

SOFT620020.01  
Advanced Software  
Engineering

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# Course Outline

Date	Topic	Date	Topic
Sep. 09	Introduction	Nov. 04	Mobile Testing
Sep. 16	Testing Overview	Nov. 11	Delta Debugging
Sep. 23	Guided Random Testing	Nov. 18	Bug Localization
Sep. 30	Search-Based Testing	Nov. 25	Presentation 2
Oct. 12	Performance Analysis	Dec. 02	Automatic Repair
Oct. 14	Presentation 1	Dec. 09	<b>Symbolic Execution</b>
Oct. 21	Security Testing	Dec. 16	Big Code Analysis
Oct. 28	Compiler Testing	Dec. 23	Presentation 3

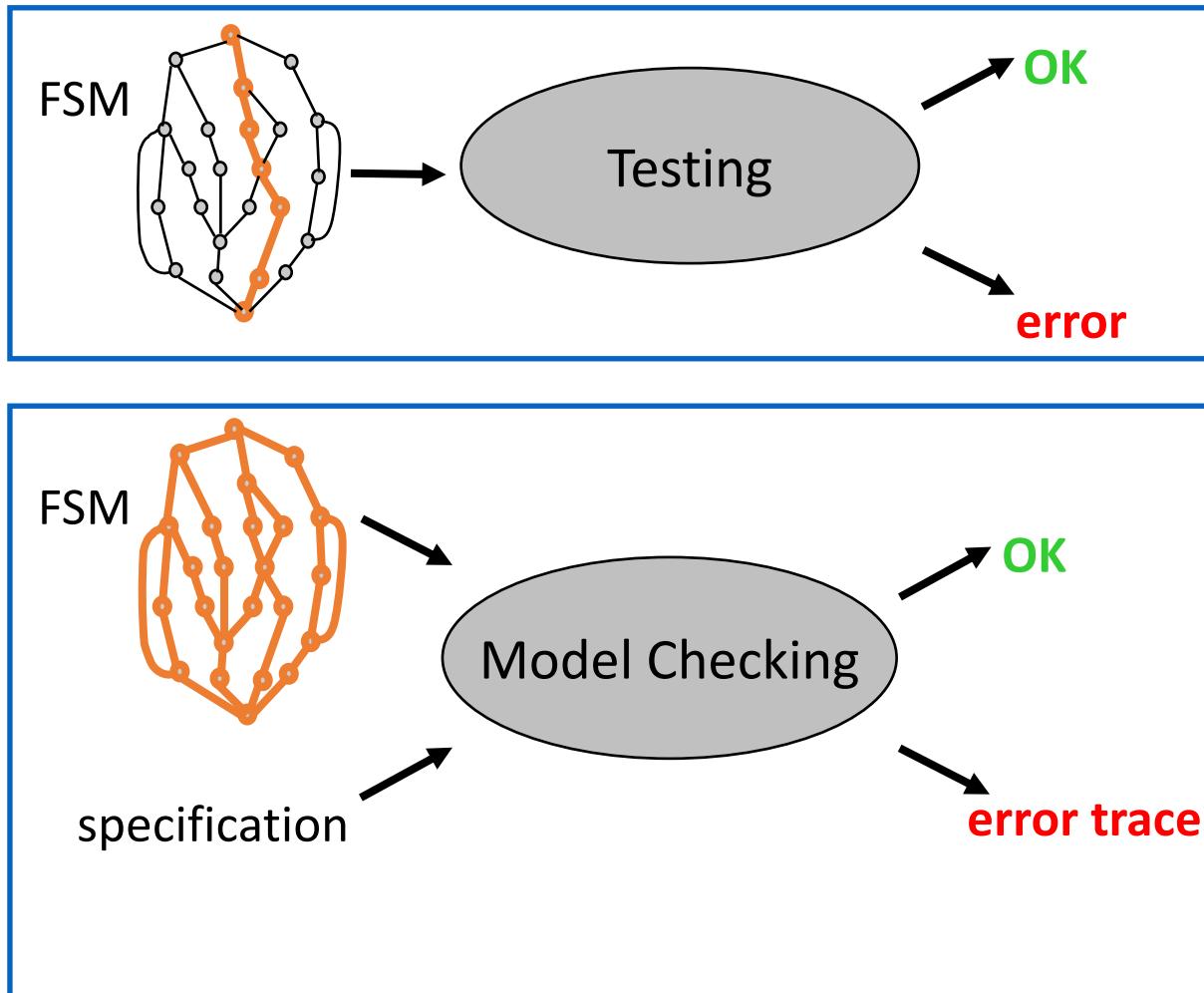
# Discussion – Will the Assertion Fail?

```
0. int x, y;  
1. if (x>0) {  
2.     assert(x>=0);  
3. }
```

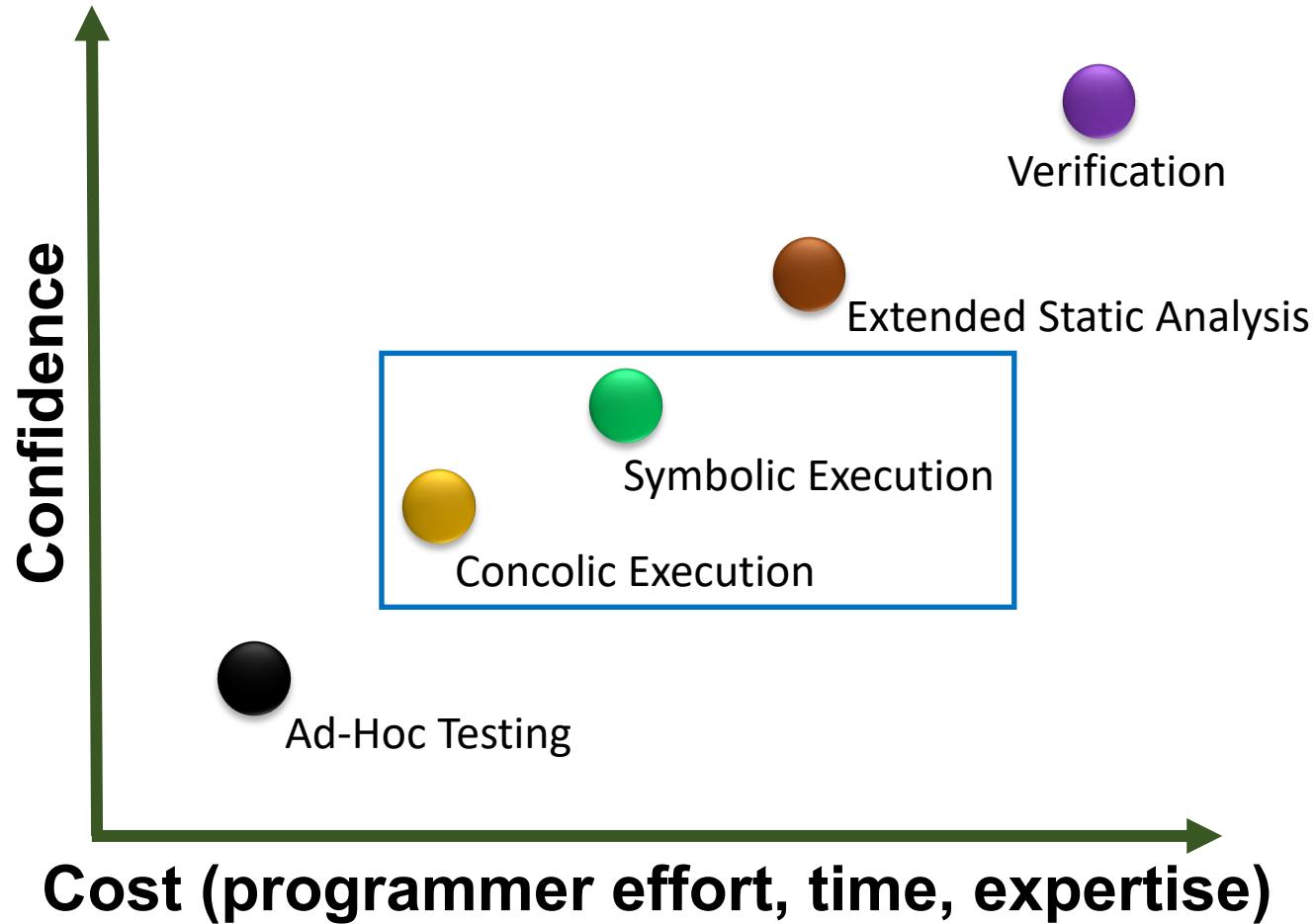
```
0. int x, y;  
1. if (x>y) {  
2.     x = x + y;  
3.     y = x - y;  
4.     x = x - y;  
5.     if (x-y>0) {  
6.         assert(false);  
7.     }  
8. }
```

- Can automatic testing prove whether the assertion fails?
  - **No!** Why?
- Can we automatically prove whether the assertion fails?
  - **Partially Yes.** How?

# Testing vs. Model Checking

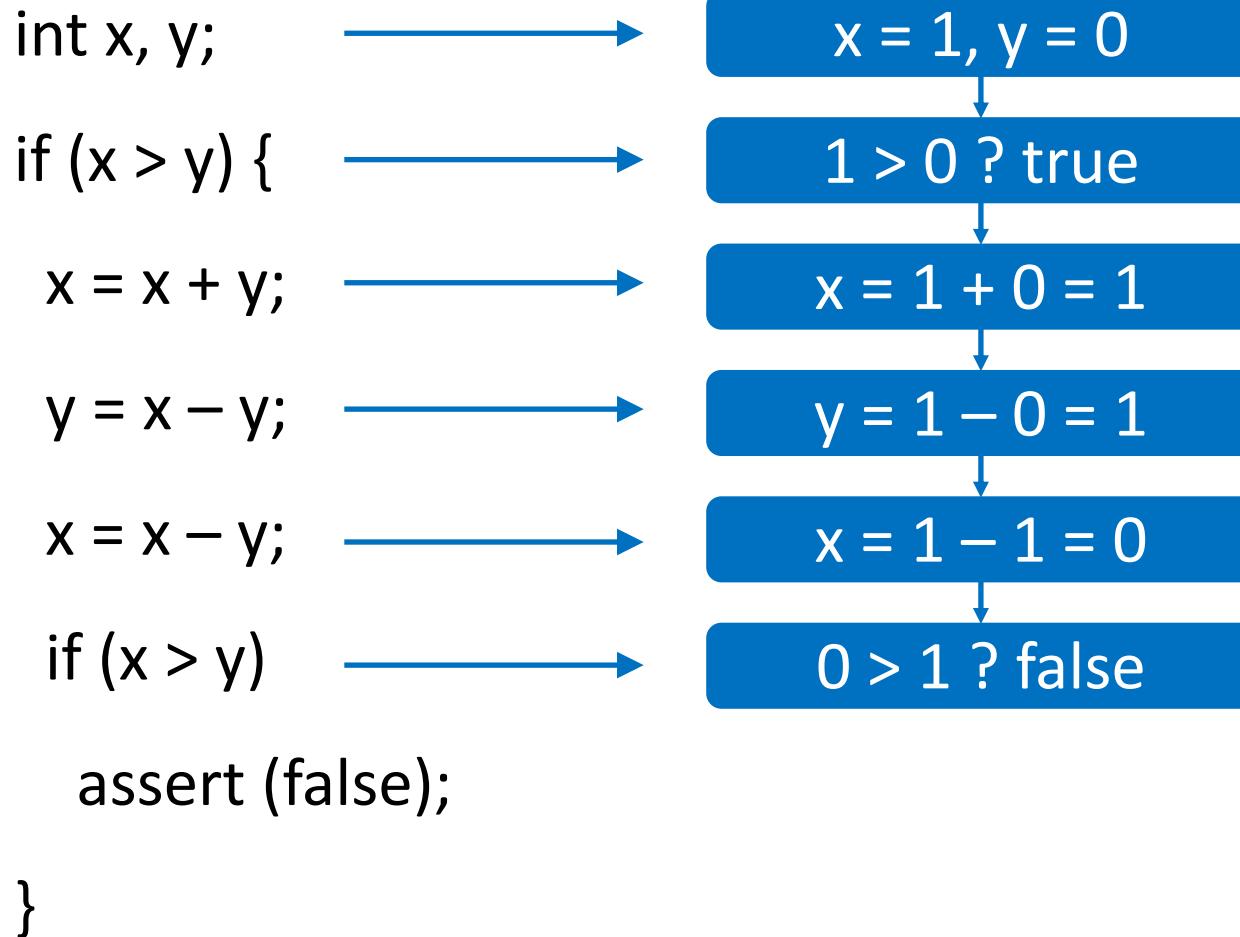


# Program Validation Approaches



# Symbolic Execution

# Example of Standard (Concrete) Execution

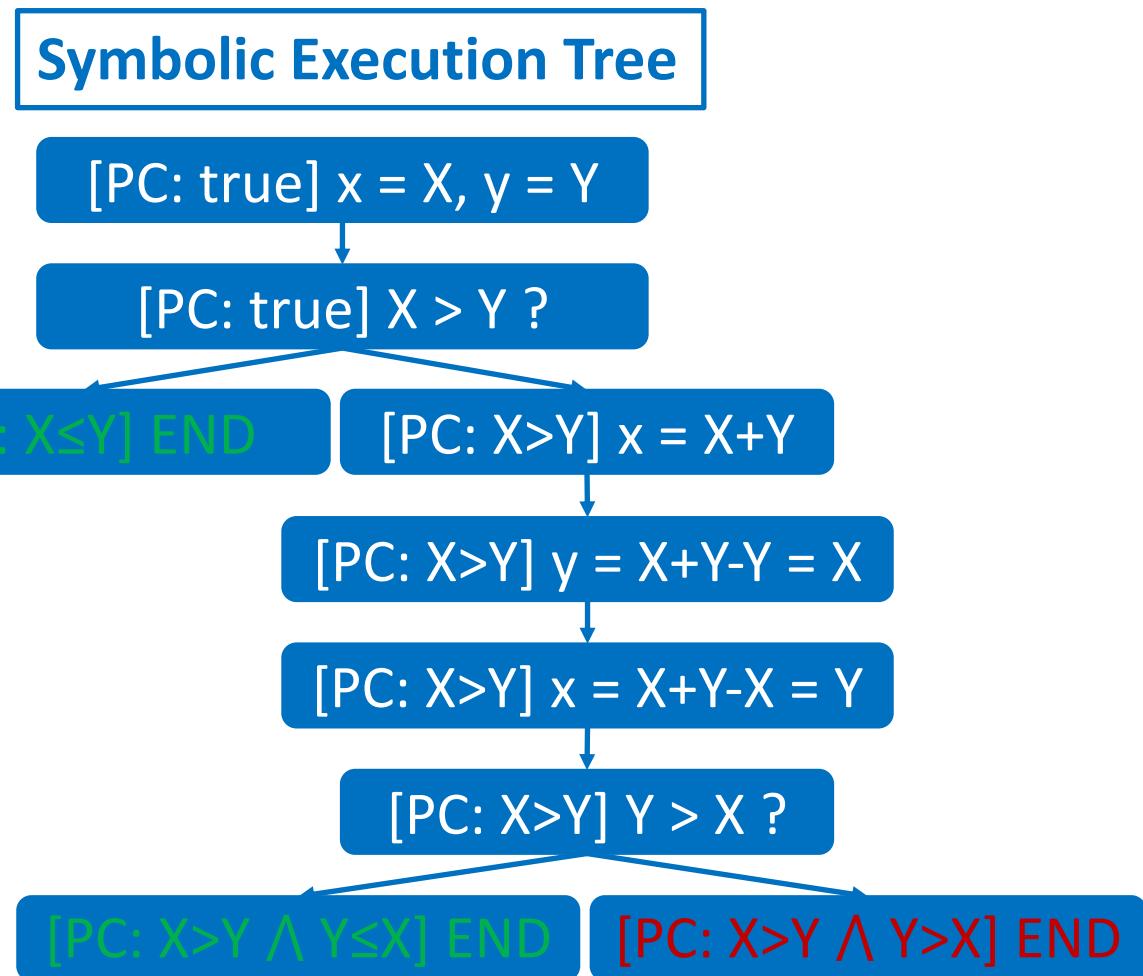


# Symbolic Execution

- Analysis of programs with unspecified inputs
  - Execute a program on **symbolic** inputs
- Symbolic states represent **sets** of concrete states
- For each path, build a **path condition**
  - Condition on inputs – for the execution to follow that path
  - Check path condition satisfiability – explore only feasible paths
- Symbolic state
  - Symbolic expressions for variables
  - Path condition
  - Program counter
- Tools: KLEE (C), JavaPath Finder (Java), ...

