Brandon Chen
MATH 445 HW 7
Due Friday 5/18/18
16 A, E, K, N, P

16A Construction Problem 16.5 (Perpendicular Bisector)

Given a segment, construct its perpendicular bisector.

The given segment has end points, call them A, B

Draw a circle with radius AB and center A, call it circle $\mathscr{A}$

Draw a circle with radius AB and center B, call it circle $\mathscr{B}$

Take a point on the interior of the segment $\overline{AB}$, call it P

Then the circle $\mathscr{A}$ has point P on its interior, and $\mathscr{B}$ has point P on its interior

Then by theorem 14.10, $\mathscr{A}, \mathscr{B}$ must intersect at exactly two points, call them C, D

Since C, D on $\mathscr{A}, \mathscr{B}$, which are circles with radius AB, then $AB = AC = BC = AD = BD$

Then by definition, $ACBD$ is a kite, it has diagonals $\overline{CD}, \overline{AB}$

Then by problem 9F, its diagonals are perpendicular and one of them bisects the other.

16E Construction Problem 16.9 (Copying a Triangle to a Given Segment)

Given a triangle ΔABC, a segment $\overline{DE}$ congruent to $\overline{AB}$, and a side of $\overleftrightarrow{DE}$, construct a point F on the given side such that $\Delta DEF \cong \Delta ABC$

Draw circle $\mathscr{D}$ with center D and radius AC

Draw circle $\mathscr{E}$ with center E, radius BC

Then by two circles theorem, $\mathscr{E}$ intersects $\mathscr{D}$ at exactly two points on each side of $\overleftrightarrow{DE}$

Take the point of intersection on the side of $\overleftrightarrow{DE}$ given, call it F

Then draw triangle ΔDEF

By hypothesis, we have that $\overline{DE} \cong \overline{AB}$

By construction, DF has length AC

And EF has length BC

Then by SSS, $\Delta DEF \cong \Delta ABC$

16K Construction Problem 16.15 (Cutting Segment into n Equal Parts)

Given a segment $\overline{AB}$ and an integer $n \geq 2$, construct points $C_1, ..., C_n \in Int\overline{AB}$ such that $A * C_1 * ... * C_{n-1} * B$ and $AC_1 = C_1C_2 = ... = C_{n-1}B$

We are given $\overline{AB}$

Draw a ray $\overrightarrow{r}$ from point A such that the angle between $\overrightarrow{r}$ and $\overline{AB}$ is acute.

Choose a positive radius m

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Draw line segment $\overline{P_n}B$

Draw line segments through points P_i parallel to $\overline{P_n}B$, it intersects $\overline{AB}$ at a point, call it C_i

By construction of the circles of radius $i * m$, $A * P_1 * ... * P_n$

And since points $C_1..C_{n-1}$ are formed as an intersection of parallel lines, then $A{*}C_1{*}..C_{n-1}{*}B$

Show that $AC_1 = C_1C_2 = ... = C_{n-1}B$

We will show that each length is the same by examining the triangles formed by A, P_i, C_i

For $n = 1$, there will only be one triangle, then it will have only one side length AB, it is equal.

For $n > 2$, compare two consecutive triangles

First, compare the first two: ΔAC_1P_1 is a triangle, ΔAC_2P_2 is a triangle

By construction of parallel segments $\overline{P_1}C_1$, $\overline{P_2}C_2$, $\angle AC_1P_1 \cong \angle AC_2P_2$ and $\angle AP_1C_1 \cong \angle AP_2C_2$

Then by AAA similarity, they are similar

By construction, $AP_2 = 2 * AP_1$

Then $AC_2 = 2 * AC_1$

We know that $AC_2 = AC_1 + C_1C_2$

Then $AC_1 = C_1C_2$

Assume that for i, $AC_i = AC_1 * i$, $C_{i-2}C_{i-1} = C_{i-1}C_i$

Show that for $i + 1$, $C_i = Ci + 1$

Distance AP_{i+1} is $(i + 1) * m$, by construction

Since $\overline{P_{i+1}C_{i+1}}$ is parallel to $\overline{P_1C_1}$, then $\Delta AC_1P_1 \sim AC_{i+1}P_{i+1}$

Then for $\frac{AP_{i+1}}{AP_1} = i + 1$, it must be the case that $\frac{AC_{i+1}}{AC_1} = i + 1$

Then AC_{i+1} has length $AC_1 * (i + 1)$

By inductive hypothesis, AC_i has length $AC_1 * i$

We know that $AC_{i+1} = AC_i + C_iC_{i+1}$

Then $C_iC_{i+1} = AC_1 = C_{i-1}C_i$

Then each segment is equal length

16N Construction Problem 16.23 (Doubling a square)

Given a square, construct a new square whose area is twice that of the original one

We are given a square, it has length l

Construct a right triangle with 2 side lengths $AB = l, BC = l$

Then it has hypotenuse $AC = \sqrt{2}l$

Draw a circle with center A and radius $2l$

It intersects the line $\overleftrightarrow{AB}$ on the same side as B at one point, call it D

Draw a circle with center C and radius $2l$

It intersects the line $\overleftrightarrow{BC}$ on the same side as B at one point, call it E

By construction, $AD = 2l$, then $BD = l$

Similarly, $BE = l$

By 4 right angles theorem, $\angle ABC, \angle CBD, \angle DBE, \angle EBA$ are right

Then by SAS, $\Delta ABC \cong \Delta CBD \cong \Delta DBE \cong \Delta EBA$

Then $AC = CD = ED = AE = \sqrt{2}l$

Then $ACDE$ is a square with side length $\sqrt{2}l$

Then $ACDE$ has area $2l$

16P Construction Problem 16.25 (Inscribed Circle)

Given a triangle, construct its inscribed circle.

We are given a triangle ΔABC

Draw lines bisecting $\angle ABC, \angle ACB$

They intersect at a point inside the triangle, call it D

From D, drop perpendiculars $DE \perp AB, DG \perp AC, DF \perp BC$

$\angle ABD = \angle CBD$, by construction of angle bisector $\overleftrightarrow{BD}$

By construction of perpendicular, $\angle BED = \angle BFD = 90$

Then by ASA, $\Delta EBD \cong \Delta FBD$

Then $DE = DF$

Similarly, we can show that $\Delta CGD \cong \Delta CDF$

Then $DG = DF$

Then $DE = DG = DF$

So if we draw a circle $\mathscr{C}$ with center D and radius DF, it will contain three points E, F, G on the triangle.

Then $\mathscr{C}$ is an inscribed circle in the triangle ΔABC.