NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

CE2101/CZ2101 – ALGORITHM DESIGN AND ANALYSIS

Apr/May 2021 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer **ALL** questions.
3. This is a closed-book examination.
4. All questions carry equal marks.

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1. (a) When the Mergesort algorithm is applied to an input array [5, 2, 8, 5, 3, 4], what is the total number of key comparisons performed by the implementation of Mergesort learnt in our lectures? What is the number of key shiftings by one position to the right performed in the **last call** of the merge function? Here we count each key shifting instead of block shifting. Briefly explain your answer.

(12 marks)

(b) Consider a hybrid sorting algorithm that combines Quicksort with Insertion Sort for better efficiency. Once the size of a subarray in a recursive call of Quicksort is less than or equal to 16, the algorithm will switch to Insertion Sort. What is the number of key comparisons performed by this hybrid algorithm in the worst case when running on an input array of size n? Briefly justify your answer. You could assume $n = 2^k$, for some integer $k > 4$.

(6 marks)

(c) An array A of n elements is said to have a majority element if more than half of its elements are the same. Given an input array, the task is to find the majority element if it exists and return NIL if it doesn't exist. The only operation you can perform is the equality test, which checks if two

Note: Question No. 1 continues on Page 2

elements are equal. Use the divide-and-conquer approach to design an algorithm to solve the problem with O(n log n) equality tests. Describe the algorithm sketch and analyze its complexity. You could assume $n = 2^k$, for some integer k.

(7 marks)

2. (a) A weighted graph H is shown in Figure Q2. Execute the Prim's algorithm on H starting at vertex B. Use alphabetical order to break a tie in choosing the next vertex. Draw the minimum spanning tree obtained and calculate its total weight.

(9 marks)

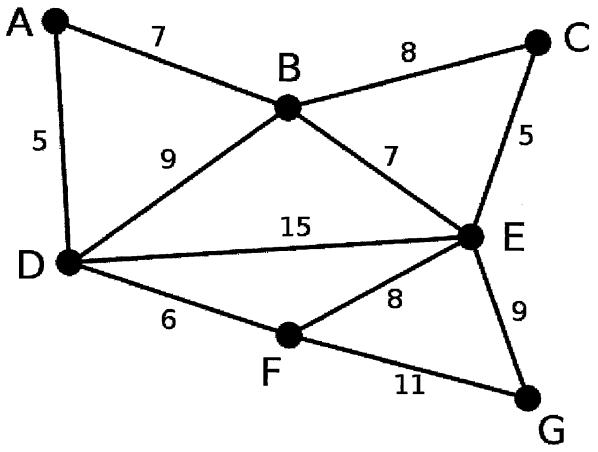

Figure Q2

(b) Given a directed weighted graph G with non-negative edge weights, the Dijkstra's algorithm is used to compute the shortest path P from a source vertex s to a target vertex t. If the weight of every edge in G is multiplied by 5, does P always remain the shortest path from s to t in the modified graph? Explain why or why not.

(8 marks)

(c) Do the Prim's algorithm and the Kruskal's algorithm always obtain the same minimum spanning tree (MST) on a given input graph? If yes, provide a proof. Otherwise, describe when they generate different MSTs.

(8 marks)

3. (a) Determine which of the following recurrences can be solved by the master method and justify your answer. Solve the one(s) using the master method that can.

(i) $W(n) = W(n/2) + n\lg n$

(5 marks)

(ii) $W(n) = 2W(n/2) + n\lg n$

(5 marks)

(b) (i) Show how the Boyer-Moore algorithm performs the preprocessing to compute the two arrays charJump and matchJump for the string pattern "PAPAYA".

(6 marks)

(ii) Show how the simple Boyer-Moore algorithm and the Boyer-Moore algorithm find the string pattern "PAPAYA" in the text "BANANA KAYA PAPAYA…". Note there is only one space character in between the words in this text. How many character comparisons are done by each algorithm in searching for the pattern in the text?

(9 marks)

4. (a) Consider an unlimited supply of coins of denominations d_1, d_2, … d_m dollars. Let change(n) compute the minimum number of coins that add up to n dollars. We want to find the minimum number of coins that add up to a given N dollars.

(i) Give a recursive definition of the function change(n).

(7 marks)

(ii) Draw the subproblem graph for change(7) where the denominations of coins are \$1, \$3, \$5.

(4 marks)

(iii) Design a dynamic programming algorithm of change(n) using the bottom-up approach.

(6 marks)

(b) What is a P problem, an NP problem, an NP-complete problem? The following problems can be solved by dynamic programming. Are they P problems? Justify your answers.

Note: Question No. 4 continues on Page 4

- Longest common subsequence of 2 strings of length n
- 0/1 knapsack with n objects

(8 marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.