Report on panel model results

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July 27, 2021

Abstract

These note is to relate several models that can use panel data.

The objective of these note is to give a broad overview of the possible models that can use panel data. There are several usual features to consider in a model with panel data, for example, changes on parameters for time or individual. Also, specification on error term is relevant for interpretation.

The note is based on Hsiao (2014). It goes from the theory in the text, to the application.

1 Data plots

Here is a summary of the data available for the analysis.

Figure 1 presents the initial data points, these are used for the analysis.

I made a decision on which brands to include based on the number of observations on the period previous to the tax implementation, the tax started in january 2021, I made an exploratory analysis on the december 2020 data. This would ease the inclusion of brand explicitly in the analysis, brands with few observations, with difficulties to calculate most estimates could be analyzed after applying some criteria to make groups of brands out of the individual ones.

Figure 2 only considers the 7 most frequent brands. The graph provides some guidance on what to consider for the proposed descriptive model. In particular, there is clear trend over time and there are price adjustments in january.

Except for prices of other products, there are no other potential regressors to consider at the same level of the data.

2 Dummies for each level: city, brand, time

Estimations using areg, fixed effects are imposed. Using this method there is one category with parameters "absorbed", which are not estimated as a result of the procedure.

Specification with indicators for city, brand and trend for time. Same tax effect on all the brands.

$$y_{ctm} = \alpha_i^* + \gamma_m^* + \lambda * t + \beta_0' jan + \beta_1' tax 2020 + \beta_1' tax 2021 + u_{ctm}; \qquad (2.1)$$

$$c = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M.$$

Specification with indicators for city, brand and trend for time. Effect interacted for each brand.

$$y_{ctm} = \alpha_{i}^{*} + \gamma_{m}^{*} + \lambda * t + \beta_{0}^{'} jan + \beta_{1m}^{'} tax2020 + \beta_{2m}^{'} tax2021 + u_{itm}; \quad (2.2)$$

$$c = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M.$$
Results with data for the 7 brands:

Table 1: Fixed Effects for city-brand combination

	(1)	(2)	(3)	(4)
VARIABLES	ppu	ppu	ppu	ppu
2.marca	-0.012***	-0.011***	-0.006**	-0.006**
	(0.003)	(0.003)	(0.003)	(0.003)
3.marca	-0.595***	-0.593***	-0.592***	-0.591***
	(0.004)	(0.004)	(0.003)	(0.003)
4.marca	-0.270***	-0.268***	-0.268***	-0.266***
	(0.003)	(0.003)	(0.003)	(0.003)
5.marca	0.011***	0.010***	0.011***	0.010***
_	(0.003)	(0.003)	(0.002)	(0.002)
6.marca	-0.524***	-0.521***	-0.527***	-0.524***
	(0.005)	(0.005)	(0.004)	(0.004)
7.marca	-0.435***	-0.435***	-0.441***	-0.440***
	(0.003)	(0.003)	(0.002)	(0.002)
$1.m1_{-}20$		0.238***		
4 4 20 1/2		(0.024)		0.040
$1.\text{m}1_20\#2.\text{marca}$		-0.007		-0.012
1 1 20 119		(0.042)		(0.032)
$1.m1_{-}20#3.marca$		-0.111**		-0.112***
4 4 20 11 4		(0.043)		(0.033)
1.m1_20#4.marca		-0.147***		-0.150***
4 4 00 115		(0.039)		(0.030)
1.m1_20#5.marca		0.052		0.051**
1 1 20 1/2		(0.032)		(0.024)
$1.\text{m}1_20\#6.\text{marca}$		-0.239***		-0.235***
1 1 00 // 7		(0.062)		(0.048)
$1.\text{m}1_20\#7.\text{marca}$		-0.028		-0.024
1 1 01		(0.034)		(0.026)
1.m1_21		0.285***		
		(0.024)		

$1.m1_21#2.marca$		-0.035		-0.040
		(0.040)		(0.031)
$1.m1_21#3.marca$		-0.116**		-0.118***
		(0.046)		(0.035)
$1.m1_21\#4.marca$		-0.132***		-0.134***
		(0.038)		(0.029)
$1.m1_21\#5.marca$		0.022		0.021
		(0.032)		(0.024)
$1.m1_21\#6.marca$		-0.361***		-0.357***
		(0.063)		(0.048)
$1.\text{m}1_21\#7.\text{marca}$		-0.036		-0.031
		(0.034)		(0.026)
m1	-0.024***	-0.024***	-0.069***	-0.069***
	(0.004)	(0.004)	(0.004)	(0.004)
ym	0.009***	0.009***		
	(0.000)	(0.000)		
$m1_{-}20$	0.209***	,	-0.017*	0.012
	(0.011)		(0.009)	(0.018)
$m1_21$	0.242***		-0.011	0.032*
	(0.011)		(0.010)	(0.019)
Constant	-3.925***	-3.926***	1.991***	1.990***
	(0.018)	(0.018)	(0.004)	(0.004)
Observations	24,010	24,010	24,010	24,010
R-squared	0.897	0.897	0.940	0.940
				0.010

The columns 1 and 3 consider the same effect for each brand, the columns 2 and 4 estimate a different effect for each brand. The columns 1 and 2 consider a trend, columns 3 and 4 use a combination of dummy variables for year and month.