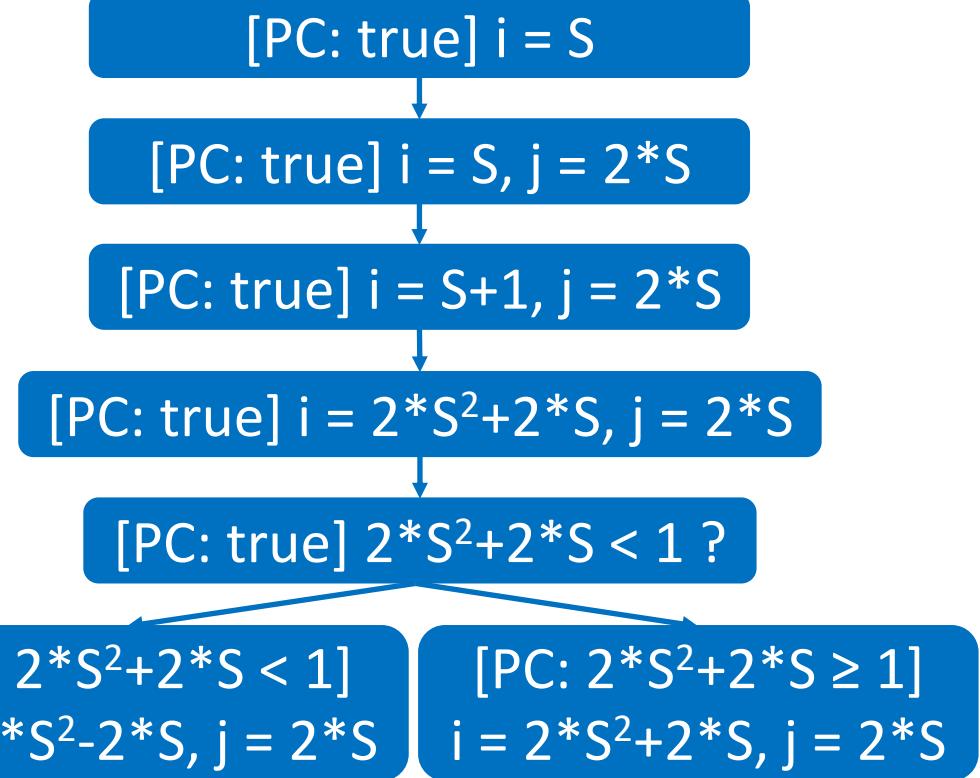
# Example of Symbolic Execution

```
int x, y;  
if (x > y) {  
    x = x + y;  
    y = x - y;  
    x = x - y;  
    if (x > y)  
        assert (false);  
}
```



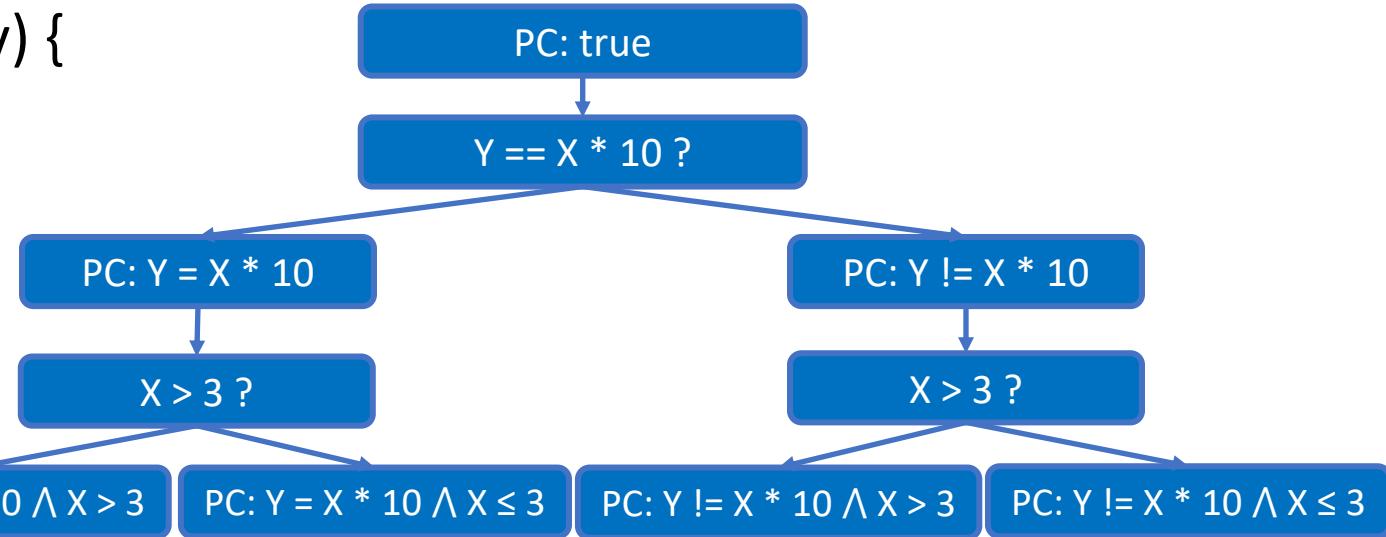
# Exercise: Symbolically Execute the Program

int i; →  
int j = 2 \* i; →  
i = i++; →  
i = i \* j →  
if (i < 1) →  
i = -i; →



# Exercise: Symbolically Execute the Program

```
void test(int x, int y) {  
    if (y == x*10) ... ;  
    else ... ;  
    if (x > 3) ... ;  
    else ... ;  
}
```



- How can we check the satisfiability of path conditions?
  - Constraint Solving
- How can we explore the large space of symbolic execution tree?
  - Searching Strategy

# Constraint Solving

- Symbolic execution was proposed in 1976
  - L. A. Clarke, “A System to Generate Test Data and Symbolically Execute Programs”, IEEE Transactions on Software Engineering
- Become popular only in recent years due to advancement in constraint solving techniques
  - SAT Solver
  - SMT Solver
  - ...

# Boolean Satisfiability Problem

- **Boolean Satisfiability** (often abbreviated **SAT**) is the problem of determining if there exists an assignment that satisfies a given Boolean formula
- Consider the formula  $(a \vee b) \wedge (\neg a \vee \neg c)$ 
  - The assignment **b = true** and **c = false** satisfies the formula
- History
  - SAT is shown to be NP-complete in 1971 (Stephen Cook)
  - Breakthrough occurred in 1990s
  - Advanced SAT solver handles problem instances with millions of Boolean variables
  - Annual competition: <http://www.satcompetition.org/>

# SAT Solver Progress

Solvers have continually improved over time



Instance	Posit' 94
ssa2670-136	40.66s
bf1355-638	1805.21s
pret150_25	>3000s
dubois100	>3000s
aim200-2_0-no-1	>3000s
2dix_..._bug005	>3000s
c6288	>3000s

# Exercise: Applying SAT Solving

- Consider the following constraints
  - John can only meet either on Monday, Wednesday or Thursday; Catherine cannot meet on Wednesday; Anne cannot meet on Friday; Peter cannot meet neither on Tuesday nor on Thursday
- Question: when can the meeting take place?
  - John =  $(\text{Mon} \vee \text{Wed} \vee \text{Thu}) \wedge \neg \text{Tue} \wedge \neg \text{Fri}$
  - Catherine =  $(\text{Mon} \vee \text{Tue} \vee \text{Thu} \vee \text{Fri}) \wedge \neg \text{Wed}$
  - Anne =  $(\text{Mon} \vee \text{Tue} \vee \text{Wed} \vee \text{Thu}) \wedge \neg \text{Fri}$
  - Peter =  $(\text{Mon} \vee \text{Wed} \vee \text{Fri}) \wedge \neg \text{Tue} \wedge \neg \text{Thu}$
  - John  $\wedge$  Catherine  $\wedge$  Anne  $\wedge$  Peter ?
  - **Monday!**

# SAT Extension: SMT

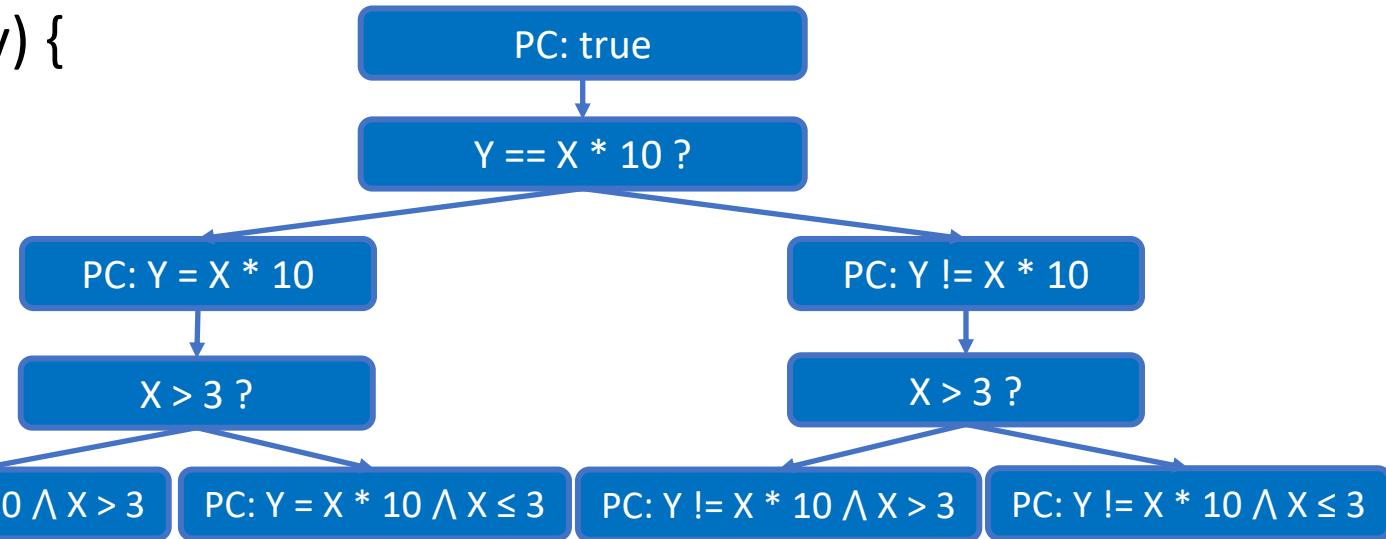
- **Satisfiability Modulo Theories (SMT)** enrich quantified Boolean formulas (QBF) with linear constraints, uninterpreted functions, etc.
  - QBF: Is there a solution  $\{x, y, z\}$  satisfying  $\forall x \forall y \exists z (x \vee y \vee z) \wedge (\neg x \vee \neg y \vee \neg z)$ ?
  - Linear constraints: Is there a solution  $\{x, y, z\}$  satisfying  $3x+2y \geq 5z \wedge 5z = 2x$ ?
- Very efficient SMT solvers are now available that can handle many such kinds of constraints.
- Annual competition: <http://www.smtcomp.org/>

# Optimizing SMT Queries

- Expression rewriting
  - Simple arithmetic simplifications ( $x * 0 \rightarrow 0$ )
  - Linear simplification ( $2 * x - x \rightarrow x$ )
- Constraint set simplification
  - $x < 10 \wedge x = 5 \rightarrow x = 5$
- Implied value concretization
  - $x + 1 = 10 \rightarrow x = 9$
- Constraint independence
  - $c > a \wedge b \leq 5 \wedge a \geq d + 10 \rightarrow c > a \wedge a \geq d + 10, b \leq 5$

# Search Strategy – Breadth First Search

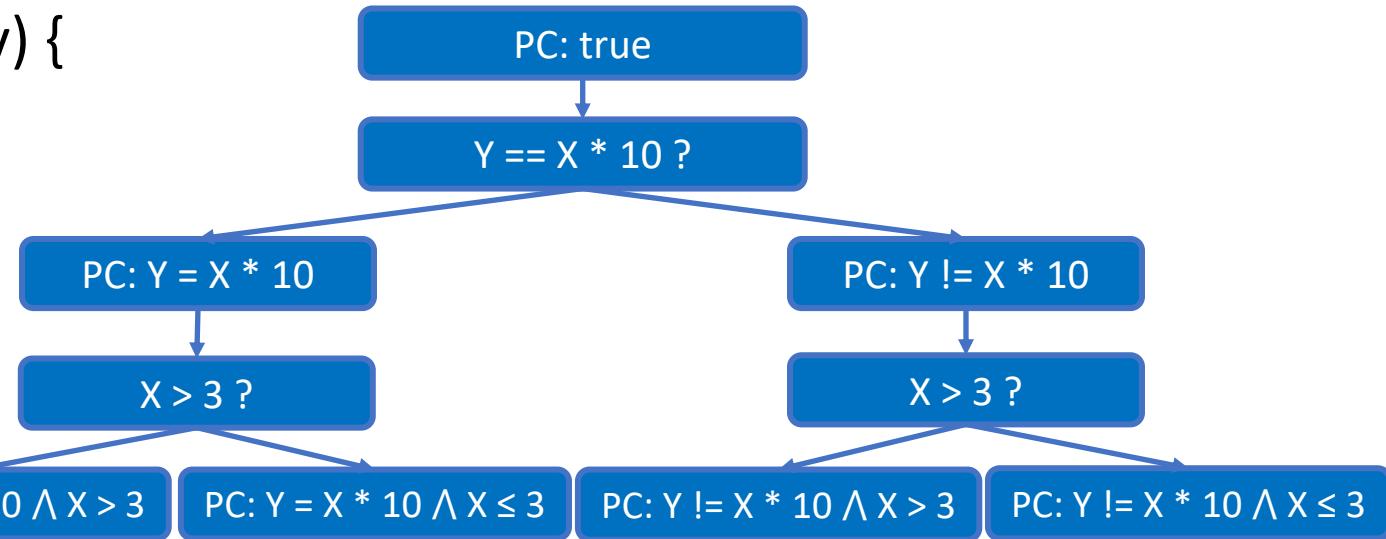
```
void test(int x, int y) {  
    if (y == x*10) ... ;  
    else ... ;  
    if (x > 3) ... ;  
    else ... ;  
}
```



**Breadth first search is very slow to determine properties for a path if there are many branches**

# Search Strategy – Depth First Search

```
void test(int x, int y) {  
    if (y == x*10) ... ;  
    else ... ;  
    if (x > 3) ... ;  
    else ... ;  
}
```



**Depth first search can stuck at somewhere in a loop**

# More Search Strategies

- Random Search
  - Pick next path to explore uniformly at random
  - Hard to reproduce, probably good to use pseudo-randomness based on seed
- Coverage Guided Search
  - Select paths likely to hit the new statements
  - Favor paths on recently covering new statements
  - Errors are often in hard-to-reach parts of the program, this strategy tries to reach everywhere
- Generational Search
  - Generation 0: pick one path at random, run to completion
  - Generation 1: take the path from generation 0, negate one branch condition on the path to yield a new path prefix, find a solution for that path prefix, and then take the resulting path
  - ...

# Applications of Symbolic Execution

- Generating test inputs
- Detecting infeasible paths
- Finding bugs and vulnerabilities
  - E.g., divide by zero bugs
- Proving two code segments are equivalent
- Generating program invariants
- Debugging
- Repairing programs
- ...

# Challenges – Path Explosion

- Exponential in branch structures

```
1. int a = input(), b = input();  
2. Int c = input(), d = input();  
3. if (a) {...} else {...}  
4. if (b) {...} else {...}  
4. if (c) {...} else {...}  
4. if (d) {...} else {...}
```

4 variables and  $2^4 = 16$  program paths!

- Even worse, loops on symbolic variables

```
1. int x = input();  
2. while (x > 0) {  
3.     x++;  
4.     assert(...);  
5. }
```

Potentially  $2^{31}$  paths through the loop!

