PLAY OPTIMAL POKER

Practical Game Theory For Every Poker Player

ANDREW BROKOS

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by Andrew Brokos

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Introduction

You are playing No-Limit Texas Hold 'Em. The river card has just been dealt, and it's a fourth club. You double-check your cards, but unfortunately, they have not changed: you've got no flush, no pair, and, you figure, no shot of winning this pot. Unless you bluff.

As you gather your chips, you watch your opponent for a glimmer of fear or excitement, but he's stone-faced. The clock is ticking. It's decision time. Are you all-in?

Maybe, before you answer, you want more information. What exactly is on the board? What was the action leading up to the river? Who is your opponent? Is his playing style wild, conservative, or somewhere in between? How many chips do you have? How much is in the pot?

I could answer all these questions and more - I could even tell you your opponent's hand! - and you still might not know whether you should bluff. The critical bit of information you don't have is what your opponent will do after you bet, whether he will call or fold. In most cases, you won't be able to determine that with certainty no matter how much other information you have.

This is the essence of what makes poker fun and interesting in a way that other gambling games are not. Unlike in craps or blackjack, you do not make decisions in a vacuum. Your *payoff*, what you win or lose, depends not only on the choices that you make but also on the choices that other players make.

Because you and your opponents have opposing interests - every dollar you win is a dollar someone else loses - there is conflict and deception. To get the outcomes you want, you need your opponent to take the actions that are best for you. But typically, what’s good for you is bad for him, and so he will try *not* to take the actions you want him to take. Your opponent will attempt to deceive you about what he plans to do, just as you will attempt to deceive him about what you *want* him to do. He will study you carefully, attempting to predict your future actions just as you attempt to predict his.

The frustrating thing about poker is that you can never be sure about your opponent’s intentions, which means that in most cases you cannot be sure about your own best play. Even after the fact, you won’t know whether your play was a mistake.

As human beings, we’re used to learning from experience. If we do something and get a good result, we do it again. If we get a bad result, we try something different.

Poker doesn’t work that way, which makes it impossible to learn through experience alone. Sure, you’ll get better with practice, but you’ll also pick up some bad habits.

Not only are results not a reliable guide to whether you made a mistake, but *there is no reliable guide*. I spent the first decade of my career grasping in the dark, trying to grab something firm but coming away with handfuls of sand.

Like most people, I started by looking to the results. If I got a fold, then my bluff was a good one, and if I didn’t, then it was a mistake.

Then I learned about ranges, and I realized that sometimes, when my bluffs are called, it’s just bad luck. If I go all-in on the river and my opponent calls with quads, that doesn’t tell me anything, because that wasn’t the hand I was expecting him to fold. If he could have had many weaker hands that would have folded, then my bluffs might have been good a good one that just got unlucky.

But how are you supposed to know what your opponent could have had or would have done? You can ask, but he might not tell you. Or he might lie. Or he might just not know the answer himself. Plenty of players have no self-awareness about their own ranges.

So, I flailed around, like pretty much everyone was doing in those days. I struck upon some things that worked, but I couldn’t always tell you *why* they worked, and that made it hard to distinguish between a bad idea and a good idea that just didn’t work this time.

My quest for firm footing eventually led to game theory. While game theory doesn’t exactly offer an escape from uncertainty, it does offer a way through, which is the best you’re going to get in this cruel game of ours.

Game theory is about making decisions under conditions of uncertainty. We don’t know what our opponents have, but we know what they *could* have. We don’t know what they’re going to do, but we know what they *could* do.

The good news is that your opponents must grapple with the same uncertainty. They don’t know what you have, and they don’t know what you’re going to do. Uncertainty is as much their enemy as it is yours. When you get right down to it, poker isn’t a battle of guts or wits, it’s a contest of who can make better decisions under conditions of uncertainty, and game theory can be a valuable ally.

This realization revolutionized how I thought about the game. Now, when I play poker, my objective isn’t to play tight or to play circles around my opponents or even to win money. My objective is to give my opponents difficult decisions, to put them in spots where they have no idea what the right play is. I can’t force them to make mistakes, but I can create the conditions that lead to mistakes, and it turns out that that’s both profitable and fun.

Why *You* Need Game Theory

There's a pernicious myth out there that game theory is only for elite players in tough games. Not true. Game theory is for anyone who's ever been unsure about what to do at the poker table. If that doesn't describe you, then please get in touch – I want to read your book.

Oxford Dictionaries describe game theory as "the branch of mathematics concerned with the analysis of strategies for dealing with competitive situations where the outcome of a participant's choice of action depends critically on the actions of other participants." In short, game theory helps us formulate strategies that will be reasonably good regardless of what actions our opponents take.

Once, only the most mathematically astute poker players spent much time thinking about game theory. It helped them better understand some broad strategic principles, but the complexity of poker necessitated a lot of guesswork when it came to the details.

Now, however, dedicated software and ever-cheaper processing power enable any player to apply game theoretic principles to poker with rigor and precision. Even if you don't personally use software to study poker, if you've read a poker book or watched an instructional video in the last few years, then you've benefited indirectly from the insights that such software provides.

Game theory is not a panacea. It is not the only lens through which one should view poker, but it is an increasingly indispensable one. If you don't understand the basic game theory that underlies poker, then you don't really understand the game at all. At best, you've developed, through trial and error, strategies that work against certain types of opponents. You may be comfortable in situations that you encounter commonly, but you probably struggle in less familiar situations, and as your opponents become more adept with game theory, they will increasingly put you in difficult spots.

Game theory solver software generates extremely complex strategies, but you do not have to understand them in-depth or implement them precisely in order to learn from them. Indeed, unless you are competing at the very highest level, your goal when studying game theory should not be to memorize the details for any specific situation. Rather, your goal should be to extract broad principles that can guide you in a variety of situations when you are unsure about the best strategy.

This book will get you well on your way. Starting from simple scenarios, we'll tease out the most important theoretical concepts that underlie poker strategy. At each stage, you will acquire tools that will be useful immediately and analytical skills that will enable you to investigate more complex situations. You will learn how to ask the right questions, draw the right conclusions, and apply these concepts in real time as you play.

We will focus on identifying trends and understanding principles rather than memorizing frequencies. In some cases, we may call into question what you think you know about poker strategy. In other cases, we'll simply shed light on the theory that underlies what you already do well. This will enable you to better understand why the things you already do effectively work and under what conditions they might not work so well.

Ultimately, understanding game theory will make you more comfortable at the poker table. You will be better equipped to exploit the mistakes of less-skilled opponents and to hold your own against those more skilled than you. Best of all, you'll feel more confident and better prepared to handle any situation that our beloved game might throw at you.

The Game Theory Approach

Most of what you know about poker strategy probably comes from an exploitative framework. In other words, it depends on certain assumptions about how your opponents will play. You may or may not be aware of those assumptions, but they are there.

For example, you've probably heard that if you raised before the flop, you should make a “continuation bet” on the flop even if you whiffed it entirely. That’s not actually an iron-clad law of poker. The profitability of such a bet depends on certain assumptions about your opponents, namely that they call too much pre-flop, give up too easily on the flop, and rarely raise as a bluff.

Depending on the environment in which you're playing, these may well be safe assumptions. The thing about exploitative play, though, is that your opponents may be trying to figure out and counter-exploit your strategy. Any time you adapt your own play in order to exploit a mistake you expect your opponent to make, you also open yourself up to exploitation. If your assumption turns out to be wrong, or if your opponent sees you coming and deliberately does the opposite of what you expect, then you are the one getting exploited.

For example, if you decide to bluff at a high frequency because you think your opponent will fold too much, that's a great strategy if your assumption is correct. But if your assumption is wrong and your opponent will in fact call too much, then you're making an expensive mistake.

Game theory offers a completely different framework for thinking about poker. It assumes that any time there is a mathematically correct play, your opponent will make it. Your goal is to create situations where there is no correct play. If your opponent calls with a hand that can only beat a bluff, you'll have the winning hand just often enough to make him regret it. If he folds that same hand, you'll have a bluff just often enough to make him regret that too.

You're probably rolling your eyes and thinking about someone specific right now, which raises an important point about when to use game theory. You should treat it as a set of tools that you take out when needed. If you believe a certain opponent will make a certain mistake, then play accordingly – you may not need game theory to take advantage of that situation. But you probably don't know how that player will behave in every possible situation, and that's when game theory comes in handy.

To be clear, we do not assume that your opponent knows your hand, but we do assume that he knows your strategy. When you go all-in with a flush on the river, we don't assume that he knows you have a flush, but we do assume that he knows you could have a flush. If you would never go all-in in that spot unless you had a flush, then we assume he knows that too. And if it's true that you would never go all-in without a flush, then there's a clear counter-strategy for him: he folds when he can't beat a flush and calls when he can.

To avoid giving your opponent the opportunity to employ this easy counter-strategy, you must balance your bet with some bluffs. Later, we'll talk about how exactly to determine which hands to bluff with, but for now you just need to understand that in order to avoid giving your opponent profitable opportunities, your bets - most of your actions, actually - must be made with a variety of hands.

River bets make for easy examples, but balance is relevant everywhere. When you raise pre-flop, you should typically do so with a mix of hands, some of which are eager to get re-raised, some of which will begrudgingly call a re-raise, and some of which will fold to a re-raise. If you get the mix right, then your opponents will never be able to predict how you will respond to a re-raise, which makes it difficult for them to determine whether they should re-raise you in any given situation.

A common misunderstanding is that game theory requires you to throw your reads out the window and pretend that your opponents will play perfectly. But game theory doesn't require you to do anything. It's a tool, and as with any tool, it's up to you to decide whether it's suited to the task at hand.

When you know, or at least think you can guess, what your opponent's strategy will be, that's the time to craft an exploitative strategy. When you aren't comfortable making such an assumption, either because your opponent is a strong player or simply because you aren't sure about how he’ll play in this particular situation, then reach into your game theory toolbox.

The game theory approach can even open your eyes to new opportunities for exploitation. For example, there are many situations where it is not obvious which hands would make for good bluffs, and as a result many people simply don’t bluff in these spots. Yet precisely because it's so rare to see bluffs in these situations, many of your opponents probably fold too much when faced with a such a bet. Thus, if you can identify these non-intuitive bluffs, you can make a lot of money with them. Game theory can you help recognize these spots and identify which hands would make the best candidates for bluffing.

We'll start by looking at equilibrium solutions to simple games and common poker situations. Then, we'll take the lessons that we learn from studying these equilibria and practice deriving exploitative strategies. By the end of this book, you should be comfortable thinking in terms of both equilibrium and exploitation and better equipped to decide when each approach is best.

How to Use This Book

Play Optimal Poker is organized around "scenarios" designed to highlight specific concepts. We'll start by looking at some toy games, which are simple games that bear some resemblance to poker but that are much less complex. These will be relatively easy to solve and will enable us to focus on one or two concepts at a time, without all the competing priorities that arise in real poker situations. Each scenario adds a new layer of complexity, enabling us to see how all the pieces fit together. Eventually, we'll work our way up to applying concepts from these toy games to real poker situations.

Our analysis of each scenario begins by prompting you to make predictions about the optimal strategies for that scenario. Some of these will be easier than others, and I don't recommend beating your head against the wall trying to get the perfect answers. You should, however, take some time to think about these questions independently before you read the answers and explanations. You will learn and retain more by challenging yourself in this way.

These scenarios build on each other, so it's important that you grasp the central concepts from each chapter before you move on to the next. The overviews and challenge questions highlight the main takeaways and test your understanding of them. A good gut check is to review the questions for each scenario and ensure that you understand the answer to each before moving on to the next.

Depending on how much you already know about game theory, the first few scenarios might be quite simple for you. You may even already be familiar with the solutions. The questions at the beginning of each section are an opportunity to test how well you really know the relevant material. If you get them all correct, then you can skim or even skip the rest of that section and move on to the next scenario. Be sure to check yourself on this, though, as even those relatively knowledgeable about game theory get some of these details wrong. I certainly discovered, in writing this book, that I didn't understand all these concepts as well as I thought I did!

After examining the equilibrium solutions for these scenarios, we will consider how to play them exploitatively. That is, we will think about possible mistakes that a player could make and how best to take advantage of those mistakes. Even when playing exploitatively, understanding the equilibrium is a valuable first step toward finding opportunities for exploitation.

These scenarios mostly feature the same two players, Ivan and Opal. Ivan is always in position, and Opal is always out of position. In other texts about poker and game theory, you might see players referred to as IP and OP, for In Position and Out of Position. I hope that human names that start with the relevant letters will make it easier for you to visualize the scenarios.

The lessons we take from these games will be most immediately applicable in pots contested by only two players. Even at a ten-handed table, many real poker hands end up with just two players still contesting the pot by the river, and the general lessons learned from game theoretical analysis of two-player scenarios mostly do apply in multi-way pots, even though the math might be a little different.

Yes, we will do some math, but don’t get hung up on or intimidated by that. The exact details are rarely important. Instead, focus on understanding what's going on at a conceptual level: the relationship between bet size and bluffing frequency, for instance, or why bluffing with certain hands is more valuable than bluffing with others.

These scenarios should be fun, interesting, and above all useful. They are sometimes abstract, and I’ve taken pains to explain their relevance to actual decisions you make at the poker table. In the end, you should take what is useful to you and ignore the rest. This book, like game theory itself, should be just another tool in your toolbox.

Chapter 1: Understanding Equilibrium

Overview & Objectives

The concept of *equilibrium* is foundational for understanding game theory, so we're going to dedicate an entire chapter to it before we start considering poker explicitly. We'll consider instead a few "games" so simple that, although you have played them may times, you probably never thought of as exercises in game theory. They are, though!

By investigating how equilibrium applies in these simple cases, we'll build your understanding of the concept in general, leaving you better equipped to make use of it in the more complicated scenarios that poker presents. We'll also talk a bit about other important game theoretic concepts such as *strategy*, and about how computers can assist us in solving the equilibria of more complex games.

By the end of this chapter, you should be able to:

- ♠ Define "equilibrium" and "strategy" as these terms are used in game theory.
- ♠ Understand what it means for two strategies to be in equilibrium.
- ♠ Recognize when a pair of strategies is not an equilibrium.
- ♠ Explain the difference between pure strategies and mixed strategies and know when each arises.
- ♠ Randomize your play in an unpredictable and unexploitable fashion.

What is a Strategy?

Strategy is a term with a very specific meaning in game theory. Wikipedia defines it as "a complete algorithm for playing the game, telling a player what to do for every possible situation throughout the game." For our purposes, a poker strategy will describe not just a vague description of style (“tight and aggressive”, for instance) but rather exactly what you will do in every possible situation, with every hand you could possibly have in that situation.

For example, my pre-flop strategy for playing in first position at a nine-handed table might be to raise three times the big blind with a pair of 7s or better, AQo or better, and any suited broadway hand, to raise 20% of the time that I have suited connectors T9s through 76s, and to fold everything else. To really be complete, I'd also need to describe how I'd respond to three-bets of various sizes from each opponent.

This is more information than most human brains can store and process at once, which is why computers are such valuable tools for studying game theory rigorously. When actually playing poker, we won't be able to formulate strategies with nearly the same precision, but we still must consider how we will handle future situations we could encounter, both before and after the flop, in order to make good decisions in the current situation.

We'll also need to have some idea about our opponent's likely strategy, or at least about what strategies will be available to him. Knowing that we'll be vulnerable to bluffs on certain rivers, for instance, might inform which hands we call with on the turn.

Nash Equilibrium

An equilibrium, sometimes called a Nash Equilibrium after mathematician John Nash, is a set of strategies for all players in a game such that no player has incentive to deviate unilaterally from his strategy. For example, if you and I were playing equilibrium strategies in a heads up poker game, I could tell you exactly how I would play every hand in every possible situation, and you could give me the same information, and neither of us would want to change anything about our strategies even after we had perfect information about how the other would play. That formal definition makes it sound more complicated than it is, so let's look at a simple example.

Two cars approach each other on a road, traveling in opposite directions. Each driver can choose to stay to the right side of the road (from his own perspective), to stay to the left side of the road, or to proceed straight down the middle. Neither driver has a preference for which part of the street he drives on, but both would strongly prefer to avoid a collision.

This is not a game in the ordinary sense, but for our purposes it is. There are multiple players, each can choose from multiple *moves*, and the *payoff* for each player depends on both his own choice and that of the other player. In other words, without knowing what the other player will do, a player cannot predict what the result of his own choice will be. If he moves to the right, he might continue safely on his way, or he might get a collision - the result depends on what the other driver chooses.

The grid below represents the entire game. The options along the left side are the choices that the first driver can make, and the options along the top are the choices that the second driver can make. The box at the intersection of two options represents the payoffs for both players if they choose those options. The first number in each pair represents the payoff for the first driver, and the second number the payoff for the second driver. Successfully passing each other is scored as a 1, while a collision, which would be very bad news for both drivers, is scored as a -10.

	Driver 2		
Driver 1	Right	Middle	Left
Right	(1,1)	(-10,-10)	(-10,-10)
Middle	(-10,-10)	(-10,-10)	(-10,-10)
Left	(-10,-10)	(-10,-10)	(1,1)

The highlighted boxes represent two different *strategy pairs* that yield equally good results for both players: each can drive on the right or the left side of the road. If both players consistently make the same choice, then that's an equilibrium. They both get what they want, which is to keep on driving without a collision, and neither has any incentive to do anything different.

There's no intrinsic reason why consistently staying to the right is any better or worse than consistently staying to the left, and indeed in the real world there are places where both equilibria prevail. In the United States, the convention is to drive on the right. In the UK and some of its former colonies, the convention is to drive on the left. It doesn't really matter which you choose, as long as everyone is on the same page about what to do when the situation arises.

A rational player might choose to drive either to the right or to the left, but she would never drive straight down the middle. That can only lead to collision, which is not in either player's interest. Although it's technically a move in the game, there's no reason to ever use it. In the vocabulary of game theory, this is called a *dominated strategy*.

Proving an Equilibrium

These two equilibria - both players stay to the right or both players stay to the left - are so clear and intuitive that you probably don't need me to prove them to you. It's good to know how to prove or disprove that a certain strategy pair is an equilibrium, though, because in the future it won't always be so obvious.

Recall that the definition of equilibrium requires that neither player can unilaterally deviate from his strategy and achieve a better result. To prove that two strategies are at equilibrium, then, we must experiment with changing one player's strategy at a time to see whether that player can get a higher payoff. If both players change strategies at the same time, that could lead to a higher payoff, but it wouldn’t prove anything about whether the original strategies were at equilibrium.

Looking at the grid, we can see that it is not possible for any player to achieve a payoff higher than 1. Thus, any box where both players have a payoff of 1 is an equilibrium.

Now let's consider the case where Driver 1 stays to his right while Driver 2 stays to his left. This produces a payoff of -10 for both players. We know that a higher payoff is possible, but that by itself isn't enough to disprove equilibrium. We must demonstrate that one player can achieve a higher payoff solely by changing his own strategy. That is, we will keep the other player's strategy fixed and try to find a way for just one player to change his own strategy and achiever a higher payoff as a result.

In this case, either player can do so. Driver 1 could change his strategy and drive on the left. That would improve his payoff (and his opponent's, though that isn't important for this purpose) to 1. That right there is enough to prove that this is not an equilibrium, but of course Driver 2 could also unilaterally improve his payout by driving on the right. If *both* players changed their strategies, there would be a problem, but that's not the test of equilibrium.

Scenario: Going to the Movies

Now we're going to look at some more simple scenarios drawn from everyday interactions (though they may require a little suspension of disbelief). I'll explain each scenario to you, then ask you to answer some questions about it, based on what you've learned about the concept of equilibrium so far.

Two friends, Andrew and Blanca, want to go to a movie together. Andrew would prefer to see a drama, and Blanca would prefer to see a comedy, but the most important thing to both of them is that whatever they see, they see it together. That is, Andrew would rather see a comedy with Blanca than a drama by himself, and vice versa.

Like the previous example, this may not fit the colloquial idea of a “game”, but it has multiple players, each of whom can choose from two different *moves* (go to a comedy or go to a drama), and the payoff for each player depends what that the other chooses.

One small suspension of disbelief is required to make this example work: you must imagine that each player chooses which movie to see independently, without consulting the other. Andrew may know Blanca's *strategy* for choosing a movie, but he will not know her actual choice. That is, if Blanca's strategy were to flip a coin, you can assume Andrew knows she will flip a coin but not the result of the flip. (If it helps, you could imagine the two agreed to go to the movies without deciding exactly what to see, and that Andrew subsequently lost his phone, thereby preventing them from coordinating their choices.)

We'll want to do some simple math with this scenario, so we need to put numeric values on our players' preferences. For Andrew, going to a drama with Blanca is the best-case scenario, so we'll make that worth five points. Going to a comedy with Blanca is nearly as good, call it four points. Going to a drama by himself is one point, and going to a comedy by himself is the worst outcome for him, so we'll call that zero points. For Blanca, it's the opposite: going to a comedy with Andrew is five points, and going to a drama with Andrew is four points. Going to a comedy alone is one point, and going to a drama alone is zero points.

Now we can represent the entire game, including both players' strategic options and their payoffs, in a grid:

	Blanca	
Andrew	Go To Comedy	Go To Drama
Go to Comedy	(4,5)	(0,0)
Go to Drama	(1,1)	(5,4)

The upper left-hand box represents the payoffs if both players choose to go to a comedy. The first number in the pair is Andrew's payoff, and the second is Blanca's.

Questions

Do your best to answer these questions independently, but don't worry if you can't figure out how to approach some or all of them. Just take a crack at it, then proceed to the next section, where we'll discuss how to answer each.

1. Suppose both players play a pure strategy of always going to the comedy. Is this an equilibrium? How can you tell? (Hint: Remember that the definition of equilibrium is a set of strategies for all players in a game such that no player has incentive to deviate unilaterally from his strategy.)

2. What if Andrew's strategy is always to go to the drama and Blanca's is to always go to the comedy? Is this an equilibrium? How can you tell?

Answers & Explanation

1. Suppose both players play a strategy of always going to the comedy. Is this an equilibrium? How can you tell?

Yes, this is an equilibrium. To test it, we check whether either player can *unilaterally* change his or her strategy to achiever a higher payoff.

Blanca already has her highest possible payout with this set of strategies, so she's got no incentive to change.

What about Andrew? He was more interested in the drama than the comedy, so let's see what happens if he changes his strategy and goes to see the drama. Then the players end up in in the lower left quadrant, where Andrew has a payoff of 1, which is lower than what it would be if he stuck with the comedy. Even though Andrew wanted to see the drama, what he wanted most of all was to go to the movies with Blanca, and that won't happen if he unilaterally changes his strategy.

The "unilaterally" qualifier is important here. Even though this set of strategies does not give Andrew his highest possible payoff, the only way he can achieve a better outcome is if *both* players change their strategies, so it is an equilibrium, and he has no incentive to deviate from it.

2. What if Andrew's strategy is always to go to the drama and Blanca's is to always go to the comedy? Is this an equilibrium? How can you tell?

This is not an equilibrium, because either player could achieve a better payoff by changing his or her strategy. If Andrew changes strategies and goes to the comedy, his payoff increases from 1 to 4. If Blanca changes strategies and goes to the drama, her payoff increases from 1 to 4. The only way they wouldn't achieve a better outcome would be if they *both* changed strategies, but that wouldn't be a unilateral change.

Equity and Expected Value

Poker is full of uncertainty; you don't need a book to tell you that. We're going to talk about how to deal with that uncertainty, how to make good decisions despite it, and even how to quantify the value of the choices available to you when you cannot be sure about the results of those choices.

Many poker players have unrealistic expectations of certainty. They do not want to bluff unless they are sure that their opponent will fold. They do not want to value bet the river unless they are sure that their hand is best. Some do not even want to get all-in with KK before they see the flop to be sure it does not contain an Ace!

Probably you recognize the fallacy of at least some of this logic already. Poker is not about certainty; it is about learning to make good decisions under conditions of uncertainty. The most successful players accept - embrace, even - the fact that they will frequently be unsure of whether a given decision will result in a win, or even that it will have a positive *expectation*.

Uncertainty in poker comes in many forms. Most fundamentally, before the river, you do not know which cards will come. Nor do you know the cards that your opponents hold. Even if you could determine an opponent’s *range* - the set of all hands that he might hold given his actions so far - you wouldn’t know which hand he actually holds. And even if you knew an opponent’s exact hand, you *still* might not be able to predict his future actions, such as whether he will call a large bet, or whether he will bet if you check.

Equity and *Expected Value* are two different ways of quantifying uncertainty in poker, putting a price on a situation where we don't know which cards are going to come or what actions other players will take.

When choosing between different options, it helps to put a dollar value on each to enable a direct comparison. However, when your payoff depends on the choices that another player makes, you can't know what your payoff will be. You may know the possibilities - going to a drama could yield a payoff of either 1 or 5 for Andrew, depending on what Blanca chooses - but that by itself is not enough to determine what it is worth to Andrew to choose the drama.

Equity and Expected Value can help us to work around this uncertainty and determine what a choice is worth to a player even when we don't have perfect information about future cards or actions.

Equity Quantifies Uncertainty About Future Board Cards

Equity, at least as the term is widely used in the poker world, is a measure of how much of the pot a given hand or range would win if there were no further betting.

The advantage of equity as a means of estimating value is that it can be calculated easily and precisely with the help of *equity calculator* software. An equity calculator takes two or more hands and/or ranges as inputs, generates 100,000 or more possible boards, and tracks how often each wins, loses, and ties. In less than a second, it returns the average win or loss per trial for each hand or range.

For example, an equity calculator can tell us that before the flop, AKo has about 30% equity versus KK. Versus a *range* of {AK,KK,AA}, it has about 37% equity. If you were to get all-in before the flop with AKo versus KK, you would win about 30% of the final pot, on average. If you were all-in against a range of {AK,KK,AA}, you'd average about 37% of the pot.

By measuring our hand's equity, we remove uncertainty about what the final board will be. Of course, we don't know whether our AKo will win or lose in any particular instance, but we can still put a price on what it is worth to get all-in against a particular hand or range.

Even if you are not all-in, it can still be useful to know how much of the pot you could expect to win in the absence of additional betting. Because there *will* be additional betting, however, equity is only a very rough measure of what your hand will actually win. You may end up folding to a bluff, or successfully bluffing yourself, or paying off a value bet, or making a value bet. All of these possibilities and more will affect what it's actually worth to you to hold AKo versus KK in a given pre-flop situation.

Expected Value Quantifies Uncertainty About Future Actions

Expected Value, or *EV*, is a measure that attempts to take future actions by both you and your opponents into account. For this reason, though, it requires more estimation and is harder to calculate than equity.

For example, if I bluff $50 into a $100 pot on the river, I don't know whether you are going to call or fold. However, I can express the value of this bluff as a function of your raising, calling, and folding frequencies. When you fold, I win $100. When you call or raise, I lose $50. So, my EV is equal to ($100 * %Fold) - ($50 * %Call) - ($50 * %Raise).

With the help of this equation, if I can predict your folding frequency, then I can predict the EV of this bluff. If you were to fold 40% of the time, my EV would be ($100 * .4) - ($50 * .6) = $10. I collapsed the %Call and %Raise variables into a single 60%, because when you choose either of these, the result is the same: I lose $50. Even if I don't know how often you'll raise rather than call, it won't make a difference in this case.

With the help of a little algebra, an equation like this is also useful to get a sense of how often I need you to fold if I am to have an EV greater than $0 - to show a profit, in other words. We'll see how exactly that works in the next chapter.

Test Yourself

Test your understanding of the concepts above by answering the following questions. Explanations are on the next page, but you should try to come up with the answers yourself first, with the help of an equity calculator. There are many available on the internet or as mobile applications. One good free one is EquiLab, available at www.pokerstrategy.com/poker-software-tools/equilab-holdem/.

1. How much equity does T♥ 9♥ have against A♠ K♣ pre-flop?

2. How much equity does T♥ 9♥ have against a range of {55+,AJ+,KQ}?

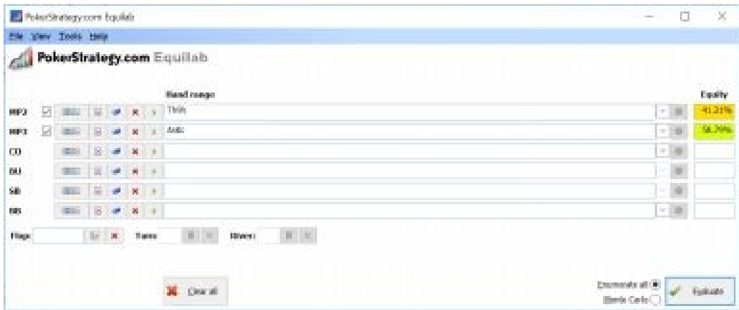
3. How much equity does T♥ 9♥ have against A♠ A♣ on an 8♥ 7♥ 3♠ flop?

4. Suppose you make a $100 bluff into a $100 pot on the river. If we assume your opponent will fold 40% of the time and you will lose any time he does not fold, what is the EV of your bet?

Answers

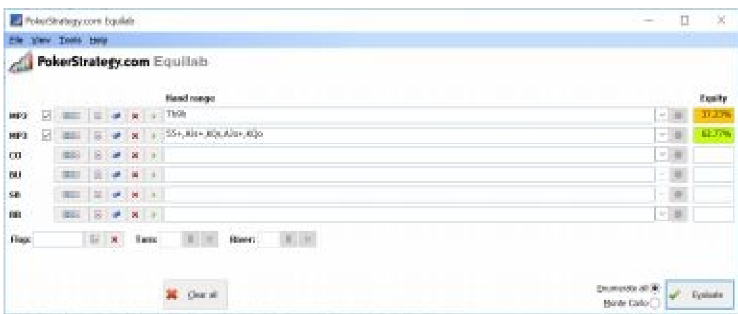
1. How much equity does T♥ 9♥ have against A♠ K♣ pre-flop?

It has about 41% equity. Here's the Equilab input that will give you the correct result:



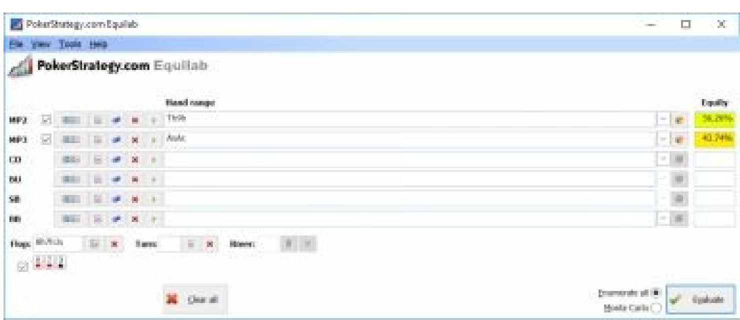
2. How much equity does T♥ 9♥ have against a range of {55+,AJ+,KQ}?

It has about 37% equity. Here's the Equilab input:



3. How much equity does T♥ 9♥ have against A♠ A♣ on an
8♥ 7♥ 3♠ flop?

It has about 56% equity. Here's the Equilab input:



4. Suppose that you make a $100 bluff into a $100 pot on the river. If we assume that your opponent will fold 40% of the time and that you will lose any time he does not fold, what is the EV of this bet?

It's -$20, meaning you would lose an average of $20 each time you attempted this bluff.

To arrive at this answer, we can use the same equation as the example above. We just need to change the amount that you will lose if your opponent calls or raises from $50 to $100. That gives us

EV = $100 * %Fold - $100 * %Call - $100 * %Raise.

Once again, we can combine %Call and %Raise into 60%, since we don't care what your opponent does if he doesn't fold. So,

EV = $100 * .4 - $100 * .6 = -$20.

If your opponent will only fold 40% of the time, then you would lose an average of $20 every time you made this bet, even though you would sometimes get lucky and win the pot.

Scenario: Dodging a Debt

Christina owes David $100. It's Saturday night, and they are both going to one of two poker rooms, Room A or Room B. If they show up at the same room, Christina will have to pay David the $100 she owes him. David hopes this will happen, while Christina hopes it will not. Here's the grid showing their strategic options and their payoffs:

	David	
Christina	Room A	Room B
Room A	(-$100,$100)	($0,$0)
Room B	($0,$0)	(-$100,$100)

Questions

These questions challenge you to work out some problems we haven't explicitly covered yet. Do your best, but don't worry if you aren't sure how to answer them yet. Just give it a shot, then continue to the next section, where we'll discuss the solutions.

1. Explain or demonstrate why none of the boxes in the grid above represents an equilibrium.

2. What is the equilibrium for this game?

3. What is each player's EV at equilibrium?

4. Prove that neither player has incentive to deviate from his or her equilibrium strategy.

Answers & Explanation

1. Explain or demonstrate why none of the boxes in the grid above represents an equilibrium.

If Christina goes to Room A and David goes to Room B, then David's payoff would be $0. This would not be an equilibrium, because David could unilaterally increase his payoff by going to Room A to collect his $100. But both players going to Room A is also not an equilibrium, as Christina could get a better payoff by changing her strategy and going to Room B so as not to pay the $100.

Any *pure strategy* pair would result in the players always going to the same room or always going to a different room, and in each case there would be a player who could change his or her strategy to get a better payoff.

Unlike in the other scenarios we've looked at, pure strategies will never produce an equilibrium for this game. The players need to mislead each other about their intentions, and that requires *mixed strategies*, some method of choosing a room that the opponent cannot predict.

2. What is the equilibrium for this game?

The equilibrium is for each player to choose randomly, whether by flipping a coin or some other method, with a 1/2 chance of going to Room A and a 1/2 chance of going to Room B.

The equilibrium requires mixed strategies because the available pure strategies are predictable. If Christina just picked a room and went there, she would be guessing about where David was likely to show up. If she guesses wrong - if David correctly predicts where she will be - she gets a bad outcome. Likewise if David tries to just pick a room.

In the scenarios we looked at previously, predictability was a good thing. When you're sharing a road with other drivers or going out with friends, you typically want them to know what your strategy is going to be, because it is possible to arrive at win-win solutions. Some outcomes are better for everyone involved and some are worse for everyone involved, and all players are incentivized to arrive at the former.

The Dodging a Debt scenario, like most two-player poker situations, is a *zero-sum game*. A gain for one player is a loss for another, so an outcome that is good for one player is not good for the other. Christina is actively trying to avoid David, so she needs to be unpredictable. That's where a mixed strategy comes in handy.

As for why each player's strategy is to choose each room with ½ probability, this has to do with the player's payoffs. Christina's only interest is in dodging David; she doesn't otherwise care which room she goes to. Nor does David have any preference other than trying catch up with Christina. By being unpredictable in exactly this way, each player creates a situation where there is no good strategy for his or her opponent. Whatever the opponent chooses, he or she will have only a ½ chance of choosing the right room. If David went to either room with greater than ½ frequency, this would be *exploitable*. That is, Christina could have a better than ½ chance of dodging him if she could predict what his preference would be.

In the next chapter, we'll focus more on how to find the exact frequencies for mixed strategies. Right now, you just need to understand what a mixed strategy is and why they are often necessary to arrive at an equilibrium.

3. What is each player's EV at equilibrium?

To find a player's EV, you must look at all possible outcomes of his or her action, how likely each is, and what each is worth.

This game has four possible outcomes: 1) Both players go to Room A, David gets $100; 2) Christina goes to A and David goes to B, David gets $0; 3) Both players go to B, David gets $100; 4) Christina goes to B and David goes to A, David gets $0.

The probability of each of these outcomes is the probability of Christina going to the given room multiplied by the probability of David going to the given room. In this example, those probabilities are all 50%, so the equation for David's EV is

$$(.5 * .5 * \$100) + (.5 * .5 * \$0) + (.5 * .5 * \$100) + (.5 * .5 * \$0)$$
$$= \$50$$

Essentially, David has a 50% chance of choosing the same room as Christina, whichever room that happens to be, and collecting his $100. That amounts to an EV of $50.

Because this is a zero-sum game, Christina's EV must be -$50. For David to make money, she must lose an equivalent amount. There is a 1/2 chance that she will have to pay her debt to David and a 1/2 chance that she dodges him.

4. Prove that neither player has incentive to deviate from his or her equilibrium strategy.

To prove an equilibrium, we can experiment with changing a single player's strategy and see whether any change that player makes could improve his payoff relative to the supposed equilibrium that we are testing. Let's try keeping Christina's strategy the same - she'll still go to each room with 1/2 probability - and changing David's.

If Christina flips a coin to decide which room to play at, while David plays a pure strategy of going to Room A, then the probabilities of each outcome change, and David's new equation looks like this:

(.5 * 1 * $100) + (.5 * 1 * $0) + (.5 * 0 * $100) + (.5 * 0 * $0) = $50

In this case, David always goes to Room A, and there is a 50% chance that Christina also chooses Room A, in which case David gets his $100. His EV is still $50, so this change does not improve his payoff.

