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T3 ________________	Problem Chosen	F3 ________________
T4 ________________	B	F4 ________________

2017

MCM/ICM

Summary Sheet

How to complete a “half-formed trapezoid”

The world is witnessing sharp increase of vehicles and naissance of driverless cars. A reasonable design of a toll plaza could ease heavy traffic pressure on highways and modeling is especially crucial to tackle this problem in the process of determining the fan-in area of a toll plaza. We hold the opinion that the basic shape of merging section is a ‘’half-formed trapezoid’’ and our job is to fix the length of baselines as well as the shape of legs and complete the trapezoid. Therefore, our research is divided into four steps.

In the first place, we fix the upper line and baseline of this “trapezoid” by coming up with the optimal number of tollbooths. We break this model into two stages. In stage One, vehicles enter into the toll plaza area. The total traffic stream is evenly fanned out into all tollbooths. We can regard each tollbooth as an independent M/M/1 queue model. In Stage Two, vehicles leave from tollbooth egress lanes to the smaller number of regular travel lanes. Considering high traffic density in the emerging area, we compare traffic flow to a kind of fluid and establish our objective function of delay time on base of hydromechanics theory. We choose minimal average delay time as the objective function to decide the number of tollbooth lane. When the average total delay time is minimized, the number of toll booths is optimized, which means that we can fix our baselines.

Secondly, we focus on the “trapezoid legs”. The merging mode and the boundary of transition area are specifically determined in this part. When we design merging mode, we consider three factors: the speed difference between ETC lanes and the MTC and STC, The speed difference between outside and inside, the proportion of ETC lanes. When we design boundary of transition area, there are mainly three factors that should be taken into consideration: accident prevention, throughput and cost. Consequently, we need to separately establish three objective functions. For accident prevention, to keep the safety distance between two moving vehicles, we build up the objective function of the coefficient concerning about the angle. For throughput, We still choose minimal average delay time as the objective function. For cost, by calculating the geometric area, we can identify the cost of merging area. After establishing three functions to quantify three goals we want to achieve, we use a three-dimensional GE-Matrix to determine the optimal coefficient and then we can determine the accurate boundary length. Now, we have already complete our ‘’trapezoid’’.

After that, we make some further discussions and effects from external factors are considered in our model. When the traffic flow is not as dense as we assume in our model, the applicability of our model reduces because traffic flow fits fluid mechanics only when it is heavy enough; Besides, when there are more driverless cars on road, which guarantees high safety of traffic, the bottom angle of our trapezoid could be diminished, and our model can reduce the number of MTC and STC;

Finally, we analyze the sensitivity of our model by disturbing the variables in our model and its result demonstrates that our model can undergo disturbance in a certain extent. Besides, we conclude the strength and weakness of our model. We also contrast our results with the actual statistics and validate the feasibility and rationality of our models

CONTENTS

Introduction 1

1.1 Background and Restatement of the Problem

The collection of highway tolls and the construction of tollbooths are crucial problems, involving with fiscal policies and public goods. During a long period of time, a majority of nations and regions adopt the linear design to build the toll stations, as it is shown in figure1. In recent decades, however, with the continuous development of technology and productivity, the pressure imposed on the toll collection is increasing. Especially after the emergence of self-driving cars, the demand of new design of toll plazas for an unhindered flow of traffic , including the appropriate fan-in area after the toll barrier, is rather looming.

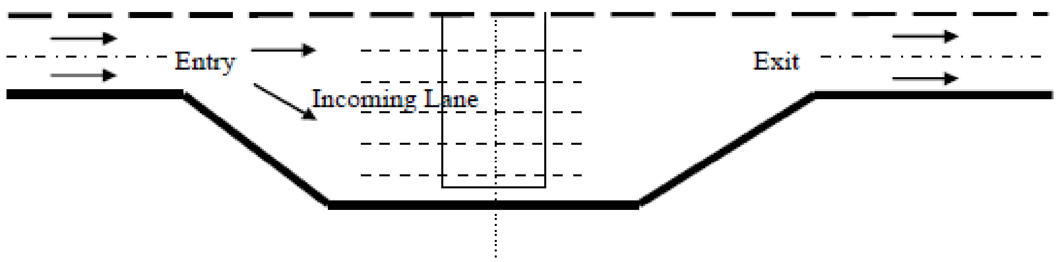

Figure 1 the common linear design

To create a fan-in area and toll plaza adapting the present situation， the following tasks are to be accomplished:

- Build a model that can determine the shape, size, and merging pattern of the fan-in area and toll plazas .By considering various factors like accident prevention, throughput and cost, the model should find a better solution extinguished from the ones in common use.
- Determine the performance of our solution in light and heavy traffic.
- Discuss how our model can be applied with more autonomous vehicles adding into the traffic mix.
- Analyze the influence of proportions of different types of tollbooths .
- Write our proposals to the New Jersey Turnpike Authority for modifying the toll plaza.

1.2 Previous Research

According to existent materials, various sets of models have been created by different countries organizations, as well as researchers.

Under the assumption of linear design of toll island, Suli stress the importance of mobile contingency charging tollbooths in keeping the fluent flow of traffic, but she doesn’t analyze if that is an economical option [1].Other studies also consider wider perspectives. For example, Albert E.Schaufle focus on the cost of operation and maintenance of toll plazas and include how to determine the shape of toll collection lane in his book[2]. A delay time model is established with hydromechanics theory and queuing theory by WuChunlei in 2008[3] .In his model, traffic flow is compared to fluid, making use of the relationship between traffic volume, traffic density and traffic speed to discuss how the traffic disperses and gathers at the transition area. Nico M.van Mark D.Hermans studies how to determine the size of toll plazas, toll systems and the number of tollbooths by using Queuing Theory [4].

