Week 1 Homework

1. Prove that $w'_{n+1}(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$ where w_{n+1} is the nodal polynomial

where $w_{n+1}(x) = \prod_{i=0}^{n} (x - x_i)$

<Pf> $w_{n+1}(x) = \prod_{i=0}^{n} (x - x_i)$

$\Rightarrow w'_{n+1}(x) = [(x-x_0)(x-x_1)\cdots(x-x_n)]'$

$= (x-x_1)\cdots(x-x_n) + (x-x_0)(x-x_2)\cdots(x-x_n) + \cdots + (x-x_0)(x-x_1)\cdots(x-x_{n-1})$

for $0 \le i \le n$

$w'_{n+1}(x_i) = \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)$ #

2. Show that $1 = \sum_{i=0}^{n} l_i(x)$, where $l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}$

<Pf> Let $S(x) = \sum_{i=0}^{n} l_i(x)$ claim $S(x) = 1$

for $i \neq k \Rightarrow l_k(x_k) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x_k - x_j}{x_i - x_j}$ $\because i \neq k \therefore$ there exist a tern $(x_k - x_k) = 0$

$= 0$

for $i = k \Rightarrow l_k(x_k) = \prod_{j\neq k} \frac{x_k - x_j}{x_k - x_j} = 1$

Hence $S(x_k) = \sum_{i=0}^{n} l_i(x_k) = 1 \Rightarrow S(x)$ has $n+1$ nodes $(x_0, x_1, \cdots, x_n)$ that make $S(x) = 1$

define $Q(x) = S(x) - 1 \Rightarrow Q(x_k) = S(x_k) - 1 = 0$ So $Q(x)$ has $n+1$ roots

but $Q(x)$ is a polynomial with degree at most n

$\Rightarrow Q(x) \equiv 0 \Rightarrow S(x) - 1 \equiv 0 \Rightarrow S(x) \equiv 1 \Rightarrow \sum_{i=0}^{n} l_i(x) = 1$