What if David played a more complicated strategy, with an 80% chance of going to Room A and a 20% chance of going to Room B? Then the equation would look like this:

(.5 *.8 * $100) + (.5 * .8 * $0) + (.5 * .2 * $100) + (.5 * .2 * $0) = $50

David could change his strategy in lots of ways: he could play a pure strategy of going to Room B, he could flip a coin, or he could randomize in some other proportion. None of these improves his payoff, though. No matter what he does, he has a 1/2 chance of choosing the room where Christina will be.

We'd get the same result by keeping David's coin flipping strategy static and experimenting with different strategies for Christina. No matter what change she made, if David played his half of the equilibrium, she would have an EV of -$50.

It's important to note that even though the EVs remain the same when we change one player's strategy, there is no longer an equilibrium. If Christina flips a coin while David consistently goes to Room A, their EVs remain the same, but now David's strategy is exploitable. The risk he takes is that if Christina also changes her strategy, she might end up with a better than 1/2 chance of evading him. There are infinitely many strategy pairs that would yield EVs of (-$50, $50) in this game, but only one of them is an equilibrium.

The crucial thing is that neither player can do better than the equilibrium strategy by unilaterally deviating from it. It's quite common that a change will produce an equally good payoff for one player, as it does here.

A player can even *decrease* his payoff by unilaterally deviating from the equilibrium; think about Andrew going to a drama by himself instead of seeing a comedy with Blanca. In real poker situations, it's quite common that a balanced strategy will do a good deal better against a weak opponent than it would against an opponent playing a balanced strategy, even if it doesn't do quite as well as a strategy specifically crafted to exploit that player's weaknesses. In fact, understanding the equilibrium can help us to craft more effective exploitative strategies, a goal that we'll return to each time we explore new concepts.

Indifference Means Giving Your Opponents No Good Options

When a player has no preference between two or more of his strategic options, then he is *indifferent* between them.

Christina's equilibrium strategy is to randomize her choice of card room, with a 50% chance of ending up at either Room A or Room B, because this is the only way to avoid being predictable, or exploitable. If she shifted that frequency even a little bit, going to Room A at a 51% frequency, for instance, then there would be a clear best strategy for David: he should always go to Room A, as he'd have the best chance of catching her there.

By getting that frequency exactly right, Christina creates a situation where David's choice is not meaningful. Whether he always goes to A, always goes to B, or employs any mixed strategy, his payoff remains the same. He has a 50% of getting his $100, for an expected value, or average return, of $50.

The technical way of saying this is that at equilibrium, David is indifferent between the two rooms. Neither is better or worse for him. Not only that - he's indifferent between all possible strategies of choosing a room. Whether he always goes to A, goes to A 50% of the time, or goes to A 39.37568% of the time, his EV remains the same. By playing her equilibrium strategy, Christina makes David indifferent between all his potential strategies. The same is true if David plays his half of the equilibrium: no matter which strategy Christina chooses, she will have an EV of -$50.

Real poker players have lots of weird preferences. They'll tell you that their main goal is to make money, but when you watch them play and hear them talk about their decision-making, they often reveal other preferences of which they may or may not be aware. Many are risk-averse, meaning they try to avoid big losses even if it means they will make a little less money overall. Some are risk-loving; they’re there to gamble, and they’re willing to play losing strategies because, like people chasing jackpots at slot machines, they enjoy trying to hit hands and win big pots. Some want to be regarded as good players by their peers and so will not pursue unconventional strategies even if they are likely to be profitable.

Despite all of this, when we talk about game theory and equilibrium, we assume that all players’ only preference is to maximize their expected value. This is not to say that other preferences are invalid, only that they are beyond the scope of this book. My goal here is to help you identify the most profitable choice at any decision point. How you weigh that against other considerations is entirely up to you.

Regardless of your own preferences, you absolutely should consider your opponents’ preferences. Identifying whether an opponent is risk-averse, interested in gambling, or obsessed with protecting her hand will help you determine how best to exploit that player.

In a game theoretic sense, though, a player would have no preference between two options only if they had the same EV for him. We will assume that when presented with a choice between two or more strategic options in a poker game, a player will choose the one with the highest EV. While it isn't always easy to determine which option has the highest EV, this is what most players are trying to do most of the time. For our purposes, indifference arises only when two or more options have the same EV.

Equilibrium strategies typically involve quite a bit of indifference. In fact, you can think of indifference as the goal of poker. You can't force your opponents to make mistakes - to call when they ought to fold, for instance, or to fold when they ought to raise. What you can do is put them in situations where there is no right play, or where the right play is difficult to identify because it is barely better than the wrong play. This not only denies them the opportunity to make good decisions but also increases the likelihood that they will make mistakes.

This may require some adjustment to how you currently think about poker. Most poker players are accustomed to thinking first about mistakes that they want or expect their opponents to make and then focusing on how to induce those mistakes. The game theoretic approach is to focus instead on putting opponents in difficult situations where they will have trouble identifying the correct play. The mistakes will follow.

Your Brain Is Not Capable of Randomization

Suppose that you have a bluff-catcher on the river, and you believe that you should call 1/2 the time with it. You need to find some way of generating a random number. You could look at the final digit of a digital watch or clock and call if it is less than 5. If you're playing online, you could flip a coin or roll a die or use a website such as Random.org to generate a random number.

What you should *not* do is say, "Well, I folded the last time this player bet the river, so I'll call this time. That way I'll be calling 1/2 the time."

For one thing, the last time you folded was a different situation. The board was different, the action was different, and your hand was different. From an exploitative perspective, you might be able to guess whether the fact that you folded that previous hand would make your opponent more or less likely to bluff this time, but if you're aiming for randomness, then what happened in that other hand should have no bearing on how you play this one. After all, your opponent knows that you folded to his last bet, so he may also try to predict how that will influence *your* decision this time around.

Even if by some miracle of probability you were to find yourself in precisely the same situation a second time, though, you'd still need a random number generator. There must be a 50% chance that you will call *each time* you are in this situation. Whatever you did last time should have no bearing on the current situation. If it did, then an opponent who knows that you folded the last time could predict that you would call this time. That's not random.

Consider flipping a fair coin. There is a 50% chance that any given flip will be heads, but just because you got heads on the first flip doesn't mean that the second will be tails. It's an independent event, and you can't predict anything about the next flip based on the previous flips.

Solving Complex Poker Scenarios Requires Computer Assistance

Human brains aren't capable of true randomness, but they face an even bigger challenge applying game theory to poker. Finding equilibrium strategies for real poker games is, to put it mildly, a computationally intensive task. In the Dodging a Debt scenario, we worked with a neat and tidy 2x2 grid, but in no limit hold 'em, the *game space* - the number of potential situations in which you could find yourself - is millions of times larger.

The only equilibria that humans can solve without the aid of computers are for extremely simplified toy games like the ones we've looked at so far. Hopefully, solving those has helped you better understand what equilibria are, why they are relevant, and how to find them.

When it comes to solving actual poker situations, we need the assistance of computers and specialized software. These programs, called *game theory solvers* or *equilibrium solvers* or just *solvers*, enable us to approximate equilibrium solutions to these much more complex games.

I say "approximate" because even when working with solvers, we're still forced to restrict the game space by limiting the number of available bets. Even a computer can’t handle the hundreds of different betting options available to you in a real game. After all, in a $1/$2 game with $200 effective stacks, you could bet $2 or $3 or $4 or any amount you wish up to $200. That's nearly two hundred strategic options! Then your opponent could respond to that bet with a raise of many different sizes, you'd have the choice to re-raise, etc. The number of possible game situations explodes quickly.

When working with a solver, you must decide how to limit the bet sizes. You could, for instance, allow bets of 50% of the pot on the flop, 50% or 100% on the turn, and 50%, 100%, or all-in on the river. Then, a solver could give you an equilibrium solution within these constraints. Restricting players to three bet sizes probably won't result in an output that's too far off from the true equilibrium, but strictly speaking, it isn't a solution to a no-limit game, it's a solution to a closely related game with limited betting options.

Solvers have other limitations besides bet size restrictions. If you're solving for a situation on the flop, for instance, you'll still have to do some guesswork to input the ranges with which each player will see the flop. There actually are now consumer-facing solvers capable of solving pre-flop situations, but these are *extremely* computationally intensive, often requiring hundreds if not thousands of dollars in processing power to do substantive work.

Choosing the right inputs when working with a solver is a skill unto itself. Think of a scientist designing a laboratory experiment: if she designs it poorly, she will end up testing something other than the hypothesis she intended, and any conclusions she draws from the results will be flawed.

You won't need access to a solver to use this book. We will look at some solver solutions, but I've already done the set-up for you; we’ll just analyze the outputs. After studying the material in this book, however, you should be capable of using a solver independently should you wish to do so.

Chapter 2: Polarized Versus Condensed Ranges

Overview & Objectives

Poker is at its simplest on the river. Hand values are fixed, and we mostly don't have to factor future betting into our decisions. Reasons for betting on the flop and turn are more complicated, but on the river most bets should be either value bets or bluffs.

This chapter focuses on the fundamentals of betting and calling on the river, with the goal of building a solid foundation for understanding poker strategy more generally. This enables us to think about value betting and bluffing in isolation.

The concepts discussed here are relevant on earlier streets as well, but they are complicated by other factors. For our most rigorous analysis, we'll assume that hands have fixed value, as they do on the river, but when we talk about real-world applications of these concepts, we'll consider how to use them on earlier streets as well.

We'll start by learning about the differences between polarized and condensed ranges. Then, we'll work through a toy game that highlights the dynamic of a polarized range playing against a condensed range and find the optimal strategy for each.

By the end of this chapter, you should be able to:

- ♠ Distinguish between polarized and condensed ranges.
- ♠ Identify which hands are best for betting and which for checking on the river.
- ♠ Approximate an unexploitable bluffing frequency on the river.
- ♠ Approximate an unexploitable bluff-catching frequency on the river.
- ♠ Appreciate the value of holding a polarized range and the penalty for holding a condensed one.
- ♠ Apply the above concepts to real poker decisions.

Recognizing Polarized And Condensed Ranges

Polarized Ranges Consist of Strong and Weak Hands

A polarized range consists of strong and weak hands but nothing in between. Polarized ranges are sometimes described as "nuts or air", although they can certainly contain strong hands beyond the literal nuts, and the weak hands don't have to be complete air.

The key is that hands in a polarized range should be clearly identifiable as either value bets or bluffs. When you are value betting, you should be able to clearly articulate the hands worse than yours that you expect your opponent to call with. When you are bluffing, you should be able to identify hands stronger than yours that you expect your opponent to fold. This is a good test to give yourself *before* you bet. If you can't pass this test, you probably shouldn't bet your hand. "I want to take it down now" and "Checking would look weak" are generally not sufficient reasons to bet.

Betting and raising ranges tend to be polarized, with larger bets being more polarized. That is, the larger a bet, the closer the bettor’s range should be to the extreme of "nuts or air". It's hard for ranges to be completely polarized before the river, because even very strong hands are rarely 100% to win nor are very weak hands 100% to lose.

A common misunderstanding about polarized ranges is that they are necessarily weak. Although it's usually easier to have weak hands than strong hands, nothing compels a player to bet all his weak hands. A player who arrives at the river with many weak hands and few strong ones could still construct a perfectly balanced, polarized range by checking most of his weak hands and bluffing with just a few of them. Many real-world players don't have the discipline to check and accept a sure loss with a weak hand, which is why polarized ranges are often presumed to be weak. In theory, however, there's no reason they must be.

Condensed Ranges Consist of Medium-Strength Hands

A condensed range consists of medium-strength hands that will lose to strong hands but win against weak ones at showdown. Because these hands beat bluffs but lose to value-bets, a condensed range is sometimes called a *bluff-catching* range. That doesn't mean that they *should* necessarily call a bet, just that inducing bluffs is the only way they can cause weaker hands to put money into the pot. The hands in a condensed range may still be ahead of an opponent's range, but unlike stronger holdings, they cannot expect to be ahead if the opponent calls a bet or raise. Thus, checking and calling ranges tend to be condensed.

Because medium-strength hands are vulnerable and facing bets from a polarized range is uncomfortable, it may be tempting to play medium-strength hands aggressively. As we'll see later in this chapter, however, betting or raising these hands is usually a mistake.

Test Yourself

On the river, the board is A♦ 8♣ 4♣ J♠ 3♥. There is $50 in the pot and $200 in the effective stacks. Opal checks, Ivan bets $50, and Opal calls.

1. Which player should have a polarized range and which a condensed range?

2. Which player is more likely to have KJ? Why?

3. Which player is more likely to have JJ? Why?

4. Which player is more likely to have A5? Why?

5. Which player is more likely to have 6♣ 5♣? Why?

Answers & Explanation

Ivan's range should be polarized when he makes a large river bet. That means he should either be value betting a very strong hand such as a set of Jacks or bluffing a very weak hand such as 65. The A5 and KJ make more sense for Opal.

Opal should have a condensed range consisting of medium-strength hands. With a very strong hand like JJ, she should either bet or check-raise. With a very weak hand like 65, she should probably fold. She could consider betting or check-raising as a bluff, but she certainly shouldn't call with it. When she checks and calls, she should have medium-strength hands like second pair (KJ) or top pair with a weak kicker (A5). These hands can beat bluffs but probably aren't strong enough to bet for value.

If Ivan had a medium-strength hand like KJ or A5, he should check. These hands may not win every time, but because they can beat weak pairs or missed draws that Opal may hold, they have too much showdown value to bluff.

Ivan can't bet them for value, however, especially not for a large bet, because Opal will not call with many of the hands that KJ or A5 is ahead of. These hands can win at showdown if they check, but they probably won't be favorites against the range with which Opal is willing to call a large bet.

Note that this true even though they may beat *some* of the hands Opal would call with. For instance, if Ivan bets A5 and Opal calls with KJ, he would win. However, he would also be lucky to run into one of the weakest hands she would call with. More often, he should expect to get called by an Ace with a better kicker or maybe even a weak two pair. It's not enough for Ivan to be ahead of a few of the hands in his opponent's calling range; he must expect to win more than half the time that he is called to warrant betting.

Scenario: The Clairvoyance Game (Opal's Strategy)

The rest of this chapter will focus on the interplay between polarized and condensed ranges, so we'll start with a toy game that pits these ranges against each other. I first encountered this game in Bill Chen and Jerrod Ankenman's excellent book *The Mathematics of Poker*. Chen and Ankenman refer to it as The Clairvoyance Game, so I'll stick with that name in homage to them.

Each player is dealt a single card from a three-card deck consisting of one A, one K, and one Q. The objective of the game is simply to hold a better card than your opponent at showdown, with an A being the best card and a Q the worst. There are no community cards and no draws, just a single round of betting followed by a showdown.

The out-of-position player, Opal, is always dealt a K. The in-position player, Ivan, is dealt an A 50% of the time and a Q 50% of the time. In other words, he has a range consisting of {A, Q}. Each player knows the other's range, so Opal knows that Ivan has either an A or a Q, and he knows that she always has a K.

Each player antes $1. Opal acts first and may either bet $1 or check. Facing a check, Ivan may bet $1 or check. Facing a bet, a player may only call or fold; there is no raising permitted. If there is no bet or if a bet is called, then there is a showdown and the high card wins the pot.

Before we determine the equilibrium for this game, let's try to apply some concepts we've already learned. Step into Opal's shoes for a moment and make a few predictions about what her equilibrium strategy will look like. Remember that when thinking about equilibrium strategy, you should assume that your opponent knows your range in any given situation and will not make any mistakes.

Questions

1. Does Opal have a polarized range or a condensed range?

2. What is Opal's equity in this game?

3. Is Opal's equilibrium expected value in this game greater than, less than, or equal to her equity? Why?

4. Will Opal bet sometimes, always, or never with her K?

5. If Opal checks and Ivan bets, will she call sometimes, always, or never?

6. If we switched the positions so that the Opal still had the condensed range but got to act last instead of first, would her equilibrium EV increase, decrease, or stay the same?

Answers & Explanation

1. Does Opal have a polarized range or a condensed range?

Opal has a condensed range. Of the three cards in this game, hers is in the middle. If she goes to showdown, she will always win against Ivan's Qs but always lose to his As.

Ivan has a polarized range, consisting of one extremely strong hand that is guaranteed to win at showdown and one extremely weak hand that is guaranteed to lose at showdown.

2. What is Opal's equity in this game?

Opal’s equity is 50% of the pot, or $1. Recall that equity is a measure of how much a player would win if there were no betting. If the players just turned their cards over to see whose is higher, Opal would win when Ivan turned over a Q and lose when he turned over an A. Because Ivan has a 1/2 or 50% chance of being dealt each of these cards, then Opal has a 50% chance of winning the $2 pot.

3. Is Opal's equilibrium expected value in this game greater than, less than, or equal to her equity? Why?

At equilibrium, Opal's EV is lower than her equity.

To understand why, we need to think about how EV differs from equity. Equity tells us how much each player would win if there were no betting. EV tells us what each player will actually win, on average, if they play out the game. In other words, the fact that this game *does* contain betting is bad for Opal. Even though both players have the option to bet, Ivan is better equipped to profit from this option.

Ivan's polarized range is the source of his advantage, which we'll quantify later. For now, let’s understand *why* he has an advantage.

No matter which card he is dealt, Ivan has a chance to profit from betting. If he has an A, he can value bet and profit if Opal calls. If he has a Q, he can bluff and profit if she folds.

Opal, who always holds a K, does not have anything to gain from betting. If she were to bet, it would be trivial for Ivan to play perfectly by calling when he has an A, which is the nuts, and folding when he has a Q, which is the nut low. Opal would not profit from betting, because the only hands that would fold would be those she was going to beat anyway, and the only hands that would call would be hands that beat her.

When Ivan bets, however, Opal's decision is not so simple. If she folds, she loses a $3 pot that she might have won by calling. But if she calls, she risks losing an extra $1 when Ivan has an A.

A polarized range consists of hands that can profit from betting, while a condensed range consists of hands that do not want either to bet or to face a bet. Thus, the introduction of betting to the game enables the player with the polarized range, Ivan in this case, to profit at his opponent's expense.

4. Will Opal bet sometimes, always, or never with her K?

Never. Looking at the above, Opal has nothing to gain by betting. If she bets when Ivan has a Q, he'll just fold and she'll win $1 that she probably would have won anyway. If she bets when he has an A, then he will call and she'll lose an extra $1. Unless Ivan makes a truly grievous mistake and either calls with a Q or folds an A, then Opal gains nothing from betting. An equilibrium strategy assumes the opponent will not make mistakes, so Opal will always check.

5. If Opal checks and Ivan bets, will she call sometimes, always, or never?

Sometimes. This is the trickiest question yet, and it gets to the heart of what equilibrium means.

When two strategies are at equilibrium, neither player can unilaterally change his strategy to gain additional EV. Even if Ivan told Opal his exact betting strategy, there would be nothing she could do to exploit it. We can test whether a strategy could be part of an equilibrium by looking for ways that an opponent could counter that strategy if he knew what it was.

If Opal played a pure strategy of always folding, Ivan could exploit her by bluffing and profiting from those folds. If Opal played a pure strategy of always calling, Ivan could exploit *that* by never bluffing but winning an extra $1 when he bets an A. To be unexploitable, Opal must be unpredictable. She must employ a mixed strategy of sometimes calling and sometimes folding.

6. If we switched the positions so that the Opal still had the condensed range but acted last instead of first, would her equilibrium EV increase, decrease, or stay the same?

It would stay the same.

You might have guessed that Ivan would have the advantage in the original scenario because he was in position. Although you were right that he has the advantage, position is not the source of his advantage. In fact, if we swapped the positions of the players, nothing about the equilibrium strategy would change. Ivan would always bet his As, he'd bluff his Qs at the same frequency, and Opal would call a bet at the same frequency but never bet herself. Consequently, the EV of the game would remain exactly the same for both players.

We can make sense of this intuitively. In the original scenario, Ivan doesn't gain any useful information from his position. He already knows that Opal has a K, so he doesn't learn anything from seeing her check.

Technically, if the players switch positions, Opal does gain information when Ivan checks, but she can't do anything useful with it. If Ivan checks, then he must have a Q. Opal could bet if she wanted to, but it wouldn't gain her any EV, because he would fold and she would win the same $2 pot that she would have won anyway by checking.

Scenario: The Clairvoyance Game (Ivan's Strategy)

Questions

Now we'll flip things around and put you in Ivan's shoes. We've already established that if Opal were to bet, his decisions would be trivial: he'd call with the nuts and fold the nut low. Take a moment to consider his betting strategy if Opal checks.

1. Facing a check, will Ivan bet sometimes, always, or never when he holds an A?

2. Facing a check, will Ivan bet sometimes, always, or never when he holds a Q?

Answers & Explanation

1. Facing a check, will Ivan bet sometimes, always, or never when he holds an A?

Always. He's got the nuts, and if he checks, then the betting is over and it's time for showdown. This is his last opportunity to put money in the pot. There's no guarantee that Opal will call, but Ivan must give her the chance, because he's certainly not going to win anything more by checking.

2. Facing a check, will Ivan bet sometimes, always, or never when he holds a Q?

Sometimes. We can think about Ivan's bluffing strategy in much the same way that we thought about Opal's calling strategy.

If Ivan played a pure strategy of never betting a Q, Opal could exploit him by always folding and refusing to pay off his value bets. If Ivan always bluffed his Qs, then Opal could exploit him by always calling. Even though she would sometimes pay off value bets, she'd still come out ahead. If Ivan bet 100% of the time, no matter his hand, then Opal would win $3 1/2 the time that she called and lose $1 the other half. That means that calling would have an EV of $1 for her, making it strictly better than folding, which would have an EV of $0. So, any pure strategy Ivan could play with a Q would be exploitable. Against real-life opponents, exploitable strategies might be desirable, but they won't be part of an equilibrium.

Mixed Strategies Arise Only When a Player is Indifferent

We've worked with mixed strategies already, but there's an important point to make about them now: *mixed strategies arise only when a player is indifferent between two or more options*. If one option has a higher EV than any other, then it is correct to exercise that option 100% of the time, and that's what we'll see at equilibrium.

The common misconception here is that sometimes you must make a sub-optimal play for the sake of being deceptive, to "mix it up" and keep your opponents guessing. For example, they may treat bluffing as a form of "advertising" that, though it loses you money in the short term, causes you to eventually win more with your value bets.

This puts the cart before the horse and is not game theoretically sound. Although "mixing it up" is the motivation for adopting mixed strategies, you shouldn't do so if you believe one of the options to be sub-optimal.

In the Clairvoyance Game, Ivan doesn't mix it up when he has an A. Betting is strictly better than checking, so that's what he does 100% of the time. He does mix it up with a Q, but that's because there is no clearly best option. If Opal plays optimally, then betting and checking will both have $0 EV and it won't matter what Ivan chooses. Against Opal’s equilibrium strategy, he sacrifices nothing by playing a mixed strategy, and getting the mix right ensures that he does not leave himself open to exploitation should Opal deviate from equilibrium.

Mixed strategies are most relevant when an opponent is not playing optimally (it's basically impossible for human players to do so), but you don't know which mistakes, exactly, he is prone to make. If you were Ivan in the Clairvoyance Game, you might be sure that Opal will not play optimally, but you might not be able to predict specific mistakes that she will make, in which case neither betting nor checking is strictly better for you. You could do one or the other with 100% frequency, but that would just be a wild guess, and if you guessed wrong, it would cost you. The mixed strategy guarantees that you will leverage your polarized range for some additional EV without having to do any guessing.

I think the logic that leads to error here starts with thinking along the lines of, "This player calls too much, so I don't really want to bluff him, but if I don't ever bluff, he won't pay off my value bets. So, I have to let him catch me bluffing sometimes. It's an investment in getting future value bets paid off."

This may sound compelling, but it’s ultimately contradictory. You assume your opponent will call too much if you bluff, but you also worry that he might not call enough when you have strong hands. You can't logically maintain both assumptions, so you have two options: either you accept that you don't know what his strategy will be, in which case betting and checking have the same EV for you and you should employ a mixed strategy, or you commit to a belief that he'll call either too much or too little and act accordingly.

This doesn't have to be a lifelong commitment. Dynamics between players evolve, and it's entirely possible that your opponent is planning to call too much in this situation, but that if he sees you decline to bluff, then the next time you bet the river, he'll plan to fold. If you can predict that, great, but it's not an argument for a mixed strategy. It's an argument for never bluffing in the current situation and then always bluffing the next time.

At equilibrium, we will only ever encounter mixed strategies when a player is indifferent. It won't always be the case that both options have $0 EV, but they will always have the *same* EV, or else there would be no incentive to mix.

Indifference is the Goal of a Balanced Betting Range

In this section, we'll look at how to determine the exact frequency at which Ivan should bluff with his Qs in the Clairvoyance Game. There's going to be some math, but the mathematical details are not the important part. You don't need to memorize these solutions, and you certainly don't need to do algebra at the poker table. What's important is to understand, at a conceptual level, what's going on: Where do these equations come from? What do the variables represent? How would changes in those variables change the optimal strategy?

To determine how often Ivan should bluff with a Q, we need to clarify exactly what his betting range is designed to accomplish. The intuitive answer is that when he bets a Q, he is trying to get his opponent to fold a K. Though that's technically correct, it's not an answer to what his *range* is trying to accomplish. Ivan's betting range consists of both As and Qs. Sure, he'd like to get a fold when he has a Q, but he'd like to get a call when he has an A. What is his overall range, which consists of both As and Qs, trying to accomplish?

The answer is *indifference*. The goal of a balanced betting range is to make an opponent indifferent between calling and folding with a bluff-catcher, in this case a K.

No other strategy could be part of an equilibrium. If Ivan tries to make Opal fold when he has a Q and make her call when he has an A, he would have to play the two hands differently. Perhaps he would bet more when he had a Q than when he had an A, or perhaps he would try some reverse psychology and do the opposite. But whatever he did, he would run the risk of her figuring out his strategy and exploiting it. Once Opal deduces that Ivan wants her to fold, she's going to call, and vice versa. Only by playing both hands in the same way can Ivan guarantee that he will not leak that critical information.

There are many real-world situations where the best strategy will be to play your bluffs differently than your value bets. If you believe that you can predict how your opponent will respond to a bet, then you should absolutely try to take advantage of that, even though it opens you up to potential counter-exploitation. The method described here is for situations where you don't know how your opponent will respond. An approximately balanced betting strategy will be better than a wild guess, so it's important to know how to construct one.

To make Opal indifferent between calling and folding, we need to ensure that both options have the same EV for her. Otherwise, she would not be indifferent, she would prefer the play with the higher EV.

The EV of folding is $0. If Opal folds, she neither wins nor loses anything.

Yes, she contributed $1 to the pot, but that money is no longer hers. Once it's in the pot, it no longer counts as a loss for her.

To make Opal indifferent, then, Ivan needs to find a betting strategy such that calling also has an EV of $0 for her. We'll start by writing an equation for the EV of calling. When Opal calls and catches Ivan with a Q (a bluff), she wins the $2 in the pot plus the $1 bet for a total of $3. When she calls and Ivan has an A (a value bet), she loses the $1 bet. So,

EV = (%Bluff) * $3 - (%Value) * $1.

We also know that %Bluff and %Value must sum to 1. That is, all of Ivan's bets are either bluffs or value bets, either Qs or As. There is no third option, nor is there any way for a bet to be both. So, we can replace %Value in the equation with (1-%Bluff). This gives us

EV = (%Bluff * $3) - ((1-%Bluff) * $1).

Now we set the EV to $0, because that's the outcome we want, and we solve for %Bluff. That yields a result of 1/4. If Ivan's betting range consists of ¼ bluffs, then Opal will break-even on the call. She will do no better or worse by calling than she would by folding. For every four bets that Ivan makes, he should average three value bets and one bluff.

The simpler way to express this is that you want a bluff-to-value ratio equal to the size of the bet divided by the bet plus the pot, or Bet/(Bet + Pot). That's a ratio worth remembering and using in real poker situations. You don't need to get this ratio exactly right for it to be useful, you just need to understand the two factors that determine roughly how often you get to bluff:

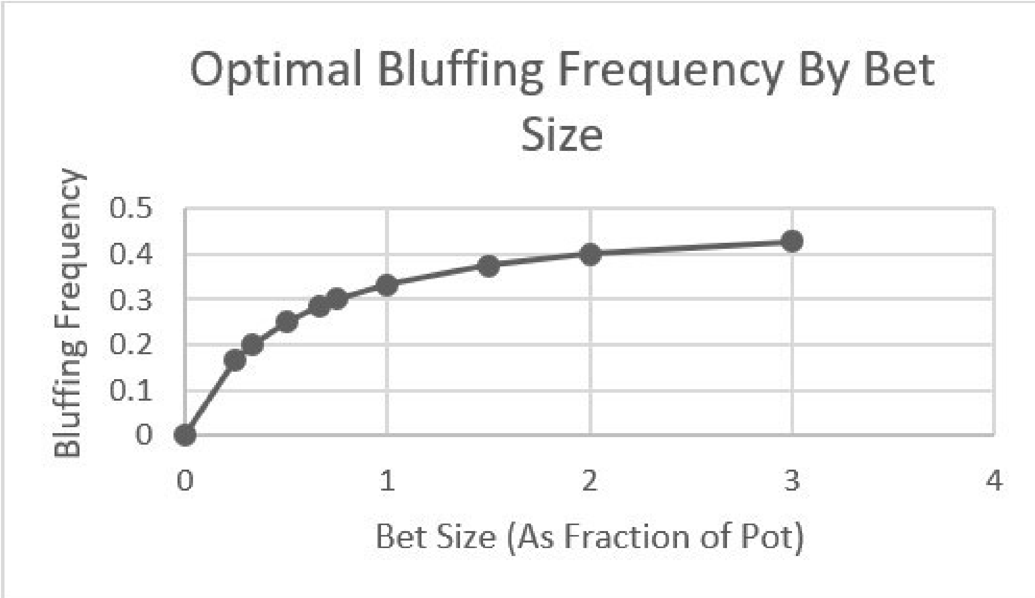
1. Your value bets. The more hands you could have that would value bet in a given situation, the more bluffing you get to do. In spots where you could easily have many strong hands, you should be more inclined to bluff.

2. The size of your bet relative to the pot. Big bets mean more bluffing. If that seems counter-intuitive, think of it this way: big bets offer less good pot odds to your opponent. You should want to make big bets with your strong hands, but you need to give your opponent some incentive to call them. The possibility of beating a bluff is that incentive. When you bet small, your opponents are supposed to call often even though they will probably lose because they are getting such a good price. When you bet big, they aren't getting such a good price, so they must have a better chance of winning (by catching a bluff) to compensate.

This is not to say that you should only bluff with big bets, or that big bets should always be bluffs. The balanced approach is to use the same bet size with both value bets and bluffs, but to include more bluffs in your betting range when the bet is larger. If your large bets were always bluffs, that would be a very easy strategy for a savvy opponent to counter!

The chart and graph below show the equilibrium bluffing frequency for polarized river bets of various sizes. Note how bluffing frequency increases along with bet size but never quite reaches 1/2. No matter how big your bet, a bluffing frequency of 1/2 or higher would always make calling with a bluff-catcher +EV for your opponent.

Bet Size (Relative to Pot)	Bluffing Frequency
1/4	1/6
33/100	33/166
1/2	1/4
67/100	67/234
3/4	3/10
1	1/3
1 1/2	3/8
2	2/5
3	3/7
10	10/21
100	100/201



Optimal Bluffing Frequency By Bet Size
Bluffing Frequency
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
Bet Size (As Fraction of Pot)

Optimal Calling Frequency Depends on Bet Size and Pot Size

After checking and facing a bet, how often should Opal call? The process for answering this question resembles the process for determining Ivan's bluffing frequency. We start by thinking about what Opal's calling strategy is designed to accomplish.

The objective of Ivan's betting strategy was to make Opal indifferent between calling and folding a K. Opal's calling strategy will similarly try to make her opponent indifferent between two options. Can you predict which two? Take a moment to try before you read on.

If Ivan has an A, there's nothing Opal can do to give him a difficult decision or to deter him from betting. There’s nothing for Ivan to fear, no downside to betting when he has the nuts.

Thus, Opal's calling strategy must revolve around making her opponent indifferent between betting and checking a Q. Checking a Q has an EV of $0 for Ivan: he'll always lose the pot, but he won't put in any more money. To make him indifferent to betting, Opal’s calling strategy must cause betting a Q to also have an EV of $0.

Once again, we'll write an EV equation, set EV equal to $0, and solve for a variable. This time, the equation will be for Ivan's Expected Value when betting a Q, and the variable will be Opal’s calling frequency. When Opal calls, Ivan loses the $1 bet. When Opal folds, Ivan wins the $2 pot (note that the $1 bet is a loss if called, but it is not profit if the bluff succeeds - we are measuring the net change for Ivan as a result of the bet, which will be either +$2 or -$1 depending on whether Opal calls). So,

EV = %Fold * $2 - %Call * $1.

Call and Fold are the only two options for Opal, so %Fold = (1-%Call). This enables us to rewrite the equation as

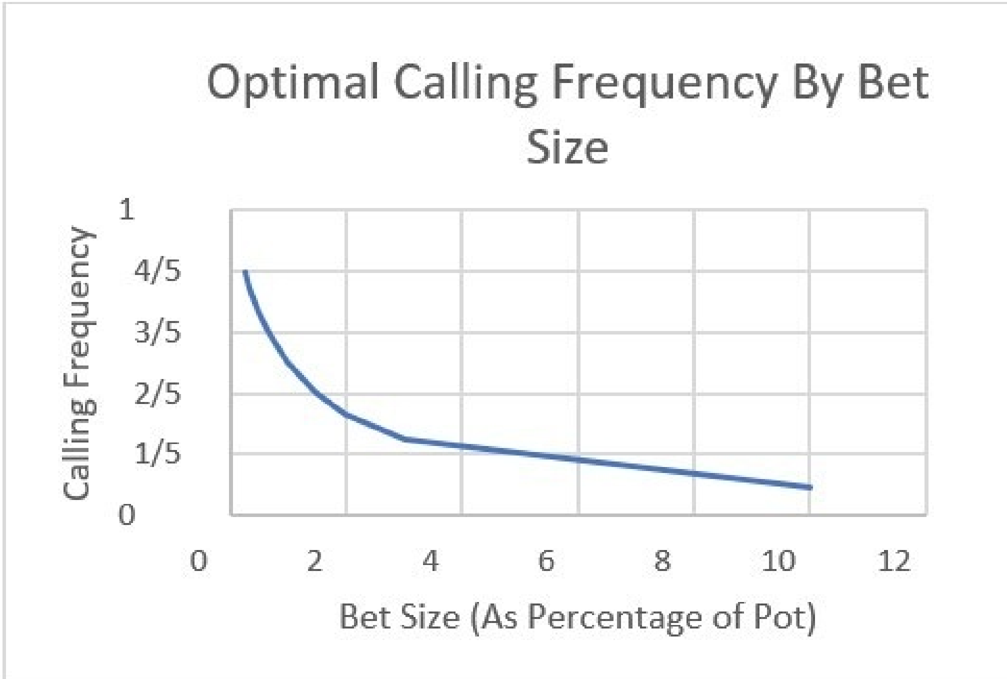
EV = (1-%Call) * 2 - (%Call * $1).

If we set EV to $0 and solve for %Call, we get 2/3. If Opal calls 2/3 of the time, then Ivan is indifferent between betting and checking with a Q.

As before, you don't have to do algebra at the table. The more general solution is 1 - (Bet/(Bet + Pot)), and simply using this equation as a rough guide to determine your strategy should be helpful. Just remember that the larger your opponent's bet relative to the pot, the less you need to call to make him indifferent to bluffing. The more expensive his bluffs are, the safer it is to let him get away with them because of how much he loses when you do call.

The chart and graph below show the equilibrium bluff-catching frequency facing bets of various sizes. Note how calling frequency decreases as bet size relative to the pot increases.

Bet Size (Relative to Pot)	**Calling Frequency**
1/4	4/5
1/3	3/4
1/2	2/3
2/3	3/5
3/4	4/7
1	1/2
3/2	2/5
2	1/3
3	1/4
10	1/11
100	1/101


Optimal Calling Frequency By Bet Size
Calling Frequency
1
4/5
3/5
2/5
1/5
0
0
2
4
6
8
10
12
Bet Size (As Percentage of Pot)

What's This Game Worth?

Suppose that you are Ivan, the player with the polarized range in this game. Instead of $1, however, you have the option of betting any amount up to $1000. You can only pick one bet size, but you can construct your betting range however you wish. How much would you bet, and why?

Take a moment to answer that question yourself. We'll return to answer it together at the end of this section. First, let's calculate the EV of the game as originally conceived.

It's easiest to do this if we look at things from Ivan's perspective. We know that betting with a balanced range makes Opal indifferent between calling and folding. That means that, if Ivan bets a balanced range, both players' EV will remain the same whether Opal always folds, always calls, or chooses any combination of the two plays.

To make the math easy, let's find Ivan's EV if Opal always folds. In that case, Ivan wins $2 every time he bets, whether he's holding an A or a Q. He'll never check an A, so if he checks, he always holds a Q and therefore wins $0. That means that the EV of this game for Ivan, if he employs his equilibrium betting strategy, is equal to his total betting frequency times the size of the pot, which is what he wins when Opal folds.

