String Matching

Liu Ziwei

References: Introduction to Algorithms. Cormen, T.H., C.E. Leiserson. R.L. Rivest, Chapter 34

Computer Algorithms. Sara Baase & Allen Van Gelder, Chapter 11

The problem: Given a text T of n characters and a pattern P of m characters, find the first occurrence of P in T.

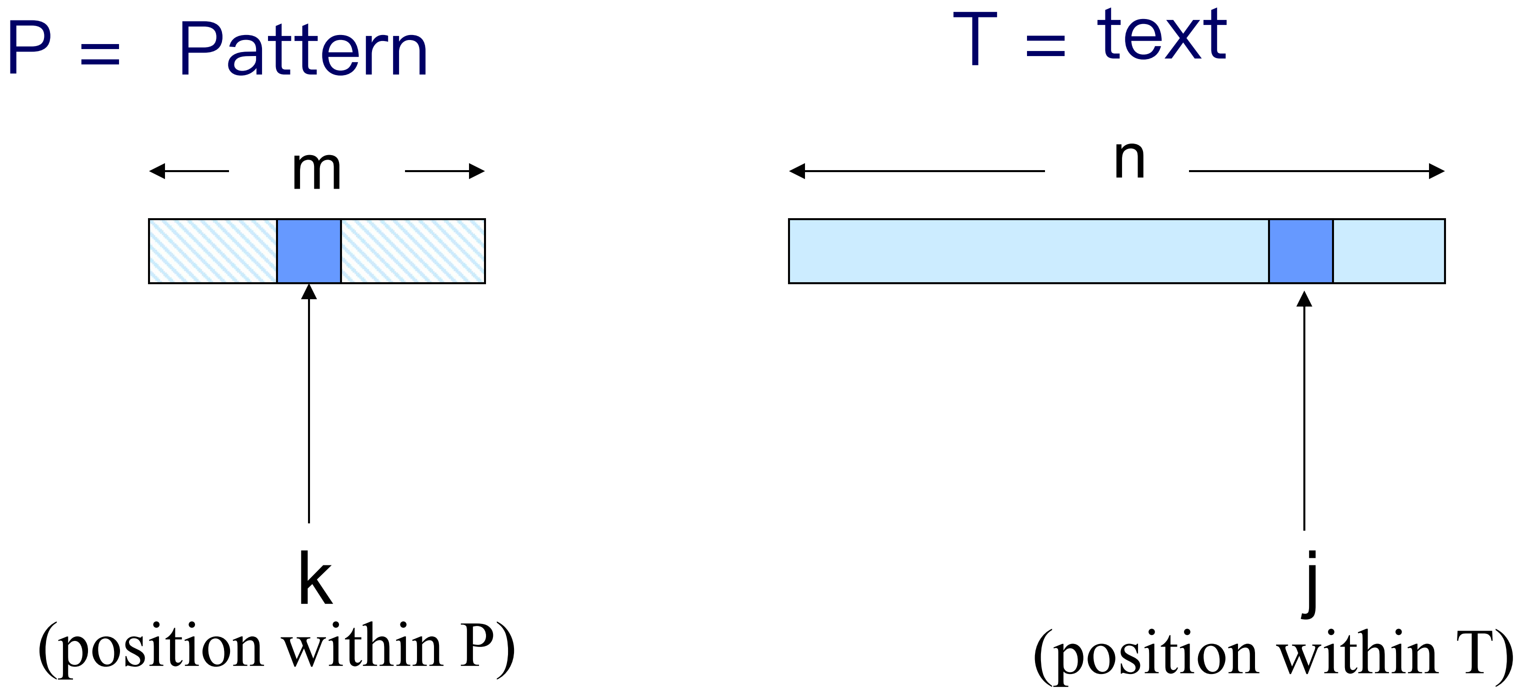
We may be looking for

- A character string in text;
- A pattern in DNA sequences;
- A piece of coded information representing graphical, audio data, or machine code;
- A sublist in linked list……….

We will study ----

- A straightforward solution
- The Rabin-Karp Algorithm
- The Boyer-Moore Algorithm

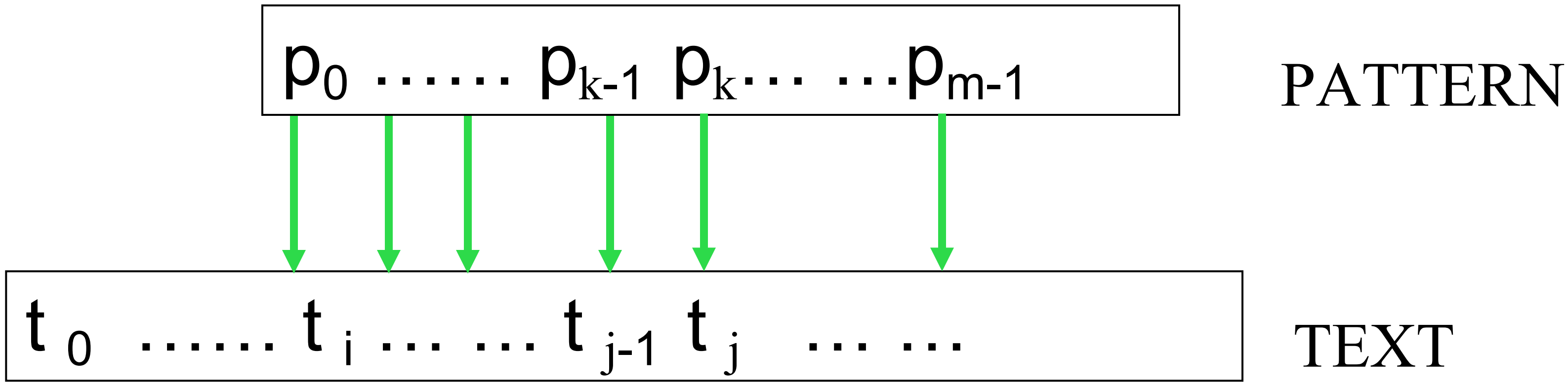
Conventions used:



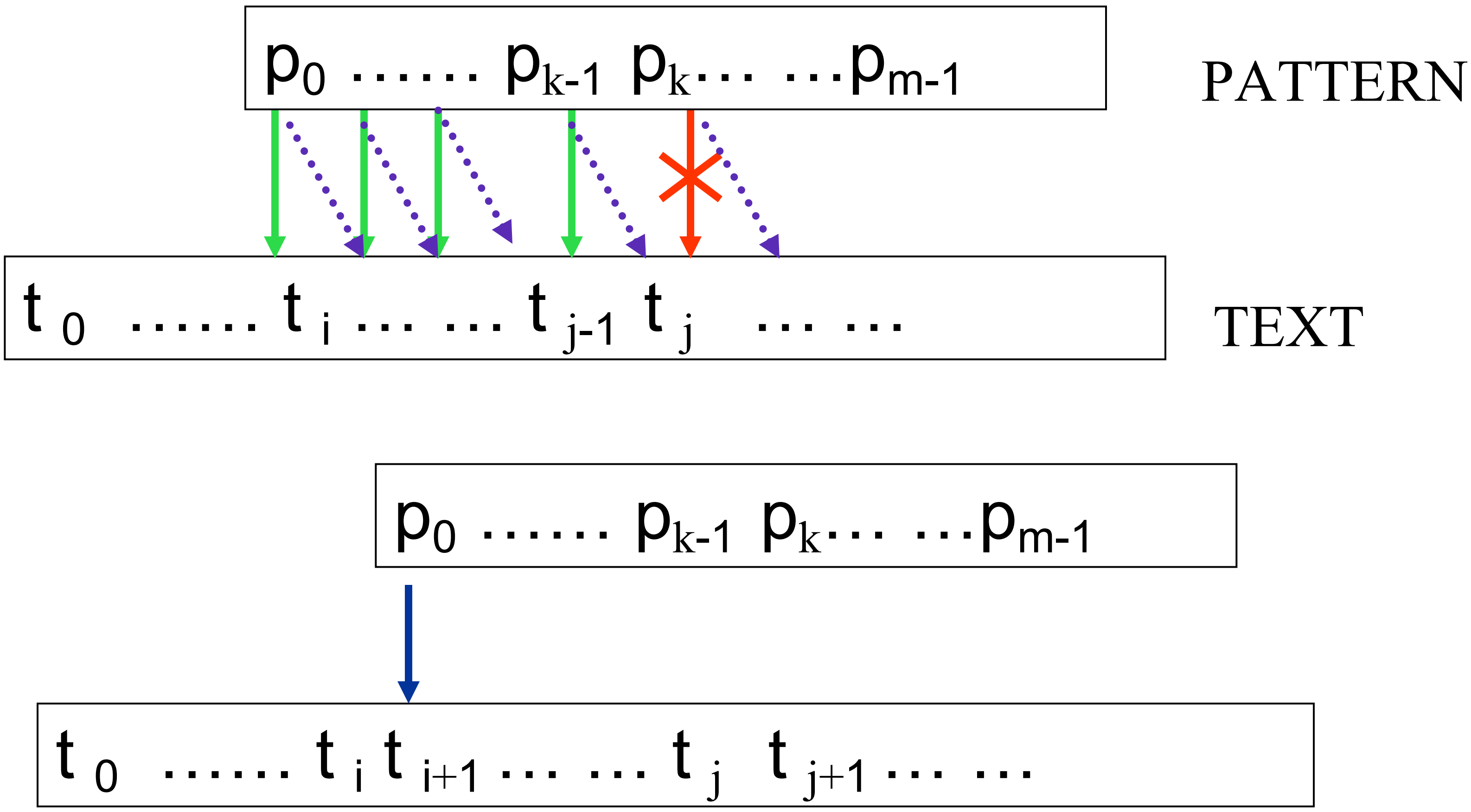
A straightforward solution

```
int SimpleScan (char [] P, char [] T, int m)
{
    int i, j, k;
   // i is the current guess of where P begins in T;
    // j is the index of the current character in T;
    // k is the index of the current character in P;
  j = k = 0;
  i = 0;
```

```
  while (j < n) {
      if (T[j] != P[k]) {
          j = ++i;
          if (j > n-m)  break;
          k = 0; }
      else {
           j++;
          k++;
          if (k == m)   return i;   }
  return -1;
}
```



Comparison starts with k=0 and j=i. When k reaches m, all characters have been compared and matched.



When a mismatch happens, shift the pattern right one position: j=++i, k=0

Example

P = **ABABC**

T = **ABABABCCAC**

ABABC

ABABABCCAC

ABABC

Match Successful!

ABABABCCAC

Worst case

P = **AAAAC** → **AAAAC**

T = **AAAAAAAAAAAA**   **AAAAAAAAAAAA**

AAAAC

AAAAAAAAAAAA

From the 1st character to the 5th last character of the text T, 5 comparisons are done before a mismatch. Total is m(n-m+1) comparisons

Worst case complexity is O(*mn*) where *m* is the length of the pattern and *n* is the length of the text

The Rabin-Karp Algorithm

- Outline of the steps of Rabin-Karp Algorithm
  1) Convert the pattern (length m) to a number, p
  2) Convert the first *m*-characters (the first text window) to a number, t
  3) If p and t are equal, pattern found and exit
  4) If not end-of-text, shift the text window one character right and convert the string in it to a number t, go to step 3); else pattern not found and exit

P:

0	0	1	0	1

$m = 5, \ p = 5$

T:

0	0	1	1	0	0	1	0	0	0	0	1	0	1	1	1	0	1	0	0

t = **6 12 25 18 4 8 16 1 2 5**

Found!

To compute the number for the pattern and the number for the first *m*-character text window,

- The set of possible characters is referred to as an alphabet and denoted with sigma Σ. e.g. $\Sigma = \{0, 1\}$ or $\Sigma = \{0, 1, 2, \ldots, 9\}$ or $\Sigma = \{a, b, c, \ldots, z\}$
- Let $d = |\Sigma|$

- The number p of the pattern and the number t of the first *m*-character text window, are calculated iteratively.

