RRY025-- Image Processing

Exercises 3 – Lecture 3 – Fourier I

EX 1. Visualize the 1D Fourier transform.

Open the website: http://nain.oso.chalmers.se:8501

The page shows an applet demonstrating Fourier transforms. The top left quadrant shows the input function and the bottom right shows the real part of its Fourier transform. The other quadrants show the kernel function and the kernel multiplied by the input function. Inspect especially the top left, top right, and bottom left frames, and relate them to the theory of the 1D Fourier transform.

Do you understand how the real part of the Fourier transform (bottom right) relates to the other frames? Try to figure it out. Use the 'select u-frequency' slider on the left-side panel to change the frequency that is being inspected.

Question: Choose values of u of 1, 2, 3 Hz. Why is the real part of the Fourier transform zero at these frequencies?

Question: Why does the Fourier transform have maxima and minima at u = 1.5, 2.5, 3.5 Hz?.

Question: Do you understand why the imaginary part of the tophat function is zero? Change the function f(t) from 'tophat' to 'ramp'. Do you understand why now the real part of the Fourier transform is zero and imaginary part is not?

Additional insight: the difference between the Fourier transforms of even and odd functions.

EX 2. Basics of 2D Fourier transform basics in Matlab

Construct a 255 x 255 pixel sized image (*I*) that contains a centered, 2D circular tophat function. Construct it so that you can set the radius of the tophat function as an input parameter, *tophat_radius*.

Hint: The function bwdist() in matlab is likely helpful in constructing the tophat function.

Take the Fourier transform *F*(*I*) and inspect the amplitude image of *F*(*I*). How can you get it to be well visible (and why)? Display and visually inspect also the phase image of *F*(*I*).

Experiment with fftshift() and ifftshift(), and make sure you understand what is meant by "centering" of the Fourier transform.

How do you relate what you see to Exercise 1?

Change the *tophat_radius* to be larger and smaller, and study how those changes affect the amplitude image of F(I). Can you describe *why* the amplitude image changes the way it does?

EX 3. … continued…

Repeat EX 2, but for a tophat function that is box-shaped (instead of circular).

Make the tophat function rectangular, instead of a box.

Can you develop an understanding of how the frequency domain image (amplitude) responses to the changes in the *I*?

Finally, create a smoother version of the tophat function by convolving it with some Gaussian kernel. Study the effect of smoothing to the amplitude image of $F(I)$. Any insight?

EX 4. Continuing 2D Fourier basics…

Load the cameraman.tif image and display it (the image is a default Matlab stock image).

Take the 2D DFT of the image. Display the amplitude and phase. How can you get them to be well visible (and why)?

What features in the amplitude and phase images can you relate to the properties of the original image, if any?

Reconstruct the original image using the Fourier transformed image. Is all well?

Reconstruct the image using only the amplitude information of the Fourier transform. Then do the same using only the phase information. Which component seems to be more important in capturing human-recognizable features of the image?