Brandon Chen
MATH 394 HW 5
Chapter 4 pg 163 43, 45, 46, 50, 51, 60, 64, 66, 70, 74, 77, 78 and 84

43 Telecommunications either 0 or 1

P(incorrect) = 0.2

The receiver uses majority decoding (that is, it sends 5 of the sequence, and it chooses the one that has more bits flipped)

Suppose the message is intended to be 1, which would be coded as 11111

This would be misinterpreted when 3, 4, or 5 bits are wrong.

There is a 0.2 chance for any given bit to be incorrect

There are $\binom{5}{3}, \binom{5}{4}, \binom{5}{5}$ arrangements of correct and incorrect bits

Then the probability is $\binom{5}{3}(0.2)^3 + \binom{5}{4}(0.2)^4 + \binom{5}{5}(0.2)^5 = \frac{276}{3125} \approx 0.08832$

The independence assumptions I am making is that the probability of having any index flipped to the wrong bit is independent of the probability of having other indexes wrong, and that the probability of having 3, 4, or 5 bits wrong are mutually exclusive.

45 $P(off) = \frac{2}{3}, P(on) = \frac{1}{3}$

$P(pass|off) = 0.4$

$P(pass|on) = 0.8$

Let EX be the expected value for the student passing the exam given 3 examiners

Let EY be the expected value for the student passing the exam given 5 examiners

If the student has 3 exams, they will pass if they pass 2 or 3 out of 3 exams.

If the student has 5 exams, they will pass if they have 3, 4, or 5 out of 5 exams.

$EX = \frac{2}{3}[\binom{3}{2}(0.4)^2(0.6) + \binom{3}{3}(0.4)^3] + \frac{1}{3}[\binom{3}{2}(0.8)^2(0.2) + \binom{3}{3}(0.8)^3] \approx 0.533$

$EY = \frac{2}{3}[\binom{5}{3}(0.4)^3(0.6)^2 + \binom{5}{4}(0.4)^4(0.6) + \binom{5}{5}(0.4)^5] + \frac{1}{3}[\binom{5}{3}(0.8)^3(0.2)^2 + \binom{5}{4}(0.8)^4(0.2) + \binom{5}{5}(0.8)^5 \approx 0.5256$

Since the expected value for passing with 3 examiners is greater than the expected value for passing with 5 examiners, the student should request 5 examiners.

So it is more advantageous for the student to request an examination with 3 examiners.

46 9 votes of 12 needed

P(free but guilty) = 0.2

P(innocent but convicted) = 0.1

P(guilty) = 0.65

What is the probability that the jury renders a correct decision

What percentage of defendants is convicted?

4 cases:

free and guilty

free and innocent

convicted and guilty

convicted and innocent

P(correct) = P(convicted and guilty) + P(free and innocent)

$P(convicted and guilty) = 0.65[\binom{12}{9}(0.8)^9(0.2)^3+\binom{12}{10}(0.8)^{10}(0.2)^2+\binom{12}{11}(0.8)^{11}(0.2)^1+\binom{12}{12}(0.8)^{12} \approx 0.516469$

$P(free and innocent) = 0.35[\binom{12}{9}(0.9)^9(0.1)^3+\binom{12}{10}(0.9)^{10}(0.1)^2+\binom{12}{11}(0.9)^{11}(0.1)+\binom{12}{12}(0.9)^{12} \approx 0.3410$

P(correct) $\approx$ 0.85749

P(convicted) = P(convicted and guilty) + P(convicted and innocent)

P(convicted and innocent) = $0.35[\binom{12}{9}(0.1)^9(0.1)^3 + \binom{12}{10}(0.1)^{10}(0.9)^2 + \binom{12}{11}(0.1)^{11}(0.9) + \binom{12}{12}(0.1)^{12} \approx 5.804225 * 10^{-8}$

P(convicted) $\approx 5.804225 * 10^{-8} + 0.5164 \approx 0.5164$

50 Biased coin.

P(heads) = p

Flip 10 times

Given that total count of heads = 6

P(6 heads) = $P(X = 6) = \binom{10}{6}(p)^6(1-p)^{10-6}$

a) Find P(h,t,t given 6 heads)

$= \frac{p(1-p)^2\binom{7}{5}p^5(1-p)^2}{\binom{10}{6}p^6(1-p)^4} = \frac{1}{10}$

b) Find P(t,h,t given 6 heads)

This should be equal to the probability found in part a, since only one head is found in the first 3, and the probability of getting a head in any index is the same.

