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MATH 445 HW 4
Due Wednesday 4/25
12 A,D,E
13 A,C

12A Prove theorem 12.5 (The SSS similarity theorem). [Hint: Use the similar triangle construction theorem to construct triangle $\Delta D'E'F'$ that is similar to ΔABC, but with $\overline{D'E'} \cong \overline{DE}$. Then use the hypothesis and a little algebra to show that $\Delta D'E'F' \cong \Delta DEF$

Theorem 12.5: If ΔABC and ΔDEF are triangles such that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, then $\Delta ABC\ \Delta DEF$

By theorem 12.4, we can construct a triangle $\Delta D'E'F'$ similar to ΔABC with side length $D'E' \cong DE$.

Then by construction of similar triangle, $\frac{AB}{D'E'} = \frac{BC}{E'F'} = \frac{BC}{E'F'}$

By transitive property of similarity, $\Delta DEF\ \Delta D'E'F'$

Then by definition of similar triangle, $\frac{DE}{D'E'} = \frac{D'F'}{D'F'} = \frac{E'F'}{E'F'}$

But we know that $D'E' = DE$

So $DE = D'E', D'F' = D'F', E'F' = E'F'$

Then $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

12D CONVERSE TO THE ANGLE BISECTOR PROPORTION THEOREM: Suppose ΔABC is a triangle and D is a point in the interior of $\overline{BC}$ such that $\frac{BD}{DC} = \frac{AD}{AC}$. Prove that $\overrightarrow{AD}$ is the bisector of $\angle BAC$

12E Prove that the three midsegments of a triangle yield an admissible decomposition of the triangle into four congruent triangles, each of which is similar to the original one and has one quarter the area.

Construct a triangle ΔABC

Let D be the midpoint of $\overline{AB}$

Let E be the midpoint of $\overline{BC}$

Let F be the midpoint of $\overline{AC}$

$\frac{AD}{AB} = \frac{1}{2} = \frac{AF}{AC}$

For triangle ΔADF, it has angle $\angle BAC \cong \angle DAF$

So by theorem 12.6, SAS similarity, $\Delta ADF\ \Delta ABC$

Using a similar argument for $\Delta DBE, \Delta EFC$, we find that

$\Delta DBE\ \Delta ABC$, and $\Delta EFC\ \Delta ABC$

So $\Delta ADF\ \Delta DBE\ \Delta EFC$

So by definition of similar triangles $\frac{AD}{DB} = \frac{AF}{DE} = \frac{DF}{BE}$

Since length $AD = DB$, then $AF = DE$ and $DF = BE$

So by SSS congruence, $\Delta ADF \cong \Delta DBE$

Similarly, we show that $\Delta EFC \cong \Delta ADF \cong DBE$

For triangle ΔDEF, it is formed by shared side lengths with the other triangles.

We get that $DF \cong DF$, $DE \cong DE \cong AF$, and $FE \cong FE \cong AD$

Then $\Delta DEF \cong \Delta ADF \cong DBE \cong EFC$

And ΔDEF ΔADF ΔDBE ΔEFC ΔABC

13A

13C