2.1 Comparisons by segment

Results by brand type: premium, columns 1 and 2; medium, columns 3 and 4; low, columns 5 and 6. The omitted identifier corresponds to the reference brand.

Table 2: Fixed Effects by brand type

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ppu	ppu	ppu	ppu	ppu	ppu
	0 000	0 00 - 444				
2.marca		-0.007***				
	(0.003)	(0.003)				

5.marca	0.008***	0.007***				
1.m1_20	(0.002)	(0.002) $0.198***$		0.119***		0.225***
$1.m1_20\#2.marca$		(0.019) -0.009		(0.033)		(0.040)
$1.m1_20\#5.marca$		(0.032) $0.051**$				
1.m1_21		(0.025) $0.235***$		0.188***		0.310*** (0.042)
$1.m1_21\#2.marca$		(0.019) -0.037 (0.031)		(0.033)		(0.042)
$1.m1_21\#5.marca$		0.022 (0.025)				
m1	-0.018*** (0.004)	-0.018*** (0.004)	-0.040*** (0.007)	-0.040*** (0.007)	-0.013 (0.009)	-0.013 (0.009)
ym	0.010*** (0.000)	0.010*** (0.000)	0.007) 0.009*** (0.000)	0.007) 0.009*** (0.000)	0.009) 0.007*** (0.000)	0.009) 0.007*** (0.000)
m1_20	0.219*** (0.012)	(0.000)	0.182*** (0.021)	(0.000)	0.191*** (0.034)	(0.000)
m1_21	0.238*** (0.012)		0.235*** (0.021)		0.231*** (0.036)	
7.marca	(0.012)		-0.165*** (0.004)	-0.166*** (0.004)	(0.050)	
$0b.m1_20\#7o.marca$			(0.004)	0.004) 0.000 (0.000)		
$1.m1_20\#7.marca$				0.102** (0.041)		
$1.m1_21\#7.marca$				0.076* (0.040)		
6.marca				(0.040)	0.101*** (0.007)	0.102*** (0.007)
$1.m1_20\#6.marca$					(0.007)	(0.007) $-0.119*$ (0.072)
$1.m1_21\#6.marca$						-0.252***
Constant	-4.429***	-4.429***	-4.040***	-4.041***	-2.892***	(0.073) $-2.895***$
	(0.019)	(0.019)	(0.037)	(0.037)	(0.051)	(0.051)
Observations R-squared	$13,396 \\ 0.917$	$13,396 \\ 0.918$	$6,700 \\ 0.846$	$6,700 \\ 0.846$	$3,914 \\ 0.760$	$3,914 \\ 0.761$
*		1	• 11			

Table with results to test for difference of coeficients in brands. The first

two rows correspond to the complete sample. Rows third and fourth test for difference in premium brands. Rows fifth and sixth test for difference in medium brands. Rows seventh and eighth test for difference in lower priced brands. Columns 1 to 3 indicate the parameters and result for the test of fixed effects or individual effects by brand. Columns 4 to 6 show the values to test for the equality of the effect by brand of the tax in 2020. Columns 7 to 9 show the values and test result for the equality of the effect by brand of the tax in 2021.

Table 3: F tests for equality of coefficients

(6)		Ξı		9.14		1.88		3.52		11.8	
(8)	Equality of Tax 2021	Numerator Denominator		23942		13340		6647		3865	
(7)	Equal	Numerator		9		2		1		1	
(9)		ഥ		8.15		2.89		60.9		2.74	Ţ,
(5)	Equality of $Tax 2020$	Immerator Denominator		23942		13340		6647		3865	t 1 percent level
(4)	Equali	Numerator		9		2		Н		1	All tests are significant at 1
(3)	t.	뇐	10163.44	10006.08	19.05	16.81	1604.66	1613.81	187	193.13	All tests an
(2)	Equality of Intercept	Denominator	23954	23942	13344	13340	6649	6647	3867	3865	
(1)	Eqι	Numerator	9	9	2	2	П	П	П	1	
		Equation	(2.1)	(2.2)	(2.1)	(2.2)	(2.1)	(2.2)	(2.1)	(2.2)	

