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MATH 394 HW 6
Ch 5 pg 212: 1, 3, 6, 7, 10, 12, 21, 23, 29, 31, 32, 34, 38, 40 and 41

1 $f(x) = c(1-x^2), -1 < x < 1, 0$ otherwise

a) Find c

Know that $\int_{-\infty}^{\infty} f(x)dx = 1$

$f(x)$ is only non zero on $-1 < x < 1$, so integral is $\int_{-1}^{1} c(1-x^2)dx$

This is equal to $c\int_{-1}^{1}(1-x^2)dx = c(x - \frac{x^3}{3})\Big|_{-1}^{1} = c(1-\frac{1}{3}) - c(-1+\frac{1}{3} = \frac{4c}{3}$

So $\frac{4c}{3} = 1$, so $c = \frac{3}{4}$

b) What is the cumulative distribution function of X?

The cumulative distribution function of X is $P(X \leq x)$

This is equal to $\int_{-\infty}^{x} f(x)dx$

Know that integral is $\int_{-1}^{x} \frac{3}{4}(1-x^2)dx$

This is equal to $\frac{3}{4}(x - \frac{x^3}{3})\Big|_{-1}^{x}$

This is equal to $\frac{3}{4}[(x - \frac{x^3}{3}) - (-1 + \frac{1}{3})]$

Which is equal to $\frac{3x - x^3 + 2}{4}$

3 $f(x) = C(2x - x^3)$ for $0 < x < \frac{5}{2}$, 0 otherwise

Could f be a probability density function? If so, determine C.

In both cases, they cannot be density functions, since there is a point where $f(x)$ will be negative

In the first function, when $x = 5/2$, it is the opposite sign of when $x = 1$, regardless of C, so one of the must be negative, so it cannot be a density function

In the second function, when $x = 5/2$, it is the opposite sign of when $x = 1$, regardless of the value of C, so it cannot be a density function.

6 We know that E[X] $= \int_{-\infty}^{\infty} xf(x)dx$

a) E[X] $= 0 + \int_{0}^{\infty} x\frac{1}{4}xe^{-x/2}dx$

Substitution, let $t = \frac{x}{2}, dx = 2dt$

E[X] $= \int_{0}^{\infty}(2t)^2 e^t(2dt)$

E[X] $= 2\int_{0}^{\infty} t^2 e^{-t}dt$

Know gamma function defined by $\Gamma(n) = \int_{0}^{\infty} x^{n-1}e^{-}xdx, \Gamma(n) = (n-1)!$

E[X] $= 2\Gamma(3) = 4$

b) We found $c = \frac{3}{4}$ in exercise 1 part a

E[X] $= 0 + [\int_{-1}^{1} x\frac{3}{4}(1-x^2)dx] + 0$

E[X] $= \frac{3}{4}\int_{-1}^{1} x - x^3 dx$

E[X] $= \frac{3}{4}[\frac{1}{2}x^2 - \frac{1}{4}x^4]\Big|_{-1}^{1}$

E[X] $= \frac{3}{4}[(0.5 - 0.25) - (0.5 - 0.25)] = 0$

c) E[X] $= 0 + \int_5^{\infty} x\frac{5}{x^2}dx$

E[X] $= 5\int_5^{\infty} \frac{1}{x}dx = 5ln(x)\Big|_5^{\infty} = \infty$

7 $f(x) = a + bx^2, 0 \leq x \leq 1$, 0 otherwise

E[X] $= \frac{3}{5}$, find a, b

Know that $\int_{-\infty}^{\infty} f(x) = 1$

So $\int_0^1 a + bx^2 dx = 1$

So $ax + \frac{b}{3}x^3\Big|_0^1 = a + \frac{b}{3} = 1$

Know E[X] $= \frac{3}{5}$, so $\int_0^1 xa + bx^3 dx = \frac{3}{5}$

So $\frac{1}{2}ax^2 + \frac{b}{4}x^4\Big|_0^1 = \frac{a}{2} + \frac{b}{4} = \frac{3}{5}$

Solving system of equations for a, b,

So $a = \frac{3}{5}, b = \frac{6}{5}$

10 a)

The person can only ride from 7:05 - 7:15, 7:20 - 7:30, 7:35 - 7:45, or 7:50 - 8:00

Then the total number of minutes that this is possible is 40, with total minutes possible 60

Then $p(A) = \frac{40}{60} = \frac{2}{3}$

b) This is the same as part a, since the times to ride will still total 40 minutes from 7:10 to 8:10, and the total possible minutes will still be 60, for $p(b) = \frac{2}{3}$

12 The bus can break down anywhere with equal probability

Then E[X] = 50

Since it is uniform, breakdowns will be spread around the expected value

So it is more advantageous to have stations between the midpoint and end points.

