IERG 4160 – Lecture 4

Image Restoration & Reconstruction

Restoration vs. Reconstruction

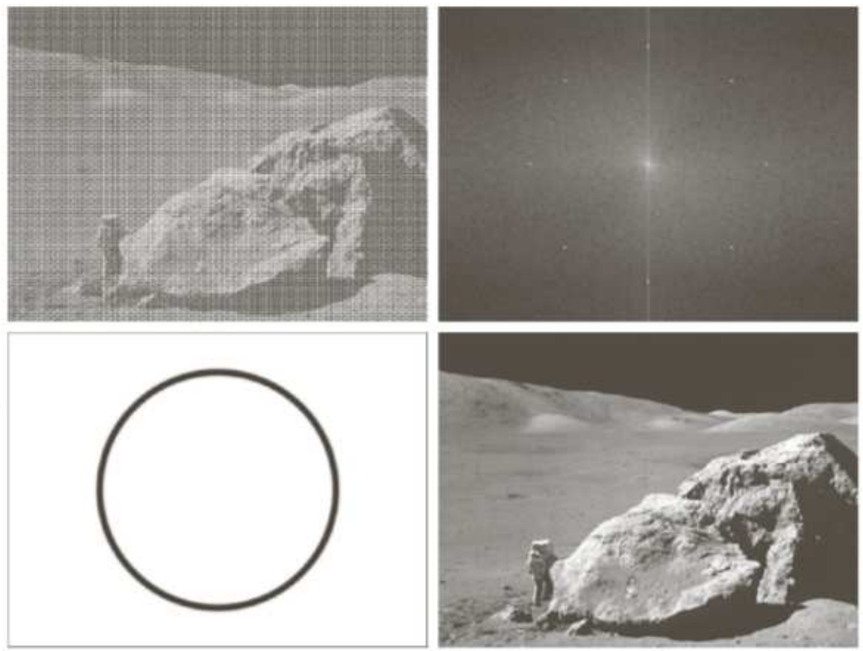
- *Image restoration* and *image enhancement* share a common goal: to improve image for human perception
- Image enhancement is mainly a subjective process in which individuals' opinions are involved in process design.
  - For instance: Image sharpening

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Restoration vs. Reconstruction

- Image restoration is mostly an objective process which
  - utilizes a prior knowledge of degradation phenomenon to recover image.
  - models the degradation and then to recover the original image.
    - For instance: Image denoising

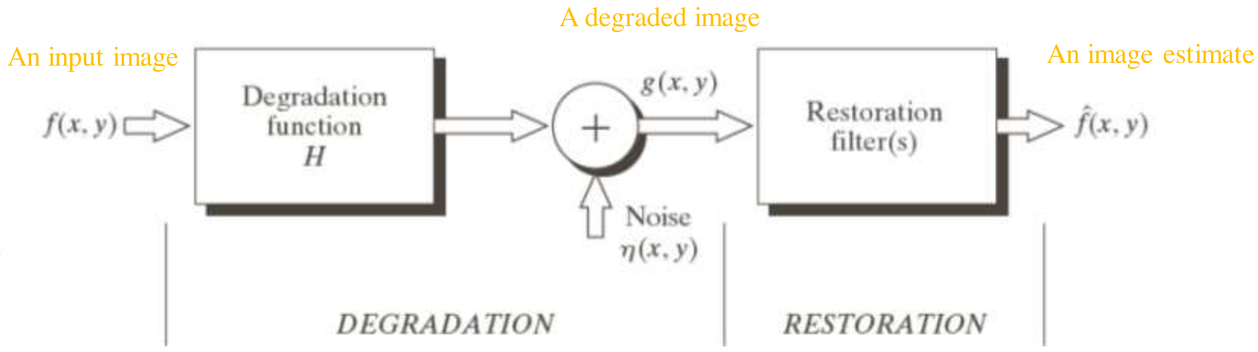


a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Model of Degradation/Restoration Process

FIGURE 5.1 A model of the image degradation/restoration process.

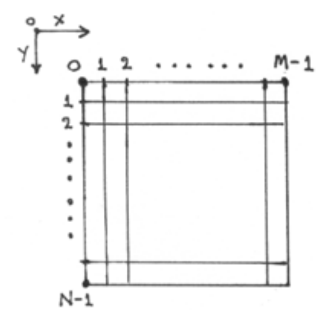


- If H is a linear, position-invariant process (filter), the degraded image is given in the spatial domain by

$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)$$

- whose equivalent frequency domain representation is

$$G(u,v) = H(u,v) \bullet F(u,v) + N(u,v)$$



The frame of reference

Objective of Restoration

- The objective of restoration is to obtain an image estimate which is as close as possible to the original input image.
- A typical difference measurement is the *mean square error (MSE)*:

$$MSE = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

- Generally, the more H and noise are known, the lower MSE will become.

The Source of Noises

The principal sources of noise in digital images arise during:

- Image <u>acquisition</u>
  - For instance, with a CCD camera, light levels and sensor temperature introduce noise to the resulting image.
- Image <u>transmission</u>
  - For instance, an image transmitted over a wireless network might be corrupted as a result of lighting or other atmospheric disturbance.

Spatial & Frequency Properties

- Spatial properties
  - We assume the noises are independent of spatial locations
  - We assume there is no correlation between pixel values and the noise components

- Frequency properties
  - The frequency content of noise in Fourier sense
  - If the Fourier spectrum is constant, the noise is usually called **white noise**.

