The Way of Machine Thinking

Volume 1:
Rules of Universal Language and Fundamental Math

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1 Preface

The purpose of this book is to create a universal language for machine thinking. The language is completely independent of human natural language, and it is a closed system that recognizes self-definitions and self-explanations. In fact, this book does not rely on natural language, but only introduces the basic structure of universal language through natural language in the first chapter. In the following chapters, the universal language will be used entirely. Therefore, readers need to think in terms of universal language.

Definitions, axioms, theorems, and proofs are the entire content of this first volume. To simplify, easier theorems will omit proofs. At the same time, theorems that can be proved using similar methods are no longer listed. Finally, in the more complex proof process, frequently occurring steps, especially the recovery process, will be omitted.

For the arrangement of the chapters, some basic propositional concepts are defined to explain the basic principles of data structure and operation, at the same time, to prove a series of important theorems. Later chapters will define the concepts of number, addition, and multiplication, and demonstrate the fundamental laws of arithmetic. Finally, the last chapter defines paradox and proves that a paradox cannot lead to a contradiction.

The concepts of infinity, abstraction, composition, and deeper mathematical concepts will be covered in the next volume.

2 Introduction

2.1 General

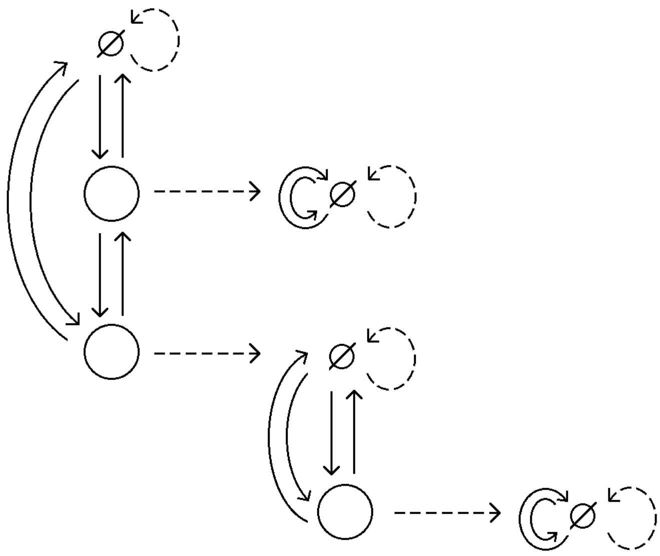
Universal language is a tool for machine thinking. It can not only execute code, but also infer rules. These processes are equivalent. Rules and inferences are the foundation of machine thinking. Firstly, they can define concepts, and secondly, they can explain principles of universal language and mathematics.

Universal language structure:
data structure → operations → language system → logic system → propositions system → axioms system → theorems system.

Theorems system structure:
basic theorems system → mathematics system → virtual world system → physical world system → society.

2.2 Data structure

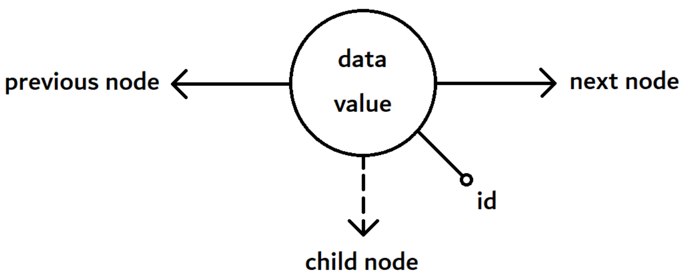
Universal language is based on data structure. The basic element of data structure is node. This data structure is tree-like multidimensional structure, and it is doubly linked circular in one dimension.



Data structure

2.2.1 Node

A node consists of:
1. Data value.
2. Link, pointing to the next node.
3. Link, pointing to the previous node.
4. Link, pointing to the child node.
5. Unique node id.



Node

2.2.2 Empty node

The empty node is ϕ. There is exactly one empty node in one dimension. An empty node is used to identify the start and end of a one-dimensional loop. An empty node has no child node, so it points to itself.

2.3 Operations

Operator is a Operation instruction. There are 11 operators:
⊙, ⊚, ⧂, ⅄⃝, ⧃, ⊜, ⇒⃝, ⊕, ⊖, ⤓⃝, ⊗.

Operand is a variable expressed in conjunction with operators. An operand can be interpreted as a pointer to a node within a data structure.

An operation consists of an operator and several operands. Example:
i⅄⃝j is an operation, i and j is an operand, ⅄⃝ is an operator.

Operations:
⊙i : Create a new operand i, pointing to a unique global data structure.

⊚i : Create a new operand i, pointing to a temporary newly allocated data structure.

i⅄⃝j : Create a new operand j. i and j point to the same node.

i⧂j : Create a new operand j pointing to a temporary newly allocated data structure. The data value of the node pointed to by j is the id of the node pointed to by i.

i⧃j : Create a new operand j. j points to a child of the node pointed to by i.

i⤓⃝ : Release operand i. If i is the last operand that points to a temporary data structure, free the temporary data structure.

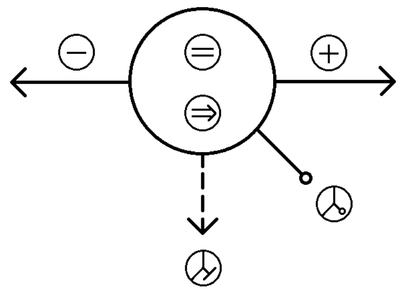
$i\oplus$: Move i to the next node.

$i\ominus$: Move i to the previous node.

i⊜$j \left[\begin{array}{l} , codeA, \\ \\ , codeB, \end{array} \right]$: Compare the value of the node pointed to by i with the value of the node pointed to by j. If equal, codeA executes, otherwise codeB executes.

i⇒⃝j : Insert a new non-empty node or delete a non-empty node.

$\otimes$: Mean logic error and halt the program.



Node operation

2.4 Language

2.4.1 Code

Code consists of operations, functions, and ",". "," is not only a connector for multiple operations but also empty code. Code variables are represented as © and alphanumerics, it represents anyone of the set of all code.
The syntax of the code:
Operand names cannot conflict. An existing operand cannot be created until it is released. A released operand can't be used until it is created.

2.4.2 Function

Functions are how concepts are defined.

Syntax of the function:

";" is the delimiter for operand parameters.

$fn(L) \Leftrightarrow codeA$. The input operands of codeA are the operands in the parameter list L, any operands created in codeA must be released in codeA.

$fn(L) : r \Leftrightarrow codeB$. The input operands of codeB are the operands in the parameter list L, any operands created in codeB must be released in codeB except operand r. This is to ensure the closure of the function and to ensure consistency in the inference replacement process.

The difference in functions:

The name of the function: R(i), T(i) are different.

The number of parameters: Fn(i), Fn(i;j) are different.

The differences in parameter names: Fn(i;j),Fn(j;i) are the same. Fn(i;j), Fn(i;i) are different. $i{\to}j, i{\to}i$ are different.

Type of functions:

General function:

$$,Cpo(r), \iff ,r☮m, m⊝r, m⏚,$$

Branch function:

$$,if(i=j)\!-\!\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \iff ,i⊜j\!-\!\left[\begin{array}{l}, \\ \\ ,\end{array}\right.$$

Proposition function:

$$,i=j, \iff ,if(i=j)\!-\!\left[\begin{array}{l}, \\ \\ ,\otimes,\end{array}\right]\!-$$

Recursive function:

$$,R(i), \iff ,if(i=\varnothing)\!-\!\left[\begin{array}{l}, \\ \\ ,i\oplus, R(i),\end{array}\right]\!-,$$

Return function:

$$,i+j:r, \iff ,\circledcirc r, i☮i_0, j☮j_0, r☮r_0, Rcpo(i_0;r_0), Rcpo(j_0;r_0), i_0⏚, j_0⏚, r_0⏚,$$

r is return operand.

2.4.3 Flag object

Flag objects are named by "&" and alphanumerics , and are used to represent special properties of data structures, operations, and codes. Flag objects are defined by rules. Flag objects can combine the symbol and operand parameters.Example:
$\&SHi⚲i$, $\&SHi{\to}i$, $\&Fam(i)$.

2.5 Logic system

2.5.1 rule text

A rule text consists of code, code variables, and flag objects. Rule text variables are named by ⓣ and alphanumerics. It represents anyone of the set of all rule text.

2.5.2 rule

Given that A, B are rule text. The rule format : $A \Leftrightarrow B$.

A rule is used to represent two equivalent rule text, which can replace each other. A, B must start and end with ",".

When a code variable exists in a rule, it represents the set of all rules that replace the code variable with a code constant. Example:

$$\left.\begin{matrix},\\,\end{matrix}\right],\copyright code, \Leftrightarrow \left.\begin{matrix},\copyright code,\\,\copyright code,\end{matrix}\right],$$

means:

$$\left.\begin{matrix},\\,\end{matrix}\right],\otimes, \Leftrightarrow \left.\begin{matrix},\otimes,\\,\otimes,\end{matrix}\right],$$

$$\left.\begin{matrix},\\,\end{matrix}\right],\circledcirc i, i\textcircled{$\downarrow$}, \Leftrightarrow \left.\begin{matrix},\circledcirc i,i\textcircled{$\downarrow$},\\,\circledcirc i,i\oplus,\end{matrix}\right],$$

$$\left.\begin{matrix},\\,\end{matrix}\right],i\oplus, \Leftrightarrow \left.\begin{matrix},i\oplus,\\,i\oplus,\end{matrix}\right],$$

... ...

A rule has nothing to do with the naming of the operands in the rule, as long as the names do not conflict. Example:

$$,i\oplus,i\ominus, \Leftrightarrow ,i\ominus,i\oplus,$$

$$,j\oplus,j\ominus, \Leftrightarrow ,j\ominus,j\oplus,$$

the same rule.

$$, \Leftrightarrow ,\circledcirc i, i\textcircled{$\downarrow$},$$

$$, \Leftrightarrow ,\circledcirc j, j\textcircled{$\downarrow$},$$

the same rule.

$$,i\oplus,i\ominus, \Leftrightarrow ,i\ominus,i\oplus,$$

$$,i\oplus,j\ominus, \Leftrightarrow ,j\ominus,i\oplus,$$

different rules.

We simplify rule ($A \Leftrightarrow A, B$) to rule($A \Leftrightarrow \sim, B$).

2.5.3 inference

Inference format: premise $\Rightarrow$ conclusion.

Inference: if the premise exists, then the conclusion exists. Inference can be axiom or theorem.

Premise can be one of more rules or inferences. Conclusion is a rule.

When there is a rule text variable in an inference, it represents the set of all inferences that replace the rule text variable with the rule text constant.

How to infer?

If inference (premise $\implies$ conclusion) exists and premise exists, then conclusion exists.

How to get an inference?

If the premise is assumed to exist, conclusion can be inferred. Then we can get an inference (premise $\implies$ conclusion). If conclusion exists, then inference (any premise $\implies$ conclusion) always exists.

2.5.4 Type of rule or inference

Axiom

Axioms describe the natural properties of data structure, operations, code, and rule. Axioms do not need to be proved.

Definition

Concepts are defined by means of rules. A function is a definition. Flag objects can be defined by commutative rules.

Theorem

A theorem is a conclusion of inference. A theorem requires proof.

2.5.5 inference axiom of rule equivalence

Given that A, B, M, N are rule text.

Equivalent commutativity:

$$A \Leftrightarrow B \implies B \Leftrightarrow A$$

Equivalent transitivity:

$$\left\{ \begin{matrix} A \Leftrightarrow B \\ B \Leftrightarrow C \end{matrix} \right\} \implies A \Leftrightarrow C$$

Equivalent substitution:

$$A \Leftrightarrow B \implies MAN \Leftrightarrow MBN$$

Rule text(M A N) and rule text(M B N) must not have naming conflicts. rule text A and rule text B must be in the same “,” start position and “,” end position.

naming conflict:

$$, \Leftrightarrow ,i\text{☮}j,j\text{⏚}, \implies ,j\oplus, \Leftrightarrow ,j\oplus,i\text{☮}j,j\text{⏚},$$

should be:

$$, \Leftrightarrow ,i\text{☮}t,t\text{⏚}, \implies ,j\oplus, \Leftrightarrow ,j\oplus,i\text{☮}t,t\text{⏚},$$

naming conflict:

$$,\circledcirc i, \Leftrightarrow ,\circledcirc i,i!\text{♂}j, \implies ,j\text{⏚},\circledcirc i, \Leftrightarrow ,j\text{⏚},\circledcirc i,i!\text{♂}j,$$

should be:

$$,\circledcirc i, \ \Leftrightarrow \ ,\circledcirc i, i!\text{♂}t, \ \Rightarrow \ ,j\text{⊕}, \circledcirc i, \ \Leftrightarrow \ ,j\text{⊕}, \circledcirc i, i!\text{♂}t,$$

2.5.6 contradiction rule

Contradiction rule:

$$, \ \Leftrightarrow \ ,\otimes,$$

If inference(premise $\Rightarrow$, $\Leftrightarrow$ $,\otimes,$) exists, then premise is not compatible with existing system. Premise can be axiom or definition. The contradiction rule is not compatible with existing system.

2.5.7 proof

Example:

$$,i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, i\oplus,$$

proof:
$inference : Equivalent\,substitution.$
$premise : , \ \Leftrightarrow \ ,j\oplus, j\ominus, (axiom)$
$conclusion : ,i\oplus, j\ominus, \ \Leftrightarrow \ ,j\oplus, j\ominus, i\oplus, j\ominus,$

$inference : Equivalent\,substitution.$
$premise : ,j\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, j\oplus, (axiom)$
$conclusion : ,j\oplus, j\ominus, i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, j\oplus, i\oplus, j\ominus,$

$inference : Equivalent\,transitivity.$
$premise1 : ,i\oplus, j\ominus, \ \Leftrightarrow \ ,j\oplus, j\ominus, i\oplus, j\ominus,$
$premise2 : ,j\oplus, j\ominus, i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, j\oplus, i\oplus, j\ominus,$
$conclusion : ,i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, j\oplus, i\oplus, j\ominus,$

$inference : Equivalent\,substitution.$
$premise : ,j\oplus, i\oplus, \ \Leftrightarrow \ ,i\oplus, j\oplus, (axiom)$
$conclusion : ,j\ominus, j\oplus, i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, i\oplus, j\oplus, j\ominus,$

$inference : Equivalent\,transitivity.$
$premise1 : ,i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, j\oplus, i\oplus, j\ominus,$
$premise2 : ,j\ominus, j\oplus, i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, i\oplus, j\oplus, j\ominus,$
$conclusion : ,i\oplus, j\ominus, \ \Leftrightarrow \ ,j\ominus, i\oplus, j\oplus, j\ominus,$

$inference : Equivalent\,commutativity.$
$premise : , \ \Leftrightarrow \ ,j\oplus, j\ominus, (axiom)$
$conclusion : ,j\oplus, j\ominus, \ \Leftrightarrow \ ,$

$inference : Equivalent\,substitution.$

$premise:,j\oplus,j\ominus,\ \Leftrightarrow\ ,(proved)$
$conclusion:,j\ominus,i\oplus,j\oplus,j\ominus,\ \Leftrightarrow\ ,j\ominus,i\oplus,$

$inference: Equivalent\,transitivity.$
$premise1:,i\oplus,j\ominus,\ \Leftrightarrow\ ,j\ominus,i\oplus,j\oplus,j\ominus,$
$premise2:,j\ominus,i\oplus,j\oplus,j\ominus,\ \Leftrightarrow\ ,j\ominus,i\oplus,$
$conclusion:,i\oplus,j\ominus,\ \Leftrightarrow\ ,j\ominus,i\oplus,$

$simplify:$
$,i\oplus,j\ominus,$

$\Leftrightarrow\ ,j\oplus,j\ominus,i\oplus,j\ominus,(,\ \Leftrightarrow\ ,j\oplus,j\ominus,)$

$\Leftrightarrow\ ,j\ominus,j\oplus,i\oplus,j\ominus,(,j\oplus,j\ominus,\ \Leftrightarrow\ ,j\ominus,j\oplus,)$

$\Leftrightarrow\ ,j\ominus,i\oplus,j\oplus,j\ominus,(,j\oplus,i\oplus,\ \Leftrightarrow\ ,i\oplus,j\oplus,)$

$\Leftrightarrow\ ,j\ominus,i\oplus,(,\ \Leftrightarrow\ ,j\oplus,j\ominus,)$

$Minimize:$
$,i\oplus,j\ominus,$

$\Leftrightarrow\ ,j\oplus,j\ominus,i\oplus,j\ominus,$

$\Leftrightarrow\ ,j\ominus,j\oplus,i\oplus,j\ominus,$

$\Leftrightarrow\ ,j\ominus,i\oplus,j\oplus,j\ominus,$

$\Leftrightarrow\ ,j\ominus,i\oplus,$

2.6 Propositions system

We describe laws and properties through propositions. Propositions come from operator of equal comparison($i⊜j\left[\begin{smallmatrix}, \\ ,\end{smallmatrix}\right.$), but only one branch can be executed depending on the axioms.

Before defining propositions , we should define branch function($if(p)\left[\begin{smallmatrix}, \\ ,\end{smallmatrix}\right.$).