3 Different parameters for each brand

Uses xtsur, user-defined, command. One estimate of each parameter for each brand. The intention was to make a unique model of Seemingly Unrelated Regressions to test the coefficients of the tax change for equality. Unfortunately, it is impossible (using the xtsur routine, in a 4th gen i7 with 16ram) to make the estimation based on the complete sample, with 7 brands. I present the test based on three groups of brands.

$$y_{itm} = \alpha_i^* + \lambda_1 * t + \beta_0' janDummy_m + \beta_1' taxDummy_m + u_{itm};$$

$$i = 1, \dots, N; t = 1, \dots, T.m = 1, 2, \dots, 7$$

3.1 Comparisons by segment

Results for premium brands

	(1)	(2)	(3)
VARIABLES	ppu1	ppu2	ppu5
$m1_{-}20$	0.156***	0.135***	0.151***
	(0.018)	(0.018)	(0.012)
$m1_{-}21$	0.077	0.027	0.070
	(0.000)	(0.000)	(0.000)
m1	0.024***	0.000	0.003
	(0.004)	(0.004)	(0.002)
ym	0.009***	0.009***	0.009***
	(0.000)	(0.000)	(0.000)
Observations	2,543	2,543	2,543
Number of cve_ciudad	45	45	45

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Results for lower segment brands

	(1)	(2)
VARIABLES	ppu3	ppu6
	PPGS	PPGG
$m1_{-}20$	0.322***	0.204***
	(0.047)	(0.030)
$m1_{-}21$	0.364***	0.446***
	(0.047)	(0.030)
m1	0.009	-0.035***
	(0.015)	(0.010)
ym	0.007***	0.005***
	(0.000)	(0.000)
Observations	614	614
Number of cve_ciudad	43	43
Standard errors		
*** p<0.01, ** p		
Results for mid-range seg	gment branc	
	(1)	(2)
VARIABLES	ppu4	ppu7
$m1_{-}20$	0.150***	0.071***
	(0.018)	(0.018)
$m1_{-}21$	0.050***	0.191***
	(0.018)	(0.018)
m1	-0.013***	-0.018***
	(0.005)	(0.005)
ym	0.009***	0.010***
ym		0.010*** (0.000)
·	0.009*** (0.000)	0.010*** (0.000)
ym Observations Number of cve_ciudad	0.009***	0.010***

4 Constant Parameters over time

Estimations using xtreg, first static estimations, second dynamic estimates. Separate regression for each brand.

The estimation routine has the possibility to distinguish between fixed or random individual coefficients.

Separate regression for each individual defined by city and brand.

$$y_{it} = \alpha_i^* + \beta_i' x_{it} + u_{it}; i = 1, \dots, N; t = 1, \dots, T.$$

Where i represents a combination of city and brand.

4.1 Static models

The proposed model only uses fixed regressors, the effect of the price change in every january, january 2020 and january 2021,

$$y_{it} = \alpha_i^* + \gamma_m^* + \lambda * t + \beta_0' jan + \beta_1' tax 2020 + \beta_1' tax 2021 + u_{it}; \tag{4.1}$$

$$c = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M.$$

It includes interactions, for the effect of the price change in january 2020 and january 2021, for different brand-types.

$$y_{it} = \alpha_i^* + \gamma_m^* + \lambda * t + \beta_0' jan + \beta_{1m}' tax 2020 + \beta_{2m}' tax 2021 + u_{it}; \tag{4.2}$$

$$i = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M.$$

Because there are many omitted variables captured in the individual effects, there is the question of the relevance of them as fixed or random.

The result of the Hausman test for fixed effects does not rule out the non systematic difference in coefficients, this is in favour of the random effects model: $\text{Chi2}(4) = 0.60, Prob \ge chi2 = 0.9628$

Similarly the Breusch-Pagan test for random effects does rule out the alternative of OLS : chibar2(01) = 7.5e+05, $Prob \ge chibar2 = 0.0000$

The test of unit root, using the Fischer type estimation from Choi (2001): Inverse chi-squared (500) = 1112.8056, $Prob \ge chi2 = 0.0000$, does not rule out the presence of unit root for any panel (defined as a combination of city and brand), except for the model that includes a drift. The result suggests to consider different trends for each brand or city, there is an estimation by brand to test for unit roots by specifications of the panel.

Next are the results from the complete sample. First two columns and last two columns are estimated using random effects. The third column using fixed effects, since the next estimates by brand suggest that most brands have fixed effects for city.