1.3 Our Work

Considering the existent researches, our researches focus on two aspects, which firstly are optimizing the shape and size of fan-in area by taking factors like the merging mode, the cost of construction as well as the safety of drivers and then considering other external effects. Detail work is as follow:

1) We create a model to fix the baselines of the our merging area .Considering the two stage s including vehicles entering the waiting area and leaving the merging area after payment, we determine the width of baselines by determing the number of tollbooths first.
2) We also design a new merging methods after counting various factors.
3) We conduct a way to decide the specific boundary length in our new merging area.
4) Further work in our research focuses on effects from exogenous influence of our solution.

General Assumptions 2

- The width of tollbooth egress lanes and lanes of traffic are already given by High Way Capacity Manual complied by National Research Council [5]
- The traffic flow is huge
- It is a three-lane highway

Model for building a reasonable fan-in area 3

According to the existent design of toll plaza and merging area, we hold the opinion that the basic shape of merging section should be a ‘’half-formed trapezoid’’. Our job is to adjust the baselines and the legs and complete the trapezoid.

Therefore, in this section, we decompose our model into two submodels. In the first place, we fix the upper line and baseline of this “trapezoid” by coming up with the optimal number of tollbooths. Next, we focus on the “trapezoid legs”. The shape of transition area and the methods for vehicles to fan in are specifically determined in this part. Finally, after the two steps of modeling, we are supposed to complete our “trapezoid” transition zone.

3.1 Model I： Draw the baseline

In order to obtain the length of the two parallel baseline, the optimal number of tollbooth lanes under the given traffic lanes shall be decided, which we can use to calculate the baseline figure by referring to the next. We choose minimal average delay time as the objective function to decide the number of tollbooth lane. When the average total delay time is minimized, the number of toll booths is optimized. We get our answer with the aid of queuing theory and hydromechanics theory. Due to the symmetrical distribution of highway and toll plaza area, only a one-way toll plaza need to be built.

3.1.1 Notations

Table 1 Symbol Table–Variables

variable	Definition	unit
T_w	Average time to wait for the vacancy of the tollbooth when it is occupied.	h
T_u	Average time to leave transition area per vehicle	h
T	Average time to pass the toll plaza in total per vehicle	h
q_0	the total number of vehicles getting into the toll plaza per hour	veh/h
q_1	the total number of vehicles getting out of the toll plaza per hour	veh /h
c	The number of tollbooth lane	
λ	the average number of vehicles arriving at the toll plaza per hour	veh/h
μ	Average served vehicle number per hour per tollbooth lane	veh/h
W_s	Average serving time	h
W_q	Average queuing time	h
x	How far vehicles drive away from toll station	km
v_0	Average speed of vehicles driving into toll plaza	km/h
v_1	Average speed of vehicles driving away from toolbooth	km/h
$v(x)$	Average speed at position x	km/h

k_1	The traffic density when vehicles leaving tollbooth	veh/km
$k(x)$	The traffic density function on x	veh/km
w_1	The width of merging area in the side of entering this area	km
w_2	The width of merging area in the direction of leaving this area	km
$w(x)$	The width in position x in the merging area	km
l_n	The length of transition area）	km
d	Width of utilateral traffic lane	km

3.1.2 Stage One: Vehicles Pass the Tollbooth

Assumptions of Stage One

- The flow is infinite; the arrival of vehicle is separate ,among which there is no correlation;
- Vehicles arrive with gaps determined by Poisson distribution; the process is smooth;
- Considering the stability of the model, we assume $\lambda>\mu$ in general;

Establish the Objective Function of Stage One

Under our assumptions, when vehicles entering into the toll plaza area, the total traffic stream q_0 coming from the entrance of the toll plaza are evenly fanned out into all tollbooths, as it is shown in figure 2 and each tollbooth will receive a stream whose inter arrival time obeys exponential distribution. Furthermore, the service time also has an exponential distribution. Thus, we can regard each tollbooth as an independent *M/M/1* queue. According to queuing theory, we can obtain a function of the average consumption wait time, which is:

$$T_w = W_s + W_q = \frac{1}{\mu-\lambda} + \frac{\rho}{\mu-\lambda} \qquad (\lambda = \frac{q_0}{c}, \rho = \frac{q_0}{c\mu})$$

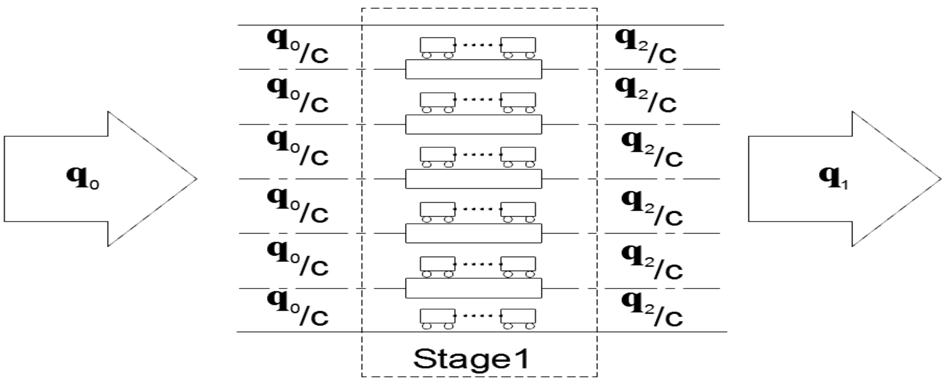

Figure 2

3.1.3 Stage Two: Vehicles Leave the Toll station

As soon as the service is over, vehicles leave the station at speed of in traffic flow of, as it shown in figure3

In this leaving stage, if the total booth-exiting flow reaches or exceeds the downstream capacity, congestion is created in the merging area. And according to our assumptions that all vehicle flows are merging in one direction, traffic congestion is more likely to take place.

Considering the high traffic density in the emerging area, we compare the traffic flow to fluid in order to approach the maximum capacity and establish our objective function on base of hydromechanics theory. Since highway traffic flow has the properties of fluid such as liquidity and diffusion, we therefore, are able to turn a problem concerning highway traffic flow into a problem concerning fluid model.

