RRY025-- Image Processing

Exercises 4 – Lecture 4 – Fourier II

EX 1. 2D Fourier filtering basics.

Load the cameraman.tif image. Filter the image in the frequency domain using an ideal low-pass filter with cutoff equal to one-eight of the maximum frequency along each axis, without zero padding.

HELP: Among the Matlab files of the course, there is a file raduv.m. Have a look and see if it is useful. You can use it, or program your own way of doing achieving the same.

Display the filtered image in the spatial domain. Apart from excessive smoothing, there is ringing. Can you explain why? HELP: Display the filter in the spatial domain...

Test also filtering the original image using an ideal high pass filter.

ANSWER: Example given as a Matlab file.

The excessive ringing is a result of the ideal filter and its abrupt shape (either 0 or 1). As learned during the lessons, the fourier transform of a tophat function (windowed function) is the sinc function. Similarly, the inverse fourier transform of the tophat filter is a sinc function. Thus, the multiplication of the tophat filter with DFT(I) equals to a convolution of I with a sinc function. This causes the ringing pattern.

EX 2. … continued…

Same as Ex 1, but using Butterworth filters. What are the main differences to the ideal filter in Ex 1? Can you recognise where those differences come from?

ANSWER: The ringing is considerably lessened. Depending on the exact choices of the parameters for the Butterworth filter, some ringing is still present.

EX 3. … continued…

Same as Ex 1, but using Gaussian filters. What are the main differences to the ideal filter in Ex 1? Can you recognise where those differences come from?

ANSWER: There should be least ringing in this case. This comes from the fact that the Gaussian shape remains during the Fourier transform. Convolution of an image with Gaussian equals to smoothing the image.

EX 4. Convolution theorem

Seek to prove the convolution theorem empirically, by convolving an image in both image domain (with filter2) and in Fourier domain using multiplication of the Fourier transform of the image and kernel.

Select an image to represent *f*.

Construct a 2D Gaussian kernel function, *g*, that has the window size of the image f; the sigma of the Gaussian is given as an input parameter.

Smooth (=convolve) the image using the convolution theorem. Compare with the direct calculation of $g \star\star f$ in the image domain.

Hint: Remember that even though the Fourier transform of a Gaussian is a Gaussian, the sigma of that Gaussian changes during the Fourier transform. Be careful to use the right sigma in the right situation.

ANSWER: Example given as a Matlab script.

EX 5. Fourier filtering experiment from the scratch.

Use an image of your choosing (HINT: Astro pictures available at https://www.eso.org/public/images/). Try to first plan and then execute the following scenario.

Choose an image of your liking that has some recurrent features . This could be, for example, pattern in clothing, leafs of a tree, ants on a ground, stars on a sky, or nebulae in astro images.

Hypothesise which filtering technique could work well in emphasising those features. Write down a few lines for a hypothesis.

Experiment with highpass, lowpass, unsharp masking, or bandpass/reject filters on the image. See if you can find out a filter that emphasises the features you chose.

Go back and compare the experiments with your hypothesis. Consider what you understood or learned from filtering during this exercise. What remained unclear?

ANSWER: The answer to this depends on the task one chose. One example is the “cat’s eyes” demonstration given during the FourierII lesson.