# Challenges – Constraint Solving

- The state-of-the-art: support theories on linear integer arithmetic, bit vectors, string, etc., but have limited support for complex constraints and are not particularly scalable
  - Non-linear constraints, e.g.,  $\sin()$ ,  $\cos()$
  - Float-point constraints, e.g.,  $2.5 * x - 2.9 * y > 1.1$
  - ...

```
1. int x = input(), y = input(), z = input();  
2. if (5x^63 + 7x^12 = 78y^2 + z) {  
3.     assert(false);  
4. }
```

# Challenges – Complex Code and Environment Dependencies

- Complex code
  - Data structures and pointers
- Environment dependencies
  - Native calls
  - System calls
  - Library calls
  - File operations
  - Network events
  - ...

# Potential Solutions

- Path explosion
  - Heuristic-based search strategies
  - Limit the exploration depth (but lead to incompleteness)
  - ...
- Constraint solving
  - Meta-heuristic-based solvers (but lead to imprecise)
  - ...
- Complex code and environment dependencies
  - Build simple versions of library calls
  - Simulate system calls
  - Concretization
  - ...

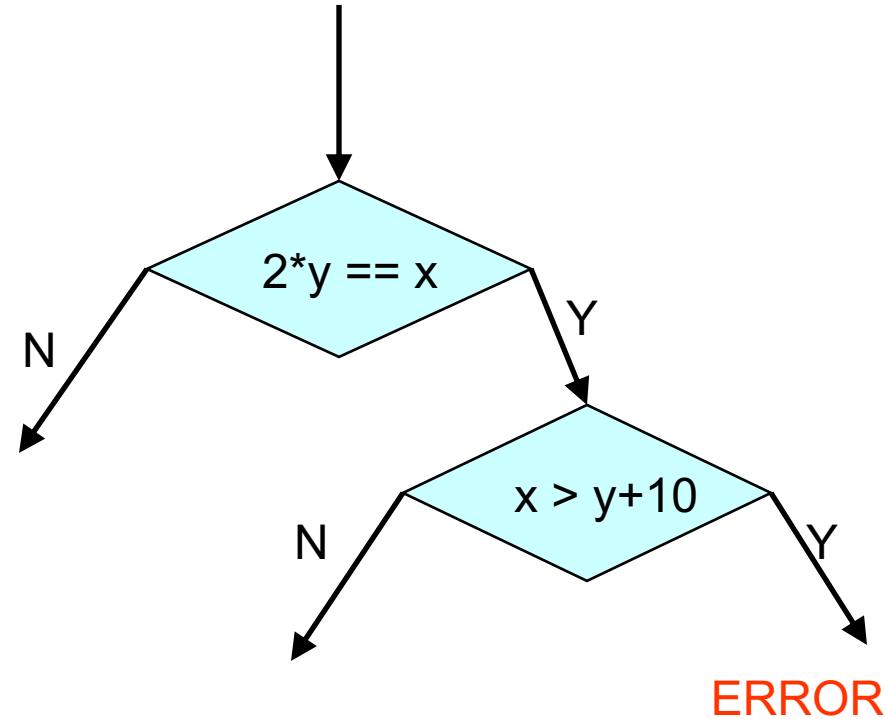
# Concolic Execution

# Concolic (Concrete + Symbolic) Execution

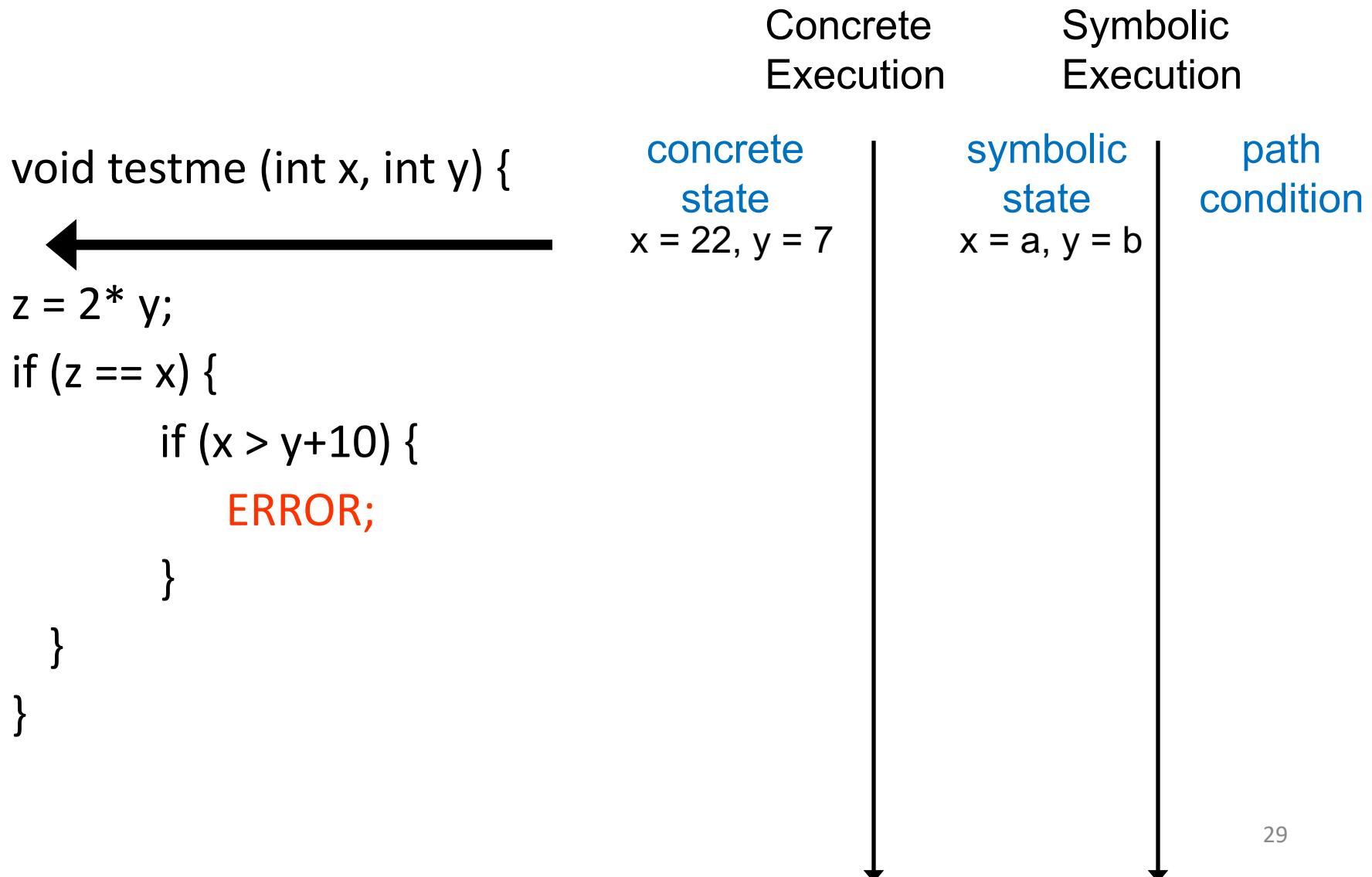
- Execute program with both concrete and symbolic inputs (also known as dynamic symbolic execution)
- The intention is to visit deep into the program execution tree
- Steps
  - Generate a random test input
  - Execute the program with the random test input and the symbolic input to collect the path constraint
  - Negate the last conjunct to obtain a constraint
  - Solve it to get test input to another path and execute the input

# Example

```
void testme (int x, int y)
{
    z = 2*y;
    if (z == x) {
        if (x > y+10) {
            ERROR;
        }
    }
}
```



# Example of Concolic Execution

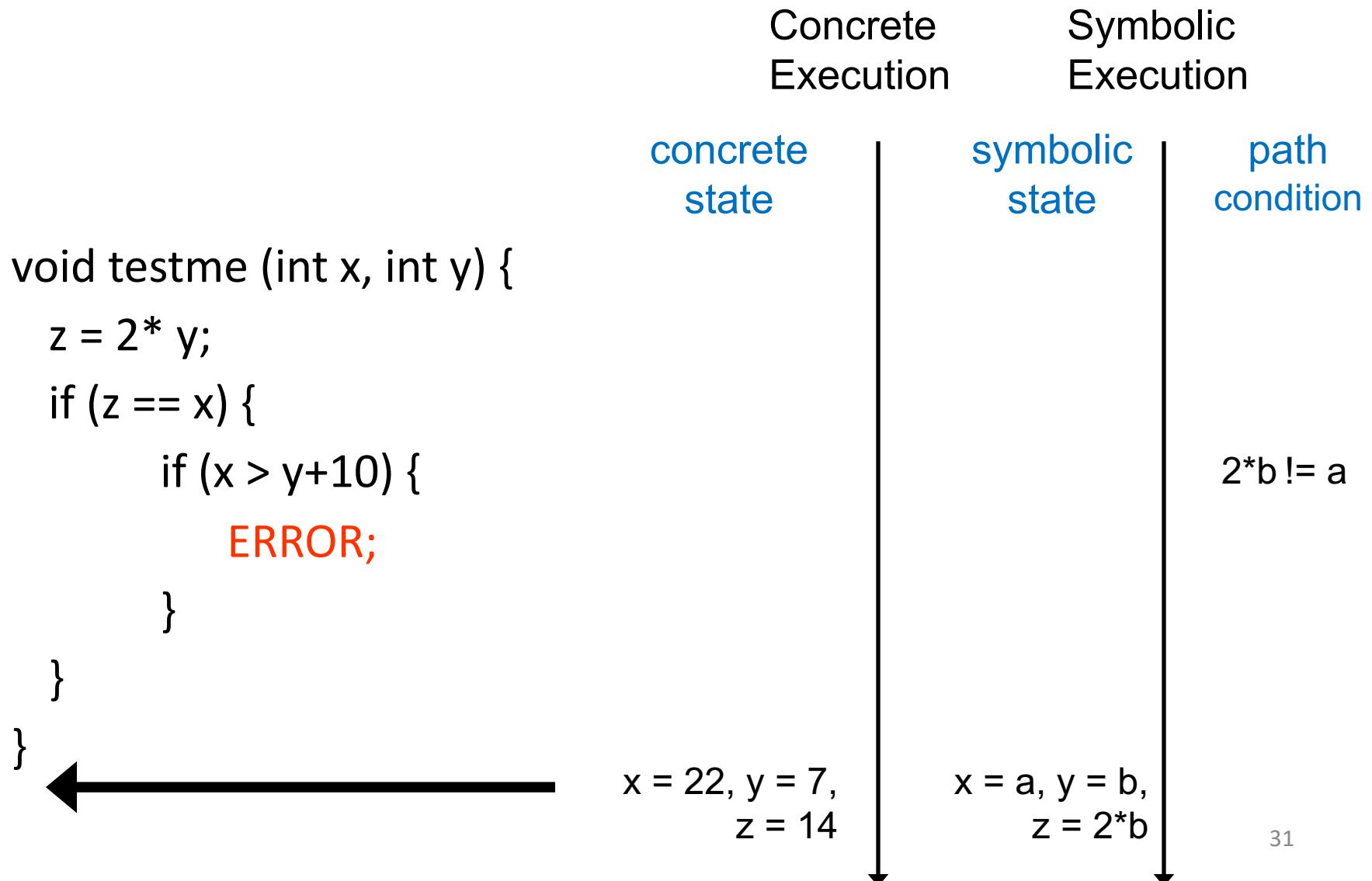


# Example of Concolic Execution

```
void testme (int x, int y) {  
    z = 2*y;  
    ←—————  
    if (z == x) {  
        if (x > y+10) {  
            ERROR;  
        }  
    }  
}
```

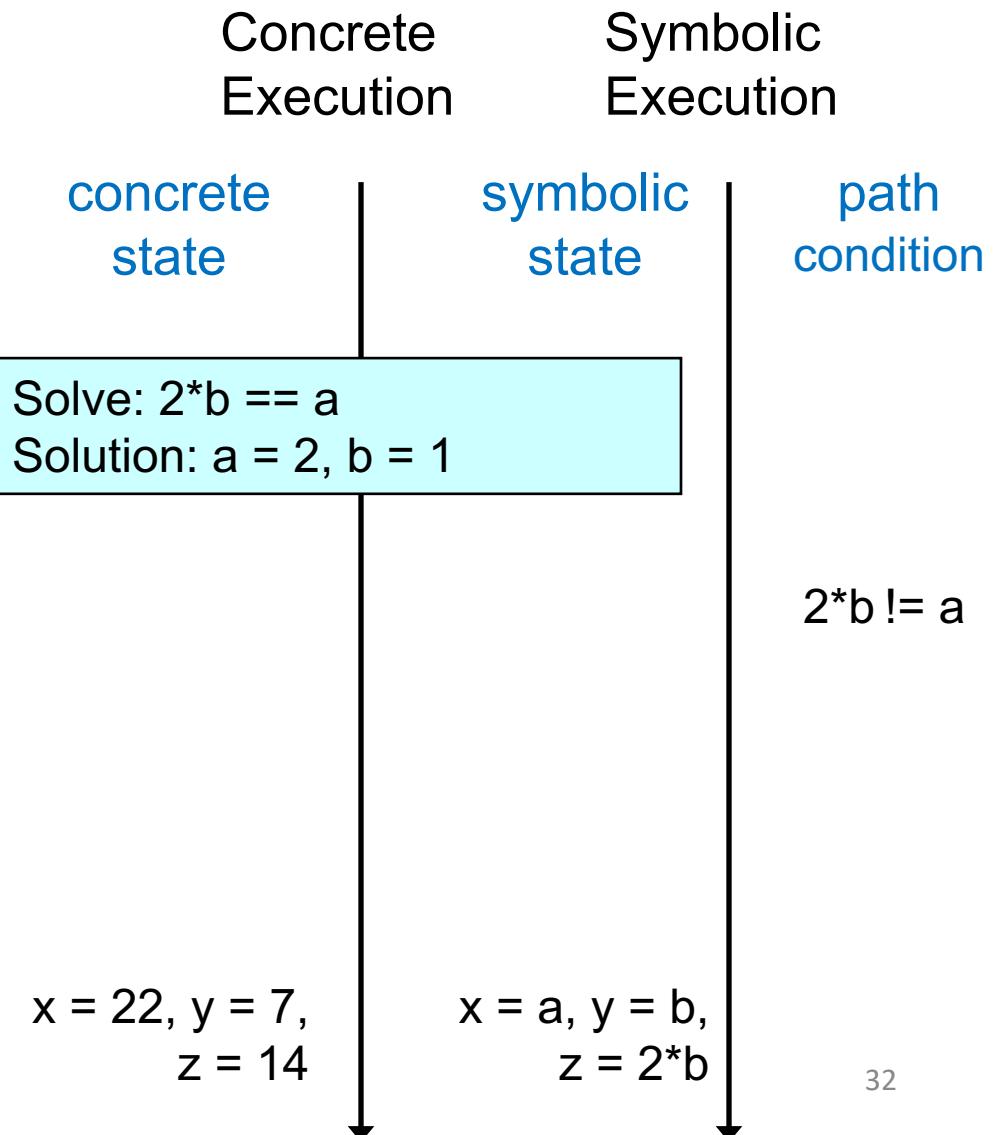
Concrete Execution	Symbolic Execution	path condition
concrete state  $x = 22, y = 7,$ $z = 14$	symbolic state  $x = a, y = b,$ $z = 2^*b$	