Table 4: Fixed Effects for brand and city by brand type

	(1)	(2)	(3)	(4)	(5)
VARIABLES	ppu	ppu	ppu	ppu	ppu
2.marca	-0.013	-0.013		-0.009	-0.008
	(0.024)	(0.022)		(0.018)	(0.017)
3.marca	-0.605***	-0.604***		-0.603***	-0.602***
	(0.024)	(0.022)		(0.018)	(0.017)
4.marca	-0.274***	-0.272***		-0.269***	-0.268***
	(0.024)	(0.021)		(0.017)	(0.017)
5.marca	$0.004^{'}$	$0.004^{'}$		$0.007^{'}$	$0.007^{'}$
	(0.022)	(0.020)		(0.016)	(0.016)

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-0.503***
6.marca
                              -0.502***
                                           -0.500***
                                                                   -0.505***
                                            (0.026)
                                                                                 (0.020)
                                (0.028)
                                                                     (0.021)
                                           -0.425***
                                                                   -0.443***
                                                                               -0.442***
7.marca
                              -0.426***
                                            (0.021)
                                                                                 (0.017)
                                (0.023)
                                                                     (0.017)
                                           0.230***
                               0.202***
m1_{-}20
                                                       0.230***
                                                                    -0.015**
                                                                                 0.012
                                (0.010)
                                            (0.021)
                                                        (0.021)
                                                                    (0.007)
                                                                                 (0.015)
                               0.232***
                                           0.276***
m1_{-}21
                                                       0.275***
                                                                     -0.012
                                                                                 0.027*
                                (0.010)
                                            (0.021)
                                                        (0.021)
                                                                     (0.008)
                                                                                 (0.016)
1.m1_20#2.marca
                                                                                 -0.011
                                                                                 (0.026)
1.m1_20#3.marca
                                                                               -0.103***
                                                                                 (0.027)
1.m1_20#4.marca
                                                                               -0.129***
                                                                                 (0.024)
1.m1_20#5.marca
                                                                                0.052***
                                                                                 (0.020)
1.m1\_20\#6.marca
                                                                               -0.195***
                                                                                 (0.039)
1.m1\_20\#7.marca
                                                                                 -0.040*
                                                                                 (0.021)
1.m1\_21\#2.marca
                                                                                 -0.039
                                                                                 (0.025)
                                                                               -0.100***
1.m1\_21\#3.marca
                                                                                 (0.029)
                                                                               -0.113***
1.m1_{-}21#4.marca
                                                                                 (0.024)
1.m1_21#5.marca
                                                                                 0.022
                                                                                 (0.020)
                                                                               -0.360***
1.m1_21\#6.marca
                                                                                 (0.039)
1.m1_21#7.marca
                                                                                 -0.033
                                                                                 (0.021)
                              -0.023***
                                           -0.023***
                                                                   -0.070***
                                                                               -0.070***
                                                       -0.023***
m1
                                (0.003)
                                            (0.003)
                                                        (0.003)
                                                                     (0.003)
                                                                                 (0.003)
                               0.009***
                                           0.009***
                                                       0.009***
ym
                                (0.000)
                                            (0.000)
                                                        (0.000)
2.marca#1.m1_20
                                            -0.012
                                                         -0.011
                                            (0.037)
                                                        (0.037)
                                           -0.109***
                                                       -0.108***
3.marca#1.m1_20
                                            (0.039)
                                                        (0.039)
4.marca#1.m1_20
                                           -0.125***
                                                       -0.124***
                                                        (0.034)
                                            (0.034)
5.marca#1.m1_20
                                           0.056**
                                                        0.056**
                                            (0.028)
                                                        (0.028)
                                           -0.194***
                                                       -0.191***
6.\text{marca}\#1.\text{m}1\_20
                                            (0.056)
                                                        (0.056)
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$7.\text{marca}\#1.\text{m}1_20$		-0.052*	-0.053*		
		(0.030)	(0.030)		
$2.\text{marca}\#1.\text{m}1_21$		-0.047	-0.051		
		(0.036)	(0.036)		
$3.\text{marca}\#1.\text{m}1_21$		-0.101**	-0.101**		
		(0.041)	(0.041)		
$4.\text{marca}\#1.\text{m}1_21$		-0.117***	-0.118***		
		(0.034)	(0.034)		
$5.\text{marca}\#1.\text{m}1_21$		0.027	0.027		
			(0.028)		
$6.\text{marca}\#1.\text{m}1_21$		-0.364***			
		(0.056)	(0.056)		
$7.\text{marca}\#1.\text{m}1_21$		-0.053*			
		(0.030)	(0.030)		
Constant	-3.940***	-3.940***	-4.137***	1.989***	1.989***
	(0.023)	(0.022)	(0.017)	(0.012)	(0.012)
Observations	23,926	23,926	23,926	23,926	23,926
Number of gr_marca_ciudad	263	263	263	263	263
R-squared			0.866		

F tests show in the specification in this stage does reject equality of coeficients.