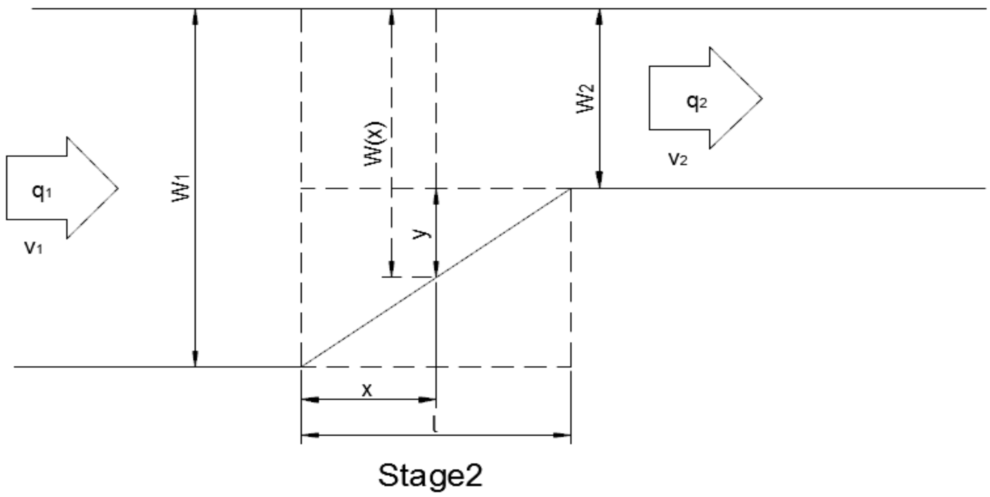

Figure 3

Assumptions

- The traffic flow in cross-section is a constant
- We assume that traffic flow keeps stable in the unit time

Establish the Objective Function of Stage Two

Referring to the *Traffic Flow Theory* which is also called *L-W theory*, we get the quotation as below:

$$q_1 = v_1 k_1$$

According to our assumptions, the traffic flow in cross-section is a constant. At a certain point of transition stage, the following equation is established:

$$q_1 = v(x) \cdot k(x), \quad k_1 w_1 = k(x) \cdot w(x)$$

Utilizing geometry in the figure 3, we have:

$$\frac{y}{l_n-x}=\frac{w_1-w_2}{l_n},\quad w_2+y=w(x)$$

Then we can calculate the algebraic relationship between the width of transition area and the number of tollbooths:

$$w_1=cd,\quad l_n=na_l$$

(a_l is a coefficient which is used to calculate l_n, value range of 0.01~0.025, we define a_l as 0.02 now)

Then the below equation come out:

$$v(x)=\frac{v_1}{w_1\cdot l_n}\cdot[x(w_2-w_1)+w_1l_n]$$

According to *L-W theory*：

$$\mathrm{d}T=\frac{dx}{v(x)},\quad T_u=\int_0^{l_n}dT$$

Hence,

$$T_u=\int_0^{l_n}\frac{w_1\cdot l_n}{v_1}\cdot\frac{1}{x(w_2-w_1)+w_1l_n}dx=-\frac{c^2da_l}{v_1(cd-w_2)}ln\,(\frac{w_2}{cd})$$

3.1.4 The Optimal Number of Tollbooths

Now, we combine our two models to calculate the objective function of time, including both the time delay in toll station and the time delay in merging area considering the large traffic flow. (Figure 4) Hence, the average time to pass the toll plaza in total per vehicle is:

$$\mathrm{T}=T_w+T_u=\frac{1}{\mu-\lambda}+\frac{\rho}{\mu-\lambda}-\frac{c^2da_l}{v_1(cd-w_2)}ln\,(\frac{w_2}{cd})$$

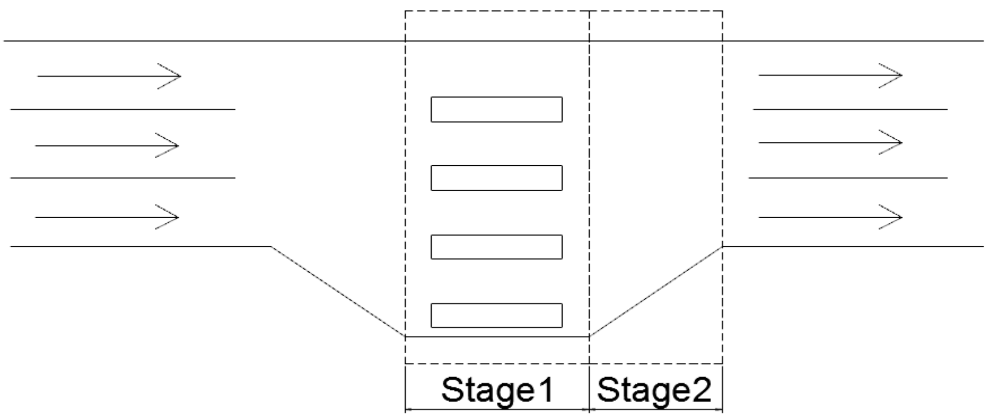

Figure 4

Our optimal goal is to minimize the average delay time for both two processes.

Referring to Handbook of highway operation management in America ,the width of single lane is 3.6 m and the width of each barrier toll(including the width of tollbooth and the width of tollbooth lane) is 5.2 m. Since we have assumed that it is a three-lane highway, We can identify the upper line ***W_2=0.0036*3=0.0108.*** We also assume: the average serving rate is $800\ veh/h$, the average arriving rate is $2800 veh/h$ and the speed of vehicles leaving the toll plaza is $20km/h$, as we can see in table2.

Table 2 value of parameter

w_2/km	d/km	$E(\mu)/(veh\cdot h^{-1})$	$E(q_0)/(veh\cdot h^{-1})$	$v_1/(km\cdot h^{-1})$
0.0108	0.0052	800	2800	20

Since c should vary in a reasonable range (not too large or too small) and must be an integer, we can calculate T_w, T_u, T by adjusting the value of c. We use Matlab to compute T_w, T_u, T for c varying from 4 to 15 and the results are shown as table 2. Figure 5 shows the average delay time for different value of c.

