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In-Class Presentation
Construction Problem 16.28

16.28 Let $\mathscr{C}$ be a circle, and A be a point on the circle.

Draw the line $\overleftrightarrow{OA}$, and take point P on $\mathscr{C}$ different from A

Take point G on $\overline{OP}$ with $\frac{PO}{PG}$ equal to the golden ratio

Draw circle $\mathscr{D}$ with center P containing point G.

Then by theorem 14.10, $\mathscr{D}$ contains an interior point in $\mathscr{C}$, so it must intersect at exactly two other points, call them C, D

Draw circle with center C containing point D, call the other point of intersection B

Draw circle with center D containing point E, call the other point of intersection E

Draw pentagon $ABCDE$

We know $\overline{PC} \cong \overline{PG} \cong \overline{PD}$, since both are radii of same circle $\mathscr{C}$

Similary $\overline{OP} \cong \overline{OC}$

We know $\frac{PO}{PC} = \frac{PO}{PG}$, then ΔPOC is a golden triangle.

Then $\angle POC = 36$

Similarly, $\angle POD = 36$

Since D, C on opposite sides of $\overleftrightarrow{OP}$, and by two circles theorem, $\angle POC, \angle POD$ are adjacent angles, so $\angle COD = 72$

$\Delta COD \cong \Delta BOC \cong \Delta DOE$ by SSS

Then $\angle BOC = \angle DOE = 72$

Apply linear triple theorem for angles $\angle AOB, \angle BOC, \angle POC$

Then $\angle AOB$ must be 72

Apply linear triple theorem for $\angle AOE, \angle EOD, \angle POD$, then $\angle AOE = 72$

Then by SAS, triangles $\Delta AOB, \Delta AOE$ are congruent to triangles $\Delta BOC, \Delta COD, \Delta DOE$

Then $AB = BC = CD = DE = EA$

Then $ABCDE$ is equilateral.