Definition of propositions:

$$,p,\ \Leftrightarrow\ ,if(p)\left[\begin{matrix}, & \\ & \\ , & \otimes,\end{matrix}\right],$$

and

$$, !p, \;\Leftrightarrow\; , if(p) \left[\begin{array}{l} , \otimes , \\ \\ , \end{array} \right] ,$$

3 Rules of Operators

3.1 Axioms of Operators

3.1.1 swap axioms of operator

id operator:

$$, i\idop m, i\idop n, \iff , i\idop n, i\idop m,$$

$$, i\idop m, j\idop n, \iff , j\idop n, i\idop m,$$

$$, i\idop m, j\copyop n, \iff , j\copyop n, i\idop m,$$

$$, i\idop m, j\swapop n, \iff , j\swapop n, i\idop m,$$

$$, i\idop m, \odot n, \iff , \odot n, i\idop m,$$

$$, i\idop m, \circledcirc n, \iff , \circledcirc n, i\idop m,$$

$$, i\idop m, j\oplus, \iff , j\oplus, i\idop m,$$

$$, i\idop m, j\obot, \iff , j\obot, i\idop m,$$

$$, i\idop m, j\circledequal t\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\circledequal t\!-\!\left[\begin{array}{l} , i\idop m, \\ \\ , i\idop m, \end{array}\right.$$

copy operator:

$$, i\copyop m, j\copyop n, \iff , j\copyop n, i\copyop m,$$

$$, i\copyop m, j\swapop n, \iff , j\swapop n, i\copyop m,$$

$$, i\copyop m, \odot n, \iff , \odot n, i\copyop m,$$

$$, i\copyop m, \circledcirc n, \iff , \circledcirc n, i\copyop m,$$

$$, i\copyop m, j\oplus, \iff , j\oplus, i\copyop m,$$

$$, i\text{☮}m, j\text{⏚}, \iff , j\text{⏚}, i\text{☮}m,$$

$$, i\text{☮}m, j\text{⊜}t\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\text{⊜}t\!-\!\left[\begin{array}{l} , i\text{☮}m, \\ \\ , i\text{☮}m, \end{array}\right.$$

subnode operator:

$$, i\text{⦸}m, j\text{⦸}n, \iff , j\text{⦸}n, i\text{⦸}m,$$

$$, i\text{⦸}m, \odot n, \iff , \odot n, i\text{⦸}m,$$

$$, i\text{⦸}m, \circledcirc n, \iff , \circledcirc n, i\text{⦸}m,$$

$$, i\text{⦸}m, j\oplus, \iff , j\oplus, i\text{⦸}m,$$

$$, i\text{⦸}m, j\text{⏚}, \iff , j\text{⏚}, i\text{⦸}m,$$

$$, i\text{⦸}m, j\text{⊜}t\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\text{⊜}t\!-\!\left[\begin{array}{l} , i\text{⦸}m, \\ \\ , i\text{⦸}m, \end{array}\right.$$

temporary space operator:

$$, \circledcirc m, \circledcirc n, \iff , \circledcirc n, \circledcirc m,$$

$$, \circledcirc m, \odot n, \iff , \odot n, \circledcirc m,$$

$$, \circledcirc m, j\oplus, \iff , j\oplus, \circledcirc m,$$

$$, \circledcirc m, j\text{⏚}, \iff , j\text{⏚}, \circledcirc m,$$

$$, \circledcirc m, j\text{⊜}t\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\text{⊜}t\!-\!\left[\begin{array}{l} , \circledcirc m, \\ \\ , \circledcirc m, \end{array}\right.$$

global space operator:

$$,\odot m, \odot n, \iff , \odot n, \odot m,$$

$$,\odot m, j\oplus, \iff , j\oplus, \odot m,$$

$$,\odot m, j\textcircled{$\downarrow$}, \iff , j\textcircled{$\downarrow$}, \odot m,$$

$$,\odot m, j\textcircled{$=$}t - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\textcircled{$=$}t - \left[\begin{array}{l} ,\odot m, \\ \\ ,\odot m, \end{array}\right.$$

next node operator:

$$,i\oplus, j\oplus, \iff , j\oplus, i\oplus,$$

$$,i\oplus, i\ominus, \iff , i\ominus, i\oplus,$$

$$,i\oplus, j\textcircled{$\downarrow$}, \iff , j\textcircled{$\downarrow$}, i\oplus,$$

$$,i\oplus, j\textcircled{$=$}t - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\textcircled{$=$}t - \left[\begin{array}{l} ,i\oplus, \\ \\ ,i\oplus, \end{array}\right.$$

release operator:

$$,i\textcircled{$\downarrow$}, j\textcircled{$\downarrow$}, \iff , j\textcircled{$\downarrow$}, i\textcircled{$\downarrow$},$$

$$,i\textcircled{$\downarrow$}, j\textcircled{$=$}t - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\textcircled{$=$}t - \left[\begin{array}{l} ,i\textcircled{$\downarrow$}, \\ \\ ,i\textcircled{$\downarrow$}, \end{array}\right.$$

$$,i\textcircled{$\downarrow$}, j\textcircled{$=$}j - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \iff , j\textcircled{$=$}j - \left[\begin{array}{l} ,i\textcircled{$\downarrow$}, \\ \\ ,i\textcircled{$\downarrow$}, \end{array}\right.$$

3.1.2 fundamental axioms of logic error operator

$$,\otimes, \iff ,\otimes,\copyright code,$$

$$,i\oplus,\otimes, \iff ,\otimes,$$

$$,i\textcircled{$\underline{\downarrow}$},\otimes, \iff ,\otimes,$$

$$,i\textcircled{$\Rightarrow$}j,\otimes, \iff ,\otimes,$$

3.1.3 fundamental axioms of equivalent comparison operator

$$\left.\begin{array}{l},\\,\end{array}\right]-,\copyright c, \iff \left.\begin{array}{l},\copyright c,\\,\copyright c,\end{array}\right]-,$$

$$,i\circledequal i-\left[\begin{array}{l},\\ \\,\end{array}\right. \iff ,i\circledequal i-\left[\begin{array}{l},\\ \\,\otimes,\end{array}\right.$$

$$,i\circledequal j-\left[\begin{array}{l},\\ \\,\end{array}\right. \iff ,j\circledequal i-\left[\begin{array}{l},\\ \\,\end{array}\right.$$

$$,i\circledequal j-\left[\begin{array}{l},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right]-, \iff ,i\circledequal j-\left[\begin{array}{l},i\circledequal j-\left[\begin{array}{l},\copyright c_1,\\ \\,\copyright c_3,\end{array}\right]-,\\,\copyright c_2,\end{array}\right]-,$$

$$,i\circledequal j-\left[\begin{array}{l},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right]-, \iff ,i\circledequal j-\left[\begin{array}{l},\copyright c_1,\\,i\circledequal j-\left[\begin{array}{l},\copyright c_3,\\ \\,\copyright c_2,\end{array}\right]-,\end{array}\right]-,$$

$$,i\circledequal j-\left[\begin{array}{l}i\circledequal m-\left[\begin{array}{l},\\ \\,\end{array}\right.\\ \\,\end{array}\right. \iff ,i\circledequal j-\left[\begin{array}{l},j\circledequal m-\left[\begin{array}{l},\\ \\,\end{array}\right.\\ \\,\end{array}\right.$$

$$,i\circledequal j-\left[\begin{array}{l},m\circledequal n-\left[\begin{array}{l},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right]-,\\,m\circledequal n-\left[\begin{array}{l},\copyright c_3,\\ \\,\copyright c_4,\end{array}\right]-,\end{array}\right]-, \iff ,m\circledequal n-\left[\begin{array}{l},i\circledequal j-\left[\begin{array}{l},\copyright c_1,\\ \\,\copyright c_3,\end{array}\right]-,\\,i\circledequal j-\left[\begin{array}{l},\copyright c_2,\\ \\,\copyright c_4,\end{array}\right]-,\end{array}\right]-,$$

3.1.4 fundamental axioms of release operator

$$, i\oplus, i⏚, \;\Leftrightarrow\; , i⏚,$$

3.1.5 axioms of creativity

$$, \;\Leftrightarrow\; , i\circeq j\text{-}\left[\begin{smallmatrix} ' \\ , \end{smallmatrix}\right]\text{-},$$

$$, \;\Leftrightarrow\; , i\circledast m, m⏚,$$

$$, \;\Leftrightarrow\; , i\circledast m, m⏚,$$

$$, \;\Leftrightarrow\; , i\circledast m, m⏚,$$

$$, \;\Leftrightarrow\; , \odot m, m⏚,$$

$$, \;\Leftrightarrow\; , \circledcirc m, m⏚,$$

$$, \;\Leftrightarrow\; , i\oplus, i\ominus,$$

$$, \;\Leftrightarrow\; , ,$$

3.2 Theorems of Operators

3.2.1 theorems of previous node operator

$$, i\circledast m, j\ominus, \iff , j\ominus, i\circledast m,$$

proof:

$, i\circledast m, j\ominus,$

$\iff , j\oplus, j\ominus, i\circledast m, j\ominus,$

$\iff , j\ominus, j\oplus, i\circledast m, j\ominus,$

$\iff , j\ominus, i\circledast m, j\oplus, j\ominus,$

$\iff , j\ominus, i\circledast m,$

$$, i\circledast m, j\ominus, \iff , j\ominus, i\circledast m,$$
$$, i\circledast m, j\ominus, \iff , j\ominus, i\circledast m,$$

$$, \odot m, j\ominus, \iff , j\ominus, \odot m,$$
$$, \circledcirc m, j\ominus, \iff , j\ominus, \circledcirc m,$$

$$, i\oplus, j\ominus, \iff , j\ominus, i\oplus,$$

proof:

$, i\oplus, j\ominus,$

$\iff , j\oplus, j\ominus, i\oplus, j\ominus,$

$\iff , j\ominus, j\oplus, i\oplus, j\ominus,$

$\iff , j\ominus, i\oplus, j\oplus, j\ominus,$

$\iff , j\ominus, i\oplus,$

$$, i\ominus, j\ominus, \iff , j\ominus, i\ominus,$$

proof:

$, i\ominus, j\ominus,$

$\Leftrightarrow\ , j\oplus, j\ominus, i\ominus, j\ominus,$

$\Leftrightarrow\ , j\ominus, j\oplus, i\ominus, j\ominus,$

$\Leftrightarrow\ , j\ominus, i\ominus, j\oplus, j\ominus,$

$\Leftrightarrow\ , j\ominus, i\ominus,$

$$, i\ominus, j\text{\textcircled{$\downarrow$}}, \ \Leftrightarrow\ , j\text{\textcircled{$\downarrow$}}, i\ominus,$$

proof:

$, i\ominus, j\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\ominus, j\text{\textcircled{$\downarrow$}}, i\oplus, i\ominus,$

$\Leftrightarrow\ , i\ominus, i\oplus, j\text{\textcircled{$\downarrow$}}, i\ominus,$

$\Leftrightarrow\ , i\oplus, i\ominus, j\text{\textcircled{$\downarrow$}}, i\ominus,$

$\Leftrightarrow\ , j\text{\textcircled{$\downarrow$}}, i\ominus,$

$$, i\ominus, j\text{\textcircled{=}}t\left[\begin{array}{l} , \\ \\ , \end{array}\right. \ \Leftrightarrow\ , j\text{\textcircled{=}}t\left[\begin{array}{l} , i\ominus, \\ \\ , i\ominus, \end{array}\right.$$

proof:

$, i\ominus, j\text{\textcircled{=}}t\left[\begin{array}{l} , \\ \\ , \end{array}\right.$

$\Leftrightarrow\ , i\ominus, j\text{\textcircled{=}}t\left[\begin{array}{l} , i\oplus, i\ominus, \\ \\ , i\oplus, i\ominus \end{array}\right.$

$\Leftrightarrow\ , i\ominus, i\oplus, j\text{\textcircled{=}}t\left[\begin{array}{l} , i\ominus, \\ \\ , i\ominus, \end{array}\right.$

$\Leftrightarrow\ , i\oplus, i\ominus, j\text{\textcircled{=}}t\left[\begin{array}{l} , i\ominus, \\ \\ , i\ominus, \end{array}\right.$

$$\Leftrightarrow\ , j\textcircled{$=$}t\left[\begin{array}{l} , i\ominus, \\ \\ , i\ominus, \end{array}\right.$$

$$, i\ominus, i\textcircled{$\downarrow$}, \ \Leftrightarrow\ , i\textcircled{$\downarrow$},$$

proof:

$$, i\ominus, i\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , i\ominus, i\oplus, i\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , i\oplus, i\ominus, i\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , i\textcircled{$\downarrow$},$$

$$, i\ominus, \otimes, \ \Leftrightarrow\ , \otimes,$$

proof:

$$, i\ominus, \otimes,$$

$$\Leftrightarrow\ , i\ominus, i\oplus, \otimes,$$

$$\Leftrightarrow\ , i\oplus, i\ominus, \otimes,$$

$$\Leftrightarrow\ , \otimes,$$

3.2.2 theorems of logic error operator

$$, \otimes, \ \Leftrightarrow\ , \otimes, \otimes,$$

$$\left.\begin{array}{l} , \copyright c, \\ , \otimes, \end{array}\right], \ \Leftrightarrow\ \left.\begin{array}{l} , \\ , \otimes, \end{array}\right], \copyright c,$$

proof:

$$\left.\begin{array}{l} , \copyright c, \\ , \otimes, \end{array}\right],$$

$$\Leftrightarrow\ \left.\begin{array}{l} , \copyright c, \\ , \otimes, \copyright c, \end{array}\right],$$

$$\Leftrightarrow\ \left.\begin{array}{l} , \\ , \otimes, \end{array}\right], \copyright c,$$

$$\left.\begin{array}{l} , \otimes, \\ , \copyright c, \end{array}\right], \ \Leftrightarrow\ \left.\begin{array}{l} , \otimes, \\ , \end{array}\right], \copyright c,$$

$$, i \text{☮} t, \otimes, \quad \Leftrightarrow \quad , \otimes,$$

proof:

$, i \text{☮} t, \otimes,$

$\Leftrightarrow \ , i \text{☮} t, t \circledast, \otimes,$

$\Leftrightarrow \ , \otimes,$

$$, i \text{☮} t, \otimes, \quad \Leftrightarrow \quad , \otimes,$$

$$, i \text{☮} t, \otimes, \quad \Leftrightarrow \quad , \otimes,$$

$$, \odot i, \otimes, \quad \Leftrightarrow \quad , \otimes,$$

$$, \circledcirc i, \otimes, \quad \Leftrightarrow \quad , \otimes,$$

4 Rules of Three Fundamental Relationships

4.1 Definition of Relationships

4.1.1 Definition of node value comparison

Branch function:

$$,if(i=i)-\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ ,i\circleddash i-\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

Propositions:

$$,i=i, \ \Leftrightarrow \ ,if(i=i)-\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right]-,$$

Branch function:

$$,if(i=j)-\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ ,i\circleddash j-\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

Propositions:

$$,i=j, \ \Leftrightarrow \ ,if(i=j)-\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right]-,$$

$$,i \mathrel{!=} j, \ \Leftrightarrow \ ,if(i=j)-\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right]-,$$

4.1.2 Definition of node null comparison

Branch function:

$$, if(i = \varnothing) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \; , \circledcirc j, if(i = j) \left[\begin{array}{l} , j\textcircled{$\downarrow$}, \\ \\ , j\textcircled{$\downarrow$}, \end{array}\right.$$

Propositions:

$$, i = \varnothing, \;\Leftrightarrow\; , if(i = \varnothing) \left[\begin{array}{l} , \\ \\ , \otimes, \end{array}\right],$$

$$, i \,!\!= \varnothing, \;\Leftrightarrow\; , if(i = \varnothing) \left[\begin{array}{l} , \otimes, \\ \\ , \end{array}\right],$$

4.1.3 Definition of identical node comparison

Branch function:

$$, if(i \mathring{\bigcirc} j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \; , i \textcircled{$\curlywedge$} m, j \textcircled{$\curlywedge$} n, if(m = n) \left[\begin{array}{l} , m\textcircled{$\downarrow$}, n\textcircled{$\downarrow$}, \\ \\ , m\textcircled{$\downarrow$}, n\textcircled{$\downarrow$}, \end{array}\right.$$

Propositions:

$$, i \mathring{\bigcirc} j, \;\Leftrightarrow\; , if(i \mathring{\bigcirc} j) \left[\begin{array}{l} , \\ \\ , \otimes, \end{array}\right],$$

$$, i ! \mathring{\bigcirc} j, \;\Leftrightarrow\; , if(i \mathring{\bigcirc} j) \left[\begin{array}{l} , \otimes, \\ \\ , \end{array}\right],$$

4.2 Axioms of Relationships

4.2.1 Substitution axioms of identical node comparison

$$
\begin{aligned}
,i\overset{\circ}{\bigcirc}j, i\circledcirc t, &\iff ,i\overset{\circ}{\bigcirc}j, j\circledcirc t,\\
,i\overset{\circ}{\bigcirc}j, i\circledast t, &\iff ,i\overset{\circ}{\bigcirc}j, j\circledast t,\\
,i\overset{\circ}{\bigcirc}j, i\otimes t, &\iff ,i\overset{\circ}{\bigcirc}j, j\otimes t,
\end{aligned}
$$

$$
,i\overset{\circ}{\bigcirc}j, if(i=j)\!-\!\left[\begin{array}{l},\\ \\ ,\end{array}\right. \iff ,i\overset{\circ}{\bigcirc}j, if(i=i)\!-\!\left[\begin{array}{l},\\ \\ ,\end{array}\right.
$$

4.2.2 Axioms of node id operator and propositions

$$
\begin{aligned}
,i\circledcirc m, &\iff ,i\circledcirc m, m\,!\!=\varnothing,\\
,i\circledcirc m, i\circledcirc n, &\iff ,i\circledcirc m, i\circledcirc n, m=n,
\end{aligned}
$$

4.2.3 Axioms of copy operator and propositions

$$
,i\circledast j, \iff ,i\circledast j, i\overset{\circ}{\bigcirc}j,
$$

4.2.4 Axioms of subnode operator and propositions

$$
,i=\varnothing, i\otimes t, \iff ,i=\varnothing, i\circledast t,
$$

$$
,i\otimes t, \iff ,i\circledcirc t, t=\varnothing,
$$

$$
,i_1\,!\!=\varnothing, i_2\,!\!=\varnothing, i_1\otimes t_1, i_2\otimes t_2, if(i_1\overset{\circ}{\bigcirc}i_2)\!\left[\begin{array}{l},\\ ,\end{array}\right. \iff ,i_1\,!\!=\varnothing, i_2\,!\!=\varnothing, i_1\otimes t_1, i_2\otimes t_2, if(t_1\overset{\circ}{\bigcirc}t_2)\!\left[\begin{array}{l},\\ ,\end{array}\right.
$$

4.2.5 Axioms of global space operator and propositions

$$,\odot i, \odot j, \;\Leftrightarrow\; ,\odot i, i \text{☮} j,$$

$$,\odot i, \;\Leftrightarrow\; ,\odot i, i=\varnothing,$$

4.2.6 Axioms of temporary space operator and propositions

$$,\circledcirc i, \circledcirc j, \;\Leftrightarrow\; ,\circledcirc i, \circledcirc j, i=j,$$

$$,\circledcirc i, i\oplus, \;\Leftrightarrow\; ,\circledcirc i, i\oplus, i=\varnothing,$$

$$,\circledcirc i, \;\Leftrightarrow\; ,\circledcirc i, i!\overset{\circ}{\bigcirc} j,$$

4.2.7 Axioms of next node operator and propositions

$$,i\oplus, j\oplus, if(i\overset{\circ}{\bigcirc} j)\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \;\Leftrightarrow\; ,if(i\overset{\circ}{\bigcirc} j)\!-\!\left[\begin{array}{l} ,i\oplus, j\oplus, \\ \\ ,i\oplus, j\oplus, \end{array}\right.$$