Table 2 split time and total time through toll plaza

c	T_w/h	T_u/h	T/h
4	0.0188	0.0055	0.0242
5	0.0071	0.0075	0.0146
6	0.0048	0.0097	0.0145
7	0.0038	0.0121	0.0158
8	0.0032	0.0146	0.0178
9	0.0028	0.0172	0.0200
10	0.0026	0.0198	0.0224
11	0.0024	0.0226	0.0250
12	0.0023	0.0255	0.0277
13	0.0022	0.0284	0.0305
14	0.0021	0.0314	0.0335
15	0.0020	0.0344	0.0364

The curve of total wasted time is composed by the curves of time delay in tollbooth and merging. These two curves indicate that with the number of tollbooths increasing, the tollbooth delay decreases while merging delay increases.

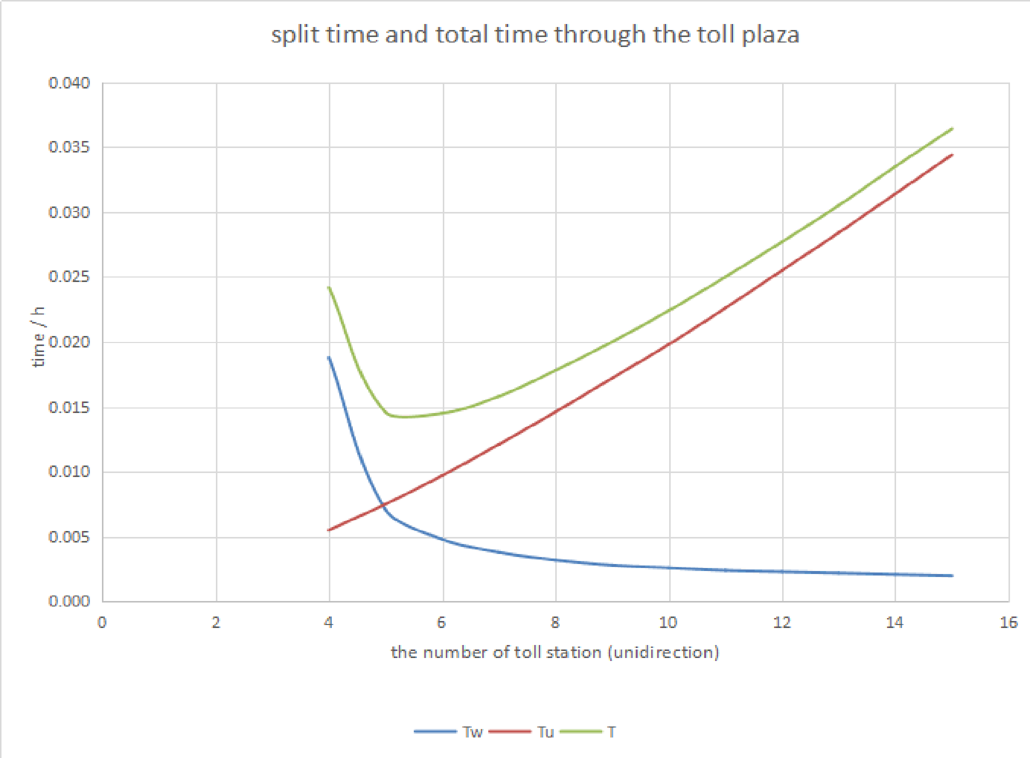

Figure 5 split time and total time through the toll plaza

As we can see from figure 5 and table 2,the minimum value of total average delay time is 0.0145 hour, which happens when c = 6,so 6tollbooths is optimal for this three-lane highway configuration.

3.1.5 Results of Model I

Now, we have derived the optimized number of tollbooths6, under the given configuration of 3 traffic lanes. We can draw our baselines on the basis of it.

$$\begin{cases} Upper\ line\ \ W_2 = 0.0036 * 3 = 0.0108\ (\mathrm{km}) \\ Baseline\ \ W_1 = 0.0052 * 6 = 0.0312(\mathrm{km}) \end{cases}$$

3.2 Model Ⅱ： Fix the Lateral Sides

After we fix the length of baseline of our trapezoid, we discuss the design of lateral sides. In this model, we need to come up with methods to decide the merging pattern and the length of merging area and then we can complete the “trapezoid”.

3.2.1 Assumptions

- The average traffic flow is relatively larger than the average flow in *Model I*
- The proportion of electronic toll collection booths is between 70%~80%
- Only consider the merging area after the barrier toll

● Assume the velocity of vehicles passing through ETC lanes is and the velocity of vehicles passing through ETC lanes is

3.2.2 Notations

Table 3 Symbol Table–Variables

variable	Definition
m	The average cost of highway construction per square kilometer (including the average land price and the average cost of laying the pavement in this region ($/km^2)
d	The width of single highway lane (km)
W_f	The total width of ETC lanes (km)
W_e	The total width of other toll lanes (km)
W_n	The total width of merging ETC lanes (km)
W_u	The total width of merging non-ETC lanes (km)
B	The number of tollbooths
L	The number of traffic lanes
T_n	Time passing through *I region* （h）
T_u	Time passing through *II region* (h)
T	The total time delay in merging area (h)
W	The construction cost of merging area （$）
θ	The bottom Angle of trapezoidal area in the transitional zone

3.2.3 New Merging Zone

In *Model I*, we regard as the average time and we do not pay special attention to the different contribution of traffic flow in various speeds. For example, the average speed of traffic flow passing through ETC is generally higher than the group passing through the MTC. The throughput in toll plaza will relatively larger traffic flow under circumstances with more ETC. At this time, the determined in Model One is no longer proper. We need to consider new determinants.