Noise Distributions

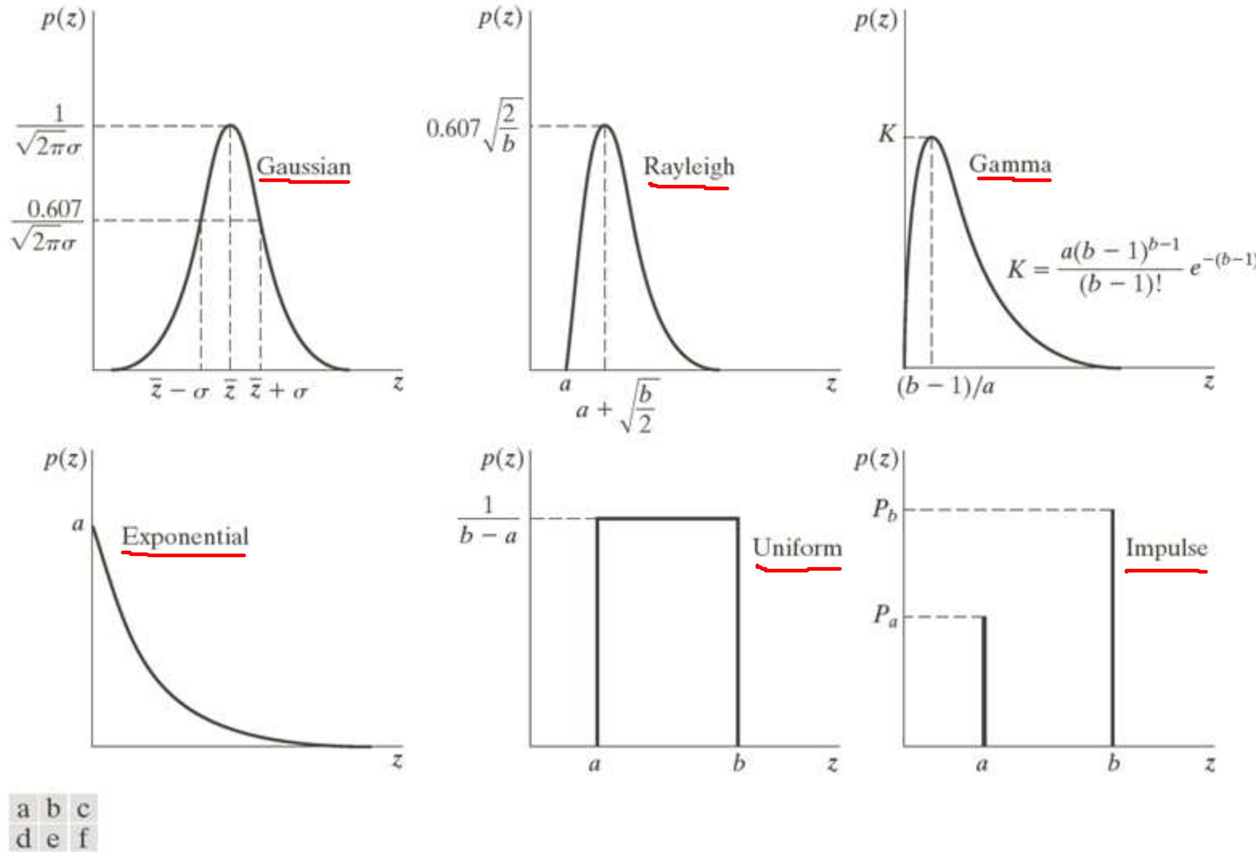
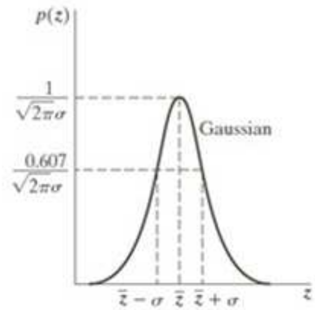


FIGURE 5.2 Some important probability density functions.

Gaussian Noise

<u>Gaussian (normal)</u> noise



$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

z represents intensity

$\bar{z}$ is the mean of z

σ is the standard deviation of z

σ^2 is the variance of z

–frequently used in practice since it is mathematically tractable in both the spatial and frequency domains

–70% of z's values fall into the range $[(\bar{z}-\sigma),(\bar{z}+\sigma)]$

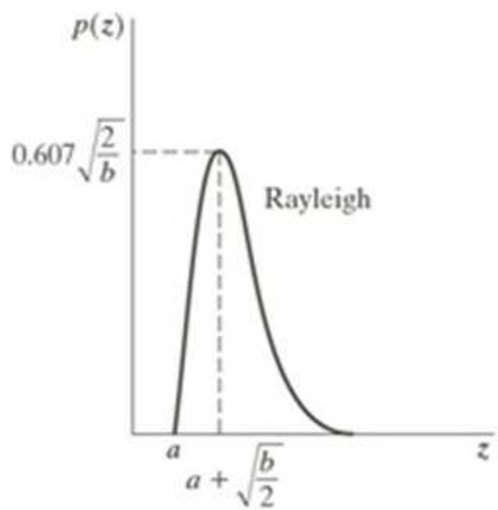
–95% of z's values fall into the range $[(\bar{z}-2\sigma),(\bar{z}+2\sigma)]$

–arising in an image due to factors such as <u>electronic circuit noise</u> and <u>sensor noise</u> due to poor <u>illumination</u> and/or high <u>temperature</u>

–<u>Central limit theorem</u>

Rayleigh Noise

Raleigh noise



$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \ge a \\ 0 & \text{for } z < a \end{cases}$$

$$\bar{z} = a + \sqrt{\pi b/4}$$

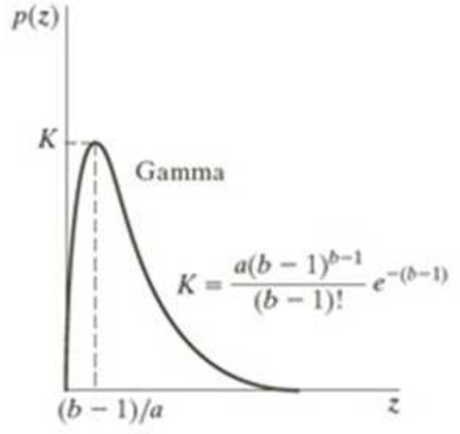
$$\sigma^2 = \frac{b(4-\pi)}{4}$$

–Displacement from origin; and skewed to the right; useful for approximating skewed histograms

–characterizing noise phenomena in range imaging

Erlang (Gamma) Noise

<u>Erlang</u> (<u>Gamma</u>) noise



$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \ge 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad \bar{z} = \frac{b}{a}, \qquad \sigma^2 = \frac{b}{a^2} \qquad a \in R^+, b \in Z^+$$

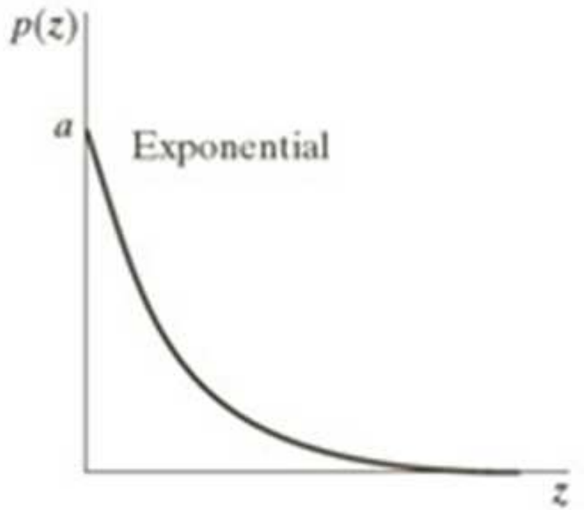
–developed by Erlang to model telephone traffics