For example, P = “36415”, d = 10

3	6	4	1	5	2

$$\begin{aligned} p &= 3 * 10^4 + 6 * 10^3 + 4 * 10^2 + 1 * 10^1 + 5 \\ &= (3 * 10^3 + 6 * 10^2 + 4 * 10^1 + 1) * 10 + 5 \\ &= ((3 * 10^2 + 6 * 10^1 + 4) * 10 + 1) * 10 + 5 \\ &= (((3 * 10 + 6)*10 + 4) * 10 + 1) * 10 + 5 \end{aligned}$$

- p = P[0]*d^(m-1) + P[1]*d^(m-2) + … + P[m-2]*d + P[m-1]

  = (((P[0]*d + P[1])*d + P[2])*d + … P[m-2])*d + P[m-1]

```
p = P[0];
For j = 1 to m-1
    p = p*d + P[j]
```

The numbers p and t can be computed in $\theta(m)$ time.

- To compute the number t after shifting the text window, it can be done in constant time based on the number of the previous text window

For example:

In general,
new = (old – MSB * d^{m-1}) * d + LSB
d^{m-1} is pre-calculated as below

36415

3	6	4	1	5	2

i

$$(36415 - 3 * 10^4) * 10 + 2$$

```
dM = 1;
For j = 1 to m-1
    dM = dM*d
// dM = d^(m-1)
```

```
t = (t – T[i]*dM)*d + T[i+m]
// t before this is the number for T[i .. i+m-1]
// t after this is the number for T[i+1 .. i+m]
```

- If the pattern is long (e.g. m = 100), then the resulting number will be enormous. Overflow may occur.
- For this reason, we hash the value by taking it mod a prime number *q*. This prime number should be large.

1) **Hash** the pattern to a number, hp
2) **Hash** the first *m*-character text window to a number, ht
3) If hp and ht are equal, compare the pattern with the text window. If equal, pattern found and exit
4) If not end-of-text, shift the text window one character right and **(re)hash** it to a number ht, go to step 3); else pattern not found and exit

Note: if hp = ht, it does not necessarily imply that T[i..i+m-1] = P[0..m-1].
However, if hp $\neq$ ht, definitely T[i..i+m-1] $\neq$ P[0..m-1]

P:

0	0	1	0	1

m = 5, q = 13, hp = 5

T:

0	0	1	1	0	0	1	0	0	0	0	1	0	1	1	1	0	1	0	0

ht =6 4 8 3 1 2

ht = 12

ht = 25%13=12

ht = 18%13=5 compare P and T

ht = 5 compare P and T

Found!

- The mod function (% in Java) is particularly useful in this case due to several of its inherent properties:
  - (x+y) mod q = [(x mod q) + (y mod q)] mod q
  - (x mod q) mod q = x mod q
  - xy mod q = [(x mod q)(y mod q)] mod q

  Example:

  21*15 mod 13 = 315 mod 13 = 3

  21*15 mod 13 = ((21 mod 13) * (15 mod 13)) mod 13

  = (8 * 2) mod 13 = 3

- To calculate hp, the hash value for P[0..m-1], call hash(P, m, base). The hash function is also used to compute the value of the first text window

```
int hash(Txt, m, d)
{
    int h = Txt[0] % q;
    for (int i =  1; i < m; i++)
        h = (h * d + Txt[i] )% q;
    return h;
}
```

E.g. P = "36415", **q = 7**
h = 3
i = 1, h = 1
i = 2, h = 0
i = 3, h = 1
i = 4, h = 1
hash("36415", 5, 10) = 1

Ref:
```
p = P[0];
For j = 1 to m-1
    p = p*d + P[j]
```

The numbers hp and ht can be computed in $\theta(m)$ time.

- After finding ht for T[i .. m-1], ht for T[i+1 .. m] can be calculated by rehash(T, i, m, ht) in constant time θ(1).

```
int rehash(T, i, m, ht)
{    oldest = (T[i] * dM) % q;
     oldest_removed = ((ht + q) - oldest) % q;
     return (oldest_removed * d + T[i+m]) % q;
}
```

- Compare with no hashing:

```
t = (t – T[i]*dM)*d + T[i+m]
// dM is d^(m-1)
// t before this is the number for T[i .. i+m-1]
// t after this is the number for T[i+1 .. i+m]
```

Example

```
int rehash(T, i, m, ht)
{    oldest = (T[i] * dM) % q;
     oldest_removed = ((ht + q) - oldest) % q;
     return (oldest_removed * d + T[i+m]) % q;
}
```

ht=1, dM = 4, q = 7

	i	i+1	i+2	i+3	i+4	i+5
T	3	6	4	1	5	2

Rehash(T, i, 5, 1) =

Oldest = 5
Oldest_removed = 3
Return 32 % 7 = 4

```
// computed once
dM = 1;
For j = 1 to m-1
     dM = dM*d % q
```

```
int RKscan(P, T)
{    m = Length(P);
     n = Length(T);
     dM = 1;
     For j = 1 to m-1     dM = dM*d % q;    // d =
        |Σ|

     hp = hash(P, m, d);
     ht = hash(T, m, d);

     for (j = 0; j <= n - m; j++) {
          if (hp == ht && equal_string(P, T, 0, j,
m))
               return j;
          if (j < n-m)   ht = rehash(T, j, m, ht);
     }
     return -1;  // pattern not found
}
```

Maximum n-m+1 iterations

- The running time of Rabin-Karp algorithm in the worst case is $\theta((n - m + 1)m)$
- However, in many applications, the expected running time is $O(n+m)$ plus the time required to process spurious hits.
  - $O(m)$ time for the 2 hash() calls
  - Close to $O(n)$ time on the for loop
- The number of spurious hits can be kept low by using a large prime number q for the hash functions

The Boyer-Moore Algorithm

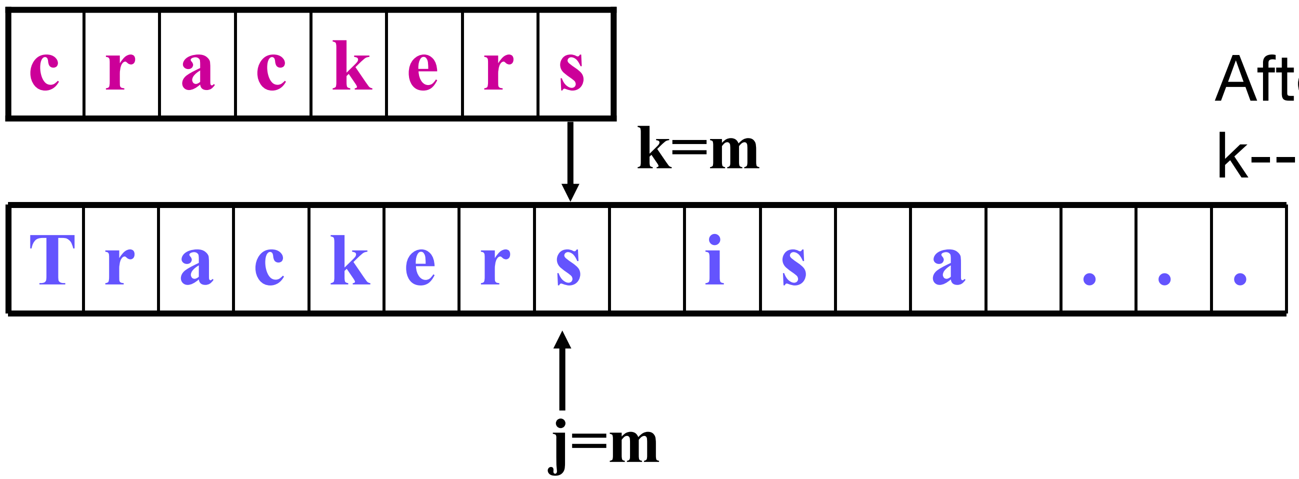
- It is a very efficient algorithm for string searching
- The text being scanned is T with *n* characters
- The pattern we are looking for is P with *m* characters
- Process the text T[1..*n*] from left to right
- Scan the pattern P[1..*m*] <u>from *right to left*</u>
- Preprocessing to generate two tables based on which to slide the pattern as much as possible after a mismatch
- It performs even better with long patterns

```
int BMscan(char[]P char[]T, int m,
                    int[]charJump, int []matchJump )
{ int j;   int k;
  j = m;  k = m;
  while (j <= n) {
        if (k < 1)  return j + 1;    //match found
        if (T[j] == P[k])    {   j--;  k--;  }
        else {     j += max(charJump[T[j]], matchJump[k]);
                   k = m;      }
  }
  return -1;    // match not found
}
```

charJump and matchJump are the 2 tables generated in a preprocessing step

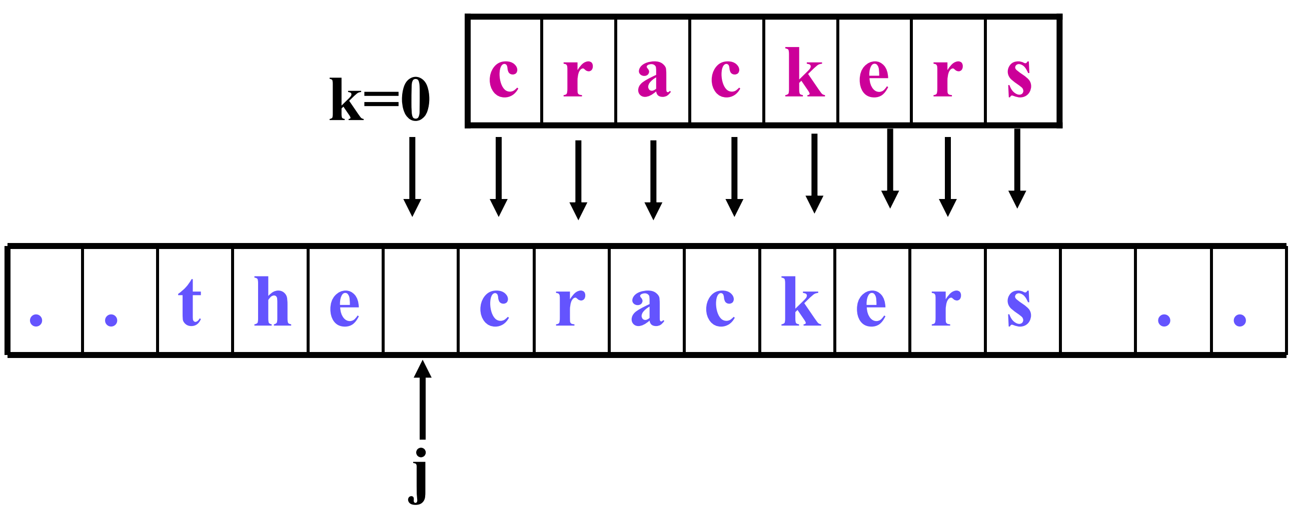
Assume the 1st character is at P[1] and T[1] respectively