● New Toll Plaza and Merging Pattern

According to *Model I*, when, the optimized number of tollbooths B=6.We design the following merge new toll plaza (figure 6) for 3 reasons:

1) We refer to the official website of New Jersey highway administration, finding out that the average service rate of ETC is approximately 70%, so we assume the number of our ETC lanes is 4.

2) As we have explained, the average speed of traffic flow passing through ETC is generally higher than the group passing through the MTC and STC. Due to safety concerns, we separate the ETC lanes from non-ETC lanes, as the bold line of figure 6 shows.

3) The speed of vehicles outside is typically slower than those running inside. So we choose to put ETC lanes in the inner position.

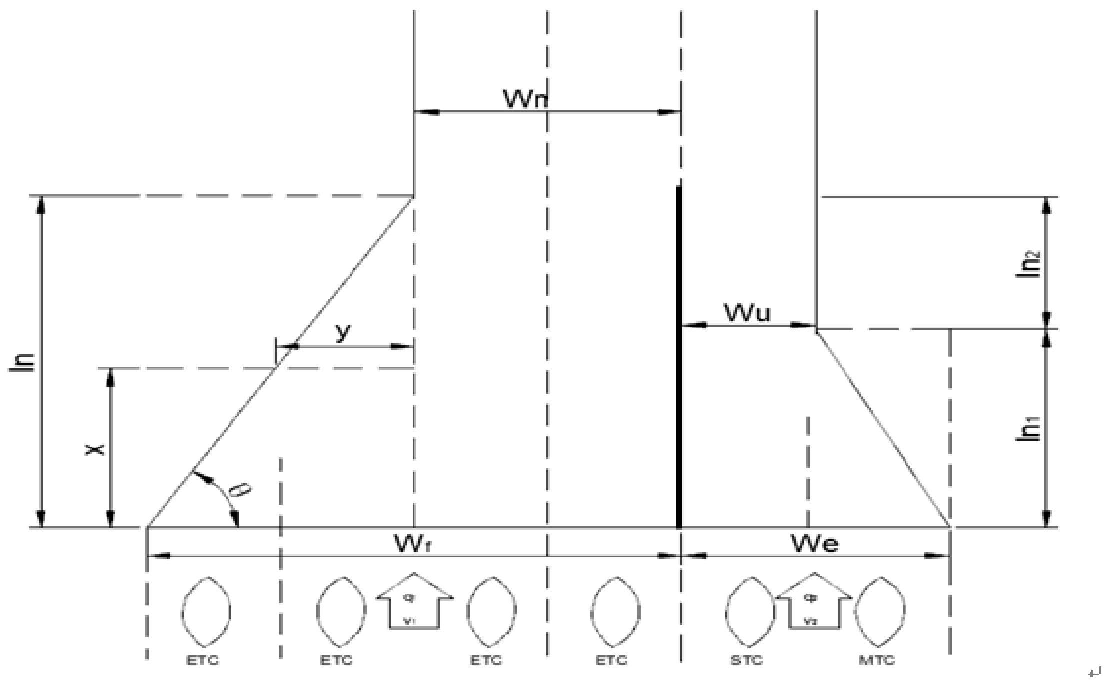

Figure 6

- **Establish Three Separate Objective Functions**

There are mainly three factors that should be taken into consideration: accident prevention, throughput and cost. Consequently, in the first step, we need to separately establish three objective functions .Secondly, we build up a multi-objective programming system to combine the three functions and come up with the comprehensive optimal design.

1）Accident Prevention

If the bottom angle, as it is marked in figure 6, is too small, the slope will be too steep for drivers to turn into the merging lanes, and the car accidents and congestion are created. To keep the safety distance between two moving vehicles, we building up the objective function of the coefficient concerning about the angle. And we try to find the largest in a reasonable range considering other two goals.

Utilizing geometry in the figure 6, we have:

$$tan\theta = \frac{2a_l}{d}$$

(we can easily prove that the bottom angle in both sides are the same)

2）Throughput

The less time vehicles spend on the merging area, the more fluent the traffic is and the number of vehicles passing through the toll plaza will be increasing when other things equal. We use the same method as Stage Two, *Model I*, but we calculate delay time in trapezoid area in different toll collection system. Referring to the *Traffic Flow Theory* which is also called *L-W theory*, we get the quotation for *Zone Ⅰ*:

$$T_n = -\frac{c_1 d \cdot l_n}{v_1(c_1 d - w_n)} ln\left(\frac{w_n}{c_1 d}\right)$$

And since the traffic flow of MTC and STC are both small compared to ETC, we regard them as the same group when we discuss .We get the quotation for *Zone Ⅱ*

$$T_u = -\frac{c_2 d \cdot l_{n1}}{v_2(c_2 d - w_u)} ln\left(\frac{w_u}{c_2 d}\right)$$

Hence,

$$T = T_n + T_u$$

3）Cost function

We assume that the price land is ***4,365,000 $/km***, and the area of three-lane highway ***0.018 km²*** per kilometer, so we can identify the average cost of highway construction per square kilometer=***242,500,000 $/km²***.

Calculate the area of merging stage and then we can establish the cost function of fan-in area.

$$W = \left[\frac{(w_n + w_f) \cdot l_n}{2} + (l_n - l_{n1}) \cdot d + \frac{(w_u + w_e) \cdot l_{n1}}{2}\right] \cdot m$$

3.2.4 A three-dimensional GE-Matrix to Determine Optimal Results

After establishing three functions to quantify three goals we want to achieve, we need to build up a multi-objective programming system to combine these three functions and come up with the comprehensive optimal design.

The general principle is to minimize the delay time T and the cost W while maximizing the angle θ. By giving different value of a_1 ,the corresponding T,W,tan θ are displayed in table 4.