–called Gamma noise if the denominator is the gamma function, $\Gamma(b)$

–useful in <u>laser imaging</u>

Exponential Noise

Exponential noise

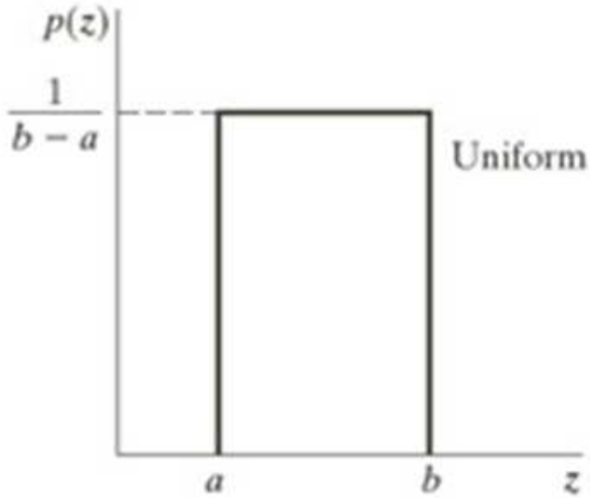


$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad \bar{z} = \frac{1}{a} \qquad \sigma^2 = \frac{1}{a^2}$$

– a special case of the Erlang density, with b=1

Uniform Noise

Uniform noise



$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{z} = \frac{a+b}{2}$$

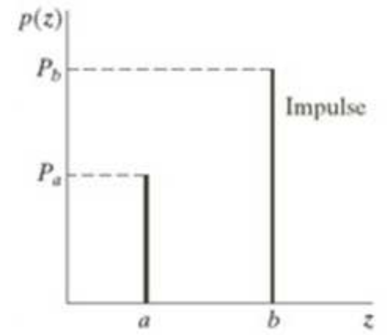
$$\sigma^2 = \frac{(b-a)^2}{12}$$

–each noise intensity being equally probable

Impulse Noise

Impulse (salt-and-pepper) noise

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



–bipolar if neither P_a or P_b is zero; in practice, for an 8-bit image, b=255 (white) and a = 0 (black)
–bipolar one, also known as salt-and-pepper, data-drop-out and spike noise
–called unipolar if either P_a or P_b is zero
–caused by either sensors' failure to respond (**pepper**, *black*) or sensors' saturation in color (**salt**, *white*)

Test Pattern

Figure 5.3 shows a test pattern well suited for illustrating the noise models just discussed,

- composed of simple constant areas that span the gray scale from black to near white in only three increments
- facilitating visual analysis of the characteristics of the various noise components added to the image

