

Brandon Chen
MATH 395 HW 8

7: 58, 69ab

8: 2, 4, 7, 8

7.58 A coin having probability p of coming up heads is continually flipped until both heads and tails have appeared. Find

- a) The expected number of flips;
- b) The probability that the last flip lands of heads.

7.69ab The number of accidents that a person has in a given year is a Poisson random variable with mean λ . However, suppose that the value of λ changes from person to person, being less than or equal to $1 - e^{-x}$. If a person a person is chosen at random, what is the probability that he will have

- a) 0 accidents
- b) exactly 3 accidents in a certain year?

8.2 From past experience, a professor knows tha the test score of a student taking her final examination is a random variable with mean 75.

- a) Give an upper bound for the probability that a student's test score will exceed 85. Suppose, in addition, that the professor knows that the variance of a student's test score is equal to 25.
- b) What can be said about the probability that a student will score between 65 and 85?
- c) How many students would have to take the examination to ensure with a probability at least 0.9 that the class average would be within 5 of 75? Do not use the central limit theorem.

8.4 Let X_1, \dots, x_{20} be independent Poisson random variables with mean 1.

- a) Use the Markov inequality to obtain a bound on $P(\sum_1^{20} X_i > 15)$
- b) Use the central limit theorem to approximate $P(\sum_1^{20} X_i > 15)$

8.7 A person has 100 lightbulbs whose lifetimes are independent exponentials with mean 5 hours. If the lightbulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.

8.8 In problem 8.7, suppose that it takes a random time, uniformly distributed over $(0, 0.5)$, to replace a failed bulb. Approximate the probability that all bulbs have failed by time 550.