Tabel 4 Data of variables

al	*Tn*	*Tu*	*T*	*w*	*tanθ*
0.0115	0.0016	0.0016	0.0032	170671.5	6.3889
0.0130	0.0018	0.0018	0.0036	192933.0	7.2222
0.0145	0.0020	0.0020	0.0040	215194.5	8.0556
0.0160	0.0022	0.0022	0.0044	237456.0	8.8889
0.0175	0.0024	0.0024	0.0049	259717.5	9.7222
0.0190	0.0026	0.0026	0.0053	281979.0	10.5556
0.0205	0.0028	0.0028	0.0057	304240.5	11.3889
0.0220	0.0030	0.0030	0.0061	326502.0	12.2222
0.0235	0.0033	0.0033	0.0065	348763.5	13.0556

The *GE-Matrix (McKinsey Matrix)* is originally used in evaluating competitiveness and attractiveness of a company in a two-dimensional frame. The matrix can tell the manager how move next. Although there are many differences between toll collection and company operation, especially in the extension in three-dimension. We believe we can still borrow its idea to decide which point is in the best position.

We set T and W as the variables put in the horizontal surface and set tanθ as vertical axis .And then we divide the space into a few cubes. (9*9*9). According to the central idea of *GE-Matrix*, different cube represents different comprehensive levels. We plot our data as point (T, W, tanθ) and according to the principle, we first filter our data by comparing the value of tanθ when C and W lie in the some coordinate range. After this progress, some data were filtered and then we compare T and W at the fixed level of tanθ.

Hence, a GE-Matrix is adopted. Figure 7 and Figure 8 show the matrix we adopt.

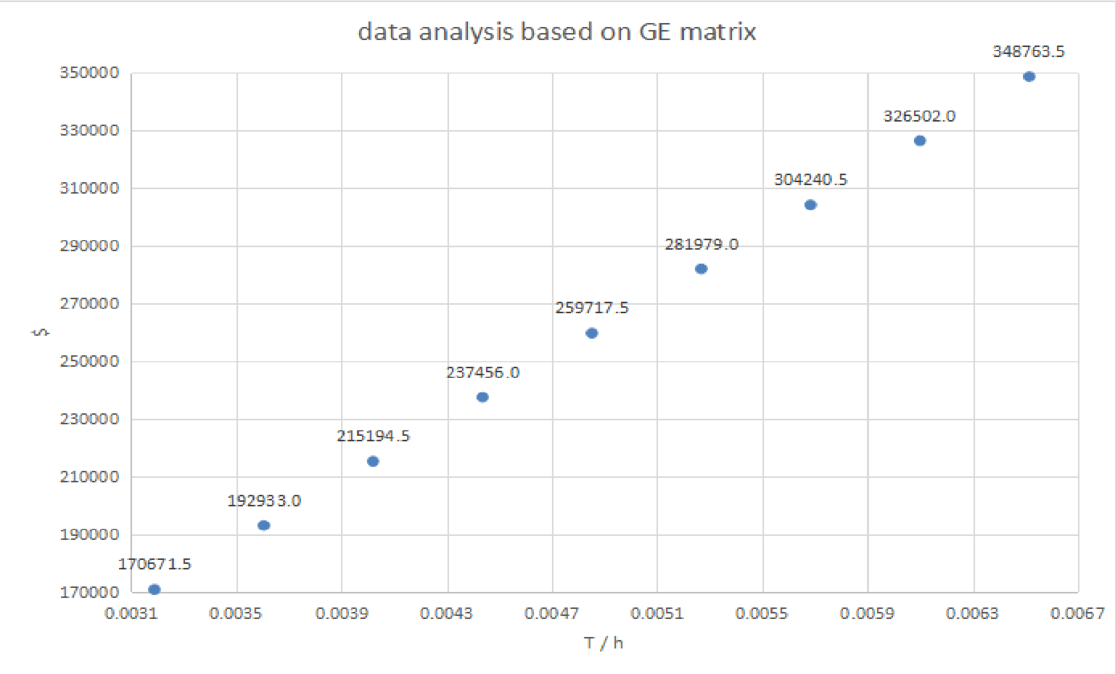

Figure 7

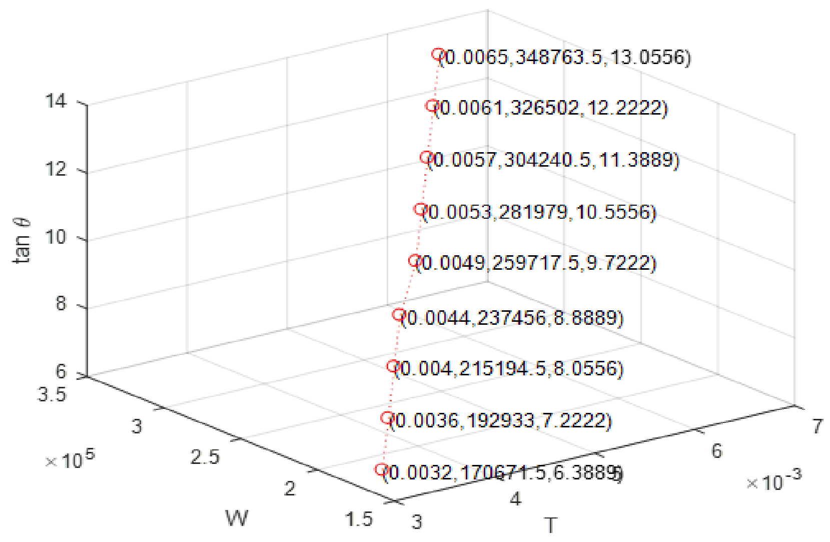

Figure 8

We finally calculate that the optimal point is (0.0049, 259717.5, 9.7222) and a_1=0.0175 at this time.

3.2.5 Our result

bring the above data into

$$l_n = 2d \cdot tan\theta \ , l_{n1} = d \cdot tan\theta \ , l_{n2} = l_n - l_{n1}$$

Hence:

$$l_n = 0.1011km, l_{n1} = l_{n2} = 0.05055km$$

Further Discussion 4

4.1 What if there are more self-driving vehicles?

Self-driving car, which is also known as autonomous car or driverless car, is a vehicle that is capable of sensing its environment and navigating without human input.

