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MATH 444 HW 7
5B, 5C, 5D, 5E, 5F

5B Prove Corollary 5.3 (to Pasch's theorem)

Theorem 5.2 (Pasch's theorem): Suppose $\Delta ABC$ is a triangle and $\ell$ is a line that does not contain any of the points $A, b, C$. If $\ell$ intersects one of the sides of $\Delta ABC$, then italso intersects another side.

Corollary 5.3: If $\Delta ABC$ is a triangle and $\ell$ is a line that does not contain any of the points $A, B, C$, then either $\ell$ intersects exactly two sides of $\Delta ABC$ or it intersects none of them.

$\ell$ is a line. It does not contain any of the points $A, B, C$, so it is not collinear to any of the segments $\overline{AB}, \overline{BC}, \overline{AC}$

Case: $\ell$ does not intersect any of the line segments.

Then $\ell$ does not intersect any of the line segments.

Case: $\ell$ intersects one of the line segments. Then by theorem 5.2, it intersects another side.

Then $\ell$ intersects exactly two sides of $\Delta ABC$

5C Suppose $\Delta ABC$ is a triangle and $\ell$ is a line (which might or might not contain one or more vertices). Is it possible for $\ell$ to intersect exactly one side of $\Delta ABC$? Exactly two? All three? In each case, either give an example or prove that it is impossible.

Case 1: Intersects exactly one side of $\Delta ABC$

If $\ell$ intersects at a vertice, for example $A$, then since $A \in \overline{AB}, \overline{AC}$, then it intersects more than one side.

Then assume that $A$ does not intersect at a vertice, and intersects at one side.

But by theorem 5.2, we know that if it intersects at one side, it must intersect another side.

Then it is not possible for $\ell$ to intersect the triangle on exactly one side.

Case 2: Intersects exactly two sides of $\Delta ABC$

Yes, it is possible: example picture provided (Picture is of triangle $\Delta ABC$, and line $\ell$, where $\ell$ intersects through sides $\overline{AB}, \overline{AC}$, and touches none of the vertices.)

Case 3: Intersects exactly three sides of $\Delta ABC$

Yes, it is possible: example picture provided (Picture is of triangle $\Delta ABC$, and line $\ell$, where $\ell$ is collinear to $\overline{AB}$, and touches both vertices $A, B$)

5D Prove Theorem 5.8 (The converse to the isoceles triangle theorem). [Hint: One way to proceed is to construct an indirect proof, like Euclid's proof of proposition 1.6. Another is to mimic Pappus's proof of the isosceles triangle theorem.]

Theorem 5.7: Isosceles Triangle theorem: If two sides of a triangle are congruent to each other, then the angles opposite those sides are congruent.

Theorem 5.8: If two angles of a triangle are congruent to each other, then the sides opposite those angles are congruent.

Let $\Delta ABC$ be a triangle with $\angle B \cong \angle C$

Then draw the mirrored image $\Delta A'B'C'$ under the correspondence $A \leftrightarrow B$, $B \leftrightarrow C$, and $C \leftrightarrow B$

The triangles $\Delta ABC, \Delta A'B'C'$ satisfy the hypothesis set by ASA congruence, since $\overline{BC} \cong \overline{B'C'}$

and $\angle B \equiv \angle B', \angle C \cong \angle C'$

Thus the corresponding sides $\overline{AB}, \overline{AC}$ are congruent

The figure shown is the same as Fig 5.7 from the book, mimicing Pappus's proof.

5E Prove Theorem 5.10 (The triangle copying theorem)

Theorem 5.10: Suppose $\Delta ABC$ is a triangle and $\overline{DE}$ is a segment congruent to $\overline{AB}$. On each side of $\overleftrightarrow{DE}$ there is a point $F$ such that $\Delta DEF \cong \Delta ABC$

$\overline{DE} \cong \overline{AB}$

Take $F$ on one side of $\overline{DE}$ such that $\angle ABC \cong \angle FDE$ and $\overline{FD} \cong \overline{AB}$

Then by SAS postulate, we have the the triangle $\Delta FDE \cong \Delta ABC$

On the other side $\overleftrightarrow{DE}$, take $F'$ similarly.

Then we have $\angle ABC \cong \angle F'DE$ and $\overline{F'D} \cong \overline{AB}$

And we know$\overline{DE} \cong \overline{AB}$

Then by SAS postulate $\Delta F'DE \cong \Delta ABC$

5F Prove Theorem 5.18 (The triangle inequality)

If $A, B, C$ are noncollinear points, then $AC < AB + BC$

Construct a triangle with vertices $A, B, C$, call it $\Delta ABC$

Take $D$ on $\overleftrightarrow{AB}$ such that $B$ is between $A, D$ and $BD = DC$

Then by isosceles triangle theorem, $\Delta BCD$ is an isosceles triangle, with $\angle BDC \cong \angle BCD$

We know that $\angle ACD = \angle ACB + \angle BCD$

Then $\angle ACD > \angle BCD = \angle CDB = \angle CDA$

So by the scalene inequality, $\overline{AD} > \overline{AC}$

Therefore, since $B$ is between $A, D$, $BC + AB = BD + AB = AD > AC$

Then we have shown that the sum of the length of two sides is strictly greater than the length of the third.