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 MATH 395 HW 8
 7: 58, 69ab
 8: 2, 4, 7, 8

- 7.58 A coin having probability p of coming up heads is continually flipped until both heads and tails have appeared. Find
- The expected number of flips;
 - The probability that the last flip lands of heads.

- 7.69ab The number of accidents that a person has in a given year is a Poisson random variable with mean λ . However, suppose that the value of λ changes from person to person, being less than or equal to $1 - e^{-x}$. If a person a person is chosen at random, what is the probability that he will have
- 0 accidents
 - exactly 3 accidents in a certain year?

- 8.2 From past experience, a professor knows tha the test score of a student taking her final examination is a random variable with mean 75.

- Give an upper bound for the probability that a student's test score will exceed 85. Suppose, in addition, that the professor knows that the variance of a student's test score is equal to 25.

$$P(X \geq a) \leq \frac{E[x]}{a}$$

$$\text{Upper bound } P(X \geq 85) \leq \frac{75}{85}$$

- What can be said about the probability that a student will score between 65 and 85?

$$P(|X - \mu| \geq b) \leq \frac{\sigma^2}{b^2}$$

$$P(|X - 75| \geq 10) \leq \frac{25}{100}$$

- How many students would have to take the examination to ensure with a probability at least 0.9 that the class average would be within 5 of 75? Do not use the central limit theorem.

$$P(|\frac{1}{n} \sum_1^n X_i - \mu| \geq \epsilon) \leq \frac{\sigma^2}{n\epsilon^2}$$

$$P(|\frac{1}{n} \sum_1^n X_i - 75| \geq 5) = \frac{25}{n25} = \frac{1}{n}$$

$$P(|\frac{1}{n} \sum_1^n X_i - 75| < 5) \geq 1 - \frac{1}{n}$$

So for $p \geq 0.9$, this is $n \geq 10$

- 8.4 Let X_1, \dots, x_{20} be independent Poisson random variables with mean 1.

- Use the Markov inequality to obtain a bound on $P(\sum_1^{20} X_i > 15)$
- Use the central limit theorem to approximate $P(\sum_1^{20} X_i > 15)$

- 8.7 A person has 100 lightbulbs whose lifetimes are independent exponentials with mean 5 hours. If the lightbulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.

- 8.8 In problem 8.7, suppose that it takes a random time, uniformly distributed over $(0, 0.5)$, to replace a failed bulb. Approximate the probability that all bulbs have failed by time 550.