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MATH 445 HW 1
7: B, D, E, F, G

7B Prove Theorem 7.7 (Existence and uniqueness of perpendicular bisectors)

Theorem 7.7: Every segment has a unique perpendicular bisector

7D Prove Theorem 7.10 (Existence and uniqueness of a reflected point)

Theorem 7.10: Let ℓ be a line and let A be a point not on ℓ. Then there is a unique point A', called the reflection of A across ℓ, such that ℓ is hte perpendicular bisector of $\overline{AA'}$.

7E Prove Lemma 7.12 (Properties of closest points)

Let P be a point and let S be any set of points.

a) If C is a closest point to P in S, then $C' \in S$ is also a closest point to P if and only if $PC' = PC$

b) If C is a point in S such that $PX > PC$ for every point $X \in S$ other than C, then C ius the unique closest point to P in S.

7F Prove Theorem 7.13 (The closest point on a line)

Suppose ℓ is a line, P is a point not on ℓ, and F is the foot of the perpendicular from P to ℓ

a) F is the unique closest point to P on ℓ

b) If A, B are points on ℓ such that $F * A * B$, then $PB > PA$

7G Prove theorem 7.14 (The lcosest point on a segment) [Hint: consider separately the cases in which $P \in \overleftrightarrow{AB}$ and $P \notin \overleftrightarrow{AB}$, and divide the second case into two subcases depending on whether the foot of the perpendicular from P to $\overleftrightarrow{AB}$ does or does not lie in $\overline{AB}$].

Theorem 7.14: Suppose $\overleftrightarrow{AB}$ is a segment and P is any point. Then there is a unique closest point to P in $\overleftrightarrow{AB}$