Examples



To start.
k = m, j = m

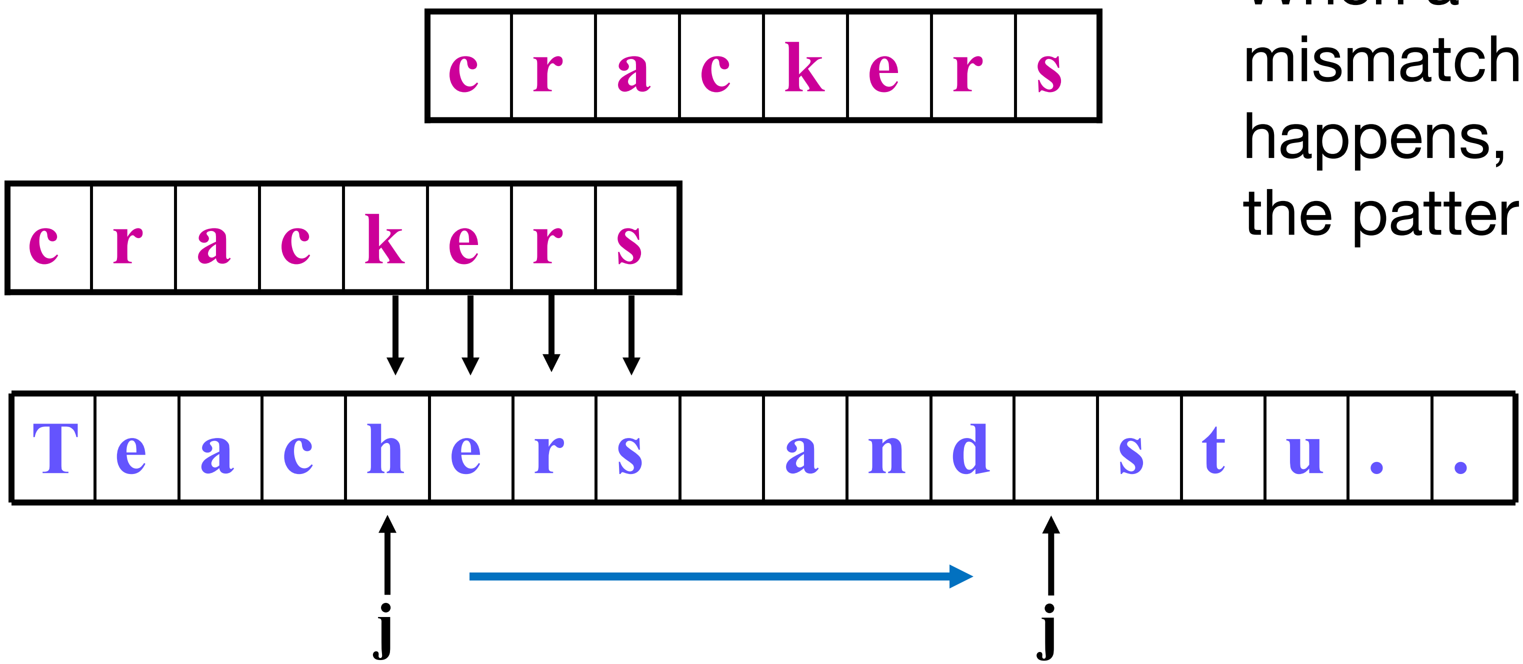
After a match,
k--, j--



When k reaches 0, all characters have been matched:
Return j+1

Example

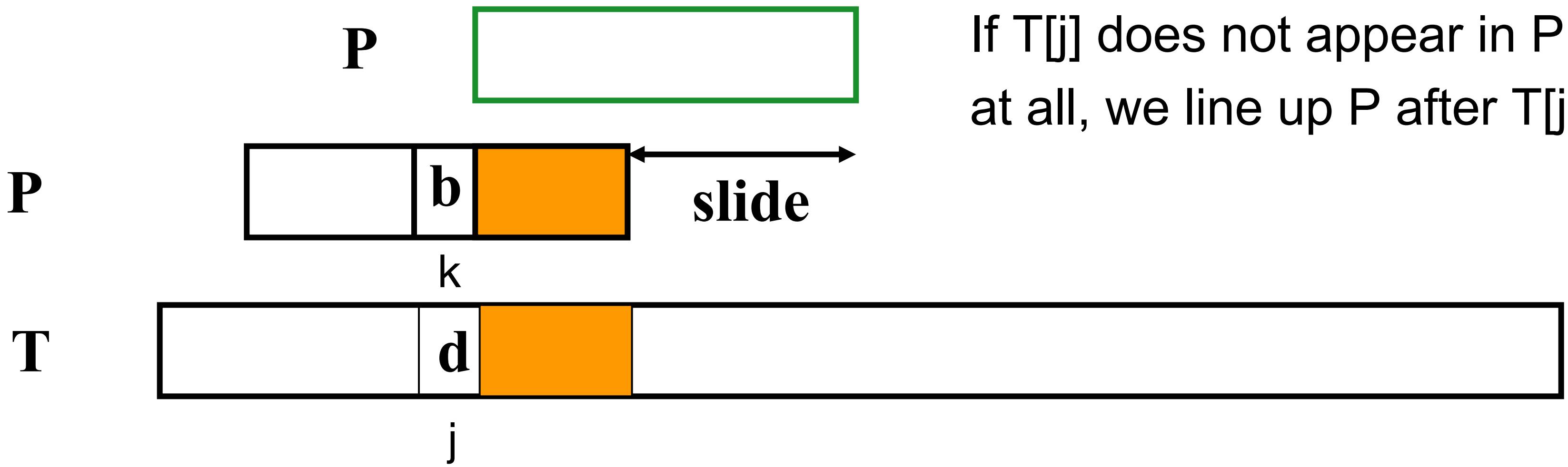
When a mismatch happens, shift the pattern



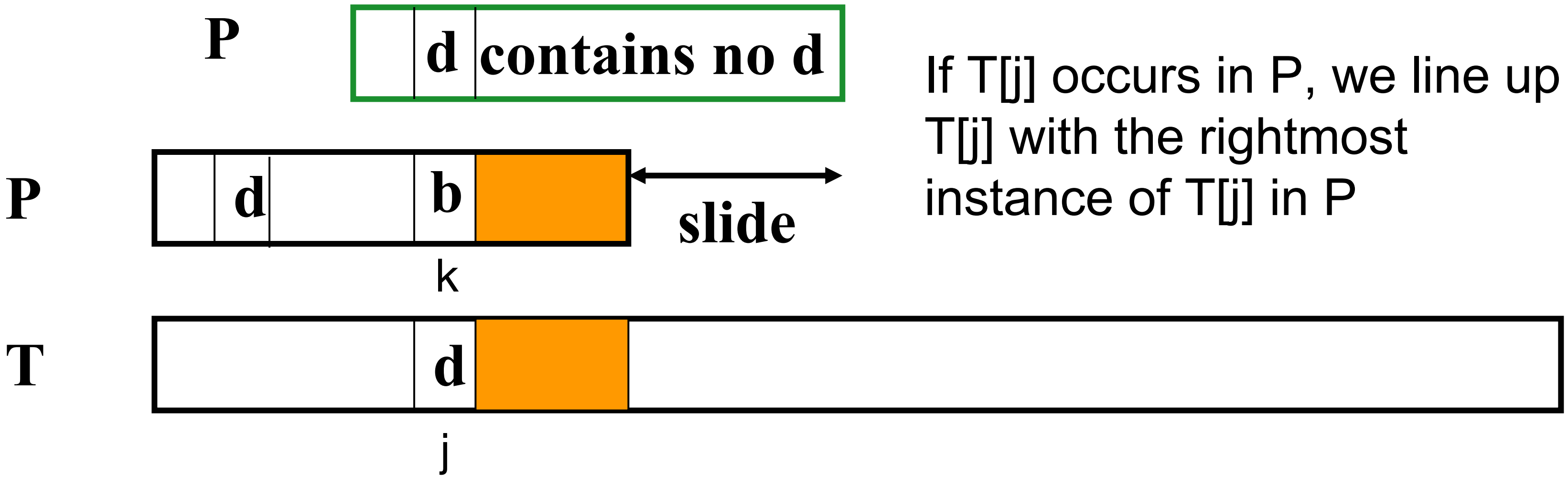
Shift the pattern as much as possible – increment j as much as possible for the next comparison:

```
{    j += max(charJump[T[j]], matchJump[k]);
          k = m;    }
```

Preprocessing to compute charJump

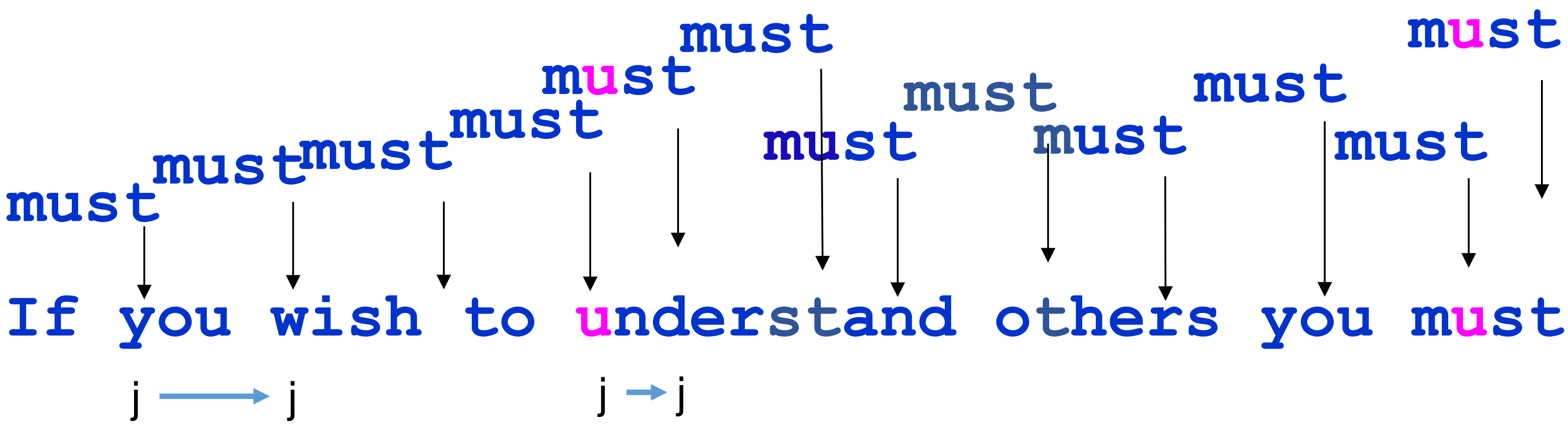


If T[j] does not appear in P at all, we line up P after T[j]

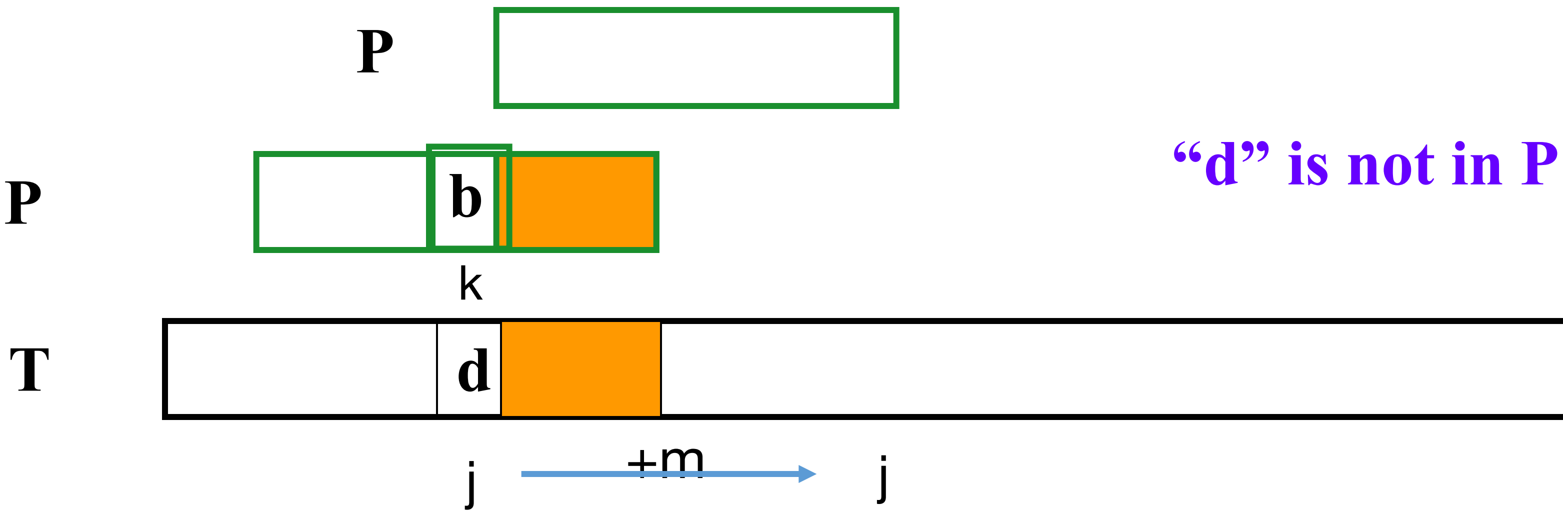


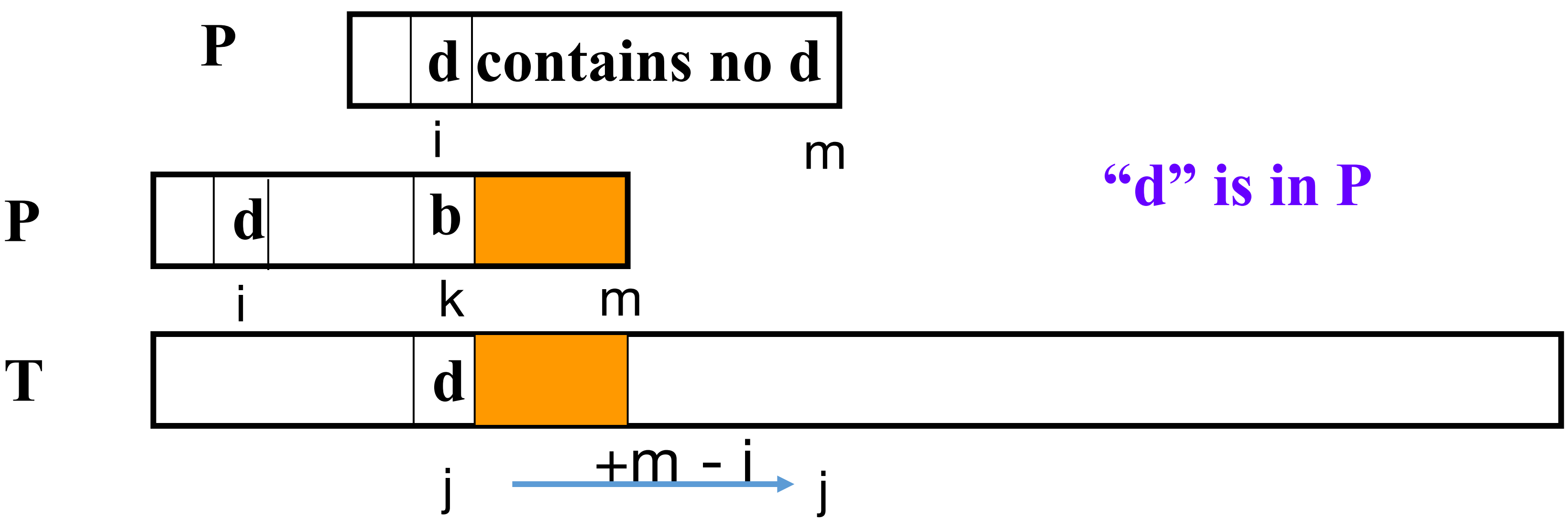
If T[j] occurs in P, we line up T[j] with the rightmost instance of T[j] in P