So P = $\frac{1}{10}$

51 Expected number of errors is 0. 2

a) Find P(0 errors)

We know that the probability of typos can be modeled with Poisson distribution.

In this case, $\lambda = 0.2$, so $P(0) = \frac{0.2^0 e^{-0.2}}{0!} = e^{-0.2}$

b) Find P(2 or more errors) = 1 - P(1 error) - P(0 error)

P(1 error) = $\frac{0.2^1 e^{-0.2}}{1!} = 0.2e^{-0.2}$

So P(2 or more errors) = $1 - 0.2e^{-0.2} - e^{-0.2} = 1 - 1.2e^{-0.2}$

60 $\lambda = 5$

For 0.75 of the population, $\lambda = 3$ with drug

Other 0.25, no effect

If an individual tries the drug for a year and has 2 colds in that time, how likely is it that the drug is beneficial?

Let P(B) = beneficial

Let P(C) = 2 colds

Then $P(B|C) = \frac{P(C|B)*\frac{3}{4}}{P(C|B)*\frac{3}{4}+P(C|B^c)*\frac{1}{4}}$

$= \frac{\frac{3^2e^{-3}}{2!}\frac{3}{4}}{\frac{3^2e^{-3}}{2!}\frac{3}{4}+\frac{5^2e^{-5}}{2!}\frac{1}{4}}$

≈ 0.88864

64 P(S) = $\frac{1}{100000}$

a) Find the probability that in a city of 400,000 inhabitants, there are 8 or more suicides in a given month.

$\lambda = 400000 * \frac{1}{100000} = 4$, so we can use Poisson distribution to model

P(8 or more) = $1 - \sum_0^7 \frac{4^i e^{-4}}{i!} \approx 0.0511336$

b) P(at least 2 months with 8 or more suicides)?

For any month, there can either be 8 or more suicides with p = 0.0511, or less than 8 suicides, with probability (1-p)

There are 12 months, and we want the probability of at least 2 or more months

This is equal to

1 - P(1 month) - P(0 months)

$= 1 - \binom{12}{0} * (1-0.0511)^{12} - \binom{12}{1}0.0511(1-0.0511)^{11}$

≈ 0.12272

c) What is the probability that the first month to have 8 or more suicides will be month number i, $i \geq 1$. What assumptions are you making?

Geometric random variable X, $P(X = k) = (1-p)^{k-1}p$

So $P(X = i) = (1-0.0511)^{i-1}(0.0511)$

66 2n people, n married couples

Randomly seated at a round table

C_i is the event that the members of couples i are seated next to each other

i = 1, ... n

a) Find $P(C_i)$

For C_i to happen, the two must sit next to each other, with 2! permutations.

Treat the pair i as a single object

There are now $(2n-1)$ objects, to be permuted around a round table with $(2n-1-1)$ permutations

Total possible permutations around round table = $(2n-1)$

$P(C_i) = \frac{2!(2n-2)}{(2n-1)!} = \frac{2}{2n-1}$

b) For $j \neq i$, find $P(C_j|C_i)$

$P(C_j|C_i) = \frac{P(C_j \cap C_i)}{P(C_i)}$

$P(C_j \cap C_i)$ happens when the pair j sits together with 2! perms and i sits together with 2! perms

The pair i also sits together with 2! permutations

Treat pairs i and j as objects, there are now $(2n-2)$ unique objects.