21 We know that the height of a 25 year old man is a normal random variable with parameters $\mu = 71, \sigma^2 = 6.25$

a) Find $P(X > 74)$

$P(X > 74) = P(\frac{x-71}{\sqrt{6.25}} > \frac{3}{\sqrt{6.25}})$

$= P(x > \frac{3}{\sqrt{6.25}})$

$= P(z < \frac{3}{\sqrt{6.25}}$

$= 1 - \Phi(1.2) \approx 0.1151$

b) Find $P(X > 77|x \geq 72)$

$= \frac{P(x>77)}{P(x>72)}$

$= \frac{1-P(z<\frac{6}{\sqrt{6.25}})}{1-P(z<\frac{6}{\sqrt{6.25}}}$

$= \frac{1-\Phi(2.4)}{1-\Phi(0.4)}$

≈ 0.0238

23 Let X be a binomial random variable for number of 6's with probability 1/6, n = 1000

then EX = $\frac{1000}{6} \approx 166.6$

And Var(X) = $\frac{1000}{6}\frac{5}{6} = \frac{5000}{36} \approx 138.8$

a) Find $P(149.5 \leq x \leq 200.5)$

$= P(\frac{149.5-166.6}{\sqrt{138.8}} < z < \frac{200.5-166.6}{\sqrt{138.8}}$

$= \Phi(2.87) - 1 + \Phi(1.46) \approx 0.92$

29 $u = 1.012, d = 0.990, p = 0.52$

Find probability that stocks will be up at least 30 percent after 1000 periods

Let X be the number of time periods with stock increases

Stock price = $s[u^x d^{1000-x}]$

$= sd^{1000}(\frac{u}{d})^x$

Want $d^{1000}(\frac{u}{d})^x > 1.3$

Which is $x > \frac{ln(1.3-1000ln(d)}{ln\frac{u}{d}} \approx 469.2$

So x needs to rise at least 470 times

So we need to find $P(X > 469.5) = P(\frac{x-520}{\sqrt{249.6}} > \frac{469.5-520}{\sqrt{249.6}}$

$= P(z > -3.19)$

≈ 0.99

31 a) The least travel for a uniform distribution would occur at the midpoint

This is equal to $\frac{A}{2}$

b) E[X - a] = $\int_0^A |x-a|\lambda e^{-\lambda x} dx$

$= \int_0^a (a-x)\lambda e^{-\lambda x} dx + \int_a^\infty (x-a)\lambda e^{-\lambda x} dx$

$= a + \frac{2}{\lambda}e^{-\lambda a} - \frac{1}{\lambda}$

Differentiating and setting to 0, we get $a = fracln(2)\lambda$

32 Exponentially distributed with random variable parameter $\lambda = \frac{1}{2}$

a) $P(x > 2) =?$

$P(x > 2) = \int_2^\infty \frac{1}{2}e^{\frac{-1}{2}t} dt$

$= e^{-1}$

b) $P(x > 10|x > 9)$?

This is equal to $P(x > 10 - 9) = P(x > 1)$

This is equal to $1 - P(x < 1) = 1 - (1 - e^{-1/2}) = e^{-\frac{1}{2}}$

34 Exponential random variable with parameter $\frac{1}{20}$

a) Find $P(x > 30000|x > 10000)$

This is equal to $P(X > 20000)$

$= \int_{20}^{\infty} e^{\frac{-1}{20}x} dx$

$= e^{-1}$

b) Let X $U(0, 40)$

Want $P(X > 30|x > 10)$

This is equal to $\frac{P(X>30)}{P(X>10)}$

$= \frac{\int_{30}^{40} 1/40dx}{\int_{10}^{40} 1/40dx}$

$= \frac{1}{3}$

38 For roots to be real, discriminant must be nonnegative

For $4x^2 + 4xy + y + 2 = 0$

Discriminant is $(4y)^2 - 4(4(y + 2)) \geq 0$

So $(y - 2)(y + 1) \geq 0$

So $y \geq 2$

Uniform over (0,5), need Y greater than or equal to 2

So $P(y \geq 2) = \frac{3}{5}$

40 $Y = e^x$

$F_y(y) = P(e^x \leq y) = P(x \leq lny)$

$= F_x(lny)$

$F_y(y) = f_x(lny)\frac{1}{y} = \frac{1}{y}$ for $1 < y < e$

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