5 Theorems of Relationship of Node Value Comparison

5.1 Branch function to propositions

$$,if(i=j)\left[\begin{array}{l} ,\copyright c, \\ \\ ,\otimes, \end{array}\right], \;\Leftrightarrow\; ,i=j,\copyright c,$$

$$,if(i=j)\left[\begin{array}{l} ,\otimes, \\ \\ ,\copyright c, \end{array}\right], \;\Leftrightarrow\; ,i\,!{=}j,\copyright c,$$

5.2 Unity

$$, \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} , \\ , \end{array}\right],$$

$$,i=j,\otimes, \;\Leftrightarrow\; ,\otimes,$$

proof:

$,i=j,\otimes,$

$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],\otimes,$

$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,\otimes, \\ \\ ,\otimes, \end{array}\right],$

$$\Leftrightarrow\ ,if(i=j)\left[\begin{matrix},\\ \\ ,\end{matrix}\right],\otimes$$

$$\Leftrightarrow\ ,\otimes,$$

$$,i\,!\!=j,\otimes,\ \Leftrightarrow\ ,\otimes,$$

5.3 Symmetry

$$,if(i=j)\left[\begin{matrix},\\ ,\end{matrix}\right. \Leftrightarrow\ ,if(j=i)\left[\begin{matrix},\\ ,\end{matrix}\right.$$

$$,i=j,\ \Leftrightarrow\ ,j=i,$$

$$,i\,!\!=j,\ \Leftrightarrow\ ,j\,!\!=i,$$

5.4 Swap

5.4.1 Branch function and operator

$$,\circledcirc m,if(i=j)\left[\begin{matrix},\\ \\ ,\end{matrix}\right. \Leftrightarrow\ ,if(i=j)\left[\begin{matrix},\circledcirc m,\\ \\ ,\circledcirc m,\end{matrix}\right.$$

$$,\odot m,if(i=j)\left[\begin{matrix},\\ \\ ,\end{matrix}\right. \Leftrightarrow\ ,if(i=j)\left[\begin{matrix},\odot m,\\ \\ ,\odot m,\end{matrix}\right.$$

$$,m\text{☮}n,if(i=j)\left[\begin{matrix},\\ \\ ,\end{matrix}\right. \Leftrightarrow\ ,if(i=j)\left[\begin{matrix},m\text{☮}n,\\ \\ ,m\text{☮}n,\end{matrix}\right.$$

$$,m\text{☮}n,if(i=j)\left[\begin{matrix},\\ \\ ,\end{matrix}\right. \Leftrightarrow\ ,if(i=j)\left[\begin{matrix},m\text{☮}n,\\ \\ ,m\text{☮}n,\end{matrix}\right.$$

$$,m\text{☮}n,if(i=j)\left[\begin{matrix},\\ \\ ,\end{matrix}\right. \Leftrightarrow\ ,if(i=j)\left[\begin{matrix},m\text{☮}n,\\ \\ ,m\text{☮}n,\end{matrix}\right.$$

$$, m\textcircled{$\downarrow$}, if(i=j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i=j)\left[\begin{array}{l} , m\textcircled{$\downarrow$}, \\ \\ , m\textcircled{$\downarrow$}, \end{array}\right.$$

$$, m\oplus, if(i=j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i=j)\left[\begin{array}{l} , m\oplus, \\ \\ , m\oplus, \end{array}\right.$$

$$, m\ominus, if(i=j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i=j)\left[\begin{array}{l} , m\ominus, \\ \\ , m\ominus, \end{array}\right.$$

5.4.2 Branch function and Branch function

$$, if(i=j)\left[\begin{array}{l} , if(m=n)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \\ , if(m=n)\left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right], \ \Leftrightarrow \ , if(m=n)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} , \copyright c_2, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

5.4.3 Branch function and Propositions

$$, m=n, if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i=j)\left[\begin{array}{l} , m=n, \copyright c_1, \\ \\ , m=n, \copyright c_2, \end{array}\right],$$

proof:

$$, m=n, if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , if(m=n)\left[\begin{array}{l} , \\ \\ , \otimes, \end{array}\right], if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(m{=}n)\left[\begin{array}{l} ,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(m{=}n)\left[\begin{array}{l} ,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(i{=}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right],\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(m{=}n)\left[\begin{array}{l} ,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(i{=}j)\left[\begin{array}{l} ,\otimes, \\ \\ ,\otimes, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} ,if(m{=}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\otimes, \end{array}\right], \\ ,if(m{=}n)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\otimes, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} ,m{=}n,\copyright c_1, \\ \\ ,m{=}n,\copyright c_2, \end{array}\right],$$

$$,m\,!{=}n,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\ \Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} ,m\,!{=}n,\copyright c_1, \\ \\ ,m\,!{=}n,\copyright c_2, \end{array}\right],$$

5.4.4 Propositions and operator

$$,i{=}j,\circledcirc m,\ \Leftrightarrow\ ,\circledcirc m,i{=}j,$$

proof:

$, i = j, \circledcirc m,$

$$\Leftrightarrow\ , if(i = j) \left[\begin{array}{l} , \\ \\ , \otimes, \end{array}\right], \circledcirc m,$$

$$\Leftrightarrow\ , if(i = j) \left[\begin{array}{l} , \circledcirc m, \\ \\ , \otimes, \end{array}\right],$$

$$\Leftrightarrow\ , if(i = j) \left[\begin{array}{l} , \circledcirc m, \\ \\ , \circledcirc m, \otimes, \end{array}\right],$$

$$\Leftrightarrow\ , \circledcirc m, if(i = j) \left[\begin{array}{l} , \\ \\ , \otimes, \end{array}\right],$$

$$\Leftrightarrow\ , \circledcirc m, i = j,$$

$$, i = j, \odot m, \quad \Leftrightarrow \quad , \odot m, i = j,$$

$$, i = j, m ☮ n, \quad \Leftrightarrow \quad , m ☮ n, i = j,$$

$$, i = j, m ☮ n, \quad \Leftrightarrow \quad , m ☮ n, i = j,$$

$$, i = j, m ☮ n, \quad \Leftrightarrow \quad , m ☮ n, i = j,$$

$$, i = j, m ⏚, \quad \Leftrightarrow \quad , m ⏚, i = j,$$

$$, i = j, m \oplus, \quad \Leftrightarrow \quad , m \oplus, i = j,$$

$$, i = j, m \ominus, \quad \Leftrightarrow \quad , m \ominus, i = j,$$

$$, i \,!{=}\, j, \circledcirc m, \quad \Leftrightarrow \quad , \circledcirc m, i \,!{=}\, j,$$

$$, i \,!{=}\, j, \odot m, \quad \Leftrightarrow \quad , \odot m, i \,!{=}\, j,$$

$$, i \,!{=}\, j, m ☮ n, \quad \Leftrightarrow \quad , m ☮ n, i \,!{=}\, j,$$

$$, i \,!{=}\, j, m ☮ n, \quad \Leftrightarrow \quad , m ☮ n, i \,!{=}\, j,$$

$$, i \,!{=}\, j, m ☮ n, \quad \Leftrightarrow \quad , m ☮ n, i \,!{=}\, j,$$

$$
\begin{aligned}
,i\,!\!=j,m\textcircled{$\downarrow$}, \;&\Leftrightarrow\; ,m\textcircled{$\downarrow$},i\,!\!=j,\\
,i\,!\!=j,m\oplus, \;&\Leftrightarrow\; ,m\oplus,i\,!\!=j,\\
,i\,!\!=j,m\ominus, \;&\Leftrightarrow\; ,m\ominus,i\,!\!=j,
\end{aligned}
$$

5.4.5 Propositions and Propositions

$$,i=j,m=n, \;\Leftrightarrow\; ,m=n,i=j,$$

proof:

$,i=j,m=n,$

$$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],m=n,$$

$$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,m=n, \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,m=n, \\ \\ ,m=n,\otimes, \end{array}\right],$$

$$\Leftrightarrow\; ,m=n,if(i=j)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\; ,m=n,i=j,$$

$$
\begin{aligned}
,i=j,m\,!\!=n, \;&\Leftrightarrow\; ,m\,!\!=n,i=j,\\
,i\,!\!=j,m\,!\!=n, \;&\Leftrightarrow\; ,m\,!\!=n,i\,!\!=j,
\end{aligned}
$$

5.4.6 Propositions to Propositions with branch function

$$,if(i=j)\left[\begin{array}{l} ,m\,!{=}n, \\ \\ , \end{array}\right], \iff ,if(m=n)\left[\begin{array}{l} ,i\,!{=}j, \\ \\ , \end{array}\right],$$

proof:

$$,if(i=j)\left[\begin{array}{l} ,m\,!{=}n, \\ \\ , \end{array}\right],$$

$$\iff ,if(i=j)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right], \\ \\ , \end{array}\right],$$

$$\iff ,if(i=j)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right], \\ \\ ,if(m=n)\left[\begin{array}{l} , \\ \\ , \end{array}\right], \end{array}\right],$$

$$\iff ,if(m=n)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right], \\ \\ ,if(i=j)\left[\begin{array}{l} , \\ \\ , \end{array}\right], \end{array}\right],$$

$$\iff ,if(m=n)\left[\begin{array}{l} ,i\,!{=}j, \\ \\ , \end{array}\right],$$

$$,if(i=j)\left[\begin{array}{l} , \\ \\ ,m=n, \end{array}\right], \iff ,if(m=n)\left[\begin{array}{l} , \\ \\ ,i=j, \end{array}\right],$$

5.5 Transitivity

5.5.1 Branch function with branch function

$$,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ ,if(i=j)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

5.5.2 Branch function with propositions

$$,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,i=j,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

proof:

$$,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\otimes, \end{array}\right], \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,i=j,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,i\,!\!=j,\copyright c_2, \end{array}\right],$$

5.5.3 Propositions with branch function

$$,i\!=\!j, if(i\!=\!j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,i\!=\!j, \copyright c_1,$$

proof:

$,i\!=\!j, if(i\!=\!j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$

$\Leftrightarrow\; , if(i\!=\!j)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right], if(i\!=\!j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$

$\Leftrightarrow\; , if(i\!=\!j)\left[\begin{array}{l} , if(i\!=\!j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,\otimes, \end{array}\right],$

$\Leftrightarrow\; , if(i\!=\!j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\otimes, \end{array}\right],$

$\Leftrightarrow\; ,i\!=\!j, \copyright c_1,$

$$,i\,!\!=\!j, if(i\!=\!j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,i\,!\!=\!j, \copyright c_2,$$

5.5.4 Propositions with propositions

$$,i\!=\!j, \;\Leftrightarrow\; ,i\!=\!j, i\!=\!j,$$

proof:

$,i\!=\!j,$

$\Leftrightarrow\ ,i\!=\!j,,,$

$\Leftrightarrow\ ,i\!=\!j,if(i\!=\!j)\!-\!\left[\begin{array}{l},,, \\ \\ ,\otimes,\end{array}\right]\!-,$

$\Leftrightarrow\ ,i\!=\!j,if(i\!=\!j)\!-\!\left[\begin{array}{l}, \\ \\ ,\otimes,\end{array}\right]\!-,$

$\Leftrightarrow\ ,i\!=\!j,i\!=\!j,$

$$,i\,!\!=j,\ \Leftrightarrow\ ,i\,!\!=j,i\,!\!=j,$$

5.6 Substitution

5.6.1 Branch function with branch function

$$,if(i\!=\!j)\!-\!\left[\begin{array}{l},if(j\!=\!m)\!-\!\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i\!=\!j)\!-\!\left[\begin{array}{l},if(i\!=\!m)\!-\!\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \\ \\ ,\end{array}\right.$$

5.6.2 Propositions with branch function

$$,i\!=\!j,if(j\!=\!m)\!-\!\left[\begin{array}{l},\copyright c_1, \\ \\ ,\copyright c_2,\end{array}\right]\!-,\ \Leftrightarrow\ ,i\!=\!j,if(i\!=\!m)\!-\!\left[\begin{array}{l},\copyright c_1, \\ \\ ,\copyright c_2,\end{array}\right]\!-,$$

proof:

$,i\!=\!j,if(j\!=\!m)\!-\!\left[\begin{array}{l},\copyright c_1, \\ \\ ,\copyright c_2,\end{array}\right]\!-,$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],if(j{=}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} ,if(j{=}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} ,if(i{=}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],if(i{=}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i{=}j,if(i{=}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

5.6.3 Propositions with propositions

$$,i{=}j,j{=}m,\ \Leftrightarrow\ ,i{=}j,i{=}m,$$

proof:
$,i{=}j,j{=}m,$

$$\Leftrightarrow\ ,i{=}j,if(j{=}m)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i{=}j,if(i{=}m)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],$$

$\Leftrightarrow\ ,i\!=\!j,i\!=\!m,$

$$,i\!=\!j,j\,!\!=\!m,\ \Leftrightarrow\ ,i\!=\!j,i\,!\!=\!m,$$

5.7 Opposition

$$,i\!=\!j,i\,!\!=\!j,\ \Leftrightarrow\ ,\otimes,$$

proof:
$,i\!=\!j,i\,!\!=\!j,$

$\Leftrightarrow\ ,i\!=\!j,if(i\!=\!j)\!-\!\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right]\!-,$

$\Leftrightarrow\ ,i\!=\!j,\otimes,$

$\Leftrightarrow\ ,\otimes,$

$$,i\,!\!=\!j,i\!=\!j,\ \Leftrightarrow\ ,\otimes,$$

proof :
$,i\,!\!=\!j,i\!=\!j,$

$\Leftrightarrow\ ,if(i\!=\!j)\!-\!\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right]\!-,i\!=\!j,$

$\Leftrightarrow\ ,if(i\!=\!j)\!-\!\left[\begin{array}{l} ,\otimes, \\ \\ ,i\!=\!j, \end{array}\right]\!-,$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} ,\otimes, \\ ,if(i{=}j\left[\begin{array}{l} , \\ ,\otimes, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} ,\otimes, \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i{=}j)\left[\begin{array}{l} , \\ , \end{array}\right],\otimes,$$

$$\Leftrightarrow\ ,\otimes,$$

6 Theorems of Relationship of Node Null Comparison

6.1 Node null comparison propositions to Node value comparison propositions

$$,i\!=\!\varnothing, \;\Leftrightarrow\; ,\circledcirc m,i\!=\!m,m\textcircled{\downarrow},$$

proof:

$,i\!=\!\varnothing,$

$$\Leftrightarrow\; ,if(i\!=\!\varnothing)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc m,if(i\!=\!m)\left[\begin{array}{l} ,m\textcircled{\downarrow}, \\ \\ ,m\textcircled{\downarrow},\otimes, \end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc m,if(i\!=\!m)\left[\begin{array}{l} ,m\textcircled{\downarrow}, \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc m,if(i\!=\!m)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],m\textcircled{\downarrow},$$

$$\Leftrightarrow\; ,\circledcirc m,i\!=\!m,m\textcircled{\downarrow},$$

$$,i\,!\!=\!\varnothing, \;\Leftrightarrow\; ,\circledcirc m,i\,!\!=\!m,m\textcircled{\downarrow},$$

6.2 Branch function to propositions

$$,if(i\!=\!\varnothing)\left[\begin{array}{l} ,\copyright c, \\ \\ ,\otimes, \end{array}\right], \;\Leftrightarrow\; ,i\!=\!\varnothing,\copyright c,$$

$$,if(i=\varnothing)\text{-}\left[\begin{array}{c} ,\otimes, \\ \\ ,\copyright c, \end{array}\right]\text{-}, \iff ,i\,!\!=\varnothing,\copyright c,$$

6.3 Unity

$$, \iff ,if(i=\varnothing)\text{-}\left[\begin{array}{c} , \\ , \end{array}\right]\text{-},$$

proof:

$,$

$\iff ,\circledcirc j,\textcircled{\downarrow} j,$

$\iff ,\circledcirc j,if(i=j)\text{-}\left[\begin{array}{c} , \\ \\ , \end{array}\right]\text{-},\textcircled{\downarrow} j,$

$\iff ,\circledcirc j,if(i=j)\text{-}\left[\begin{array}{c} ,\textcircled{\downarrow} j, \\ \\ ,\textcircled{\downarrow} j, \end{array}\right]\text{-},$

$\iff ,if(i=\varnothing)\text{-}\left[\begin{array}{c} , \\ \\ , \end{array}\right]\text{-},$

$$,i=\varnothing,\otimes, \iff ,\otimes,$$

proof:

$,i=\varnothing,\otimes,$

$\iff ,if(i=\varnothing)\text{-}\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right]\text{-},\otimes,$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right],\otimes,$$

$$\Leftrightarrow\ ,\otimes,$$

$$,i\,!\!=\varnothing,\otimes,\ \Leftrightarrow\ ,\otimes,$$

6.4 Swap

6.4.1 Branch function and operator

$$,\circledcirc m,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,\circledcirc m, \\ \\ ,\circledcirc m, \end{array}\right.$$

proof:

$$,\circledcirc m,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

$$\Leftrightarrow\ ,\circledcirc m,\circledcirc j,if(i=j)\left[\begin{array}{l} ,j\text{\textcircled{$\downarrow$}}, \\ \\ ,j\text{\textcircled{$\downarrow$}}, \end{array}\right.$$

$$\Leftrightarrow\ ,\circledcirc j,\circledcirc m,if(i=j)\left[\begin{array}{l} ,j\text{\textcircled{$\downarrow$}}, \\ \\ ,j\text{\textcircled{$\downarrow$}}, \end{array}\right.$$

$$\Leftrightarrow\ ,\circledcirc j,if(i=j)\left[\begin{array}{l} ,\circledcirc m,j\text{\textcircled{$\downarrow$}}, \\ \\ ,\circledcirc m,j\text{\textcircled{$\downarrow$}}, \end{array}\right.$$

$$\Leftrightarrow\ ,\circledcirc j,if(i=j)\left[\begin{array}{l} ,j\text{\textcircled{$\downarrow$}},\circledcirc m, \\ \\ ,j\text{\textcircled{$\downarrow$}},\circledcirc m, \end{array}\right.$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},\circledcirc m,\\ \\ ,\circledcirc m,\end{array}\right.$$

$$,\odot m,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},\odot m,\\ \\ ,\odot m,\end{array}\right.$$

$$,m\text{☮}n,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},m\text{☮}n,\\ \\ ,m\text{☮}n,\end{array}\right.$$

$$,m\text{☮}n,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},m\text{☮}n,\\ \\ ,m\text{☮}n,\end{array}\right.$$

$$,m\text{☮}n,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},m\text{☮}n,\\ \\ ,m\text{☮}n,\end{array}\right.$$

$$,m\text{⏚},if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},m\text{⏚},\\ \\ ,m\text{⏚},\end{array}\right.$$

$$,m\oplus,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},m\oplus,\\ \\ ,m\oplus,\end{array}\right.$$

$$,m\ominus,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right.\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},m\ominus,\\ \\ ,m\ominus,\end{array}\right.$$