Half the time, Ivan has an Ace, and he always bets that. The other half of the time, he has a Q, and he will bet 1/3 of those. His overall betting frequency is 1/2 + (1/2 * 1/3) = 4/6, which reduces to 2/3. So, 2/3 of the time, Ivan bets and wins $2, for an EV of 2/3 * $2 = $1.33. He had to ante $1 in order to play the game, so his overall EV is $0.33. If you could always be the player with the polarized range, you would average a profit of $0.33 each time you played the Clairvoyance Game.

Recall that when no betting was allowed, no player had an advantage, so both had an EV of $0. What's happening, essentially, is that betting enables Ivan to win the pot not only when he has an A - he wins those pots even when there is no betting - but also some of the time when he has a Q.

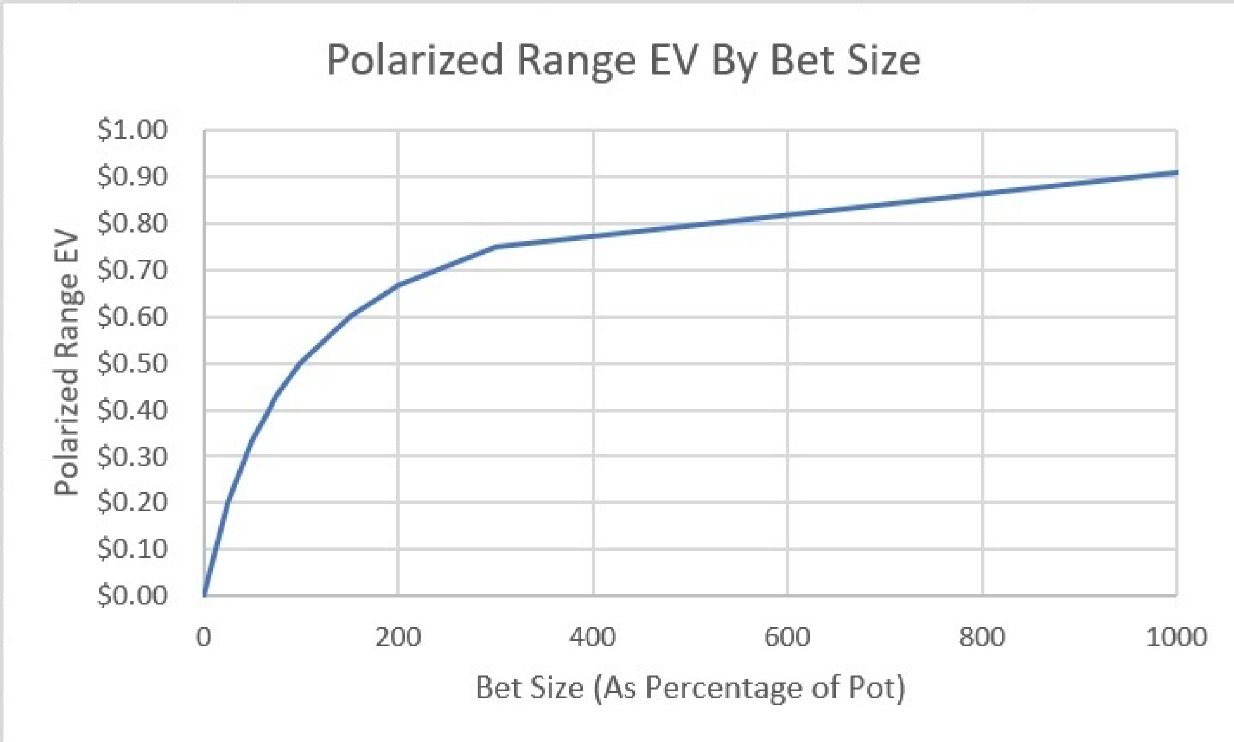
In other words, he gets to do some amount of bluffing, and *there's nothing his opponent can do about it!* Ivan could literally tell her that his strategy will be to bet all his As and 1/3 of his Qs, and Opal still couldn't stop him from winning that $0.33. If she called more often, she'd catch some extra bluffs, but she'd also pay off more value bets. If she folded more often, she'd save more on value bets but lose more to bluffs.

This profit is only guaranteed if Ivan sticks to a balanced betting range, though. If he gets greedy and starts bluffing more often, then Opal's calls become profitable. In other words, Opal can exploit this strategy by calling. If Ivan always bluffs and Opal always calls, then Ivan loses his edge and the game goes back to being $0 EV for both players. Only the right balance of bluffs and value bets guarantees a profit to the polarized range.

Now that we have this intuitive understanding of why exactly betting a polarized range is profitable, let's return to the question of what the optimal bet size would be. We've learned that betting is good for Ivan because it enables him to sometimes win the pot with bluffs and/or to get value bets paid off, depending on Opal's calling strategy. We've also learned that the larger the bet, the more bluffs can be included in a polarized range.

It follows from this that bigger is better. In fact, the optimal strategy for a polarized range is to make the largest possible bet, and then to include as many bluffs as necessary to make a bluff-catcher indifferent to calling. This is true even if means betting $1000 into a $2 pot! Here's a chart showing how the value of the game increases for the polarized range as the bet size gets larger:

Bet Size (% of Pot)	Polarized Range EV
25	$0.20
33	$0.25
50	$0.33
67	$0.40
75	$0.43
100	$0.50
150	$0.60
200	$0.67
300	$0.75
1000	$0.91
10000	$0.99



Recall that as the bet size gets larger, Ivan's bluffing frequency gets closer and closer to 50%, or a 1:1 ratio of value bets to bluffs. It will never reach 50%, but it eventually gets quite close. Even though she knows that her opponent is bluffing nearly half the time, there's nothing that Opal can do about it when the bet is so large. At some very large bet size, she ends up folding virtually 100% of the time and giving the entire pot to Ivan.

You might be wondering why, if such large bets are optimal, you so rarely see them in real poker games. One reason is that a lot of poker players, even experienced ones, don’t understand theory well and don’t always make the best possible plays, especially when they fly in the face of convention.

However, it’s also important to recognize that huge bets are only optimal when your range is as perfectly polarized, as it is in this example. In the next chapter, we’ll revisit this game but give each player the same range, and it will be interesting to see how the optimal strategy changes.

First, though, we’ll look how you can use what we’ve already learned to make better decisions in real poker situations.

Real World Applications

We just put a lot of work into solving a game you'll never play. Let's talk about how you can use these lessons at the poker table! What follows is a brief discussion of how concepts explored in this chapter apply to common poker situations. It's not meant to be an exhaustive consideration of these topics, more like a prompt to get you thinking about the connections between theory and practice.

Checking to the Raiser

We've seen that polarized ranges benefit from betting, growing pots and putting pressure on bluff-catchers, while condensed ranges prefer not to put money into the pot unless they must due to the risk of bluffs. It's important, then, to be aware of whether you are the player more likely to have a polarized range.

Although polarized ranges consist of both strong and weak hands, strong hands are usually the limiting factor. As we all know, it isn't hard to make weak hands. So, the player with more nut or near-nut hands in his range is the one who should typically do the betting.

I sometimes use the term *nuts advantage* to describe a situation where one player has more of the strongest hands in his range. It's important to recognize that the player with the nuts advantage is not always the player whose range has the most equity. The two often go together - nutty hands have a lot of equity, after all - but they don't have to.

Some awareness of this concept is baked into commonly received poker wisdom. This, for instance, is the reason for checking to the raiser. The player who was the last aggressor on the previous street probably has more strong hands in his range, unless the newest community card(s) dramatically change the board texture.

For example, if you call a raise from the big blind, you probably don't have AA, KK, or AK in your range, as you would have re-raised with such strong hands. The pre-flop raiser, however, could have any of those. So, you should typically check to him on the flop. The same is true if you checked and called a bet on the flop or turn. Your default on the next street should be to check to the aggressor, regardless of the strength of your hand.

One common mistake is for a player who slowplayed a strong hand on the flop and turn, or who rivered a strong hand on an apparently innocuous card, to suddenly take the lead and bet. Presumably he is worried that his opponent will not bet again, which is understandable, but he should also be worried about the strength that this bet will show. Causing your opponent to fold in this situation is just as bad as causing him to check behind, and is probably more likely.

When in doubt, if you are out of position, you should start by checking to the player who was the aggressor on the previous street. This is true regardless of whether you have a strong hand, a weak hand, a vulnerable hand, or anything else. If you decide to bet simply because you have a certain type of hand, then you risk revealing to your opponent what type of hand you have, and that’s not going to end well for you.

Donk Betting

Because checking to the aggressor is so frequently correct, players who did not understand this concept were once derided as “donkeys”, and such bets came to be known as “donk bets”. The unflattering name has stuck, but don’t let it fool you: there are situations where it is correct for the previously-passive player to take the betting lead.

Sometimes a flop, turn, or river will dramatically shift the balance of power, making it more likely for the passive player on the previous street to have strong hands. By definition, this doesn't occur too often, but it does occur. In such cases, the out-of-position player should expect that her opponent will often check behind if she checks, and she should consider taking the lead and betting a polarized range of her own.

A classic example is if you call a raise from the big blind, you check and call a continuation bet on the flop, and then the turn pairs the middle or bottom card on the board. There are good reasons why you are more likely than the pre-flop raiser to have turned trips. For one thing, a big blind calling range is more likely than a pre-flop raiser's range to contain smaller cards, the kind that often make middle or bottom pair on the flop.

Even if the pre-flop raiser were to flop middle or bottom pair, he would have some incentive to check the flop. Remember that very strong and very weak hands are the ones that benefit most from betting, while medium-strength hands benefit the least. Thus, after betting the flop, your opponent is less likely to hold medium-strength hands like middle pair. This is not to say that the pre-flop raiser should never continuation bet middle or bottom pair, only that it is one factor making these hands more likely to be in your range than his.

The upshot of all of this is that you should consider betting when you turn trips. "But if I bet, it will be obvious that I have trips!" is the natural objection here. But it will be obvious that you *may* have turned trips as soon as your opponent looks at the turn card. The best way to create deception is to bet your strong hands and balance them with bluffs. In this same situation, you should also consider betting weak hands such as draws that you checked and called on the flop.

When the Aggressor Checks

You should typically check to the raiser as long as he continues to show aggression. Once he stops betting, however, this is a sign that he probably has either a very weak hand that is giving up or a medium-strength hand that is trying to get to showdown cheaply. This means that he is not likely to continue growing the pot, and you should start betting a polarized range at your next opportunity.

Of course, traps are also possible, but we've seen that your opponent has a lot of incentive to bet very strong hands when he already has the betting lead, so you shouldn't worry overly much about this possibility. Even the risk of an occasional slowplayed monster hand is probably not enough to deter you from betting a polarized range.

To be clear, a check from the aggressor is not a green light to bet with just anything. In fact, you might still end up checking the majority of your range. It is, however, an opportunity to bet some strong hands and some bluffs.

Bluffing

It's a bit counter-intuitive, but the best situations for value betting also tend to be the best situations for bluffing. After all, it's the credible threat of running into a strong hand that gives your opponent incentive to fold when you are bluffing, and the threat of a bluff that gives him incentive to pay off your value bets.

Many players make decisions about whether to bluff based largely on a guess about whether they expect their opponent to fold. While it's great to bluff when you expect a fold, it is often correct to bluff when you are not sure how your opponent will respond. Bluffing can be profitable even when your opponent is more likely to call than to fold! A half-pot bluff, for example, only needs to succeed a bit more than a third of the time to show a profit.

Without a strong read to guide you, there should be some hands that you would bluff with in any situation where you could have value bets. The question to ask is not, "Is this a good spot to bluff?" or "Do I think my opponent will fold?" but rather "*Which hands* will be best for bluffing in this situation?"

You should bluff in proportion to the number of strong hands that you could have in a given situation. The more easily you could have a strong hand, the more willing you should be to bluff. As your bet size gets larger, your bluffing frequency approaches, but never quite reaches, 50%.

Realistically, you aren't going to do precise calculations at the table. What you can do is assess how easily you could value bet and then be more willing to bluff in situations where you have more potential value bets.

Slowplaying and Deception

Trapping with strong hands is not usually necessary or desirable. It’s nice to be deceptive, but you also want to grow the pot when you have strong hands. In the Clairvoyance game, the player with the polarized range creates deception not by checking - he bets 100% of the time that he has the nuts - but by balancing his value bets with bluffs. It is the threat of bluffs that provides his opponent with the incentive to call with hands that can't beat his value bets.

It's a common mistake to try to trick opponents into betting when you hold a strong hand. You should generally focus on inducing calls instead, and they can't call if you don't bet.

Defending the Big Blind

You will often find yourself in situations where you are getting a very good price to call with a very bad hand. On early streets, especially in big bet games like no-limit hold 'em and pot limit Omaha, immediate pot odds tend to be less important than how well your hand will play on later streets, but that does not make them negligible.

For a long time, the conventional wisdom was simply to fold in these situations. For example, suppose you're playing a no-limit tournament with antes, someone makes a small raise, and the action folds to you in the big blind with K7o. Sure, you're getting a good price, but it's such a bad hand, why not just fold rather than risk getting into a tough spot?

The answer is that it's OK to get into tough spots if you can play them with some competence. You can accept that you're going to suffer from a positional disadvantage and from having a poor hand, and it can still be correct to call simply because the price is so good. You just don't need to win very often when you are getting 5:1 odds, as long as you don't make huge mistakes later in the hand.

Game theory is an invaluable tool for avoiding huge mistakes. When you make a pot odds call from the big blind, it's rare that you're going to flop a strong hand. Even on a K62 flop, K7 is hardly the nuts, unless you are very shallow.

What you're really signing up for when you make these calls is playing a condensed range on future streets.

We’ve seen that playing a condensed range isn't ideal, but it's not the end of the world, either. In our Clairvoyance Game, we found that the player with the condensed range can play a strategy that guarantees an EV of $0.67. That's just playing a balanced bluff-catching strategy, not making any attempt to outplay her opponent or do anything exploitative. That's a bad deal if it costs you $1 to play the game, but it's a good deal if it only costs you $0.50 because you were already forced to put the other $0.50 in blind.

Essentially, this is what you are doing when you call, before or after the flop, with a bad hand getting a good price: you're voluntarily entering into a disadvantageous situation because you're well-compensated for the risk. If you understand the relevant game theory, there's a limit to just how disadvantageous the situation can be.

This concept is most applicable in tournaments or other games played with an ante, though it has some application in traditional no-limit cash games.

Betting to "Take It Down"

Condensed ranges do best by checking and then finding a calling strategy that minimizes the profitability of betting for the polarized range. You'll sometimes see players make big bets with medium-strength hands because they know that they are vulnerable and they want to just lock up the pot now. Although that's an understandable impulse, it's typically a mistake.

What we see clearly in the Clairvoyance game is that betting a medium-strength hand enables your opponent to play perfectly. It guarantees that you lose a bet when he has a strong hand while preventing him from putting that same bet into the pot with the hands that you beat.

If you're going to put a bet in with a medium-strength hand, it's often better to do it by checking and calling rather than by betting. Either way you lose the bet when your opponent has a strong hand, but at least calling gives you the chance to win a bet from weak hands. Bluff-catching is never a pleasant situation to be in, but a proper understanding of game theory will minimize the discomfort by enabling you to get these tough decisions right more consistently.

Overbetting

Big bets, including those larger than the size of the pot, are the best way to leverage a polarized range. Many players focus on the downside of big bets (they might not get paid on strong hands) and not enough on the upside (more bluffs get through, big hands win more when they are called).

You're more likely to have a polarized range late in a hand, on the river or perhaps the turn. On the flop, even very weak hands tend to have significantly more than 0% equity, and very strong hands significantly less than 100%. So, you'll do most of your biggest betting on turns and especially rivers.

It's common to see players bet half the pot on every street, but that's usually not optimal. It will often be more profitable to bet 25% to 33% of the pot with a wide range on the flop, followed by a much larger bet of 75% to 200% of the pot on the turn with a more polarized range.

Test Yourself

Time to put your learning to the test! I'm going to drop you into some real poker situations so that you can practice applying the concepts from this chapter. The examples below all assume that you are playing no-limit hold 'em at a nine-handed table, with $2 and $5 blinds and $500 effective stacks.

1. The first player to act pre-flop raises to $20, and you call from the big blind with 7♥ 7♠. The flop comes Q♦ 7♣ 3♥. Do you bet or check?

Check.

As a BB caller, you are virtually guaranteed to have the more condensed range, especially when playing against an early position raiser who should have a very strong range. In this case, you got lucky to flop a hand that might as well be the nuts, but your *range* - the set of all the hands that you could hold in this situation - is still much weaker than your opponent’s. Thus, you should play the role of the condensed range, checking and giving him the opportunity to bet a polarized range.

This is the most deceptive thing that you can do, because it is most consistent with how you would play other types of hands, including very weak hands you intend to fold and medium-strength hands you intend to call (whether you should call or raise your set after checking is a discussion we won’t get into yet). Checking gives your opponent the opportunity to bet hands that he perceives as strong, even though they will mostly be weaker than yours, as well as bluff with weak hands. He should bet fairly often in this situation, including with most of the hands that would have called if you had bet.

If the aggressor checks behind on the flop, then you might consider betting the turn, but a good default strategy is to “check to the raiser” as long as that player continues to show aggression. The same is true if you checked and called a bet on the flop or turn: your default on the next street should be to check to the aggressor, regardless of the strength of your hand.

2. Now let’s flip it around and make you the pre-flop raiser. You raise in first position with 7♥7♠, and the big blind calls. The flop comes Q♦ 7♣ 3♥. Your opponent checks. Do you bet or check?

Bet.

You have a strong hand, and you want to build a big pot. The only downside of betting is that it will show strength and could cause your opponent to fold.

However, because you were the aggressor on the previous street, there is nothing suspicious about you betting the flop. In fact, checking might be suspicious, especially if you then raised on a future street.

Betting is the most deceptive thing you can do, because it's consistent with how you should play so many other types of hands, such as 99 or 87 or KJ. Thus, betting effectively disguises your strongest hands and gives your opponent incentive to call with bluff-catchers. As we saw in the Clairvoyance Game, even when your opponent knows that you would bet every single time that you have a strong hand, she still has incentive to call in order to avoid losing too much to bluffs.

All of that said, it's entirely possible that your opponent will fold. That doesn't mean that you did anything wrong, it just means that she didn't have much of a hand and probably wasn't going to put money into the pot no matter what you did. If you check the flop and then bet or raise later, that's far more likely to raise a red flag than if you just bet straight away.

3. Take a moment to consider the difference between questions 1 and 2. You hold the same hand on the same board with the same stacks, but in the first case the best play is to check, and in the second it's to bet. What's the difference?

When you are the big blind, you are playing a mostly condensed range that benefits from checking. In the rare case that you have a very strong hand like 77, you can effectively conceal that strength by checking.

When you are an early position raiser, you are better equipped to bet a polarized range consisting of both strong hands and bluffs. Because you will bet so many bluffs, the best way to conceal your strongest hands is to bet them. The fact that you could easily be bluffing will give your opponent incentive to call or raise you, which when you have 77 is exactly what you want.

There's nothing inherently deceptive about checking a strong hand. What's deceptive is to play strong hands the same way you'd play weaker hands. For the big blind, that means checking the flop. For an early position raiser, it means betting.

4. The first player to act pre-flop raises to $20 and you call from the big blind with 8♥ 8♠. The flop comes Q♦ 7♣ 3♥. You check, and your opponent bets $25. Do you fold, call, or raise?

Call.

As a big blind caller, you have the more condensed range, and you got off to a good start by checking the flop. Raising would be an extremely aggressive action that would put you well on your play to playing a large pot. That's OK if you have a hand that *wants* to play a large pot, or a hand that wants your opponent to *think* you want to play a large pot. You can have a small raising range in this situation, but when you do raise, it should be with a polarized range, mostly a Queen or better for value plus a few bluffs.

In this case, you have a decidedly medium-strength hand. It's not strong enough to raise for value, but it’s far too strong to fold.

Think of it like playing a K in the Clairvoyance Game: if you raise with 88, nothing good happens. Hands better than yours won’t fold, and hands worse than yours won’t call. Your hand is a bluff-catcher, and the way to make money with it is to check and give your opponent room to bet hands worse than yours. Of course, she could easily have a better hand, but raising won’t change that, it will just cause you to lose even more when you’re behind.

A common concern about checking and calling is that your hand will have trouble standing up to bets on later streets. In other words, you may call the flop just to end up folding later.

Not only is that OK, it’s correct! This is a valid concern, but it’s important to recognize that your opponent continuing to bet is a worst-case scenario. It *may* happen, but it isn’t *guaranteed* to happen. Your opponent doesn’t know that you have 88. For all he knows, you have 77 or KQ or some other very strong hand that will happily call or even raise a turn bet. You know how I keep saying that you can conceal your strong hands by playing them the same way you'd play weaker hands? Well, that works both ways! Because you could very plausibly have a strong hand after checking and calling the flop, your opponent can't easily profit by just blindly firing away at the turn and river.

Don’t assume that you will always face pressure on later streets after calling the flop. Sometimes you will just check to showdown and win. You might even squeeze out a value bet on the river!

5. The first player to act pre-flop raises to $20, and you call from the big blind with 8♥8♠. The flop comes Q♦ 7♣ 3♥. You check, your opponent bets $25, and you call. The turn is the 2♣. You check, your opponent bets $50, and you call. The river is the 8♦. Do you bet or check?

Check.

Although you just rivered the second nuts, your range is very much condensed. By consistently checking and calling, you’ve represented a medium-strength bluff-catcher, which is exactly what you had until that miracle river. You should keep telling the same story by checking again.

Your opponent, who has shown consistent aggression, has the more polarized range and should drive the betting on the river. She should both bluff and value bet hands that are worse than yours, after which you can decide whether you want to call or raise.

This does not mean that she will *definitely* bet, only that she should be more likely to put money into the pot if you check than if you bet. She has more incentive to bet, with different types of hands, than she would have to call if you bet.

6. The first player to act pre-flop raises to $20, and you call from the big blind with 8♦ 7♦. The flop is J♥ 7♣ 3♥. You check, your opponent bets $25, and you call. The turn is the 7♠. Do you bet or check?

Checking is OK, but this is one of those rare cases where the player with the more polarized range on the flop has a lot less incentive to bet the turn. With strong hands such as overpairs, many players will fear that you just turned trips and will - correctly - check behind if you give them that opportunity. Or if they do bet the turn, it will be with the intention of checking back the river. So, unless you take the lead at some point, you aren't likely to win more than one more bet from even your opponent's strongest second-best holdings.

You can't make the risk of trips go away. The two sevens are right there on the board for your opponent to see. What you can do is present him with a different risk to worry about: the risk of folding the best hand to a bluff. The way to do that is to bet the turn.

7. The first player to act pre-flop raises to $20, and you call from the big blind with T♣ 9♣. The flop is J♥ 7♣ 3♥. You check, your opponent bets $25, and you call. The turn is the 7♠. Do you bet or check?

Again, you should at least consider betting. You are more likely to have turned trips than your opponent. That entitles you to bet out with a polarized range consisting of some very strong hands and some bluffs, and this is a reasonable bluffing candidate.

You shouldn’t expect your opponent to fold anything as strong as a Jack, but he could easily have unpaired hands such as AK or KQ that, though seemingly weak, are nevertheless favorites to beat you. While you wouldn’t mind seeing a free river, which is what will likely happen if you check, winning the pot immediately is worth a lot more to you than your roughly 23% chance of improving.

8. The first player to act pre-flop raises to $20, and you call from the big blind with Q♠ J♠. The flop comes Q♦ 7♣ 3♥. You check, and your opponent checks behind. The turn is the 2♣. Do you bet or check?

Bet. When the previous aggressor stops betting, that's your cue to start growing the pot with a polarized range of your own.

Although QJ is a long way from the nuts, your opponent’s flop check should give you confidence. It could be a trap, but hands stronger than QJ are hard to come by on this board. Your opponent will need to call with many weaker hands, including pocket pairs and even strong unpaired hands like AK or AJ, to ensure that bluffing is not profitable for you. After all, you could easily hold a weak hand that would love to bluff in this situation. You should bet the turn for value and plan to bet many rivers as well.

9. The first player to act pre-flop raises to $20 and you call from the big blind with A♥ 6♥. The flop comes Q♦ 7♣ 3♥. You check, and your opponent checks behind. The turn is the 2♣. Do you bet or check?

Check. Although your opponent has shown some weakness by not betting the flop, it’s not a green light for you to bet any two cards.

His check is a sign that he probably has either a very weak hand that he declined to bluff or a medium-strength hand that does not want to build a large pot. Those medium strength hands are not necessarily going to fold to a bet, though. They are bluff-catchers, after all, so they may well choose to call in the hopes of catching a bluff.

There certainly are hands that your opponent will fold to a bet, and this is a good spot for you to do *some* bluffing. Just as in the Clairvoyance Game, though, you should bet a polarized range that includes only a fraction of your potential bluffing hands, nowhere near all of them. With the majority of your range, including many of your weakest hands and bluff-catchers, you should check.

You don’t need to worry about the exact frequency, just recognize that after the pre-flop raiser checks behind, you get to do some bluffing but not too much. The best candidates tend to be draws with a significant chance of improving. A♥ 6♥ has three or fewer outs against anything that would call a bet, which makes it unappealing as a bluff.

The other problem with betting this hand is that even when your opponent does fold, he’s probably folding a hand worse than yours. That’s still worth something to you, because those hands could beat you on the river, but it's not worth nearly as much as making him fold when he's ahead.

Better hands for bluffing here would be draws such as 5♦ 4♦ or 8♣ 6♣, which you could easily have.

Conclusion

In the Clairvoyance Game, one player has a polarized range consisting exclusively of strong and weak hands, while the other has a condensed range consisting exclusively of bluff-catchers. The player with the condensed range has nothing to gain from betting; she always checks to the player with the polarized range, and if he bets, the best she can do is call at a frequency that makes him indifferent to bluffing. Although this is not ideal - the player with the condensed range has a substantially lower EV in this game than she would have if betting were not allowed - it's the best she can do with the hand that she's been dealt. Betting would only make things worse.

The player with the polarized range has the advantage in this game, and he leverages that advantage by betting a balanced range of value bets and bluffs. His optimal strategy is to bet all his strong hands and just enough bluffs to make his opponent indifferent to calling. The larger the bet, the more bluffing he gets to do, and the higher his EV.

Key Lessons

Real poker situations are typically more complicated than this, but boiling the game down to this one dynamic reveals some lessons that should be useful over the felt.

- ♠ **Aggression is good, but not all hands benefit from it.** Aggression is not the *source* of the advantage enjoyed by the player with the polarized range. Having a polarized range is inherently advantageous; aggression is simply the means of leveraging that advantage. A player with a condensed range is at a disadvantage, but she can't undo that by playing aggressively. In fact, aggressive action will likely make matters worse for her.
- ♠ **Know your role.** In most poker situations, one player is more likely than his opponent (or opponents - this is true even in multiway pots) to hold very strong hands. Before deciding how to play your *hand*, you should consider your *range*. If you are the player with the more polarized range, aggressive action should be your default. If you are the player with the more condensed range, passive action should be your default. This is a question you need to ask yourself on every street. Just because you had the condensed range on the flop doesn't mean you will still have the condensed range on the river.

- ♠ **You don't have to know where you stand.** Many poker players hate ending up in spots where they face bets from an undefined range that includes both strong and weak hands. This is understandable - there's a real disadvantage to being in such a spot - but it's not the end of the world, and when you're getting a good price, calling can easily be better than folding even though it's likely to land you in this spot. As long as you can approximate an unexploitable calling frequency, there's a limit to just how bad this spot can be for you.
- ♠ **When faced with a bet, some part of your range should call, and some part should fold.** This may sound obvious, but it can be tricky to implement in game. A good trick is to ask yourself, before folding, "If I don't call with this, what would I call with?" If you can name a few hands that you could plausibly hold in this situation that you would call with, then you can feel better about folding your current hand. The reverse works as well: before calling, ask yourself what hands you could hold in this spot that would fold?
- ♠ **Not bluffing is exploitable.** Some players are reluctant to bluff until they have experience with a player, or unless they are "sure" he will fold. But poker isn't about being sure. Most players, even so-called fish, have folding ranges. Whenever you decide that you simply aren't going to bluff, you get exploited by a player who was going to fold. You can plug this leak with the same trick I suggested above: before checking a weak hand, ask yourself what other weak hands you could hold that you would bluff with? If you can't think of any, reconsider whether you shouldn't bluff with your current holding. The reverse can help if you bluff too much. Before you bluff with a weak hand, force yourself to identify a few hands that you would give up with.
- ♠ **Bet bigger.** Not all bets will be made with polarized ranges. Particularly on early streets, there may be reasons to bet more "medium" hands. But when you are betting a polarized range, as you often are on the turn and river, experiment with betting the pot, or even more than the pot, with both nutty hands and bluffs. If that makes you uncomfortable, just think about how uncomfortable it will make your opponent!

Caveats

These lessons assume a polarized range playing against a condensed range. Such situations do arise in real poker games, but more commonly, both players hold a variety of hands ranging from very weak to very strong, with much in between. In future chapters, we'll look at basic strategy for more complex ranges. The above lessons will still apply, but they will require some adaptation for ranges that are somewhat polarized but not as extremely as in this example. For now, let's get clear on when these lessons may *not* apply.

- ♠ **Weak hands can get stronger.** Pre-flop and flop ranges tend not to be polarized, because it's rare to hold hands with anywhere near 0% or 100% equity. Even 72o has 20% equity against the strongest pre-flop hands. On the flop, bluffs may have outs to straights, flushes, or big pairs. By the turn, hand values are better defined as either strong or weak, and of course there are no draws on the river. So, lessons learned from the Clairvoyance Game are most relevant on later streets.
- ♠ **Ranges may be similar.** When one player has been betting and another calling, the polarized versus condensed range dynamic emerges most starkly. When the level of aggression has been low, especially if both players started in similar positions and therefore had similar pre-flop ranges, then they are likely to end up with similar ranges on later streets. In the next chapter, we'll investigate how such situations differ from the polarized-versus-condensed dynamic.
- ♠ **A condensed range may contain a few strong hands.** A player with a mostly condensed range might still harbor the occasional slow-played set or rivered two-pair. This doesn't significantly change the polarized versus condensed dynamic, but, as we'll see in the next chapter, it may make overbetting less appealing.

Chapter 3: Reciprocal Ranges

Overview & Objectives

The Clairvoyance Game taught us that, when one player has a polarized range and the other a condensed range, the player with the polarized range has an advantage, and there's nothing his opponent can do to prevent that. At best, the player with the condensed range can mitigate her disadvantage.

If you're one of those poker players who likes having an advantage over his opponents, then you may be wondering a few things right now, like, “How do I get one of those polarized ranges?” and “Why on Earth would I ever play a condensed range?”

As it happens, these two questions have the same answer. In order to develop a polarized range, you must also create a condensed range. Suppose that you raised before the flop, and the big blind called. Now she's checked to you on the flop, and you, having studied the Clairvoyance Game, decide that betting with a polarized range sounds like a pretty great thing to do. So, you bet your strong hands and some bluffs.

The thing is, your range for seeing the flop does not consist exclusively of strong hands and weak hands. Even if you started with only strong pre-flop hands, the flop still has a significant randomizing effect. Kings is a great hand pre-flop, but you probably aren't going to make multiple big value bets with it if an Ace comes on the flop. And we all know how Ace-King shrivels when it doesn’t flop a pair. In order to bet only strong and weak hands, you must check your medium-strength hands and probably some weak ones as well, as you usually have too many to bluff with all of them.

Because you only ever play one hand at a time, this is really a matter of incentives. Betting strong hands will often be more profitable than checking them, even though that will mean that when you do check, your opponent can correctly deduce that you have a condensed range and pressure you accordingly. Likewise, even though checking medium-strength hands makes you vulnerable to bets from a polarized range, it's often more profitable to check these hands and deal with the consequences than to bet them.

In this chapter, we're going to look at how the game plays out when both players start with identical ranges. First, we'll look at a game where Opal is forced to check, in order to more closely mirror the Clairvoyance Game. Then, we'll allow Opal the option to bet, which will give us a chance to look at playing from out of position.

By the end of this chapter, you should be able to:

♠ Split a range into polarized and condensed regions.
♠ Use both passive and aggressive actions to present opponents with difficult decisions.
♠ Articulate why, exactly, position has value.

Scenario: Reciprocal Ranges, Half-Street Game

To better model real poker situations, we're going to play the same game again, but this time both players will have identical ranges. Each player antes $1 and is dealt a single card from a three-card deck consisting of one A, one K, and one Q, but this time either player could be dealt any card. The only information a player has about his opponent is that the card in his hand can't be in his opponent's hand. In other words, if Opal has an A, then she knows that Ivan's range is {K,Q}, and if Ivan has a Q, he knows that Opal's range is {A,K}.

The objective of the game is simply to hold the higher card at showdown, with an A being the highest card and a Q the lowest. There are no community cards and no draws, just a single round of betting followed by a showdown.

We'll start by analyzing this game with only a half-street of betting. That is, the out-of-position player (Opal) is forced to check, while the in-position player (Ivan) may either bet $1 or check. There's no raising allowed, so if Ivan bets, Opal may only call or fold.

Questions

The questions below are a mix of concepts we've already covered and predictions about what the equilibrium for this game will look like. You might not feel fully equipped to answer the latter, but give it your best shot before you move on to the answers and explanations in the next section.

1. What is Opal's equity in this game?

2. Which player has a higher EV in this game? Why?

3. Will Ivan's bluffing frequency with a Q be higher, lower, or the same as it was in the Clairvoyance Game? Why?

4. Will Opal's calling frequency with a K be higher, lower, or the same as it was in the Clairvoyance game? Why?

5. Suppose that we changed the size of the bet, such that Ivan's options were either to bet $2 or check. Which player would benefit from this change, and why?

Answers & Explanations

1. What is Opal's equity in this game?

Both players have 50% equity. They have the same range, so each would have an equal chance of winning the pot if there were no betting.

2. Which player has higher EV in this game? Why?

Ivan has the higher EV. Although both players' ranges are the same, only Ivan can bet. We know that both players' ranges contain hands that can benefit from betting and hands that would prefer not to face a bet. Ivan will never face a bet when he holds a K, but he will be able to value bet his As and, if he chooses, bluffs his Qs. These opportunities have value.

3. Will Ivan's bluffing frequency with a Q be higher, lower, or the same as it was in the Clairvoyance Game? Why?

It will be the same. That may be a bit counter-intuitive, but think about what exactly Ivan is trying to accomplish when he bets a Q. Remember that he can't force Opal to fold. The best he can do is put her in a tough spot when she holds a K. That's still the reason to bluff with a Q in the Reciprocal Ranges Game. The difference is that, because a K is no longer the strongest hand in Opal's range, we must first determine whether he *can* put her in a difficult spot by bluffing.

When we looked at bluffing in the Clairvoyance Game, Ivan knew that Opal had a K. He didn't know whether she would call, but he knew that she would have a tough decision if he bet. He also knew that, by making the same bet that he would make with an A, he could guarantee himself an increased share of the pot regardless of Opal's calling strategy.

In this new variant of the game, he must consider the possibility that Opal holds an A and will have a trivial call if he bluffs. If anything, it would seem that the risk of running into the nuts might cause Ivan to lower his bluffing frequency or deter him from attempting bluffs at all. Let's start by looking at whether bluffing even makes sense for Ivan.

We know that Opal must call 1- (Bet/(Bet + Pot)) or 2/3 of her range to make Ivan indifferent to bluffing with a half-pot bet. When Ivan holds a Q, Opal's range consists of 1/2 As and 1/2 Ks. If her strategy were to call with an A and fold with a K, then she'd only call 1/2 of the time, and a bluff would be profitable. That doesn't prove that Ivan can profitably bluff, but it does guarantee a profitable *strategy*: either Opal folds all her Ks, in which case Ivan's bluffs make money, or she calls with some, in which case Ivan's value bets make money.

A common misconception is that when a player is *uncapped*, meaning that she could have very strong hands including the nuts, then bluffing that player will not be profitable. In fact, the critical question to ask is not, "Could she have the nuts?", but rather "*How often* will she have the nuts (or some other very strong hand that will surely call)?"

Once we determine that bluffing plays a role in Ivan's equilibrium strategy, we actually don't need to change his bluffing frequency from the Clairvoyance game. When Ivan is bluffing in either game, he should expect to lose more often than not. After all, Opal is supposed to call 2/3 of the time. It doesn't matter to Ivan whether he gets called by an A or a K, as he loses the same amount either way.

The purpose of bluffing is to give difficult decisions to the bluff-catchers in Opal's range. When Opal holds a K, the EV of a call is a function of the bet size, the pot size, and Ivan's bluffing frequency. The bet size and the pot size are the same as they were in the Clairvoyance Game, so Ivan's bluffing frequency will be the same as well. That half of his bluffs are guaranteed to lose because they run into an A reduces the overall *value* of betting a polarized range, but the math and theory behind it remain the same: Ivan bluffs in proportion to his value bets so as to make his opponent indifferent to calling with a K.