They are to be permuted around a round table, with $(2n-2-1)$ unique permutations

Then the probability is $\frac{2!2!(2n-3)!}{(2n-1)!}$

Then $P(C_j|C_i) = \frac{\frac{2!2!(2n-3)!}{(2n-1)!}}{\frac{2}{(2n-1)}} = \frac{1}{n-1}$

c) Approximate the probability, for n large, that there are no married couples who are seated next to each other

Let EX be the expected value for number of couples sitting together

$EX = n * P(C_i) = \frac{2n}{2n-1}$

So for n large, $EX \approx 1$

Since n large, p small, and np moderate, then $\lambda = EX \approx 1$

Let Y be a random variable for number of couples sitting together

Then P(none sit together) = P(Y = 0) = $\frac{1^0 e^{-1}}{0!} \approx e^{-1}$

70 $P(H|\text{no additional flips}) = 1$

$P(H|\text{additional flips}) = p$

$P(H) = P(H|\text{no additional flips}) * P(\text{no additional flips}) + P(H|\text{additional flips}) * P(\text{additional flips})$

Poisson process with rate λ, over time interval $0, t$

Then $P(H) = 1 * (e^{-\lambda t}) + p * (1 - e^{-\lambda t})$

This is equal to $p + (1-p)e^{-\lambda t}$

74 5 needed

a) All 5 of the candidates must accept, with a probability $\frac{2}{3}$ each.

So EX = $\frac{2}{3}^5 = \frac{32}{243} \approx 0.131687$

b) There are 8, so we can pick 5, 6, 7, or 8 of them to be interviewed, with probability $\frac{2}{3}$ each

$EX = \binom{8}{5}\frac{2}{3}^5\frac{1}{3}^3 + \binom{8}{6}\frac{2}{3}^6\frac{1}{3}^2 + \binom{8}{7}\frac{2}{3}^7\frac{1}{3} + \binom{8}{8}\frac{2}{3}^8 = \frac{4864}{6561} \approx 0.741$

c) If need exactly 6 people,

Negative binomial random variable P(X = n) = $\binom{n-1}{r-1}p^r(1-p)^{n-r}, n = r, r+1, ...$

In this case, n = 6, r = 5, p = $\frac{2}{3}$

Then P(X = 6) = $\binom{6-1}{5-1}\frac{2}{3}^5\frac{1}{3}^{6-5} = \frac{160}{729}$

d) If need exactly 7,

Negative binomial random variable, n = 7, r = 5, p = $\frac{2}{3}$

P(X = 7) = $\binom{7-1}{5-1}\frac{2}{3}^5\frac{1}{3}^2 = \frac{160}{729}$

77 Both hands start with N matches.

Assume that the left hand has no more matches, and k matches are in the right hand.

For this event to occur, the N^{th} choice of the left hand is on the $(N + (N-k))^{th}$ trial.

We know that the probabilities of choosing either the left or right hand are equally likely.

So the event has a probability $\binom{N+N-k-1}{N-1} * (\frac{1}{2})^{N+N-k}$

Assume that the right hand has no more matches, and k matches are in the left hand.

This has the same probability as for the left hand.

Then the probability that any one hand will run out of matches with the other hand having k matches remaining is $2 * \binom{2N-k-1}{N-1} * \frac{1}{2}^{2N-k}$

78 Urn contains 4 white, 4 black

Choose 4 random

If 2 white 2 black, stop

If not, replace and repeat

Probability of 2 white and 2 black selected $= \frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}} = \frac{18}{35}$

Want success after exactly n trials. This is a geometric random variable.

$P(X = n) = p(1-p)^{n-1} = \frac{17}{35}^{n-1} * \frac{18}{35}$

84 10 balls, 5 boxes, each independent in box i with probability $p_i, \sum_{i=1}^{5} p_i = 1$

a) For any given box, there is a probability p_i that the ball goes in, so there is a $(1-p)^{10}$ probability that none of the 10 balls go into that specific box i.

There are 5 boxes, and are all independent, so the probability that none of the 5 boxes get a ball is equal to the sum of the probabilities that any box i will get no balls.

This is equal to

$EX = \sum_{i=1}^{5}(1-p_i)^{10}$