# Example of Concolic Execution

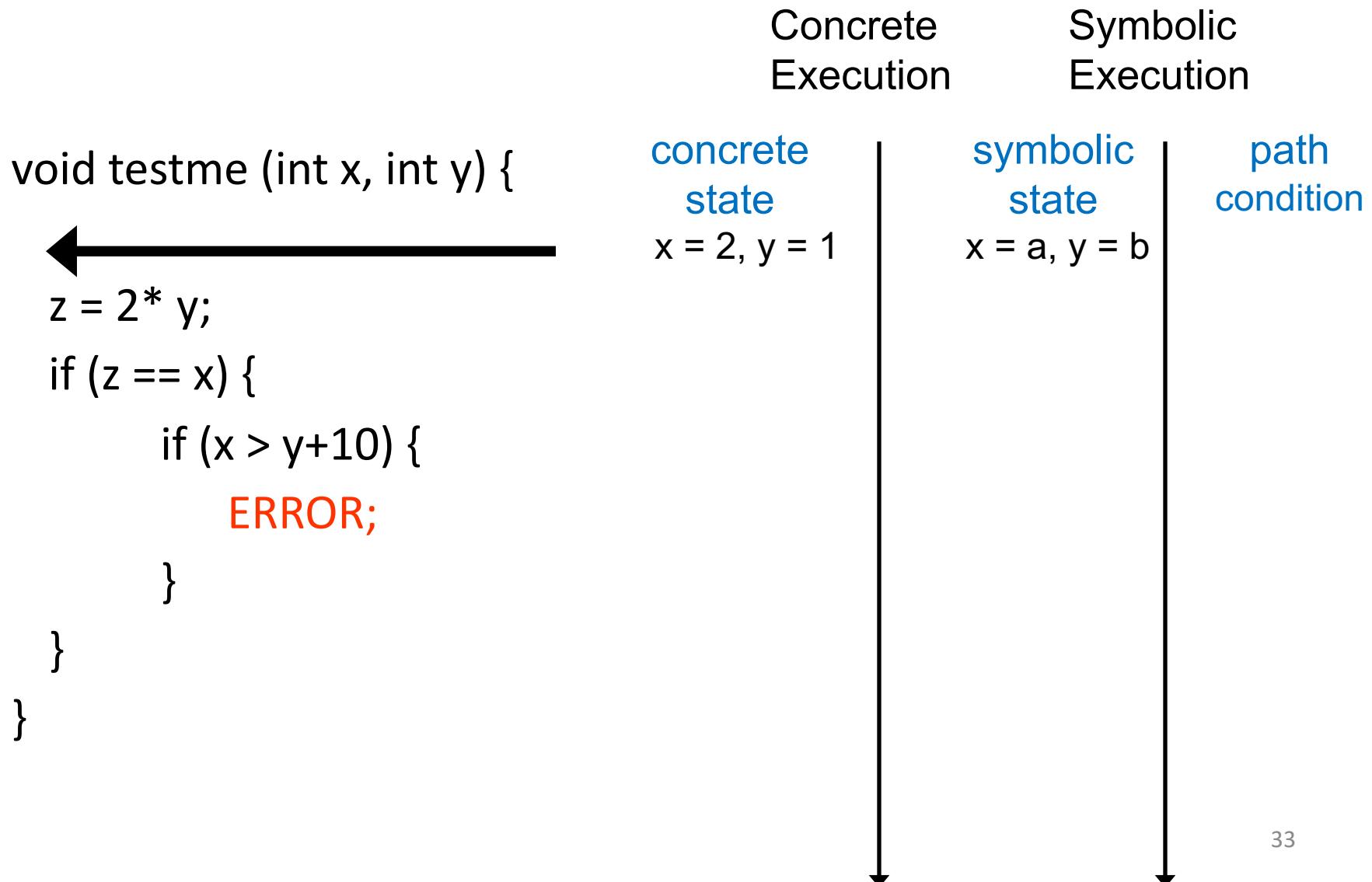


# Example of Concolic Execution

```
void testme (int x, int y) {  
    z = 2*y;  
    if (z == x) {  
        if (x > y+10) {  
            ERROR;  
        }  
    }  
}
```



# Example of Concolic Execution



# Example of Concolic Execution

```
void testme (int x, int y) {  
    z = 2*y;  
    ←—————  
    if (z == x) {  
        if (x > y+10) {  
            ERROR;  
        }  
    }  
}
```

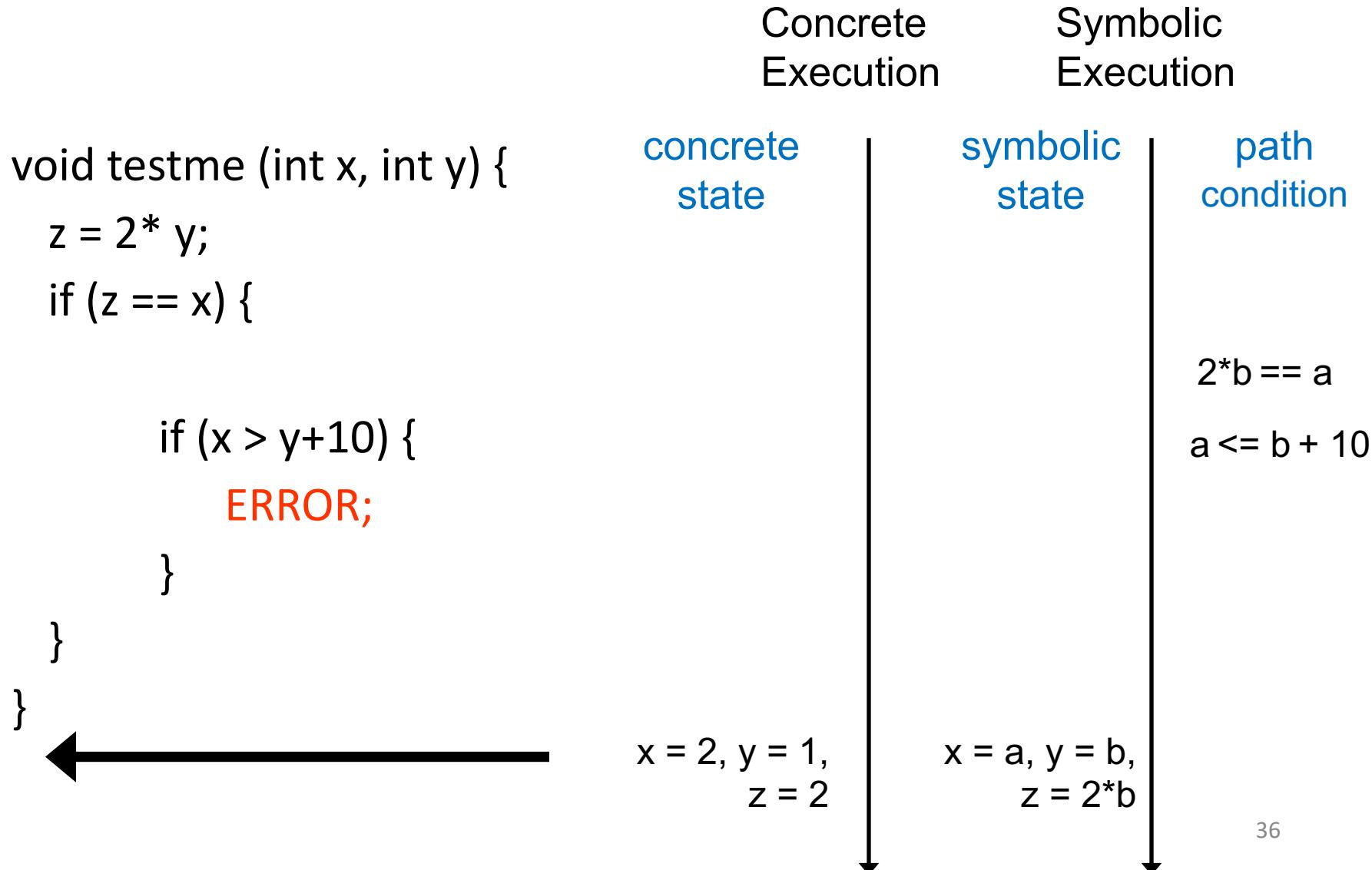
Concrete Execution	Symbolic Execution	path condition
concrete state $x = 2, y = 1, z = 2$	symbolic state $x = a, y = b, z = 2^*b$	

# Example of Concolic Execution

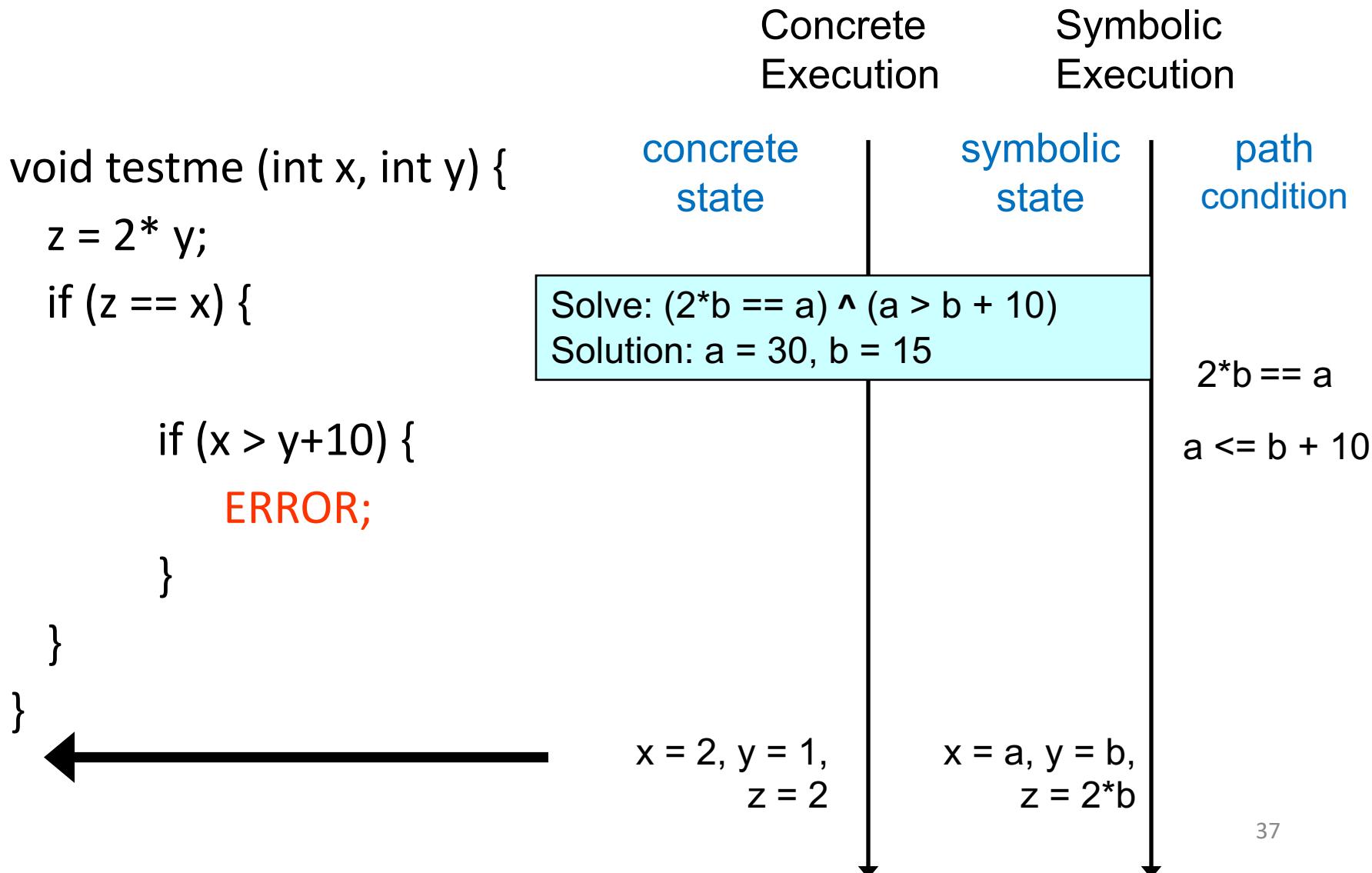
```
void testme (int x, int y) {  
    z = 2*y;  
    if (z == x) {  
        ←  
        if (x > y+10) {  
            ERROR;  
        }  
    }  
}
```

Concrete Execution	Symbolic Execution	path condition
concrete state  $x = 2, y = 1, z = 2$	symbolic state  $x = a, y = b, z = 2^b$	$2^b == a$