4. Will Opal's calling frequency with a K be higher, lower, or the same as it was in the Clairvoyance Game? Why?

It will be lower. Essentially, the As in Opal's range pick up some of the slack when it comes to making Ivan indifferent to bluffing.

As before, Opal's goal will be to call at a frequency that makes Ivan indifferent to bluffing with a Q, which in this case will require calling 2/3 or 4/6 of her range (the math is a bit more intuitive if we don't simplify the fractions). The difference from the previous game is that now, Opal’s range when Ivan holds a Q is {A, K}, not just {K}.

It will be trivial for her to call when she has an A, which is 3/6 of her range. To get up to a 4/6 calling frequency, she needs to call with another 1/6 of her range. Because Ks are 1/2 of her range, she needs to call 1/3 of the time that she has a K (1/3 * 1/2 = 1/6). That means she's calling less often with a K than she was in the Clairvoyance Game. The As in her range are doing a lot of the work towards making Ivan indifferent to bluffing, and Ks only have to make up the difference.

5. Suppose that we changed the size of the bet, such that Ivan's options were either to bet $2 or check. Which player would benefit from this change, and why?

Opal would benefit.

I found this to be an extremely counter-intuitive result. It's a big change from the Clairvoyance Game, and I got it wrong when I first started working with this scenario. Finding surprising results and then working to discover the explanations for them is a great way to learn about game theory, so let's dig in!

First, we'll determine the value of the game when Ivan bets $1. To do that, we can look at the equilibrium EV of each hand in Ivan's range and then divide by the number of hands in his range, which in this case is three.

When Ivan has a Q, his equilibrium EV is $0. Either he checks and loses, or he bets and Opal calls at a frequency that makes the EV of the bet $0. Remember, this is the meaning of indifference: no matter what Ivan does with a Q, if Opal plays an equilibrium strategy, then Ivan has an EV of $0.

With a K, Ivan has an EV of $1. He never bets with it, and at showdown he wins $2 half the time and $0 the other half of the time, depending on whether Opal holds an A or a Q.

With an A, it's a bit trickier. When Ivan has an A, Opal has a range of {K,Q}. The Q never calls a bet, and the K calls 1/3 of the time. Overall, Ivan wins $2 5/6 of the time and $3 1/6 of the time when he bets an A, for an EV of about $2.17. His EV for the game, then, is ($0 + $1 + $2.17)/3, or about $1.06. That's a profit of 6% on the $1 that he invests in the game.

What happens if we change the bet size to $2? For starters, we'll need to reconsider Opal's calling frequency. The value of 1 - Bet/(Bet + Pot) is now 1/2 instead of 1/3. That's a big problem for Ivan, because when he holds a Q, As make up 1/2 of Opal's range. Because of the larger bet, Opal no longer has to worry about calling with a K at all! Calling with just the nuts is enough to make bluffing unprofitable for Ivan, which means that even if she always folds a K, he has no incentive to bluff.

The reason to bluff is to present medium-strength hands with a difficult decision. When Opal can only have medium-strength hands, then bigger bets make for tougher decisions. When Opal can have the nuts, though, then Ivan must size his bet in a way that forces Opal either to pay off some value bets or yield the pot to some bluffs.

Although Opal could have the nuts, she will not have the nuts often enough to deter Ivan from bluffing with a small bet. A large bet, however, makes bluffing more expensive for Ivan. The risk of running into the nuts becomes great enough that Opal no longer has to call with a K to prevent Ivan from bluffing profitably. In other words, Opal never has a difficult decision when faced with a large bet: she simply folds Ks and Qs and calls with As.

Ivan has an advantage as long as he can bet less than the pot. Once the bet size reaches $2, then the EV of the game goes to $0 for both players. No one has incentive to put money into the pot without the nuts, because the risk of running into the nuts is too great.

Ultimately, this is to Opal's benefit. When the game is played with a $1 bet, she gives up some EV, but with a $2 bet, the game is break-even for both players.

Scenario: Reciprocal Ranges, Full Street Game

In the half-street variant of this game, only Ivan could bet. Opal was forced to check and then either call or fold. We found that even without a range advantage, Ivan could claim more than his share of the pot by betting a polarized range and giving Opal difficult decisions when she had a K.

Will Opal be able to do the same to him from out of position? Let's tweak the game to allow either player to bet. As before, each is dealt a single card from a three-card deck consisting of one A, one K, and one Q, and either player could have any card. The only information a player has about his opponent is that the card in his hand is not in his opponent's hand. Either player can bet $1 or check. There's still no raising allowed, so once there's a bet, the only options are call or fold.

Questions

Do your best to predict what the equilibrium for this game will look like, then read on for the answers and explanation.

1. Which player will have a higher EV? Why?

2. If Opal checks, will Ivan's betting strategy differ from what it was in the half-street game? Why or why not?

3. Will Opal's betting strategy differ from Ivan’s equilibrium strategy in the half-street game? Why or why not?

Answers & Explanation

1. Which player will have a higher EV? Why?

Even with reciprocal ranges and betting opportunities, position still enables Ivan to show a profit, though it will be smaller than when only he could bet.

2. If Opal checks, will Ivan's betting strategy differ from what it was in the half-street game? Why or why not?

No, it will not. Remember that Ivan's betting strategy "targets" the Ks in Opal's range, and even when Opal is allowed to bet these, she has no incentive to do so. The prevalence of Qs and As in Opal's checking range may change, as she has some incentive to bet those hands. That will have no influence on Ivan's betting strategy, though, because when he bets, he targets only Opal's Ks.

3. Will Opal's betting strategy differ from Ivan's equilibrium strategy in the half-street game? Why or why not?

Trick question! The answer is actually "maybe."

In every variant of this game we've looked at so far, it was always trivial to bet an A, because until now checking would mean going to straight to showdown. The new option available to Opal is checking to induce a bluff when she has an A. Of course betting with an A will be profitable, but that's not enough to determine that it's the best course of action. We must consider whether betting is *more profitable* than checking and calling.

This will depend on Ivan's strategy. When Opal bets an A, she gets value when Ivan calls with a K. When she check-calls with an A, she gets value when Ivan bluffs a Q. The question, then, is whether Ivan is more likely to call with a K or to bluff with a Q.

If Opal bets a polarized range of {A, Q} into Ivan's uncapped range of {A, K, Q}, we know that Ivan's equilibrium calling frequency with a K will be 1/3; this is what we solved in the previous scenario, where we also determined that the equilibrium EV of betting an A in such a scenario is $2.17. What we must investigate now is whether checking an A would yield a higher EV.

An equilibrium strategy will not presume any kind of exploitable play on the opponent's part, so we will consider the value of checking and calling with an A against Ivan's equilibrium betting strategy. We already know that Ivan will check all his Ks and bets 1/3 of his Qs. Opal's As win $2 5/6 of the time, when Ivan checks, and $3 1/6 of the time, when Ivan bluffs, for an overall EV of $2.17, which is exactly what betting an A was worth. It turns out that at equilibrium, Opal is indifferent between betting and checking with an A.

Not coincidentally, the frequency with which Ivan calls with a K is exactly the same as the frequency with which he bluffs a Q: 1/3. If it weren't, then Opal would be incentivized always to play her As in whatever way would benefit from Ivan's imbalance. In other words, if Ivan were more likely to bet a Q than call with a K, then Opal would always check her As to induce those bluffs. If Ivan were more likely to call a with a K than bluff with a Q, then Opal would prefer to bet her As to induce those calls.

Not only is Opal indifferent between betting and checking an A at equilibrium, but in fact, any mix can be part of an equilibrium strategy. Whatever Opal does with her As has no bearing on Ivan's strategy, and whether Opal always checks them, always bets them, or does anything in between, there's nothing Ivan can do to exploit that if she plays the rest of her range correctly. The more Opal bets with her As, the more she must also bet her Qs, to make Ivan indifferent between calling or folding a K, and check-call with her Ks, to make Ivan indifferent between betting or checking a Q.

It may help to think of it this way: no matter how often Opal checks an A, Ivan is still going to bet a Q at the same frequency. And no matter how often Opal bets her As, Ivan still calls with just enough Ks to make her indifferent to bluffing. Essentially, if Opal bets an A more frequently, she gets more value from Ivan's Ks but loses more value with her own Ks, as she no longer has as many As to take up the bluff-catching slack once she checks.

This is why the answer to the question is "maybe." A betting strategy identical to Ivan's, where Opal bets all her As and 1/3 of her Qs, is one of many viable equilibrium strategies.

Real World Applications

Putting Opponents To The Test

We have said that your profit in poker comes from your opponents' mistakes. We have also said that game theory assumes opponents will not make mistakes unless they are forced into situations where there is no correct choice.

When you bet a polarized range, you create such a situation for your opponent's medium-strength hands. They can either call and risk paying off your value bets or fold and risk losing the full pot to your bluffs. If you get the ratio of value bets to bluffs just right, then there is no correct choice for your opponent, and any calling strategy will result in some losses. This accords with the commonly received poker wisdom that it is desirable to put an opponent "to the test" by betting.

Yet we also found that in the Reciprocal Ranges Game, Opal does just as well checking her strongest hands as she does betting them. How does checking present Ivan with a difficult decision? Is there a sense in which she puts her opponent to the test by checking?

Yes! Checking and being prepared to bluff-catch at the appropriate frequency presents your opponent with a difficult decision about whether to bluff when he has a weak hand. That's not enough to make the Reciprocal Ranges Game profitable for the out-of-position player, but it significantly reduces her losses. As we saw in the half-street variant of the game, when a player is at a structural disadvantage, either because of her range or position, she may not be able to show a profit. Difficult spots are inevitable in poker. Your goal should not be to avoid them but rather to make the best of them by keeping your potential mistakes to a minimum.

We found that at equilibrium, it makes no difference what Opal does when she has an A (as long as she doesn't fold!). Against human opponents, though, you would probably want to think about which "test" your opponent is more likely to fail. That is, is he more likely to call with medium-strength hands or to bluff with weak ones? Armed with the answer to that question, you could then either strictly bet or strictly check your strongest hands accordingly.

The Value of Position

The full version of the Reciprocal Ranges Game helps us to isolate the strategic value of position. After all, everything else about the game is reciprocal: the players have the same ranges and the same betting options available to them. Ivan's advantage in this game must therefore come from his position. So, what exactly is it that position enables him to do?

The in-position player essentially gets to use his strong hands twice, once as bluff-catchers and once as value bets, while the out-of-position player must choose between these functions. As we saw, she can choose purely value bets or purely bluff-catchers or some of both, but she can't do both 100% of the time the way that her opponent can. When Ivan faces a bet, he has all his strongest hands in his range and so doesn't have to call with too many medium-strength hands to make bluffing unprofitable for his opponent. Yet when he faces a check, he still has all those strong hands available for value betting.

The usual explanation for the value of position is that it provides an informational advantage, which in some situations it surely does. Yet this scenario demonstrates that there is more to it. In this case, Ivan doesn’t get any information from Opal's check. Even at equilibrium, she can check as many or as few As as she pleases.

Without making any assumptions about his opponent's strategy or adjusting his own strategy in any way, the in-position player profits simply by acting last. This calls to mind an old lesson from poker guru Tommy Angelo: “Acting last is like taking a drink of water. We don’t have to understand why it’s good for us to know that it is."

Test Yourself

Just like before, I'll give you some real poker situations so that you can practice applying concepts from this chapter. The examples below assume that you are playing no-limit hold 'em at a nine-handed table, with $2 and $5 blinds and effective stacks of $500.

1. The action folds to you on the button, and you raise to $15 with J♥ J♦. The big blind calls, and the flop comes K♠ Q♠ 2♣. He checks. What's your play?

Check. Although you are the player with the more polarized range and the one who should drive the action on this board, that does not mean that betting your entire range is the optimal strategy. Jacks is a decidedly medium-strength hand here, and it has virtually nothing to gain from betting. Your betting range should be polarized, consisting of strong hands like AA, AK, and 22, and weak hands such as flush draws, straight draws, and backdoor draws. Other medium-strength hands that benefit from checking include most Queens and even some of your weaker Kings.

2. You check behind the flop with your Jacks. The turn is the 7♦, and your opponent checks again. What's your play?

Check. Very little has changed from the flop. You still have a medium-strength, and it still does best by checking. Your betting range should still be polarized. The strong hands you bet should be anything you may have slowplayed on the flop plus hands like 77 or Q7 that were not terribly strong on the flop but improved on the turn. Weak hands that didn't bluff the flop are all candidates for bluffing the turn, but JJ doesn't fit that bill. Sure, it would be nice to take the pot down now, but you have a good chance of winning even if you check, and there is still a substantial risk that your opponent holds a hand better than yours that he will not fold.

3. You check behind the turn. The river is the 7♣, and your opponent makes a pot-sized bet. What's your play?

Don't raise. Calling and folding are both reasonable options. You have a medium-strength hand facing a large bet, which means that at equilibrium, you're probably indifferent between calling and folding. It's not a sexy answer, but it's the truth.

Just because you have a difficult decision here doesn't mean that you made a mistake. Difficult decisions are part of poker; you played your hand as well as you could have up to this point. It's possible that, facing this bet, you'll make a mistake. But betting on an earlier street would have been a guaranteed mistake. When you put it that way, pre-emptively making a mistake on the turn just to avoid a risk of making a mistake on the river doesn't sound so appealing.

It may feel like the relative strength of your hand is obvious after you checked the flop and turn, but is it really? How does your opponent know that you don't have K6s or QTo or even a 7 for that matter? Or, how does he know that you don't have 65s and have simply declined to bluff with it? The point of balance is that it shouldn't be obvious to your opponent what you have or, more importantly, what you're going to do. Even if you never have a full house here, you'll have some hands that are trivial calls (K6s), some that are tough calls but more appealing than JJ (QTo), and some that are trivial folds (65s). Your opponent has no way of knowing which you hold, because your play up to this point is perfectly consistent with all of them. In some sense, he just got lucky that you happened to hold one of a few hands that will have a truly tough decision about whether to call.

When a decision is close, it's tempting to obsess about it, even long after the hand is over. These are often the spots players want to discuss with their friends (or anyone who will listen). Because there are strong arguments in favor of both calling and folding, it's hard to be sure you made the right choice.

Precisely because the decision is close, however, it's also not terribly important. Indeed, at equilibrium it may literally not matter what you do. Admittedly, your actual opponent is not a calculating poker robot but a real, live human who probably is weighted either a bit too much towards bluffs or a bit too much towards value bets, so go ahead and take a guess. If you think your opponent seems like the sort to bluff more than is good for him, then call. If he seems the opposite, then fold. Even if you guess wrong, you're not going to make a big mistake.

Some decisions are just tough, and you have to accept that. There isn't always a best play, at least not with the information you have available to you at the time. Focus on getting the important decisions right, the ones where the difference in EV between the right decision and the wrong one is large. Don't beat yourself up over small stuff like this river decision.

4. The action folds to you on the button, and you raise to $15 with A♥ Q♦ The big blind calls, and the flop comes K♠ Q♠ 2♣. He checks. What's your play?

Check. Although this hand is considerably stronger than JJ, it's still not something you want to play a large pot with.

5. You check behind the flop with your pair of Queens. The turn is the 7♦ and your opponent checks again. What's your play?

This is a closer decision than the flop. Against more straight-forward opponents who will reliably bet the turn with any King, your hand is strong enough to start betting for value as part of a polarized range. Against trickier or more passive opponents, checking again is best.

6. You check behind the turn. The river is the 7♣, and your opponent checks. What's your play?

Now your hand is definitely strong enough to bet, but the bet should not be particularly large, probably in the neighborhood of 1/3 to 2/3 of the pot. With so much checking up until the river, you and your opponent should have similar ranges. Just as in the Reciprocal Ranges Game, you can't be sure that your opponent isn't checking hands stronger than yours a third time, which means he may not have incentive to call a large bet with anything other than strong hands. Even if you were value betting a stronger hand like a pair of Kings or trip 7s, you should keep your bet on the small side.

Conclusion

In the Reciprocal Ranges Game, the out-of-position player must make a choice. She can either check her entire range, including her strongest hands, or she can bet a polarized range and check a more condensed range that may still include some strong hands. Whatever her checking strategy, she must call at a frequency that makes her opponent indifferent to bluffing.

Neither of these options is inherently better than the other. At equilibrium, she is indifferent between them, though if she believes her opponent prone to a particular mistake, she does best with the strategy that gives him the most opportunities to make that mistake.

Being in position, Ivan does not have to make this tradeoff. Checking guarantees him a free showdown, so for him there is no cost to checking a condensed range. His optimal strategy is exactly what it was in the Clairvoyance game: he bets all his As and just enough Qs to make his opponent indifferent to calling with a K, and he always checks his Ks. This is his optimal strategy regardless of whether Opal checks her As sometimes, always, or never.

Key Lessons

- ♠ **"Splitting your range" can be profitable even though it reveals information.** In the Reciprocal Ranges Game, Opal is indifferent between checking her entire range or betting a polarized range and checking a more condensed range, but in real poker situations this kind of splitting will often be downright profitable. That it creates opportunities for opponents to pressure the more condensed portion of your range is simply a cost that must be weighed against the benefits of betting a more polarized range.
- ♠ **The reason to bluff is to present medium-strength hands with difficult decisions.** The risk of running into a very strong hand, even the nuts, makes bluffing *less* profitable, but it does not necessarily make it *un*profitable.
- ♠ **Small bet sizes compel calling with wider ranges.** Because wider ranges are weaker ranges, smaller bets enable you to bet weaker hands for value.
- ♠ **Slowplaying is more important with deep stacks**. A condensed range is a greater liability when the effective stacks are large relative to the pot. The larger the bet your opponent can make, the more pressure he can apply to your medium-strength hands. The upside, however, is that if he

perceives your range as condensed and makes a large bet after you check a very strong hand, then your reward for slowplaying is greater.

- ♠ **"Pot control" is only an option for the in-position player.** Only the player who is last to act can ensure that no bet goes into the pot on the current street. When the out-of-position player checks, her opponent still has the option of betting a polarized range to pressure her medium-strength hands. This is one of the main reasons why being in position is valuable.

Caveats

- ♠ **Board textures and hand values change from street to street.** In real poker situations, balancing a checking range requires not just checking hands that are currently strong but anticipating what the board may look like on future streets. For example, if you never check a flush draw on the flop, then your checking range may be vulnerable to big bets from a polarized range should the turn or river bring a third card of a suit.
- ♠ **Checking is an imperfect means of pot control before the river.** The player who is last to act can guarantee a look at the next card by checking, which is itself valuable, but he still may face pressure from polarized betting ranges on future streets.

Chapter 4: Get Real!

Overview & Objectives

In this chapter, we're going to take a break from toy games and zoom way out in order to apply what we've learned so far to a real hold ‘em situation. We aren't going to solve this scenario with anywhere near the precision that we used for the toy games. Instead, we're going to talk about how to use the theory we've learned to work through real poker decisions on the fly.

The goal is just to get a rough sense of what optimal strategy looks like for both players and to practice applying theory in a way that helps you take advantage of the highest-value opportunities. It's not a big deal if you bet a hand that would be slightly more profitable as a check, but you want to avoid betting when checking would be much more profitable and vice versa.

By the end of this chapter, you should be able to:

- ♠ Appreciate how the polarized versus condensed range dynamic plays out in real hold ‘em situations.
- ♠ Identify which player in a heads up hold ‘em situation is better equipped to bet a polarized range.
- ♠ Split a real hold ‘em range into polarized and condensed regions by recognizing which hands benefit most and least from betting.
- ♠ Recognize when a previously passive player should change course and take an aggressive action.

Scenario: UTG vs BB

The first player to act before the flop (Under-the-Gun, or UTG) at a nine-handed table raises, and only the big blind (BB) calls. The flop comes A♠ 9♥ 6♠. Effective stacks are roughly 100 big blinds, and players start with the ranges shown below:

UTG's Opening Range

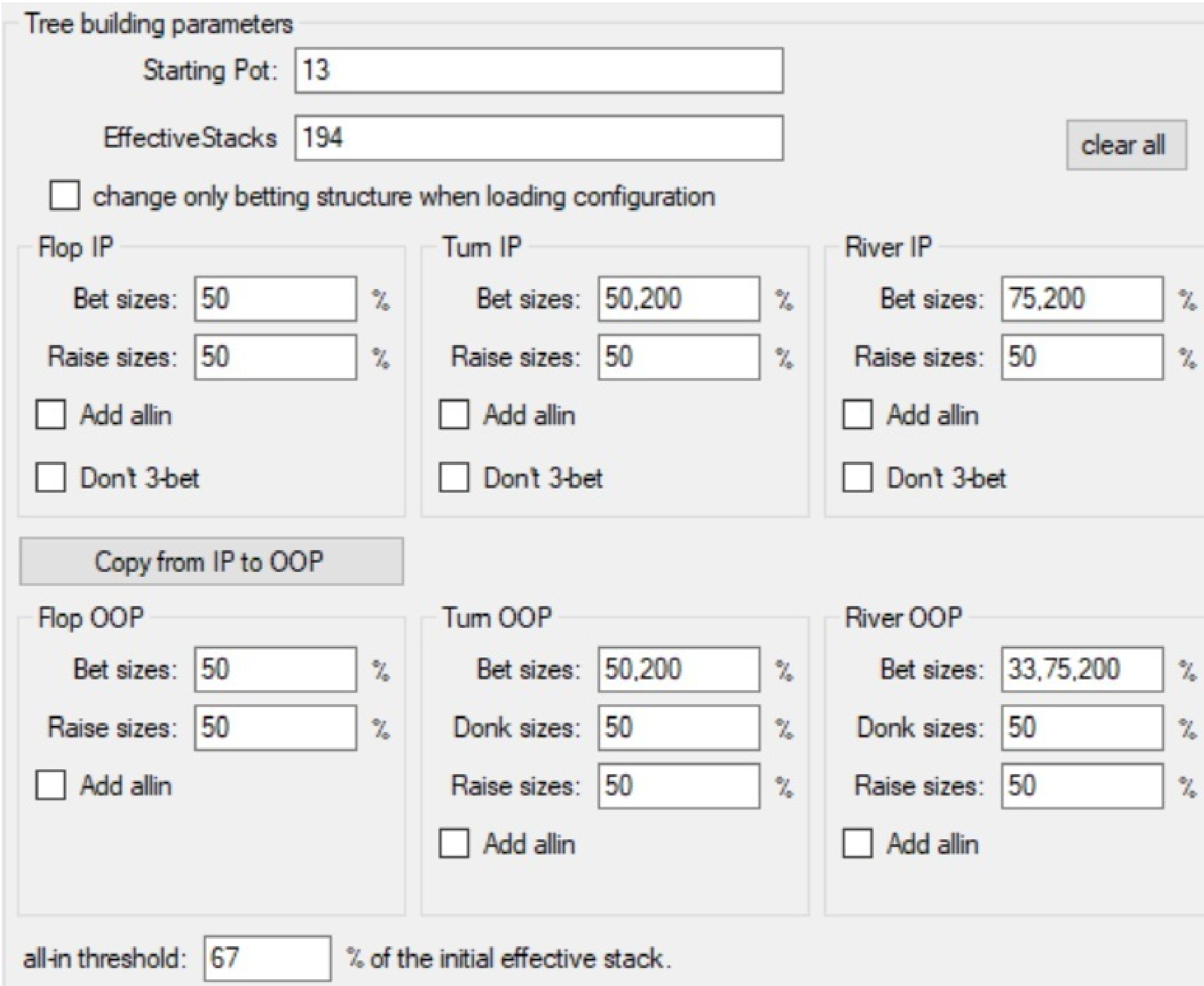
AA 1	AKs 1	AQs 1	AJs 1	ATs 1	A9s 0	A8s 0	A7s 0	A6s 0	A5s 1	A4s 1	A3s 1	A2s 1
AKo 1	KK 1	KQs 1	KJs 1	KTs 1	K9s 0	K8s 0	K7s 0	K6s 0	K5s 0	K4s 0	K3s 0	K2s 0
AQo 1	KQo 1	QQ 1	QJs 1	QTs 1	Q9s 0	Q8s 0	Q7s 0	Q6s 0	Q5s 0	Q4s 0	Q3s 0	Q2s 0
AJo 1	KJo 0	QJo 0	JJ 1	JTs 1	J9s 0	J8s 0	J7s 0	J6s 0	J5s 0	J4s 0	J3s 0	J2s 0
ATo 0	KTo 0	QTo 0	JTo 0	TT 1	T9s 1	T8s 0	T7s 0	T6s 0	T5s 0	T4s 0	T3s 0	T2s 0
A9o 0	K9o 0	Q9o 0	J9o 0	T9o 0	99 1	98s 1	97s 0	96s 0	95s 0	94s 0	93s 0	92s 0
A8o 0	K8o 0	Q8o 0	J8o 0	T8o 0	98o 0	88 1	87s 0	86s 0	85s 0	84s 0	83s 0	82s 0
A7o 0	K7o 0	Q7o 0	J7o 0	T7o 0	97o 0	87o 0	77 1	76s 0	75s 0	74s 0	73s 0	72s 0
A6o 0	K6o 0	Q6o 0	J6o 0	T6o 0	96o 0	86o 0	76o 0	66 0	65s 0	64s 0	63s 0	62s 0
A5o 0	K5o 0	Q5o 0	J5o 0	T5o 0	95o 0	85o 0	75o 0	65o 0	55 0	54s 0	53s 0	52s 0
A4o 0	K4o 0	Q4o 0	J4o 0	T4o 0	94o 0	84o 0	74o 0	64o 0	54o 0	44 0	43s 0	42s 0
A3o 0	K3o 0	Q3o 0	J3o 0	T3o 0	93o 0	83o 0	73o 0	63o 0	53o 0	43o 0	33 0	32s 0
A2o 0	K2o 0	Q2o 0	J2o 0	T2o 0	92o 0	82o 0	72o 0	62o 0	52o 0	42o 0	32o 0	22 0

BB's Calling Range

AA 0	AKs 0	AQs 1	AJs 1	ATs 1	A9s 1	A8s 0	A7s 0	A6s 0	A5s 0	A4s 0	A3s 0	A2s 0
AKo 1	KK 0	KQs 1	KJs 1	KTs 1	K9s 1	K8s 0	K7s 0	K6s 0	K5s 0	K4s 0	K3s 0	K2s 0
AQo 1	KQo 1	QQ 1	QJs 1	QTs 1	Q9s 1	Q8s 0	Q7s 0	Q6s 0	Q5s 0	Q4s 0	Q3s 0	Q2s 0
AJo 1	KJo 1	QJo 0	JJ 1	JTs 1	J9s 1	J8s 0	J7s 0	J6s 0	J5s 0	J4s 0	J3s 0	J2s 0
ATo 1	KTo 0	QTo 0	JTo 0	TT 1	T9s 1	T8s 1	T7s 0	T6s 0	T5s 0	T4s 0	T3s 0	T2s 0
A9o 0	K9o 0	Q9o 0	J9o 0	T9o 0	99 1	98s 1	97s 1	96s 0	95s 0	94s 0	93s 0	92s 0
A8o 0	K8o 0	Q8o 0	J8o 0	T8o 0	98o 0	88 1	87s 1	86s 0	85s 0	84s 0	83s 0	82s 0
A7o 0	K7o 0	Q7o 0	J7o 0	T7o 0	97o 0	87o 0	77 1	76s 1	75s 0	74s 0	73s 0	72s 0
A6o 0	K6o 0	Q6o 0	J6o 0	T6o 0	96o 0	86o 0	76o 0	66 1	65s 1	64s 0	63s 0	62s 0
A5o 0	K5o 0	Q5o 0	J5o 0	T5o 0	95o 0	85o 0	75o 0	65o 0	55 1	54s 0	53s 0	52s 0
A4o 0	K4o 0	Q4o 0	J4o 0	T4o 0	94o 0	84o 0	74o 0	64o 0	54o 0	44 1	43s 0	42s 0
A3o 0	K3o 0	Q3o 0	J3o 0	T3o 0	93o 0	83o 0	73o 0	63o 0	53o 0	43o 0	33 1	32s 0
A2o 0	K2o 0	Q2o 0	J2o 0	T2o 0	92o 0	82o 0	72o 0	62o 0	52o 0	42o 0	32o 0	22 1

When interpreting solver outputs, it's important to pay close attention to the starting range and betting option inputs. In this case, I allowed only checking or betting 50% of the pot on the flop, betting of 50% or 200% of the pot on the turn (or checking), and betting 75% or 200% of the pot on the river.

For the out-of-position player, I added an additional option of betting 33% of the pot on the river. A small "blocking bet" tends to play a more important role in the out-of-position player's strategy than in the in-position player's strategy. That's not something we're going to discuss in this scenario, but the options available to players later in the hand do have some effect on how earlier streets play out, so I wanted to note it. Those interested can see the exact parameters that I used in the below screenshot from PioSolver:



There's plenty of room to quibble with these ranges and bet size options. I tried to keep things simple so it would be easier to wrap your head around the differences between the player's ranges. I'm not necessarily saying that this is what an UTG raising range or a BB calling range *should* look like, and our primary objective here isn't to figure out how to play this exact situation. Rather, we want to understand what factors cause a shift in the polarized versus condensed ranges dynamic that we've been studying. So, I'm deliberately putting my thumb on the scale to make certain points more dramatically.

Questions

With that in mind, do your best to answer the following questions, in light of what we've learned so far.

1. Which player’s range has more equity on this flop?

2. Which player is better equipped to bet a polarized range on this flop? Why?

3. If BB checks, what should UTG's strategy should look like? Remember, in order to keep this scenario simple, his only options are to bet half the pot or check. Is there one action, either bet or check, that he should take with his entire range? If not, should his betting range be polarized or condensed?

4. If BB checks and calls a flop bet, should she plan to mostly check or mostly bet after the turn is revealed? Why?

5. On which turn cards should BB be most likely to deviate from the strategy described in Question 4? Why?

6. If BB checks and calls the flop and then checks a *blank* turn card, one that does not pair the board or complete any obvious draw, what kind of range should UTG bet? If he had to pick just one bet size to use for his entire betting range, roughly how much should he bet, relative to the pot? Why?

Answers & Explanation

The first player to act before the flop (UTG) at a nine-handed table raises, and the big blind (BB) calls. The flop comes A♠ 9♥ 6♠.

1. Which player's range has more equity on this flop?

UTG has a little over 57% equity. There's no reason why you would have come up with that exact number, but you should recognize both that UTG's starting range is much stronger than BB's and that this is a particularly favorable flop for him. In fact, UTG's pre-flop range is so much stronger than BB's that there is probably no flop on which BB would be a favorite, though there are many on which the equity distribution would be closer than on this one.

Why is UTG's range so much stronger than the BB's? Although our analysis begins once the players are heads up on the flop, UTG raised into eight players before the flop. When he raised, he didn't know that seven people were going to fold. To raise in first position, he must be prepared for the possibility of one or more players who have position on him calling or even re-raising him. Consequently, he can only justify entering the pot with very strong hands, mostly pairs and Aces with big kickers.

When only the big blind calls, the raiser has already gotten an above-average outcome. He is guaranteed to be in position, with just a single opponent to beat. If he knew that this would be the result of his raise, then he could include weaker hands in his range, but he had no way of knowing that everyone else would fold when he raised.

The big blind, however, did know that if she called, she would be guaranteed to see the flop against a single opponent. Because she was last to act, she did not have to worry about a re-raise. Plus, she already had a substantial fraction of the raise in the pot, so she could afford to call with some hands that it would not make sense for her opponent to raise. Although BB has strong hands in her calling range, they are diluted by the many weaker hands that she has incentive to play.

It is also significant that BB declined to re-raise. This weakens her range, because she has a lot of incentive to raise with her very strongest hands. In this scenario, I removed only AA, KK, and AKs from BB's range for this reason, but often just calling before the flop will remove even more strong hands from a player's range. Thus, the BB caller ends up with a range containing slightly fewer strong hands and many more weak hands than the UTG raiser.

UTG's advantage is not specific to this flop texture. It may be less dramatic on other flops, but his pre-flop range is so much stronger than BB’s that he will retain an equity advantage on any flop.

A pre-flop raiser from any position will typically have a stronger range than a big blind caller, though the disparity gets less dramatic as the raiser's position improves.

2. Which player is better equipped to bet a polarized range on this flop? Why?

We've seen that polarized ranges benefit from betting, while condensed ranges prefer checking. Thus, your first thought on seeing a flop - or any new card, really - should be an assessment of which player is better equipped to bet a polarized range on this board.

In this instance, it's the pre-flop raiser, as will typically be the case. If you ever have any doubt about this, assume that the big blind should check. As discussed above, an early position raiser starts with such a strong pre-flop range that it's difficult for any flop to shift the advantage to a big blind caller.

The A♠ 9♥ 6♠ flop is a particularly favorable one for the early position raiser. The big blind had a lot of incentive to re-raise before the flop with pocket Aces and perhaps even Ace-King, so several of the strongest possible hands are more likely to be in UTG's range than in BB's range.

It's also difficult for UTG to have very weak hands on this flop, of the sort that would be comparable to a Q in our toy games. Most of his range either started as a pair before the flop or just flopped top pair. Even his unpaired hands are among the best possible high cards and have substantial equity against BB's wider calling range.

All of this is to say that BB is very much the player with the weaker range in this situation, as she will be on virtually any flop against an early position raiser. So, she should check. Depending on your inputs, a solver might suggest a very small betting range for the BB, something like 5-10% of his hands. The EV gained by betting this narrow range as opposed to strictly checking is quite small, however, and in practice it's likely that you will make mistakes identifying which hands to bet, balancing both your betting and checking ranges, etc. The safe play is just to check to the pre-flop raiser 100% of the time.

3. If BB checks, what should UTG's strategy should look like? Remember that, in order to keep this scenario simple, his only options are to bet half the pot or check. Is there one action, either bet or check, that he should take with his entire range? If not, should his betting range be polarized or condensed?

After BB checks, UTG should bet a polarized range. This should include strong hands like AQ, AK, 99, and even AA. Any of his unpaired hands can work as a bluff, though spade draws and backdoor heart draws are especially appealing.

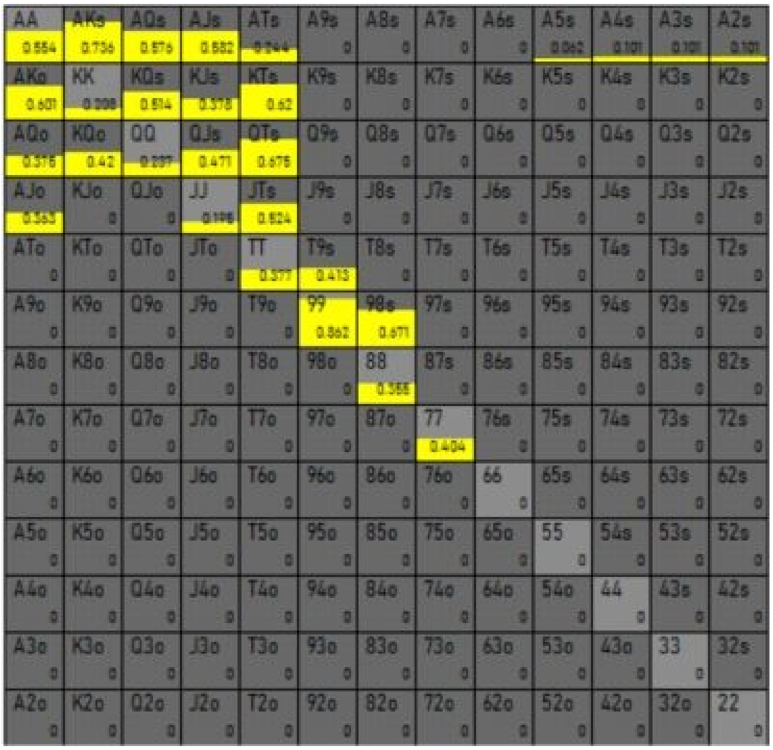
UTG should not necessarily bet his entire range, though, especially if the bet is going to be large. With the ranges that I assigned to BB and UTG, the solver advises UTG to bet half the pot with a bit less than 50% of his range. If a smaller bet of 1/4 or 1/3 of the pot were permitted, we'd see a higher continuation betting frequency with a less polarized range.

For a half-pot bet on this board, UTG's range should be mostly polarized, and the range he checks mostly condensed. The best checking hands are medium-strength holdings that don't much benefit from bluffing but that also don't want to play large pots. The most obvious candidates are big pocket pairs, especially KK, and top pair with a bad kicker. The weakest Aces in UTG's range check nearly 100% of the time.

A few strong hands go into this checking range, but it's not the norm. Slowplaying AA is defensible, because it's hard for the BB to give action when UTG holds so many of the Aces. Just the opposite is true of 99, though: when UTG flops middle set, it's very easy for BB to hold top pair. As a result, a set of 9s is a great betting hand.

