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MATH 445 HW 2
7: J, K
10: A, B, H

7J Prove Corollary 7.20 (the corresponding angles theorem)

Corollary 7.20: If two lines are cut by a transversal making a pair of congruent corresponding angles, then they are parallel.

ℓ and ℓ' are distinct lines, cut by a transversal t

t intersects ℓ at a point A

t intersects ℓ' at a point A'

Forming angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$

Without loss of generality, suppose the congruent corresponding angles are $\angle 2, \angle 6$

We know that $\angle 2, \angle 3$ are vertical angles

Then $\angle 2 \cong \angle 3$

So $\angle 3 \cong \angle 6$

And we know that $\angle 3, \angle 6$ are alternate interior angles

Then by theorem 7.19, ℓ, ℓ' are parallel.

7K Prove Corollary 7.21 (the consecutive interior angles theorem)

Corollary 7.21: If two lines are cut by a transversal making a pair of supplementary consecutive interior angles, then they are parallel

ℓ and ℓ' are distinct lines, cut by a transversal t

t intersects ℓ at a point A

t intersects ℓ' at a point A'

Forming angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$

Without loss of generality, assume $\angle 4, \angle 6$ are supplementary consecutive interior angles

So $\angle 4 + \angle 6 = 180$

$\angle 4, \angle 3$ form a linear pair, so $\angle 4 + \angle 3 = 180$

$\angle 3 = \angle 6$

So $\angle 3, \angle 6$ congruent

And we know that $\angle 3, \angle 6$ are alternate interior angles

So by theorem 7.19, ℓ, ℓ' are parallel

10A Prove corollary 10.2 (the converse to the corresponding angles theorem)

Corollary 10.2: If two parallel lines are cut by a transversal, then all four pairs of corresponding angles are congruent.

ℓ and ℓ' are distinct lines, cut by a transversal t

t intersects ℓ at a point A

t intersects ℓ' at a point A'

Forming angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$

ℓ, ℓ' are parallel, by assumption.

Then by theorem 10.1, $\angle 3, \angle 6$ are congruent, and $\angle 4, \angle 5$ are congruent

We know that $\angle 2, \angle 3$ are vertical angles, so they are congruent

$\angle 1, \angle 4$ are vertical angles and congruent

And $\angle 7, \angle 6$ are vertical angles, so they are congruent

$\angle 5, \angle 8$ are vertical angles, congruent

So $\angle 2 \cong \angle 6$

$\angle 4 \cong \angle 8$

$\angle 1 \cong \angle 5$

$\angle 3 \cong \angle 7$

Then all four pairs of corresponding angles are congruent

10B Prove corollary 10.3 (the converse ot the consecutive interior angles theorem)

Corollary 10.3: If two parallel lines are cut by a transversal, then both pairs of consecutive interior angles are supplementary

ℓ and ℓ' are distinct lines, cut by a transversal t

t intersects ℓ at a point A

t intersects ℓ' at a point A'

Forming angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$

ℓ, ℓ' are parallel, by assumption.

Then by theorem 10.1, $\angle 3, \angle 6$ are congruent, and $\angle 4, \angle 5$ are congruent

$\angle 3, \angle 4$ form a linear pair, so $\angle 3 + \angle 4 = 180$

and $\angle 5, \angle 6$ form a linear pair, so $\angle 5 + \angle 6 = 180$

Since $\angle 3, \angle 6$ congruent, then $\angle 5 + \angle 3 = 180$

And $\angle 4, \angle 5$ congruent, so $\angle 4 + \angle 6 = 180$

Then $\angle 4, \angle 6$ and $\angle 3, \angle 5$ are pairs of interior angles, and are supplementary

10H Prove theorem 10.17 (the AAA construction theorem)

Theorem 10.17: Suppose $\overline{AB}$ is a segment and α, β, γ are three positive real numbers whose sum is 180. On each side of $\overleftrightarrow{AB}$, there is a point C such that ΔABC has the following angle measures $m\angle A = \alpha, m\angle B = \beta, m\angle C = \gamma$.

$\overline{AB}$ is a segment with unique points A, B

We can construct a line ℓ through the point A, such that the angle between $\ell, \overline{AB}$, call it A, is α

We can construct a line ℓ' through at point B, such that the angle between $\ell', \overline{AB}$, call it B, is β

A, B distinct, α, β positive, less than 180, so ℓ, ℓ' distinct

ℓ, ℓ' are cut by transversal $\overleftrightarrow{AB}$

and angles A, B are positive, angles add up to less than 180

Then by theorem 10.16, ℓ, ℓ' intersect on the same side of the transversal $\overleftrightarrow{AB}$ as the two angles

Call this intersection C

This forms a triangle ΔABC

We know by theorem 10.11 that every triangle has angle sum 180

$\angle A = \alpha, \angle B = \beta$ by construction

Then $\angle A + \angle B + \angle C = 180$

And we know $\alpha + \beta + \gamma = 180$

Then $m\angle C = \gamma$