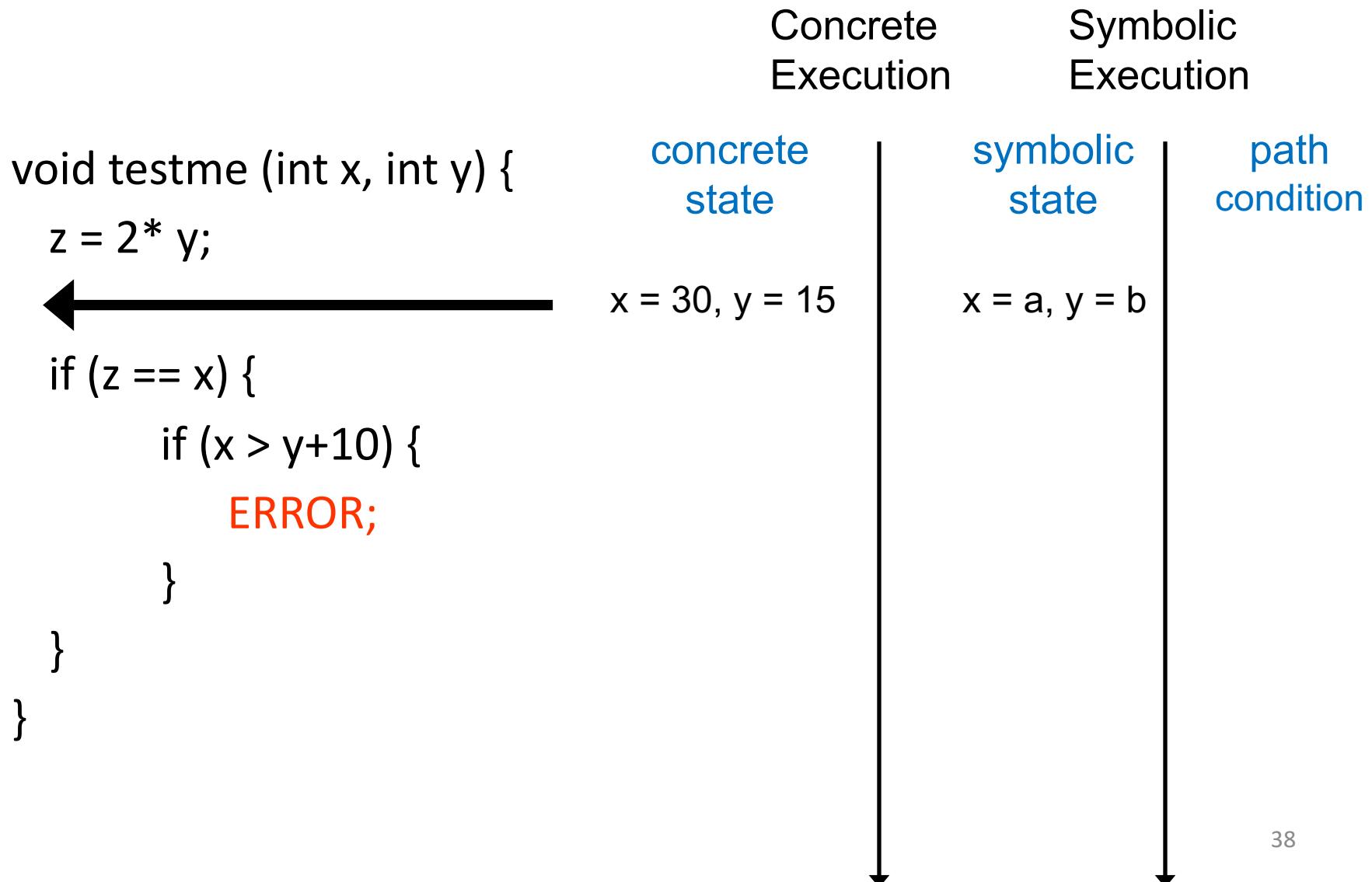
# Example of Concolic Execution



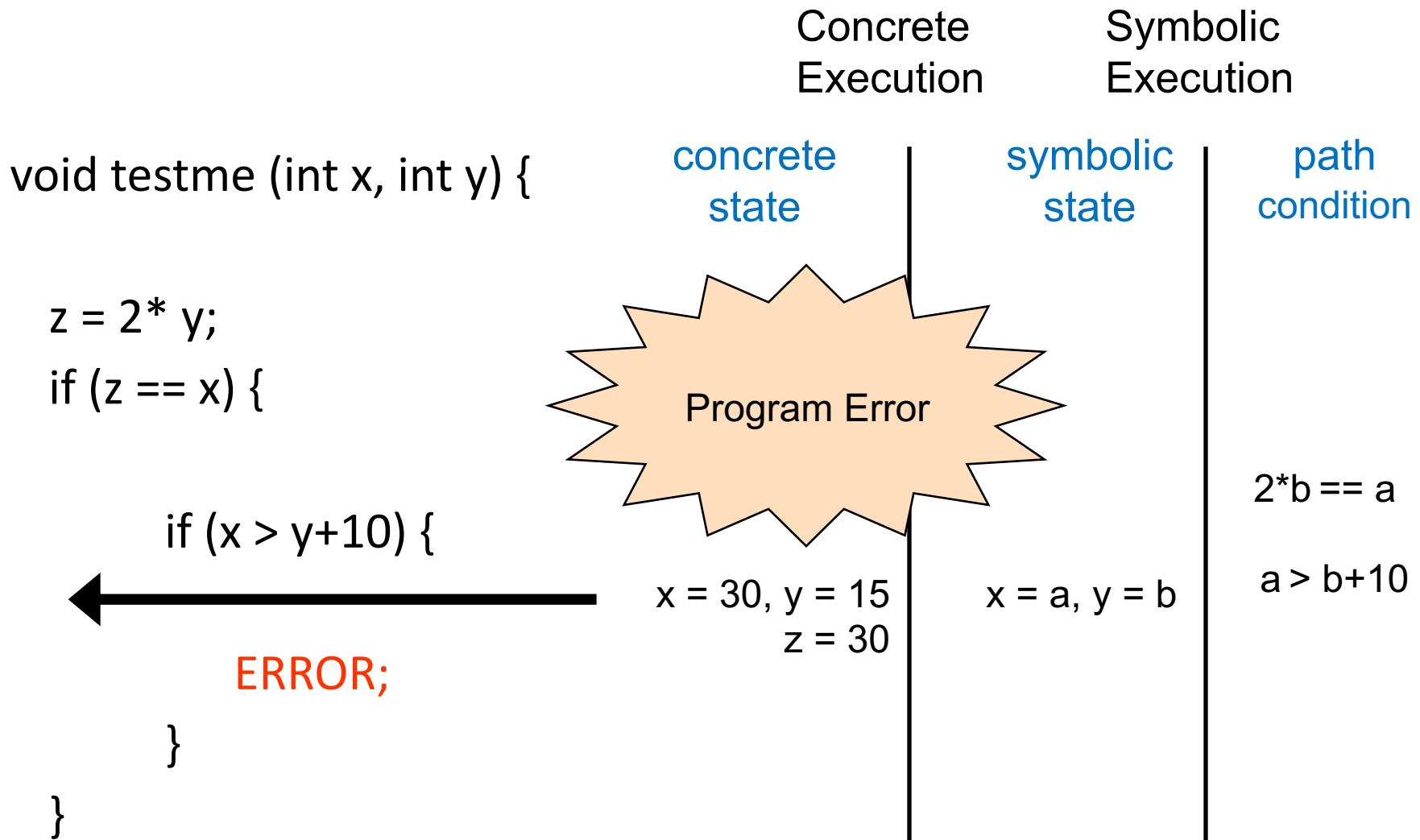
# Example of Concolic Execution



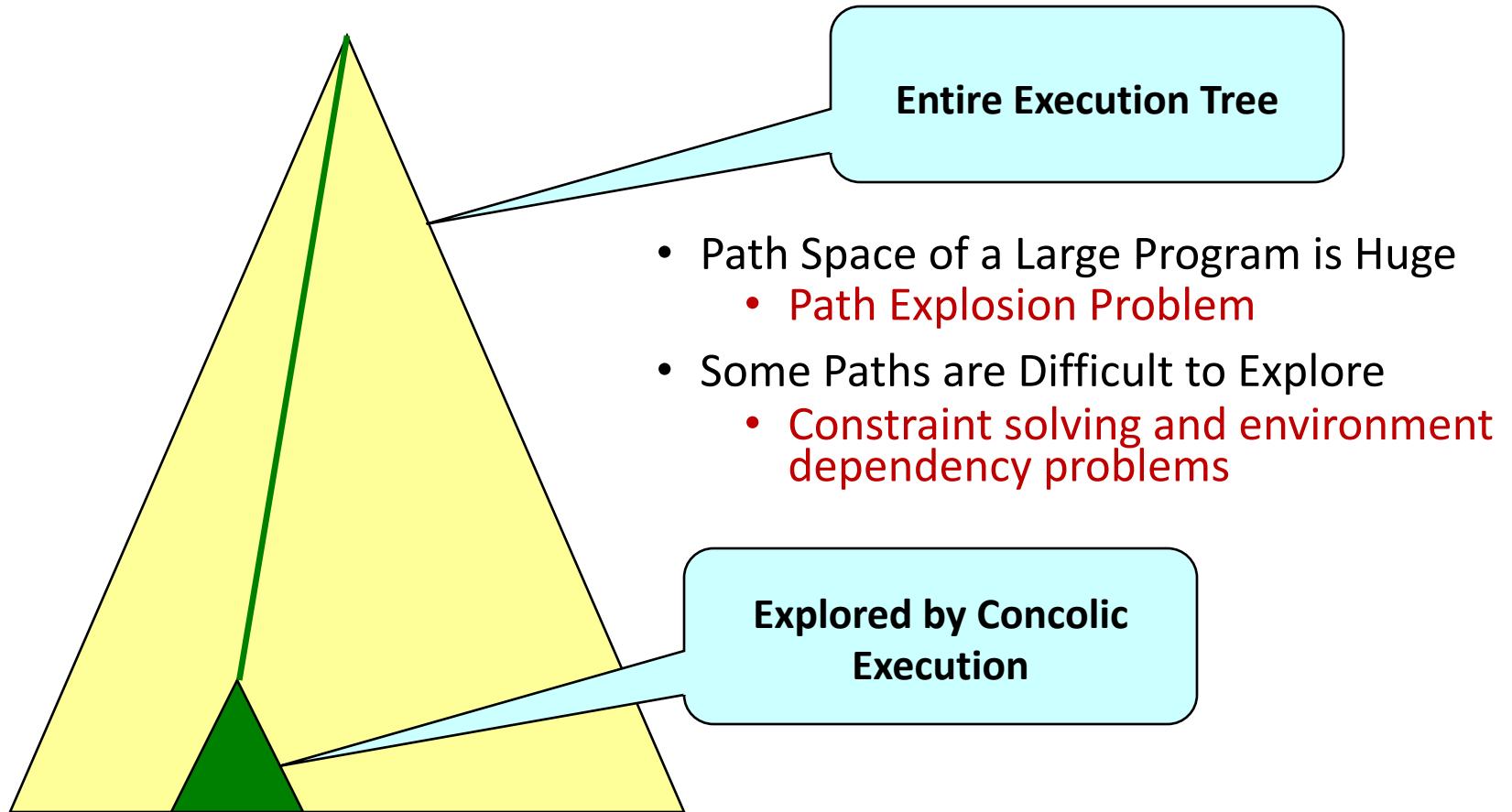
# Example of Concolic Execution



# Example of Concolic Execution



# Bird's View

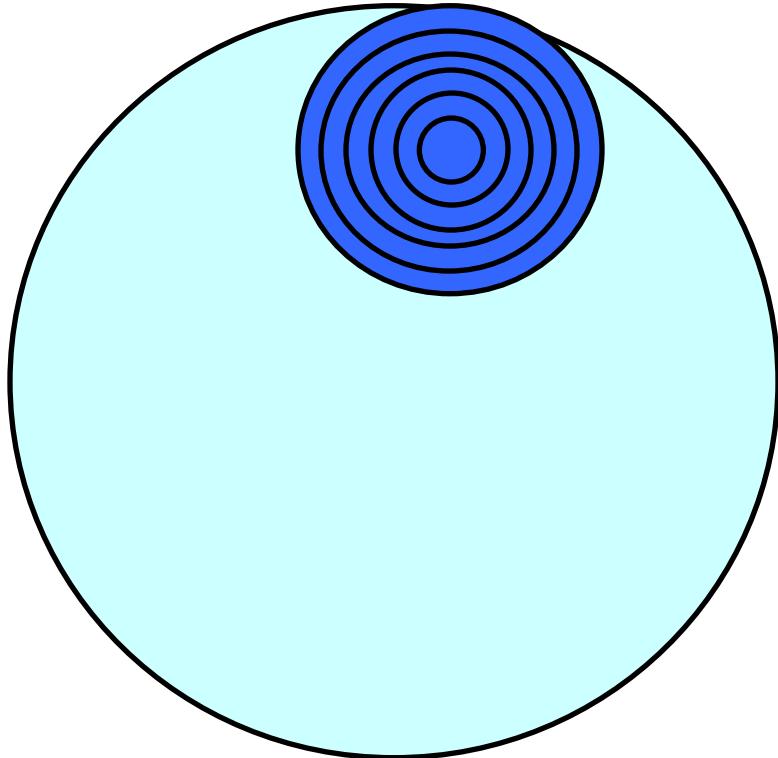


# An Example

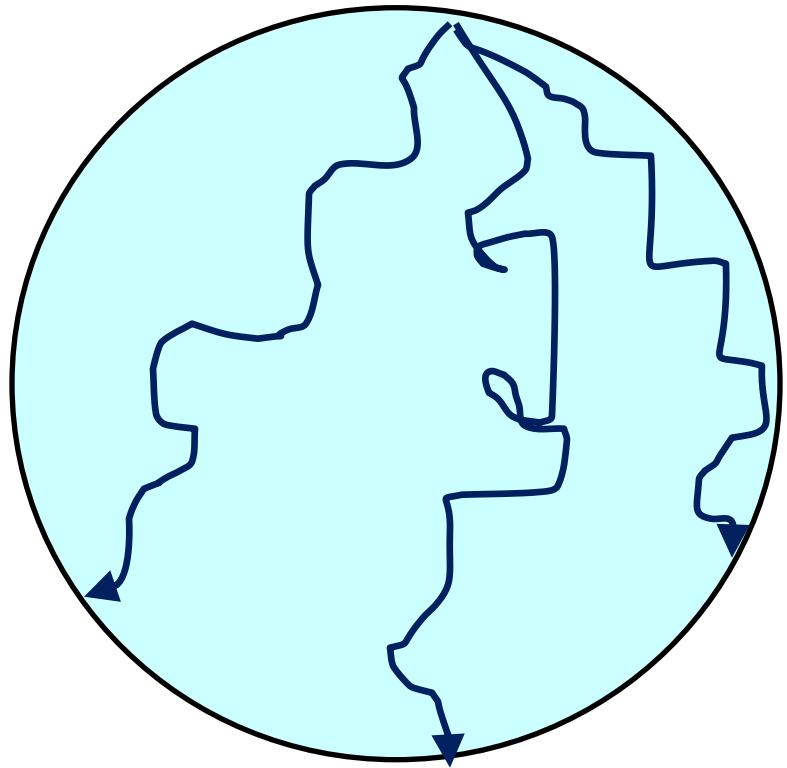
```
1. foobar(int x, int y){  
2.   if (x*x*x > 0){  
3.     if (x>0 && y==10){  
4.       abort();  
5.     }  
6.   } else {  
7.     if (x>0 && y==20){  
8.       abort();  
9.     }  
10. }  
11. }
```

- Random testing gets stuck at line 3
- Symbolic execution gets stuck at line 2
- Concolic execution finds the only error at line 4
  - Use concrete value for x

# Concolic Execution vs. Random Testing



**Concolic: Broad, Shallow**



**Random: Narrow, Deep**

# An Example

```
Example ( ) {  
1: state = 0;  
2: while(1) {  
3:   s = input();  
4:   c = input();  
5:   if(c==':' && state==0)  
       state=1;  
6:   else if(c=='\n' && state==1)  
       state=2;  
7:   else if (s[0]=='I' &&  
             s[1]=='C' &&  
             s[2]=='S' &&  
             s[3]=='E' &&  
             state==2) {  
       COVER_ME;  
    }  
  }  
}
```

COVER\_ME can be hit on an input

- s = "ICSE"
- c = ':' '\n'

Similar code in

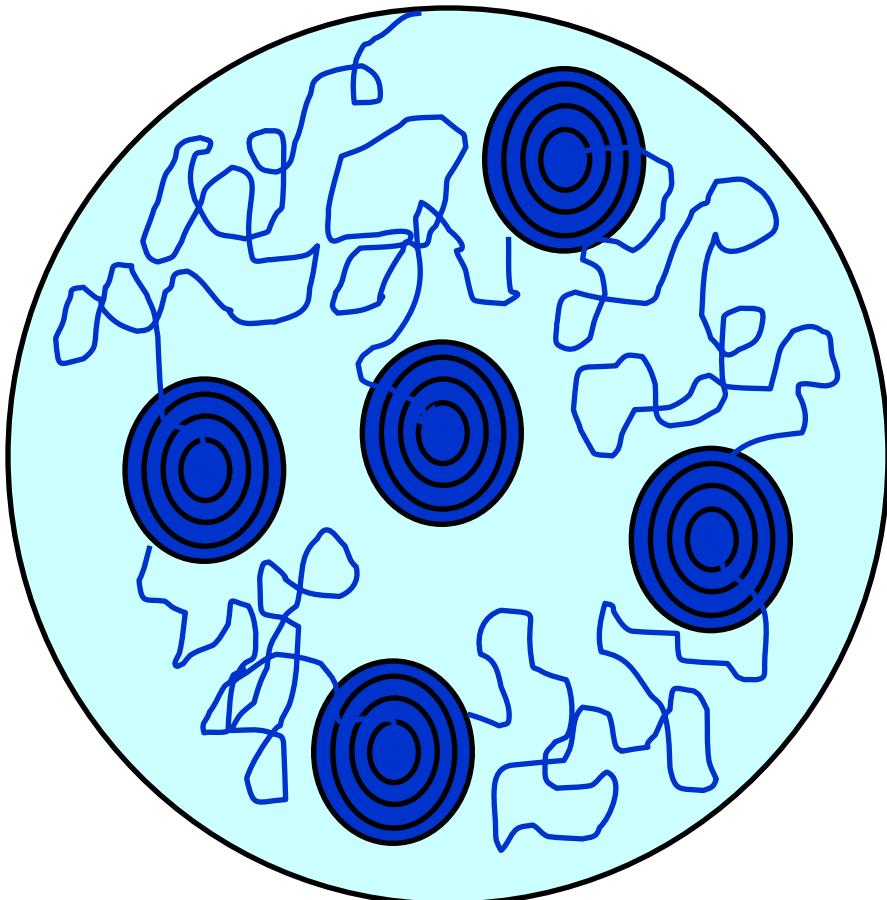
- Text editors (vi)
- Parsers (lexer)
- Event-driven programs (GUI)

# An Example

```
Example ( ) {  
1: state = 0;  
2: while(1) {  
3:   s = input();  
4:   c = input();  
5:   if(c==':' && state==0)  
       state=1;  
6:   else if(c=='\n' && state==1)  
       state=2;  
7:   else if (s[0]=='I' &&  
             s[1]=='C' &&  
             s[2]=='S' &&  
             s[3]=='E' &&  
             state==2) {  
       COVER_ME;  
    }  
  }  
}
```

- **Pure random testing** can get to state = 2, but difficult to get 'ICSE'
- Probability  $1/(2^8)^6 = 3*10^{-15}$
- **Concolic testing** can generate 'ICSE' but explores many paths to get to state = 2

# Hybrid Concolic Testing



```
while (not required coverage) {  
    while (not saturation)  
        perform random testing;  
    Checkpoint;  
    while (not increase in coverage)  
        perform concolic testing;  
    Restore;  
}
```

Interleave random testing and  
concolic testing to increase coverage

Deep and broad search

# Hybrid Concolic Testing

```
Example ( ) {  
1: state = 0;  
2: while(1) {  
3:   s = input();  
4:   c = input();  
5:   if(c==':' && state==0)  
       state=1;  
6:   else if(c=='\n' && state==1)  
       state=2;  
7:   else if (s[0]=='I' &&  
             s[1]=='C' &&  
             s[2]=='S' &&  
             s[3]=='E' &&  
             state==2) {  
       COVER_ME;  
    }  
}
```

