NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CX1107 Data Structures and Algorithms **2020/21 Semester 2**

Tutorial 4: Algorithm Analysis

School of Computer Science and Engineering *Nanyang Technological University*

Q1 Given the precedence of some operators,

Operators	Precedence
*, /, %	highest
+, -	
<<, >>	
&&	
=	lowest

(a) convert an infix expression, $x = a + b * c\%d >> e$, to a postfix expression

(b) convert a prefix expression, $= y\&\& << ab >> c + de$, to an infix expression

(c) convert a postfix expression, $xabc * d\% + e >>=$, to a prefix expression

Q2 The function subset() below takes two linked lists of integers and determines whether the first is a subset of the second. Give the worst-case running time of subset as a function of the lengths of the two lists. When will this worst case happen?

```
1  typedef struct _listnode{
2    int item;
3    struct _listnode *next;
4  } ListNode;
5
6  //Check whether integer X is an element of linked list Q
7  int element (int X, ListNode* Q)
8  {
9    int found;  //Flag whether X has been found
10   found = 0;
11   while ( Q != NULL && !found) {
12      found = Q->item == X;
13      Q = Q->next;
14   }
15   return found;
16 }
17
18 // Check whether L is a subset of M
19 int subset (ListNode* L, ListNode* M)
20 {
21   int success; // Flag whether L is a subset so far
22   success = 1;
23   while ( L != NULL && success) {
24       success = element(L->item, M);
25       L = L->next;
26   }
27   return success;
28 }
```

Q3 Find the number of printf used in the following functions. Write down its time complexity in Θ notation in terms of N.

```
1  void Q3a (int N)
2  {
3      int j, k;
4      for (j=1; j<=N;j*=3)
5          for(k=1;k<=N; k*=2)
6              printf("CZ1107\n");
7  }
```

$\log_3 N$

1, 3, 9, ..., N

$\log_2 N$

1, 2, 4, 8, ..., N

$\Theta(f(N)) = \log_3 N \times \log_2 N$

$= (\log_2 N)^2$

```
1   void Q3b (int N)
2   {
3       int i;
4       if(N>0)
5       {
6           for(i=0;i<N;i++)
7               printf("CZ1107\n");
8           Q3b(N-1);
9           Q3b(N-1);
10      }
11  }
```

$W(N) = 2W(N-1) + N$

$= 2[2W(N-2) + N-1] + N$

$= 2^2 W(N-2) + 2N - 2 + N$

$= 2^2 [2W(N-3) + N-2] + 3N - 2$

$= 2^3 W(N-3) + 2^2 N + 2N + N - 2^2 - 2 - 1$

$= 2^n W(N-N) + (2^{n-1} + 2^{n-2} \cdots + 1) \times (N-1)$

$= 0 + (2^n - 1) \times (N-1)$

$\Theta(W(N)) = (2^N - 1)(N-1)$

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Solution 4: Algorithm Analysis

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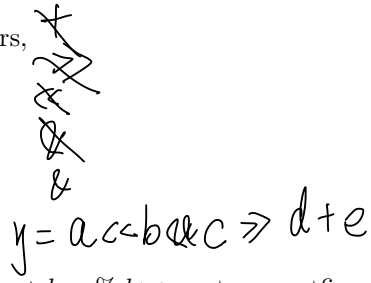

(a) convert an infix expression, $x = a + b * c\%d >> e$, to a postfix expression

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(c) convert a postfix expression, $xabc * d\% + e >>=$, to a prefix expression

S1 **(a)** $xabc * d\% + e >>=$

(b) $y = a << b\&\&c >> d + e$

(c) $= x >> +a\% * bcde$

Q2 The function subset() below takes two linked lists of integers and determines whether the first is a subset of the second. Give the worst-case running time of subset as a function of the lengths of the two lists. When will this worst case happen?

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```

S2 Let $|L|$ and $|M|$ indicate the length of the linked lists, L and M, respectively. The worst-case running time of subset:

- the first $|L|-1$ elements of L from the last $|L|-1$ elements of M in reverse order.
- the last element of L not in M

$\therefore$ Total number of comparisons between elements of L and M

$$\begin{aligned}
&=|M|+(|M|-1)+(|M|-2)+\ldots+(|M|-(|L|-2))+|M| \\
&=|L||M|-(1+2+3+\ldots+(|L|-2)) \\
&=|L||M|-\frac{(|L|-2)}{2}(1+|L|-2) \\
&=|L||M|-\frac{(|L|-2)(|L|-1)}{2} \\
&=\Theta(|L||M|)
\end{aligned}$$

Here we assume that $|L|<|M|$

Q3 Find the number of printf used in the following functions. Write down its time complexity in Θ notation in terms of N.

```
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2  {
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4     for (j=1; j<=N;j*=3)
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6               printf("CZ1107\n");
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```

```
1   void Q3b (int N)
2  {
3     int i;
4     if(N>0)
5     {
6          for(i=0;i<N;i++)
7               printf("CZ1107\n");
8          Q3b(N-1);
9          Q3b(N-1);
10    }
11 }
```

S3a let K denote the number of iterations for the inner loop and J denote the number of iteration for the outer loop.
For the inner loop, we have

$$\begin{aligned}
2^{K-1} &\leq N < 2^K \\
(K-1) &\leq \log_2 N < K \\
K &\leq \log_2 N + 1 < K+1 \\
K &= \lfloor \log_2 N + 1 \rfloor = \lfloor \log_2 N \rfloor + 1
\end{aligned}$$

For the outer loop, we have

$$\begin{aligned} 3^{J-1} &\leq N < 3^J \\ (J-1) &\leq \log_3 N < J \\ J &\leq \log_3 N + 1 < J + 1 \\ J &= \lfloor \log_3 N + 1 \rfloor = \lfloor \log_3 N \rfloor + 1 \end{aligned}$$

$\therefore$ The number of printf is $JK = (\lfloor \log_3 N \rfloor + 1)(\lfloor \log_2 N \rfloor + 1) = \Theta((logN)^2)$

S3b let W(N) denote the number of printf used in the function with problem size of N

$$\begin{aligned} W(N) &= 2W(N-1) + N \\ &= 2(2W(N-2) + (N-1)) + N \\ &= 2^2 W(N-2) + 2(N-1) + N \\ &= 2^2(2W(N-3) + (N-2)) + 2(N-1) + N \\ &= \cdots \\ &= 2^{N-1}(1) + 2^{N-2}(2) + \cdots + 2^3(N-3) + 2^2(N-2) + 2(N-1) + N \\ &= \sum_{t=0}^{n-1} 2^t(n-t) \\ &= 2^{N+1} - 2 - N \end{aligned}$$

$\therefore$ The number of printf is $\Theta(2^N)$