CX2101 Algorithm Design and Analysis

Tutorial 6: Introduction to NP

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1. Problem: Given a network of cities G and a positive integer k. Are the shortest paths between all pairs of cities not longer than k? Is this problem in the class of P or NP? Justify your answers.

2. Problem: Given a graph G = (V, E), where V is the set of vertices and E is the set of edges, and a positive integer k. Is there a way to colour the vertices of the graph using k colours or less such that adjacent vertices have different colours? Show that this K-colouring problem is in NP.

3. Why do we say NP-Complete problems are the hardest problems in NP?

4. Implement the shortestLinkTSP() algorithm below (slide 29 of lecture notes) to find a TSP tour in graph G. You may consider using a minimizing heap, a union-find data structure and other data structures in your implementation of the algorithm.

```
shortestLinkTSP(V, E, W)
{   R = E;
    C = empty;   // C is a forest

    while (no. of edges in C < |V| - 1)  {
                remove the lightest edge vw from R;
                if (vw does not form a cycle in C and
                    vw would not be the third edge in C incident on v or w)
                  add edge vw to C;   }

   add edge connecting the end points to C;
   return C;
}
```

5. Greedy heuristic algorithms are often used to solve problems because of its simplicity. Design a greedy heuristic method to solve the chain matrix multiplication problem where array d is used to store the dimensions of *n* matrices.