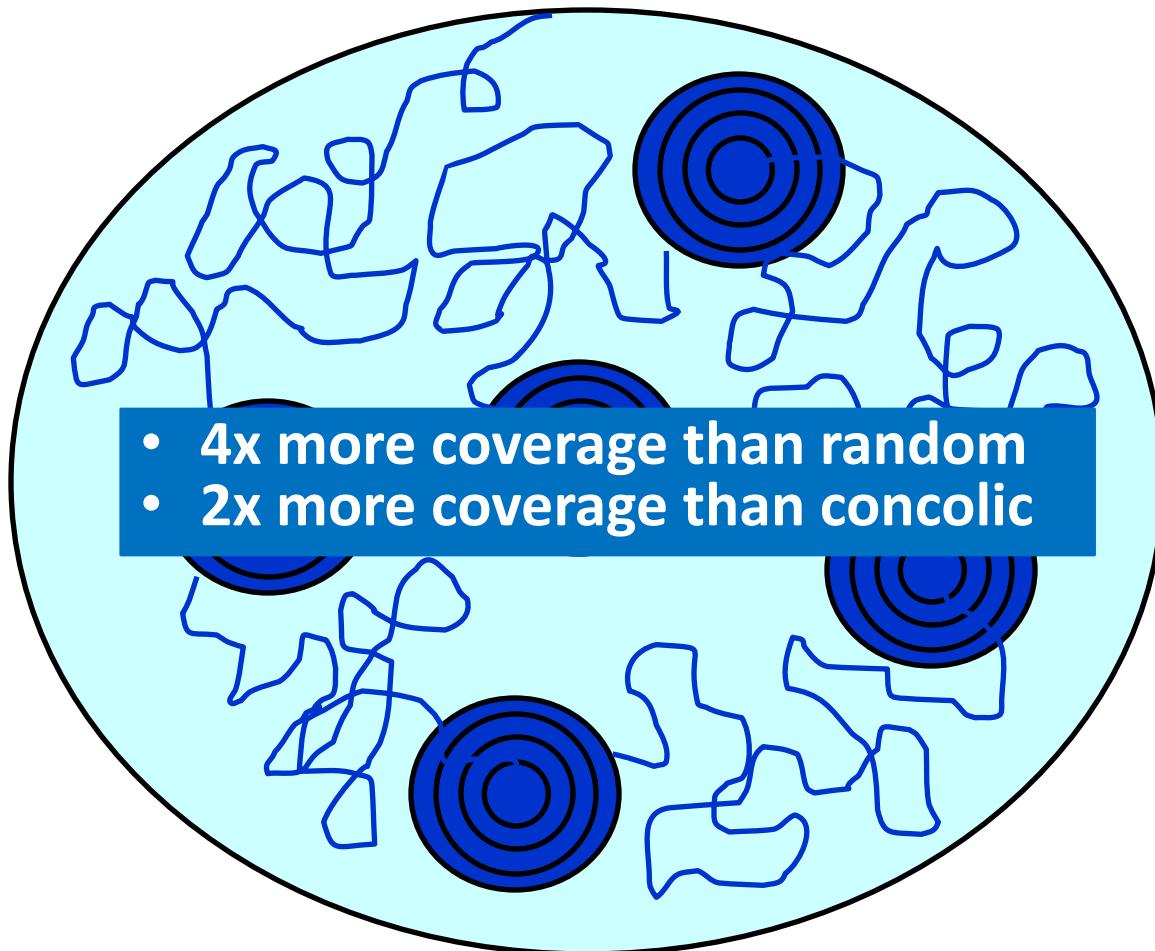
## Random Testing Phase

- '\$', '&', '!', '6', ':', '%', '^', '\n', 'x', '~' ...
- Saturate after many (~10000) iterations
- In less than 1 second
- **COVER\_ME** is not reached

## Concolic Testing Phase

- s[0]='I', s[1]='C', s[2]='S', s[3]='E'
- Reach **COVER\_ME**

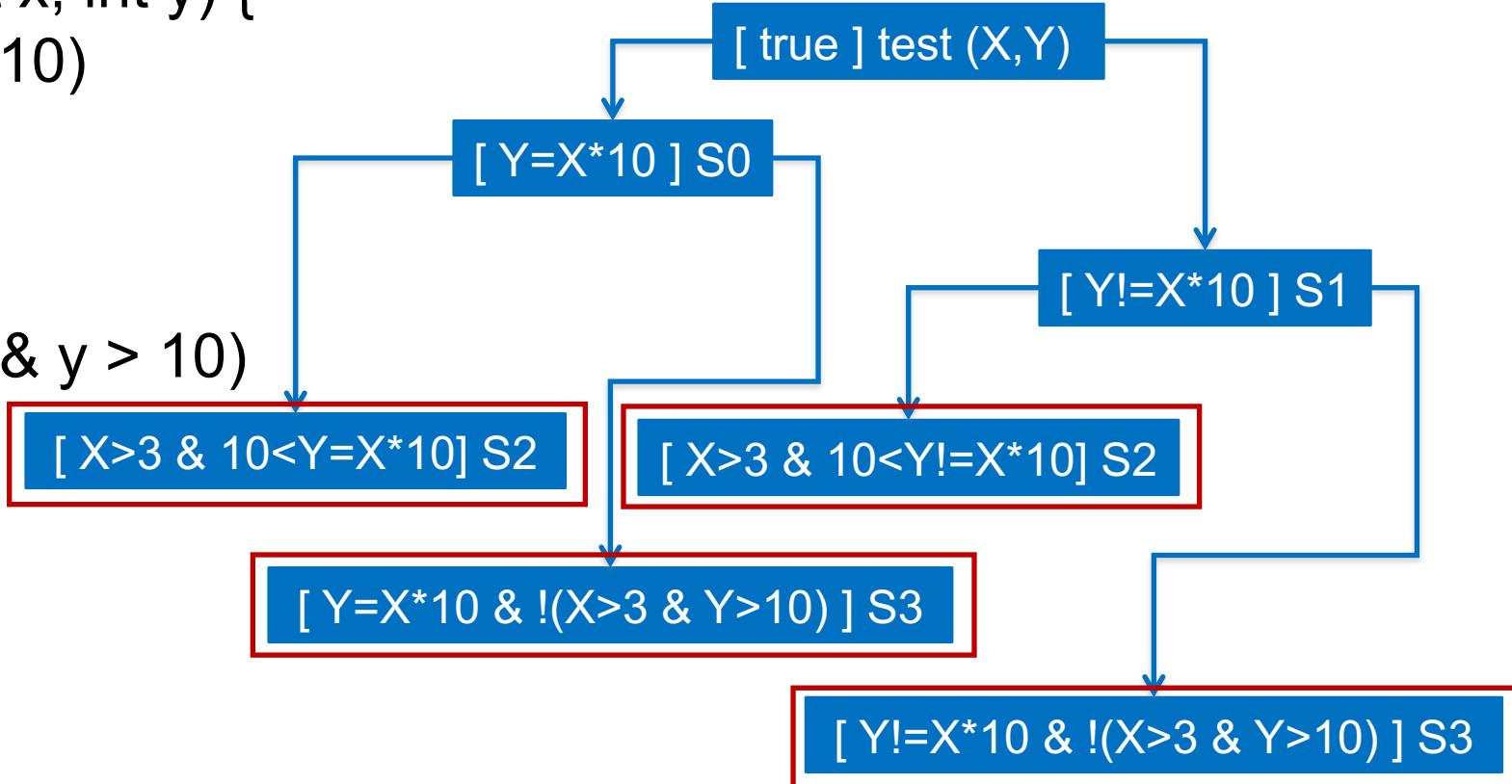
# Hybrid Concolic Testing



# Probabilistic Symbolic Execution

# Symbolic Execution

```
void test(int x, int y) {  
    if (y == x*10)  
        S0;  
    else  
        S1;  
    if (x > 3 && y > 10)  
        S2;  
    else  
        S3;  
}
```



What is the probability of reaching these state?

# Probabilistic Symbolic Execution



$\gamma =$



$$PC = C_1 \ \& \ C_2 \ \& \ \dots \ \& \ C_n$$

PC solutions

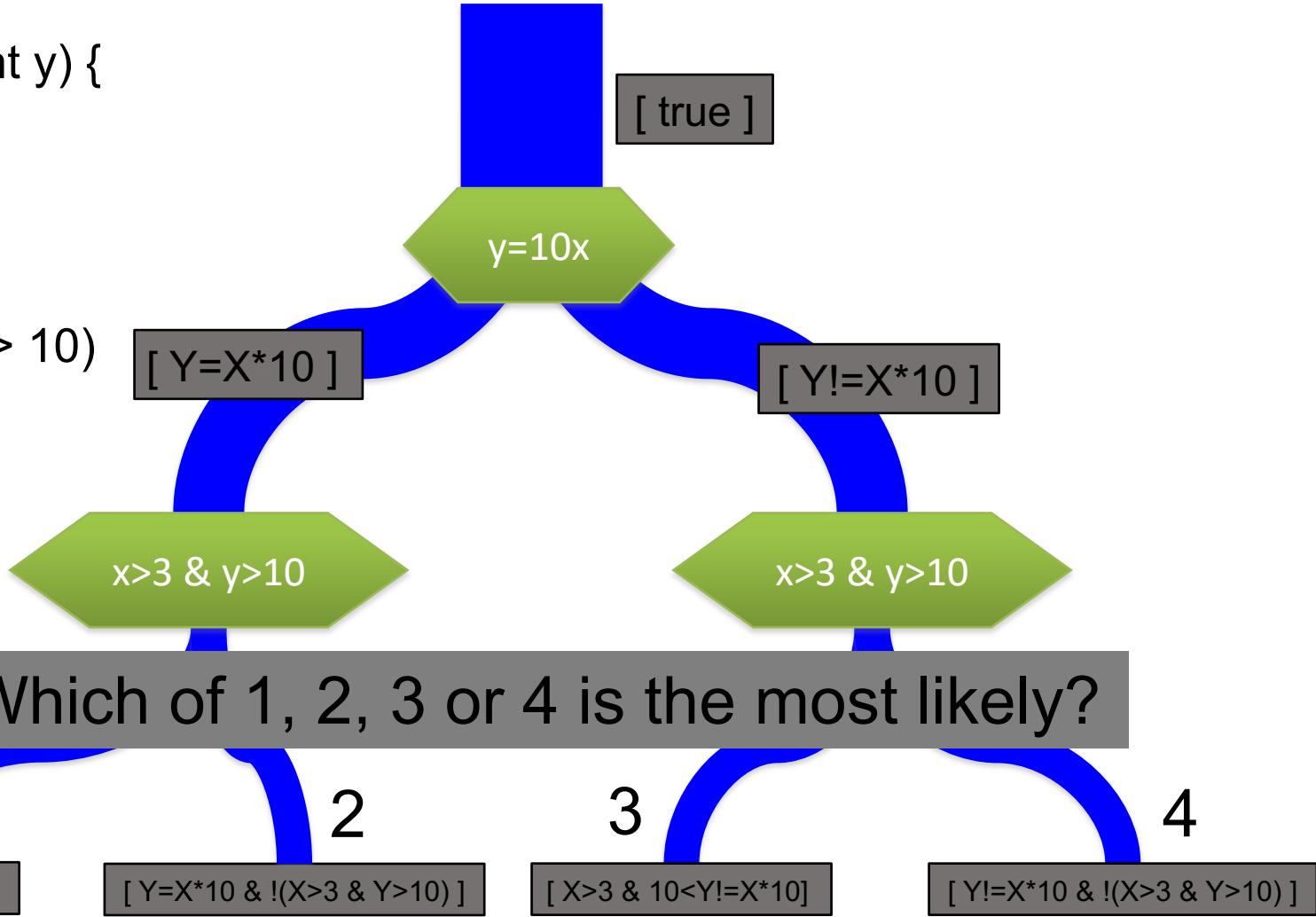
PC feasibility



$> 0$

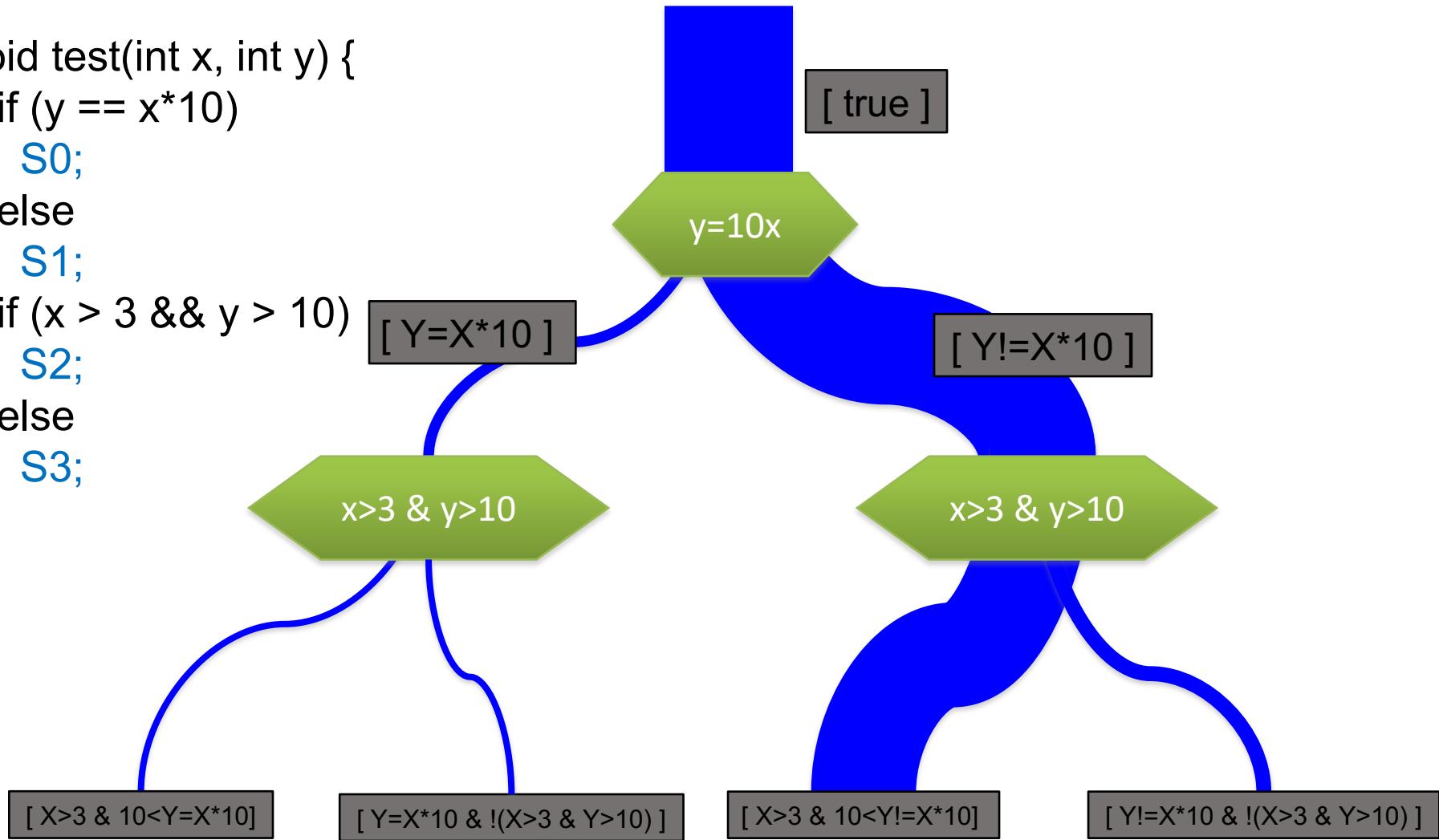
# Paths and Rivers

```
void test(int x, int y) {  
    if (y == x*10)  
        S0;  
    else  
        S1;  
    if (x > 3 && y > 10)  
        S2;  
    else  
        S3;  
}
```