Example

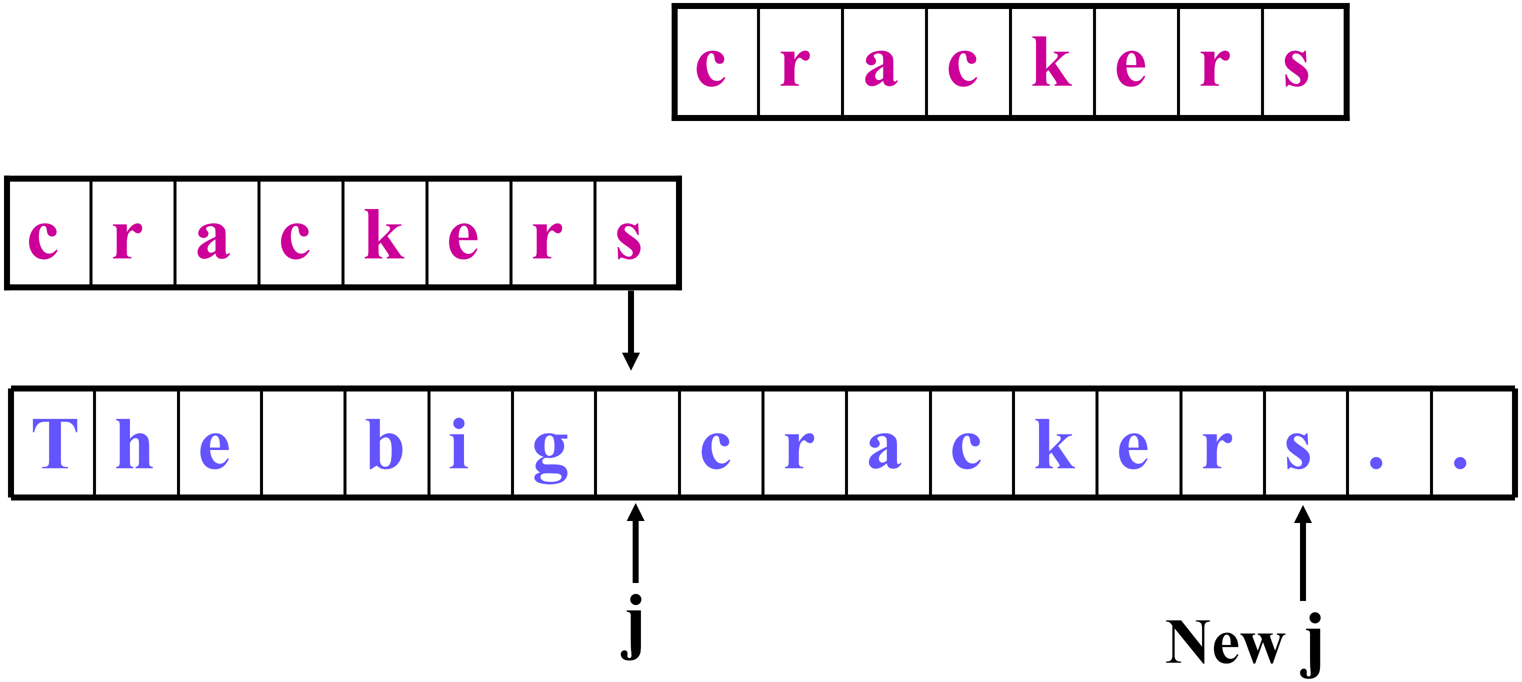


- Many of the *n* characters in the text are never compared – sublinear complexity
- We need to calculate how the text index j should be incremented to begin the next right-to-left scan of the pattern


P
P
b
k
T
d
j
+m
j
"d" is not in P

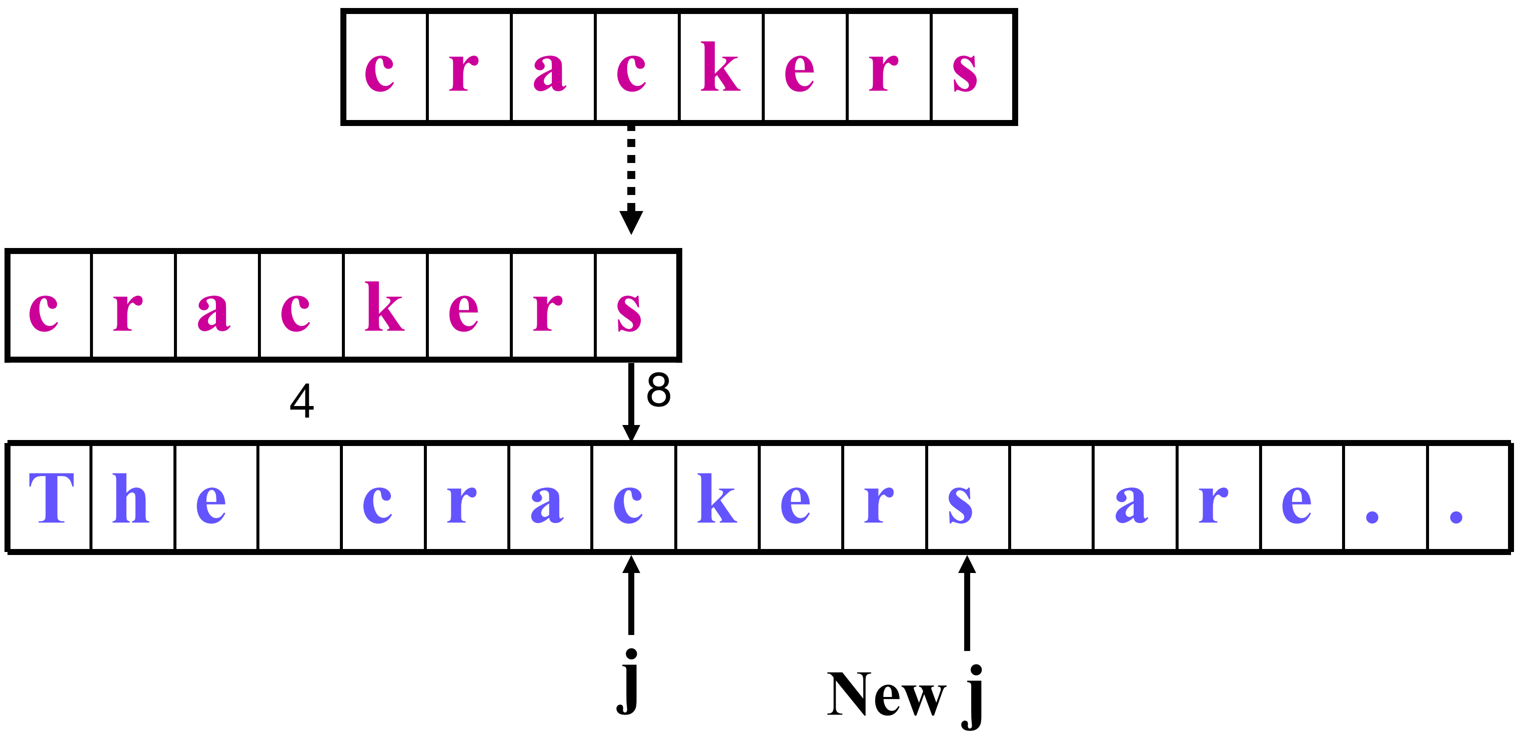

P
d
contains no d
i
m
P
d
b
i
k
m
T
d
j
+m - i
j
"d" is in P

Example



To line up P after T[j], e.g. ‘ ’, P is slid 8 places to the right: j = j + 8

Example



To line up T[j], e.g. ‘c’, with the rightmost ‘c’ in P, P is slid 4 places to the right: j = j + 8 - 4

- Computing the jumps for all the characters:

```
void computeJumps(char [] P, int m,
            int alpha, int [] charJump)
{   char ch;  int k;
      for (ch = 0; ch < alpha; ch++)
          charJump[ch] = m;
      for (k = 1; k <= m; k++)
            charJump[ P[k] ] = m - k;
}
```

Number of characters in character set

Position from the end

Notice that if a character appears more than once, we take the right-most occurrence.

Complexity is $O(|\Sigma| + m)$

E.g. for

r	a	t	s	a	n	d	c	a	t	s
1	2	3	4	5	6	7	8	9	10	11

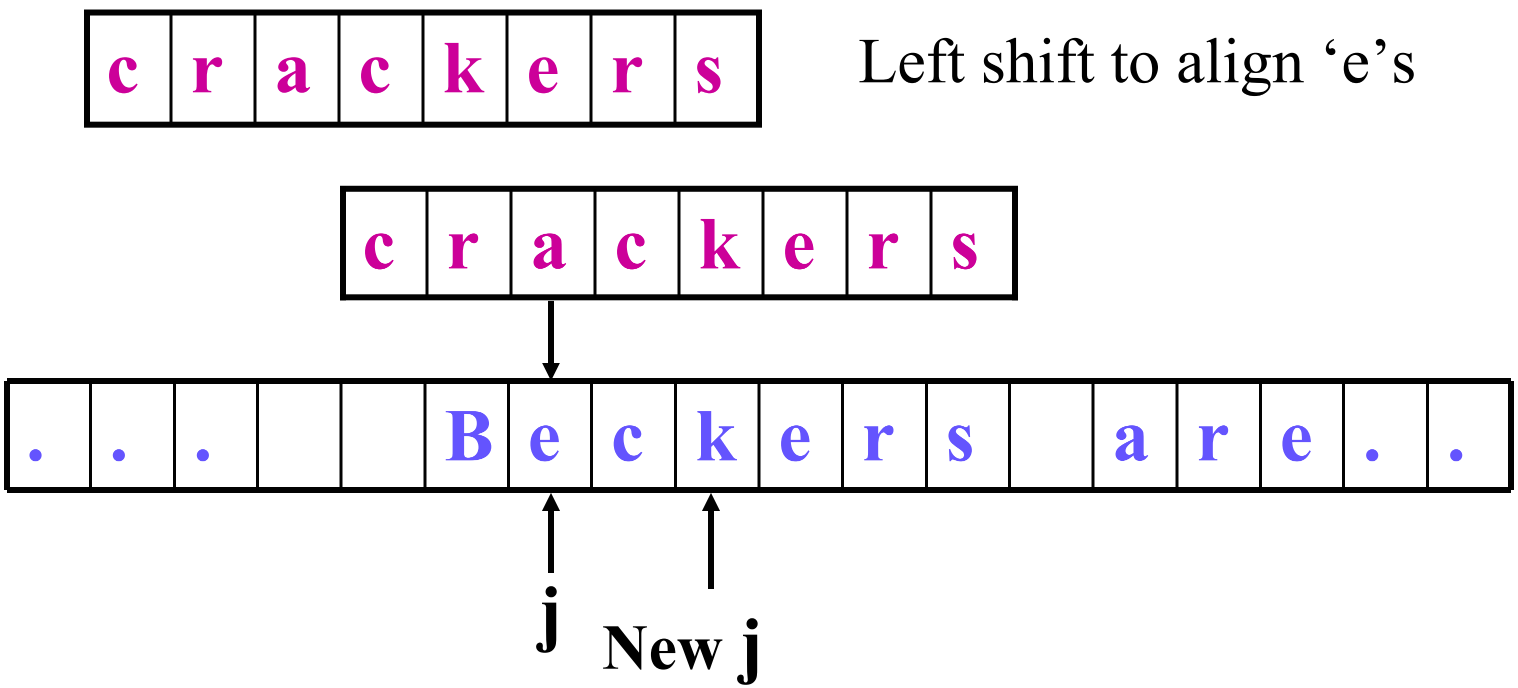
$m = 11$,

charJump

2	11	3	4	…	5	…	10	0	1	…
a	b	c	d	…	n	…	r	s	t	…

Sometimes this heuristic fails, for example,

Left shift to align ‘e’s



Simplified Boyer-Moore (using charJump only)

```
int simpleBMscan(char[]P char[]T, int m,  int[]charJump)
{ int j;   int k;
  j = m;  k = m;
  while (j <= n) {
        if (k < 1)  return j + 1;    //match found
        if (T[j] == P[k])    {   j--;  k--;  }
        else {     j += max(charJump[T[j]], m-k+1);
                   k = m;      }
  }
  return -1;    // match not found
 }
```

