

5B Prove Corollary 5.3 (to Pasch's theorem)

Theorem 5.2 (Pasch's theorem): Suppose  $\triangle ABC$  is a triangle and  $\ell$  is a line that does not contain any of the points  $A, B, C$ . If  $\ell$  intersects one of the sides of  $\triangle ABC$ , then it also intersects another side.

Corollary 5.3: If  $\triangle ABC$  is a triangle and  $\ell$  is a line that does not contain any of the points  $A, B, C$ , then either  $\ell$  intersects exactly two sides of  $\triangle ABC$  or it intersects none of them.

$\ell$  is a line. It does not contain any of the points  $A, B, C$ , so it is not collinear to any of the segments  $\overline{AB}, \overline{BC}, \overline{AC}$

Case:  $\ell$  does not intersect any of the line segments.

Then  $\ell$  does not intersect any of the line segments.

Case:  $\ell$  intersects one of the line segments. Then by theorem 5.2, it intersects another side.

Then  $\ell$  intersects exactly two sides of  $\triangle ABC$

5C Suppose  $\triangle ABC$  is a triangle and  $\ell$  is a line (which might or might not contain one or more vertices). Is it possible for  $\ell$  to intersect exactly one side of  $\triangle ABC$ ? Exactly two? All three? In each case, either give an example or prove that it is impossible.

5D Prove Theorem 5.8 (The converse to the isosceles triangle theorem). [Hint: One way to proceed is to construct an indirect proof, like Euclid's proof of proposition 1.6. Another is to mimic Pappus's proof of the isosceles triangle theorem.]

5E Prove Theorem 5.10 (The triangle copying theorem)

5F Prove Theorem 5.18 (The triangle inequality)