# Paths and Rivers

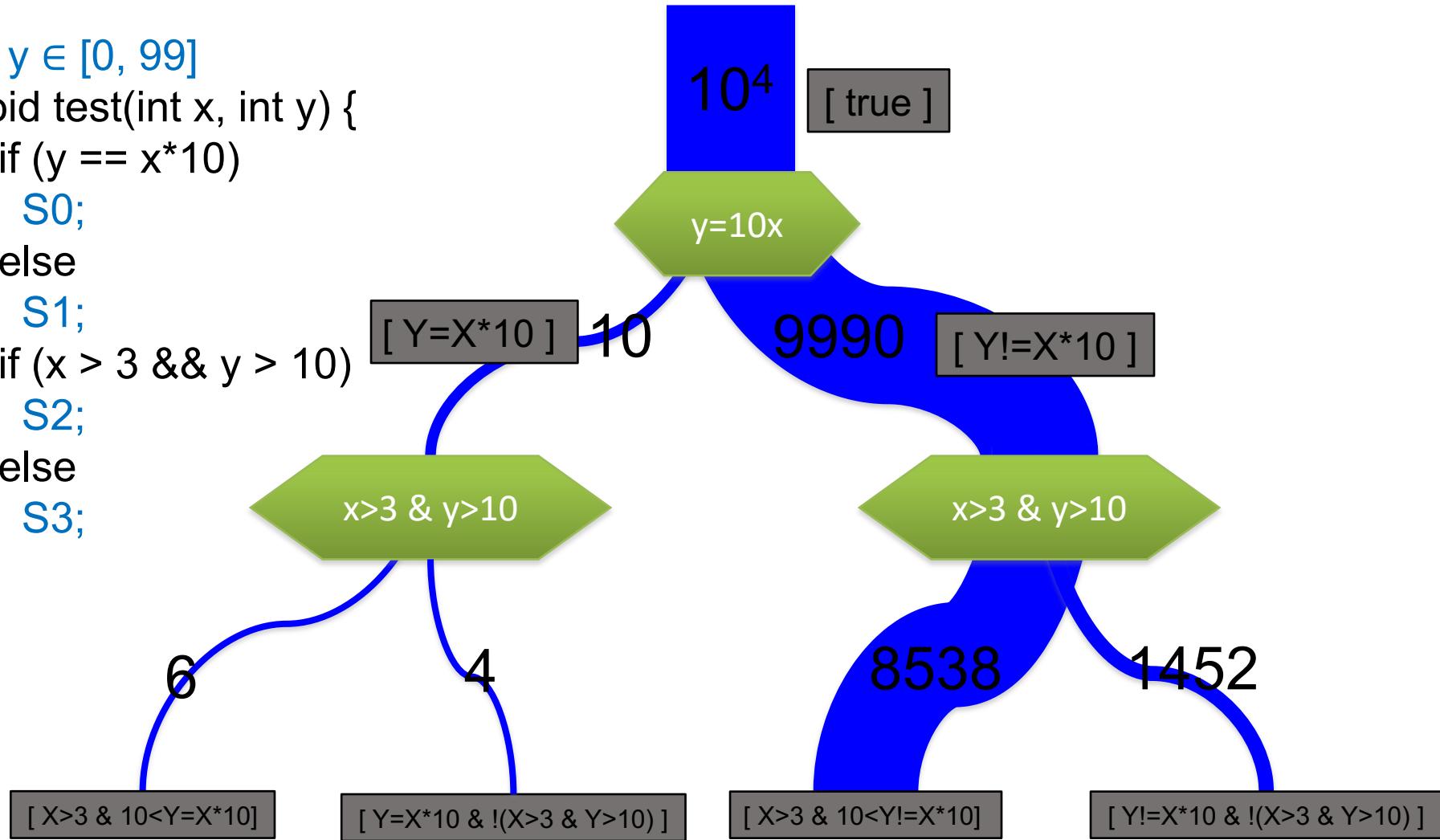
```
void test(int x, int y) {  
    if (y == x*10)  
        S0;  
    else  
        S1;  
    if (x > 3 && y > 10)  
        S2;  
    else  
        S3;  
}
```



# Model Counting (e.g., LattE)

$x, y \in [0, 99]$

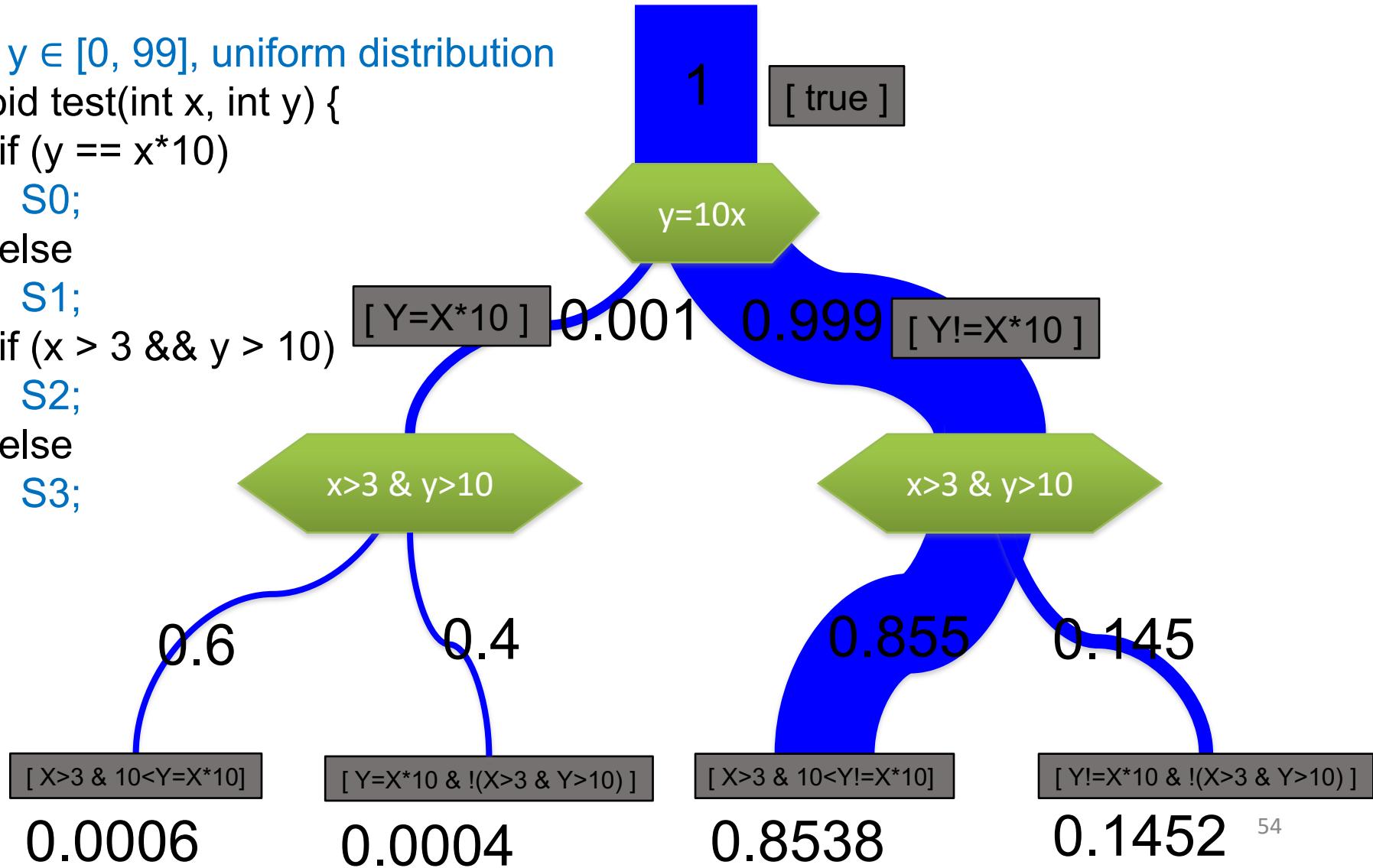
```
void test(int x, int y) {  
    if (y == x*10)  
        S0;  
    else  
        S1;  
    if (x > 3 && y > 10)  
        S2;  
    else  
        S3;  
}
```



# Probabilities

$x, y \in [0, 99]$ , uniform distribution

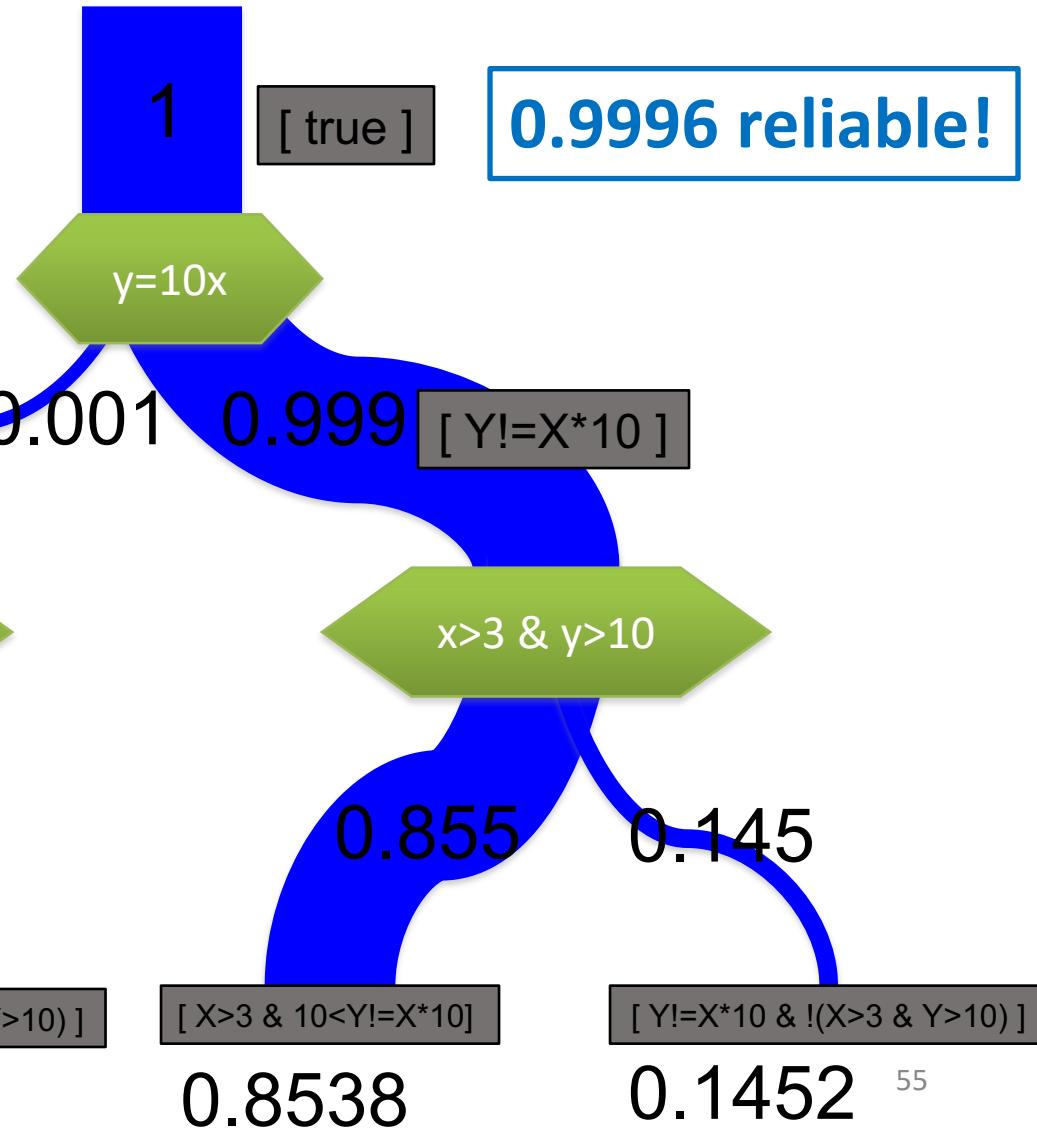
```
void test(int x, int y) {  
    if (y == x*10)  
        S0;  
    else  
        S1;  
    if (x > 3 && y > 10)  
        S2;  
    else  
        S3;  
}
```



# Probabilities

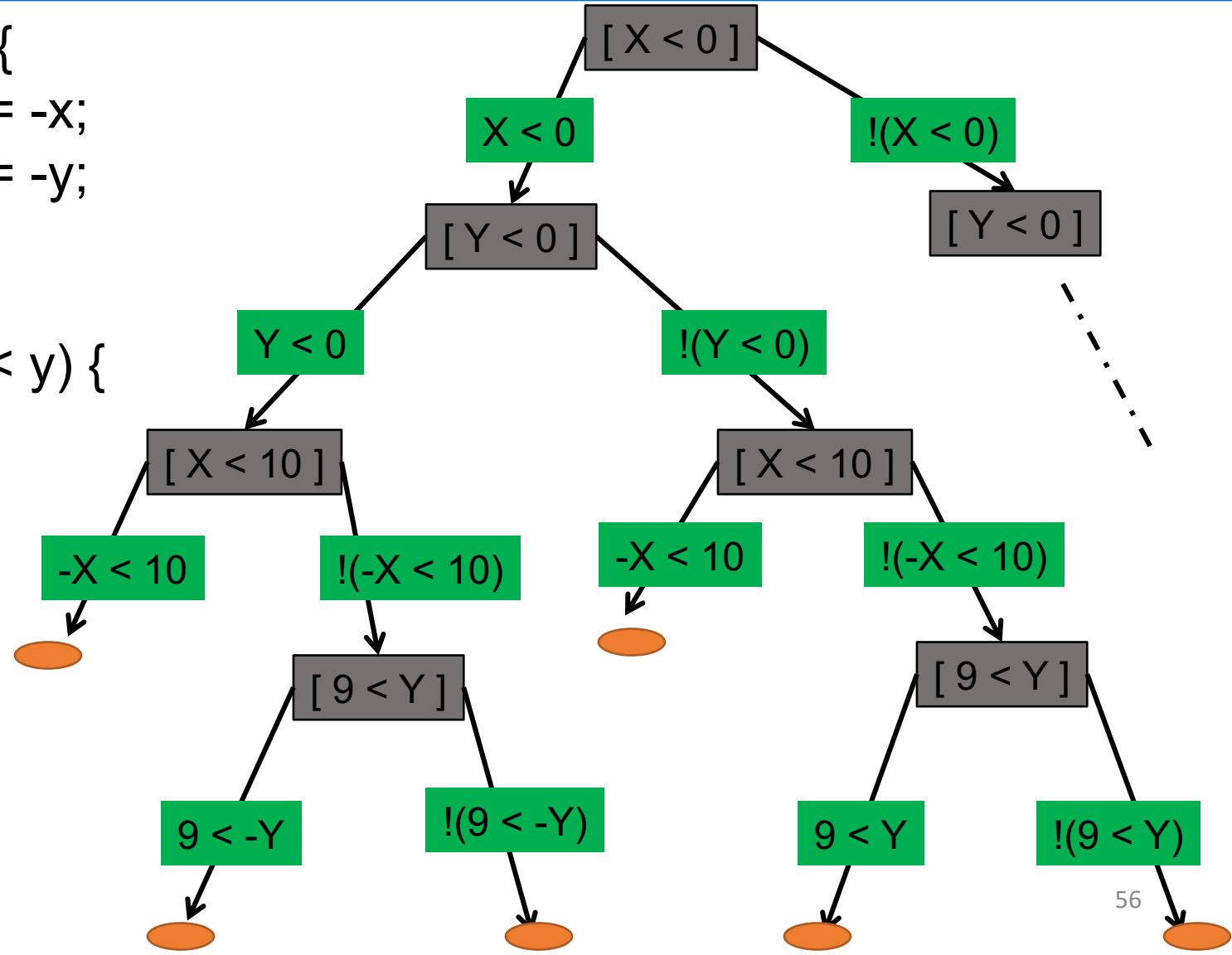
$x, y \in [0, 99]$ , uniform distribution

```
void test(int x, int y) {  
    if (y == x*10)  
        S0;  
    else  
        S1;  
    if (x > 3 && y > 10)  
        S2;  
    else  
        S3;  
}
```