E.g.

c	r	a	c	k	e	r	s

charJump

5	8	4	8	2	…	3	…	1	0	…
a	b	c	d	e	…	k	…	r	s	…

c	r	a	c	k	e	r	s

Shift m-k+1 places

c	r	a	c	k	e	r	s

k=3

.	.	.			B	e	c	k	e	r	s		a	r	e	.	.

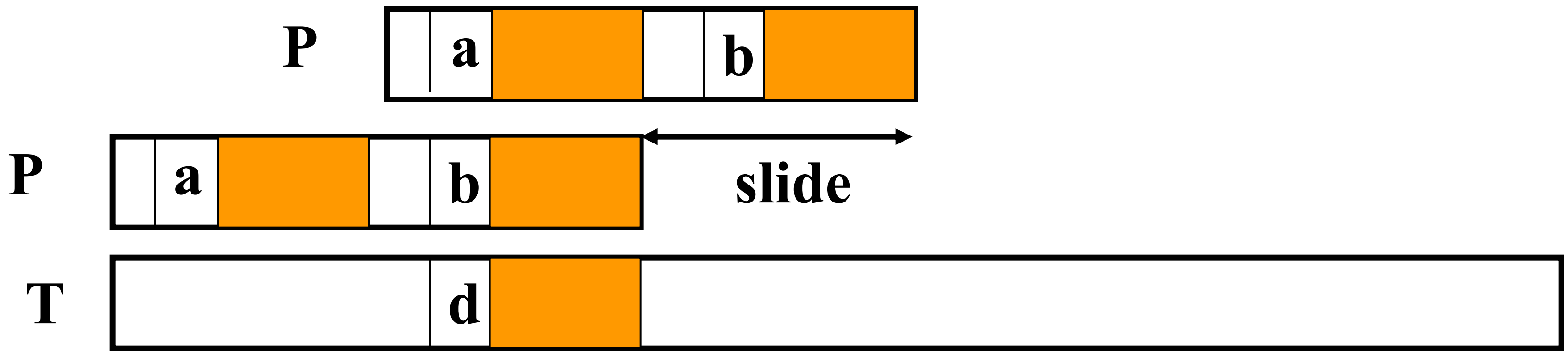
j

New j = j + 8 – 3 + 1

Preporocessing to compute matchJump

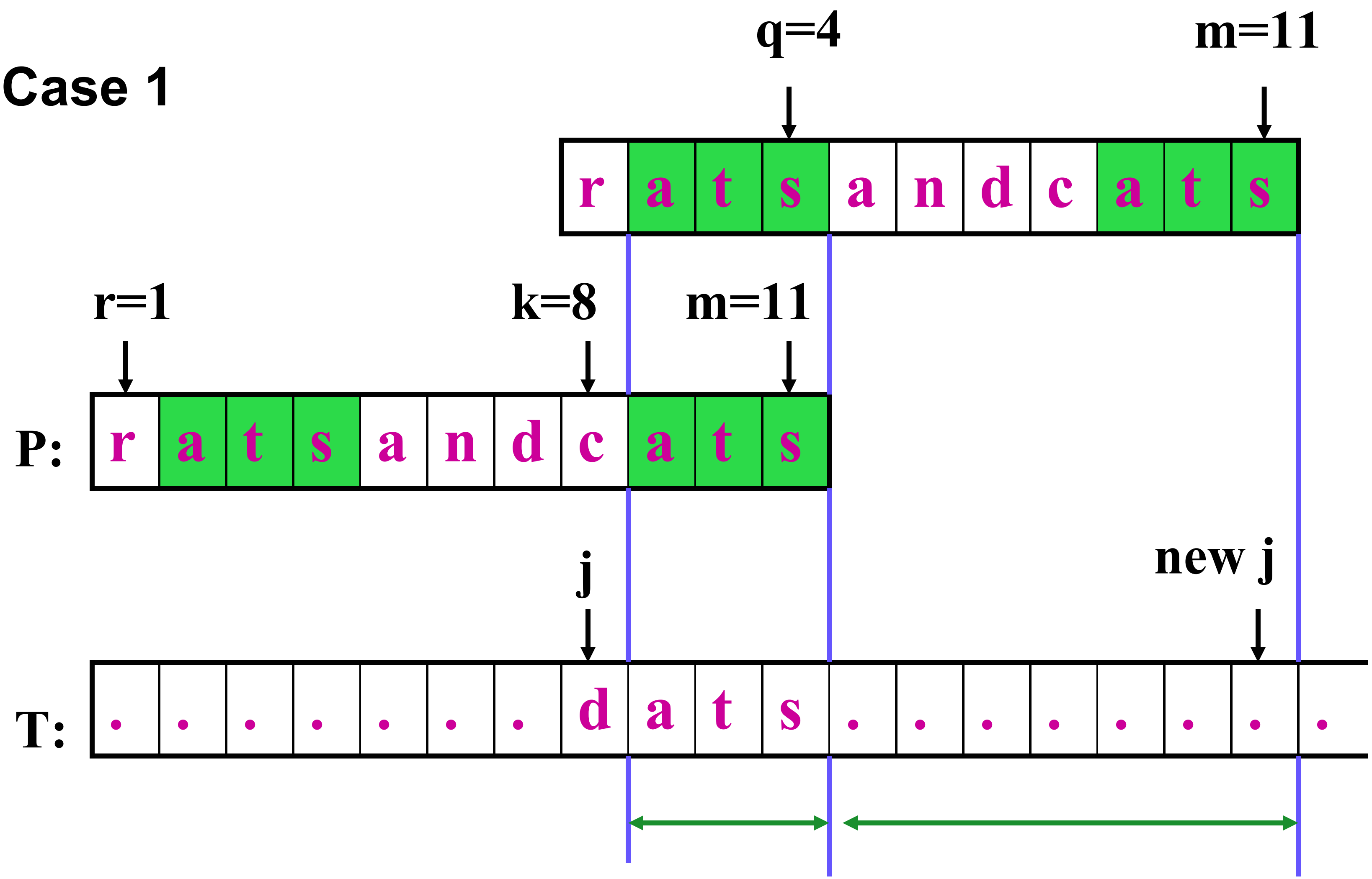
This heuristic tries to derive the maximum shift from the structure of the pattern. It is defined for each of the characters in P.

Case 1: The matching suffix occurs earlier in the pattern, but preceded by a different character



We line up the earlier occurrence of the suffix in P with the matched substring in T

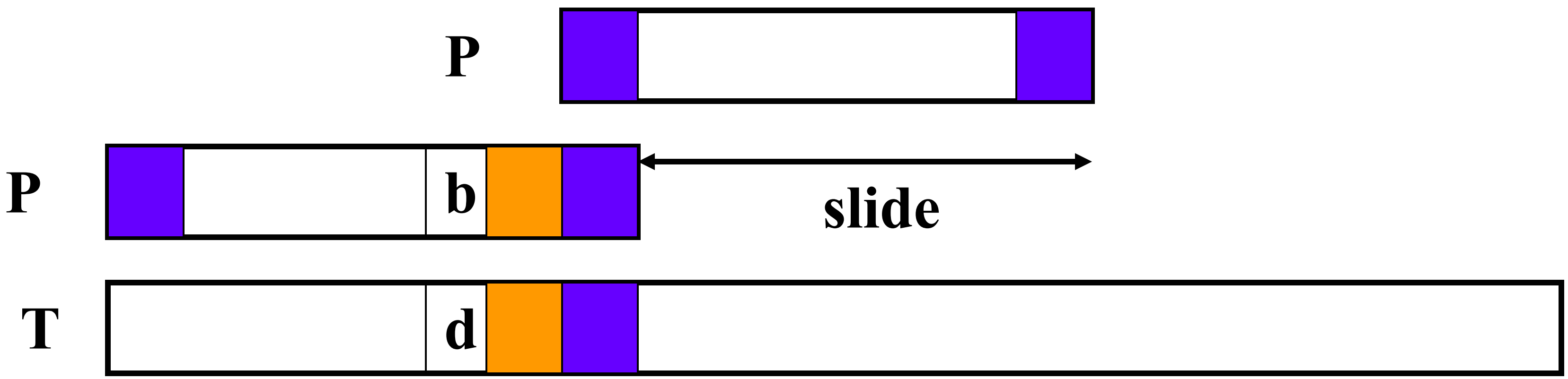
Case 1



matchJump[k] = m-k + slide[k]

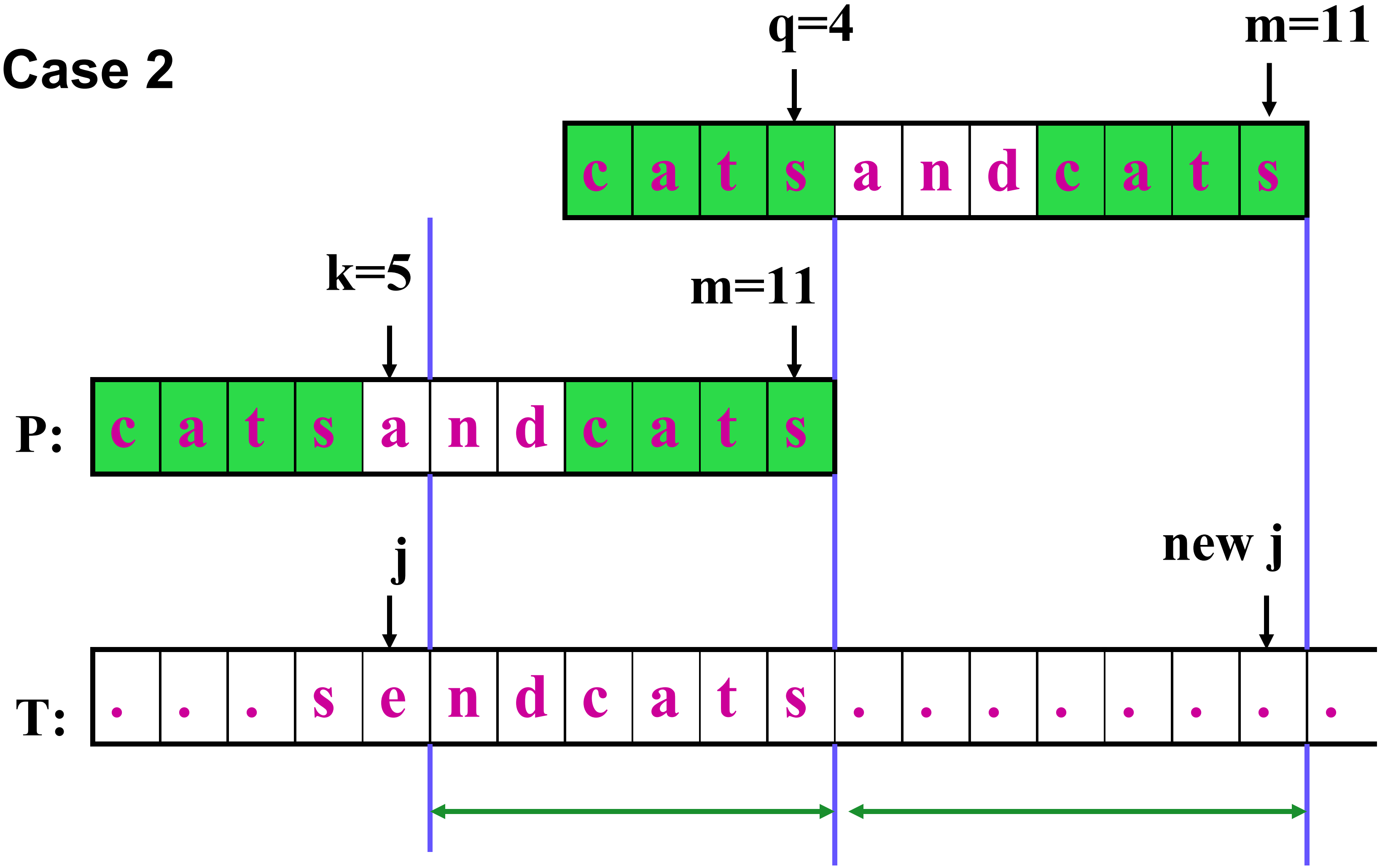
slide[k] = m - q (P[r] ≠ P[k])

Case 2: Only part of the matching suffix occurs at the beginning of the pattern (a prefix).