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Noise Examples

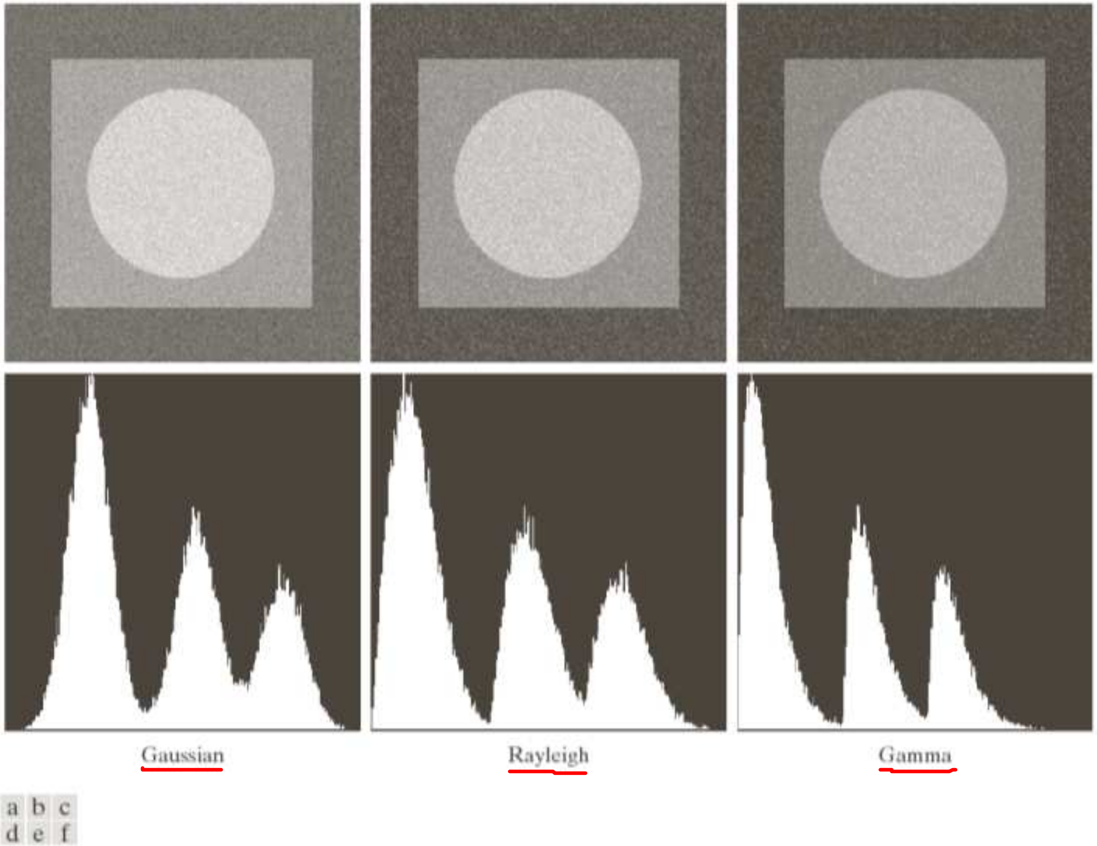


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Noise Examples

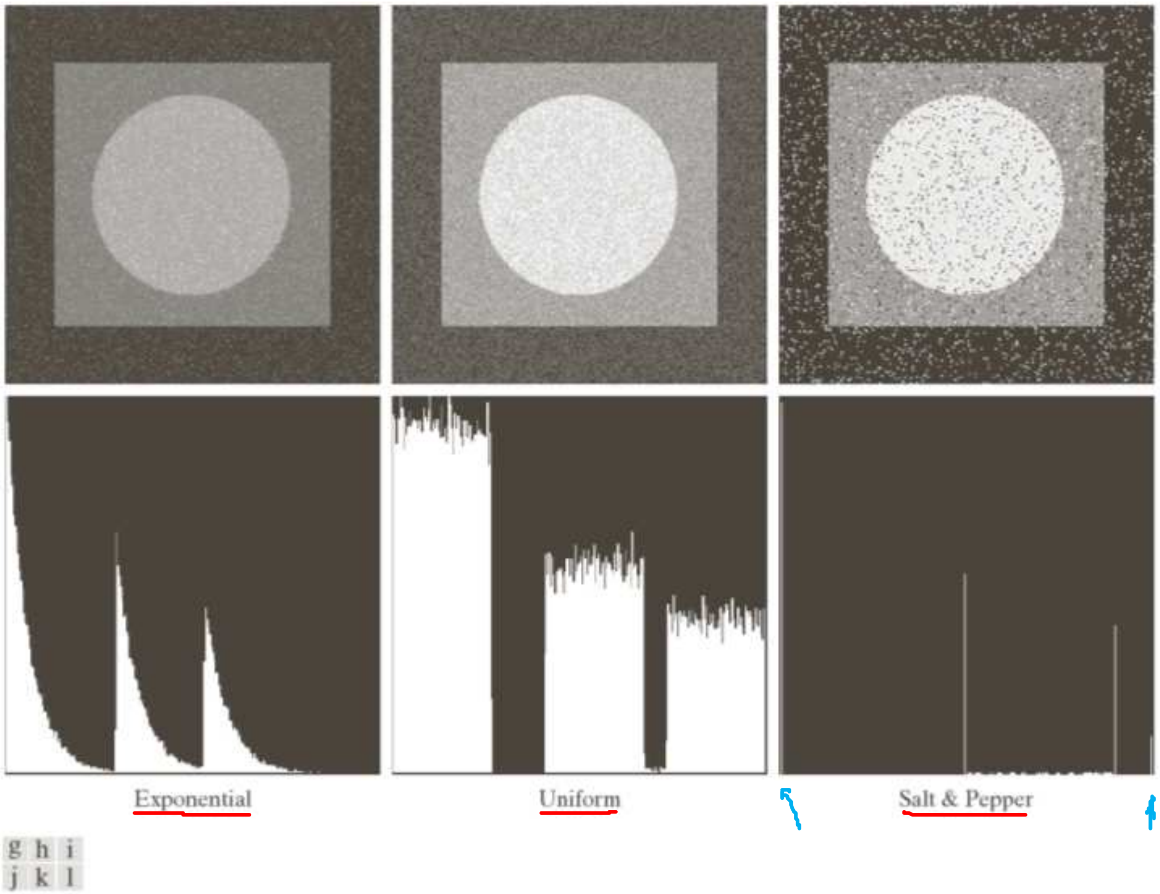


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Estimation of Noise Parameters

- The parameters of noise PDFs
  maybe known partially from sensor specifications
  often necessary to be estimated for a particular imaging arrangement
  - capturing a set of images of "flat" environments
- Possible to be estimated from small patches of *reasonably constant background intensity*, when only images already generated by a sensor are available
  - e.g., the vertical strips of 150x20 pixels

Noise-only Degradation

Spatial domain model

$$g(x, y) = f(x, y) + \eta(x, y)$$

Frequency domain model

$$G(u, v) = F(u, v) + N(u, v)$$

We often use <u>spatial filtering</u> for image restoration when <u>only additive noise is present</u>.

Filters for Noise Removal

- Mean Filters
    - Arithmetic mean filter
    - Geometric mean filter
    - Harmonic mean filter
    - Contraharmonic mean filter

- Order-Statistic Filters
    - Median filter
    - Max and min filter
    - Midpoint filter
    - Alpha-trimmed mean filter

- Adaptive Filters
    - Adaptive, local noise reduction filter
    - Adaptive median filter

Mean Filters

Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

S_{xy}, so-called a filter window, represents a rectangular sub-image of size $m \times n$, centered at (x,y)

- the simplest mean filters
- representing the restored pixel value at (x,y) by the arithmetic mean computed within the filter window
- smoothing local variations in an image → blurring
- noise-reducing as a by-product of blurring

Mean Filters (cont'd)

Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

• each restored pixel value given by the product of all the pixel values in the filter window, raised to the power 1/mn
• achieving smoothing comparable to the arithmetic mean filter, but tending to lose less image detail in the process

Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

• working well for salt and Gaussian noises
• but failing for pepper noise

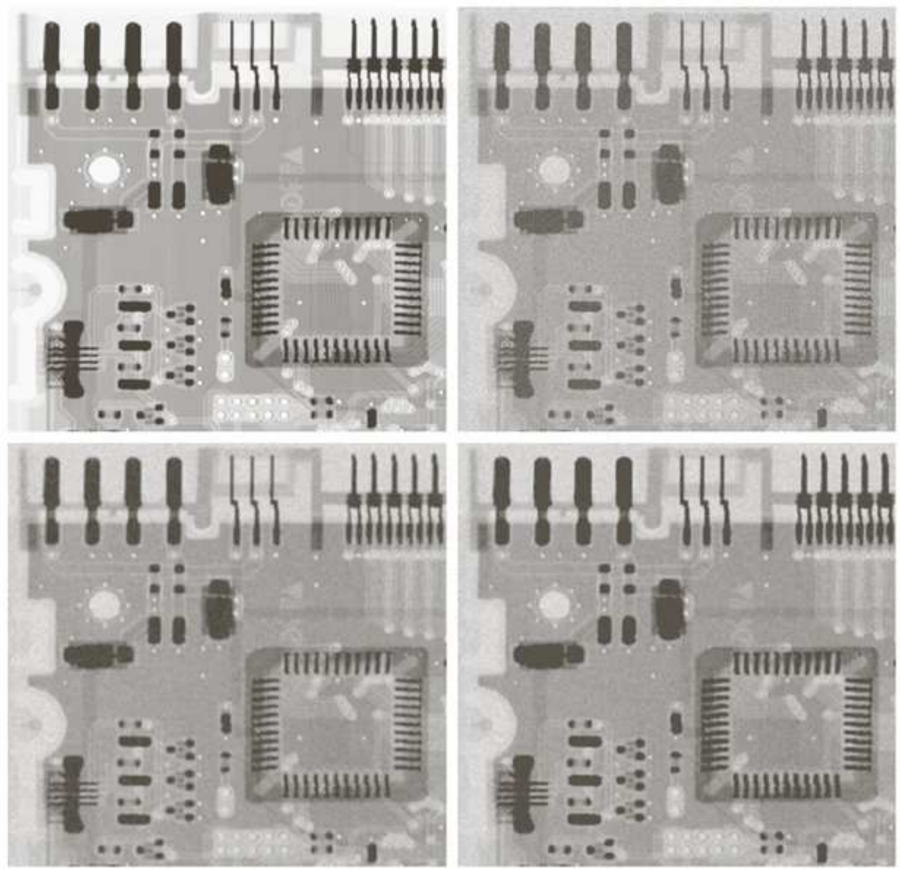
Mean Filters (cont'd)

Contraharmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

- Q called the order of the filter and $Q \in R$
- well handling or virtually eliminating the effects of salt-and-pepper noise.
- however, unable to eliminate both salt and pepper noises simultaneously
  - eliminating pepper noise when $Q \in R^{+}$
  - eliminating salt noise when $Q \in R^{-}$

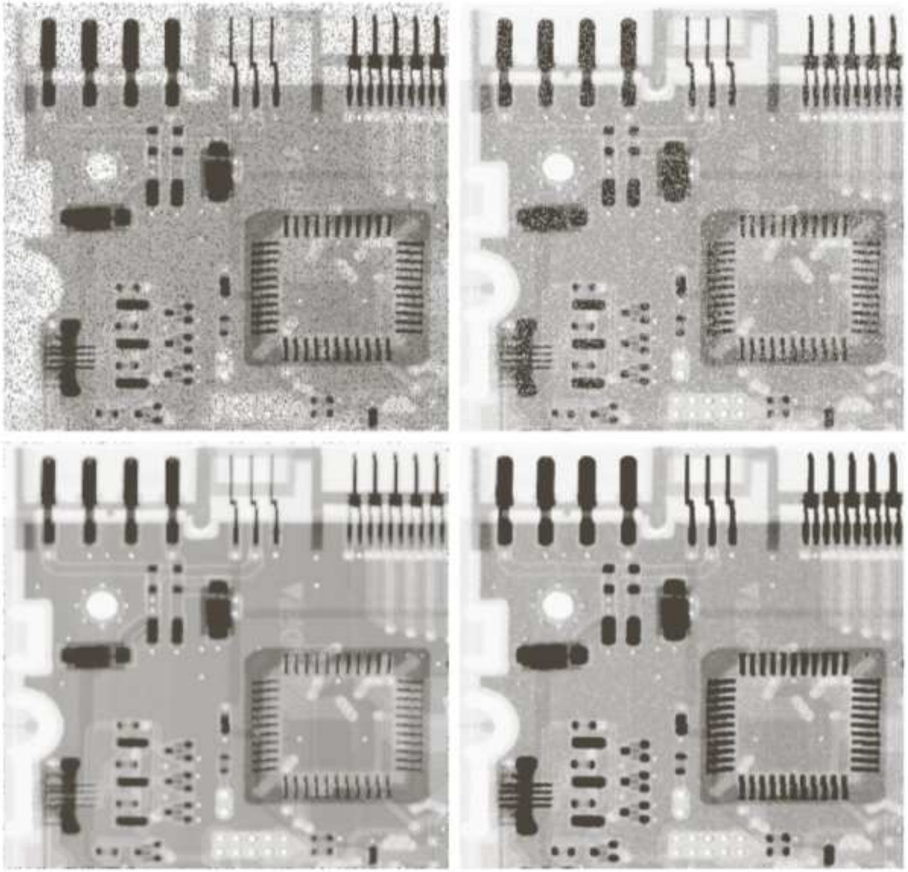
Mean Filters: Examples



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

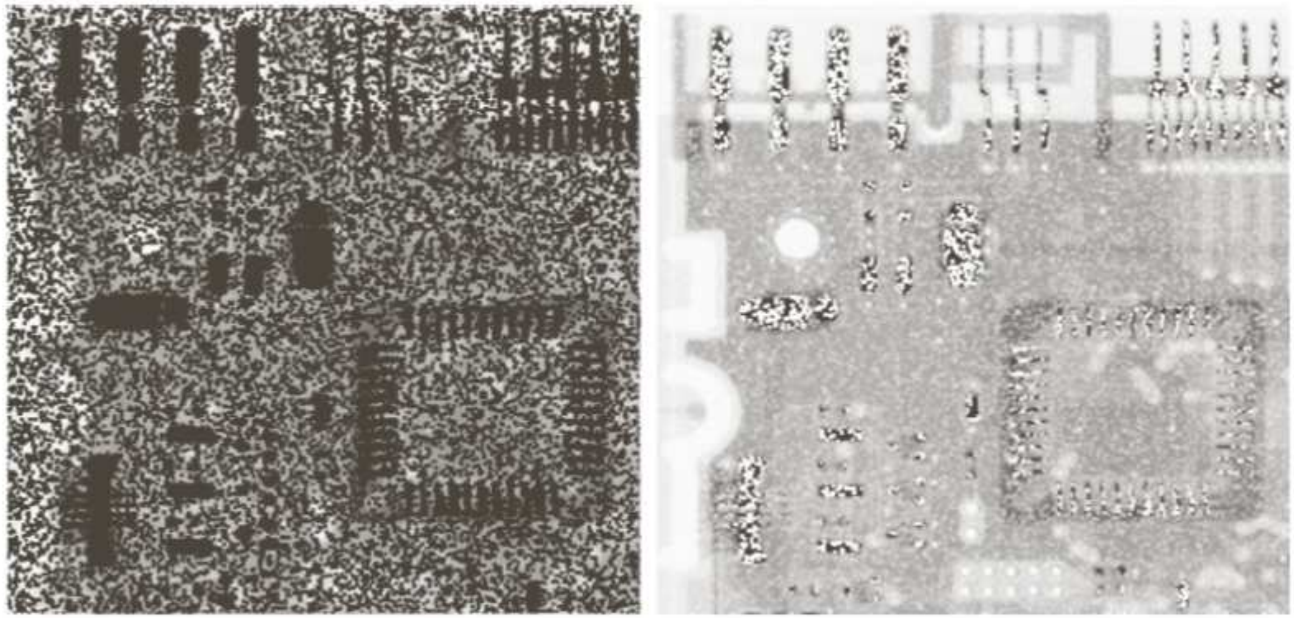
Mean Filters: Examples



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Mean Filters: Examples



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Order-Statistic Filters (OSF)

- Order-statistic filters are spatial filters whose response is based on ordering (ranking) the pixel values contained in certain neighborhoods.