Next are the results for brand type.

Table 5: Fixed Effects/Random Effects for brand and city by brand type

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ppu	ppu	ppu	ppu	ppu	ppu
4.marca			-4.013***	0.150***		
1.m1_20		0.196***	(0.041)	(0.028) $0.132***$		0.212***
$1.\text{m}1_20\#7.\text{marca}$		(0.018)		$(0.030) \\ 0.071*$		(0.037)
1.m1_21		0.233***		(0.037) $0.251***$		0.295***
1.m1_21#4.marca		(0.018)		(0.024) $-0.062*$		(0.040)
	0.010***	0.010***	0.020***	(0.037)	0.010	0.010
m1	-0.018*** (0.004)	-0.018*** (0.004)	-0.039*** (0.006)	-0.039*** (0.006)	-0.012 (0.008)	-0.012 (0.008)
ym	0.010*** (0.000)	0.010*** (0.000)	0.009*** (0.000)	0.009*** (0.000)	0.007*** (0.000)	0.007*** (0.000)

$1.m1_20\#2.marca$		-0.012				
		(0.031)				
$1.\text{m}1_20\#5.\text{marca}$		0.053**				
		(0.024)				
$1.\text{m}1_21\#2.\text{marca}$		-0.049				
		(0.030)				
$1.\text{m}1_21\#5.\text{marca}$		0.024				
		(0.024)				
$m1_{-}20$	0.218***		0.176***		0.190***	
	(0.011)		(0.019)		(0.032)	
$m1_{-}21$	0.235***		0.228***		0.217***	
	(0.011)		(0.019)		(0.034)	
7.marca			-4.162***			
			(0.041)			
$1.m1_20\#6.marca$						-0.078
						(0.068)
$1.m1_21\#6.marca$						-0.245***
						(0.069)
Constant	-4.417***	-4.416***		-4.164***	-2.961***	-2.963***
	(0.019)	(0.019)		(0.041)	(0.050)	(0.049)
Observations	13,396	13,396	6,620	6,620	3,910	3,910
R-squared	0.916	0.916			0.720	0.721
Number of gr_marca_ciudad	126	126	80	80	57	57

By brand type, for premium(1) and low(3) brand types the Hausman test rejects the alternative of random effects in favour of fixed individual effects. The test for type 1 and 3 are Chi2(4) = 0.60, $Prob \ge chi2 = 0.9628$ and Chi2(4) = 0.60, $Prob \ge chi2 = 0.9628$, respectively.

Similarly the Breusch-Pagan test for random effects does rule out the alternative of OLS: chibar2(01) = 7.5e+05, Prob > chibar2 = 0.0000

The test of unit root, using the Fischer type estimation from Choi (2001): Inverse chi-squared(500) = 1112.8056 , $Prob \geq chi2 = 0.0000$, does not rule out the presence of unit root for any panel (defined as a combination of city and brand), except for the model that includes a drift. The result suggests to consider different trends for each brand or city, there is an estimation by brand to test for unit roots by specifications of the panel.

The next table presents F or Chi-squared tests for equality of coeficients, F corresponds to models by fixed effects, Chi-squared corresponds to models estimated by random effects.

The distribution is the same used in Table 3, first two rows correspond to the complete sample. Rows third and fourth test for difference in premium brands. Rows fifth and sixth test for difference in medium brands. Rows seventh and eighth test for difference in lower priced brands. Columns 1 to 3 indicate the

parameters and result for the test of fixed effects or individual effects by brand. Columns 4 to 6 show the values to test for the equality of the effect by brand of the tax in 2020. Columns 7 to 9 show the values and test result for the equality of the effect by brand of the tax in 2021.

Table 6: F tests for equality of coefficients

(6)	2021	${ m F/Chi2}$		67.31***		3.08***		2.85*		12.43*	
(8)	of Tax	Den				13262				3847	
(7)	Equality	Num/gl-chi2 Den F/Chi2		9		2		1		1	
(9)	2020	${ m F/Chi2}$		51.86***		3.51**		3.62*		1.32	0.1
(2)	of Tax	Den				13262				3847	5, * p<(
(4)	Equality	Num/gl-chi2 Den F/Chi2		9		2		1		1	*** p<0.01, ** p<0.05, * p<0.1
(3)	ercept	${ m F/Chi2}$	1287.39***	1547.03***	22.32***	22.32***	11224.48***	28.63***	41.93***	42.17***	d ***
(2)	Equality of Intercept	Den			13266	13262			3849	3847	
(1)	Equali	Num/gl-chi2 I	9	9	125	125	2	1	26	26	
		Equation	(4.1)	(4.2)	(4.1)	(4.2)	(4.1)	(4.2)	(4.1)	(4.2)	

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The next table presents the static results by brand. Considering the Hausman test, the model was estimated using random or fixed effects, for each brand. The column number corresponds to the brand label.