We line up the prefix in P with part of the matched substring in T

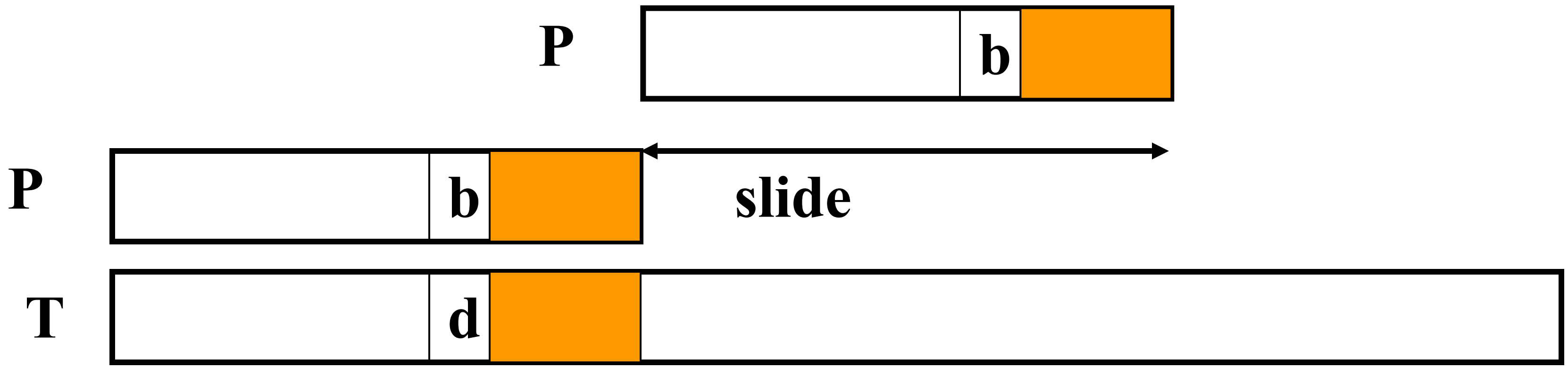
Case 2



matchJump[k] = m-k + slide[k]

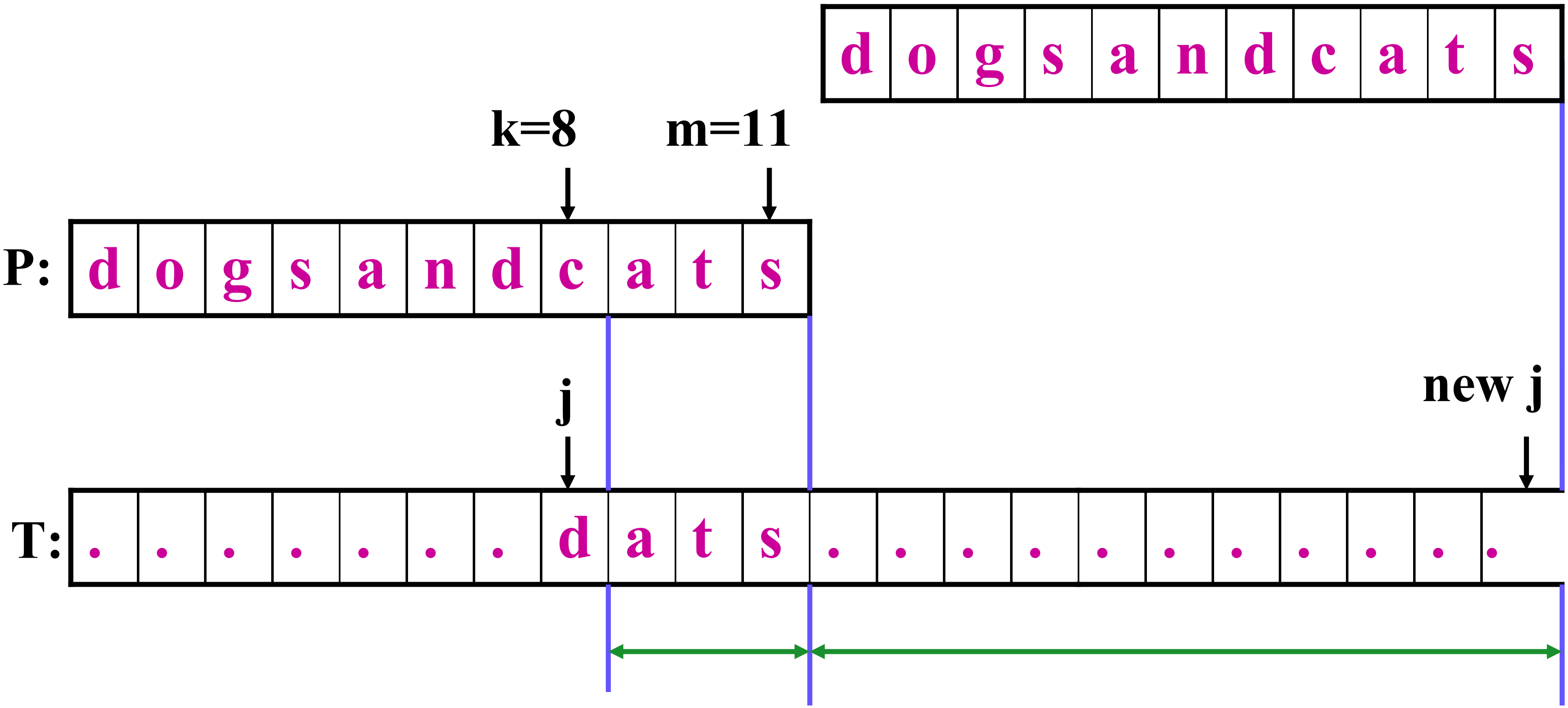
slide[k] = m - q

Case 3: There is no other occurrence of the matching suffix in the pattern. (Case 1 and Case 2 do not happen)



We line up P after the matched substring in T

Case 3

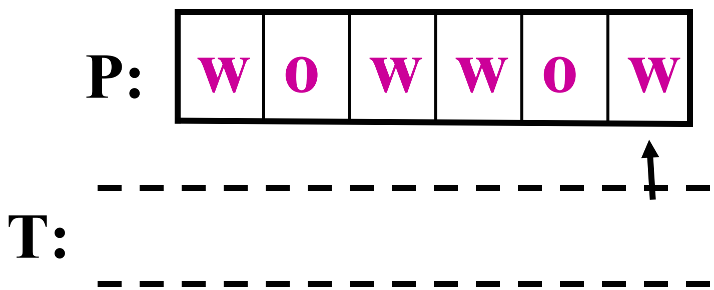


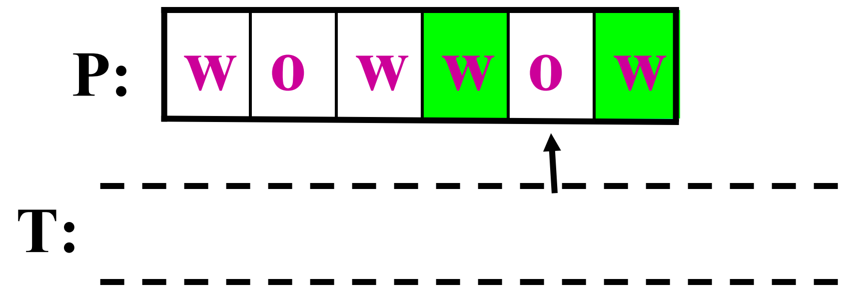
matchJump[k] = m-k + slide[k]

slide[k] = m (= m – q where q is 0)

What should the jumps be?