On the one hand, since autonomous cars can detect environments using a variety of techniques such as radar, GPS, and computer vision, one of the significant benefits of automated cars is the reduction of traffic accidents, and major increases in roadway capacity. Thus, the average traffic flow Q should both be raised .On the other hand , the self-driving car put the safety in the first position. So if it encounters high traffic density in the merging area, it will slow down the speed to keep the safe distance.

As for our model, there are mainly two types of effect. First, because of the high safety of driverless cars the angel θ could be diminished with more autonomous vehicles added to the traffic mix and the cost could be saved in this way. Second, as U.S. consulting firm IHS forecasts , smart cars sales will reach 11.8 million in 2035 , accounting for 9% of the same period the global automotive market. Correspondingly, our model can reduce the number of MTC and STC.

4.2 What will change when the proportion of ETC/STC/MTC change?

As we have discussed in *Model Ⅱ*,

4.3How is thing going when the traffic flow is rather small?

Both the Stage Two of *Model Ⅰ* and *Model Ⅱ* have used the large traffic flow theory in which we assume we encounter large traffic flow. So when the flow is small, our model can not simulate the reality anymore and we must consider the car as an individual at this time.

4.4 Discussion on the specific type of ETC

ETC electronic toll collection system can be divided into electronic toll collection system and hybrid electronic toll collection system, which is shown in the figure9. *Free flow ETC systems* allow vehicles to pass through in high-speed but also have high costs, which is suitable for large flow multi-Lane Toll stations. *Hybrid system*, on contrast, suitable for smaller traffic sites.

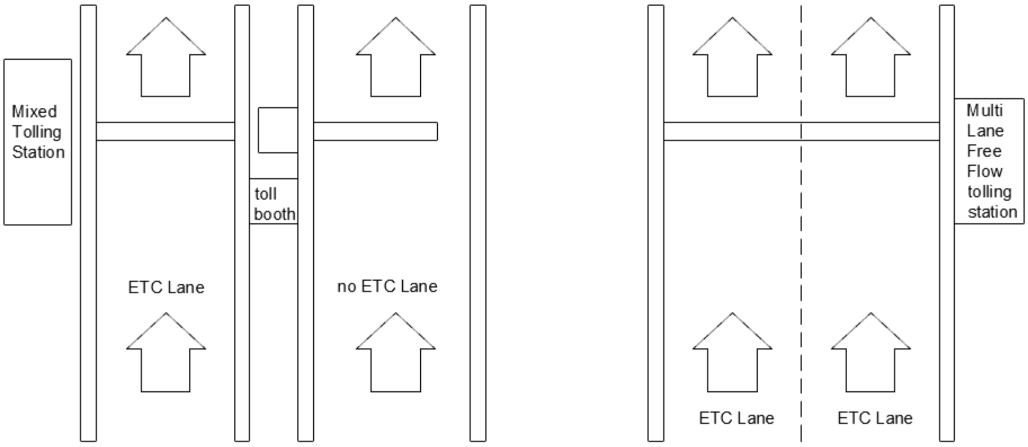

Figure 9

There has been a part of the highway in New Jersey with E-Zpass automatic electronic, scanning charges

Sensitivity Analysis 5

5.1 According to general data, the length of transition area ranging from 50m to 150m[8], the result we get is reasonable.

5.2 The scale coefficient of the length of transition area and the number of tollbooth can be changed within limits to influence the quantity including the total construction cost of merging area, the total time delay in merging area and angle θ .

5.3 The total time of vehicle passing the two areas of the toll plaza can also be influenced by changing the number of different types of tollbooth in *Zone* I and *Zone* II

5.4 Besides, the variables defined in this model are assigned a value from particular statistics, we can further modify our model according to highway and the toll plaza under different circumstances.

5.5 There are three objective functions of W,T in our model, When we change the value of some variable, we only get the Calculating data to find the changing, it's difficult for us to get the precise relationship between the variables and functions.
But we can get a relatively optimal solution to deal with the data
by GE matrix.

Strengths and Weaknesses 6

6.1 Strength

6.1.1 Since there are different parameters in our model to simulate all kinds of reality of the toll station, we hold the belief that it can be used in a variety of cases.

6.1.2 The Model One is divided into two model stages. Applying the queuing theory and assuming traffic flow as a kind of hydromechanics model, it's easy to calculate how long it takes to queue and accomplish the toll collection progress.

6.1.3 Considering the speed of ETC traffic flow is higher than others in model No.2, so we optimize its location setting.

6.1.4 When it comes to the objective function, GE matrix has been applied to solve the optimization problem in the Model Two

6.1.5 In the process of selecting the parameters needed, we make it more sensible by looking up relevant information.

6.2 Weakness

6.2.1 Owing to the lack of genuine local data base, we set some factor values, such as traffic flow, road width and service time approximately, not accurately.

6.2.2 In different kinds of toll collection, we only compare cars' speed of passing the toll station, ignoring the difference of time that it takes to pay the fare.

6.3.3 Besides, traffic flow can be analyzed by a kind of hydromechanics model only in case that there is light traffic flow. Otherwise, modeling can't promise an optimized and accurate result.

6.3.4 In process of modeling, we focus on traffic flow, numbers and area shape of toll collection, road safety, and ETC ratio but neglect other sides.

6.3.5 Although we get the optimized result by applying GE matrix, it's just a kind of trial, which we cannot make sure its suitability

Possible Improvement 7

7.1 The probability distribution in theory of probability can be used to evaluate traffic flow of every toll collection road, like Poisson's distribution.

7.2 We can further our modeling by comparing average-service time and ETC proportion under different toll collection system.

7.3 Cellular automaton can be applied to simulate the traffic flow to testify our models.

Recommendation Letter to the New Jersey Turnpike Authority

Dear New Jersey Turnpike Authority:

With the rapid development of economic society, traffic flow on highways becomes increasingly heavier, toll collection plaza undertakes significant functions of control, and we suppose that some positive goals can be achieved by applying relevant models in our team works to evaluate and research toll station and its reconstruction methods.