Here’s the betting strategy suggested by the solver. The lighter portion of each box indicates the frequency with which UTG bets that hand at equilibrium. The exact frequencies aren't the important part, though. Rather, pay attention to which hands do the most betting and which do the most checking.

UTG’s Flop Betting Strategy



4. If BB checks and calls a flop bet, should she plan to mostly check or mostly bet after the turn is revealed? Why?

If BB checks and calls a bet on the flop, she should check again on almost any turn card. We've already determined that BB started with the more condensed range, and this dynamic becomes more pronounced after the flop action. UTG bet, which should further polarize his range, and BB just called, which should further condense her range. That is, BB should fold many of her weakest hands and perhaps raise some of her strongest (and weakest), so what she's left with when she calls is a range consisting mostly of medium-strength hands that will prefer to play small pots when possible.

5. On which turn cards should BB be most likely to deviate from the strategy described in Question 4? Why?

The cards on which BB does the most betting are any T or non-spade 9. Checking really predominates in BB’s turn strategy, though. The best turns for her are the T♣ and T♦, and even on those she bets only about 35% of her range. On a 9, she bets about 16%. On most turns, she does not bet at all.

We've already discussed that the middle card pairing tends to be better for the BB than for the pre-flop raiser. In this case, I have hands like J9s and 97s in BB's range but not in UTG's, which is part of why this is a more favorable card for her. The other reason is that even if UTG did have some of these hands, he might choose not to bet them on the flop.

I also have 87s in BB's range but not UTG's, which is what makes the T such a favorable turn card. If 87s were in UTG's range, then there wouldn't be such a dramatic shift in terms of which player could more easily hold the nuts.

When BB has a betting range at all, it should be mostly polarized.

6. If BB checks and calls the flop and then checks a *blank* turn card, one that does not pair the board or complete any obvious draw, what kind of range should UTG bet? If he had to pick just one bet size to use for his entire betting range, roughly how much should he bet, relative to the pot? Why?

On most turns, if UTG bets at all, it should be an overbet! There are very few turns where his betting frequency exceeds 50%, but if the turn is not a spade, then his strategy is basically to overbet a very polarized range and check a condensed range.

Overbetting in this situation is not primarily about protecting against draws, it's about winning bigger pots with your strong hands and/or winning more pots with bluffs. In short, it's about pressuring the mostly medium-strength hands in BB's range. We may not know how she'll respond to an overbet, but we don't need to know. Putting her to a difficult decision is a strategy that's guaranteed to benefit some portion of UTG's betting range, even if we don't know which hands exactly will benefit.

Spades have come up as an exception a few times now, so let's take a moment to talk about them. Spades don't give a nuts advantage to either player; both will make plenty of flushes on these turn cards. What spades really do is *neutralize* the nuts advantage. Either player could easily have a flush, so neither has an advantage that enables him to exert extreme pressure with a polarized range.

There are also strong blocker effects with three spades on the board. If you have two spades in your hand, then it's hard for your opponent to have a spade, and if he doesn't have at least one spade, then he's unlikely to play a big pot with you. So, it's hard to get big value bets paid off.

Paradoxically, though, it's also hard to make big bluffs profitable. This is because if you don't have any spades, which is when you'd most like to bluff, then your opponent is more likely to have them, which means your bluffs are less likely to succeed. There will be some semi-bluffing when a player has one big spade, but in general equilibrium strategies on three-flush boards tend to involve surprisingly low levels of aggression from both players.

Chapter 5: Crafting Exploitative Strategies

Overview & Objectives

I've been promising for a while now that you can use insights from game theory to craft better exploitative strategies. In this chapter, we're going to talk about how to do that.

Exploitative strategies approach poker in a completely different way than equilibrium ones. Instead of assuming that other players will play optimally, exploitative strategies assume that an opponent will make one or more specific mistakes and then try to increase EV by deviating from the equilibrium in order to take advantage of those mistakes.

Contrary to popular belief, equilibrium strategies can profit from an opponent’s mistakes, provided those mistakes are large enough, but they don’t profit as much as exploitative strategies. The risk of pursuing exploitative strategies is that any deviation from equilibrium is itself exploitable. When you make guesses about how your opponent will play, there’s a chance that you will guess wrong. If you do, then you'll end up playing straight into his hands - bluffing when he plans to call, for instance - and you will lose EV relative to an equilibrium strategy. There are always judgment calls involved in pursuing exploitative strategies, and we’ll talk about how to make those decisions.

Some exploits are obvious. You don't need a book to tell you that if your opponent is folding too much, you should bluff more often. That's just the lowest-hanging fruit, though; there are often other potential exploits as well. Besides, it isn't always obvious which hands, exactly, you should bluff *with*.

A *maximally exploitative strategy* is one that will have the highest possible EV if your assumptions are correct. Maximally exploitative strategies typically involve large deviations from the equilibrium and seek to take advantage of many different opportunities that may arise from a single, broad strategic mistake.

We’ll start by looking at a four-step process that should help you get more precise, rigorous, and creative with your exploitative strategies. After looking at this process in-depth, we'll practice applying it to the Reciprocal Ranges Game.

By the end of this chapter, you should be able to:

♠ Make good choices about when to employ equilibrium strategies and when to employ exploitative ones.
♠ Develop reads that are specific and actionable.
♠ Distinguish between small and large deviations from equilibrium strategy.
♠ Craft strategies that maximally exploit specific mistakes from your

opponents.

- ♠ Determine the degree of adaptation that is appropriate given the magnitude of your opponent's mistakes and your level of confidence in your read.

The Four-Step Exploitative Process

Making good reads and acting on them is exhilarating. Arguably, it's what poker is all about.

For that reason, though, it's easy to get carried away. Just because your opponent is folding too much doesn't mean you should bet with anything. Start by thinking about your default play, what you would do in the absence of a read (at equilibrium, in other words), then consider whether and how to deviate exploitatively from that default.

Reads should most commonly be used to settle what would otherwise be close decisions. This means you must first identify whether a decision would be close at equilibrium. The bigger your opponent's likely mistake and the more confident you are that he's actually making that mistake, the more you can deviate from your equilibrium strategy.

Too many players try to jump straight to reads. When facing a bet with anything less than the nuts on the river, for instance, the first and perhaps only thing they think about is how aggressive their opponent is. You might be surprised by how rarely that's even a relevant consideration. Plenty of hands just aren't good enough to call even against an aggressive opponent, and plenty of others are too strong to fold even to a passive one.

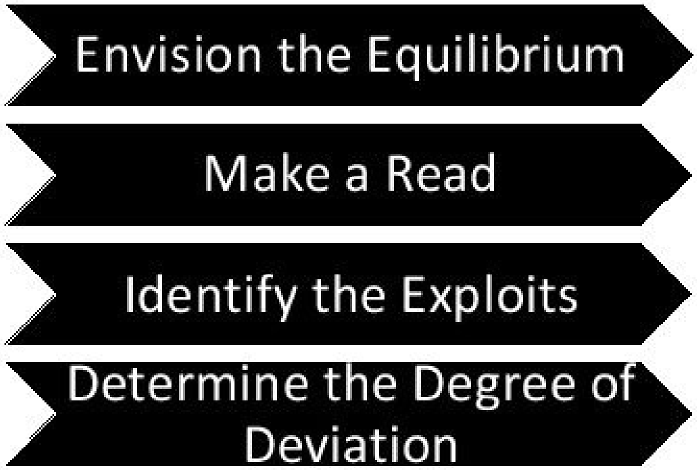
The other danger of this thought process is that it leaves you lost at sea when you don't have a read. One of my pet peeves is when a player asks for advice about a hand on a poker forum, and someone responds that they can't answer his question until they have more information about the opponent.

Can't? That kind of information is nice to have, but you've got to be able to act without it! You'll frequently find yourself in significant pots with players about whom you know nothing, or maybe even players who are so good that you don't want to try to do anything exploitative against them. You need to know how to handle these situations when you don't have reads. Start there, and then make a deliberate choice about whether you have a read that would be relevant given the cards you hold.

I recommend a four-step process for integrating reads into your decision-making process. With experience, you may find that you can take some of these steps without consciously thinking about them. For now, though, I encourage you to think about each stage of the process separately. Even when your decision seems trivial, it's good to practice going through the process.

I'll give you all four steps at once, then we'll discuss each in more detail.

The Four-Step Exploitative Process



Envision the Equilibrium

Ask yourself, “How would this hand play out if my opponent and I were replaced by game theoretically optimal supercomputers?” You aren’t a game theoretically optimal supercomputer, so you won’t be able to answer this precisely, but you should consider factors like:

Which player should be more inclined to bet?

What kind of range should each player bet?

Which are the best hands for each player to value bet?

If I bet, what should my opponent’s raising range look like?

What are the weakest hands with which my opponent ought to commit his entire stack?

Once you have a sense of what your opponent’s equilibrium strategy ought to look like, it will be easier to envision how he is likely to deviate from it. Then, you can think about how to take advantage of those deviations.

I often use the term "mistake" to mean any deviation from equilibrium. In truth, there are many good reasons why a player might deliberately deviate from equilibrium, especially if he underestimates your understanding of poker. So, these may not be “mistakes” in that they might be the best plays given the incomplete information that he has about you. However, if you adapt correctly, you can ensure that they are indeed mistakes that will cost him EV.

If you’re new to game theory, this will probably be the hardest step in the entire process. The good news is that it’s not an all-or-nothing proposition: even if you get only the rough contours correct (“I should mostly check, and my opponent should mostly bet”) that’s still a big step up from not thinking about the equilibrium at all. With practice, experience, and study, you’ll get better and better at envisioning equilibria.

Make a Read

Ask yourself, "How is my opponent deviating from his equilibrium strategy?" Be explicit about this, and as specific as you can. "He's calling too often with pure bluff-catchers on the river," is much more useful than "He's too loose," or "He's a fish."

It's OK if all you have are hunches. It's rare to have a surefire read, just as it's rare to have a surefire winning hand. Poker is fundamentally a game about making decisions under conditions of uncertainty. Later in the process, we'll account for how confident you are in your read.

If you're playing online, you can use the “Notes” feature offered by virtually all sites to record your reads and then change and refine them as you gain more information over time. This also enables you to keep track of reads across sessions, so that if you encounter a player again, you can remind yourself how he played. Just bear in mind that people change over time, both because they make deliberate changes to their play and because of nebulous factors like mood. Old reads are better than nothing, but they aren't as good as fresh reads.

If you're playing live, I still recommend writing down reads. You can use a pen and a paper or an app on one of those tiny computers that you probably carry with you. You won't be able to consult your notes during a hand, but writing them down will encourage you to be explicit and specific, and it will likely aid with your recall as well. I like to review my live notes each time I return from a break, in order to refresh my memory and prompt myself to think actively about how I want to approach my current group of opponents.

Identify the Exploits

Once you identify a likely deviation from equilibrium strategy, ask yourself, "What can I do to exploit this mistake?" Once again, you want to be as precise and specific as possible. You also want to be creative, as there is often more than one potential exploit for a particular mistake.

If an opponent doesn't bluff enough on the river, for instance, the most obvious exploit would be to fold your bluff-catchers when he does bet. However, you should probably also be less inclined to check strong hands to him, and perhaps more inclined to call turn bets with bluff-catchers, knowing that you won't face as much pressure as you could on the river.

Solvers can be quite useful in this regard because of a feature called *node locking*. This enables you to lock in a strategy for a player at a particular decision point, preventing him from check-raising the flop or requiring him to bet only certain hands on the river. The solver can then find new equilibrium strategies for both players, given that constraint. It can provide, for instance, an otherwise unexploitable strategy that assumes your opponent will never check-raise the flop.

This is an advanced use of solvers that this book will not address, but it’s a feature worth learning if you do get familiar with the software, and it’s how I arrived at or verified many of the exploits discussed in this book.

Determine the Degree of Deviation

Ask yourself, "How far should I deviate from the equilibrium?"

Suppose your opponent doesn't seem to bluff enough on the river, and you plan to exploit him by folding more often. The next step is to determine *how much* more often. Does this just mean folding when the decision would otherwise be close? Is it as extreme as folding the second nuts? Or is it somewhere in between?

There's a strong situational component to this - it's harder to find bluffs on some rivers than on others - but two factors that you can consider in advance are the magnitude of your opponent's mistake and the degree of confidence you have in your read.

An opponent who never bluffs with hands that are indifferent at equilibrium is probably only making a small mistake. An opponent who never bluffs with the bare Ace of hearts when a third heart comes on the river is making a much larger mistake, and you should fold stronger hands to him than you would to the first player.

The other factor is your confidence in your read. Even a hunch can justify folding a hand that's mixed between calling and folding at equilibrium. You need a much stronger read to fold the second nuts.

Remember that in order to exploit an opponent who seems to be deviating from his equilibrium strategy, you must deviate from your own equilibrium. This, in turn, raises the possibility that you will be exploited if you are wrong about your read. The larger your deviation from the equilibrium, the greater the penalty for being wrong, which is why larger deviations typically require stronger reads.

You should be particularly careful when playing with highly skilled opponents. You may think, for instance, that you have a read that such an opponent will fold too much on the river because you've seen him tank several times recently before folding. What you must realize, however, is that this player knows that you saw that. While you are thinking about how to exploit him, he is thinking about how you might adapt and how he can get one step ahead of you.

It's dangerous to play these kinds of mind games with tough opponents. The good news is that you don't have to. Armed with your knowledge of game theory, you should be able to craft strategies that are only minimally exploitable when squaring off against such players.

That's an overview of the process. Don't worry if you're not clear on how exactly to make some of these judgment calls right now. The rest of this chapter - indeed, the rest of this book - will help you build the necessary skills.

Scenario: Exploitation in the Reciprocal Ranges Game

So far, we've assumed that both players in our scenarios will play as well as possible. In some cases, that was a reasonable assumption because finding the right play was trivial. Only the most clueless players would fold the nuts to a river bet or call that same bet with the nut low. In other cases, the decision was not trivial, but we did not have any insight into what the other player's strategy might be.

When thinking about whether to bluff with a Q, we used game theory to develop a robust strategy that guaranteed a reasonably good outcome regardless of what our opponent did. There's no obvious right play for him when he's facing a bet holding a K, and we didn't have any insight into what strategy he might choose, so we developed a strategy that presented him with no good options.

Now we'll consider how to exploit an opponent whose play we *can* predict.

Questions

We're playing the full-street version of the Reciprocal Ranges Game: each player antes $1 and is dealt a single card from a three-card deck consisting of one A, one K, and one Q. Either player may bet $1 or check. Raising is not allowed, so once one player bets, his opponent's only options are to call or fold.

The questions below ask you to craft maximally exploitative strategies for mistakes that a player in this game might make. Do your best to apply the Four-Step Exploitative Process from the previous section, and then check yourself against the explanations that follow.

1. Suppose that you are Opal and you have a read that Ivan will call with all his As but only 1/4 of his Ks. In all other ways, he plays an equilibrium strategy. With which hands would you deviate from your equilibrium strategy to exploit this player? How dramatic would the deviations be? Would you adjust your frequencies only slightly, or would you make big changes?

2. What if Ivan fundamentally misunderstood the game and would fold 100% of the time, even if he had an A? How would this change your strategy as Opal?

3. Suppose that you are Opal and you encounter an opponent who bets with all his As and 1/2 of his Qs. In all other ways, he plays an equilibrium strategy. With which hands would you deviate from the equilibrium strategy to exploit this player? How dramatic would the deviations be?

4. Suppose that you are Ivan and you encounter an opponent who always bets when she has an A. In all other ways, she plays an equilibrium strategy. With which hands would you deviate from the equilibrium strategy to exploit this player? How dramatic would the deviations be?

Answers & Explanation

1. Suppose that you are Opal and you have a read that your opponent will call with all his As but only 1/4 of his Ks. In all other ways, he plays an equilibrium strategy. With which hands would you deviate from your equilibrium strategy to exploit this player? How dramatic would the deviations be? Would you adjust your frequencies only slightly, or would you make big changes?

There are two adjustments to make: bluff with 100% of your Qs and check 100% of your As. To see why, let’s walk through the Four-Step Exploitative Process.

Step One: Envision the Equilibrium. When facing a bet, Ivan's equilibrium strategy is to call with all his As and 1/3 of his Ks. This makes Opal indifferent to bluffing with a Q and checking with an A.

Step Two: Make a Read. This player is calling only ¼ of his Ks. That means he is folding slightly too often.

Step Three: Identify the Exploits. If your opponent folds too much, then you should increase your bluffing frequency - that’s fairly obvious. But you should also be less inclined to bet strong hands hoping to get called by a bluff-catcher. So, the adjustments to make here are both to bluff more often and to "trap" more often by checking strong hands.

If you identified the first but not the second of these adjustments, keep practicing Step Three in the exploitative process. You may need to slow down and force yourself to think outside of the box a bit, which can be especially challenging in the heat of the moment. That's all the more reason to practice away from the table.

Step Four: Determine the Degree of Deviation. Your opponent’s mistake is a small one. At equilibrium, he’s indifferent between calling and folding a K, so even if he does one or the other exclusively, it’s unlikely to be a huge mistake. Also, the difference between his equilibrium and actual calling frequencies is not large.

Given that Ivan’s mistake is a small one, it may seem excessive to triple your betting frequency, from 33% to 100%, when you have a Q. When you are indifferent at equilibrium, though, your strategy is extremely sensitive to small changes in your opponent’s play. Remember the metaphor of the precisely balanced scale: once the balance shifts even a bit, with slightly too much or too little calling, then the scale tips and your best response with a hand that was previously indifferent changes dramatically.

As we’ll see, this is not the most aggressive bluffing strategy that you could possibly implement. But for now, let’s look at why it makes sense to bluff so much more often.

When Ivan plays an equilibrium strategy, it makes no difference whether you bet or check with a Q. The EV is the same, $0, either way. The only reason to choose the mixed strategy of 1/3 bets and 2/3 checks is if you don't know what Ivan's strategy will be. In that case, this precise mix guarantees some profit and protects you from exploitation.

If Ivan folds only slightly too much, then bluffing a Q will be only slightly more profitable than checking. But it will *always* be more profitable. You are no longer indifferent, and you have no reason ever to check a Q.

Similar logic applies when you hold an A. We know that the EV of betting and checking are the same at equilibrium, because Ivan calls with a K exactly as often as he bluffs with a Q. But now we are not at equilibrium. Ivan's bluffing frequency with a Q remains the same, but his calling frequency with a K has gone down. That means that you're not indifferent: you'll win slightly more by checking than by betting, so that's what you should do, every single time.

Finally, let's recognize that you should still check 100% of the time that you have a K. Your only read relates to your opponent's strategy when *he* holds a K, which he can't if you hold it. So, you'll start by checking, and then if he bets, you'll have to find a calling frequency that makes his bluffs $0 EV. Because you're checking all your As, you'll only need to call 1/3 of your Ks to make him indifferent to bluffing.

2. What if Ivan fundamentally misunderstood the game and would fold 100% of the time, even if he had an A? How would this change your strategy as Opal?

Just as when Ivan's folding mistake was less extreme, you'd want to bet all your Qs and check all your As. Now, though, you actually do best by also bluffing with 100% of your Ks!

Envision the equilibrium: Ivan calls with all his As and 1/3 of his Ks for a total of 2/3 of his range. This makes Opal indifferent to bluffing with a Q.

Make a read: Ivan is folding too much.

Identify the exploits: As before, we’ll want to bluff more and check fewer strong hands.

Determine the degree of deviation: This time, your opponent’s mistake is a huge one. With an A, he is not indifferent to calling at equilibrium; in fact, he makes a good deal of money by calling. If he is folding As, then he is making a very large mistake, and your maximally exploitative strategy may be dramatically different from the equilibrium.

We already maxed out our bluffing opportunities with Qs to exploit an opponent who folded a K too often, so what new exploit is available to profit from a player who is folding an A? Turning Ks into bluffs!

When you bet a K, you win the $2 pot 100% of the time, so the EV of betting is $2. When you check, you open yourself up to a bet from a polarized range, against which you will have no good options. We’ve seen that such hands underperform their equity, which in this case is only 50% of the pot, or $1, to begin with. In fact, even if your opponent employed the worst possible strategy, betting every time he had a Q and checking every time he had an A, and even if you always called those bets, you’d *still* make less by checking than by betting. You’d win $3 ½ the time and $0 ½ the time, for an EV of $1.50.

Bluffing with a hand that has so much showdown value is an extreme deviation from your equilibrium strategy, but that's to be expected. Ivan's mistake is extreme, so your resulting exploit should be extreme as well.

Ivan's mistake is so extreme that even an equilibrium strategy would profit from it to some degree. That is, if Opal were just to play an equilibrium strategy while Ivan made the huge mistake of folding an A to a bet, then Opal would have a higher EV than she would if Ivan were also playing an equilibrium strategy (unless she were unlucky enough to choose the one equilibrium strategy that didn't involve any betting).

This was not the case with the opponent in Question 1. If Ivan calls too much or too little with his Ks, Opal's EV doesn't change unless she deviates from her equilibrium betting strategy. That's because Ivan is indifferent to calling with a K at equilibrium.

Not all hands are indifferent at equilibrium, though. Calling with an A is +EV at equilibrium. Calling with a Q would be -EV. If Ivan were to do either of these things, Opal would profit from them even if she did not deviate from her own equilibrium strategy. She wouldn't profit as much as if she actively changed her play to exploit these mistakes, but she would profit to some degree.

3. Suppose that you are Opal and you encounter an opponent who bets with all his As and 1/2 of his Qs. In all other ways, he plays an equilibrium strategy. With which hands would you deviate from the equilibrium strategy to exploit this player? How dramatic would the deviations be?

You should call with 100% of your Ks, and you should also check 100% of your As in order to give your opponent maximum opportunity to make this mistake. Your strategy with a Q will remain unchanged.

Envision the equilibrium: When facing a check, Ivan's equilibrium strategy is to bet all his As and 1/3 of his Qs. This makes Opal indifferent to calling with a K.

Make a read: This player is bluffing too much, betting 1/2 of his Qs instead of 1/3 of them.

Identify the exploits: The logic here is basically the same as in Question 1. Where you were previously indifferent, you now have preferences because of the mistake your opponent is going to make. At equilibrium, your As can either check to induce bluffs or bet to induce bluff-catches. Against this opponent, you strictly prefer the former, because he will bluff too much. Similarly, your Ks are indifferent between folding and bluff-catching at equilibrium, but once your opponent starts bluffing too much, even if it's only slightly too much, then you strictly prefer calling to folding.

Determine the degree of deviation: Your opponent's mistake is only a small one, compared to folding the nuts as he did in Question 2, so your exploit is less dramatic. None of your pure strategies change. The only deviations you make from equilibrium are to shift from mixed strategies to pure strategies that exploit the small mistake your opponent is making.

4. Suppose that you are Ivan and you encounter an opponent who always bets when she has an A. In all other ways, she plays an equilibrium strategy. With which hands would you deviate from the equilibrium strategy to exploit this player? How dramatic would the deviations be?

You should not deviate at all!

Envision the equilibrium: Opal is indifferent between betting and checking an A at equilibrium.

Make a read: Remember that this is a game with many equilibria. Any mix of bets and checks is a viable equilibrium strategy for Opal, including a "mix" of 0% checks and 100% bets.

Identify the exploits. As long as Opal bluffs with Qs and bluff-catches with Ks at the appropriate frequencies, which we stipulated that she will, then there's no way for you to exploit her, even though she is predictable. That's the magic of equilibrium!

Real World Applications

Picking Your Spots

Think back to the first two questions from the previous scenario, the ones about adapting to an opponent who folded too much on the river. In the first case, he folded slightly too much with his Ks but always called with his As. In the second case, he folded 100%, even when he had an A.

The maximally exploitative strategy against these two players was different. Against the first player, we adapted by always betting with a Q, but we still checked our Ks. Against the second, even bluffing with a K was more profitable than checking.

What we're seeing here are two different levels of adaptation, depending on just how severe our opponent's likely mistakes are. With some hands, the difference between betting and checking at equilibrium is nonexistent. In those cases, even small changes in our opponent's strategy can shift our strategy from a mix of bets and checks to a strict preference for betting or checking depending on the opponent's presumed mistake, as with a Q in the Reciprocal Ranges Game.

In other cases, the difference between betting and checking at equilibrium is significant, and our equilibrium strategy is not mixed. In the Reciprocal Ranges Game, a K always gets checked at equilibrium. With those hands, we must anticipate much more extreme deviations on the opponent's part before it makes sense to change our own strategy. Only against the extremely unlikely and counter-intuitive strategy of folding an A would it make sense for us to bet a K.

This highlights an important point about making reads that are as specific as possible and recognizing the magnitude of your opponent's mistakes. If you were playing online and found an old note on a player that said, "Folds too much on the river," that wouldn't tell you everything you need to know to craft a maximally exploitative strategy.

Probably the adaptation you should make, with that limited information, would be the more conservative one of always bluffing with a Q but not with a K. If you have the opportunity to make a more specific read, though, then you'd do yourself a disservice by recording so little information. While the strategy of betting Qs and checking Ks will profit to some degree from your opponent's mistake, it leaves a fair bit of money on the table relative to the maximally exploitative strategy.

This is one of the reasons why it's helpful to understand poker at a theoretical level even when you intend to play exploitatively: not all hands are equally good candidates for a particular exploit. Having a sense of how close a decision would be at equilibrium will help you to determine how strong of a read you need to have or how large of a mistake you need to anticipate to warrant an exploitative adjustment.

Suppose that you are considering a re-raise before the flop, and it seems to you that AQ is the sort of hand that might be mixed between raising and calling at equilibrium. Against an opponent whom you expect to fold slightly too often, it would probably be profitable to re-raise 100% of the time with AQ and even with similar, slightly weaker hands such as AJ. However, that does not imply that re-raising with 72 would be profitable. It certainly could be, against the right opponent, but you'd need to anticipate a much larger mistake before you could expect such a play to show a profit.

Bluffing is probably the part of your game that you could most improve by thinking in these terms. Many people bluff with little regard for their cards, choosing instead to focus on factors such as whether their "story makes sense" or what they think their opponent has. Both are important considerations, but as we've seen, there's more to the decision than that. When you're holding a Q in the Reciprocal Ranges Game, betting is perfectly consistent with how you would play the nuts, but even when you're sure that your opponent does not have the nuts, that's still not enough information to determine whether or not you should bet.

More importantly, it matters a good deal which hand you choose to bluff with. Bluffing with a Q is unlikely to be a big mistake even if you get the frequency wrong, but bluffing with a K is typically a disaster. That's an unlikely mistake in the Reciprocal Ranges Game, but in real-world poker situations, it's quite common for people to choose poor bluffing candidates.

Game Flow & Table Dynamics

When I talk about exploitative strategies or make claims about how an opponent is playing, it is always in reference to a specific hand or decision. To say that an opponent is folding 100% is not to say that he will fold every time he finds himself in this situation for the rest of this life. Rather, it is to say that *in this instance* he will fold 100% of the hands that he could hold. Likewise, to say that you should bet all of your bluffs against this player is not to say that you should bluff him at every opportunity for the rest of your life, but rather that your strategy *in this hand* ought to be to bet if you have a bluffing candidate.

People's strategies change over time, for a variety of reasons. Sometimes these changes are relatively permanent - a player realizes that he bluffs too much and resolves to restrain himself. Sometimes they are fleeting - a player who typically bluffs too much gets caught in several large bluffs and does not attempt another one for the rest of that session.

As a result of your opponents' changes, your exploits, and indeed your choice of whether to play exploitatively at all, must change as well. When I say that the best way to exploit an opponent who will fold all of his Ks and all of his As in the Reciprocal Ranges Game is to bluff with all of your Ks and Qs and to check your As, that is assuming that our only goal is to maximize our EV in a single instance of the game, or against an opponent who will never catch on and play differently. It is entirely possible that if you knew you would play this game thousands of times against the same opponent, you might maximize your EV across those games by being less blatant about your exploits. You might deliberately let him win a pot now and again, or bet an A and show it to him when he folds, in order to prevent him from catching on to your exploitative strategy and adapting to it.

In poker jargon, considerations about how strategies change over time are called *game flow* or *table dynamics*. These considerations are important, but we aren't going to spend a lot of time on them because there's no way to solve them mathematically; you just have to make judgment calls.

That said, you absolutely *should* think about these things explicitly, and the tools in this book will help you do so more effectively. Essentially, every time you do something exploitative, there is a risk that your opponent will realize you are exploiting him and change how he plays against you in the future. You have to decide whether the value of exploiting him in this instance is worth that risk. This requires estimating the value of your current exploitative opportunity, how likely your opponent is to change his strategy as a result of you taking that exploit, how much you expect to make from exploiting this mistake in the future if your opponent does not adapt.

In practice, my philosophy is, "A bird in the hand is worth two in the bush." I usually take an exploitative opportunity when I see one and then do my best to anticipate the consequences of that decision, in terms of how my opponent perceives me and how he'll play against me going forwards. There are many reasons why I think this approach is generally best.

For one, opportunities to exploit an opponent in a particular way come around only so often, especially at a nine-handed table. Even if you decide to bluff him relentlessly on the river, that still requires you to find yourself in a pot with him, on the river, holding a hand that would benefit from bluffing. That situation might arise once or twice an hour, at which rate it's going to take him a long time to pick up on a pattern. And by the time he does, the table might break or one of you might quit for the night.

Additionally, people are not good at changing their behavior. In most cases, poker mistakes are rooted in psychology, not misunderstanding of strategy. A player who calls too often, for example, isn't necessarily playing that way because he thinks it's maximally +EV. Some loose players love action, others hate the idea of getting bluffed, and still others are just curious and want to see your cards. Those psychological "mistakes" aren't going to go away forever just because someone realizes he's a calling station. At worst, this player might tighten up a bit after a few expensive mistakes, but he'll probably still be too loose, and he'll probably struggle to maintain that discipline. Most likely, he'll slip back into old habits when the pain of his calling mistakes is no longer fresh in his mind.

In many cases, though, your opponent may not even realize that you've exploited him. If, in the Reciprocal Ranges Game, you check a Q to an overly loose player, and he checks back and wins with a K, what can he conclude from that? Your equilibrium strategy would be to check a Q more often than not, so even if your strategy against him were to never bluff, he would need to see that Q go to showdown several times before he started to get suspicious.

Other exploits are more obvious. A player who caught you bluffing with a K might recognize that such a play is an extreme deviation from equilibrium, and whether or not he realizes that it was something you were doing deliberately to exploit him, he might well be tempted to play differently against you in the future as a result.

Perhaps most importantly, though, it's not necessarily bad for you if your opponent changes his strategy. It's entirely possible that a player who starts out as overly tight might become frustrated with your constant bluffing and eventually start playing overly loose against you. That would be bad if you kept trying to bluff him, but if you're alert to signs of his frustration, then you might be able to anticipate his change in strategy and exploit his new mistake just as profitably as you were exploiting the old one. Amazingly, many players will not be shy about telling you that they are sick of your bluffing, or, conversely, that they are frustrated at themselves after several losing river calls. Pay attention to this information, think about how it will affect game flow, and you should be able to anticipate many profitable exploits against your opponents, even if they aren't the same exploits time and time again.

You must ask yourself, at each decision point, how do I expect my opponent to play *now*, based on all the information I have about him, including his innate tendencies and his current mood? That will tell you how to exploit him this time, even if that's not how you'll exploit him next time.

Test Yourself

In the examples below, assume that you are playing no-limit hold 'em at a nine-handed table, with $2 and $5 blinds and $500 effective stacks.

1. You are first to act pre-flop and raise to $15 with A♦ J♦. The big blind calls. Your read on this player is that he is a typical “splashy” recreational player, which is to say he plays too many hands pre-flop but is both tight and passive when it comes to putting big bets into the pot. The flop comes A♥ 9♠ 6♠, and the big blind checks. Do you bet or check?

Bet. At equilibrium, you would likely be indifferent, but if your read is correct, then he has more draws and dominated Aces in his range than he should, and you want to get value from those. Importantly, it sounds like this player is unlikely to check-raise, which would really put you in a tough spot. AJ is a strong value hand, but it’s not so strong that it can stand up to pressure from a well-balanced, polarized range. The risk of a check-raise from such a range is one of the main reasons why you’d be indifferent at equilibrium. Against a loose and passive opponent, betting is strictly best.

2. You bet $20 with your AJ, and the big blind calls. The turn is the 8♠. The big blind checks. Do you bet or check?

This is a legitimately close decision, even with your reads. You still have a pretty good hand, and there are many hands that you’d like to either charge or fold out. However, there’s also a real risk that you’re already behind and drawing slim or dead. The more confident you are that your opponent will never raise the turn without a hand that’s crushing you, the more inclined you should be to bet and fold if raised. Checking is perfectly fine here, though.

3. You check, and the river is the 3♦, making the final board
A♥ 9♠ 6♠ 8♠ 3♦. The big blind bets $60, which is just a bit less than the size of the pot. What’s your play?

Fold.

The first thing to recognize is that your hand is a bluff-catcher. Your read suggests that your opponent isn’t going to make a big value bet with a worse Ace than yours. That means that either he can beat an Ace or he’s bluffing. He certainly could be bluffing, but that’s not enough to make this a call.

As we know, a bluff-catcher facing a bet from a polarized range is roughly indifferent at equilibrium. In other words, if he bluffs at optimal frequency, then it makes no difference whether you call or fold. The question to ask, then, is not whether this bet could be a bluff but whether this opponent is likely to bluff out of proportion to his value bets. Given the number of potential value hands here, that's a tall order, and your read suggests that he's passive when it comes to big bets. So, while you may be indifferent at equilibrium, you should fold against this opponent.

4. You are first to act pre-flop and raise to $15 with K♠ J♠. The big blind calls. Your read on this player is that he is very loose. The flop comes A♥ 9♠ 6♠. The big blind checks. Do you bet or check?

Bet. Although this is not a great spot to bluff against this opponent, who will often have top pair or some other piece of the board, your hand is very good for bluffing. In fact, this bet is not even really a bluff, as it will be ahead of a decent portion of your opponent's calling range. Besides, a loose player still probably won't call the flop with absolutely nothing, and even though you're a favorite against hands like Q♥ 8♥, you don't mind folding them off of their equity.

5. You are first to act pre-flop and raise to $15 with J♥ T♥. The big blind calls. Your read on this player is that he is very tight and straightforward, both pre- and post-flop. He is afraid to slowplay for fear of getting drawn out on, he never bluffs, and he always assumes that his opponents hit the board. The flop comes A♥ 9♠ 6♠. The big blind checks. Do you bet or check?

Check. Although betting would surely be profitable, checking is probably the most profitable way to exploit this opponent.

Tight opponents have strong pre-flop ranges. They are actually not the best targets for big bluffs, because they fold before the flop with some of the weaker hands that others would call. This means they tend to have overly strong hands when they contest pots. It's easy to steal small pots from them, but they usually "have it" when the pot is large.

In this case, your read is not just that this player is tight but that he is straightforward and afraid to bluff or slowplay. When you bet the flop, you have no information about his hand, as he's probably just checking his entire range to an early position raiser.

By checking, you get to see what he does on the turn. If he bets, you can fold, confident that he had a hand that wouldn't have folded the flop anyway. In that case, you save a bet.

If he checks, then it’s much more likely that he plans to fold, and you can make the same bluff that you would have made on the flop with a higher chance of success.

This is a good example of finding value in less-obvious exploits. While bluffing is the obvious exploit against a player who will fold too much, it behooves you to think about exactly how and when you can bluff him most profitably.

On boards where the turn card could more easily improve him, it might be best to bluff immediately. On this board, however, he either has an Ace or a draw, in which case he’s not folding, or he doesn’t, in which case the turn isn’t likely to change how he feels about his hand.

6. You are first to act pre-flop and raise to $15 with 9♦ 9♥. The big blind calls. Your read on this player is that he is very tight and straightforward, both pre- and post-flop. He is afraid to slowplay for fear of getting drawn out on, he never bluffs, and he always assumes that his opponents hit the board. The flop comes A♥ 9♠ 6♠. The big blind checks. Do you bet or check?

Bet.

Even though your opponent will fold more than he should, checking isn’t likely to change that. If he has an Ace or a draw, he’ll give you action on the flop, and you want to get that action right away, before the board gets so scary that he starts thinking about folding an A.

If he doesn’t have anything, then you’re unlikely to win anything extra by checking. He’s not going to bluff, and even if he turns a pair, he’s going to worry about the A on the flop.

Conclusion

Choosing an exploitative strategy over an equilibrium one is a gamble. That doesn't make it wrong - after all, poker is about gambling with an edge - but it does mean that the choice to play exploitatively should be a deliberate one.

Any time you deviate from equilibrium in an attempt to exploit an opponent's tendencies, you run the risk of being exploited yourself. If you are right about your read, then you'll have an EV higher than what the equilibrium strategy would yield. But if you're wrong in your read - if, for instance, your opponent anticipates your adjustment and adjusts his own play accordingly - then you'll have a lower EV than you would with the equilibrium strategy.