6.4.2 Branch function and Branch function

$$,if(i=\varnothing)\left[\begin{array}{l},if(m=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(m=\varnothing)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],\ \Leftrightarrow\ ,if(m=\varnothing)\left[\begin{array}{l},if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i=\varnothing)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i=\varnothing)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

proof:

$$,if(i=\varnothing)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, if(i=j)\left[\begin{array}{l} ,\text{\textcircled{$\downarrow$}}j, if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,\text{\textcircled{$\downarrow$}}j, if(m=n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, if(i=j)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\text{\textcircled{$\downarrow$}}j, \copyright c_1, \\ \\ ,\text{\textcircled{$\downarrow$}}j, \copyright c_2, \end{array}\right], \\ ,if(m=n)\left[\begin{array}{l} ,\text{\textcircled{$\downarrow$}}j, \copyright c_3, \\ \\ ,\text{\textcircled{$\downarrow$}}j, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, if(m=n)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,\text{\textcircled{$\downarrow$}}j, \copyright c_1, \\ \\ ,\text{\textcircled{$\downarrow$}}j, \copyright c_3, \end{array}\right], \\ ,if(i=j)\left[\begin{array}{l} ,\text{\textcircled{$\downarrow$}}j, \copyright c_2, \\ \\ ,\text{\textcircled{$\downarrow$}}j, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(m=n)-\left[\begin{array}{l} ,\circledcirc j, if(i=j)-\left[\begin{array}{l} ,\textcircled{$\downarrow$}j,\ \copyright c_1, \\ \\ ,\textcircled{$\downarrow$}j,\ \copyright c_3, \end{array}\right]-, \\ ,\circledcirc j, if(i=j)-\left[\begin{array}{l} ,\textcircled{$\downarrow$}j,\ \copyright c_2, \\ \\ ,\textcircled{$\downarrow$}j,\ \copyright c_4, \end{array}\right]-, \end{array}\right]-,$$

$$\Leftrightarrow\ ,if(m=n)-\left[\begin{array}{l} ,if(i=\varnothing)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_3, \end{array}\right]-, \\ ,if(i=\varnothing)-\left[\begin{array}{l} ,\ \copyright c_2, \\ \\ ,\ \copyright c_4, \end{array}\right]-, \end{array}\right]-,$$

6.4.3 Branch function and Propositions

$$,m=n, if(i=\varnothing)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l} ,m=n,\ \copyright c_1, \\ \\ ,m=n,\ \copyright c_2, \end{array}\right]-,$$

$$,m\,!\!=n, if(i=\varnothing)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l} ,m\,!\!=n,\ \copyright c_1, \\ \\ ,m\,!\!=n,\ \copyright c_2, \end{array}\right]-,$$

$$,j=\varnothing, if(i=\varnothing)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l} ,j=\varnothing,\ \copyright c_1, \\ \\ ,j=\varnothing,\ \copyright c_2, \end{array}\right]-,$$

proof:

$$,j=\varnothing, if(i=\varnothing)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-,$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right],\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(i=\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ ,\otimes, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,if(j=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\otimes, \end{array}\right], \\ ,if(j=\varnothing)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\otimes, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,j=\varnothing,\copyright c_1, \\ \\ ,j=\varnothing,\copyright c_2, \end{array}\right],$$

$$,j\,!{=}\varnothing,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,j\,!{=}\varnothing,\copyright c_1, \\ \\ ,j\,!{=}\varnothing,\copyright c_2, \end{array}\right],$$

$$,m=\varnothing,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\ \Leftrightarrow\ ,if(i=j)\left[\begin{array}{l} ,m=\varnothing,\copyright c_1, \\ \\ ,m=\varnothing,\copyright c_2, \end{array}\right],$$

$$,m\,!{=}\varnothing,if(i{=}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l},m\,!{=}\varnothing,\copyright c_1,\\ \\ ,m\,!{=}\varnothing,\copyright c_2,\end{array}\right],$$

6.4.4 Propositions and operator

$$,i{=}\varnothing,\circledcirc m, \;\Leftrightarrow\; ,\circledcirc m,i{=}\varnothing,$$

proof:

$,i{=}\varnothing,\circledcirc m,$

$$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],\circledcirc m,$$

$$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l},\circledcirc m,\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l},\circledcirc m,\\ \\ ,\circledcirc m,\otimes,\end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc m,if(i{=}\varnothing)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc m,i{=}\varnothing,$$

$$,i{=}\varnothing,\odot m, \;\Leftrightarrow\; ,\odot m,i{=}\varnothing,$$

$$,i{=}\varnothing,m \mathbin{\text{Ⓨ}} n, \;\Leftrightarrow\; ,m \mathbin{\text{Ⓨ}} n,i{=}\varnothing,$$

$$,i{=}\varnothing,m \mathbin{\text{Ⓨ}^{\circ}} n, \;\Leftrightarrow\; ,m \mathbin{\text{Ⓨ}^{\circ}} n,i{=}\varnothing,$$

$$,i\!=\!\varnothing, m☮n, \;\Leftrightarrow\; ,m☮n, i\!=\!\varnothing,$$

$$,i\!=\!\varnothing, m⏚, \;\Leftrightarrow\; ,m⏚, i\!=\!\varnothing,$$
$$,i\!=\!\varnothing, m\oplus, \;\Leftrightarrow\; ,m\oplus, i\!=\!\varnothing,$$
$$,i\!=\!\varnothing, m\ominus, \;\Leftrightarrow\; ,m\ominus, i\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing, \circledcirc m, \;\Leftrightarrow\; ,\circledcirc m, i\,!\!=\!\varnothing,$$
$$,i\,!\!=\!\varnothing, \odot m, \;\Leftrightarrow\; ,\odot m, i\,!\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing, m☮n, \;\Leftrightarrow\; ,m☮n, i\,!\!=\!\varnothing,$$
$$,i\,!\!=\!\varnothing, m☮n, \;\Leftrightarrow\; ,m☮n, i\,!\!=\!\varnothing,$$
$$,i\,!\!=\!\varnothing, m☮n, \;\Leftrightarrow\; ,m☮n, i\,!\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing, m⏚, \;\Leftrightarrow\; ,m⏚, i\,!\!=\!\varnothing,$$
$$,i\,!\!=\!\varnothing, m\oplus, \;\Leftrightarrow\; ,m\oplus, i\,!\!=\!\varnothing,$$
$$,i\,!\!=\!\varnothing, m\ominus, \;\Leftrightarrow\; ,m\ominus, i\,!\!=\!\varnothing,$$

6.4.5 Propositions and Propositions

$$,i\!=\!\varnothing, m\!=\!n, \;\Leftrightarrow\; ,m\!=\!n, i\!=\!\varnothing,$$

$$,i\!=\!\varnothing, m\,!\!=\!n, \;\Leftrightarrow\; ,m\,!\!=\!n, i\!=\!\varnothing,$$
$$,i\,!\!=\!\varnothing, m\!=\!n, \;\Leftrightarrow\; ,m\!=\!n, i\,!\!=\!\varnothing,$$
$$,i\,!\!=\!\varnothing, m\,!\!=\!n, \;\Leftrightarrow\; ,m\,!\!=\!n, i\,!\!=\!\varnothing,$$

$$,i\!=\!\varnothing, m\!=\!\varnothing, \;\Leftrightarrow\; ,m\!=\!\varnothing, i\!=\!\varnothing,$$

proof:

$,i\!=\!\varnothing, m\!=\!\varnothing,$

$$\Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,if(m=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l},if(m=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,\\ \\ ,\otimes,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l},if(m=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,\\ ,if(m=\varnothing)-\left[\begin{array}{l},\\ \\ ,\end{array}\right]-,\otimes,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l},if(m=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,\\ ,if(m=\varnothing)-\left[\begin{array}{l},\otimes,\\ \\ ,\otimes,\end{array}\right]-,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(m=\varnothing)-\left[\begin{array}{l},if(i=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,\\ ,if(i=\varnothing)-\left[\begin{array}{l},\otimes,\\ \\ ,\otimes,\end{array}\right]-,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(m=\varnothing)-\left[\begin{array}{l},if(i=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,\\ ,if(i=\varnothing)-\left[\begin{array}{l},\\ \\ ,\end{array}\right]-,\otimes,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(m=\varnothing)-\left[\begin{array}{l},if(i=\varnothing)-\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]-,\\ \\ ,\otimes,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(m=\varnothing)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],if(i=\varnothing)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,m=\varnothing,i=\varnothing,$$

$$,i=\varnothing,m\,!\!=\varnothing,\ \Leftrightarrow\ ,m\,!\!=\varnothing,i=\varnothing,$$

$$,i\,!\!=\varnothing,m\,!\!=\varnothing,\ \Leftrightarrow\ ,m\,!\!=\varnothing,i\,!\!=\varnothing,$$

6.4.6 Propositions to Propositions with branch function

$$,if(i=\varnothing)\left[\begin{array}{l} ,m\,!\!=n, \\ \\ , \end{array}\right],\ \Leftrightarrow\ ,if(m=n)\left[\begin{array}{l} ,i\,!\!=\varnothing, \\ \\ , \end{array}\right],$$

$$,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ ,m=n, \end{array}\right],\ \Leftrightarrow\ ,if(m=n)\left[\begin{array}{l} , \\ \\ ,i=\varnothing, \end{array}\right],$$

$$,if(i=\varnothing)\left[\begin{array}{l} ,m\,!\!=\varnothing, \\ \\ , \end{array}\right],\ \Leftrightarrow\ ,if(m=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, \\ \\ , \end{array}\right],$$

$$,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ ,m=\varnothing, \end{array}\right],\ \Leftrightarrow\ ,if(m=\varnothing)\left[\begin{array}{l} , \\ \\ ,i=\varnothing, \end{array}\right],$$

6.5 Transitivity

6.5.1 Branch function with branch function

$$,if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{l},if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ \\ ,\copyright c_2,\end{array}\right],$$

proof:

$$,if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, if(i=j)\left[\begin{array}{l},j\textcircled{\downarrow}, \copyright c_1,\\ \\ ,j\textcircled{\downarrow}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, if(i=j)\left[\begin{array}{l},if(i=j)\left[\begin{array}{l},j\textcircled{\downarrow}, \copyright c_1,\\ \\ ,j\textcircled{\downarrow}, \copyright c_3,\end{array}\right],\\ \\ ,j\textcircled{\downarrow}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, \circledcirc m, m\textcircled{\downarrow}, if(i=j)\left[\begin{array}{l},if(i=j)\left[\begin{array}{l},j\textcircled{\downarrow}, \copyright c_1,\\ \\ ,j\textcircled{\downarrow}, \copyright c_3,\end{array}\right],\\ \\ ,j\textcircled{\downarrow}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, \circledcirc m, if(i=j)\left[\begin{array}{l},m\textcircled{\downarrow}, if(i=j)\left[\begin{array}{l},j\textcircled{\downarrow}, \copyright c_1,\\ \\ ,j\textcircled{\downarrow}, \copyright c_3,\end{array}\right],\\ \\ ,m\textcircled{\downarrow}, j\textcircled{\downarrow}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,\circledcirc j, \circledcirc m, j=m, if(i=j)\left[\begin{array}{l},m\textcircled{\downarrow}, if(i=j)\left[\begin{array}{l},j\textcircled{\downarrow}, \copyright c_1,\\ \\ ,j\textcircled{\downarrow}, \copyright c_3,\end{array}\right],\\ \\ ,m\textcircled{\downarrow}, j\textcircled{\downarrow}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc j,\circledcirc m, j=m, if(i=m)\left[\begin{array}{l} ,m\textcircled{$\downarrow$}, if(i=j)\left[\begin{array}{l} ,j\textcircled{$\downarrow$},\ \copyright c_1, \\ \\ ,j\textcircled{$\downarrow$},\ \copyright c_3,\end{array}\right], \\ \\ ,m\textcircled{$\downarrow$}, j\textcircled{$\downarrow$},\ \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc j,\circledcirc m, if(i=m)\left[\begin{array}{l} ,m\textcircled{$\downarrow$}, if(i=j)\left[\begin{array}{l} ,j\textcircled{$\downarrow$},\ \copyright c_1, \\ \\ ,j\textcircled{$\downarrow$},\ \copyright c_3,\end{array}\right], \\ \\ ,m\textcircled{$\downarrow$}, j\textcircled{$\downarrow$},\ \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m,\circledcirc j, if(i=m)\left[\begin{array}{l} ,m\textcircled{$\downarrow$}, if(i=j)\left[\begin{array}{l} ,j\textcircled{$\downarrow$},\ \copyright c_1, \\ \\ ,j\textcircled{$\downarrow$},\ \copyright c_3,\end{array}\right], \\ \\ ,m\textcircled{$\downarrow$}, j\textcircled{$\downarrow$},\ \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, if(i=m)\left[\begin{array}{l} ,\circledcirc j, m\textcircled{$\downarrow$}, if(i=j)\left[\begin{array}{l} ,j\textcircled{$\downarrow$},\ \copyright c_1, \\ \\ ,j\textcircled{$\downarrow$},\ \copyright c_3,\end{array}\right], \\ \\ ,\circledcirc j, m\textcircled{$\downarrow$}, j\textcircled{$\downarrow$},\ \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, if(i=m)\left[\begin{array}{l} ,m\textcircled{$\downarrow$},\circledcirc j, if(i=j)\left[\begin{array}{l} ,j\textcircled{$\downarrow$},\ \copyright c_1, \\ \\ ,j\textcircled{$\downarrow$},\ \copyright c_3,\end{array}\right], \\ \\ ,m\textcircled{$\downarrow$},\circledcirc j, j\textcircled{$\downarrow$},\ \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, if(i=m)\left[\begin{array}{l} ,m\textcircled{$\downarrow$},\circledcirc j, if(i=j)\left[\begin{array}{l} ,j\textcircled{$\downarrow$},\ \copyright c_1, \\ \\ ,j\textcircled{$\downarrow$},\ \copyright c_3,\end{array}\right], \\ \\ ,m\textcircled{$\downarrow$},\ \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , if(i=\varnothing)\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_3,\end{array}\right], \\ \\ ,\ \copyright c_2,\end{array}\right],$$

$$,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ ,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

6.5.2 Branch function with propositions

$$,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,i{=}\varnothing,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

proof:

$$,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\otimes, \end{array}\right], \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,i{=}\varnothing,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,i\,!{=}\varnothing,\copyright c_2, \end{array}\right],$$

6.5.3 Propositions with branch function

$$,i=\varnothing,if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \iff ,i=\varnothing,\copyright c_1,$$

proof:

$$,i=\varnothing,if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\iff ,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\iff ,if(i=\varnothing)\left[\begin{array}{l},if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ \\ ,\otimes,\end{array}\right],$$

$$\iff ,if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\otimes,\end{array}\right],$$

$$\iff ,i=\varnothing,\copyright c_1,$$

$$,i\,!\!=\varnothing,if(i=\varnothing)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \iff ,i\,!\!=\varnothing,\copyright c_2,$$

6.5.4 Propositions with propositions

$$,i=\varnothing, \iff ,i=\varnothing,i=\varnothing,$$

proof:

$$,i\!=\!\varnothing,$$

$$\Leftrightarrow\ ,i\!=\!\varnothing,,,$$

$$\Leftrightarrow\ ,i\!=\!\varnothing,if(i\!=\!\varnothing)\!-\!\left[\begin{array}{l},,,\\ \\ ,\otimes,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\!=\!\varnothing,if(i\!=\!\varnothing)\!-\!\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\!=\!\varnothing,i\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing,\ \Leftrightarrow\ ,i\,!\!=\!\varnothing,i\,!\!=\!\varnothing,$$

6.6 Substitution

6.6.1 Propositions with branch function

$$,i\!=\!j,if(i\!=\!\varnothing)\!-\!\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\ \Leftrightarrow\ ,i\!=\!j,if(j\!=\!\varnothing)\!-\!\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,$$

proof:

$$,i\!=\!j,if(i\!=\!\varnothing)\!-\!\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\!=\!j,\circledcirc m,if(i\!=\!m)\!-\!\left[\begin{array}{l},m\textcircled{\downarrow},\copyright c_1,\\ \\ ,m\textcircled{\downarrow},\copyright c_2,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,\circledcirc m,i\!=\!j,if(i\!=\!m)\!-\!\left[\begin{array}{l},m\textcircled{\downarrow},\copyright c_1,\\ \\ ,m\textcircled{\downarrow},\copyright c_2,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,\circledcirc m, i=j, if(j=m)\left[\begin{array}{l}, m\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , m\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i=j, \circledcirc m, if(j=m)\left[\begin{array}{l}, m\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , m\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i=j, if(j=\varnothing)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$, i=\varnothing, if(i=j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow\ , i=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

proof:

$$, i=\varnothing, if(i=j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, i=m, m\textcircled{$\downarrow$}, if(i=j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, i=m, if(i=j)\left[\begin{array}{l}, m\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , m\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, i=m, if(m=j)\left[\begin{array}{l}, m\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , m\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, i=m, if(j=m)\left[\begin{array}{l}, m\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , m\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, \circledcirc n, n\textcircled{$\downarrow$}, i=m, if(j=m)\left[\begin{array}{l}, m\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , m\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc m, \circledcirc n, i=m, n\textcircled{$\downarrow$}, if(j=m)\left[\begin{array}{l}, m\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , m\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,\circledcirc m,\circledcirc n,m=i,n\textcircled{$\downarrow$},if(j=m)\begin{bmatrix} ,m\textcircled{$\downarrow$},\copyright c_1, \\ \\ ,m\textcircled{$\downarrow$},\copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow \ ,\circledcirc m,\circledcirc n,m=n,m=i,n\textcircled{$\downarrow$},if(j=m)\begin{bmatrix} ,m\textcircled{$\downarrow$},\copyright c_1, \\ \\ ,m\textcircled{$\downarrow$},\copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow \ ,\circledcirc m,\circledcirc n,m=n,n=i,n\textcircled{$\downarrow$},if(j=m)\begin{bmatrix} ,m\textcircled{$\downarrow$},\copyright c_1, \\ \\ ,m\textcircled{$\downarrow$},\copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow \ ,\circledcirc m,\circledcirc n,n=i,n\textcircled{$\downarrow$},if(j=m)\begin{bmatrix} ,m\textcircled{$\downarrow$},\copyright c_1, \\ \\ ,m\textcircled{$\downarrow$},\copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow \ ,\circledcirc n,n=i,n\textcircled{$\downarrow$},\circledcirc m,if(j=m)\begin{bmatrix} ,m\textcircled{$\downarrow$},\copyright c_1, \\ \\ ,m\textcircled{$\downarrow$},\copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow \ ,\circledcirc n,i=n,n\textcircled{$\downarrow$},\circledcirc m,if(j=m)\begin{bmatrix} ,m\textcircled{$\downarrow$},\copyright c_1, \\ \\ ,m\textcircled{$\downarrow$},\copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow \ ,\circledcirc n,i=n,n\textcircled{$\downarrow$},if(j=\varnothing)\begin{bmatrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow \ ,i=\varnothing,if(j=\varnothing)\begin{bmatrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{bmatrix},$$