Table 7: Fixed/Random individual effects for each brand

	(1)	(2)	(3)	(4)	(2)	(9)	(7)
VARIABLES	ndd	ndd	ndd	ndd	ndd	ndd	ndd
$m1_{-20}$	0.193***	0.202***	0.202***	0.202***	0.202***	0.202***	0.202***
	(0.018)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
m1_21	0.231***	0.232***	0.232***	0.232***	0.232***	0.232***	0.232***
	(0.018)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
m1	-0.013**	-0.023***	-0.023***	-0.023***	-0.023***	-0.023***	-0.023***
	(0.006)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
ym	0.010***	0.009***	0.009	0.009***	0.009***	0.009***	0.009***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	4,650	23,926	23,926	23,926	23,926	23,926	23,926
R-squared	0.921	0.865	0.865	0.865	0.865	0.865	0.865
Number of gr_marca_ciudad	44	263	263	263	263	263	263
	*	Standard er	Standard errors in parentheses	theses			
		p<0.01,	p<0.00,	p<0.1			

4.2 Dynamic models

Since we cannot rule the presence of unit roots for each panel, by city, except for a model with drift, an alternative is to consider dynamics in the equation, in particular the lagged dependent variable.

$$y_{it} = \delta * y_{i,t-1} + \alpha_i^* + \gamma_m^* + \lambda * t + \beta_0' jan + \beta_1' tax 2020 + \beta_1' tax 2021 + u_{it};$$
(4.3)
$$c = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M.$$

Similar to the previous section, estimations include interactions, to consider the effect of the price change in every january, january 2020 and january 2021.

$$y_{it} = \delta * y_{i,t-1} + \alpha_i^* + \gamma_m^* + \lambda * t + \beta_0' jan + \beta_{1m}' tax 2020 + \beta_{2m}' tax 2021 + u_{it};$$
 (4.4)

$$i = 1, \dots, N; t = 1, \dots, T; m = 1, \dots, M.$$

The table shows the results prefered by the Hausmann test, in this case fixed effects. It shows also the estimates by type. The first two columns correspond to the model estimation without interactions. The third and fourth include interactions by brand. The fifth and sixth columns consider interactions by brand type. Columns 2, 4 and 6 present, for reference, an estimation without lagged dependent variable.

The results show that the lag effect is captured, when ommitted, by the parameter of the linear trend and the effect of the tax in january 2021.

The estimation by brand type is extended in the next table, (1) and (2) show the estimates for premium brands, (3) and (4) show the estimates for medium brands, (5) and (6) show the estimates for lower brands. Columns (2,4,6) show results with interactions for brand and tax coefficients and columns (1,3,5) without interactions. In the next table, the dependent variable was transformed to cents.

The coefficient for m1_20 and m1_21 correspond to the value of the reference brand within a brand type. The coefficient for the other brands indicate the difference to that reference brand for the correspondent brand. The labels correspond to the alphabetical order, 1 is Benson, reference for premium brands, 2 is Camel, 3 is Chesterfield, reference for lower brands, 4 is Lucky Strike, reference for medium brands, 5 is Marlboro, 6 is Montana and 7 is Pall Mall.

With premium for the first label, it shows that the medium brands have the smallest tax impact, although, counterintuitively the lowest impact is estimated for the medium brands with a decrease of 8.9 cents while the lower brand only decreased 5 cents, both with respect to the premium brands average.

The distribution in the next table is the same used in Table 3 and Table 6. F tests for equality of coeficients, significant indicates difference between coeficients, the first two rows correspond to the complete sample. The third and fourth to the premium brands, where the equality is still relevant. Fifth and sixth rows are medium brands, where only is difference for the impact of the tax in 2021. The last two rows correspond to the lower priced brands, here there is no difference in the coefficients for each brand.

Columns 1 to 3 indicate the parameters and result for the test individual effects by brand. Columns 4 to 6 show the values to test for the equality of the effect by brand of the tax in 2020. Columns 7 to 9 show the values and test result for the equality of the effect by brand of the tax in 2021.