As long as you have a read, or even a hunch, to guide your exploitative adjustments, then they are often worthwhile gambles. The important thing is that you deviate from the equilibrium strategy deliberately and with good reason. The average poker player plays in a highly exploitable way, not as a result of a deliberate choice, but because he doesn't know any better. Often, he doesn't even realize that his strategy is exploitable, because it's simply what he's learned as "standard" or "ABC".

The point of studying game theory is not to approximate equilibrium solutions whenever possible, it is to know roughly what an equilibrium strategy would look like so that you can make wise adjustments when you have the right information to do so and avoid opening yourself up to exploitation when you don't.

Key Lessons

- ♠ **Not all exploits are obvious.** Thinking in terms of equilibrium can increase your profits by helping you find additional opportunities to exploit common errors.
- ♠ **Small deviations from equilibrium can warrant dramatic changes to your strategy.** An opponent who folds just a bit more than he should incentivizes you to adjust your betting and checking frequencies dramatically. The change to your EV may be small, but the changes in your play are not. When a choice is close at equilibrium, the best play is often to guess at which mistake your opponent is more prone to make.
- ♠ **Not all hands are equally good for all exploits.** Some are better suited to bluffing or bluff-catching than others, and your threshold for playing those hands exploitatively should be lower.

Caveats

- ♠ **You must balance immediate profit against the risk that an opponent will catch on to his mistakes**. Whenever you make an exploitative play, there is a chance that your opponent will catch on to what you are doing and play differently in the future. When in doubt, take the immediate profit, but stay alert to changes in your opponents' behavior – and not exclusively with regard to how he plays his cards!
- ♠ **Exploitative play can have unintended side effects**. Changing your play to exploit one mistake may make you vulnerable to other exploits. For example, if you think your opponent will not call often enough with bluff-catchers, you may choose to bluff more often. However, if this opponent also bluff-raises aggressively, your extra bluffs might fail to profit for this reason, even if your initial read is correct.

Chapter 6: Complex Ranges

Overview & Objectives

The Reciprocal Ranges Game is useful as an introduction to toy games, but it's hard to imagine anyone making a meaningful mistake. Sure, it would be easy to bluff a bit too much or too little, but that won't affect your EV much. In fact, if your opponent plays an equilibrium strategy, it won't affect your EV at all. A big mistake would be folding an A or calling with a Q, and that's just not very likely.

It's understandable that players who know only a little bit about game theory might get the idea that equilibrium strategies never do better than break-even and that they can't benefit from opponents' mistakes. We're going to see why this isn't true, with the help of a slightly more complex toy game where significant mistakes are more plausible.

To better approximate real poker, we'll look at a similar game that adds more cards to the deck. Decisions about which hands to value bet, which to bluff, and which to bluff-catch with will be a lot less intuitive, and there will be more opportunities to make non-trivial mistakes. First, we'll find the equilibrium for this game, then we'll talk about how to exploit various deviations from the equilibrium.

By the end of this chapter, you should be able to:

- ♠ Appreciate the strategic role played by various parts of a range.
- ♠ Construct ranges that present your opponents with difficult decisions and opportunities to make mistakes.
- ♠ Integrate information about an opponent's likely strategy into your own decisions about bluffing, bluff-catching, and value betting with less-than-ideal candidates.
- ♠ Identify appropriate targets for your bets, checks, and calls.

Scenario: The Ace-to-Five Game

Two players, Opal and Ivan, are dealt a single card from a ten-card deck consisting of one of each card from 5 to A. Each player antes $1, and the game permits only a single, $1 bet. That is, Opal may either bet $1 or check. Facing a check, Ivan may bet $1 or check. There is no raising, so facing a bet, a player's only options are to call or fold. If the hand goes to showdown, the player with the highest card wins.

Questions

We're going to do things a bit differently this time. I'll show you the equilibrium strategies for various decision points and highlight what I think is most interesting and noteworthy about them. Your job will be to explain why these solutions look the way that they do.

In my experience, the best way to approach questions like this is to ask yourself, "If the player in question changed this strategy in some way, how could his opponent exploit it?" For instance, in the Clairvoyance Game, if Opal were to call less than 2/3 with her K, then Ivan could exploit that by increasing his bluffing frequency. The equilibrium would fall apart, in other words, because Ivan would have incentive to change his strategy.

In answering the questions below, try to envision how a change in one player's strategy would create an incentive for the other player to change *her* strategy. Once you've given it your best, read on to the next section for my explanations.

1. Below is Ivan's calling strategy when facing a bet. Which of these hands are +EV calls, which are -EV (money-losing) calls, and which are $0 EV calls? How can you tell?

Ivan's Strategy Facing a Bet		
Hand	**Call**	**Fold**
A	100%	0%
K	100%	0%
Q	100%	0%
J	100%	0%
T	83%	17%

9	59%	41%
8	42%	58%
7	16%	84%
6	0%	100%
5	0%	100%

2. What is the overall calling frequency that Ivan needs to achieve to make Opal indifferent to bluffing?

3. Consider Opal's equilibrium betting strategy shown below. Notice that her betting range is polarized - it contains {A,K,Q,J} for value and {6,5} for bluffs but nothing in between - and that she is indifferent between betting and checking with all of her value hands and all of her bluffs, just as she was in the Reciprocal Ranges Game.

It's easy to see why A, K, and Q can value bet, because they will be ahead of Ivan's calling range. But why is a J a value bet? It will always lose to A, K, and Q and only sometimes get called by T, 9, 8, and 7. (When Opal bets a J, Ivan can't have one, so we're not considering it in his calling range here.) Overall, a J will lose more often than not if called, so why is Opal including it in the value portion of her polarized range?

Opal's Betting Strategy		
Hand	**Bet**	**Check**
A	81%	19%
K	73%	27%
Q	63%	37%
J	57%	43%
T	0%	100%
9	0%	100%
8	0%	100%
7	0%	100%
6	36%	64%
5	56%	44%

4. Consider Ivan's equilibrium betting strategy below. One striking difference from Opal's strategy is that Ivan is indifferent between betting and checking his Ts, but Opal never bets hers. Why is Ivan able to value bet a T while Opal is not?

Ivan's Betting Strategy		
Hand	**Bet**	**Fold**
A	100%	0%
K	100%	0%
Q	100%	0%
J	100%	0%
T	50%	50%
9	0%	100%
8	0%	100%
7	0%	100%
6	50%	50%
5	100%	0%

5. Ivan sometimes checks a 6, but he always bluffs with a 5. Why is this?

6. Consider Opal's equilibrium check-calling range shown below. Note that because she has a betting range, she does not have 100% of her As, Ks, Qs, and Js after checking. That is why, although she never folds these hands, they are not shown as calling 100%.

Unlike Ivan, Opal always calls with her Ts. Why is this?

Opal's Check-Calling Strategy		
Hand	**Call**	**Fold**
A	19%	0%
K	27%	0%
Q	37%	0%
J	43%	0%
T	100%	0%
9	49%	51%

8	42%	58%
7	40%	60%
6	0%	64%
5	0%	44%

Answers & Explanation

1. Which Ivan's calls are +EV, which are -EV, and which are $0 EV? How can you tell?

The $0 EV calls are T, 9, 8, and 7; these are all equivalent to a K in the AKQ game, which is why they play a mix of calls and folds. Opal's betting range should be polarized, meaning that she will not bet the medium-strength hands between T and 7. This is why Ivan's 8s and 7s can bluff-catch just as well as Ts and 9s. Ivan can reach equilibrium by calling with any mix of 7s through Ts as long as the overall calling frequency makes Opal indifferent to bluffing with a 5 or 6.

The +EV calls are A, K, Q, and J. You can tell these are +EV because Ivan calls with them 100% of the time.

Qs and better are +EV calling hands because Opal can bet a J for value. When Ivan calls with a Q, he beats not only Opal's bluffs but also some portion of her value range. Although a Q could lose to an A or a K, it is not a pure bluff-catcher and should therefore be a +EV call unless she deviates significantly from her equilibrium strategy.

If Opal were to play a severely suboptimal betting strategy, calling with a Q could be -EV for Ivan. However, that loss would be more than offset by benefits to other parts of his range. For instance, if Opal were to never bluff and only bet As and Ks for value, then Qs would lose EV when they call, but there would be many other cases where Ivan could check behind and win pots with hands that would have folded to bluffs or save bets with hands that would have called and lost. Overall, this would be better for him than the equilibrium, even though he'd prefer not to call with the Qs if he knew Opal's betting strategy.

A J is a +EV call for a different reason. Although it doesn't beat any hand in Opal's value range, it does *block* a portion of her value range. Opal's equilibrium strategy is to bluff in proportion to her value bets, assuming that she could be value betting a J, Q, K, or A. When Ivan has a J, however, he knows that Opal cannot have one. And because Opal does not know that Ivan holds a J, she cannot adjust her bluffing range accordingly. That means that when Ivan holds a J (or a Q, K, or A), Opal's range will not actually be balanced; it will be weighted towards bluffs, and Ivan will have a profitable rather than a breakeven call with a J.

For similar reasons, Ivan strictly prefers folding a 6, even though it can beat some of Opal's bluffs. He is not indifferent to calling with a 6, as he is with a 7, because the 6 blocks some of Opal's potential bluffs. When Ivan has a 6, Opal is less likely to be bluffing, and therefore calling with a 6 would be -EV.

2. What is the overall calling frequency that Ivan needs to achieve to make Opal indifferent to bluffing?

It's 2/3, because just as in the other games we've looked at, Opal's bet risks $1 to win $2.

When facing a bet, Ivan has nine hands in his range. He starts with ten, but we must account for the one that Opal holds. Because the objective is to make Opal indifferent to bluffing with a weak hand, we assume that she has a 5 and then figure out how Ivan can call with 2/3 or 6/9 of his possible non-5 holdings.

Ivan constructs his calling range by first adding all the hands that are profitable calls. As we saw above, there are four of them, so that's 4/9 of his range. He needs to call with the equivalent of two more hands to reach the 6/9 frequency.

He can fill out his calling range with any combination of hands from 7 through T. The most intuitive strategy would be to call with Ts and 9s and fold 8s and 7s, but calling 8s and 7s and folding Ts and 9s would work just as well, as would any other mix that brings his overall calling frequency up to 2/3. Note that in the calling strategy shown in the grid above, Ivan's calling frequency with these hands sums to 200% (17% + 41% + 58% +84%), just as it would if he played a pure strategy of always calling two of these hands and always folding two of them.

3. Why is a J a value bet for Opal? It will lose more often than not if called, so why is she including it in the value portion of her polarized range?

When Opal has a J and Ivan has something better, she is destined to lose a bet. A J is too strong of a hand to fold, so either Opal bets and gets called by a better hand, or she checks and calls a bet from the better hand. Either way, she is going to lose that bet.

The thing for Opal to focus on when she has a J, then, is not how to save a bet when behind; that bet was functionally lost as soon as the cards were dealt. Once she accepts that she's going to put a bet into the pot one way or the other, Opal should focus on how to have as much equity as possible when it goes in. In other words, she needs to figure how to get her opponent to match that bet with as many weaker hands as possible.

At equilibrium, Opal is indifferent with a J. Either she bets to squeeze what value he can from the {T,9,8,7} portion of Ivan's range, or she checks and calls to squeeze what value she can from Ts, 6s and 5s.

This is yet another illustration of how “pot control” is not really an option for the out-of-position player. Because Opal cannot simply check her medium-strength hands and guarantee a showdown as her opponent can, betting can sometimes be the “least bad” (or equally bad, in this case) option.

Against a human opponent, you should think about what sort of mistake your opponent is more likely to make. Will he be more inclined to pay off with too many bluff-catchers or to bluff with too many weak hands? Because you are indifferent between betting and checking at equilibrium, this is a safe time to try something exploitative. Even if you guess wrong, you won't end up making much of a mistake.

4. Why is Ivan able to value bet a T while Opal is not?

This is one of the advantages of position. When Opal checks, she reveals some information about her range. It is not perfect information - she could check strong hands - but she has incentive to bet a polarized range. This reduces the likelihood that Ivan will get called by a better hand when he bets a T and so increases the value of betting to the point where it is competitive with checking.

Opal’s strategy involves betting a polarized range that consists of more value hands than bluffs. In this betting structure, the ratio is three strong hands for every one weak hand. She has incentive to put many of her strongest hands into a betting range, so even though she has some “traps”, her checking range is weaker than Ivan's range, which is still his full starting range, including all the strong hands.

Because Opal bets so many strong hands, she is forced to call a bet with more of the {T,9,8,7} portion of her range than Ivan is. Otherwise, bluffing with a 6 would be profitable for Ivan. Because Ivan is less likely to run into a strong hand and more likely to run into a weak hand, he can value bet more widely than Opal.

Opal could counteract this by checking her entire range. If she did so, then Ivan would lose money betting a T. However, Opal would lose all the benefits of betting a polarized range. Overall, she does best by betting a polarized range and accepting that Ivan will get to make some thin value bets. The cost of denying him that opportunity is too high.

Although Ivan is indifferent between betting and checking with a T, this does not mean that betting is $0 EV. Indifference just means that two options have the same EV; one is no better or worse than the other. Checking with a T is +EV, as it will often win at showdown against hands that Opal checked. Betting with a T is also +EV, as it will win more than half the time against Opal's check-calling range.

Unlike in the Reciprocal Ranges Game, the frequency with which Opal checks a strong hand is not arbitrary. It doesn't matter which ones she checks, but her overall checking frequency with strong hands is just enough to make Ivan indifferent between betting and checking a T. If Opal were never to check a J or better, then Ivan would strictly prefer betting with a T (and a 9, for that matter). Opal must "trap" at the right frequency to maintain the equilibrium.

Ivan’s advantage in this game is that he does not have to worry about splitting his strongest hands between his betting and checking ranges as Opal does. Opal acts, so when she bets, Ivan still has all his strongest hands in his range. When Opal checks, Ivan can check behind with his medium-strength hands and go straight to showdown. He doesn’t have to worry about facing a bet from a polarized ranger after checking, which is why he has not incentive to check some strong hands as Opal does.

5. Ivan sometimes checks a 6, but he always bluffs with a 5. Why is this?

Because Ivan has a wider value range than Opal, he also has a wider bluffing range. Faced with a check, Ivan's equilibrium strategy is to bet 100% of the time with a 5 and 50% of the time with a 6. That means that Ivan is not indifferent to bluffing with a 5; it's actually a +EV bluff!

Bluffs that are profitable at equilibrium are not a phenomenon that we encountered in any of our toy games so far, so let's make sure we're clear on what's happening here. Opal needs a calling strategy that makes Ivan indifferent between bluffing and checking. The problem is that Ivan has two bluffing candidates, 5s and 6s, and these two hands do not have the same EV when they check. A 5 will always lose, so checking it is $0 EV, but a 6 has a small chance of beating a 5 at showdown and is therefore slightly +EV to check.

Because Ivan has a different EV when he checks with a 6 than when he checks with a 5, Opal cannot make him indifferent to both. She can either call at a frequency that would make a 6 indifferent to bluffing, which means that a 5 will be a guaranteed profitable bluff, or she can call at a slightly higher frequency that would make a 5 indifferent to bluffing.

The problem with a higher calling frequency is that it would require Opal either to check more strong hands or to call with more weak hands, both of which come with significant tradeoffs. Checking more strong hands means missing out on value bets, while calling with more weak hands would make Ivan's value bets more profitable.

Ultimately, the cost of making Ivan indifferent to bluffing with a 5 is too high. The best that Opal can do is accept that her opponent will have slightly profitable bluffs with his very worst hands and develop a check-calling frequency that makes him indifferent to bluffing with a 6.

This is another advantage of position: Ivan gets more profitable bluffs *and* more profitable value bets than Opal, even though they start the game with the same ranges.

Why is checking a 6 profitable for Ivan but not for Opal? It's because Opal will sometimes check a 5 - she can't value bet widely enough to support bluffing with 100% of them - which enables Ivan to occasionally win when he checks behind with a 6. Ivan, however, will never check a 5, and when Opal does check a 6, she can't justify calling with it. So, even when a 6 is the best hand, Opal never gets to show it down. She can either bluff with a 6 or resign herself to losing the pot one way or the other.

6. We saw that Ivan is indifferent between calling and folding a T, but Opal always calls. Why is this?

Ivan's value range is wider than Opal's and includes some of his Ts, so when Opal has a T, she blocks a portion of her opponent's value range and therefore has a slightly +EV call.

Opal constructs her calling range by first calling whenever it is +EV to do so. That includes all of her traps - the As, Ks, Qs, and Js that she checked - and also all of her Ts.

Calling with only these hands isn't enough, though. In order to make Ivan indifferent to bluffing with a 6, Opal must call with 2/3 of the portion of her checking range *that can beat a 6*. She can do so with any combination of 7s, 8s, and 9s in order to reach this frequency.

Note that Opal is *not* calling with 2/3 of her checking range. Her objective is to make Ivan indifferent to bluffing with a 6, and Ivan gains nothing from betting a 6 when Opal folds a 5. So, Opal's folding frequency is higher than 1/3, but that's OK because all of those "extra" folds are 5s, and Ivan doesn't gain anything from them.

Summary

That was a complicated solution, so let's review it. It's helpful to pay attention to the indifference thresholds, the hands with which each player has a choice, because this is where mistakes and exploitable opportunities are most likely to arise. It's unlikely that anyone would make the mistake of folding an A, for instance, but they might fold a J or a Q.

Opal checks enough strong hands to make Ivan indifferent to value betting a T. She bets a polarized range, with the rest of her Js, Qs, Ks, and As betting for value and some of her 6s and 5s betting as a bluff. Her overall ratio of value bets to bluffs is 3:1, which makes Ivan indifferent to calling with his pure bluff-catchers: 7s, 8s, 9s, and Ts.

Facing a check, Ivan bets a polarized range that is larger and more profitable than Opal's. Ivan always bets with a J or better, and he bets 50% of the time with a T. He always bluffs with a 5 - these are +EV bluffs, the first time we've seen those in an equilibrium strategy - and bets 50% of the time with a 6. His overall ratio of value bets to bluffs is 3:1.

This betting range makes Opal indifferent to calling with 7, 8, or 9. With a T or better, Opal always calls, and she also calls enough of her weaker hands to make Ivan indifferent to bluffing with a 6.

Being in position enables Ivan to both value bet and bluff more hands than his opponent. It also means he doesn't have to worry about revealing weakness by checking, because his check ends the action and takes him straight to showdown.

What exactly is position worth in this game? Opal has an EV of $0.94, and Ivan has an EV of $1.06. That means that no matter how well Opal handles a disadvantageous situation, Ivan should be able to leverage his position to improve his EV by about 6% over what his equity would be in a game with no betting.

Note that Ivan’s EV is the same as in the Reciprocal Ranges Game. We're fundamentally playing the same game here, it's just more complicated to implement the equilibrium strategies correctly, which means that it's more plausible to envision mistakes and opportunities to profit from them.

Scenario: Exploitation in the Ace-to-Five Game

The advantage of the Ace-to-Five Game over the Reciprocal Ranges Game is that because there are more close decisions, it's more realistic to imagine players making exploitable mistakes. There's also more room to make fine-grained adaptations. In the Reciprocal Ranges Game, when we wanted to expand our bluffing range beyond the most obvious candidate, we had to use a hand with a good chance of winning at showdown. In the Ace-to-Five Game, we have more choices, and we'll be able to see more clearly how the magnitude of an opponent's deviation from equilibrium determines the degree to which we should deviate ourselves to exploit him.

Questions

Consider the following mistakes that a player might make in the Ace-to-Five Game. Using the Four-Step Exploitative Process, try to predict how the other player should adapt to maximally exploit each mistake. Start by thinking broadly - "Bluff more often on the river", for instance - then try to get more specific about exactly which hands are best suited to that exploit. Once you've attempted to craft exploitative strategies on your own, read on for answers and explanations.

1. Suppose that Ivan will only call if he has a J or better. Other than that, we do not have any insight into how he will play. How should Opal adapt her strategy to exploit this mistake?

2. What if Ivan will call only if he has a Q or better? How would Opal's maximally exploitative strategy differ from her strategy in Question 1?

3. Suppose that Ivan will only bluff if he has a 5. Other than that, we do not have any insight into how he will play. How should Opal adapt her own strategy to exploit this mistake?

4. What if Ivan will never bluff? How would Opal's maximally exploitative strategy differ from her strategy in Question 3?

5. What if Opal bet a more straight-forward range of {A,K,Q,5} with 100% frequency. Note that this is a balanced and polarized range, with one bluff for every three value bets. Other than that, we do not have any insight into how she

will play. How could Ivan maximally exploit this mistake?

Answers & Explanation

1. Suppose that Ivan will only call if he has a J or better. Other than that, we do not have any insight into how we will play. How should Opal adapt her strategy to exploit this mistake?

Envision the equilibrium: Ivan calls with 6/9 of his range to keep Opal indifferent between bluffing and checking with 6s and 5s. Js or better comprise 4/9 of his range, and he also calls sometimes with Ts, 9s, 8s, and 7s. This enables Opal to value bet all hands J or better, though she makes just as much by checking them.

Make a read: Ivan is folding too much.

Identify the exploits: Opal should do more bluffing and less thin value betting. As a result, her checking range will be a lot stronger, so she should fold some hands after checking that are too strong to fold at equilibrium.

Determine the degree of deviation: Large. Ivan's mistake may not seem significant, as the hands that he's folding are all indifferent between calling or folding at equilibrium. This is analogous to a player folding all his Ks in the Reciprocal Ranges Game; it's not like he's folding the nuts or anything close to it. Collectively, though, these four hands - in whatever mix Ivan chose to call them - constitute 1/3 of his equilibrium calling range. When he folds all of them, his folding frequency goes up considerably.

The easiest adaptation to an opponent folding too much is to bluff with all hands that are indifferent at equilibrium, 6s and 5s in this case. Opal should also bluff with 9s, 8s, and 7s, which are strictly checks at equilibrium.

Because Ivan's calling range is stronger than at equilibrium, Opal does less value betting. The weakest hand that she can value bet at equilibrium is a J, but against this player a J is a pure check.

As, Ks, and Qs are still mixed between betting and checking. The EV of betting these hands is less than at equilibrium, but the value of checking them has decreased as well, which is why Opal is still indifferent. Remember, we aren't making any assumptions about Ivan's play other than that he'll fold hands worse than a J. That means that we still assume he'll play optimally against a check, which given Opal's new checking range will mean less betting than at equilibrium.

After checking and facing a bet, Opal always calls with a Q or better and is indifferent to calling with Js and Ts, both of which are pure calls at equilibrium. Because Opal no longer checks 7s, 8s, and 9s, though, Ts and Js are now much closer to the bottom of her checking range, and she only has to call with them sometimes to make Ivan indifferent to bluffing.

The net result of all this is that Opal wins less with her strong hands than she would at equilibrium. Because excessively tight and/or passive opponents make it hard to win big pots with strong hands, poker players often find them frustrating and deride them as "nits".

Overall, though, Ivan's mistakes are quite valuable to Opal as long as she adjusts appropriately. Her maximally exploitative strategy has an EV of about $1.10, which is much better than the $0.94 she gets at equilibrium. Ivan's overfolding alone is enough to make this game a slightly winning proposition for Opal rather than a slightly losing one.

The catch is that capitalizing on Ivan's mistakes requires a lot of bluffing. In my experience, the players who complain the most about "nitty" opponents are themselves excessively tight and passive. Perhaps because they are unwilling to adapt by taking on the risk of so much bluffing, they don't like playing with tight opponents.

If you want to make money at poker, then you must treat a mistake as a mistake. Rather than committing to a particular playing style and then getting frustrated with opponents who don't accommodate by throwing money at you, you should instead look at any mistake as an opportunity for profit. Strive to remain alert for anything your opponents might be doing wrong and open to any adaptation that would enable you to profit from it. Years, perhaps even weeks from now, all you're going to care about it is how much money is in your bankroll. You aren't going to remember or care whether you got it from bluffing or value betting.

2. What if Ivan will call only if he has a Q or better? How would Opal's maximally exploitative strategy differ from her strategy in Question 1?

The types of adaptations will be the same: Opal will bluff more, value bet less, and fold more after checking. Essentially, what happens is a shift in the *threshold* for each of these adaptations. Against a player calling with a J or better, the strongest hand that Opal bluffed was a 9, and the weakest hand that she value bet was a Q. Against a player calling with a Q or better, the strongest hand that Opal should bluff is a T, and the weakest hand she should value bet is a K.

After checking, Opal always calls with a Q but is indifferent to calling with a J. She never calls with a T, which she sometimes did against the player in Question 1.

The important thing to see is that some hands are better for certain purposes than others. Hands with very little showdown value, for instance, are better for bluffing than stronger hands. When Opal's bluffing frequency is low, she isn't "mixing it up" by bluffing with lots of different hands, each at very low frequencies. Rather, she's bluffing with only her weakest hands (remember, 5s and 6s are equally worthless at equilibrium).

When she wants to do more bluffing, she fills out her bluffing range by increasing the frequency with which she bets these very weakest hands until they are at 100%. Then, if she still wants to do more bluffing, she starts adding the next-weakest hand to her range until that is at 100%, then she adds the next-weakest, etc.

A decision about whether to bluff (or bluff catch, or value bet) should not be based solely on feel or even a read about whether an opponent is "too tight". Those factors determine how wide your bluffing range can be, but you still must consider whether the cards you currently hold belong in that range. Against an opponent making only small folding mistakes, you won't reach beyond the very best bluffing candidates. As your opponent's mistakes get larger, you should bluff (or bluff catch, or value bet) with less ideal hands. As we saw in the Reciprocal Ranges Game, when this is taken to the absurd extreme of an opponent who will fold everything including the nuts, then even bluffing with the second nuts is profitable.

3. Suppose that Ivan will only bluff if he has a 5. Other than that, we do not have any insight into how he will play. How should Opal adapt her own strategy to exploit this mistake?

Envision the equilibrium: Ivan bluffs with all his 5s and half his 6s. This makes Opal indifferent to calling with pure bluff-catchers - the {9,8,7} portion of her range - and also indifferent between value betting and check-calling with the {A,K,Q,J} portion of her range.

Make a read: Ivan is not bluffing enough.

Identify the exploits: The most obvious adaptation to an opponent who does not bluff enough is to do less bluff-catching. This has some secondary implications, though. Opal should also value bet more, because while her strongest hands are indifferent between betting and checking at equilibrium, checking is less profitable for them if Ivan will not bluff as much. Because Opal is value betting more, she also bluffs more.

Determine the degree of deviation: Small. Ivan is still taking his +EV bluffs, he’s just never bluffing when he’s indifferent. This will lead to a lot of changes in Opal’s strategy, but that’s only because she was indifferent with so many hands at equilibrium. Though most of those hands now play pure strategies, the changes are all at the margins. We won’t see any hands go from pure calls to pure folds.

Against this opponent, check-calling is undesirable, and Opal stops doing it entirely. 100% of the time that she checks, it is with the intention of folding. With a J or better, she value bets 100%. With a T or worse, she checks and folds if faced with a bet.

Because Opal's value betting range is wider, she also increases her bluffing frequency in order to keep Ivan indifferent to calling with bluff-catchers. Opal still isn't bluffing 100% with 5s and 6s, but she is betting them at a higher frequency than at equilibrium. Neither of these hands ever wins by checking, and despite the increased bluffing frequency, Opal is still not bluffing enough to use a hand that has a chance of winning at showdown. There would be no reason for her to start betting 7s until she was already betting 6s and 5s at 100% frequency.

Needless to say, if Opal never calls after checking, then bluffing is profitable for Ivan. That's the risk that you take when you choose an exploitative strategy: you are presenting your opponent with a profitable opportunity, and you just have to hope that your read is correct and he won't take advantage of it. This opponent just doesn't have it in him to bluff with a 6 or stronger, which is why Opal prefers this strategy.

Ivan will bluff with a 5, and he will profit by doing so. Remember, though, that bluffing a 5 is profitable for him even at equilibrium. It just isn't worth it for Opal to make the changes to her strategy that would make Ivan indifferent to bluffing with his very best bluffing candidate.

Despite some big shifts in Opal's strategy, Ivan's mistake here isn't actually a large one. His EV in the game declines from $1.06 to $1.02, but he still shows a small profit despite his suboptimal strategy.

4. What if Ivan will never bluff? How would Opal's maximally exploitative strategy differ from her strategy in Question 3?

Opal stops betting with a J. Her bluffing frequency goes down, because she's value betting less, but the decrease isn't spread across both of her bluffing candidates. Instead, she stops bluffing entirely with a 6 and bluffs 100% with a 5.

This is a tricky one. There's nothing intuitive about these adaptations, as far as I'm concerned, and the results surprised me when I worked them out with a solver.

Here's what's happening: both a 6 and J are hands that can beat a 5 at showdown. Until now, though, Ivan was never checking a 5, so Opal’s 6s had no showdown value. Even when her 6 was the best hand, Opal never got to showdown to find that out.

Against the opponent from Question 3, the one who would bluff a 5 but not a 6, a J suffered to some degree from the same problem. If it were to check, it would sometimes lose the pot to a 5 that bluffed. That made checking less appealing relative to betting. A J wasn't a favorite when called, but it could sometimes win, and one advantage of betting was that it won pots that it would have lost when Ivan held a 5.

If Opal can be sure that Ivan won't bluff with a 5, though, then she has more incentive to check both a 6 and a J. Her 6s now have some chance of winning at showdown, so they are no longer just as good for bluffing as 5s. At equilibrium, 6s are indifferent between betting and checking, but against an opponent who will never bluff, they do better by checking.

If Opal stops bluffing with 6s, though, she can't do as much value betting. A J is the worst hand that she was value betting, so that's the first hand to fall out when she shrinks her range.

Essentially, Js and 6s are the most marginal hands in Opal's equilibrium betting range. A J is a barely profitable bet, and a 6 is a break-even bluff. For both, part of the appeal of betting was that checking meant occasionally losing the pot to a bluff.

This is a common dilemma for the out-of-position player: marginal hands sometimes have no good options, and betting can be the lesser of two evils. If Opal can be sure that she won't get "punished" by a 5 for checking, then checking becomes a more appealing option, and she can remove both her thinnest value bet and her thinnest bluff from her polarized betting range.

The result is a betting range that is smaller, but still balanced. Opal bets 100% with a range of {A,K,Q,5}, a 3:1 ratio of value bets to bluffs, just as we're accustomed to seeing with a half-pot bet.

Ivan's mistake here is much bigger than his mistake in Question 3. It's enough to reduce the value of the game to $0.90, making it now a solidly losing proposition for him.

5. What if Opal bet a more straight-forward range of {A,K,Q,5} with 100% frequency. Other than that, we do not have any insight into how she will play. How could Ivan maximally exploit this mistake?

Envision the equilibrium: Opal mixes bets and checks with her strongest hands. The strong hands in her checking range make Ivan indifferent to bluffing with a 6 and value betting a T.

Make a read: Opal is checking a condensed range that contains no strong hands.

Identify the exploit: Big bets with a polarized range are the way to attack a condensed range. This game doesn't give Ivan the flexibility to change his bet size, but because he doesn't have to worry about running into Qs or better after Opal checks, he can value bet more thinly than at equilibrium, and he can balance those value bets with more bluffs.

Determine the degree of deviation: Small. The hands that Opal now strictly bets are indifferent at equilibrium, so she can't be giving up too much by checking them. Ivan's strategy should likely change only at the margins as well.

At equilibrium, Ivan is indifferent between betting and checking a T, but now betting is strictly better. At equilibrium, he has 4.5 value bets and 1.5 bluffs. Now, he has 5 value bets, so he'll need 1.67 bluffs to maintain the 3:1 ratio. He starts filling out his bluffing range with more 6s, and he only has to increase his bluffing frequency with a 6 to 2/3, so he still won't bluff with any 7s.

The other change for Ivan is that more of his hands become pure bluff-catchers when faced with a bet. Js are no longer automatic calls, and 6s no longer block any of Opal's bluffs. Ivan always calls with a Q or better, he always folds a 5, and he is indifferent between calling and folding with everything else. He still needs to call with 2/3 of his range, there are just more ways to do that now.

Note that even though Opal's betting range is properly balanced between value bets and bluffs, her overall strategy is exploitable. A balanced betting range makes Ivan indifferent between calling and folding with bluff-catchers, but Opal must also make him indifferent to thin value bets after she checks. Her equilibrium checking range makes him indifferent to value betting a T. Her exploitable checking range makes it profitable for Ivan to value bet a slightly wider polarized range that contains all his Ts and a few additional bluffs.

Real World Applications

Target Specific Hands in Your Opponent's Range

A key lesson from the Ace-to-Five Game is the importance of having a *target*. Whether you are value betting, bluffing, or bluff-catching, you should have in mind specific hands in your opponent's range from which you hope to get value. You can't force an opponent to call or fold any given hand, and you won't be able to present every hand in his range with a difficult decision. Sometimes, your opponent's cards will just play themselves. The goal is to recognize the cases where you *can* present an opponent with a difficult decision and then take advantage of as many of those opportunities as possible.

Consider the case of Opal holding a J in the Ace-to-Five Game. In comparable hold 'em situations, when out of position with marginal hands, many players reason that, "I'm not sure I'm ahead, so I'd better check." This is a decision driven by fear rather than by strategic thinking. It seeks to avoid losses rather than make maximally +EV plays and present opponents with difficult decisions.

This is also an incomplete thought process, in that it does not consider what to do after checking. If the opponent bets, then fear takes over again. Either the player says, "He could easily have a better hand, so I fold," or she says, "He could easily be bluffing, so I call." It becomes a question of whether she is more afraid of losing another bet or of getting bluffed. In either case, the motivation is fear rather than strategy.

We can see the limitations of this logic more clearly if we apply it to stronger or weaker hands. In the Ace-to-Five Game, should Opal fold a K because Ivan might have a better hand? Should she call with a 6 because Ivan might be bluffing with worse? We don't know Ivan's strategy, but he has incentive both to value bet strong hands and to bluff with weak ones, and Opal needs to formulate a strategy that accounts for all his incentives instead of focusing on just one of them.

The decision between betting and checking a J is a close one, and Opal does both at equilibrium. In either case, though, there is a target, a set of hands in Ivan's range that will have a difficult decision as a result of Opal's play. When Opal bets, she targets the {T,9,8,7} portion of Ivan's range. When she checks, she targets the Ts, 6s, and 5s in Ivan's range. The reason to check is *not* that Ivan might have a better hand - after all, Opal even checks an A sometimes at equilibrium. Besides, checking should not result in Opal saving a bet when she's behind, because Ivan should bet all better hands, and Opal should never fold.

Similarly, the reason to call after checking is not fear of a bluff - it's a deliberate strategic decision informed by Ivan's incentives. We may not know his strategy, but we do know his incentives. He has incentive not only to bluff with his weak hands but also to sometimes value bet with a T, which is what makes calling with a J definitively better than folding at equilibrium.

Opal cannot be sure that she is ahead if she value bets, nor if she checks and calls. She can't even be sure that her opponent would actually value bet a T or bluff with a 6 - Ivan is indifferent to both at equilibrium. Poker is not about certainty. To find the best play, Opal must look beyond what she would prefer to avoid and focus instead on what she can control.

Targeting becomes even more important when Opal tries to play exploitatively. Once she frames the decision as a choice between inducing calls from Ts, 9s, 8s, and 7s or inducing bets from Ts, 6s, and 5s, Opal can think about what kind of mistake her opponent is more likely to make. Against loose and passive opponents, betting will be more profitable. Against opponents who bluff too much, checking and calling will be more profitable. Against very nitty opponents who are reluctant both to bluff and to bluff-catch, checking and folding could even be the best play (recognizing that the action will often go check-check, enabling Opal to win at showdown).

It's interesting to note that when Opal bets an A, she's really targeting the same hands as when she bets a J. There are other hands that could call, such as Ks and Qs, but they aren't hands that will have difficult decisions as a result of Opal's bet, nor are they hands that Opal *needs* to bet to get value from. It should be trivial for Ivan to call with a K, Q, or J if Opal bets and also to bet those hands if Opal checks. Opal shouldn't need to do anything at all to win a bet from those hands. The hands that she targets are the ones where her action makes a difference. She won't win anything from a 9, 8, or 7 if she checks, but she might if she bets. She won't win anything from a 6 or 5 if she bets, but she might if he checks.

Threshold hands, the ones where your opponent’s decision is close at equilibrium, are typically the cases where you can present him with difficult decisions. They are the hands with which he is most likely to make a mistake and where your choice of action is most likely to affect your bottom line.

The question to ask is not what you want your opponent to do or what you hope or fear he will do. In many cases, your opponents' play will be decided by the cards he was dealt, not by how you play yours. You can manipulate your opponent's range *only at the margins*. The question to ask is, what are the hands in his range where your action will actually make a difference? These are your targets, and you should practice identifying them before you act.

Construct Ranges That Present Your Opponents with Difficult Decisions

A single hand in isolation can't really give your opponent a difficult decision. You must think in terms of how you would play a range of hands, not just the hand you currently hold. When Opal bets a J, Ivan has a difficult decision with a 9 because of the possibility that Opal holds a 5 or 6. When Opal checks, Ivan has a difficult decision with a T because Opal may hold a 9 or worse that would pay him off, but she also might be lying in wait with a J or better.