6.6.2 Propositions with propositions

$$,i=j,i=\varnothing, \ \Leftrightarrow \ ,i=j,j=\varnothing,$$

$$,i=j,i\,!{=}\varnothing, \ \Leftrightarrow \ ,i=j,j\,!{=}\varnothing,$$

proof:

$$,i=j,i\,!\!=\varnothing,$$

$$\Leftrightarrow\ ,i=j,if(i=\varnothing)\!-\!\begin{bmatrix} ,\otimes, \\ \\ , \end{bmatrix}\!-,$$

$$\Leftrightarrow\ ,i=j,if(j=\varnothing)\!-\!\begin{bmatrix} ,\otimes, \\ \\ , \end{bmatrix}\!-,$$

$$\Leftrightarrow\ ,i=j,j\,!\!=\varnothing,$$

$$,i=\varnothing,j=\varnothing,\ \Leftrightarrow\ ,i=\varnothing,i=j,$$

$$,i=\varnothing,j\,!\!=\varnothing,\ \Leftrightarrow\ ,i=\varnothing,i\,!\!=j,$$

6.7 Opposition

$$,i=\varnothing,i\,!\!=\varnothing,\ \Leftrightarrow\ ,\otimes,$$

$$,i=\varnothing,i\,!\!=\varnothing,$$

$$\Leftrightarrow\ ,i=\varnothing,if(i=\varnothing)\!-\!\begin{bmatrix} ,\otimes, \\ \\ , \end{bmatrix}\!-,$$

$$\Leftrightarrow\ ,i=\varnothing,\otimes,$$

$$\Leftrightarrow\ ,\otimes,$$

$$,i\,!\!=\varnothing,i=\varnothing,\ \Leftrightarrow\ ,\otimes,$$

7 Theorems of Relationship of Identical Node Comparison

7.1 Identical node comparison propositions to Node value comparison propositions

$$,i\mathring{\bigcirc}j, \;\Leftrightarrow\; ,i\circledast m, j\circledast n, m=n, m\circledcirc, n\circledcirc,$$

proof:

$,i\mathring{\bigcirc}j,$

$\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , \\ ,\otimes, \end{array}\right],$

$\Leftrightarrow\; ,i\circledast m, j\circledast n, if(m=n)\left[\begin{array}{l} ,m\circledcirc, n\circledcirc, \\ ,m\circledcirc, n\circledcirc, \otimes, \end{array}\right],$

$\Leftrightarrow\; ,i\circledast m, j\circledast n, if(m=n)\left[\begin{array}{l} ,m\circledcirc, n\circledcirc, \\ ,\otimes, \end{array}\right],$

$\Leftrightarrow\; ,i\circledast m, j\circledast n, if(m=n)\left[\begin{array}{l} , \\ ,\otimes, \end{array}\right], m\circledcirc, n\circledcirc,$

$\Leftrightarrow\; ,i\circledast m, j\circledast n, m=n, m\circledcirc, n\circledcirc,$

$$,i!\mathring{\bigcirc}j, \;\Leftrightarrow\; ,i\circledast m, j\circledast n, m\,!{=}\,n, m\circledcirc, n\circledcirc,$$

7.2 Branch function to propositions

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c, \\ ,\otimes, \end{array}\right], \;\Leftrightarrow\; ,i\mathring{\bigcirc}j, \copyright c,$$

$$,if(i\text{♂}j)\left[\begin{array}{l},\otimes,\\ \\ ,©c,\end{array}\right],\;\;\Leftrightarrow\;\;,i!\text{♂}j,©c,$$

7.3 Unity

$$,\;\;\Leftrightarrow\;\;,if(i\text{♂}j)\left[\begin{array}{l},\\ ,\end{array}\right],$$

proof:

$,$

$\Leftrightarrow\;\;,i\text{☮}m,m\text{⏚},$

$\Leftrightarrow\;\;,i\text{☮}m,m\text{⏚},j\text{☮}n,n\text{⏚},$

$\Leftrightarrow\;\;,i\text{☮}m,j\text{☮}n,m\text{⏚},n\text{⏚},$

$\Leftrightarrow\;\;,i\text{☮}m,j\text{☮}n,if(m=n)\left[\begin{array}{l},\\ \\ ,\end{array}\right],m\text{⏚},n\text{⏚},$

$\Leftrightarrow\;\;,i\text{☮}m,j\text{☮}n,if(m=n)\left[\begin{array}{l},m\text{⏚},n\text{⏚},\\ \\ ,m\text{⏚},n\text{⏚},\end{array}\right],$

$\Leftrightarrow\;\;,if(i\text{♂}j)\left[\begin{array}{l},\\ \\ ,\end{array}\right],$

$$,i\text{♂}j,\otimes,\;\;\Leftrightarrow\;\;,\otimes,$$

$$,i!\text{♂}j,\otimes,\;\;\Leftrightarrow\;\;,\otimes,$$

7.4 Symmetry

$$,if(i\bigcirc\!\!\!^{\circ}j)\!-\!\left[\begin{array}{l},\\,\end{array}\right. \Leftrightarrow \ ,if(j\bigcirc\!\!\!^{\circ}i)\!-\!\left[\begin{array}{l},\\,\end{array}\right.$$

proof:

$$,if(i\bigcirc\!\!\!^{\circ}j)\!-\!\left[\begin{array}{l},\\ \\,\end{array}\right.$$

$$\Leftrightarrow \ ,i\circledast t_1,j\circledast t_2,if(t_1=t_2)\!-\!\left[\begin{array}{l},t_1\circleddash,t_2\circleddash,\\ \\,t_1\circleddash,t_2\circleddash,\end{array}\right.$$

$$\Leftrightarrow \ ,j\circledast t_2,i\circledast t_1,if(t_1=t_2)\!-\!\left[\begin{array}{l},t_1\circleddash,t_2\circleddash,\\ \\,t_1\circleddash,t_2\circleddash,\end{array}\right.$$

$$\Leftrightarrow \ ,j\circledast t_2,i\circledast t_1,if(t_2=t_1)\!-\!\left[\begin{array}{l},t_1\circleddash,t_2\circleddash,\\ \\,t_1\circleddash,t_2\circleddash,\end{array}\right.$$

$$\Leftrightarrow \ ,j\circledast t_2,i\circledast t_1,if(t_2=t_1)\!-\!\left[\begin{array}{l},t_2\circleddash,t_1\circleddash,\\ \\,t_2\circleddash,t_1\circleddash,\end{array}\right.$$

$$\Leftrightarrow \ ,if(j\bigcirc\!\!\!^{\circ}i)\!-\!\left[\begin{array}{l},\\ \\,\end{array}\right.$$

$$,i\bigcirc\!\!\!^{\circ}j, \ \Leftrightarrow \ ,j\bigcirc\!\!\!^{\circ}i,$$

$$,i!\bigcirc\!\!\!^{\circ}j, \ \Leftrightarrow \ ,j!\bigcirc\!\!\!^{\circ}i,$$

7.5 Swap

7.5.1 Branch function and operator

$$,\circledcirc m,if(i\bigcirc\!\!\!^{\circ}j)\!-\!\left[\begin{array}{l},\\ \\,\end{array}\right. \Leftrightarrow \ ,if(i\bigcirc\!\!\!^{\circ}j)\!-\!\left[\begin{array}{l},\circledcirc m,\\ \\,\circledcirc m,\end{array}\right.$$

proof:

$$, \circledcirc m, if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \\ , \end{cases}$$

$$\Leftrightarrow \ , \circledcirc m, i \oslash t_1, j \oslash t_2, if(t_1 = t_2) \begin{cases} , t_1 \text{⏚}, t_2 \text{⏚}, \\ , t_1 \text{⏚}, t_2 \text{⏚}, \end{cases}$$

$$\Leftrightarrow \ , i \oslash t_1, \circledcirc m, j \oslash t_2, if(t_1 = t_2) \begin{cases} , t_1 \text{⏚}, t_2 \text{⏚}, \\ , t_1 \text{⏚}, t_2 \text{⏚}, \end{cases}$$

$$\Leftrightarrow \ , i \oslash t_1, j \oslash t_2, \circledcirc m, if(t_1 = t_2) \begin{cases} , t_1 \text{⏚}, t_2 \text{⏚}, \\ , t_1 \text{⏚}, t_2 \text{⏚}, \end{cases}$$

$$\Leftrightarrow \ , i \oslash t_1, j \oslash t_2, if(t_1 = t_2) \begin{cases} , \circledcirc m, t_1 \text{⏚}, t_2 \text{⏚}, \\ , \circledcirc m, t_1 \text{⏚}, t_2 \text{⏚}, \end{cases}$$

$$\Leftrightarrow \ , i \oslash t_1, j \oslash t_2, if(t_1 = t_2) \begin{cases} , t_1 \text{⏚}, \circledcirc m, t_2 \text{⏚}, \\ , t_1 \text{⏚}, \circledcirc m, t_2 \text{⏚}, \end{cases}$$

$$\Leftrightarrow \ , i \oslash t_1, j \oslash t_2, if(t_1 = t_2) \begin{cases} , t_1 \text{⏚}, t_2 \text{⏚}, \circledcirc m, \\ , t_1 \text{⏚}, t_2 \text{⏚}, \circledcirc m, \end{cases}$$

$$\Leftrightarrow \ , if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \circledcirc m, \\ , \circledcirc m, \end{cases}$$

$$, \odot m, if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \\ , \end{cases} \Leftrightarrow \ , if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \odot m, \\ , \odot m, \end{cases}$$

$$, m \text{☮} n, if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \\ , \end{cases} \Leftrightarrow \ , if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , m \text{☮} n, \\ , m \text{☮} n, \end{cases}$$

$$, m \oslash n, if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \\ , \end{cases} \Leftrightarrow \ , if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , m \oslash n, \\ , m \oslash n, \end{cases}$$

$$, m \obslash n, if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \\ , \end{cases} \Leftrightarrow \ , if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , m \obslash n, \\ , m \obslash n, \end{cases}$$

$$, m \text{⏚}, if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , \\ , \end{cases} \Leftrightarrow \ , if(i \bigcirc\!\!{}^{\circ} j) \begin{cases} , m \text{⏚}, \\ , m \text{⏚}, \end{cases}$$

$$, m\oplus, if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \Leftrightarrow \; , if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, m\oplus, \\ \\ , m\oplus,\end{array}\right.$$

$$, m\ominus, if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \Leftrightarrow \; , if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, m\ominus, \\ \\ , m\ominus,\end{array}\right.$$

7.5.2 Branch function and Branch function

$$, if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, if(m\mathring{\bigcirc}n)\!-\!\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \\ , if(m\mathring{\bigcirc}n)\!-\!\left[\begin{array}{l}, ©c_3, \\ \\ , ©c_4,\end{array}\right]\!-,\end{array}\right]\!-, \Leftrightarrow \; , if(m\mathring{\bigcirc}n)\!-\!\left[\begin{array}{l}, if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_3,\end{array}\right]\!-, \\ , if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, ©c_2, \\ \\ , ©c_4,\end{array}\right]\!-,\end{array}\right]\!-,$$

proof:

$$, if(i\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l}, if(m\mathring{\bigcirc}n)\!-\!\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \\ , if(m\mathring{\bigcirc}n)\!-\!\left[\begin{array}{l}, ©c_3, \\ \\ , ©c_4,\end{array}\right]\!-,\end{array}\right]\!-,$$

$$\Leftrightarrow \; , i\circledast t_1, j\circledast t_2, if(t_1=t_2)\!-\!\left[\begin{array}{l}, t_1\circledcirc, t_2\circledcirc, if(m\mathring{\bigcirc}n)\!-\!\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \\ , t_1\circledcirc, t_2\circledcirc, if(m\mathring{\bigcirc}n)\!-\!\left[\begin{array}{l}, ©c_3, \\ \\ , ©c_4,\end{array}\right]\!-,\end{array}\right]\!-,$$

$$\Leftrightarrow \; , i\circledast t_1, j\circledast t_2, if(t_1=t_2)\!-\!\left[\begin{array}{l}, t_1\circledcirc, t_2\circledcirc, m\circledast t_3, n\circledast t_4, if(t_3=t_4)\!-\!\left[\begin{array}{l}, t_3\circledcirc, t_4\circledcirc, ©c_1, \\ \\ , t_3\circledcirc, t_4\circledcirc, ©c_2,\end{array}\right]\!-, \\ , t_1\circledcirc, t_2\circledcirc, m\circledast t_3, n\circledast t_4, if(t_3=t_4)\!-\!\left[\begin{array}{l}, t_3\circledcirc, t_4\circledcirc, ©c_3, \\ \\ , t_3\circledcirc, t_4\circledcirc, ©c_4,\end{array}\right]\!-,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\text{☮}t_1,j\text{☮}t_2,m\text{☮}t_3,n\text{☮}t_4,if(t_1=t_2)\left[\begin{array}{l}
,t_1\text{⏚},t_2\text{⏚},if(t_3=t_4)\left[\begin{array}{l},t_3\text{⏚},t_4\text{⏚},\copyright c_1,\\ \\ ,t_3\text{⏚},t_4\text{⏚},\copyright c_2,\end{array}\right],\\
,t_1\text{⏚},t_2\text{⏚},if(t_3=t_4)\left[\begin{array}{l},t_3\text{⏚},t_4\text{⏚},\copyright c_3,\\ \\ ,t_3\text{⏚},t_4\text{⏚},\copyright c_4,\end{array}\right],
\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}t_1,j\text{☮}t_2,m\text{☮}t_3,n\text{☮}t_4,if(t_1=t_2)\left[\begin{array}{l}
,if(t_3=t_4)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_1,\\ \\ ,t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_2,\end{array}\right],\\
,if(t_3=t_4)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_3,\\ \\ ,t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_4,\end{array}\right],
\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}t_1,j\text{☮}t_2,m\text{☮}t_3,n\text{☮}t_4,if(t_3=t_4)\left[\begin{array}{l}
,if(t_1=t_2)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_1,\\ \\ ,t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_3,\end{array}\right],\\
,if(t_1=t_2)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_2,\\ \\ ,t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_4,\end{array}\right],
\end{array}\right],$$

$$\Leftrightarrow\ ,m\text{☮}t_3,n\text{☮}t_4,if(t_3=t_4)\left[\begin{array}{l}
,i\text{☮}t_1,j\text{☮}t_2,if(t_1=t_2)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_1,\\ \\ ,t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_3,\end{array}\right],\\
,i\text{☮}t_1,j\text{☮}t_2,if(t_1=t_2)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_2,\\ \\ ,t_1\text{⏚},t_2\text{⏚},t_3\text{⏚},t_4\text{⏚},\copyright c_4,\end{array}\right],
\end{array}\right],$$

$$\Leftrightarrow\ ,m\text{☮}t_3,n\text{☮}t_4,if(t_3=t_4)\left[\begin{array}{l}
,t_3\text{⏚},t_4\text{⏚},i\text{☮}t_1,j\text{☮}t_2,if(t_1=t_2)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},\copyright c_1,\\ \\ ,t_1\text{⏚},t_2\text{⏚},\copyright c_3,\end{array}\right],\\
,t_3\text{⏚},t_4\text{⏚},i\text{☮}t_1,j\text{☮}t_2,if(t_1=t_2)\left[\begin{array}{l},t_1\text{⏚},t_2\text{⏚},\copyright c_2,\\ \\ ,t_1\text{⏚},t_2\text{⏚},\copyright c_4,\end{array}\right],
\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \ \Leftrightarrow\ ,if(m=n)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,if(m=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=\varnothing)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \ \Leftrightarrow\ ,if(m=\varnothing)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

7.5.3 Branch function and Propositions

$$,m\mathring{\bigcirc}n,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \ \Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,m\mathring{\bigcirc}n,\copyright c_1, \\ \\ ,m\mathring{\bigcirc}n,\copyright c_2, \end{array}\right],$$

$$,m!\mathring{\bigcirc}n,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \ \Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,m!\mathring{\bigcirc}n,\copyright c_1, \\ \\ ,m!\mathring{\bigcirc}n,\copyright c_2, \end{array}\right],$$

$$,m{=}n, if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \ \Leftrightarrow \ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},m{=}n,\copyright c_1,\\ \\ ,m{=}n,\copyright c_2,\end{array}\right],$$

$$,m\,!{=}n, if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \ \Leftrightarrow \ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},m\,!{=}n,\copyright c_1,\\ \\ ,m\,!{=}n,\copyright c_2,\end{array}\right],$$

$$,m{=}\varnothing, if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \ \Leftrightarrow \ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},m{=}\varnothing,\copyright c_1,\\ \\ ,m{=}\varnothing,\copyright c_2,\end{array}\right],$$

$$,m\,!{=}\varnothing, if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \ \Leftrightarrow \ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},m\,!{=}\varnothing,\copyright c_1,\\ \\ ,m\,!{=}\varnothing,\copyright c_2,\end{array}\right],$$

$$,m\mathring{\bigcirc}n, if(i{=}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \ \Leftrightarrow \ ,if(i{=}j)\left[\begin{array}{c},m\mathring{\bigcirc}n,\copyright c_1,\\ \\ ,m\mathring{\bigcirc}n,\copyright c_2,\end{array}\right],$$

$$,m!\mathring{\bigcirc}n, if(i{=}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \ \Leftrightarrow \ ,if(i{=}j)\left[\begin{array}{c},m!\mathring{\bigcirc}n,\copyright c_1,\\ \\ ,m!\mathring{\bigcirc}n,\copyright c_2,\end{array}\right],$$

$$,m\mathring{\bigcirc}n, if(i{=}\varnothing)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \ \Leftrightarrow \ ,if(i{=}\varnothing)\left[\begin{array}{c},m\mathring{\bigcirc}n,\copyright c_1,\\ \\ ,m\mathring{\bigcirc}n,\copyright c_2,\end{array}\right],$$

$$, m!\overset{\circ}{\bigcirc}n, if(i=\varnothing)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l}, m!\overset{\circ}{\bigcirc}n, \copyright c_1,\\ \\ , m!\overset{\circ}{\bigcirc}n, \copyright c_2,\end{array}\right],$$