Order-Statistic Filters

Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\left\{g(s,t)\right\}$$

• the best-known of order-statistic filters
• representing the restored pixel value at (x,y) by the median (ranked in the 50th percentile) of intensity levels in the filter window
• for certain types of noise, providing excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters on the same basis (of similar size)
• particularly effective in the presence of both bipolar and unipolar impulse noise

Order-Statistic Filters

Max and min filters

$$\hat{f}(x,y) = \begin{cases} \max\limits_{(s,t)\in S_{xy}} \{g(s,t)\} & \text{for the max filter} \\ \min\limits_{(s,t)\in S_{xy}} \{g(s,t)\} & \text{for the min filter} \end{cases}$$

- representing the restored pixel value at (x,y) by the maximum/minimum of intensity levels in the filter window
- the max filter greatly reducing pepper noise (black dots)
- the min filter greatly reducing salt noise (white dots)

Order-Statistic Filters

Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

• representing the restored pixel value at (x,y) by the midpoint between the darkest and brightest points in the filter window
• working best for randomly distributed noise, e.g., Gaussian or uniform noise

Alpha-trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

• $g_r(x,y)$ representing the trimmed filter window of size $mn-d$ after deleting the d/2 lowest and the d/2 highest values out of the original filter window
• becoming a median filter when $d = mn-1$
• efficiently handling mixture noise, e.g., a combination of salt-and-pepper and Gaussian noise

Order-Statistic Filters: Examples

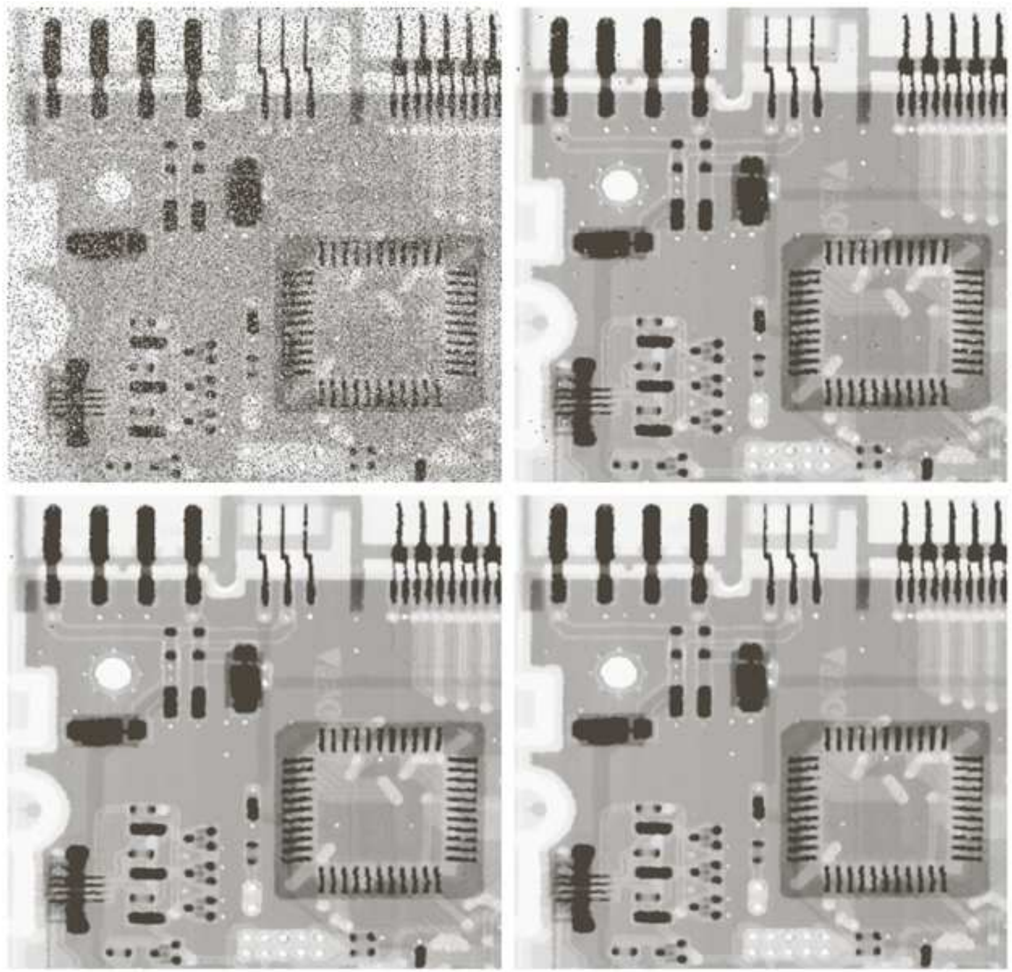
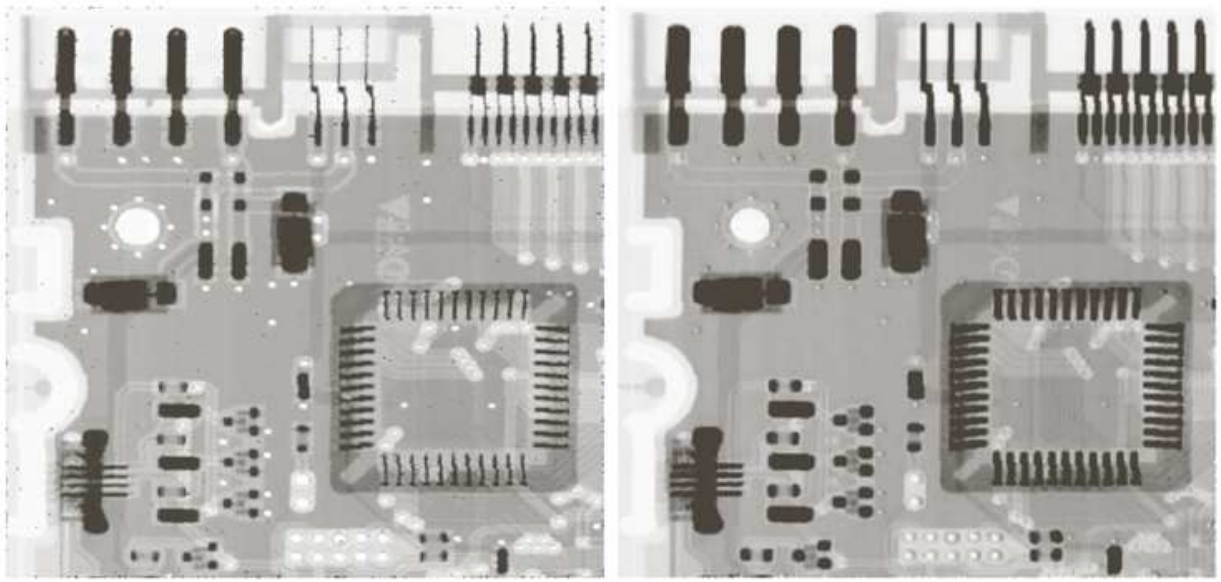


FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

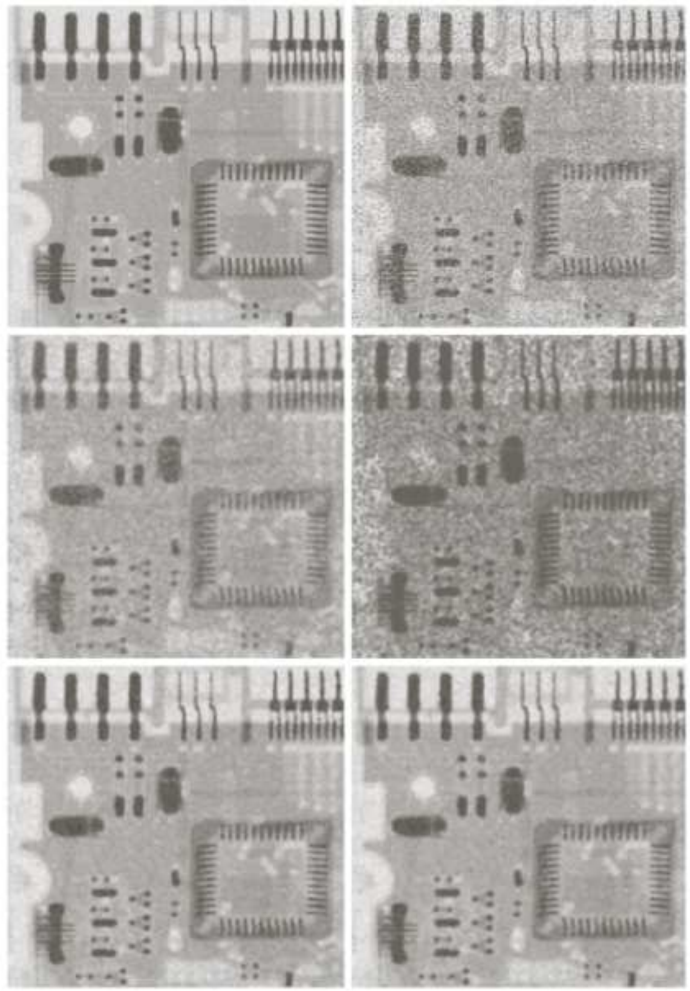
Order-Statistic Filters: Examples



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Example: Removing Mixed Noises



a	b
c	d
e	f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5 × 5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Adaptive Filters

The filters discussed thus far are non-adaptive filters.

- whose coefficients are static, collectively forming the transfer function
- applied to an image regardless of how image characteristics vary from one point to another

In this section, two adaptive filters are discussed.

- whose behavior changes according to statistical characteristics of the image inside the filter window
- whose performance is superior to that of non-adaptive filters having discussed

Adaptive Local Noise Reduction

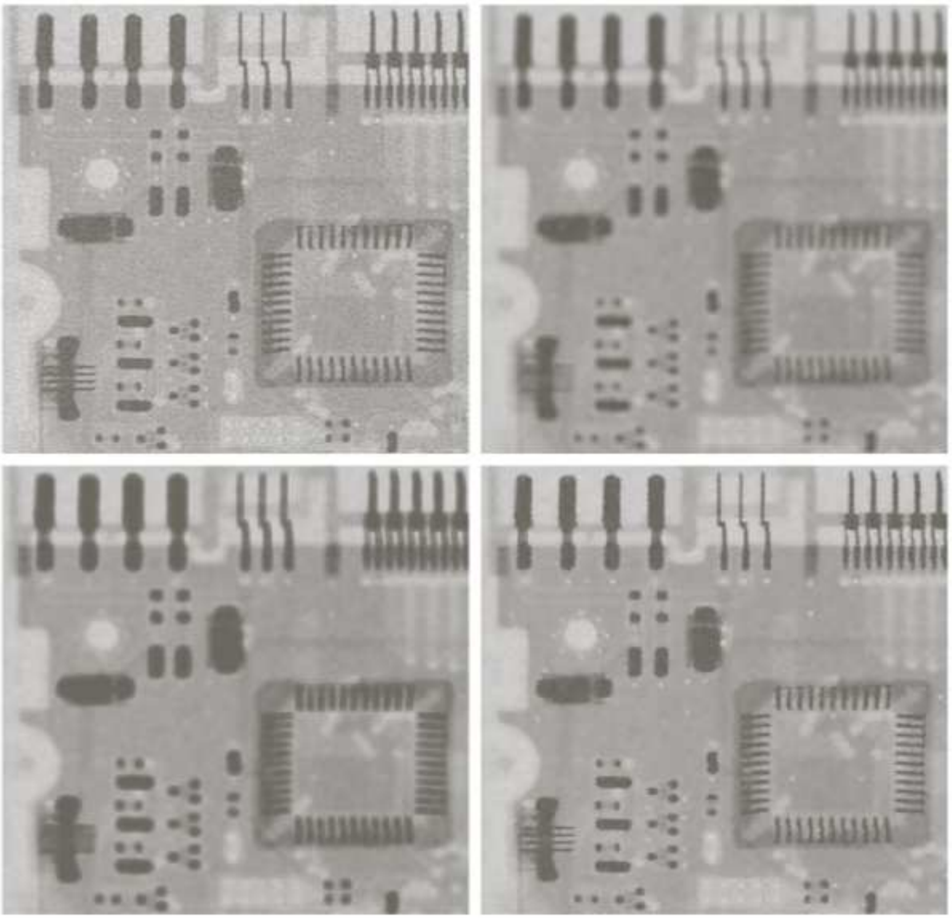
Adaptive, local noise reduction filter

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

- based on local mean (average intensity) m_L and local variance (contrast) σ_L^2
- if $\sigma_\eta^2 = 0$, no change
- if $\sigma_L^2 > \sigma_\eta^2$, edge, keep unchanged or less changed
- if $\sigma_L^2 \approx \sigma_\eta^2$, the m_L returns