When Opal constructs her equilibrium betting range, her goal is not to get folds or to get calls or to induce bluffs. Nor does she set out to maximize the value of one part of her range. Her goal is to present her opponent with as many difficult decisions as possible.

If Ivan was dealt a strong hand, he simply isn’t going to have difficult decisions. He’ll call if Opal bets, and he’ll bet if she checks. Bluffing into a strong hand or paying off a value bet from one can be frustrating, but it’s often correct. Your opponents are supposed to win something when they get strong hands. That’s the price you pay to avoid getting exploited by other parts of their range. Build ranges with the right targets in mind and try not to worry about what will happen if your opponent has a very strong handYou can’t have it all.

Especially when out of position, you have many objectives to juggle. You want to bet some strong hands in case your opponent calls with bluff-catchers. You want to bet some weak hands in case he folds bluff-catchers. You want to get the ratio of strong hands to bluffs right so that you aren’t exploited by an opponent who does too much of one or the other.

The factor that limits the size of your polarized betting range is how many strong hands you can bet, because you also want to check some strong hands in case your opponent tries to make thin value bets. You decide just how many strong hands you need to check to keep your opponent indifferent to value betting thinly (with a T, in this example), then you have the rest of them available to bet.

That gives you your value betting range. Then you look at the pot odds that your bet will offer and determine how many bluffs you need to keep your opponent indifferent to calling with bluff-catchers. Starting with your very weakest holdings, you fill out your bluffing range. If you exhaust your weakest hands and still have room to do more bluffing, then you start adding from your next-weakest hands, etc. until you hit the right ratio of value bets to bluffs.

This planning gives you the context for deciding how to play any individual hand. Even though you only ever hold one hand at a time, thinking about how you would play your entire range - every hand that you *could* hold in this situation - helps you to determine how to play the one hand you currently hold. When you have a close decision about whether to value bet, you should consider how many bluffs you could have in this situation, because that is what will give your opponent incentive to call with weak hands. When you have a close decision about whether to bluff, you should consider how many strong hands you could value bet in this situation, because that is what will give your opponent incentive to fold.

Many players are reluctant to bet when they have strong hands because they fear that betting will show strength. Planning your strategy in this way ensures that betting will not actually show strength, as it enables you to see that betting is perfectly consistent with how you would play some weak hands.

Your opponent can't see your cards, but he can understand your incentives, and he knows what cards you could be dealt. Whether or not you would bluff in a given situation is not just a matter of whim or personal style. You easily could hold weak hands, and if you did, you'd have incentive to bet with them. That's all your opponent needs to know to have incentive to call you with a bluff-catcher. Whether he *does* call is up to him, but then again you can't force him to bet by checking your strong hands, either. All you can do is play to his incentives.

This process is easier when you're playing exploitatively, because you can usually take one or more of these concerns off the table. If you know that your opponent calls too much with bluff-catchers, then you no longer need to worry about bluffing. If you know that he won't make thin value bets, then you don't have to worry about checking strong hands, and you can expand your betting range accordingly.

The reason to bluff is not to discourage your opponent from folding, nor is the reason to check strong hands to discourage him from bluffing. Rather, you bluff to *profit* from his folds, and you check strong hands to *profit* from his bluffs (and his thin value bets). If you know that he won't do these things, then you can't profit from them and should shift your attention to the places where you can profit.

Realistically, you probably will not have the time or wherewithal to plan so precisely when you have a live hand. What you can do, however, is make a quick estimate of what your ranges will look like and where your current hand is likely to fit. When you hold a weak hand, you should consider just how weak it is. Is it one of the very weakest you could hold, or will you sometimes have even weaker hands?

Then, consider roughly how wide your bluffing range will be in this situation. If you can support lots of bluffs, then you may do best by betting even if your current hand is less than ideal for the purpose. In a situation that is not so good for bluffing, you should restrict yourself to bluffing only with ideal candidates, but there usually should be some hand with which you would bluff, and it’s best to identify what that would be before you decide not to bluff with whatever you actually hold. Otherwise, how can you be sure that the hand you’re about to check isn’t the best bluffing candidate you would ever have in this situation?

You won't end up with perfectly balanced ranges this way, but as we've seen, that's not terribly important. This thought process should help you to avoid big mistakes, such as bluffing way too much or too little, that can be easily and expensively exploited.

Test Yourself

In the examples below, assume that you are playing no-limit hold 'em at a nine-handed table, with $2 and $5 blinds and $500 effective stacks.

1. You open $15 with K♥ T♥ on the button, and the big blind calls. The flop comes T♦ 6♠ 2♣. The big blind checks, you bet $15, and she calls. The turn is the Q♣. She checks, you bet $30, and she calls. The river is the 2♦. She checks. What's your play?

Bet. My default would probably be $100 or so, but if you have a read that your opponent is excessively nitty when it comes to large bets, you could size down. This is a clear value bet, though.

A common error is to check back the river simply because second pair, second kicker is pretty far from the nuts. Remember, though, that in the Ace-to-Five game, the in-position player can value bet hands as weak as a T. Position provides you with valuable information, but it's your job to capitalize on it.

In this case, your position should enable you to deduce that your hand is likely best. It's not a sure thing, but poker isn't about sure things. Your opponent has passed up three opportunities to raise. With pairs bigger than a T, she'd probably re-raise pre-flop. Had she out-flopped you, she had a lot of incentive to raise the flop or turn. Slowplaying is risky for her, because as the in-position player, you control the size of the pot. When she checks, she gives you the power to check behind with your bluff-catchers.

Still, some hands better than yours are likely in her range, such as AT, rivered trips, or the occasional Q. The question is not whether you're a lock to win, it's whether she'll have incentive to call with enough worse hands to make up for the times she calls with better.

The risk of bluffs is what gives her that incentive. A bet of $100 into $120 forces her to call with well over half of her possible holdings or else your bluffs will profit.

It's not practical to count all your combinations of bluffs and value bets in real time, but you should try to get a sense of whether you will hold many or few weak hands in this situations, as that's what determines how thinly you can value bet. We know that a pot-sized bet enables you to have two value hands for every one bluff, and with smaller bets, you get even more value hands for every bluff. KT isn't exactly the top of your range, but it doesn't need to be.

As a button opener, you should have plenty of weak hands in your pre-flop range, and you should bet plenty of them on the flop. Even if you were to bluff only with draws on the turn, you could have AK, KJ, J9, 98, 87, 54, 43, or clubs, all of which missed on the river.

Game theory can't guarantee that either betting KT or bluffing with any of the above will be profitable. It does, however, guarantee that a strategy of betting a polarized range into your opponent’s condensed range will profitable. If you think you can do better than that because of a read on your opponent, then go for it. Against loose opponents, you check the bluffs and bet the KT. Against tight opponents, you check the KT but bet the bluffs. Against an unknown opponent, the most profitable strategy will be to bet *both*, not to bet neither, which is what most people end up doing.

2. You open $15 with K♥ T♥ in early position, and the big blind calls. The flop comes T♦ 6♠ 2♣. The big blind checks, you bet $15, and she calls. The turn is the Q♣ She checks, you bet $30, and she calls. The river is the 2♦. She checks. What's your play?

Check. As an early position opener, you'll have fewer weak hands with which to bluff, which means that you can't balance as many as value bets. There's also a greater risk that your opponent holds a strong hand, as she would have less incentive to three-bet or check-raise an early position opener.

3. You open $15 with Q♥ J♥ in early position, and the big blind calls. The flop comes K♥ 9♦ 3♣. The big blind checks, you bet $15, and she calls. The turn is the A♠. She checks. What's your play?

Bet big. Really big. Like, two times the pot.

As the pre-flop raiser, you are the player who can bet a polarized range. You are far more likely than your opponent to have AA, KK, and AK. Sure, she could have 99, but so could you.

Admittedly, these “monster” hands are a small part of your range. With most of your range, even strong hands like AQ and AJ, you will not want to play such a large pot. That means that you have few hands that will make such big bets for value, so you don't get to do much bluffing.

But you do get to do some, and this is a great candidate for it. For one thing, it's one of the weakest hands you could hold. Most of your range should consist of pocket pairs, Ax, and Kx, all of which have too much showdown value to turn into bluffs. Plus, while a gutshot isn't much, it's the only draw possible on this board.

Finding an overbet here is less important than recognizing that you should bet *something*. The board is perfect for betting a polarized range, but there are no "obvious" bluffs. If you don't force yourself to consider what your betting range should look like, you're liable to miss a good bluffing opportunity.

Conclusion

In the end, the equilibrium for the Ace-to-Five Game closely resembles that of the Clairvoyance Game. The out-of-position player is indifferent to betting her strongest hands, the in-position player bets a polarized range at roughly the same frequency, and the out-of-position player calls at the same frequency to make him indifferent to bluffing.

Finding the right hands to meet these frequencies is more difficult, however, and that’s where things get interesting. You don’t often see human opponents check back the nuts on the river or call with the nut low, but people miss thin value bets and profitable bluffs all the time.

The Ace-to-Five Game helps us think about how to exploit these common mistakes, and in the next chapter, it will help us think about how the threat of a raise further complicates these decisions.

Key Lessons

- ♠ **When out-of-position, your best play is highly sensitive to your opponent's strategy.** The choice between betting and checking is not determined primarily by your degree of confidence in your hand, with the strongest hands value betting and less strong hands checking and calling. Rather, against overly loose opponents, even modest hands do better by betting. Against overly aggressive opponents, even very strong hands do better by checking.
- ♠ **When you're in position, your optimal strategy is more straight-forward**. You bet your strong hands for value, bluff with your weakest hands, and check the ones in the middle.
- ♠ **Position enables you to do everything better.** You can value bet *and* bluff more aggressively, and it's safer and more profitable for you to check your medium-strength hands.
- ♠ **Focus on targets that matter**. Whether betting or checking, you should be able to identify not only the action you are trying to induce from an opponent but with which hands you expect her to take that action. The targets to focus on are the ones where your choice can affect the outcome of the hand. In many cases, there is nothing you can do to win an additional bet when ahead or to avoid losing one when behind, and you shouldn't waste mental energy worrying about those cases.

Caveats

- ♠ **These lessons apply most straight-forwardly on the river.** They are still relevant on earlier streets, but they are complicated by other considerations.
- ♠ **The threat of a raise complicates decisions about thin value betting.** For the out-of-position player, the option to check-raise also complicates the decision about whether to bet strong hands. In the next chapter, we'll investigate how the introduction of raising changes optimal strategies for this game.

Chapter 7: Raising

Overview & Objectives

So far, we've kept our toy games simple by excluding the option to raise. Now that we know what equilibrium strategies look like without raising, we can see rising changes them. We'll look at which hands benefit from the option to raise, which are constrained by it, and how betting strategies change when a raise is possible.

We'll start by adding raising to the Reciprocal Ranges Game, then we’ll add it to the Ace-to-Five Game. Finally, we'll test your understanding of the strategy behind raising by considering how to exploit various mistakes related to raising.

By the end of this chapter, you should be able to:

- ♠ Appreciate the strategic role played by raising.
- ♠ Explain how the possibility of raising affects betting strategy.
- ♠ Adapt betting and raising strategies to exploit specific mistakes.

Scenario: Raising in the Reciprocal Ranges Game

We're returning to the Reciprocal Ranges Game: each player antes $1 and receives one card from a three-card deck containing an A, a K, and a Q. Each player may bet $1 or check. Now, however, if a player bets, his opponent has the option to raise to $3 in addition to calling or folding.

Questions

Do your best to predict how this will change the equilibrium from the Reciprocal Ranges Game:

1. Will Opal's checking frequency increase, decrease, or stay the same? Why?

2. If Opal does bet, what will Ivan's raising range be?

Answers & Explanation

1. Will Opal's checking frequency increase, decrease, or stay the same? Why?

It will stay the same. As we're about to see, adding the option to raise doesn't meaningfully change this game.

2. If Opal does bet, what will Ivan's raising range be?

Ivan should raise only when he has an A. This should be Opal's strategy after checking and facing a bet as well. Neither player ever calls a raise after betting.

That might sound exploitable - If your opponent never calls a raise, shouldn't you bluff? - but remember that the reason to bet a polarized range is to pressure medium-strength hands, and neither player ever bets a medium-strength hand in this game. That means that if Ivan is holding a Q and facing a bet, then Opal must have an A, which of course would not fold to a raise. If Ivan is holding an A and facing a bet, then Opal must have a Q. There's no harm in raising, but Opal is just going to fold, so raising doesn't make him any money.

There's an important lesson here: the more polarized your opponent's betting range, the harder it is to raise for thin value or as a bluff. The hands in the middle are the ones that face a difficult decision when raised, and polarized ranges don't have a middle.

Scenario: Raising in the Ace-to-Five Game

Equilibrium betting ranges in the Ace-to-Five Game aren't quite so polarized as in the Reciprocal Ranges Game. They contain hands that will have meaningful decisions to make when faced with a raise. So, we're going to introduce the option of raising into the Ace-to-Five Game and see how it changes the equilibrium.

Each player antes $1, and in an unopened pot, each player may bet $1 or check. After a player bets $1, his opponent has the option to raise to $3. Facing a raise, a player may only call or fold; there is no re-raising. Do your best to predict how this will change the equilibrium from the version of the game where raising is not allowed.

Questions

1. Will Opal's betting frequency increase, decrease, or stay the same? Why?

2. Will Ivan's betting frequency increase, decrease, or stay the same? Why?

3. If Opal bets, what is the worst hand that Ivan will raise for value? What hands does he bluff with?

4. How often must Opal call after betting to make Ivan indifferent to raising as a bluff?

5. What should Opal’s check-raising range look like?

6. Will the option to raise increase or decrease the value of the game for Ivan?

Answers & Explanation

1. Will Opal's betting frequency increase, decrease, or stay the same? Why?

It will decrease. Opal bets just 27% of her range, down from nearly 37% when raising was not allowed. She is still indifferent to betting an A or a Q, but she bets them less often. She strictly prefers betting a K and checking a J. Less value betting means less bluffing for her as well. Here's a chart showing the changes in her betting strategy:

Opal's Betting Strategy				
	Game Without Raising		**Game With Raising**	
Hand	**Betting Frequency**	**EV of Betting**	**Betting Frequency**	**EV of Betting**
A	81%	$2.56	20%	$2.66
K	73%	$2.11	100%	$1.97
Q	63%	$1.67	86%	$1.50
J	57%	$1.22	0%	N/A
T	0%	N/A	0%	N/A
9	0%	N/A	0%	N/A
8	0%	N/A	0%	N/A
7	0%	N/A	0%	N/A
6	36%	$0	34%	$0
5	56%	$0	34%	$0
Total	**37%**	**N/A**	**27%**	**N/A**

The threat of a raise reduces the value of the weakest hands in Opal's value betting range. With a J, she is indifferent in the game without raising, but the risk of a raise makes her strictly prefer checking.

When Opal has a J, she really just wants to get to showdown. The only reason she can value bet it in the game without raising is that even though she often ran into stronger hands, she is going to pay off those same hands by checking and calling anyway. When raising is allowed, however, betting a J could result in putting $1 into the pot and still not getting to showdown. That makes checking more appealing.

The value of betting an A increases when Ivan can raise, but the value of checking it also increases. Unlike a K, an A is strong enough to check-raise. This is why Opal remains indifferent between betting and checking an A but prefers betting a K. Opal doesn't *want* to get raised when she has a K, because it does much better against Ivan's calling range against his raising range. However, a K is strong enough to call a raise, so she doesn't have to worry about getting pushed off of her equity in the way that she would if she were to bet a J.

We have seen many times that middling hands suffer the most when they face bets from polarized ranges. Strong hands - As and Ks, in this case - have trivial calls when raised, and weak hands have trivial folds. A Q is the "middle" of Opal's betting range; it's weaker than the rest of her value bets but stronger than her bluffs, and it's indifferent between calling and folding when faced with a raise.

2. Will Ivan's betting frequency increase, decrease, or stay the same? Why?

There's a similar but less dramatic effect on Ivan's betting range, shown in the chart below. Facing a check, Ivan bets a polarized range consisting of about 45% of his hands, down from 60% in the original version of this game. Whereas before he was indifferent to betting a T for value, he's now indifferent to betting a J. He, too, is constrained from betting for thin value by the risk of a raise.

Although bluffing a 6 and a 5 have the same EV of $0.19, checking a 6 also has an EV of $0.19, while the EV of checking a 5 is $0 – it never wins at showdown. Because bluffing with a 5 is better than checking for Ivan, he does it 100% of the time. With a 6, he is indifferent.

Ivan's Betting Strategy				
	Game Without Raising		Game With Raising	
Hand	**Betting Frequency**	**EV of Betting**	**Betting Frequency**	**EV of Betting**
A	100%	$2.54	100%	$2.57
K	100%	$2.41	100%	$1.96
Q	100%	$2.22	100%	$1.94
J	100%	$1.97	37%	$1.70
T	50%	$1.53	0%	N/A
9	0%	N/A	0%	N/A

8	0%	N/A	0%	N/A
7	0%	N/A	0%	N/A
6	50%	$0.16	11%	$0.19
5	100%	$0.22	100%	$0.19
Total	**60%**	**N/A**	**45%**	**N/A**

3. If Opal bets, what is the worst hand that Ivan can raise for value? What hands does he bluff with?

The chart below compares Ivan's strategy when faced with a bet in this game, where he has the option to raise, to his strategy in the game without raising. Note that when he uses a mixed strategy, the EV is the same no matter which option he chooses. Whether he calls or raises with a K, his EV is $2.55. If this were not the case, he would not use a mixed strategy but would strictly prefer the higher-EV option.

Ivan's Strategy Facing a Bet							
	Game Without Raising			**Game With Raising**			
Hand	**Call**	**Fold**	**EV**	**Call**	**Fold**	**Raise**	**EV**
A	100%	0%	$3.00	0%	0%	100%	$3.9
K	100%	0%	$1.89	90%	0%	10%	$2.5
Q	100%	0%	$0.96	100%	0%	0%	$0.4
J	100%	0%	$0.18	87%	12%	1%	$0
T	83%	17%	$0	51%	39%	10%	$0
9	59%	41%	$0	51%	39%	10%	$0
8	42%	58%	$0	47%	42%	11%	$0
7	16%	84%	$0	20%	73%	7%	$0
6	0%	100%	$0	0%	100%	0%	$0.1
5	0%	100%	$0	0%	100%	0%	$0.1
Total	**60%**	**40%**	**N/A**	**45%**	**40%**	**15%**	**N/A**

If Opal bets, Ivan raises a polarized range consisting of all his As, some of his Ks - he's indifferent with these – and enough bluffs to make Opal indifferent to calling with a Q.

This sounds straightforward, but the composition of Ivan's bluff raising range is surprising. He bluffs with some combination of hands 7 through J but never with 6s or 5s. In every other equilibrium strategy we've looked at, the weakest hands in a player's range were always the preferred bluffs. What's going on here?

For one thing, when faced with a bet, a J is just as weak as a 5. Sure, the J could call to catch bluffs, and sometimes it does, but we know that the equilibrium EV of calling with a pure bluff-catcher is 0. So, calling or folding with a J (or a T, 9, 8, or 7) is worth $0, which is all that a 5 is worth. The difference is that raising with a J is also worth $0, because Opal should call at a frequency that makes him indifferent to bluffing, whereas raising a 5 is actually -EV.

The reason that 5s and 6s make such poor bluff raises is that they block Opal's folding range. Her betting range consists of very strong hands (A and K) that will happily call a raise, middling hands (Qs) that will be indifferent, and weak hands (5 and 6) that will strictly fold. If Ivan has a 5 or 6, then Opal is less likely to hold a hand that will fold to a raise.

This can be a little confusing, because when Ivan raises with a J and Opal folds a 5, he was bluffing with the best hand. The reason that's irrelevant is that even though he had the best hand, he was going to lose the pot anyway if he didn't raise. He sometimes calls with a J hoping to beat a bluff, but at equilibrium he also folds some of his Js. So, raising a J that he otherwise would have folded isn't trading off with a more profitable opportunity.

Calling with a J means sometimes beating a 5 and sometimes losing to a Q. Raising means sometimes winning a pot he would have won anyway by calling, but also sometimes stealing a pot from a Q and sometimes losing an extra $2 to an A, K, or stubborn Q. Calling, folding, and raising all have $0 EV, so facing a bet with a J, T, 9, 8, or 7, Ivan mixes all three options at equilibrium.

Note that Ivan’s folding frequency remains the same as in the game without raising. This exact frequency is what makes Opal indifferent to bluffing. Whether he calls or raises makes no difference to her bluffs, whose EV is determined entirely by his folding frequency.

4. How often must Opal call after betting to make Ivan indifferent to raising as a bluff?

She must call 1/2 of the time. We're used to seeing a calling frequency of 2/3, but that was for a $1 bet into a $2 pot. The general formula is 1 - Bet/(Bet+Pot). There is $3 in the pot after Opal bets, and Ivan's raise costs him $3, so Opal must call 3/6 or 1/2 of the time to make Ivan indifferent to bluffing.

5. What should Opal's check-raising range look like?

The chart below shows Opal's full strategy for responding to a bet after checking. Note that some hands do not sum to a 100% frequency because Opal sometimes bets them. For example, she only checks an A 80% of the time, but if she does check, she always raises when Ivan bets. The raising frequency of 80% represents the percentage of As that she is dealt that end up in her check-raising range. The other 20% are in her betting range.

The row labeled "Total" shows her overall calling, folding, and raising frequencies after checking. So, after checking and facing a bet, she calls 40% of her checking range, folds 45% of her checking range, and raises 15% of her checking range.

Opal's Check-Calling Strategy				
Hand	**Call**	**Fold**	**Raise**	**EV**
A	0%	0%	80%	$3.71
K	0%	0%	0%	N/A
Q	14%	0%	0%	$0.70
J	100%	0%	0%	$0.08
T	44%	49%	6.67%	$0
9	44%	49%	6.67%	$0
8	44%	49%	6.67%	$0
7	44%	49%	6.67%	$0
6	0%	67%	0%	$0
5	0%	67%	0%	$0
Total	**40%**	**45%**	**15%**	**N/A**

Her raising range is extremely polarized, consisting of only As for value and some combination of 7s through Ts as bluffs. The ratio of value to bluffs is 3:1, just as it should be – I left some decimal places on her bluff raising frequency so that you could see this relationship more clearly.

As in the game without raising, Opal folds more than 1/3 of her range to a bet. This is because her objective is to make Ivan indifferent to bluffing with a 6, not with a 5, and his 6s don't gain anything relative to checking when she folds a 5. In order to make him indifferent to bluffing with a 6, she must continue, either by calling or raising, with 2/3 of her range that can beat a 6.

6. Will the option to raise increase or decrease the value of the game for Ivan?

Technically, raising increases his EV, but the difference is extremely small. Without raising, Ivan's EV is $1.061. With raising, his EV is $1.063. At equilibrium, he wins an extra 2/10 of a penny when raising is allowed.

That assumes that both players play perfectly, though. Making the game more complex introduces more opportunities for mistakes and imbalances. Mostly that benefits the more skilled player, who is better able to balance his own ranges when appropriate and deviate from the equilibrium when he suspects that an opponent is imbalanced. All other things being equal, though, the in-position player will probably have an easier time playing unexploitably and picking up on opportunities for exploitation. Ivan doesn't have to worry about balancing his checking range, so his equilibrium betting strategy is a bit more straightforward, which means one less opportunity for error.

Scenario: Exploitation in the Ace-to-Five Game with Raising

The rules are the same as before: each player antes $1, and in an unopened pot, each player may bet $1 or check. After a player bets $1, his opponent has the option to raise to $3. Facing a raise, a player may only call or fold; there is no re-raising.

Questions

Consider the following ways in which an opponent might deviate from his equilibrium strategy, and practice using the Four-Step Exploitative Process to craft strategies that will exploit these mistakes. Try to come up with answers on your own before you read on for the full explanations.

1. Suppose that Ivan will never raise as a bluff. Other than that, we know nothing about how he will play. How should Opal adapt her equilibrium strategy to maximally exploit this mistake?

2. Suppose that Opal will only value bet with a K or better. Other than that, we know nothing about how she will play. How should Ivan adapt his equilibrium strategy to maximally exploit this mistake?

3. Suppose that Ivan has observed Opal betting some Js and believes that Opal will continue to employ the same strategy despite having shown these down. How should Ivan adapt his strategy to exploit the likely mistake here?

4. Suppose that Ivan will only call a raise with an A. Other than that, we have no insight into how he will play. How should Opal adapt her strategy to exploit this mistake?

Answers & Explanations

Each player antes $1, and in an unopened pot, each player may bet $1 or check. After a player bets $1, his opponent has the option to raise to $3. Facing a raise, a player may only call or fold; there is no re-raising.

1. Suppose that Ivan will never raise as a bluff. Other than that, we know nothing about how he will play. How should Opal adapt her equilibrium strategy to maximally exploit this mistake?

Envision the equilibrium: Ivan should raise a polarized range consisting of all his As, some of his Ks, and some bluffs. This makes Opal indifferent between betting and check-calling a Q and between betting and check-raising an A. It also makes it unprofitable for Opal to bet a J, which was a hand that she sometimes bet for value in the version of the game without raising.

Make a read: Ivan is not really using his option to raise at all. If his range contains no bluffs, then his raises never present Opal with a difficult decision.

Identify the exploits: Because Ivan never bluffs, Opal doesn't have to worry about facing difficult decisions after betting; she'll simply fold whenever she doesn't have the nuts. This increases the value of betting with hands that would ordinarily be put in a tough spot by a raise. It also decreases the value of betting with the nuts, because if Opal were to bet with an A, she would never expect to win more than $1 with it. Because Ivan never bluffs, he can't raise for thin value either - Opal has no incentive to call a raise with less than the nuts.

Determine the degree of deviation: Large. Although all the changes to Ivan's strategy are with hands that are indifferent at equilibrium, the overall effect is to eliminate raising as a meaningful part of his strategy.

This doesn't mean that Opal's betting strategy will be the same as in the game without raising. *She* is still allowed to raise, and that will influence which hands she bets. At equilibrium, she is indifferent between check-raising an A or betting and calling a raise with it. Now that she can't anticipate a raise if she bets, she strictly prefers checking As in the hopes of getting a check-raise in.

An A is the only hand strong enough for Opal to check-raise for value. She strictly prefers betting a K, just as she did at equilibrium. She also bets 100% with Qs and Js, though, which is a big change from equilibrium. Now that Opal doesn't have to worry about getting raised by a polarized range, she can value bet these hands with impunity.

In the variant of this game without raising, Opal needs some strong hands in both her checking and betting ranges, and she played a mix with As, Ks, Qs, and Js. Now, though, the option to check-raise gives her a particular interest in checking As rather than those other hands. So, she always checks As and always bets with Ks, Qs, and Js. She balances those value bets with a mix of 6s and 5s in order to make Ivan indifferent to calling with the {T,9,8,7} portion of his range. If raised, Opal folds unless she has an A.

It's quite common to encounter real poker players who are as passive as Ivan when it comes to raising, especially on the turn and river. Against these players, the best strategy is typically not to worry about "pot control" or opening yourself up to bluffs. Instead, you should bet whenever you believe you are likely to be ahead and simply fold if your opponent raises.

2. Suppose that Opal would only value bet with a K or better. Other than that, we know nothing about how she will play. How should Ivan adapt his equilibrium strategy to maximally exploit this mistake?

Envision the equilibrium: At equilibrium, Opal bets Qs at a frequency that makes Ivan indifferent between calling, folding, and raising with hands between J and 7. This also makes him indifferent between calling and raising with a K.

Make a read: Opal's betting range does not contain enough thin value bets.

Identify the exploits: A lack of thin value bets isn't quite the same as being either too strong or too weak; it's a new kind of mistake that hasn't been relevant until now. Opal's thin value bets gave Ivan incentive to raise a polarized raising range. When Opal's range contains fewer thin value bets, then Ivan will do less raising.

Determine the degree of deviation: Opal's mistake is small. She's just playing a pure strategy where she ought to be mixed. Consequently, Ivan's deviation will also be small. To some degree, he'll benefit even without adapting, just because he'll get to check and show down some hands that would have faced difficult decisions had Opal bet her wider, equilibrium range.

At equilibrium, Ivan is indifferent to raising a K for value. If there are no Qs in Opal's range, then Ivan has no incentive to raise his Ks, so they become pure calls. Facing a bet, Ivan's Qs are no longer any stronger than his 7s, and he sometimes folds them and sometimes raises them as a bluff. Even though he has a new bluffing candidate, his overall raising frequency is lower. Instead of raising all his As and some Ks for value and balancing with 1/3 as many bluffs, he now raises only the As and has proportionately fewer bluffs.

Although the EV gain for Ivan isn't large, this is an important situation to discuss because Opal's mistake is a common one. Many poker players believe that they shouldn't value bet any hand that doesn't want to face a raise, but this is incorrect. In order to make an opponent indifferent to raising as a bluff, your betting range must contain some hands that will profit from a raise and some that will suffer. If every single hand in your betting range is either so strong that it's happy to call or so weak that it's a trivial fold, then your opponent has little incentive to raise as a bluff. This is what we saw when we tried to add raising to the Reciprocal Ranges Game.

To be clear, getting raised after betting with a Q *is* really bad for Opal. What was previously a +EV hand gets turned into a $0 EV hand. This shouldn't deter her from betting, though, because she won't always face a raise. A raise is the worst case scenario, but getting called is still good for Opal when she bets a Q, and that offsets the risk of a raise.

Checking Qs doesn't even spare Opal from difficult decisions when raised. All it does is shift the decision onto her Ks, which are now her thinnest value bets and which become indifferent between calling and folding against Ivan's new, maximally exploitative strategy.

If she really wanted to spare herself difficult decisions, Opal could also stop betting her Ks and bet only an extremely polarized range of As and bluffs. By doing so, however, she would miss out on the opportunity to pressure many medium strength hands in Ivan's range. Hands that would otherwise fold to bluffs or pay off value bets would instead get to check and realize their equity at showdown. All this fearful checking would decrease Opal's EV relative to her equilibrium strategy, which requires her to bet and then find a balanced calling range if Ivan raises.

3. Suppose that Ivan has observed Opal betting some Js and believes that Opal will continue to employ the same strategy despite having shown these down. How should Ivan adapt his strategy to exploit the likely mistake here?

Envision the equilibrium: Opal's equilibrium strategy is to mix bets and checks with a Q and always to check a J. Her overall mix of bets makes Ivan indifferent between raising and calling with a K.

Make a read: Opal is value betting too thinly. We don't know whether she's balancing these bets with additional bluffs.

Identify the exploit: When an opponent is value betting too thinly, the optimal exploit isn't necessarily to value-raise more *or* to bluff-raise more; without any insight into how she'll respond to a raise, the optimal exploit is to do *both* more often. Ivan should still raise a balanced and polarized range, but it should be wider than at equilibrium.

Determine the degree of deviation: Opal's mistake isn't huge, but her deviation is more than just playing a pure strategy with a hand that's indifferent at equilibrium. She's sometimes betting a hand that's strictly more profitable as a check. This means that she's going to give up some EV even to an equilibrium strategy. Ivan's equilibrium raising strategy ends up pushing Opal off a lot of equity, because when raised she'll too often be stuck holding a hand with no good options. Ivan can really put the screws to her by widening his raising range.

Ivan is indifferent to raising a K at equilibrium, but against Opal's wider betting range, he strictly prefers raising with Ks plus an appropriate number of additional bluffs. Unlike at equilibrium, Js are not appealing as bluffs because a J is one of the hands that Ivan would like Opal to hold when he raises. Js become more appealing as bluff-catchers, though, because although they are indifferent at equilibrium, they now block some of Opal's value range. This may not be a reason to call 100% with Js, but it is a reason to prefer them over the other hands in the {J,T,9,8,7} portion of his range that need to call sometimes to make Opal indifferent to bluffing.

Raising a K by itself is not necessarily +EV for Ivan. A polarized raising range that includes all Ks and an appropriate number of bluffs, however, is definitely +EV against an opponent who is value betting too thinly. Opal could choose to call a raise only when she has an A, which would make raising -EV with a K. The EV gained by Ivan's bluffs if Opal were to choose this strategy, however, would more than make up for the value lost by raising the Ks.

This is the beauty of a polarized range: you don't have to know how your opponent will respond to a raise to craft an exploitative strategy. Once she bets an overly wide range, she's already made a mistake. Something must happen to all those overly weak hands that she bet. If she calls with them, your value raises profit. If she folds them, your bluffs profit. If you know that she'll respond by calling or folding too much, then that's an additional opportunity for exploitation, but you don't need to know that to capitalize on the error of betting an overly wide range.

4. Suppose that Ivan will only call a raise with an A. Other than that, we have no insight into how he will play. How should Opal adapt her strategy to exploit this mistake?

Envision the equilibrium: Facing a check, Ivan's equilibrium strategy is to bet all his Qs, Ks, As, and 5s along with some of his 6s and Js. He always calls a raise with an A and is indifferent with Qs, Ks, and Js. Collectively, the Qs, Ks, and Js account for half of his calling range. This makes Opal indifferent to raising as a bluff with Ts, 9s, 8s, and 7s.

Make a read: This is tricky. We know that Ivan is making one of two mistakes, but we don’t know which one. Either he folds too much after betting, or he isn't betting nearly as much as he should be. If he’s betting his equilibrium range, then he’s making a folding mistake, because he needs to call with more than just As to make Opal indifferent to bluffing. However, if uses an overly narrow betting range consisting only of As, a few Ks, and some bluffs, then calling only with As would be enough make Opal indifferent to bluffing.

Identify the exploit: Either of these strategies is exploitable, but they aren’t exploitable in the same way. Unfortunately, we don’t have any insight into which exploitable strategy Ivan is employing, so we’ll have to find a way for Opal to split the difference. She has to be careful that anything she does to exploit one of Ivan’s possible strategies wouldn’t play right into the hands of the other.

If Ivan plays the first strategy of betting his equilibrium range but folding too often to raises, then Opal would want to do more check-raise bluffing, but she wouldn’t want to check her strong hands, because they would never get raises paid off.

If Ivan plays the second strategy of betting an excessively narrow range but calling raises appropriately, then Opal would again not want to check any of her strongest hands, because she would so rarely induce bets. That settles the question of betting strong hands: it’s the best play against either of Ivan’s possible strategies, and she should do it 100% of the time.

Check-raise bluffing is more complicated, because depending on Ivan’s betting strategy, these bluffs may be +EV, breakeven, or even -EV. She’ll still do some check-raise bluffing, but she must be careful not to do so much that she rewards Ivan for betting a narrow range.

Determine the degree of deviation: Opal can't go too crazy with check-raise bluffing, because it's possible that Ivan has a balanced calling range. For that matter, it's possible that Ivan will only bet when he has an A, in which case bluff-raising would be positively lighting money on fire.

These strategies have other consequences for Ivan, though, so even though they would "punish" Opal for check-raising bluffs, they wouldn't be +EV adaptations for him unless she goes overboard with her bluffing. A modest increase in check-raise bluffing is safe for Opal, but she must ensure that if Ivan's strategy were to bet only the nuts, he would give up as much value to other parts of her range as he would gain from her check-raise bluffs.

The maximally exploitative strategy for Opal is actually quite straightforward: she bets all her As, Ks, Qs, and 5s, and checks everything else. That's a balanced, polarized betting range and a condensed checking range that would be easily exploited by an opponent betting aggressively with a polarized range. That's OK, though, because if that's what Ivan is doing, he's going to get punished for it when Opal check-raises. If Ivan isn't going to call a raise unless he has an A, he must either pass up the opportunity to exploit Opal's condensed range or open himself up to exploitation by betting with way too many hands that will fold to a raise.

After checking and facing a bet, Opal raises with about 17% of her checking range, any mix of hands in the {J,T,9,8,7} region. This is a strange-looking raising range, because it isn't polarized. Opal knows that Ivan will fold thin value bets such as Ks or Qs, so she doesn't want to have value hands in her check-raising range. They never get paid off, and she may get bluffs through without them. She also wants to avoid check-raise bluffing with 5s and 6s because they block Ivan's folding range.

Essentially, Opal’s maximally exploitative strategy makes Ivan indifferent between two different strategies, both of which are inferior to his equilibrium strategy. One possibility is that after Opal checks, Ivan bets a range that contains all his As and some thinner value bets which will incorrectly fold to a check-raising range that contains nothing but bluffs.

The other possibility is that Ivan doesn't make the thin value bets in the first place, in which case Opal loses money on her bluffs. However, she gets to show down for free with many marginal hands that would have either folded to a bluff or paid off a value bet against a more aggressive betting strategy.