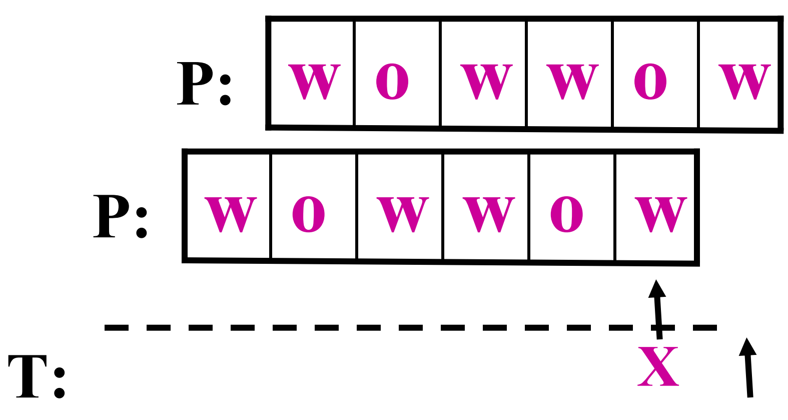
$\boxed{\text{Slide}[m] = 1}$

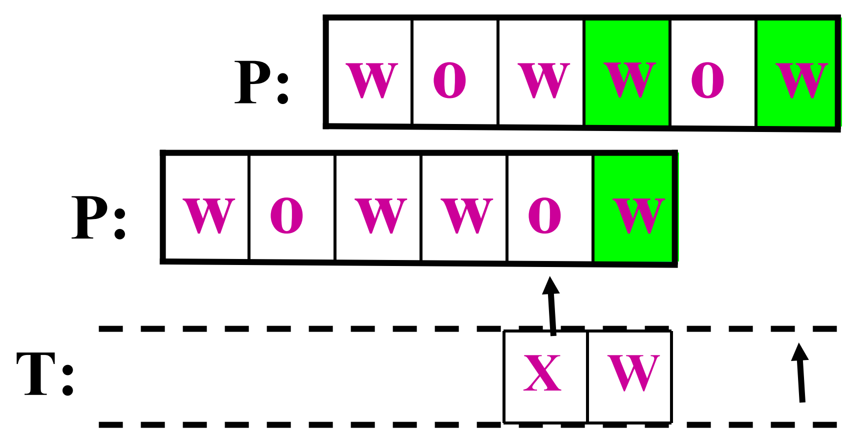


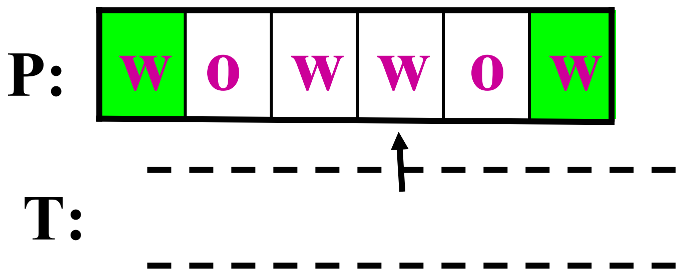


Matched = 0 (m-k)
Slide[6] = 1
matchJump[6] = 1

Matched = 1 (m-k)
Slide[5] = 2 (m-q)
matchJump[5] = 3



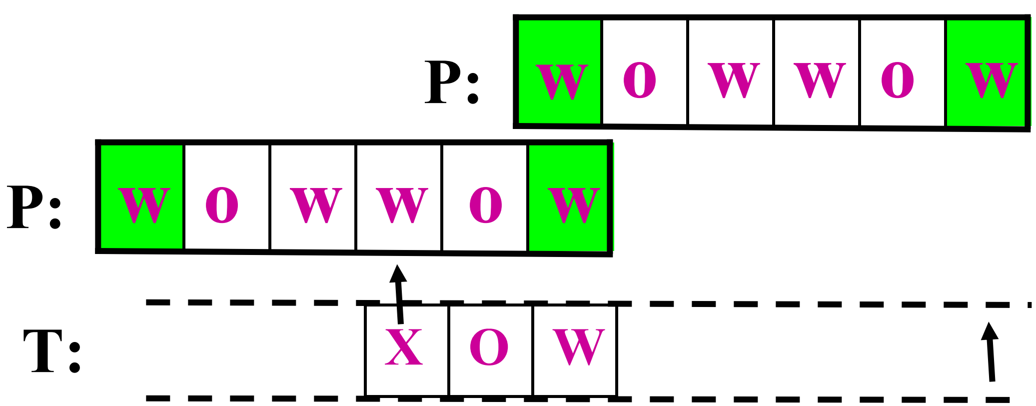


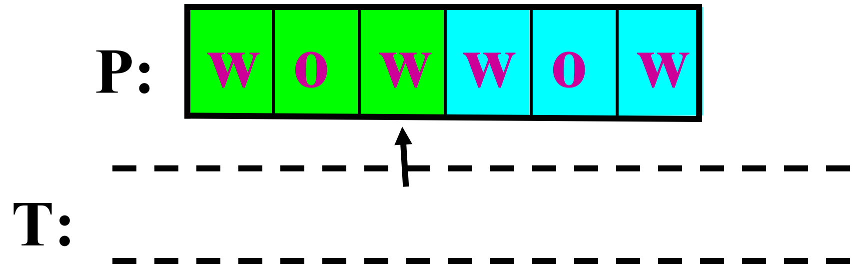


Matched = 2 (m-k)

Slide[4] = 5 (m-q)

matchJump[4] = 7

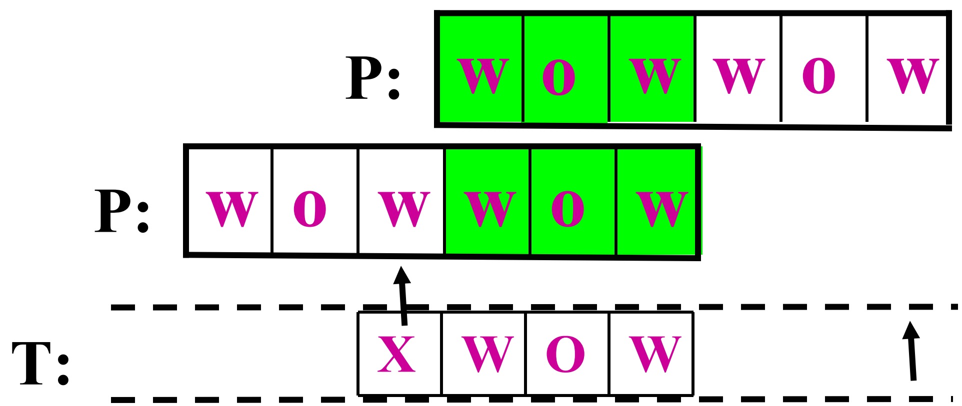


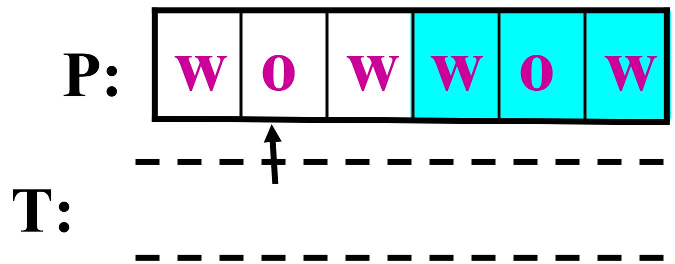


Matched = 3   (m-k)

Slide[3] = 3   (m-q)

matchJump[3] =   6

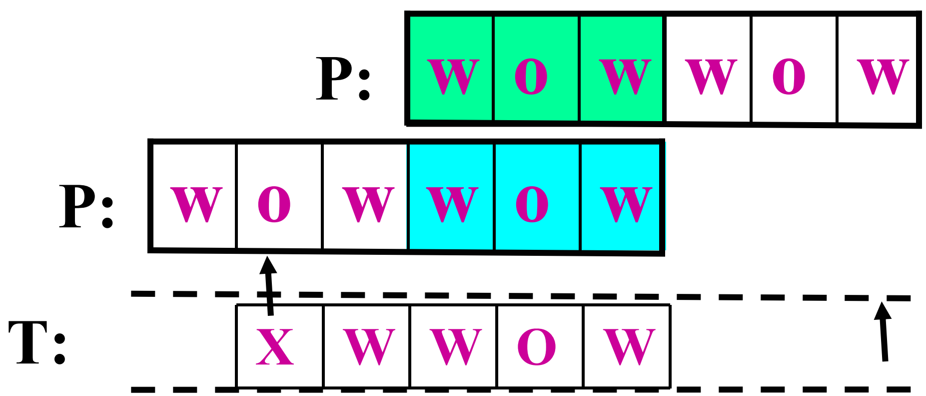


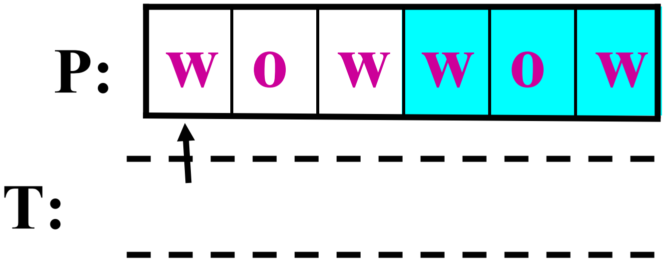


Matched = 4 (m-k)

Slide[2] = 3 (m-q)

matchJump[2] = 7





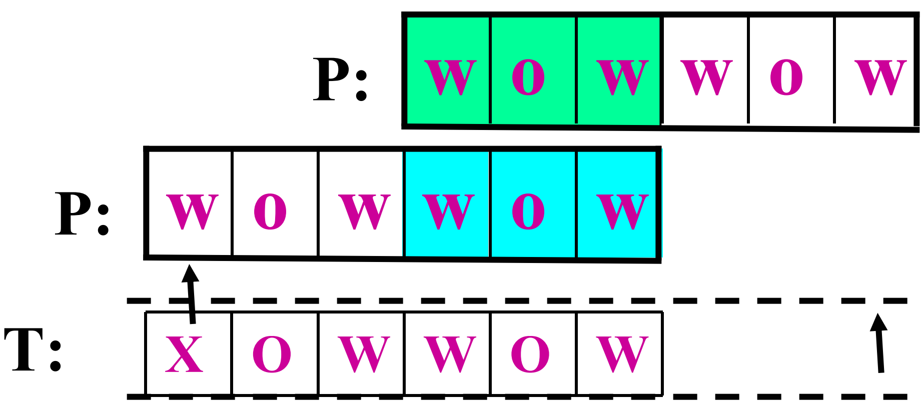
Matched = 5 (m-k)

Slide[1] = 3 (m-q)

matchJump[1] = 8

matchJump

8	7	6	7	3	1

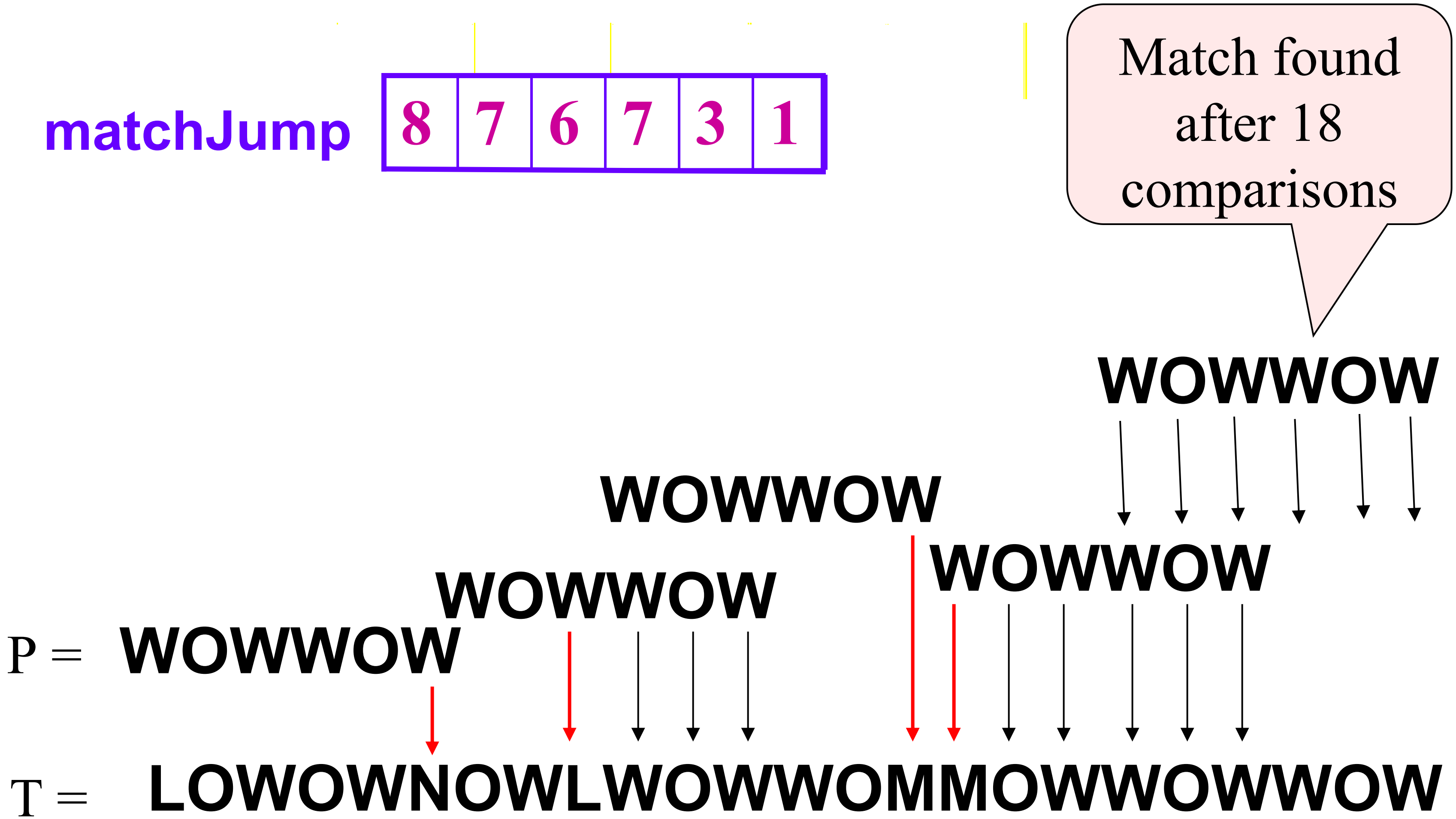


Example: Pattern is WOWWOW

charjump['o'] = 1, charjump['w'] = 0, charjump[X] = 6,

matchJump

8	7	6	7	3	1



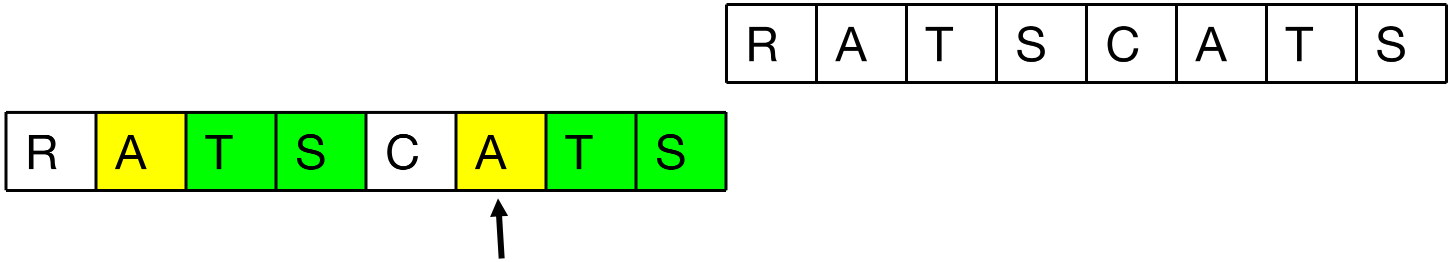
Another example

	R	A	T	S	C	A	T	S
R	A	T	S	C	A	T	S	

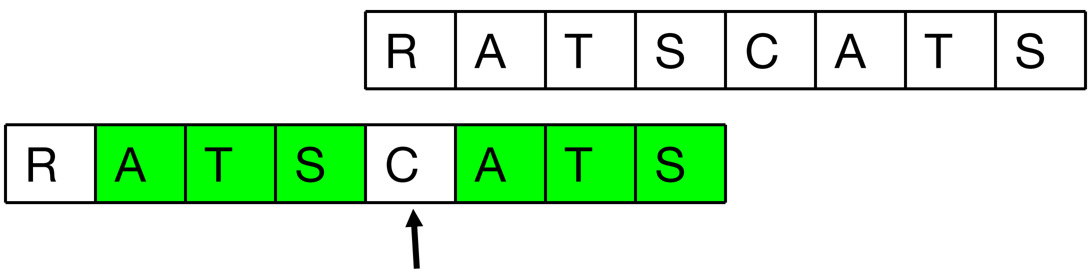
Matched = m-k = 0, slide[8] = 1
matchJump[8] = 1

