to maximize the experience of fans to maintain fans' loyalty for long term revenue. New technologies like the Internet of Things (IoT) are available to keep track of attendance rate of game and other important information like when an attendant of the game left. With more information that could be provided by new technologies like IoT, it is possible to conduct studies to optimize attendance rates for better fans experience.

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