If Opal check-raises too often as a bluff, Ivan would not be indifferent between these two options. He would strictly prefer the one where he rarely bets but picks off a lot of check-raises when he has the nuts. If Opal doesn't do enough check-raise bluffing, then Ivan would profit by betting the wider, polarized range. Even though Opal can't predict which of these strategies Ivan will pursue, she can craft an exploitative strategy that capitalizes on the observed error – never calling a raise without the nuts – regardless of how he compensates for it.

This is a tricky exploit, because it isn't as simple as always or never check-raising with bluffs. Without more information about Ivan's strategy, Opal is limited in how much she can exploit a single mistake, but she *can* still exploit it.

Test Yourself

1. You open to $15 with A♠ 8♠ in first position, and the big blind calls. The flop comes T♥ T♠ 2♦. The big blind checks. What's your play?

Against a tough opponent, this is a check. This is a medium-strength hand, and you shouldn't expect to be ahead if your opponent calls a bet. Though you have some interest in betting to deny her a free card, you must consider the risk of a raise from a polarized range.

Even though your range should flop more equity than the big blind's, she is more likely than you to hold a T. Getting check-raised by trips is not the end of the world; you simply fold and lose your bet. The real danger is that she can also check-raise with bluffs, and folding to those will cost you not just your bet but the entire pot.

As the in-position player, you have the opportunity to control the pot, and this is a spot where you want to do that. There is even a case for checking stronger hands such as pocket pairs.

Against weaker players who will not make good use of the opportunity to check-raise you, you should bet the flop.

2. You check behind and get an A♥ turn. The big blind checks. What's your play?

Believe it or not, this is still a close decision. Your hand is a lot stronger than it was on the flop, and now there are some draws for you to worry about as well.

However, you still don't want to play too large of a pot. As a first position raiser, your range is quite strong on this board. You don't have many bluffing candidates, as your range consists mostly of pocket pairs or hands that paired the Ace. That means your opponent doesn't have much incentive to pay off a bet with hands weaker than yours.

Still, if raising were not allowed, you could get away with value betting. And against your typical passive, straightforward opponent, you probably should bet. Squeezing extra value from your good-but-not-great hands is one of the main ways you exploit such players.

3. You check behind. The river is the 6♥, making the final board T♥ T♠ 2♦ A♥ 6♥. The big blind checks. What's your play?

Now it's time to bet.

Even though there's a new threat out there in the form of a possible flush, your opponent has shown nothing but weakness. The risk of a raise constrains you to some degree, but this is a profitable bet nonetheless. As the in-position player, you can value bet more thinly with the knowledge that your opponent had a lot of incentive to bet her strongest holdings earlier in the hand. You can't go too big, but you can eke out some value here.

If the big blind check-raises, you'll be indifferent between calling and folding, as is typically the case when a bluff-catcher faces a bet from a polarized range. That's a bad outcome, but you must keep in mind that it's a worse-case scenario. Sometimes your opponent will just call and you'll win an extra bet. That's the reward that compensates you for the risk of betting.

4. You open to $15 with A♠ K♦ in middle position, and the button calls. Your read is that he is loose and passive both before and after the flop. The flop comes A♥ 9♥ 6♦. You bet $20, and he calls. The turn is the 9♠, and you both check. The river is the 6♥. What’s your play?

Bet. You can probably even get away with a large bet, close to the size of the pot.

Many players instinctively check because they recognize their hand to be medium-strength and they are afraid of paying off a strong hand or getting raised by bluffs. These concerns are understandable, but as the out-of-position player, you don’t have the luxury of checking to guarantee a free showdown. If you check, your opponent will probably bet all the hands that beat you, and you’d be hard-pressed to fold a hand this strong.

This means you’ll likely pay off your opponent’s stronger hands whether you bet or check. If you’re going to put a bet in the pot one way or the other, then the question becomes whether it is more profitable for you to make that bet yourself or to check and give your opponent the option to bet when *he* wants to play a larger pot.

The advantage of betting would be to get value from your opponent’s medium-strength hands, specifically Aces with a worse kicker. The advantage of checking would be to get value from your opponent’s weakest hands, which might bluff.

At equilibrium, you might be indifferent, but when you have reads, you should use them. Your read is that your opponent will call too much and bet too rarely, so you’re better off betting to induce calls than checking to induce bluffs.

A passive opponent is also unlikely to raise you as a bluff, which is the main risk associated with thin value bets. Against this player, you should bet planning to fold if raised, confident that you will rarely be losing the pot to a bluff.

Conclusion

As we've seen in other games, medium-strength hands are the ones that suffer when facing pressure from a polarized range. In this case, that pressure comes in the form of a raise rather than a bet, but the principle is the same.

The best way to account for that is to anticipate it. Against an opponent capable of raising a balanced, polarized range, you simply cannot bet as thinly for value, and as a result you must bluff less as well.

It's also important that you exercise your option to raise a polarized range in order to punish opponents who bet too thinly. Especially when out of position, this requires careful planning and range construction in order to distinguish between hands that play best as value bets and those that play best as check-raises.

Key Lessons

- ♠ **The more polarized your opponent's betting range, the less aggressively you should raise.** Polarized ranges contain few hands that will face difficult decisions when raised, and only your very best hands will be strong enough to raise for value.
- ♠ **Raising is the way to "punish" thin value bets.** Many players respond to aggressive opponents by checking and calling, but that works only if your opponent's aggression takes the form of excessive bluffing. If he bets a wide but balanced range, then checking and calling plays right into his hands. Raising, both with bluffs and thin value bets, is the way to exploit these opponents.
- ♠ **You make more money dealing with difficult decisions than running from them.** Optimal play sometimes requires putting yourself in situations where your opponents can pressure your marginal hands. You lose value when this happens. If handled correctly, however, these spots cost you less than what you gain from getting into them. In other words, there is value in betting hands that aren't quite the nuts, even though you will face a difficult decision if raised.
- ♠ **When you need to balance a checking range, your strongest hands make the best traps**. When out of position, you frequently will need strong hands in both your betting and checking ranges. Your strongest hands tend to be best for checking because they can benefit from check-raising. Second-tier hands that aren't strong enough to check-raise, such as a K in the Ace-to-Five game, are better for betting.

Caveats

- ♠ **The threat of a re-raise constrains raising.** Our toy game did not permit re-raising, but when it's a possibility, it constrains the raiser in much the same way that the threat of a raise constrains the better.
- ♠ **On earlier streets, board coverage matters.** Because hands can change value before the river, you often have incentive to raise a somewhat less polarized range that includes a variety of draws and made hands. This ensures that your range will not be excessively strong or weak no matter how the board runs out.

Chapter 8: Putting It All Together

Overview & Objectives

The Ace-to-Five Game has prepared us to make the jump from toy games to real poker situations. If you understand what each player's strategy in that game looks like and why, then you're well on your way to thinking about equilibria in real poker situations. The only concepts that our toy games haven't fully addressed are *card removal* and *blockers*.

In a real poker game, the cards in your hand provide important information about what cards are not in your opponents' hands. We saw this to some degree in our toy games, but it's not usually an all-or-nothing issue in hold 'em, and the fact that you have two cards instead of one complicates matters considerably.

We'll take a moment to consider the strategic implications of card removal, then we'll dive into a real poker scenario. You'll have the opportunity to apply everything you've learned so far about equilibria, polarized versus condensed ranges, bluffing, raising, and more.

By the end of this chapter, you should be able to:

- ♠ Appropriately consider blockers when choosing bluffing, value betting, and bluff-catching candidates.
- ♠ Construct appropriate betting, calling, and raising ranges for bets of various sizes.
- ♠ Understand how checking makes hands stronger.
- ♠ Determine whether a particular bluffing candidate is better for betting or check-raising.

Blockers Tell You What Your Opponents Don't Have

A blocker is a card in your hand that provides information about what is *not* in your opponents' hands. This information is particularly valuable because your opponents don't know you have it.

Suppose that on the river, the board reads Q♠ J♦ 7♠ 6♦ 4♠ and you hold A♠ K♥. Everyone still in the pot should be aware that a flush draw came in, but you alone possess the proof that no one has the nuts. Lower flushes are still possible, but in most situations, players are far more likely to see the river with suited Aces than with other suited hands. You can't be certain that no one has a flush, but you know that the risk of running into a flush is greatly reduced, and you know that no one else knows this. Armed with this information, bluffing will likely be profitable.

Blockers are the be-all and end-all of no-limit strategy. Usually, they function as tiebreakers. When you know that you want to bluff or bluff-catch with just a few hands in a given situation, the hands you choose should often be defined by their blockers.

In the scenarios we've examined so far, bluffing has usually been $0 EV at equilibrium. The one exception was when Ivan bet a 5 in the Ace-to-Five Game. We'll see a similar pattern in hold 'em scenarios: players will typically be indifferent to bluffing the river at equilibrium, with two exceptions. The first is with their very weakest hands, as in the Ace-to-Five Game. The second is when they have important blockers.

Blockers can also influence value betting decisions. As with bluffing, players tend to be indifferent with their thinnest value bets - think of Ivan betting a T in the Ace-to-Five Game. If your hand blocks some of the better hands an opponent might hold, then betting is safer and more profitable.

That's a quick introduction to the concept. We'll see some good examples of blockers in action in the following scenario.

Scenario: After the Turn Checks Through

This scenario is a continuation of the one from the Get Real! Chapter, which began at a nine-handed $2/$5 no-limit hold 'em game with $500 effective stacks and no rake. The first player to act before the flop (Under-the-Gun, or UTG) raises to $15, and only the big blind (BB) calls. The flop comes A♠ 9♥ 6♠. BB checks, UTG bets half the pot, and BB calls. The turn is the 4♦, and both players check. The river is the J♦.

This is where we start our analysis. BB may check or bet 33%, 75%, or 200% of the pot. Facing a check, UTG may bet 75% or 200% of the pot. Raises of 50% of the pot are permitted.

Questions

Using everything you've learned about river strategy so far, do your best to answer the following questions about the equilibrium strategy for each player.

1. Which player would you expect to have more equity when they arrive at the river in this way? Why?

2. Which player would you expect to be better equipped to bet a polarized range? Why?

3. What are the weakest hands that BB will bet for value? Which hands should he use as bluffs to balance these bets?

4. What are the worst hands that BB calls with after checking?

5. What hands, if any, should BB check-raise for value? Which hands should she use as bluffs to balance these raises?

6. What does UTG's range for raising a bet look like?

7. What are the weakest hands that UTG will call a bet with?

8. What are the weakest hands that UTG will bet for value if BB checks? Which hands should he use as bluffs to balance these bets?

Answers & Explanation

Assume a nine-handed $2/$5 no-limit hold 'em game with $500 effective stacks and no rake. The first player to act before the flop (Under-the-Gun, or UTG) raises to $15, and only the big blind (BB) calls. The flop comes A♠ 9♥ 6♠. BB checks, UTG bets half the pot, and BB calls. The turn is the 4♦, and both players check. The river is the J♦.

This is where we start our analysis. BB may check or bet 33%, 75%, or 200% of the pot. Facing a check, UTG may bet 75% or 200% of the pot. Raises of 50% of the pot are permitted.

1. Which player would you expect to have more equity when they arrive at the river in this way? Why?

BB has more equity, by a lot. In my solver simulation, BB has about 61% equity to UTG's 39%. As with any simulation, the details are particular to assumptions about starting ranges and bet sizes, but the point remains: BB has the lion's share of the equity here.

If this surprises you, it may be because the flop heavily favors UTG, and nothing about the turn and river cards is particularly advantageous for BB. Board texture is a crucial factor in which player has the stronger range, but it's not the end of the story. The actions that each player takes provide valuable information about what kind of hand he's likely to have, and you can't afford to ignore that.

On the flop, we'd expect UTG to bet a polarized range and BB to call with a condensed range that includes some slowplays. This isn't just a matter of style or personal choice. There are significant incentives for each player to play this way, and barring some major exploitable error that a player expects his opponent to make, this is going to be the most profitable strategy for both.

That means that after the flop action, UTG's range contains mostly a mix of very strong and very weak hands, while BB's range contains mostly medium-strength hands. BB folds a lot of his weakest hands on the flop, and that goes a long way towards equalizing the equity advantage that UTG enjoyed on the flop.

On the turn, we should expect a similar dynamic, with UTG betting a polarized range and BB mostly checking and then calling or folding. UTG has little incentive to check strong hands, so when he does check, it's likely that his hand isn't that strong. He may have something with some showdown value, or he may just have a weak hand that he's declining to bluff again.

Essentially, UTG has most of the equity when both players see the flop, but a lot of that equity goes into his turn betting range. When he checks the turn, he weakens his range such that he is no longer favored to win on the river. That's not a mistake, it's a tradeoff worth making in exchange for the value of betting a polarized range on the turn.

2. Which player would you expect to be better equipped to bet a polarized range on the river? Why?

Both players will bet polarized ranges when given the opportunity, but BB's betting range will be wider. Remember: the player with the mostly condensed range keeps checking to the player with the polarized range until that player stops betting. Then, there's an opportunity for the previously passive player to bet a polarized range of her own. Once UTG declines to bet the turn, most strong hands should fall out of his range. UTG can have weak hands but not many strong ones, while BB can have strong hands but not many weak ones, because she had to have something to call the flop bet.

In order to bet a polarized range on the river, BB does need to have *some* weak hands. Indeed, the prospect of bluffing the river after the turn checks through is part of the appeal of BB's weakest flop calls, which could include hands like T♥ 8♥.

As for the strong hands in BB's polarized range, they are of three varieties. The first are hands that were strong to enough to check-raise on the flop but that BB just called as a trap. Good examples are A9, 99, and 66. Slowplaying some strong hands on the flop enables BB to bet a polarized range on the river even when the board runs out "dry", without completing any of her draws.

The second variety of hands are those that were not strong enough to raise on the flop but that improved on the turn or river. This includes hands like J9 and A4. There aren't a lot of such hands on this board, but if a draw-completing card such as a spade or a T had come, BB would have many more hands of this sort.

Finally, there are hands that were not strong enough to raise on the flop but that *are* strong enough to bet for value on the river, even though they haven't changed in rank. AT is good example of this. It isn't strong enough to play a huge pot, which is the road that check-raising the flop would put BB on. Check-raising the flop sets her up to bet the turn and river as well, ultimately putting four bets into the pot. Plenty of hands that aren't strong enough to build a pot that large will nevertheless be happy to put two bets in. They call the flop intending to call again on most turns or to bet the river for value if UTG checks the turn.

There's a similar dynamic for UTG's betting range if BB checks. He doesn't do much slowplaying on the turn, which means that on certain rivers that don't improve any part of his turn checking range, he won't have many strong hands and consequently won't do a lot of betting or raising. This particular river, however, improves AJ and JJ, both of which are hands that UTG often checks on the turn. As we'll see, these hands loom large in the strategic considerations of both players.

3. What are the weakest hands that BB will bet for value? Which hands should he use as bluffs to balance these bets?

This depends on the size of BB's bet. The smaller the bet, the wider and weaker the range with which UTG must call, and therefore the more thinly that BB can bet for value. BB needs at least a pair of Aces to bet for value, but the smaller the bet, the weaker the kicker can be. When BB bets 33% of the pot, a "blocking bet" size, she can include AT in her value range. When she bets 75% of the pot, AK is the weakest value hand in her range. When she bets twice the pot, not even J9 is strong enough - she needs AJ or better for value.

She balances these bets with bluffs drawn from the weakest hands in her range. As discussed above, it's quite difficult for her to have truly weak hands in this situation, because it's hard to justify calling the flop bet without a pair. BB's only unpaired hands are KQ of spades and T8 and 87 that missed straight draws. These hands always bet, but she needs more bluffs than these, so she also uses her weakest pairs, which are 65s and 76s.

4. What are the worst hands that BB calls with after checking?

This, too, depends on the size of the bet. Against a 75% pot bet, BB never folds an A or a J. She usually folds when she has only a pair of 9s, though she's indifferent with most of these.

Against a 200% pot bet, BB has few hands she's happy to call with. J9 and 99 always call, and AJ and JJ are mixed between calling and raising. With a pair of Aces, BB is indifferent between calling and folding regardless of her kicker, which is a hint that UTG should never have just one pair (not in his value range, anyway) when he makes this bet, or else BB's kicker would matter.

Perhaps surprisingly, BB never folds when she has a J in her hand. This is because the J blocks many of the hands that UTG would bet for value here, specifically AJ and JJ. BB sometimes calls and sometimes raises (as a bluff) when she has a J, but she never folds.

5. What hands, if any, should BB check-raise for value? Which hands should she use as bluffs to balance these raises?

BB check-raises AJ and JJ for value almost exclusively, regardless of UTG's bet size. The only exception is 99, which will raise the smaller bet but not the larger bet.

It's notable that if UTG overbets, BB can check-raise top two pair for value but not middle set. In our toy games, we never encountered a situation where a player raised with a weaker hand and called with a stronger one.

This, too, is about blockers. When BB check-raises either AJ or 99, she must worry about running into JJ. When she has a J in her hand, that risk is substantially reduced. Thus, it's more profitable for her to raise top two than to raise a set of 9s, especially against an overbet.

BB's check-raise bluffs always have a J in them. There are two reasons for this. The first is that with a J in her hand, she blocks AJ and JJ, as discussed above.

The second is that these are hands that have a chance of winning at showdown if BB checks behind. Remember how in the Ace-to-Five game, Opal check-raised Ts, 9s, 8s, and 7s as bluffs but not 6s and 5s? We're seeing something similar here. When BB has no showdown value at all, she prefers to bet as a bluff rather than check. The weakest hands that she checks have a chance of winning at showdown. If UTG bets, that's a hint to BB that perhaps her J is not good after all. With that new information, she sometimes chooses to turn her hand into a bluff by raising.

6. What does UTG's raising range look like?

A player should raise more liberally against a wider range, and BB's range ought to be wider when she bets smaller. Against a bet of 33% pot, UTG is indifferent between calling and raising with AK. Against a 75% pot bet, he needs AJ to raise for value. Against a bet of twice the pot, he also raises AJ... as a bluff! BB sometimes folds 66, 99, and AJ to this raise.

UTG's bluffs also change depending on the bet size. He has lots of hands that aren't strong enough to call a bet, so he doesn't have to choose solely on the absolute strength of the hand. Instead, he chooses based on which hands he wants to block. When he raises a small bet, it's good to block AK and AQ, so he raises hands like KQ, KJ, and even KK, which has no showdown value because it’s going to fold if it doesn't raise.

Against larger bets, it's best to block Js, so UTG’s bluffs are QJ, JT, and even AJ when faced with an overbet.

7. What are the weakest hands that UTG will call a bet with?

Against a 33% pot bet, he's indifferent to calling with TT and 77. Against 75% pot, he's indifferent with JT, A5, and AT. Against 200% pot, he's indifferent with AK, AQ, QJ, and JT.

If you look back at Question 3, you can see the interplay here. Against small bets, UTG must call (or raise) with a wider range to make BB indifferent to bluffing, and consequently BB can bet weaker hands for value.

8. What are the weakest hands that UTG will bet for value if BB checks? Which hands should he use as bluffs to balance these bets?

For a bet of 75% pot, UTG can include hands down to AQ in his value range. Note that this is a weaker hand than BB can bet for the same size, which is consistent with what we saw in the Ace-to-Five Game: the player in position can make thinner value bets than the out-of-position player. For 200% pot, he needs AJ or better.

UTG has a lot more showdown value against BB's checking range than he does against her betting range, and consequently he is more inclined to draw his bluffs from his very weakest holdings when faced with a check. This includes unpaired hands like QT, KT, and KQ, as well as his very weakest pocket pairs.

Real World Applications

Bet Sizing

There's a common misconception that betting different amounts with different hands is exploitable. Although it's true that many human players use different bet sizes exploitably - betting bigger with bluffs than with value hands, for instance - there's no theoretical reason why this must be the case. In fact, a true equilibrium solution for most no-limit river situations would involve many different bet sizes, each carefully balanced to make various hands in an opponent's range indifferent.

Before you conclude that a particular play is exploitable, ask yourself, "*How* could it be exploited?" You may find that by tweaking your ranges, you can actually make an opponent indifferent to the supposed exploit. Or, you may conclude that you are comfortable being exploitable in that way, because you do not expect your opponent to take advantage of the opportunity.

In the case of bet sizing, a player generally wishes to value bet larger with his very strongest hands than with his good-but-not great hands. Each of these value ranges could be balanced with bluffs, and it may even be the case that some hands will play better as large bluffs and some as small bluffs, usually depending on their blockers. How could this be exploited?

If a player's range for betting small contains exclusively bluffs and thin value bets, then his opponent could exploit him by check-raising a polarized range. He doesn't need a huge hand to check-raise for value, because he knows that the player would have bet bigger if he had a very strong hand.

If the opponent check-raises too much, however, then the bettor would have incentive to make the small bet with his strongest hands in order to induce the check-raise. In fact, the equilibrium solution likely involves betting big with most of his strongest hands but including a few in the small bet range in order to make his opponent indifferent to check-raising for thin value. The check-raises, in turn, make the bettor indifferent between betting big or small with very strong hands.

To summarize: the bigger your bet, the more polarized your range should be. The average hand that calls a big bet should be stronger than the average hand that calls a small bet, so you need a stronger hand to be ahead of a calling range. Smaller bets force your opponents to call with weaker hands (or lose to your bluffs), so you can bet weaker hands for value when you use a small size. The key is to *occasionally* bet small with hands that will be happy to call a check-raise as well, if you think your opponent is capable of check-raising exploitatively. If you think they aren't, then you can just go ahead and make your small, thin value bet and fold if raised (or call, if you think your opponent will bluff too much, but that's a less common mistake).

Bluffing on the River

You are probably accustomed to thinking about bluffing exploitatively. If you have reason to believe that your opponent's hand is weak (but, importantly, stronger than yours) and that he will fold it, then that is indeed a great reason to bluff. It's also true that bluffs succeed best when they "tell a story" – that is, when they credibly represent a strong hand.

The latter is not sufficient to make a bluff profitable, though, nor is the former necessary. You will often find yourself in situations where your hand is weak and you have little or no insight into whether your opponent will fold to a bet or raise. That's where game theory comes in handy.

As we've seen many times now, bluffing is a critical part of most equilibrium strategies. If you are in the habit of not bluffing when you don't have a read on a player, or in the early stages of a tournament, or when you're ahead for the session, or for any other reason, then you're playing exploitably. If your primary objective when playing poker is to make money, then bluffing is not optional, though there are some situations where, for exploitative reasons, it may be correct to never bluff.

Typically, though, in any situation where you have a value range, you should have bluffing range. The question to ask is not, "Is this a good spot to bluff?" but rather "*Which hands* will be best for bluffing in this situation?" There are many cases where, if you don't think specifically about what your bluffs should be, you'll never find them. As a result, you'll end up underbluffing, and savvy opponents who recognize situations where it's hard to find bluffs will correctly guess your strategy and exploit you by folding to your value bets.

Haphazard bluffing strategies are common on the river. When draws miss, there are lots of obvious bluffing candidates, and some people end up bluffing way too much. In other situations, they never bluff because there are no obvious bluffing candidates such as missed draws.

I find it useful to think about putting *bluffing candidates* - hands that are unlikely to win at showdown - into two "buckets", the bluffs and the no-bluffs. The size of your potential value range determines the relative sizes of the buckets. The more hands you could value bet, the more hands you get to put into the bluff bucket. This forces you to make an active decision about whether your hand is good for bluffing rather than just taking a wild guess about whether your opponent will fold, or worse, deciding based on frustration (bluffing if you’ve been losing and just want to win a pot) or complacency (not bluffing because you’re ahead on the session and don't want to risk "giving it back").

You probably know the old saying, "Play the situation, not your cards," but that's not a game theoretical approach. Even when you're bluffing, your cards matter. There are two factors that could make a hand a particularly good candidate for bluffing on the river: blockers and a lack of showdown value.

In our toy games, blockers didn't usually play a role, though they were the deciding factor for IP's bluff raises in the Ace-to-Five Game with Raising. Mostly, we saw players fill out their bluffing ranges from the bottom up, starting with their very weakest hands and then moving on to the next-weakest once those were exhausted.

In real poker situations, the two tend to work together. Often, you'll have more weak hands than you know what to do with, such that even if you only bluffed when you had no showdown value at all, you'd still end up bluffing too much. In those cases, blockers serve as tie-breakers. You need some way of choosing which hopeless hands to bluff and which to check, so you might as well choose based on blockers.

In large pots where ranges are narrow, blockers play a more significant role. In this most recent example, when the pot got large, BB mostly used hands with Js in them for his bluffs. UTG could call a check-raise all-in with only a few hands, and they mostly had a J in them. Thus, blocking a J dramatically reduced BB's risk of getting called.

Before you act on the river with a weak hand, whether you are about to bluff or to give up, take a moment to think about some other weak hands that you would play differently. In other words, if you're about to check, think about what you would bluff with. What other weak hands could you hold that would be better suited to bluffing than your current hand? If you can't come up with any, that's a sign that perhaps you should bluff with your current holding after all.

Your current hand may not belong in it, but before you give up and check, you ought to be able to identify either a bluffing range or a good exploitative reason not to have one.

Summary of Exploits

Understanding the incentives that drive the composition of ranges at equilibrium will help you to recognize opportunities to exploit imbalanced opponents. Some exploits are obvious; you probably don't need a book to tell you to call more often against opponents who bluff too much.

Others are harder to spot and require making adjustments higher up in the game tree. For instance, if your opponent will bluff too much, you may not want to bluff into him, as he may raise weak hands that an equilibrium strategy would fold.

The "balance" metaphor is quite apt. As we've seen, not all decisions are close at equilibrium, but many are. Equilibrium strategies really are very much like a delicately balanced scale; a small shift in weight one way or the other can cause dramatic changes on the other side.

Every deviation from an equilibrium strategy is an opportunity to increase your profit with exploitative play. It can be frustrating if you have a preferred style of play that does not work well against a particular opponent; you'll sometimes hear unsophisticated poker players complain that they can't win in small stakes games because their opponents don't fold often enough. Of course, an opponent who makes many mistakes should be easier to beat than an opponent who approximates an equilibrium strategy. If you want to make money at poker, your job is to find the appropriate exploits.

Remember that you can make small adjustments to an equilibrium strategy liberally, choosing pure strategies rather than mixed strategies even if your opponent is making only small mistakes or you have only minimal confidence in your read. Larger adjustments, such as purely calling with hands that would be pure folds at equilibrium, require stronger reads and bigger mistakes on your opponent's part.

Opponent Calls Too Many Weak Hands

This is the most common error in poker. Almost everyone calls too much. It's simply more fun than folding and less work than raising. So, it behooves you to put a lot of thought into how to take advantage of this error.

Many players get sloppy when faced with overly loose opponents. Because waiting for extremely strong hands is a profitable strategy, they settle for that. While it may be profitable, it is hardly the *most* profitable way to exploit a loose player. If you want to increase your win rate, you're going to have to try a little harder.

The role of calling in equilibrium strategies is to make an opponent indifferent to bluffing. Against opponents who call too much, you should not bluff with any hand that would be indifferent at equilibrium.

Do not misunderstand this to mean that you should not bluff at all! Some hands are so good for bluffing that they may be profitable even against loose opponents. This includes strong draws on early streets and the very weakest hands you could hold on the river, those that have the least chance of winning if you check. Loose opponents often hold weak hands such as missed draws on the river. If you can't even beat missed draws, you should seriously consider betting something just to make those hands fold.

For the same reasons that you bluff less, you should make more thin value bets. Any hand that's indifferent between value betting and checking at equilibrium will likely play best as a bet, unless your opponent also bets too much when checked to. For similar reasons, strong hands that are indifferent between value betting and check-raising at equilibrium will probably also play best as pure bets. Even some hands that would be too weak to value bet against a better opponent can bet into an overly loose player and make money when called.

Opponent Folds Too Often

This is a good example of a player type that many people find annoying, even though they are exploitable. Against nitty players, you simply aren't going to win as much with your strongest hands, and bluffing becomes a more important component of your strategy. Ironically, it is often nits who complain the loudest about other nits, because their own wait-for-the-nuts strategy does not perform well against others with a similar strategy.

Against these opponents, you should bluff more often but make fewer thin value bets. Hands that are indifferent between value betting and checking at equilibrium should be pure checks. In theory you would also have incentive to trap with your strongest hands, but it's common that an opponent who is reluctant to call is also reluctant to bet. So, you're probably better off betting your strongest hands anyway.

The real key to beating these players is looking for spots where they can't possibly have nutty hands. Because they are so tight, it's easier to put them on narrow, predictable ranges. When such a player raises in first position, for example, it's pretty much impossible for him to have trips on a 755 flop, and you should seriously consider representing them yourself.

Opponent Does Not Bluff Enough

The most obvious exploit is to fold all pure bluff-catchers, which would be indifferent between calling and folding at equilibrium. In extreme cases, also fold the weakest hands that would be pure calls at equilibrium.

That's easier said than done, so here's a good trick. When facing a bet from such a player, assume he will only bet if he is extremely confident in his hand. Then ask yourself whether he would feel that way about any hands weaker than yours. A common error is for the absolute strength of your hand to deceive you into thinking you are obliged to call. No such rule exists in poker, and folding top pair against players like this is frequently correct.

Make thin value bets with impunity, and simply fold if raised. You will not have to worry about losing the pot to a bluff, as would sometimes happen at equilibrium, so these bets have more value. All hands that would be mixed between bluffing and checking at equilibrium should be profitable bluffs.

Bluff-raising these opponents is usually a bad idea. If they're betting, it's a reliable indicator that they have a lot of confidence in their hand and probably don't plan to fold it.

Opponent Bluffs Too Much

The primary role of bluffs in an equilibrium strategy is to make an opponent indifferent to calling with pure bluff-catchers. Against an opponent who bets too many weak hands, you should always call with hands that would be indifferent between calling and folding at equilibrium rather than playing a mixed strategy. In extreme cases, you should also call with some of the better hands that would fold at equilibrium.

A secondary reason to bluff at the right frequency is to avoid making raises too profitable for the opponent. A range that's heavy on bluffs will struggle to defend against a raise, because the bluffs typically have to fold (though they could re-raise, which is a potential liability for the exploits discussed here). An opponent with a bluff-heavy betting range will either fold so often that bluffs show a profit or call with more of their marginal betting hands, in which case you can raise wider for value.

Against the former strategy, raise with any hand that's indifferent between raising and folding at equilibrium. In extreme cases, also raise with some hands that would be pure folds at equilibrium, particularly if they block your opponent's value range but not his bluffing range. Your bluff raises should be very small, probably just minimum raises. The profit comes from catching your opponent in a bluff, and if he is bluffing, the price that he's getting on a call won't matter. Hands that are indifferent between calling and raising as a bluff at equilibrium will play best as pure calls.

If you expect your opponent to defend by widening his calling range, then any hand that's mixed between calling and raising for value at equilibrium will play best as a raise. Note that although you won't be able to do an exploitative amount of bluffing against this strategy, there's no reason not to bluff with an equilibrium strategy. Even against the widened calling range, these bluffs won't lose money, because your opponent's folding range is also wider than it would be at equilibrium. If it turns out you're wrong about your opponent's strategy, then these bluffs will make money.

If you cannot predict how this opponent will respond to a raise, then you should widen both your bluffing and thin value raising ranges. Even if one of these adjustments ends up being a mistake, the other will more than make up for it. Remember: the way to attack marginal hands is with a polarized range. The more marginal hands in your opponent’s range, the wider your polarized range should be.

Opponent is Excessively Trappy

At equilibrium, very strong hands sometimes check in order to pick off bluffs and thin value bets. If your opponent checks strong hands too often, then you should do less bluffing and less thin value betting. The good news is that you'll save money when you check back with marginal hands that would have called had your opponent bet.

If such an opponent does bet, you can make more thin value raises without fear of running into the nuts. You may also be able to do more bluffing, depending on how well your opponent defends his weak betting range. The exploits here are quite similar to those employed against the opponent who bets too many bluffs.

On early streets, you should be especially cautious about betting draws when you are in position against this player. While such hands are usually very profitable bluffs, they hate getting check-raised, and you should be more inclined to check and take free cards when a trappy opponent gives you the opportunity. When out of position, checking for a free card isn't really an option, and an opponent who loves trapping may be less inclined to raise. Thus, you should still generally bet strong draws when out of position.

Opponent Doesn't Trap Enough

Some people just can't bring themselves to check a strong hand, especially on the river; they're too scared of losing value or getting drawn out on. Against these players, you can make thin value bets with impunity after they've checked, and probably some big bluffs as well. Your strategy here will resemble that of the player with the polarized range in the Clairvoyance Game. You may be able to exploit your opponent's calling and folding tendencies as well, but even if you can't, overbetting with a balanced, polarized range after he checks will show a nice profit.

The flip side of this is that when your opponent does bet, he is too weighted towards strong hands. You'll rarely raise, either for value or as a bluff, and hands that were indifferent between calling and folding at equilibrium may prefer to fold. It's possible for an opponent to balance more nut hands with more bluffs, but it's unlikely that he'll end up heavy on bluffs, so even if you fold "too much", it won't be a mistake. If your opponent's betting range is balanced, then it doesn't matter what you do with hands that are indifferent between calling and folding; only when he's weighted towards bluffs does folding at higher than optimal frequency cost you money.

Conclusion

If you take just one thing from this book, let it be the ability to think differently, to think in terms of equilibrium instead of jumping immediately to assumptions about how your opponent is likely to play. Even when you wish to play exploitatively, as you often should, it behooves you to consider what an equilibrium strategy might look like. Then, you can make more informed decisions about the magnitude of your opponents' errors and your options for exploiting them.

Your study of game theory will be an ongoing journey. I learned new things while writing this book, I've learned more new things since, and I know there is still much that I do not know.

You can choose to be intimidated by that, or you can find inspiration in it. When I see a complicated solver output full of precisely mixed strategies, I see opportunity: opportunity to learn new things, opportunity to understand things that my opponents do not, and even opportunities for creativity.

Yes, contrary to popular belief, solvers and game theory are not the enemies of creativity. Computers may one day ruin our beautiful game, but they haven't done so yet.

When a solver tells me to do one thing X% of the time and another Y%, I'm not concerned with what X and Y are. My goal is to understand the *why*. Just as we saw in our toy games, the purpose of this mix is to make your opponent indifferent between two or more strategies. Your job is to learn what those strategies are and, if possible, to predict which your opponent will prefer. Then, instead of mixing, you get to craft a strategy to exploit this specific mistake.

That's where the creativity comes in. That's the human element. Sizing up your opponent, spotting his weaknesses, and planning your attack has always been the beauty of poker. Game theory isn't the end of that battle, it's just the latest weapon in the arms race.

Acknowledgements

The first and biggest thanks go to Emily Abell, who not only gave me the time and space to write this but who designed the cover with the help of stock resources by kjpargeter and writerfantast.

Nate Meyvis, who co-hosts the Thinking Poker Podcast with me, is my best poker buddy. He's a big part of the reason I still find the game fun and interesting, and he's shaped the way I think and talk about it more than anyone else.

The other people who continually rekindle my passion for poker are Carlos Welch and Tommy Angelo. They are the two happiest people I know, and I'm so glad that we were able to parlay their guest appearances on my podcast into genuine friendships. They've contributed to this book in both tangible and intangible ways.

I'm grateful to the others who discussed the manuscript with me, read drafts, and offered valuable feedback: Brenna Warren, Breyer Calvert, Eric Keppler, Eric Liebeler, Graham Stuart, Kelley Slay, Josh Nixon, Sean Lango, and Steve Leahy.

The most daunting part of this process was wrapping my head around the publishing process, and Alex Fitzgerald, Barry Carter, Douglas Stewart, Ed Miller, and Zach Elwood were all very helpful with that.

It's impossible to thank by name every person who contributed to this book. I'm grateful to everyone who's ever talked poker with me, to the authors of all the books I've read and the producers of all the strategy videos I've watched, to my students, and to all the guests we've had on the Thinking Poker Podcast.

A lot of professionals complain that their families don't understand or approve of poker, but I've never known anything but support from mine. This is one of the smaller things for which I'm grateful to my parents.

My father, brother, grandmother, and grandfather were my original home game, and now I have great fun playing poker with my cousins Henry, Oliver, and Walter. My Uncle Tubby was my first backer, at a Florida dog track when I was about five years old. That doesn't make him sound like a very good uncle, but he was the best.

About the Author

Andrew Brokos's professional poker career began in 2004, when he graduated from the University of Chicago with a B.A. in philosophy. He initially played small-stakes online games to make ends meet while searching for jobs at education-oriented non-profit organizations but quickly saw that taking poker seriously could produce enough income to start his own non-profit.

Later that year, Andrew founded the Boston Debate League and served as volunteer executive director until 2008, when the organization was large enough to hire a full-time director.

Since then, Andrew has dedicated himself full-time to teaching and playing poker. He currently makes instructional videos for Tournament Poker Edge and Red Chip Poker and writes a monthly column for Two Plus Two Magazine.

In 2012, Andrew Brokos and Nate Meyvis launched the Thinking Poker Podcast, a weekly show that quickly became one of the most popular poker podcasts on the internet. You can find the podcast archives and more information about private coaching at www.thinkingpoker.net.