Since traffic jams and congestion often happens when vehicles gather into a few roads, which it's the cases of toll station, we come up with some suggestion which we do hope you can adopt some of them to upgrade your toll station service in saving vehicles' passing time, broadening ability of undertaking traffic flow, reducing construction budget and enhancing road safety.

For the fare waiting area:

- There are nearly all two-way six lane, there lanes in one direction, in New Jersey highway road, so it's sensible to build 6 toll collection to reduce the service time.
- As there is more and more vehicles instrumented ETC system, we suggest that there is at least 70%-80% ETC lane.
- As for ETC system, we add a multi-lane free flow tolling station system to it, and set it up in straight road. Combining ETC and straight road may lead to less possibility of traffic congestion and enhance the ability of road passing.
- And we hope emergence toll collection road can be set up in case that there is an emergence and need of sending people to ease traffic pressure.

For leave transition area:

This is the sketch on the shape of toll plaza construction:

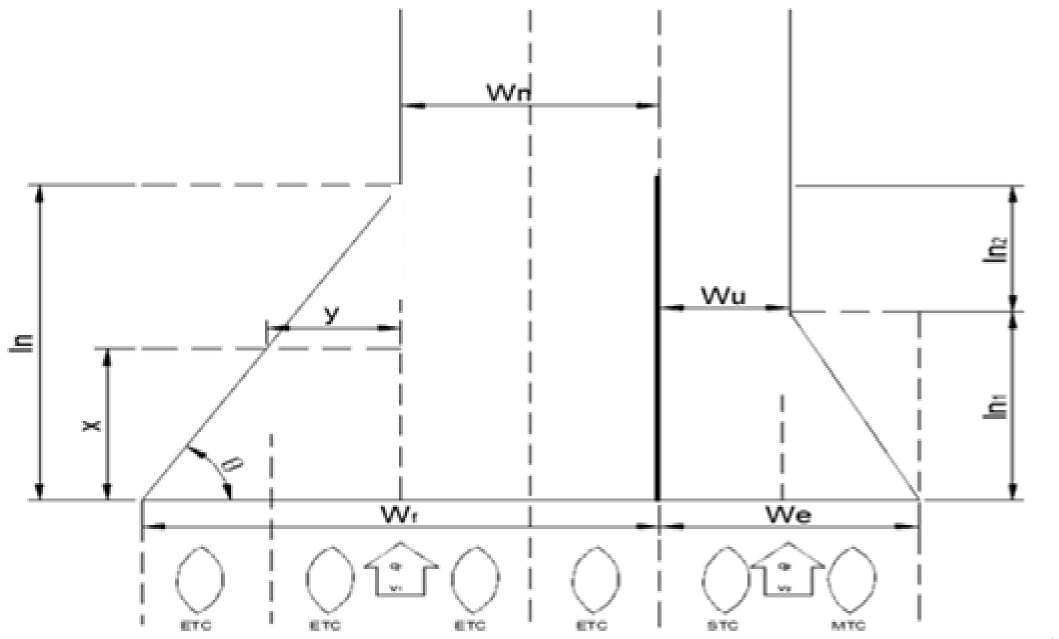

Figure 10

We suggest four ETC lane gathers into two lane, and two non-ETC lane gathers into one lane, and ETC and non-ETC are separated with each other. For 3-unidirectional-lane highway,

$l_n = l_{n1} + l_{n1}$, as shown in the picture,

the reasonable length of transition area is 101.1m when the optimal number of tollbooth is six in one direction.

For the future:

Numbers of automatic self-driving cars will rise up in the future, which can relief traffic congestion situation and reduce car accident rate. That is to say, it is wise choice to keep increasing percentage of ETC lanes, even adopt all ETC lanes toll collection system.

Yours sincerely

COMAP 69058
2017.1.23

Reference

[1] Suli: Study on the design of highway toll station (2012);pp. 40-42

[2] Albert ,Schaufler. Toll plaza design. U.S.A: Transportation Research Board,1997:8-40

[3]Wu Chunlei. Configuration Research of Toll-gate on Freeway ,2008:1-4

[4] Nico M,van Dijk, Mark D. Designing the Westerscheldetunnel Toll Plaza Using a Combination of Queuing and Simulation [C]. Proceeding of the 1999 Winter Simulation Conference, 1999: 1272-1279

[5] High Way Capacity Manual complied by National Research Council,2015

[6]Roger P Roses,William R McShane, Elena S Prassas.Traffic Engineering(2nd Edition)Prentice Hall.

[7]Special Report 209:Highway Capacity Manual.Third edition.Washington D C：TRB，Nattional Research Council,1994

[8] http://doc.mbalib.com/view/4251f2d4c306cf72ed52af2eff125579.html

Appendix

[1]

```
clc;
clear
c=15;     b=2800;
b1=b/c;
e=800;
s=b1/e;
Wq=s/(e-b1);
Ws=1/(e-b1);
T1=Wq+Ws;
Wu=0.0108;
d=0.0052;
al=0.02
v0=20;
display(Wq);
display(Ws);
Tu=-(c^2*d*al)/(v0*(c*d-Wu))*log(Wu/(c*d));
display(Tu);
T=Wq+Ws+Tu;
display(T);
```

[2]

```
a=[0.0032   170671.5    6.3889
0.0036  192933.0    7.2222
0.0040  215194.5    8.0556
0.0044  237456.0    8.8889
0.0049  259717.5    9.7222
0.0053  281979.0    10.5556
0.0057  304240.5    11.3889
0.0061  326502.0    12.2222
0.0065  348763.5    13.0556
];
```

```
plot3(a(:,1),a(:,2),a(:,3),':ro')
xlabel('T')
ylabel('W')
zlabel('tan \theta')
grid on
for i=1:length(a)
    d=['(',num2str(a(i,1)),',',num2str(a(i,2)),',',num2str(a(i,3)),')'];
    text(a(i,1),a(i,2),a(i,3),d);
end
```