7.5.4 Propositions and operator

$$
\begin{aligned}
, i\overset{\circ}{\bigcirc}j, \circledcirc m, &\;\Leftrightarrow\; , \circledcirc m, i\overset{\circ}{\bigcirc}j,\\
, i\overset{\circ}{\bigcirc}j, \odot m, &\;\Leftrightarrow\; , \odot m, i\overset{\circ}{\bigcirc}j,\\
\\
, i\overset{\circ}{\bigcirc}j, m\text{☮}n, &\;\Leftrightarrow\; , m\text{☮}n, i\overset{\circ}{\bigcirc}j,\\
, i\overset{\circ}{\bigcirc}j, m\text{☮}_{\circ}n, &\;\Leftrightarrow\; , m\text{☮}_{\circ}n, i\overset{\circ}{\bigcirc}j,\\
, i\overset{\circ}{\bigcirc}j, m\text{☮}_{\lrcorner}n, &\;\Leftrightarrow\; , m\text{☮}_{\lrcorner}n, i\overset{\circ}{\bigcirc}j,\\
\\
, i\overset{\circ}{\bigcirc}j, m\textcircled{\downarrow}, &\;\Leftrightarrow\; , m\textcircled{\downarrow}, i\overset{\circ}{\bigcirc}j,\\
, i\overset{\circ}{\bigcirc}j, m\oplus, &\;\Leftrightarrow\; , m\oplus, i\overset{\circ}{\bigcirc}j,\\
, i\overset{\circ}{\bigcirc}j, m\ominus, &\;\Leftrightarrow\; , m\ominus, i\overset{\circ}{\bigcirc}j,
\end{aligned}
$$

$$
\begin{aligned}
, i!\overset{\circ}{\bigcirc}j, \circledcirc m, &\;\Leftrightarrow\; , \circledcirc m, i!\overset{\circ}{\bigcirc}j,\\
, i!\overset{\circ}{\bigcirc}j, \odot m, &\;\Leftrightarrow\; , \odot m, i!\overset{\circ}{\bigcirc}j,\\
\\
, i!\overset{\circ}{\bigcirc}j, m\text{☮}n, &\;\Leftrightarrow\; , m\text{☮}n, i!\overset{\circ}{\bigcirc}j,\\
, i!\overset{\circ}{\bigcirc}j, m\text{☮}_{\circ}n, &\;\Leftrightarrow\; , m\text{☮}_{\circ}n, i!\overset{\circ}{\bigcirc}j,\\
, i!\overset{\circ}{\bigcirc}j, m\text{☮}_{\lrcorner}n, &\;\Leftrightarrow\; , m\text{☮}_{\lrcorner}n, i!\overset{\circ}{\bigcirc}j,\\
\\
, i!\overset{\circ}{\bigcirc}j, m\textcircled{\downarrow}, &\;\Leftrightarrow\; , m\textcircled{\downarrow}, i!\overset{\circ}{\bigcirc}j,\\
, i!\overset{\circ}{\bigcirc}j, m\oplus, &\;\Leftrightarrow\; , m\oplus, i!\overset{\circ}{\bigcirc}j,\\
, i!\overset{\circ}{\bigcirc}j, m\ominus, &\;\Leftrightarrow\; , m\ominus, i!\overset{\circ}{\bigcirc}j,
\end{aligned}
$$

7.5.5 Propositions and Propositions

$$
\begin{aligned}
,i\overset{\circ}{\bigcirc}j,m\overset{\circ}{\bigcirc}n, &\iff ,m\overset{\circ}{\bigcirc}n,i\overset{\circ}{\bigcirc}j,\\
,i\overset{\circ}{\bigcirc}j,m!\overset{\circ}{\bigcirc}n, &\iff ,m!\overset{\circ}{\bigcirc}n,i\overset{\circ}{\bigcirc}j,\\
,i!\overset{\circ}{\bigcirc}j,m!\overset{\circ}{\bigcirc}n, &\iff ,m!\overset{\circ}{\bigcirc}n,i!\overset{\circ}{\bigcirc}j,
\end{aligned}
$$

$$
\begin{aligned}
,i\overset{\circ}{\bigcirc}j,m=n, &\iff ,m=n,i\overset{\circ}{\bigcirc}j,\\
,i\overset{\circ}{\bigcirc}j,m\,!=n, &\iff ,m\,!=n,i\overset{\circ}{\bigcirc}j,\\
,i!\overset{\circ}{\bigcirc}j,m=n, &\iff ,m=n,i!\overset{\circ}{\bigcirc}j,\\
,i!\overset{\circ}{\bigcirc}j,m\,!=n, &\iff ,m\,!=n,i!\overset{\circ}{\bigcirc}j,
\end{aligned}
$$

$$
\begin{aligned}
,i\overset{\circ}{\bigcirc}j,m=\varnothing, &\iff ,m=\varnothing,i\overset{\circ}{\bigcirc}j,\\
,i\overset{\circ}{\bigcirc}j,m\,!=\varnothing, &\iff ,m\,!=\varnothing,i\overset{\circ}{\bigcirc}j,\\
,i!\overset{\circ}{\bigcirc}j,m=\varnothing, &\iff ,m=\varnothing,i!\overset{\circ}{\bigcirc}j,\\
,i!\overset{\circ}{\bigcirc}j,m\,!=\varnothing, &\iff ,m\,!=\varnothing,i!\overset{\circ}{\bigcirc}j,
\end{aligned}
$$

7.5.6 Propositions to Propositions with branch function

$$
,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,m!\overset{\circ}{\bigcirc}n, \\ \\ , \end{array}\right], \iff ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,i!\overset{\circ}{\bigcirc}j, \\ \\ , \end{array}\right],
$$

$$
,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} , \\ \\ ,m\overset{\circ}{\bigcirc}n, \end{array}\right], \iff ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} , \\ \\ ,i\overset{\circ}{\bigcirc}j, \end{array}\right],
$$

$$
,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,m\,!=n, \\ \\ , \end{array}\right], \iff ,if(m=n)\left[\begin{array}{l} ,i!\overset{\circ}{\bigcirc}j, \\ \\ , \end{array}\right],
$$

$$
,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} , \\ \\ ,m=n, \end{array}\right], \iff ,if(m=n)\left[\begin{array}{l} , \\ \\ ,i\overset{\circ}{\bigcirc}j, \end{array}\right],
$$

$$, if(i \male j) \left[\begin{array}{l} , m \mathrel{!=} \varnothing, \\ \\ , \end{array}\right], \Leftrightarrow , if(m = \varnothing) \left[\begin{array}{l} , i ! \male j, \\ \\ , \end{array}\right],$$

$$, if(i \male j) \left[\begin{array}{l} , \\ \\ , m = \varnothing, \end{array}\right], \Leftrightarrow , if(m = \varnothing) \left[\begin{array}{l} , \\ \\ , i \male j, \end{array}\right],$$

7.6 Transitivity

7.6.1 Branch function with branch function

$$, if(i \male j) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \Leftrightarrow , if(i \male j) \left[\begin{array}{l} , if(i \male j) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array}\right], \\ , \copyright c_2, \end{array}\right],$$

proof:

$$, if(i \male j) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \; , i \circledast m_1, j \circledast n_1, if(m_1 = n_1) \left[\begin{array}{l} , m_1 \textcircled{\downarrow}, n_1 \textcircled{\downarrow}, \copyright c_1, \\ \\ , m_1 \textcircled{\downarrow}, n_1 \textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \; , i \circledast m_1, j \circledast n_1, if(m_1 = n_1) \left[\begin{array}{l} , if(m_1 = n_1) \left[\begin{array}{l} , m_1 \textcircled{\downarrow}, n_1 \textcircled{\downarrow}, \copyright c_1, \\ \\ , m_1 \textcircled{\downarrow}, n_1 \textcircled{\downarrow}, \copyright c_3, \end{array}\right], \\ , m_1 \textcircled{\downarrow}, n_1 \textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash m_1, j\oslash n_1, j\oslash n_2, n_2\text{☮}, if(m_1=n_1)\left[\begin{array}{l} , if(m_1=n_1)\left[\begin{array}{l} , m_1\text{☮}, n_1\text{☮}, ©c_1, \\ \\ , m_1\text{☮}, n_1\text{☮}, ©c_3, \end{array}\right], \\ , m_1\text{☮}, n_1\text{☮}, ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash m_1, j\oslash n_1, j\oslash n_2, if(m_1=n_1)\left[\begin{array}{l} , n_2\text{☮}, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{☮}, n_1\text{☮}, ©c_1, \\ \\ , m_1\text{☮}, n_1\text{☮}, ©c_3, \end{array}\right], \\ , n_2\text{☮}, m_1\text{☮}, n_1\text{☮}, ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash m_1, j\oslash n_1, j\oslash n_2, n_1=n_2, if(m_1=n_1)\left[\begin{array}{l} , n_2\text{☮}, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{☮}, n_1\text{☮}, ©c_1, \\ \\ , m_1\text{☮}, n_1\text{☮}, ©c_3, \end{array}\right], \\ , n_2\text{☮}, m_1\text{☮}, n_1\text{☮}, ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash m_1, j\oslash n_1, j\oslash n_2, n_1=n_2, if(m_1=n_2)\left[\begin{array}{l} , n_2\text{☮}, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{☮}, n_1\text{☮}, ©c_1, \\ \\ , m_1\text{☮}, n_1\text{☮}, ©c_3, \end{array}\right], \\ , n_2\text{☮}, m_1\text{☮}, n_1\text{☮}, ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash m_1, j\oslash n_1, j\oslash n_2, if(m_1=n_2)\left[\begin{array}{l} , n_2\text{☮}, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{☮}, n_1\text{☮}, ©c_1, \\ \\ , m_1\text{☮}, n_1\text{☮}, ©c_3, \end{array}\right], \\ , n_2\text{☮}, m_1\text{☮}, n_1\text{☮}, ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash m_1, j\oslash n_2, if(m_1=n_2)\left[\begin{array}{l} , n_2\text{☮}, j\oslash n_1, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{☮}, n_1\text{☮}, ©c_1, \\ \\ , m_1\text{☮}, n_1\text{☮}, ©c_3, \end{array}\right], \\ , n_2\text{☮}, m_1\text{☮}, j\oslash n_1, n_1\text{☮}, ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}m_1, j\text{☮}n_2, if(m_1=n_2)\left[\begin{array}{l} , n_2\text{⏚}, j\text{☮}n_1, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{⏚}, n_1\text{⏚}, \copyright c_1, \\ , m_1\text{⏚}, n_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , n_2\text{⏚}, m_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , j\text{☮}n_2, i\text{☮}m_1, if(m_1=n_2)\left[\begin{array}{l} , n_2\text{⏚}, j\text{☮}n_1, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{⏚}, n_1\text{⏚}, \copyright c_1, \\ , m_1\text{⏚}, n_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , n_2\text{⏚}, m_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , j\text{☮}n_2, i\text{☮}m_1, i\text{☮}m_2, m_2\text{⏚}, if(m_1=n_2)\left[\begin{array}{l} , n_2\text{⏚}, j\text{☮}n_1, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{⏚}, n_1\text{⏚}, \copyright c_1, \\ , m_1\text{⏚}, n_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , n_2\text{⏚}, m_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , j\text{☮}n_2, i\text{☮}m_1, i\text{☮}m_2, if(m_1=n_2)\left[\begin{array}{l} , m_2\text{⏚}, n_2\text{⏚}, j\text{☮}n_1, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{⏚}, n_1\text{⏚}, \copyright c_1, \\ , m_1\text{⏚}, n_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , m_2\text{⏚}, n_2\text{⏚}, m_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , j\text{☮}n_2, i\text{☮}m_1, i\text{☮}m_2, m_1=m_2, if(m_1=n_2)\left[\begin{array}{l} , m_2\text{⏚}, n_2\text{⏚}, j\text{☮}n_1, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{⏚}, n_1\text{⏚}, \copyright c_1, \\ , m_1\text{⏚}, n_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , m_2\text{⏚}, n_2\text{⏚}, m_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , j\text{☮}n_2, i\text{☮}m_1, i\text{☮}m_2, m_1=m_2, if(m_2=n_2)\left[\begin{array}{l} , m_2\text{⏚}, n_2\text{⏚}, j\text{☮}n_1, if(m_1=n_1)\left[\begin{array}{l} , m_1\text{⏚}, n_1\text{⏚}, \copyright c_1, \\ , m_1\text{⏚}, n_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , m_2\text{⏚}, n_2\text{⏚}, m_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,j☮n_2,i☮m_1,i☮m_2,if(m_2=n_2)\left[\begin{array}{l},m_2⏚,n_2⏚,j☮n_1,if(m_1=n_1)\left[\begin{array}{l},m_1⏚,n_1⏚,©c_1,\\ ,m_1⏚,n_1⏚,©c_3,\end{array}\right],\\ ,m_2⏚,n_2⏚,m_1⏚,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,j☮n_2,i☮m_2,if(m_2=n_2)\left[\begin{array}{l},m_2⏚,n_2⏚,i☮m_1,j☮n_1,if(m_1=n_1)\left[\begin{array}{l},m_1⏚,n_1⏚,©c_1,\\ ,m_1⏚,n_1⏚,©c_3,\end{array}\right],\\ ,m_2⏚,n_2⏚,i☮m_1,m_1⏚,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,j☮n_2,i☮m_2,if(m_2=n_2)\left[\begin{array}{l},m_2⏚,n_2⏚,i☮m_1,j☮n_1,if(m_1=n_1)\left[\begin{array}{l},m_1⏚,n_1⏚,©c_1,\\ ,m_1⏚,n_1⏚,©c_3,\end{array}\right],\\ ,m_2⏚,n_2⏚,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮m_2,j☮n_2,if(m_2=n_2)\left[\begin{array}{l},m_2⏚,n_2⏚,i☮m_1,j☮n_1,if(m_1=n_1)\left[\begin{array}{l},m_1⏚,n_1⏚,©c_1,\\ ,m_1⏚,n_1⏚,©c_3,\end{array}\right],\\ ,m_2⏚,n_2⏚,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i♂j)\left[\begin{array}{l},if(i♂j)\left[\begin{array}{l},©c_1,\\ ,©c_3,\end{array}\right],\\ ,©c_2,\end{array}\right],$$

$$,if(i♂j)\left[\begin{array}{l},©c_1,\\ ,©c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i♂j)\left[\begin{array}{l},©c_1,\\ ,if(i♂j)\left[\begin{array}{l},©c_3,\\ ,©c_2,\end{array}\right],\end{array}\right],$$

7.6.2 Branch function with propositions

$$,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,i\overset{\circ}{\bigcirc}j,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,i!\overset{\circ}{\bigcirc}j,\copyright c_2, \end{array}\right],$$

7.6.3 Propositions with branch function

$$,i\overset{\circ}{\bigcirc}j,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,\copyright c_1,$$

$$,i!\overset{\circ}{\bigcirc}j,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \Leftrightarrow\ ,i!\overset{\circ}{\bigcirc}j,\copyright c_2,$$

7.6.4 Propositions with propositions

$$,i\overset{\circ}{\bigcirc}j, \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i\overset{\circ}{\bigcirc}j,$$

$$,i!\overset{\circ}{\bigcirc}j, \Leftrightarrow\ ,i!\overset{\circ}{\bigcirc}j,i!\overset{\circ}{\bigcirc}j,$$

7.7 Substitution

7.7.1 Propositions with branch function

$$, i\mathring{\bigcirc} j, if(j\mathring{\bigcirc} m)\text{-}\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right]\text{-}, \;\Leftrightarrow\; , i\mathring{\bigcirc} j, if(i\mathring{\bigcirc} m)\text{-}\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right]\text{-},$$

proof:

$$, i\mathring{\bigcirc} j, if(j\mathring{\bigcirc} m)\text{-}\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right]\text{-},$$

$$\Leftrightarrow\; , i\circledast p_1, j\circledast n_1, p_1 = n_1, p_1\circledcirc, n_1\circledcirc, if(j\mathring{\bigcirc} m)\text{-}\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right]\text{-},$$

$$\Leftrightarrow\; , i\circledast p_1, j\circledast n_1, p_1 = n_1, if(j\mathring{\bigcirc} m)\text{-}\left[\begin{matrix}, p_1\circledcirc, n_1\circledcirc, \copyright c_1, \\ \\ , p_1\circledcirc, n_1\circledcirc, \copyright c_2,\end{matrix}\right]\text{-},$$

$$\Leftrightarrow\; , i\circledast p_1, j\circledast n_1, p_1 = n_1, j\circledast n_2, m\circledast t, if(n_2 = t)\text{-}\left[\begin{matrix}, n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_1, \\ \\ , n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_2,\end{matrix}\right]\text{-},$$

$$\Leftrightarrow\; , i\circledast p_1, j\circledast n_1, j\circledast n_2, p_1 = n_1, m\circledast t, if(n_2 = t)\text{-}\left[\begin{matrix}, n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_1, \\ \\ , n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_2,\end{matrix}\right]\text{-},$$

$$\Leftrightarrow\; , i\circledast p_1, j\circledast n_1, j\circledast n_2, n_1 = n_2, p_1 = n_1, m\circledast t, if(n_2 = t)\text{-}\left[\begin{matrix}, n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_1, \\ \\ , n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_2,\end{matrix}\right]\text{-},$$

$$\Leftrightarrow\; , i\circledast p_1, j\circledast n_1, j\circledast n_2, n_1 = n_2, p_1 = n_2, m\circledast t, if(n_2 = t)\text{-}\left[\begin{matrix}, n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_1, \\ \\ , n_2\circledcirc, t\circledcirc, p_1\circledcirc, n_1\circledcirc, \copyright c_2,\end{matrix}\right]\text{-},$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, j \owedge n_2, p_1 = n_2, m \owedge t, if(n_2 = t) \left[\begin{array}{l} , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, j \owedge n_2, m \owedge t, p_1 = n_2, if(n_2 = t) \left[\begin{array}{l} , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, j \owedge n_2, m \owedge t, p_1 = n_2, if(p_1 = t) \left[\begin{array}{l} , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, j \owedge n_2, p_1 = n_2, m \owedge t, if(p_1 = t) \left[\begin{array}{l} , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, j \owedge n_2, n_1 = n_2, p_1 = n_2, m \owedge t, if(p_1 = t) \left[\begin{array}{l} , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, j \owedge n_2, n_1 = n_2, p_1 = n_1, m \owedge t, if(p_1 = t) \left[\begin{array}{l} , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, j \owedge n_2, p_1 = n_1, m \owedge t, if(p_1 = t) \left[\begin{array}{l} , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, p_1 = n_1, m \owedge t, if(p_1 = t) \left[\begin{array}{l} , j \owedge n_2, n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , j \owedge n_2, n_2 \obot, t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \owedge p_1, j \owedge n_1, p_1 = n_1, m \owedge t, if(p_1 = t) \left[\begin{array}{l} , t \obot, p_1 \obot, n_1 \obot, \copyright c_1, \\ \\ , t \obot, p_1 \obot, n_1 \obot, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, p_2\obot, i\oslash p_1, j\oslash n_1, p_1=n_1, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, i\oslash p_1, j\oslash n_1, p_1=n_1, p_2\obot, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, i\oslash p_1, p_2=p_1, j\oslash n_1, p_1=n_1, p_2\obot, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, i\oslash p_1, j\oslash n_1, p_2=p_1, p_1=n_1, p_2\obot, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, i\oslash p_1, j\oslash n_1, p_2=p_1, p_2=n_1, p_2\obot, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, i\oslash p_1, p_2=p_1, j\oslash n_1, p_2=n_1, p_2\obot, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, i\oslash p_1, j\oslash n_1, p_2=n_1, p_2\obot, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, j\oslash n_1, p_2=n_1, p_2\obot, i\oslash p_1, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, n_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, n_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\oslash p_2, j\oslash n_1, p_2=n_1, p_2\obot, n_1\obot, i\oslash p_1, m\oslash t, if(p_1=t)\left[\begin{array}{l}, t\obot, p_1\obot, \copyright c_1,\\ \\ , t\obot, p_1\obot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(i\overset{\circ}{\bigcirc}m)\!-\!\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!-,$$