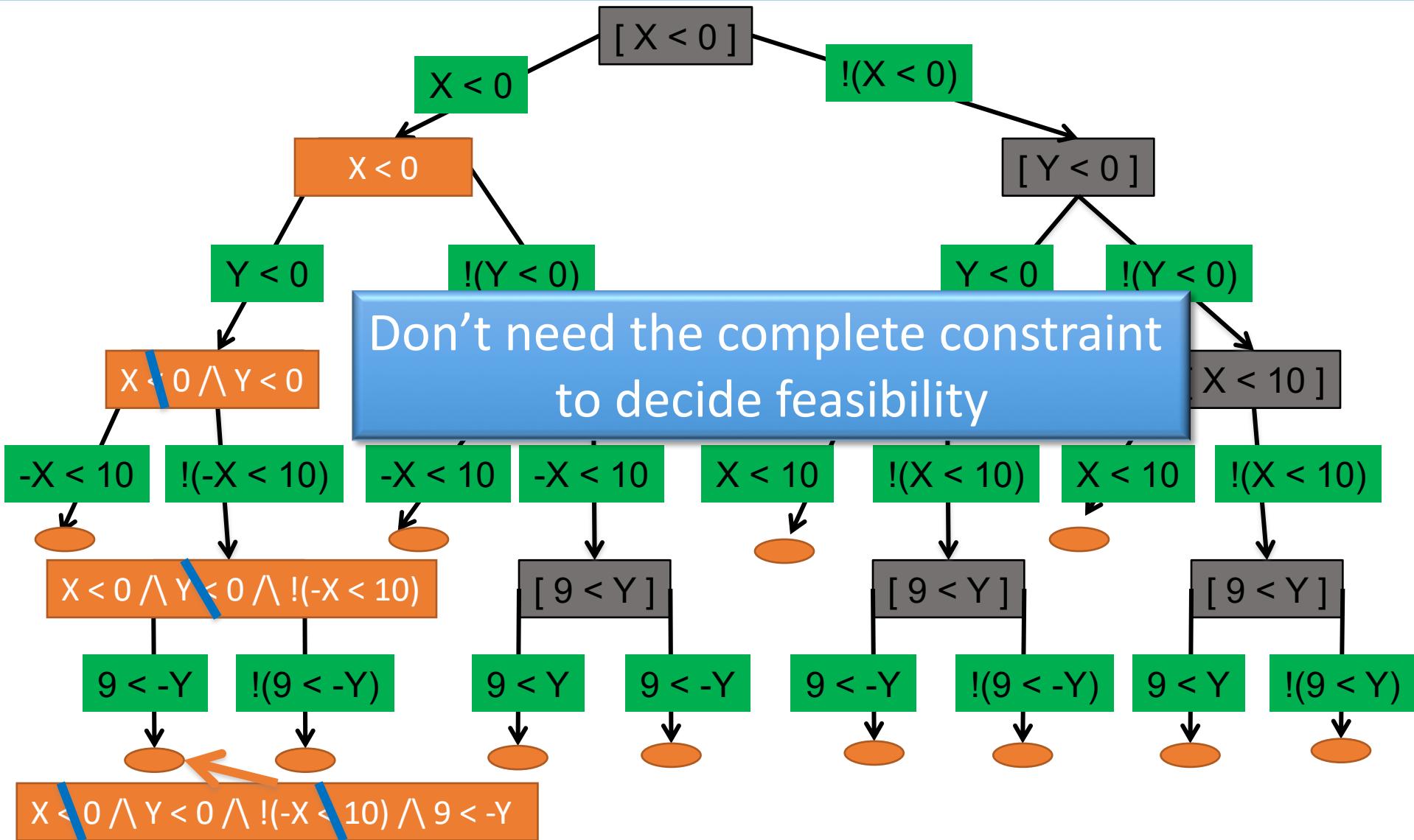


# Reduce, Reuse and Recycle Constraints

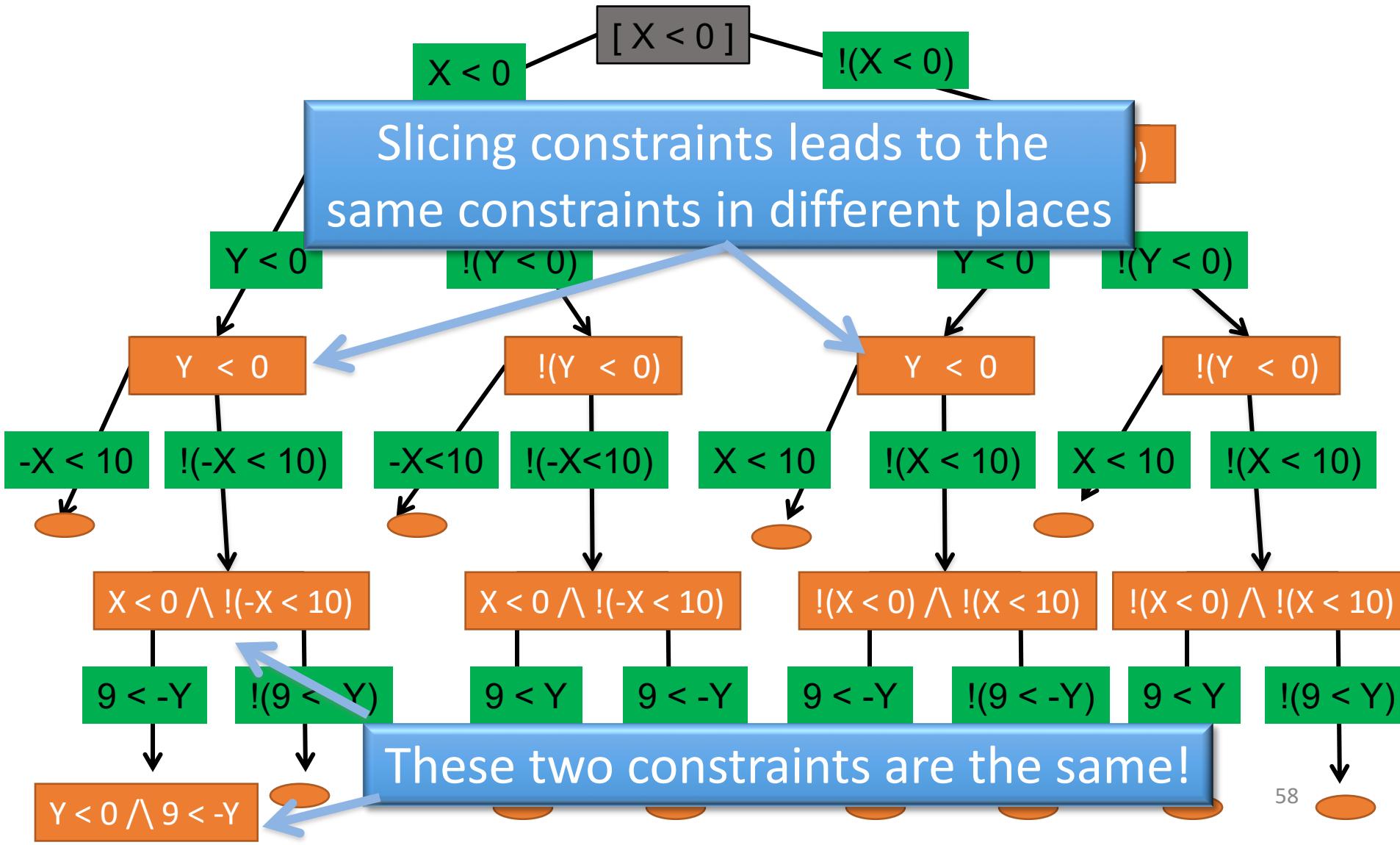
```
int m(int x,y) {  
    if (x < 0) x = -x;  
    if (y < 0) y = -y;  
    if (x < 10) {  
        return 1;  
    } else if (9 < y) {  
        return -1;  
    } else {  
        return 0;  
    }  
}
```



# Reduce, Reuse and Recycle Constraints



# Reduce, Reuse and Recycle Constraints



# Canonization of Constraints

$$X < 0 \wedge !(-X < 10)$$

$$X < 0 \wedge -X \geq 10$$

$$X < 0 \wedge X \leq -10$$

$$X + 1 \leq 0 \wedge X + 10 \leq 0$$

$$Y < 0 \wedge 9 < -Y$$

$$Y < 0 \wedge Y < -9$$

$$Y < 0 \wedge Y + 9 < 0$$

$$Y + 1 \leq 0 \wedge Y + 10 \leq 0$$

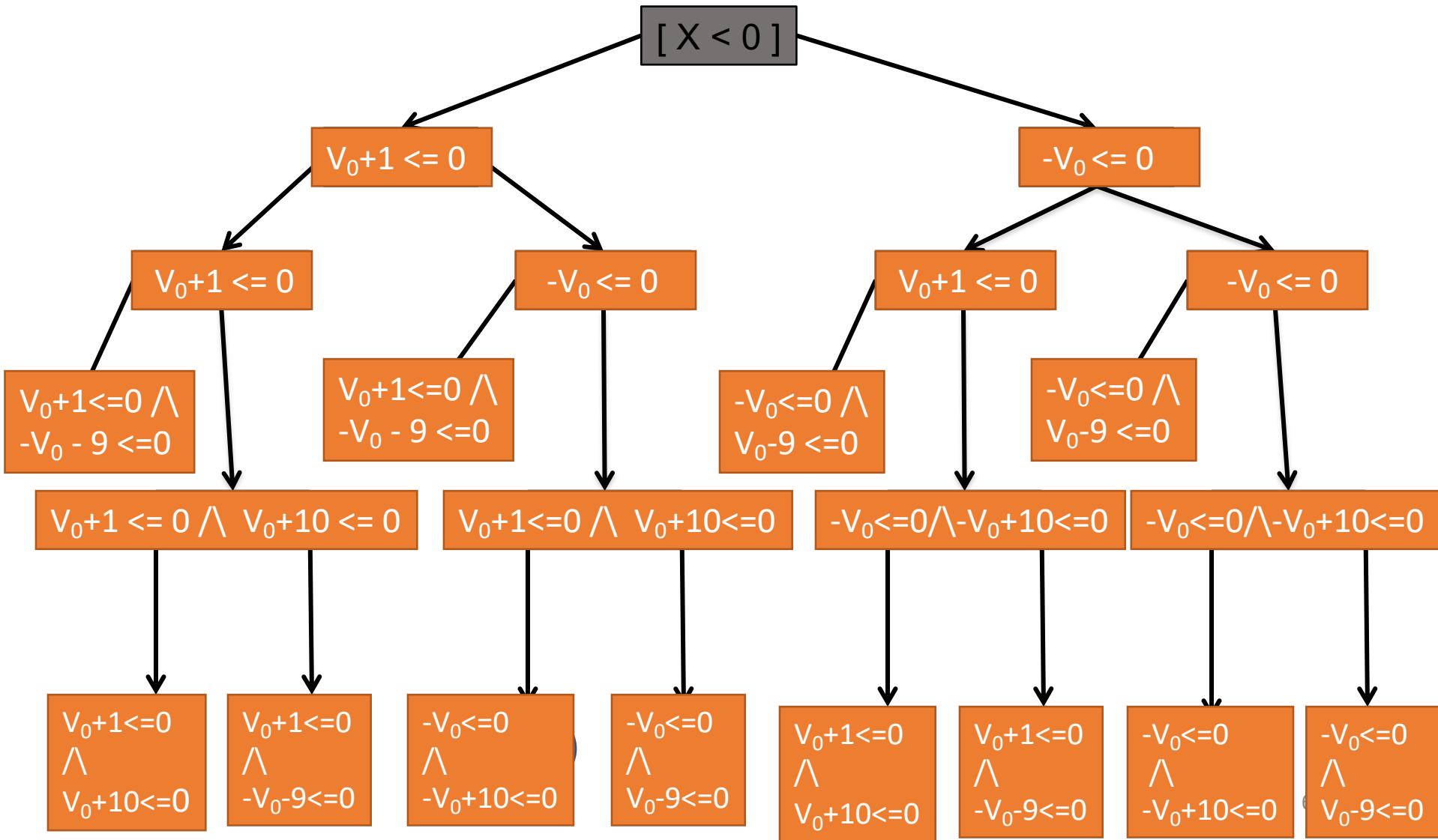
$$V_0 + 1 \leq 0 \wedge V_0 + 10 \leq 0$$

## Canonical Form

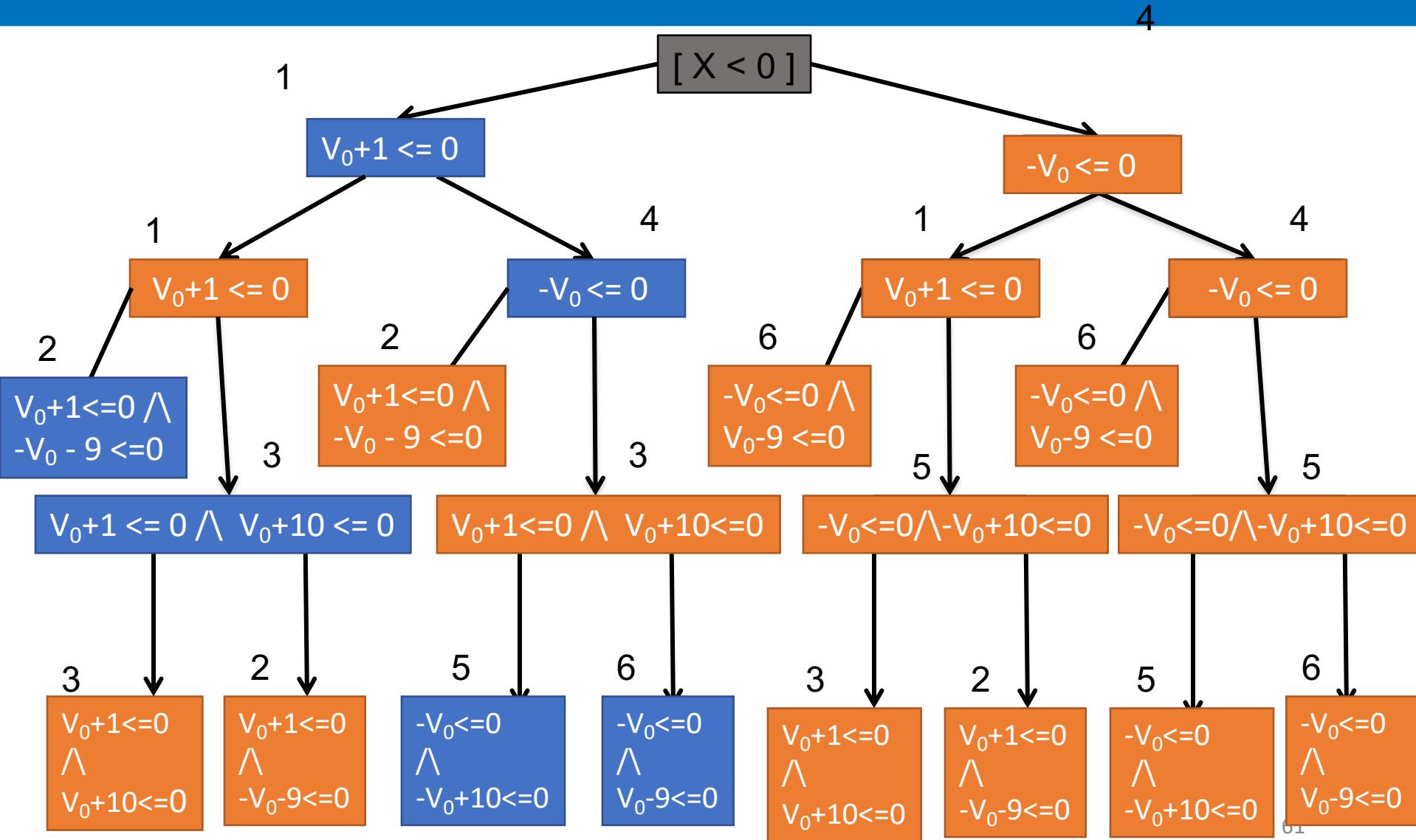
$$ax + by + cz + \dots + k \{ \leq, =, != \} 0$$

- Scale by -1 to transform  $>$  and  $\geq$  to  $<$  and  $\leq$
- Add 1 to transform  $<$  to  $\leq$

# Canonization of Constraints



# Reduce, Reuse and Recycle Constraints



# Reduce, Reuse and Recycle Constraints

```
int m(int x,y) {  
    if (x < 0) x = -x;  
    if (y < 0) y = -y;  
    if (x < 10) {  
        return 1;  
    } else if (10 < y) {  
        return -1;  
    } else {  
        return 0;  
    }  
}
```

Only the last 8 constraints are changed in the symbolic execution tree and 4 of them are reused!

Reusing the stored results from the first analysis eliminates 14 decision procedure calls!

# Reading Materials

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# Q&A?

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