Table 8: Fixed/Random individual effects

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	ppu	ppu	ppu	ppu	ppu	ppu
m1_20	0.182*** (0.003)	0.202***	0.207*** (0.006)	0.230***		0.253***
m1_21	0.044*** (0.003)	(0.010) $0.232***$ (0.010)	0.083*** (0.006)	(0.021) $0.275***$ (0.021)		(0.013) $0.278***$ (0.013)
m1	0.022*** (0.001)	-0.023*** (0.003)	0.022*** (0.001)	-0.023*** (0.003)	0.022*** (0.001)	-0.023*** (0.003)
ym	0.0001 $0.000***$ (0.000)	0.003) 0.009*** (0.000)	0.0001 $0.000***$ (0.000)	0.009*** (0.000)	0.0001 $0.000***$ (0.000)	0.009*** (0.000)
$2.\mathrm{marca}\#1.\mathrm{m}1_20$	(0.000)	(0.000)	-0.018* (0.010)	-0.011 (0.037)	(0.000)	(0.000)
$3.marca\#1.m1_20$			-0.043*** (0.010)	-0.108*** (0.039)		
$4.marca\#1.m1_20$			-0.088*** (0.009)	-0.124*** (0.034)		
$5.marca\#1.m1_20$			0.031*** (0.008)	0.056** (0.028)		
$6.\mathrm{marca}\#1.\mathrm{m}1_20$			-0.049*** (0.015)	-0.191*** (0.056)		
$7.marca\#1.m1_20$			-0.073*** (0.008)	-0.053* (0.030)		
$2.\mathrm{marca}\#1.\mathrm{m}1_21$			-0.069*** (0.010)	-0.051 (0.036)		
$3.\mathrm{marca}\#1.\mathrm{m}1_21$			-0.023** (0.011)	-0.101** (0.041)		
$4.marca\#1.m1_21$			-0.083*** (0.009)	-0.118*** (0.034)		
$5.marca\#1.m1_21$			-0.013* (0.008)	0.027 (0.028)		
$6.marca\#1.m1_21$			-0.059*** (0.015)	-0.364*** (0.056)		
$7.marca\#1.m1_21$			-0.069*** (0.008)	-0.051* (0.030)		
L.ppu	0.967*** (0.002)		0.966*** (0.002)	(0.000)	0.967*** (0.002)	
$2.tipo\#1.m1_20$	(0.002)		(0.002)		-0.089*** (0.006)	
$3.tipo\#1.m1_20$					-0.055*** (0.008)	
$2.tipo\#1.m1_21$					-0.056*** (0.006)	
$3.tipo\#1.m1_21$					-0.016* (0.009)	
Constant	-0.170*** (0.00 9)	-4.138*** (0.017)	-0.171*** (0.009)	-4.137*** (0.017)	-0.170*** (0.009)	-4.137*** (0.017)
Observations R-squared Number of gr_marca_ciudad	23,616 0.990 262	23,926 0.865 263	23,616 0.990 262	23,926 0.866 263	23,616 0.990 262	23,926 0.866 263

Table 9: Fixed individual effects by brand type, interacted

VARIABLES	(1) ppu100	(2) ppu100	(3) ppu100	(4) ppu100	(5) ppu100	(6) ppu100
m1_20	20.974***	19.904***	14.040***	13.084***	16.867***	17.046***
$1.m1_20\#2.marca$	(0.316)	(0.496) -1.762** (0.864)	(0.539)	(0.845)	(0.933)	(1.091)
$1.m1_20\#5.marca$		3.065**** (0.658)				
$1.\text{m}1_20\#6.\text{marca}$,				-0.620 (1.981)
$1.\text{m}1_20\#7.\text{marca}$				$ \begin{array}{c} 1.534 \\ (1.042) \end{array} $		
m1_21	5.700*** (0.320)	7.592*** (0.505)	1.948*** (0.544)	1.081 (0.833)	5.568*** (0.983)	6.670*** (1.175)
$1.\text{m}1_{-}21\#2.\text{marca}$		-6.874*** (0.840)				
$1.\text{m}1_21\#5.\text{marca}$		-1.311** (0.663)				
1.m1_21#6.marca						-3.465* (2.023)
1.m1_21#7.marca				1.421 (1.030)		
m1	2.960*** (0.104)	2.960*** (0.104)	0.846*** (0.191)	0.844*** (0.191)	1.862*** (0.261)	1.859*** (0.261)
ym	0.044*** (0.003)	0.044*** (0.003)	0.038*** (0.004)	0.038*** (0.004)	0.030*** (0.004)	0.030*** (0.004)
L.ppu	96.071*** (0.245)	96.065*** (0.244)	96.872*** (0.351)	96.853*** (0.351)	96.663*** (0.479)	96.618*** (0.480)
Constant	-19.521*** (1.204)	-19.513*** (1.199)	-18.151*** (1.744)	-18.251*** (1.744)	-13.425*** (2.032)	-13.580*** (2.034)
Observations	13,255	13,255	6,524	6,524	3,837	3,837
R-squared Number of gr_marca_ciudad	$0.994 \\ 125$	$0.994 \\ 125$	0.987 80	0.987 80	$0.976 \\ 57$	$0.976 \\ 57$