7.7.2 Propositions with propositions

$$,i\overset{\circ}{\bigcirc}j,j\overset{\circ}{\bigcirc}m,\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i\overset{\circ}{\bigcirc}m,$$

$$,i\overset{\circ}{\bigcirc}j,j!\overset{\circ}{\bigcirc}m,\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i!\overset{\circ}{\bigcirc}m,$$

7.7.3 Identical node comparison propositions with node value comparison propositions

$$,i\overset{\circ}{\bigcirc}j,\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i=j,$$

proof:

$,i\overset{\circ}{\bigcirc}j,$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(i=j)\!-\!\left[\begin{array}{c} , \\ \\ , \end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(i=i)\!-\!\left[\begin{array}{c} , \\ \\ , \end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i⊜i\!-\!\left[\begin{array}{c} , \\ \\ , \end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i⊜i\!-\!\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(i=i)\!-\!\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(i=j)\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i=j,$$

7.7.4 Propositions with node value comparison branch function

$$,i\overset{\circ}{\bigcirc}j,if(j=m)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(i=m)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

proof:

$$,i\overset{\circ}{\bigcirc}j,if(j=m)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,,i\overset{\circ}{\bigcirc}j,i=j,if(j=m)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,,i\overset{\circ}{\bigcirc}j,i=j,if(i=m)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,,i\overset{\circ}{\bigcirc}j,if(i=m)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

7.7.5 Propositions with node value comparison propositions

$$,i\overset{\circ}{\bigcirc}j,j=m,\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i=m,$$

$$,i\overset{\circ}{\bigcirc}j,j\,!\!=m,\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i\,!\!=m,$$

7.7.6 Propositions with node null comparison branch function

$$,i\male j, if(j=\varnothing)\left[\begin{matrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{matrix}\right], \;\Leftrightarrow\; ,i\male j, if(i=\varnothing)\left[\begin{matrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{matrix}\right],$$

proof:

$$,i\male j, if(j=\varnothing)\left[\begin{matrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{matrix}\right],$$

$$\Leftrightarrow\; ,,i\male j, i=j, if(j=\varnothing)\left[\begin{matrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{matrix}\right],$$

$$\Leftrightarrow\; ,,i\male j, i=j, if(i=\varnothing)\left[\begin{matrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{matrix}\right],$$

$$\Leftrightarrow\; ,,i\male j, if(i=\varnothing)\left[\begin{matrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{matrix}\right],$$

7.7.7 Propositions with node null comparison propositions

$$,i\male j, j=\varnothing, \;\Leftrightarrow\; ,i\male j, i=\varnothing,$$

$$,i\male j, j\,!\!=\varnothing, \;\Leftrightarrow\; ,i\male j, i\,!\!=\varnothing,$$

7.8 Opposition

$$,i\overset{\circ}{\bigcirc}j,i!\overset{\circ}{\bigcirc}j, \;\Leftrightarrow\; ,\otimes,$$

proof:

$,i\overset{\circ}{\bigcirc}j,i!\overset{\circ}{\bigcirc}j,$

$\Leftrightarrow\; ,i\overset{\circ}{\bigcirc}j,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right],$

$\Leftrightarrow\; ,i\overset{\circ}{\bigcirc}j,\otimes,$

$\Leftrightarrow\; ,\otimes,$

$$,i!\overset{\circ}{\bigcirc}j,i\overset{\circ}{\bigcirc}j, \;\Leftrightarrow\; ,\otimes,$$

8 Rules of Empty Branch Function

8.1 Definition of Empty Branch Function

$$, - \left[\begin{array}{l} ,\, i = j, \\ \\ ,\, i \,!\!= j, \end{array} \right. \Leftrightarrow \ ,\, if(i = j) - \left[\begin{array}{l} , \\ \\ , \end{array} \right.$$

8.2 Axiom of Empty Branch Function

$$,\, ©c, - \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \ , - \left[\begin{array}{l} ,\, ©c, \\ \\ ,\, ©c, \end{array} \right.$$

8.3 Theorems of Empty Branch Function

$$,\, if(i = \varnothing) - \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \ , - \left[\begin{array}{l} ,\, i = \varnothing, \\ \\ ,\, i \,!\!= \varnothing, \end{array} \right.$$

proof:

$$,\, if(i = \varnothing) - \left[\begin{array}{l} , \\ \\ , \end{array} \right.$$

$$\Leftrightarrow \ ,\, \circledcirc m,\, if(i = m) - \left[\begin{array}{l} ,\, m\textcircled{$\downarrow$}, \\ \\ ,\, m\textcircled{$\downarrow$}, \end{array} \right.$$

$$\Leftrightarrow \ ,\, \circledcirc m, - \left[\begin{array}{l} ,\, i = m,\, m\textcircled{$\downarrow$}, \\ \\ ,\, i \,!\!= m,\, m\textcircled{$\downarrow$}, \end{array} \right.$$

$$\Leftrightarrow \ , - \left[\begin{array}{l} ,\, \circledcirc m,\, i = m,\, m\textcircled{$\downarrow$}, \\ \\ ,\, \circledcirc m,\, i \,!\!= m,\, m\textcircled{$\downarrow$}, \end{array} \right.$$

$$\Leftrightarrow\ ,\left[\begin{array}{l}, i = \varnothing, \\ \\ , i\, != \varnothing,\end{array}\right.$$

$$, if(i \bigcirc^{\circ} j)\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \ \Leftrightarrow\ ,\left[\begin{array}{l}, i \bigcirc^{\circ} j, \\ \\ , i! \bigcirc^{\circ} j,\end{array}\right.$$

proof:

$$, if(i \bigcirc^{\circ} j)\left[\begin{array}{l}, \\ \\ ,\end{array}\right.$$

$$\Leftrightarrow\ , i \circledast m, j \circledast n, if(m = n)\left[\begin{array}{l}, m \odot, n \odot, \\ \\ , m \odot, n \odot,\end{array}\right.$$

$$\Leftrightarrow\ , i \circledast m, j \circledast n, \left[\begin{array}{l}, m = n, m \odot, n \odot, \\ \\ , m\, != n, m \odot, n \odot,\end{array}\right.$$

$$\Leftrightarrow\ ,\left[\begin{array}{l}, i \circledast m, j \circledast n, m = n, m \odot, n \odot, \\ \\ , i \circledast m, j \circledast n, m\, != n, m \odot, n \odot,\end{array}\right.$$

$$\Leftrightarrow\ ,\left[\begin{array}{l}, i \bigcirc^{\circ} j, \\ \\ , i! \bigcirc^{\circ} j,\end{array}\right.$$

9 Swap Theorems of the Same Operand

9.1 Identical node comparison

9.1.1 Branch function and branch function

$$, if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , if(j\mathring{\bigcirc}k)\left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_2, \end{array}\right], \\ , if(j\mathring{\bigcirc}k)\left[\begin{array}{l} , \copyright c_3, \\ , \copyright c_4, \end{array}\right], \end{array}\right], \Leftrightarrow , if(j\mathring{\bigcirc}k)\left[\begin{array}{l} , if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_3, \end{array}\right], \\ , if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , \copyright c_2, \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

proof:

$$, if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , if(j\mathring{\bigcirc}k)\left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_2, \end{array}\right], \\ , if(j\mathring{\bigcirc}k)\left[\begin{array}{l} , \copyright c_3, \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \ , i\circledast t_1, j\circledast t_2, if(t_1 = t_2)\left[\begin{array}{l} , t_1\circledcirc, t_2\circledcirc, if(j\mathring{\bigcirc}k)\left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_2, \end{array}\right], \\ , t_1\circledcirc, t_2\circledcirc, if(j\mathring{\bigcirc}k)\left[\begin{array}{l} , \copyright c_3, \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \ , i\circledast t_1, j\circledast t_2, if(t_1 = t_2)\left[\begin{array}{l} , t_1\circledcirc, t_2\circledcirc, j\circledast t_3, k\circledast t_4, if(t_3 = t_4)\left[\begin{array}{l} , t_3\circledcirc, t_4\circledcirc, \copyright c_1, \\ , t_3\circledcirc, t_4\circledcirc, \copyright c_2, \end{array}\right], \\ , t_1\circledcirc, t_2\circledcirc, j\circledast t_3, k\circledast t_4, if(t_3 = t_4)\left[\begin{array}{l} , t_3\circledcirc, t_4\circledcirc, \copyright c_3, \\ , t_3\circledcirc, t_4\circledcirc, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast t_1, j\circledast t_2, j\circledast t_3, k\circledast t_4, if(t_1=t_2)\left[\begin{array}{l} , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, if(t_3=t_4)\left[\begin{array}{l} , t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right], \\ , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, if(t_3=t_4)\left[\begin{array}{l} , t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_3, \\ \\ , t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast t_1, j\circledast t_2, j\circledast t_3, k\circledast t_4, if(t_1=t_2)\left[\begin{array}{l} , if(t_3=t_4)\left[\begin{array}{l} , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right], \\ , if(t_3=t_4)\left[\begin{array}{l} , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_3, \\ \\ , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast t_1, j\circledast t_2, j\circledast t_3, k\circledast t_4, if(t_3=t_4)\left[\begin{array}{l} , if(t_1=t_2)\left[\begin{array}{l} , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ , if(t_1=t_2)\left[\begin{array}{l} , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_2, \\ \\ , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , j\circledast t_3, k\circledast t_4, if(t_3=t_4)\left[\begin{array}{l} , t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, i\circledast t_1, j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ , t_3\textcircled{$\downarrow$}, t_4\textcircled{$\downarrow$}, i\circledast t_1, j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, \copyright c_2, \\ \\ , t_1\textcircled{$\downarrow$}, t_2\textcircled{$\downarrow$}, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , if(j\mathring{\bigcirc} k)\left[\begin{array}{l} , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array}\right], \\ , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , \copyright c_2, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

9.1.2 Branch function and propositions

$$,i\overset{\circ}{\bigcirc}j,if(j\overset{\circ}{\bigcirc}k)\text{-}\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\text{-}, \iff ,if(j\overset{\circ}{\bigcirc}k)\text{-}\left[\begin{array}{l},i\overset{\circ}{\bigcirc}j,\copyright c_1,\\ \\ ,i\overset{\circ}{\bigcirc}j,\copyright c_2,\end{array}\right]\text{-},$$

$$,i!\overset{\circ}{\bigcirc}j,if(j\overset{\circ}{\bigcirc}k)\text{-}\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\text{-}, \iff ,if(j\overset{\circ}{\bigcirc}k)\text{-}\left[\begin{array}{l},i!\overset{\circ}{\bigcirc}j,\copyright c_1,\\ \\ ,i!\overset{\circ}{\bigcirc}j,\copyright c_2,\end{array}\right]\text{-},$$

9.1.3 Propositions and propositions

$$,i\overset{\circ}{\bigcirc}j,j\overset{\circ}{\bigcirc}k, \iff ,j\overset{\circ}{\bigcirc}k,i\overset{\circ}{\bigcirc}j,$$

$$,i\overset{\circ}{\bigcirc}j,j!\overset{\circ}{\bigcirc}k, \iff ,j!\overset{\circ}{\bigcirc}k,i\overset{\circ}{\bigcirc}j,$$

$$,i!\overset{\circ}{\bigcirc}j,j\overset{\circ}{\bigcirc}k, \iff ,j\overset{\circ}{\bigcirc}k,i!\overset{\circ}{\bigcirc}j,$$

$$,i!\overset{\circ}{\bigcirc}j,j!\overset{\circ}{\bigcirc}k, \iff ,j!\overset{\circ}{\bigcirc}k,i!\overset{\circ}{\bigcirc}j,$$

9.1.4 Relationship and id operator

$$,i\otimes n,if(i\overset{\circ}{\bigcirc}j)\text{-}\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\text{-}, \iff ,if(i\overset{\circ}{\bigcirc}j)\text{-}\left[\begin{array}{l},i\otimes n,\copyright c_1,\\ \\ ,i\otimes n,\copyright c_2,\end{array}\right]\text{-},$$

proof:

$$,i\otimes n,if(i\overset{\circ}{\bigcirc}j)\text{-}\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i \circledast n, i \circledast t_1, j \circledast t_2, if(t_1 = t_2) \left[\begin{array}{l} , t_1 \circleddash, t_2 \circleddash, \copyright c_1, \\ \\ , t_1 \circleddash, t_2 \circleddash, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast t_1, i \circledast n, j \circledast t_2, if(t_1 = t_2) \left[\begin{array}{l} , t_1 \circleddash, t_2 \circleddash, \copyright c_1, \\ \\ , t_1 \circleddash, t_2 \circleddash, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast t_1, j \circledast t_2, if(t_1 = t_2) \left[\begin{array}{l} , t_1 \circleddash, t_2 \circleddash, i \circledast n, \copyright c_1, \\ \\ , t_1 \circleddash, t_2 \circleddash, i \circledast n, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i \mathbin{\overset{\circ}{\bigcirc}} j) \left[\begin{array}{l} , i \circledast n, \copyright c_1, \\ \\ , i \circledast n, \copyright c_2, \end{array}\right],$$

$$,i\text{⦲}j,i\text{⊚}n, \iff ,i\text{⊚}n,i\text{⦲}j,$$

$$,i!\text{⦲}j,i\text{⊚}n, \iff ,i\text{⊚}n,i!\text{⦲}j,$$

9.1.5 Id operator and copy operator,subnode operator

$$,i\text{⊚}m,i\text{☮}n, \iff ,i\text{☮}n,i\text{⊚}m,$$

proof:

$,i\text{⊚}m,i\text{☮}n,$

$\iff ,i\text{☮}j,j\text{⏚},i\text{⊚}m,i\text{☮}n,$

$\iff ,i\text{☮}j,i\text{⊚}m,i\text{☮}n,j\text{⏚},$

$\iff ,i\text{☮}j,i\text{⦲}j,i\text{⊚}m,i\text{☮}n,j\text{⏚},$

$\iff ,i\text{☮}j,i\text{⊚}m,i\text{⦲}j,i\text{☮}n,j\text{⏚},$

$\iff ,i\text{☮}j,i\text{⊚}m,i\text{⦲}j,j\text{☮}n,j\text{⏚},$

$\iff ,i\text{☮}j,i\text{⦲}j,i\text{⊚}m,j\text{☮}n,j\text{⏚},$

$\iff ,i\text{☮}j,i\text{⦲}j,j\text{☮}n,i\text{⊚}m,j\text{⏚},$

$\iff ,i\text{☮}j,i\text{⦲}j,i\text{☮}n,i\text{⊚}m,j\text{⏚},$

$\iff ,i\text{☮}j,i\text{☮}n,i\text{⊚}m,j\text{⏚},$

$\iff ,i\text{☮}j,j\text{⏚},i\text{☮}n,i\text{⊚}m,$

$\iff ,i\text{☮}n,i\text{⊚}m,$

$$, i\circledcirc m, i\circledast n, \ \Leftrightarrow \ , i\circledast n, i\circledcirc m,$$

9.1.6 Relationship and copy operator,subnode operator

$$, i\text{☮}n, if(i\bigcirc^{\!\circ}j) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \ \Leftrightarrow \ , if(i\bigcirc^{\!\circ}j) \left[\begin{array}{l} , i\text{☮}n, \copyright c_1, \\ \\ , i\text{☮}n, \copyright c_2, \end{array} \right],$$

proof:

$$, i\text{☮}n, if(i\bigcirc^{\!\circ}j) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right],$$

$$\Leftrightarrow \ , i\text{☮}n, i\circledcirc t_1, j\circledcirc t_2, if(t_1 = t_2) \left[\begin{array}{l} , t_1\text{⏚}, t_2\text{⏚}, \copyright c_1, \\ \\ , t_1\text{⏚}, t_2\text{⏚}, \copyright c_2, \end{array} \right],$$

$$\Leftrightarrow \ , i\circledcirc t_1, i\text{☮}n, j\circledcirc t_2, if(t_1 = t_2) \left[\begin{array}{l} , t_1\text{⏚}, t_2\text{⏚}, \copyright c_1, \\ \\ , t_1\text{⏚}, t_2\text{⏚}, \copyright c_2, \end{array} \right],$$

$$\Leftrightarrow \ , i\circledcirc t_1, j\circledcirc t_2, if(t_1 = t_2) \left[\begin{array}{l} , t_1\text{⏚}, t_2\text{⏚}, i\text{☮}n, \copyright c_1, \\ \\ , t_1\text{⏚}, t_2\text{⏚}, i\text{☮}n, \copyright c_2, \end{array} \right],$$

$$\Leftrightarrow \ , if(i\bigcirc^{\!\circ}j) \left[\begin{array}{l} , i\text{☮}n, \copyright c_1, \\ \\ , i\text{☮}n, \copyright c_2, \end{array} \right],$$

$$,i\circledast n, if(i\mathring{\bigcirc}j)\begin{bmatrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{bmatrix}, \iff ,if(i\mathring{\bigcirc}j)\begin{bmatrix} ,i\circledast n,\copyright c_1, \\ \\ ,i\circledast n,\copyright c_2, \end{bmatrix},$$

$$,i\mathring{\bigcirc}j, i\circledast n, \iff ,i\circledast n, i\mathring{\bigcirc}j,$$

$$,i\circledast m, i\circledast n, \iff ,i\circledast m, i\circledast n, m\mathring{\bigcirc}n,$$

$$,i!\mathring{\bigcirc}j, i\circledast n, \iff ,i\circledast n, i!\mathring{\bigcirc}j,$$

$$,i\mathring{\bigcirc}j, i\circledast n, \iff ,i\circledast n, i\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j, i\circledast n, \iff ,i\circledast n, i!\mathring{\bigcirc}j,$$