- only the variance of corrupting noise $\hat{f}(x,y) \geq 0$ needed to be known or estimated
- assume $\sigma_\eta^2 \leq \sigma_L^2$, otherwise, set the ratio =1

Examples



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Adaptive Median Filter

Adaptive median filter

Whether z_{med} is NOT impulse

Stage A: $A1 = z_{med} - z_{min}$
$A2 = z_{med} - z_{max}$
If $A1 > 0$ AND $A2 < 0$, go to stage B
Else increase the window size
If window size $\leq S_{max}$ repeat stage A
Else output z_{med}

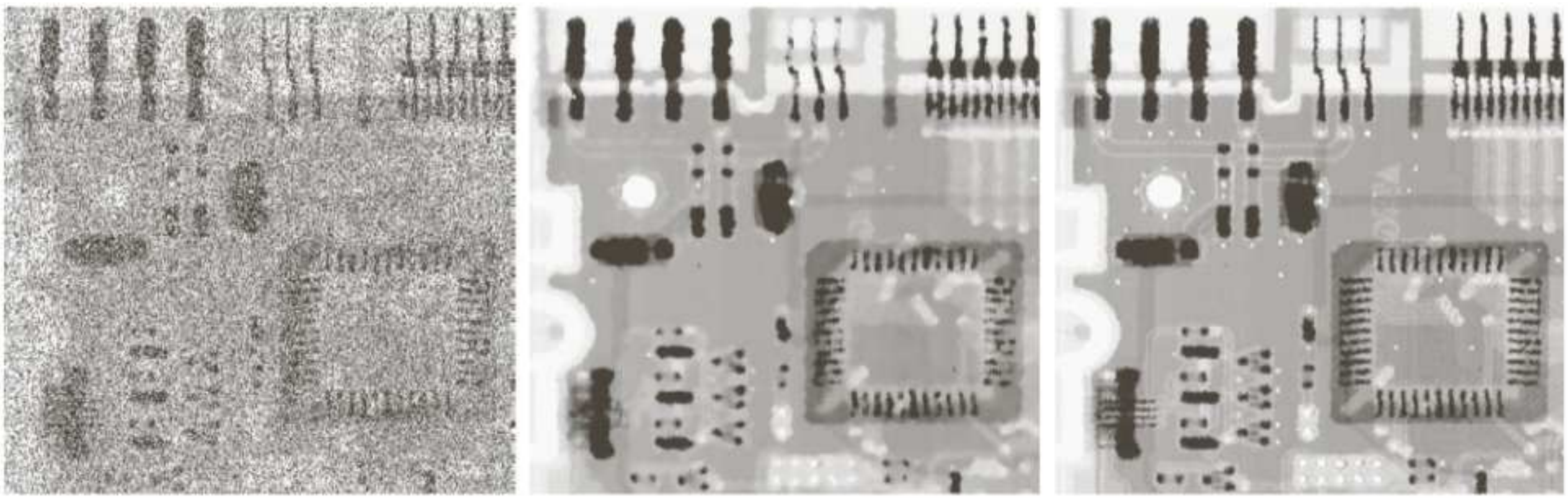
Stage B: $B1 = z_{xy} - z_{min}$
$B2 = z_{xy} - z_{max}$
If $B1 > 0$ AND $B2 < 0$, output z_{xy}
Else output z_{med}

z_{min} = minimum intensity value in S_{xy}
z_{max} = maximum intensity value in S_{xy}
z_{med} = median of intensity values in S_{xy}
z_{xy} = intensity value at coordinates (x, y)
S_{max} = maximum allowed size of S_{xy}

Whether z_{xy} is NOT an extreme value

- representing the restored pixel value at (x,y) by executing pseudocode
- the size of filter window is adaptive
- three purposes: to remove salt-and-pepper noise (capable of handling large P_a and P_b), to smooth non-impulsive noise, and to reduce distortion, e.g., excessive thinning or thickening of object boundaries
- performance is better than un-adaptive median filter

Examples



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Inverse Filtering

If H is given or estimated, the simplest approach to restoration is direct inverse filtering

$$G(u,v) = H(u,v) \bullet F(u,v) + N(u,v) \xrightarrow[\bullet 1/H(u,v)]{} \underbrace{\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}}_{\text{(Eq. 5.7-1)}} \cdots \underbrace{F(u,v) + \frac{N(u,v)}{H(u,v)}}_{\text{(Eq. 5.7-2)}}$$

- Even if H completely known, F cannot be exactly recovered because N is not known.
- If H has zero or very small values, the ratio N/H may predominate

– One way out is to limit the filter frequencies to values near the origin

Effect of Inverse Filtering

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$H(u,v) = e^{-k[(u-M/2)^2+(v-N/2)^2]^{5/6}}$$

Wiener Filtering

- Traditional inverse filtering does not explicitly deal with the noise
- Wiener Filtering considers both the degradation and the statistical characteristics of the noise
- The objective is to find an image estimate such that the mean square error (MSE) between the uncorrupted image f and the estimate $\hat{f}$ is minimized.

$$\text{minimize} \quad E\left\{(f-\hat{f})^2\right\}$$

Wiener Filter: Solution

Based on aforementioned conditions, the image estimate in the frequency domain is given by

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v) \qquad \text{(Eq. 5.8-2)}$$

$H^*(u,v) =$ complex conjugate of $H(u,v)$

$\left|H(u,v)\right|^2 = H^*(u,v)H(u,v)$

$S_\eta(u,v) = \left|N(u,v)\right|^2 =$ power spectrum density of the noise

$S_f(u,v) = \left|F(u,v)\right|^2 =$ power spectrum density of the undegraded image

Wiener Filter (cont'd)

Measures based on the power spectra of noise and of the undegraded image characterize the performance of restoration algorithms.

$$MSE = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y)-\hat{f}(x,y)]^2$$

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2} \text{ or } \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y)-\hat{f}(x,y)\right]^2}$$

Wiener Filter (cont'd)

When noise spectrum is constant (white noise), things can be considerably simplified; however, the spectrum of undegraded image seldom is known.

Under this circumstance, the image estimate, Eq. 5.8-2, may be rewritten as

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v) \qquad \text{(Eq. 5.8-6)}$$

where K is a specified constant.

This equation is often actually utilized for Wiener filtering.

Wiener Filtering: Results

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

K manually adjusted to yield the best visual results

Wiener filtering yielded a result very closer to the original image.