IMAGE PROCESSING (RRY025)
Solutions to Problem set A
Image Enhancement

1) a) Histogram equalisation. Find the transfer function $y = g(r)$ which goes from $p_r(r) = 2 - 2r$ and a flat pixel distribution $p_y(y) = constant$.

From the theory of histogram equalisation the required transfer transformation function $y = g(r)$, is the cumulative probability distribution of $p_r(r) = 2 - 2r$, hence

$$y = g(r) = \int_o^r p_{r'}(r')dr' / \int_o^1 p_{r'}(r')dr' = 2r - r^2$$

b) Histogram specification. First find $y = h(z)$ which transforms between the distribution $p_z(z) = 2z$ to a flat pixel distribution $p_y(y) = constant$
From theory this transfer function is the cumulative probability distribution of $p_z(z) = 2z$, hence

$$y = h(z) = \int_o^r p_{z'}(z')dz' / \int_o^1 p_{z'}(z')dz' = z^2$$

To go from the y to the z distribution

$$z = [h]^{-1}(y) = \sqrt{y}$$

Substituting for y as a function of r, g(r) the required transformation is $z = f(r) = \sqrt{2r - r^2}$

2)a) The first two properties, total power (sum of square of pixel values) and the entropy must be the same. Both depend only on the pixel values not the order they are arranged in the image and can be expressed as

$$Power = \sum_I H(I)I^2 \qquad Entropy = \sum_I -H(I)I \ln_2 I$$

where $H(I)$ is the histogram, and I is the gray level.
the inter-pixel covariance function however is not necessarily the same. One could take the pixels in an image of a face (high inter-pixel covariance or similarity between adjacent pixels) and move them randomly around, the image histogram would be the same but the covariance between pixels would be very small.

b) Calculate probabilities of different pixels, cumulative probability and rounded cumulative probability.

Pixel Value	Number of Pixels	Probability	Cumlative Probability	Round to nearest 1/7
0	12	0.187	0.187	0.142 = 1/7
1	16	0.250	0.437	0.426 = 3/7
2	13	0.203	0.641	0.571 = 4/7
3	10	0.156	0.796	0.856 = 6/7
4	6	0.097	0.891	0.856 = 6/7
5	4	0.0625	0.953	1.000 = 7/7
6	2	0.03125	0.984	1.000 = 7/7
7	1	0.0156	1.000	1.000 = 7/7

Hence the transfer function becomes.

Input Pixel Value	Output Pixel Value
0	1
1	3
2	4
3	6
4	6
5	7
6	7
7	7

and the output image is

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3 3 3 3 3 3 3 3
1 4 7 7 7 7 4 1
1 6 4 7 7 4 6 1
1 4 6 4 4 6 6 1
1 6 4 6 6 4 6 1
1 4 7 6 6 6 4 1
3 3 3 3 3 3 3 3
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3 a) 1-C, 2-F, 3-D., 4-A, 5-B, 6-E

b) When we do histogram equalisation we calculate a transfer function based on the cumulative integral of the image histogram (in this case histogram B) in Fig 1. Since most of the pixels are on the background rather than the coin it is the background which dominates the transfer function. After transformation the brightest gray level on the coin is equal to the fraction of pixels on the coin divided by the total number of pixels in the image, it can therefore be that after histogram equalisation the range of gray levels on the coin can actually be less than before. Another (less important) argument is that before equalisation there are two distinct peaks in the histogram, giving sharp edges in the image. After histogram equalisation the 'valley' between the two peaks is filled in and there is less of a sharp edge between coin and background.

c) One method is to do image histogram equalisation separately on and off the coin. First examine the histogram, the coin contributes the gray levels below about 160 and the background above this. First select all pixels below 160, let x= g/160, so x is in the range 0 to 1. Calculate histogram, integrate to get transfer function, y=f(x). Transfer all x to y, then multiply by 160 to get new gray level. Now select all pixels above 160 form x' = (g-160)/(255-160), so x' is in range 0 to 1. Form histogram, integrate to get transfer function y'=f'(x'), transform all x' to y' then create final gray level value g' = 160 + y' (255-160).

4) a)

$$\begin{matrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{matrix}$$

b)

$$\begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}$$

h

c) Advantage - The median filter is efficient at removing isolated pixel values very different from the the rest (so called 'shot' noise from images) like the 8 in the centre of th e'I' image, Disadvantage - Removes corners from rectangular objects.

d)

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad - \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \mathbf{A} + \mathbf{B}$$

Hence convolving with L corresponds to convolving with filter **A** (a five point local average) and then subtracting a version of the image convolved with **B**, which is just a rescaled version of the original image.