10.4 Consider the quadratic polynomial $x^2 + bx + c$, where b, c real with non negative discriminant, so that the roots r_1, r_2 real

1. Recall that $c = r_1, r_2$, $b = -(r_1 + r_2)$

2. If $c = 0$, the nature of the roots are easy to determine. Explain why

This is because the equation is now $x^2 + bx$, which factors as $x(x + b)$, so we have roots $r_1, r_2 = x, -b$

3. Assume that $c \neq 0$, show that if the roots r_1, r_2 have the same sign, then $c > 0$, and if the roots have the opposite sign, $c < 0$

r_1, r_2 have same signs:

Case 1: both positive

Then $c = r_1 * r_2$ is positive * positive = positive, $c > 0$

Case 2: both negative

Then $c = r_1 * r_2$ is negative * negative = positive, $c > 0$

r_1, r_2 have opposite signs:

Without loss of generality, assume r_1 negative, r_2 positive, then the product $c = r_1 * r_2$ is the product of a negative and a positive, so c is negative, so $c < 0$

4. Conclude that there is an odd number of positive roots when c is negative, and an even number of positive roots when c is positive

This is because when c is positive, r_1, r_2 either both positive or both negative, so there are 0 or 2 positive roots. And if exactly one of the roots is negative and the other positive, then there is an odd number of positive roots for c negative.

5. Assume that c is positive, show that the roots are positive precisely when $b < 0$, and negative when $b > 0$

$c > 0$, so $r_1 * r_2 > 0$, and r_1, r_2 must have the same sign

And $b = -(r_1 + r_2)$

Case 1: $b < 0$

Then the sum $-(r_1 + r_2)$ must be positive

So r_1, r_2 must both be positive

Case 2: $b > 0$

Then the sum $-(r_1 + r_2)$ must be negative

Then r_1, r_2 must both be negative

6. Conclude that you can use b, c to determine the signs of the roots. Describe exactly how you would do so

For nonnegative discriminant, we can use b, c to determine the signs of the roots. If $c = 0$, finding roots is trivial, they are $x, -b$. First, if c nonzero, we can determine whether the roots are the same sign (if $c > 0$) or oppposite sign ($c < 0$). And if they are the same sign, we can use b to determine whether both are positive ($b < 0$), or both negative ($b > 0$).