Table 10: F tests by brand type, interacted

Equation Num Den F Num Den F Num Den F Num Den F (4.3) 261 23349 1.6*** (4.4) 261 23349 1.6*** (4.4) 261 23337 1.62*** (4.4) 261 23337 1.62*** (4.5) 261 23337 1.62*** (4.6) 124 13125 0.59 (4.7) 79 6439 0.92 (4.8) 79 6437 0.93 (4.8) 76 6437 0.93 (4.8) 56 3775 0.94 (4.9) 56 3773 0.95 1 3773 0.1 (4.9) 8487 2.17 1 8487 1.9*** (4.1) 3773 2.93 (4.2) 2.93 (4.3) 261 23349 1.6** (5.88*** (6.3) 2.88*** (6.4) 1.3121 2.0.34** (6.3) 2.337 2.337 2.337 2.38** (7.3) 2.33 2.337 2.337 2.337 2.337 2.337 (8.4) 2.6 3775 0.94 (8.3) 2.6 3773 0.95 1 3773 0.1 1 3773 2.93		(1)	(5)	(3)	(4)	(5)	(9)	(-)	$\widehat{\infty}$	(6)
tion Num Den F Num Den F Num Den Den P Den 261 23349 $1.6***$ 6 2.3337 $47.36***$ 6 2.3337 $47.36***$ 6 2.3337 $47.36***$ 6 2.3337 $47.36***$ 6 2.3337 $47.36***$ 7 $47.36***$ 7 $47.36***$ 8 $47.36***$ 8 $47.36***$ 8 $47.36***$ 8 $47.36***$ 8 $47.36***$ 8 $47.36***$ 9 $47.36**$ 9		Eque	ality of Ir	itercept	Equ	ality of 1	lax 2020	Equ	ality of 7	ax 2021
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Equation	Num	Den	压	Num	Den	Ţ	Num	Den	ഥ
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	(4.3)	261	23349	1.6***						
124 13125 0.59 $124 13121 0.6 2 13121 20.34*** 2 13121$ $79 6439 0.92 1 6437 2.17 1 6437$ $56 3775 0.94 1 3773 0.1 1 3773$ $56 3773 0.95 1 3773 0.1 1 3773$ $56 3773 0.95 1 3773 0.1 1 3773$		261	23337	1.62***	9	23337	47.36***	9	23337	25.88***
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		124	13125	0.59						
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		124	13121	9.0	2	13121	20.34***	2	13121	35.05***
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		79	6439	0.92						
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		79	6437	0.93	П	6437	2.17	Н	6437	1.9***
0.95 1 3773 0.1 1 3773 *** p<0.01, ** p<0.05, * p<0.1		26	3775	0.94						
	(4.4)	26	3773	0.95	П	3773	0.1	Н	3773	2.93
					0.01, **	p < 0.05	* p<0.1			

5 Consistent estimation for Variable Intercept

This models are based on Andrews, et al. (2006). The initial model comes from the transformation of:

$$y_{it} = x_{it}\beta_i + w_{j(i,t)t}\gamma + u_{it}\eta + q_{j(i,t)}\rho + \alpha_i + \phi_{j(i,t)} + \mu_t + \epsilon_{i,t};$$

 $i = 1, \dots, N; t = 1, \dots, T$

Given the interest only on the fixed independent variables, we can define an heterogeneity measure on brand and city (s), take the averages at that level, and make the transformation of variables, following:

$$y_{it} - \bar{y}_s = (x_{it} - \bar{x}_s)\beta_i + (w_{j(i,t)t} - \bar{w}_s)\gamma + (\epsilon_{i,t} - \bar{\epsilon}_s);$$

 $i = 1, \dots, N; t = 1, \dots, T$

Results in the left have the same estimate for the effect. Estimates in second column correspond to the first labeled brand, Benson.

Table 11: Transformation for consistency

	(1)	(2)
VARIABLES	$\mathrm{dm_ppu_cm}$	$\mathrm{dm}_{-}\mathrm{ppu}_{-}\mathrm{cm}$
$dm_m1_20_c$	0.202***	0.230***
	(0.010)	(0.021)
$dm_m1_21_cm$	0.232***	0.275***
	(0.010)	(0.021)
dm_m1_cm	-0.023***	-0.023***
	(0.003)	(0.003)
ym	0.009***	0.009***
	(0.000)	(0.000)
Observations	23,926	23,926
R-squared	0.865	0.866
Number of gr_marca_ciudad	263	263

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1