								R	A	T	S	C	A	T	S
R	A	T	S	C	A	T	S								

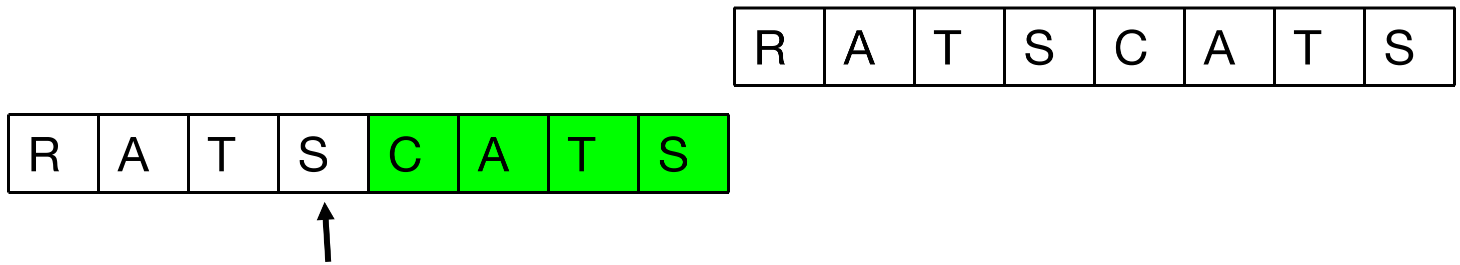
Matched = m-k = 1, slide[7] = m = 8
matchJump[7] = 9



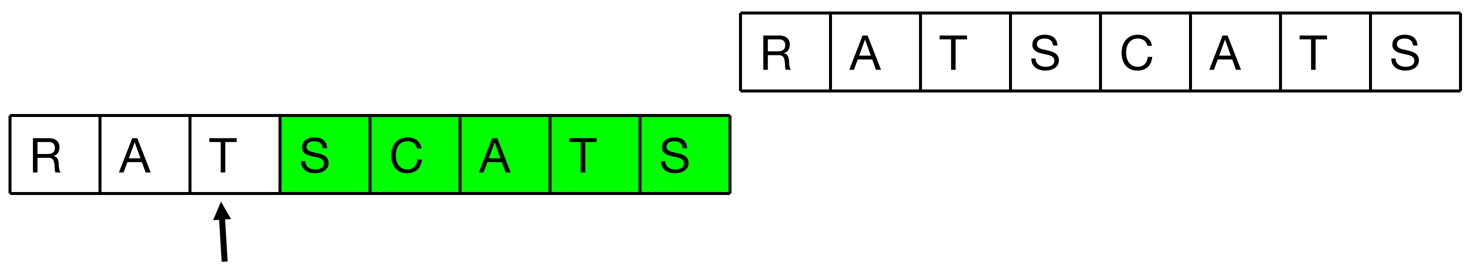
Matched = m-k = 2, slide[6] = m = 8
matchJump[6] = 10



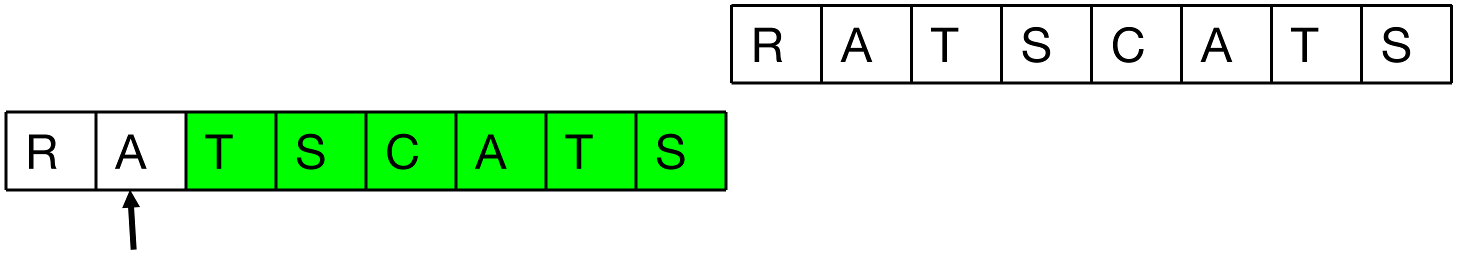
Matched = m-k = 3, slide[5] = m-q = 4
matchJump[5] = 7



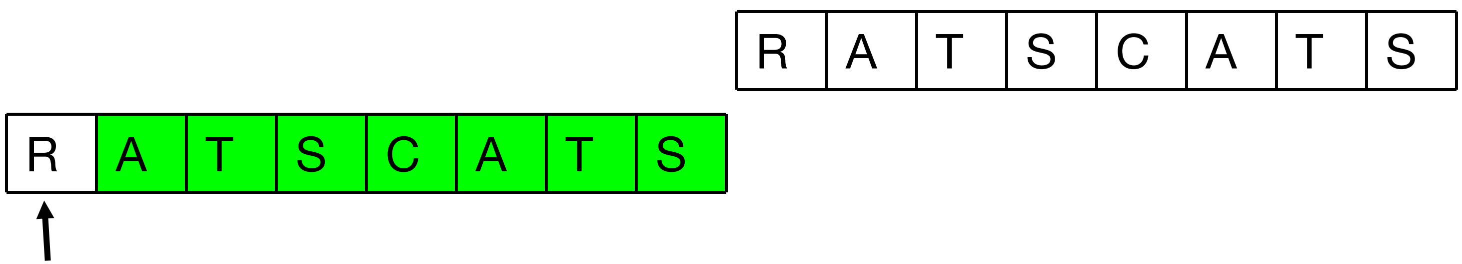
Matched = m-k = 4, slide[4] = m = 8
matchJump[4] = 12



Matched = m-k = 5, slide[3] = m = 8
matchJump[3] = 13

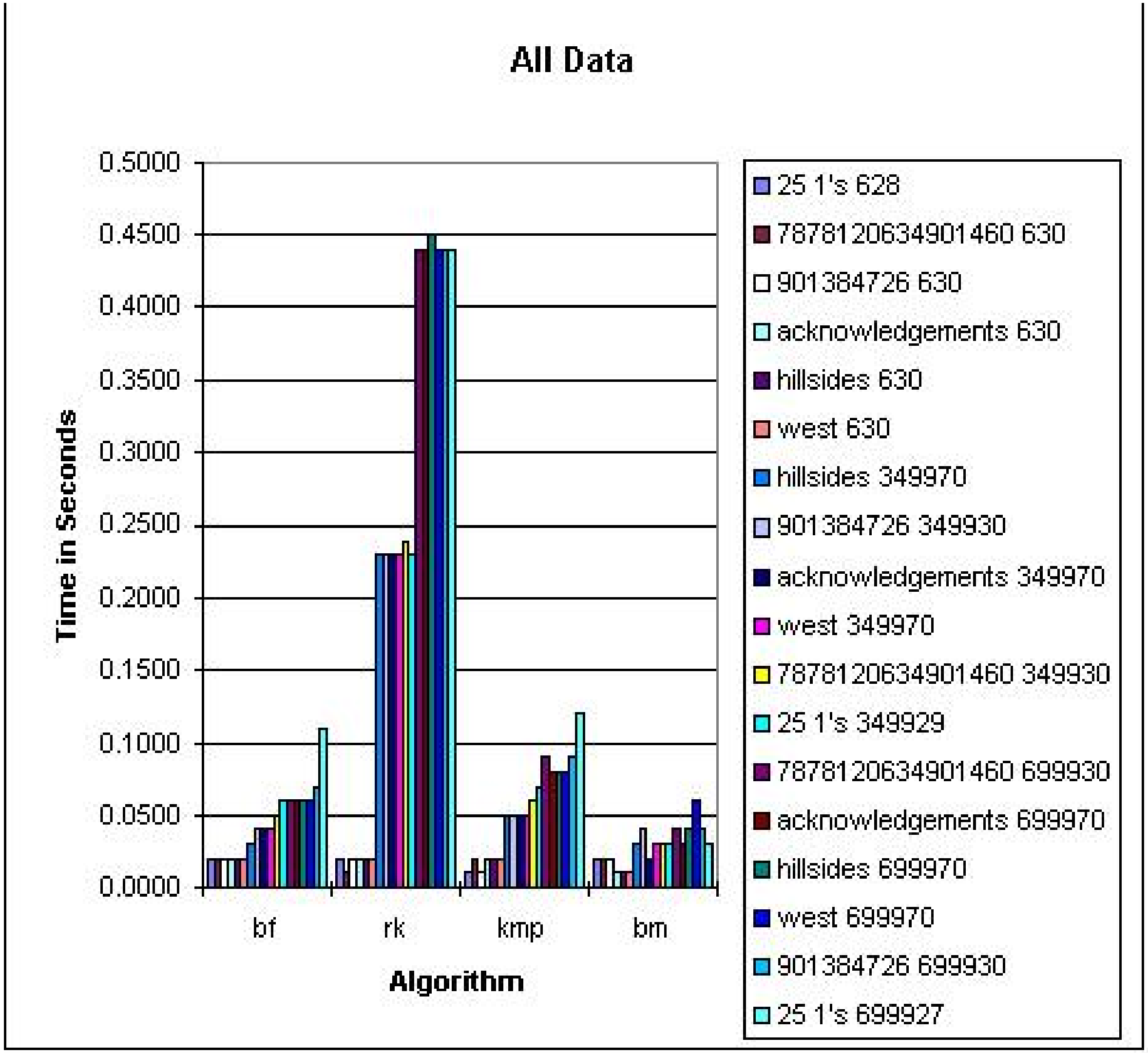


Matched = m-k = 6, slide[2] = m = 8
matchJump[2] = 14



Matched = m-k = 7, slide[1] = m = 8
matchJump[1] = 15

- Brute-Force Algorithm (bf)
- Rabin-Karp Algorithm (rk)
- Knuth-Morris-Pratt Algorithm (kmp)
- Boyer-Moore Algorithm (bm)



- Brute Force behaved better than we expected
  - because worst case is not common. Worst case would occur when the pattern and the text produced a near match.
- Rabin-Karp behaved much worse
  - Rabin-Karp has several function calls. These are expensive, timewise.
  - Any division, including mod, is time expensive.
  - The conversion from character values to numeric values takes time.

- Boyer-Moore algorithm is considered the most efficient string-matching algorithm in usual applications, for example, in text editors.
- Moore says the algorithm has the peculiar property that, roughly speaking, the longer the pattern is, the faster the algorithm goes.
- The payoff is not as for binary strings or for very short patterns.
- For binary strings Knuth-Morris-Pratt algorithm is recommended.
- For the very shortest patterns, the brute force algorithm may be better.

- What else do we learn from the BM algorithm?
  - Designing algorithms to solve problems often needs insights into a problem’s structure – analyse the problem carefully before thinking about its solution