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MATH 395 HW 8
7: 58, 69ab
8: 2, 4, 7, 8

7.58 A coin having probability p of coming up heads is continually flipped until both heads and tails have appeared. Find

a) The expected number of flips;

Want E[at least one head and at least one tails]

$= E[X|\text{first flip is Tail}] * P(\text{first flip tail}) + E[Y|\text{first flip is heads}] * P(\text{first flip head})$

This is $(1 + \frac{1}{p})(1-p) + (1 + \frac{1}{1-p})p$

b) The probability that the last flip lands on heads

If X is the event that the last flip is heads, then X is the event that the first flip is tails.

Then $P(X) = 1 - p$

7.69ab The number of accidents that a person has in a given year is a Poisson random variable with mean λ. However, suppose that the value of λ changes from person to person, being less than or equal to $1 - e^{-x}$. If a person a person is chosen at random, what is the probability that he will have

a) 0 accidents

$P(X = 0) = \int_0^\infty P(X = 0|\lambda) * f(\lambda)d\lambda$

$= \int_0^\infty e^{-\lambda}e^{-\lambda}d\lambda$

$= \frac{1}{2}\int_0^\infty 2e^{-2\lambda}d\lambda$

$= \frac{1}{2}$

b) exactly 3 accidents in a certain year?

$P(X = 3) = \int_0^\infty P(X = 3|\lambda) * f(\lambda)d\lambda$

$= \int_0^\infty \frac{\lambda^3 e^{-\lambda}}{3!}e^{-\lambda}d\lambda$

$= \frac{\Gamma(4)}{3!2^4}\int_0^\infty \frac{(2\lambda)^3 2e^{-2\lambda}}{\Gamma(4)}d\lambda$

$= \frac{1}{16}$

8.2 From past experience, a professor knows tha the test score of a student taking her final examination is a random variable with mean 75.

a) Give an upper bound for the probability that a student's test score will exceed 85. Suppose, in addition, that the professor knows that the variance of a student's test score is equal to 25.

$P(X \geq a) \leq \frac{E[x]}{a}$

Upper bound $P(X \geq 85) \leq \frac{75}{85}$

b) What can be said about the probability that a student will score between 65 and 85?

$P(|X - \mu| \geq b) \leq \frac{\sigma^2}{b^2}$

$P(|X - 75| \geq 10) \leq \frac{25}{100}$

c) How many students would have to take the examination to ensure with a probability at least 0.9 that the class average would be within 5 of 75? Do not use the central limit theorem.

$P(|\frac{1}{n}\sum_1^n X_i - \mu| \geq \epsilon) \leq \frac{\sigma^2}{n\epsilon^2}$

$P(|\frac{1}{n}\sum_1^n X_i - 75| \geq 5) = \frac{25}{n25} = \frac{1}{n}$

$P(|\frac{1}{n}\sum_1^n X_i - 75| < 5) \geq 1 - \frac{1}{n}$

So for $p \geq 0.9$, this is $n \geq 10$

8.4 Let $X_1, ..x_{20}$ be independent Poisson random variables with mean 1.

a) Use the Markov inequality to obtain a bound on $P(\sum_1^{20} X_i > 15)$

$P(\sum_1^{20} X_i > 15) = P(\frac{1}{20}\sum_1^{20} X_i > \frac{15}{20} \leq \frac{E[x]}{\frac{15}{20}}$

$P(\sum^{20} X_i > 15) \leq \frac{20}{15}$

b) Use the central limit theorem to approximate $P(\sum_1^{20} X_i > 15)$

$P(\sum^{20} X_i > 15) = P(\sum^{20} X_i > 15.5)$

$= P(Z > \frac{15.5-20}{\sqrt{20}}$

$= P(Z > -1.006) \approx 0.842$

8.7 A person has 100 lightbulbs whose lifetimes are independent exponentials with mean 5 hours. If the lightbulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.

$P(\sum^{100} X_i > 525) = P(Z > \frac{525-500}{\sqrt{2500}}$

$= P(Z > 0.5) \approx 0.3085$

8.8 In problem 8.7, suppose that it takes a random time, uniformly distributed over (0, 0.5), to replace a failed bulb. Approximate the probability that all bulbs have failed by time 550.

Light bulb fails, replace the *ith* bulb, time to replace R_i, $E[R_i] = 0.25$

$P(\sum^{100} X_i + \sum^{99} R_i \leq 550)$

$= P(Z \leq \frac{550-524.75}{\sqrt{2502}}$

$= P(Z \leq 0.504798) \approx 0.69315$