9.1.7 Copy operator and subnode operator

$$,i\circledast m, i\circledast n, \iff ,i\circledast n, i\circledast m,$$

proof:

$,i\circledast m, i\circledast n,$

$\iff ,i\circledast j, j\circleddash, i\circledast m, i\circledast n,$

$\iff ,i\circledast j, i\circledast m, i\circledast n, j\circleddash,$

$\iff ,i\circledast j, i\mathring{\bigcirc}j, i\circledast m, i\circledast n, j\circleddash,$

$\iff ,i\circledast j, i\mathring{\bigcirc}j, j\circledast m, i\circledast n, j\circleddash,$

$\iff ,i\circledast j, i\mathring{\bigcirc}j, i\circledast n, j\circledast m, j\circleddash,$

$\iff ,i\circledast j, i\circledast n, i\mathring{\bigcirc}j, j\circledast m, j\circleddash,$

$\iff ,i\circledast j, i\circledast n, i\mathring{\bigcirc}j, i\circledast m, j\circleddash,$

$\Leftrightarrow$, i☮j, i⚲j, i☮n, i☮m, j⏚,

$\Leftrightarrow$, i☮j, i☮n, i☮m, j⏚,

$\Leftrightarrow$, i☮j, j⏚, i☮n, i☮m,

$\Leftrightarrow$, i☮n, i☮m,

$$,\, i\circledast m,\, i\circledcirc n, \iff ,\, i\circledcirc n,\, i\circledast m,$$

$$,\, i\circledcirc m,\, i\circledcirc n, \iff ,\, i\circledcirc n,\, i\circledcirc m,$$

9.2 Node value comparison

9.2.1 Operators

$$,\, i\odot m,\, if(i=j)\begin{bmatrix} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{bmatrix}, \iff ,\, if(i=j)\begin{bmatrix} ,\, i\odot m,\, \copyright c_1, \\ \\ ,\, i\odot m,\, \copyright c_2, \end{bmatrix},$$

proof:

$$,\, i\odot m,\, if(i=j)\begin{bmatrix} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{bmatrix},$$

$$\iff ,\, i\circledast i_0,\, i_0\ominus,\, i\odot m,\, if(i=j)\begin{bmatrix} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{bmatrix},$$

$$\iff ,\, i\circledast i_0,\, i\odot m,\, if(i=j)\begin{bmatrix} ,\, i_0\ominus,\, \copyright c_1, \\ \\ ,\, i_0\ominus,\, \copyright c_2, \end{bmatrix},$$

$$\iff ,\, i\circledast i_0,\, i\bigcirc i_0,\, i\odot m,\, if(i=j)\begin{bmatrix} ,\, i_0\ominus,\, \copyright c_1, \\ \\ ,\, i_0\ominus,\, \copyright c_2, \end{bmatrix},$$

$$\iff ,\, i\circledast i_0,\, i\odot m,\, i\bigcirc i_0,\, if(i=j)\begin{bmatrix} ,\, i_0\ominus,\, \copyright c_1, \\ \\ ,\, i_0\ominus,\, \copyright c_2, \end{bmatrix},$$

$$\iff ,\, i\circledast i_0,\, i\odot m,\, i\bigcirc i_0,\, if(i_0=j)\begin{bmatrix} ,\, i_0\ominus,\, \copyright c_1, \\ \\ ,\, i_0\ominus,\, \copyright c_2, \end{bmatrix},$$

$$\iff ,\, i\circledast i_0,\, i\bigcirc i_0,\, i\odot m,\, if(i_0=j)\begin{bmatrix} ,\, i_0\ominus,\, \copyright c_1, \\ \\ ,\, i_0\ominus,\, \copyright c_2, \end{bmatrix},$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\overset{\circ}{\bigcirc}i_0, if(i_0=j)\left[\begin{array}{l} ,i_0\text{⏚}, i\text{☮}\!\circ m, \text{©}c_1, \\[1.5em] ,i_0\text{⏚}, i\text{☮}\!\circ m, \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\overset{\circ}{\bigcirc}i_0, if(i=j)\left[\begin{array}{l} ,i_0\text{⏚}, i\text{☮}\!\circ m, \text{©}c_1, \\[1.5em] ,i_0\text{⏚}, i\text{☮}\!\circ m, \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, if(i=j)\left[\begin{array}{l} ,i_0\text{⏚}, i\text{☮}\!\circ m, \text{©}c_1, \\[1.5em] ,i_0\text{⏚}, i\text{☮}\!\circ m, \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i_0\text{⏚}, if(i=j)\left[\begin{array}{l} ,i\text{☮}\!\circ m, \text{©}c_1, \\[1.5em] ,i\text{☮}\!\circ m, \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=j)\left[\begin{array}{l} ,i\text{☮}\!\circ m, \text{©}c_1, \\[1.5em] ,i\text{☮}\!\circ m, \text{©}c_2, \end{array}\right],$$

$$,i☮m, if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \iff ,if(i=j)\left[\begin{array}{l} ,i☮m,\copyright c_1, \\ \\ ,i☮m,\copyright c_2, \end{array}\right],$$

$$,i☮m, if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \iff ,if(i=j)\left[\begin{array}{l} ,i☮m,\copyright c_1, \\ \\ ,i☮m,\copyright c_2, \end{array}\right],$$

$$,i=j,i☮n, \iff ,i☮n,i=j,$$

$$,i=j,i☮n, \iff ,i☮n,i=j,$$

$$,i=j,i☮n, \iff ,i☮n,i=j,$$

$$,i\,!\!=j,i☮n, \iff ,i☮n,i\,!\!=j,$$

$$,i\,!\!=j,i☮n, \iff ,i☮n,i\,!\!=j,$$

$$,i\,!\!=j,i☮n, \iff ,i☮n,i\,!\!=j,$$

9.2.2 Identical node comparison

One:

$$,if(i♂j)\left[\begin{array}{l} ,if(j=m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(j=m)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(j=m)\left[\begin{array}{l} ,if(i♂j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i♂j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

proof:

$$,if(i♂j)\left[\begin{array}{l} ,if(j=m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(j=m)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast t_1, j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, if(j=m)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \\ , t_1\circleddash, t_2\circleddash, if(j=m)\left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast t_1, j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , if(j=m)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_1, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_2, \end{array}\right], \\ , if(j=m)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_3, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast t_1, j\circledast t_2, if(j=m)\left[\begin{array}{l} , if(t_1=t_2)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_1, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_3, \end{array}\right], \\ , if(t_1=t_2)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_2, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast t_1, if(j=m)\left[\begin{array}{l} , j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_1, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_3, \end{array}\right], \\ , j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_2, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , if(j=m)\left[\begin{array}{l} , i\circledast t_1, j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_1, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_3, \end{array}\right], \\ , i\circledast t_1, j\circledast t_2, if(t_1=t_2)\left[\begin{array}{l} , t_1\circleddash, t_2\circleddash, \copyright c_2, \\ \\ , t_1\circleddash, t_2\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \; ,if(j=m)\left[\begin{array}{l} ,if(i \bigcirc^{\!\!\!\circ} j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ \\ ,if(i \bigcirc^{\!\!\!\circ} j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,i\mathring{\bigcirc}j,if(j=m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(j=m)\left[\begin{array}{l},i\mathring{\bigcirc}j,\copyright c_1,\\ \\ ,i\mathring{\bigcirc}j,\copyright c_2,\end{array}\right],$$

$$,i!\mathring{\bigcirc}j,if(j=m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(j=m)\left[\begin{array}{l},i!\mathring{\bigcirc}j,\copyright c_1,\\ \\ ,i!\mathring{\bigcirc}j,\copyright c_2,\end{array}\right],$$

$$,j=m,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},j=m,\copyright c_1,\\ \\ ,j=m,\copyright c_2,\end{array}\right],$$

$$,j\,!\!=m,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},j\,!\!=m,\copyright c_1,\\ \\ ,j\,!\!=m,\copyright c_2,\end{array}\right],$$

$$,i\mathring{\bigcirc}j,j=m, \;\Leftrightarrow\; ,j=m,i\mathring{\bigcirc}j,$$

$$,i\mathring{\bigcirc}j,j\,!\!=m, \;\Leftrightarrow\; ,j\,!\!=m,i\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j,j=m, \;\Leftrightarrow\; ,j=m,i!\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j,j\,!\!=m, \;\Leftrightarrow\; ,j\,!\!=m,i!\mathring{\bigcirc}j,$$

Two:

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},if(i=j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(i=j)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l},if(i\mathring{\bigcirc}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

proof:

$$, if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i☮i_0, i_0⏚, if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i☮i_0, if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , i_0⏚, if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \\ , i_0⏚, if(i=j)\left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i☮i_0, if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} , i_0⏚, \copyright c_1, \\ \\ , i_0⏚, \copyright c_2, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} , i_0⏚, \copyright c_3, \\ \\ , i_0⏚, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i☮i_0, i\mathring{\bigcirc}i_0, if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} , i_0⏚, \copyright c_1, \\ \\ , i_0⏚, \copyright c_2, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} , i_0⏚, \copyright c_3, \\ \\ , i_0⏚, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i☮i_0, i\mathring{\bigcirc}i_0, if(i_0\mathring{\bigcirc}j)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} , i_0⏚, \copyright c_1, \\ \\ , i_0⏚, \copyright c_2, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} , i_0⏚, \copyright c_3, \\ \\ , i_0⏚, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \ , i\oslash i_0, i\mathring{\bigcirc} i_0, if(i=j) \left[\begin{array}{l} , if(i_0\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_1, \\ \\ , i_0\circledast, \copyright c_3, \end{array} \right], \\ , if(i_0\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_2, \\ \\ , i_0\circledast, \copyright c_4, \end{array} \right], \end{array} \right],$$

$$\Leftrightarrow \ , i\oslash i_0, if(i=j) \left[\begin{array}{l} , i\mathring{\bigcirc} i_0, if(i_0\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_1, \\ \\ , i_0\circledast, \copyright c_3, \end{array} \right], \\ , i\mathring{\bigcirc} i_0, if(i_0\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_2, \\ \\ , i_0\circledast, \copyright c_4, \end{array} \right], \end{array} \right],$$

$$\Leftrightarrow \ , i\oslash i_0, if(i=j) \left[\begin{array}{l} , i\mathring{\bigcirc} i_0, if(i\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_1, \\ \\ , i_0\circledast, \copyright c_3, \end{array} \right], \\ , i\mathring{\bigcirc} i_0, if(i\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_2, \\ \\ , i_0\circledast, \copyright c_4, \end{array} \right], \end{array} \right],$$

$$\Leftrightarrow \ , i\oslash i_0, i\mathring{\bigcirc} i_0, if(i=j) \left[\begin{array}{l} , if(i\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_1, \\ \\ , i_0\circledast, \copyright c_3, \end{array} \right], \\ , if(i\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_2, \\ \\ , i_0\circledast, \copyright c_4, \end{array} \right], \end{array} \right],$$

$$\Leftrightarrow \ , i\oslash i_0, if(i=j) \left[\begin{array}{l} , if(i\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_1, \\ \\ , i_0\circledast, \copyright c_3, \end{array} \right], \\ , if(i\mathring{\bigcirc} j) \left[\begin{array}{l} , i_0\circledast, \copyright c_2, \\ \\ , i_0\circledast, \copyright c_4, \end{array} \right], \end{array} \right],$$

$$\Leftrightarrow\ , i\text{☮}i_0, i_0\text{⏚}, if(i=j)\!\left[\begin{array}{l} , if(i\text{♂}j)\!\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array}\right], \\ , if(i\text{♂}j)\!\left[\begin{array}{l} , \copyright c_2, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , if(i=j)\!\left[\begin{array}{l} , if(i\text{♂}j)\!\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array}\right], \\ , if(i\text{♂}j)\!\left[\begin{array}{l} , \copyright c_2, \\ \\ , \copyright c_4, \end{array}\right], \end{array}\right],$$

$$,i\bigcirc^{\!\circ}j,if(i=j)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix}, \quad\Leftrightarrow\quad ,if(i=j)\begin{bmatrix},i\bigcirc^{\!\circ}j,\copyright c_1,\\ \\ ,i\bigcirc^{\!\circ}j,\copyright c_2,\end{bmatrix},$$

$$,i!\bigcirc^{\!\circ}j,if(i=j)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix}, \quad\Leftrightarrow\quad ,if(i=j)\begin{bmatrix},i!\bigcirc^{\!\circ}j,\copyright c_1,\\ \\ ,i!\bigcirc^{\!\circ}j,\copyright c_2,\end{bmatrix},$$

$$,i=j,if(i\bigcirc^{\!\circ}j)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix}, \quad\Leftrightarrow\quad ,if(i\bigcirc^{\!\circ}j)\begin{bmatrix},i=j,\copyright c_1,\\ \\ ,i=j,\copyright c_2,\end{bmatrix},$$

$$,i\,!\!=j,if(i\bigcirc^{\!\circ}j)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix}, \quad\Leftrightarrow\quad ,if(i\bigcirc^{\!\circ}j)\begin{bmatrix},i\,!\!=j,\copyright c_1,\\ \\ ,i\,!\!=j,\copyright c_2,\end{bmatrix},$$

$$,i\bigcirc^{\!\circ}j,i=j, \quad\Leftrightarrow\quad ,i=j,i\bigcirc^{\!\circ}j,$$

$$,i\bigcirc^{\!\circ}j,i\,!\!=j, \quad\Leftrightarrow\quad ,i\,!\!=j,i\bigcirc^{\!\circ}j,$$

$$,i!\bigcirc^{\!\circ}j,i=j, \quad\Leftrightarrow\quad ,i=j,i!\bigcirc^{\!\circ}j,$$

$$,i!\bigcirc^{\!\circ}j,i\,!\!=j, \quad\Leftrightarrow\quad ,i\,!\!=j,i!\bigcirc^{\!\circ}j,$$

9.2.3 Itself

$$,if(i=j)\left[\begin{array}{l} ,if(j=m)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(j=m)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(j=m)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_3, \end{array}\right], \\ ,if(i=j)\left[\begin{array}{l} ,\copyright c_2, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

proof:

$$,if(i=j)\left[\begin{array}{l} ,if(j=m)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(j=m)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\iff ,j\oslash j_0, j_0\ominus, if(i=j)\left[\begin{array}{l} ,if(j=m)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(j=m)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\iff ,j\oslash j_0, if(i=j)\left[\begin{array}{l} ,j_0\ominus, if(j=m)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,j_0\ominus, if(j=m)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\iff ,j\oslash j_0, if(i=j)\left[\begin{array}{l} ,if(j=m)\left[\begin{array}{l} ,j_0\ominus, \copyright c_1, \\ ,j_0\ominus, \copyright c_2, \end{array}\right], \\ ,if(j=m)\left[\begin{array}{l} ,j_0\ominus, \copyright c_3, \\ ,j_0\ominus, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_0, j\mathring{\bigcirc} j_0, if(i=j)\left[\begin{array}{l} , if(j=m)\left[\begin{array}{l} , j_0\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, \copyright c_2, \end{array}\right], \\ , if(j=m)\left[\begin{array}{l} , j_0\circleddash, \copyright c_3, \\ \\ , j_0\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_0, j\mathring{\bigcirc} j_0, if(i=j_0)\left[\begin{array}{l} , if(j=m)\left[\begin{array}{l} , j_0\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, \copyright c_2, \end{array}\right], \\ , if(j=m)\left[\begin{array}{l} , j_0\circleddash, \copyright c_3, \\ \\ , j_0\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_0, j\mathring{\bigcirc} j_0, if(j=m)\left[\begin{array}{l} , if(i=j_0)\left[\begin{array}{l} , j_0\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, \copyright c_3, \end{array}\right], \\ , if(i=j_0)\left[\begin{array}{l} , j_0\circleddash, \copyright c_2, \\ \\ , j_0\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_0, if(j=m)\left[\begin{array}{l} , j\mathring{\bigcirc} j_0, if(i=j_0)\left[\begin{array}{l} , j_0\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, \copyright c_3, \end{array}\right], \\ , j\mathring{\bigcirc} j_0, if(i=j_0)\left[\begin{array}{l} , j_0\circleddash, \copyright c_2, \\ \\ , j_0\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_0, if(j=m)\left[\begin{array}{l} , j\mathring{\bigcirc} j_0, if(i=j)\left[\begin{array}{l} , j_0\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, \copyright c_3, \end{array}\right], \\ , j\mathring{\bigcirc} j_0, if(i=j)\left[\begin{array}{l} , j_0\circleddash, \copyright c_2, \\ \\ , j_0\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_0, j\mathring{\bigcirc} j_0, if(j=m)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} , j_0\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, \copyright c_3, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} , j_0\circleddash, \copyright c_2, \\ \\ , j_0\circleddash, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,j\circledwedge j_0,if(j=m)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,j_0\textcircled{\downarrow},\copyright c_1, \\ \\ ,j_0\textcircled{\downarrow},\copyright c_3, \end{array}\right], \\ ,if(i=j)\left[\begin{array}{l} ,j_0\textcircled{\downarrow},\copyright c_2, \\ \\ ,j_0\textcircled{\downarrow},\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,j\circledwedge j_0,if(j=m)\left[\begin{array}{l} ,j_0\textcircled{\downarrow},if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,j_0\textcircled{\downarrow},if(i=j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,j\circledwedge j_0,j_0\textcircled{\downarrow},if(j=m)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i=j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=j)\left[\begin{array}{l} ,if(j=m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(j=m)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,i\!=\!j, if(j\!=\!m)\!\!-\!\!\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!\!-\!, \;\Leftrightarrow\; ,if(j\!=\!m)\!\!-\!\!\left[\begin{array}{l} ,i\!=\!j,\copyright c_1, \\ \\ ,i\!=\!j,\copyright c_2, \end{array}\right]\!\!-\!,$$

$$,i\,!\!=\!j, if(j\!=\!m)\!\!-\!\!\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!\!-\!, \;\Leftrightarrow\; ,if(j\!=\!m)\!\!-\!\!\left[\begin{array}{l} ,i\,!\!=\!j,\copyright c_1, \\ \\ ,i\,!\!=\!j,\copyright c_2, \end{array}\right]\!\!-\!,$$

$$,i\!=\!j, j\!=\!m, \;\Leftrightarrow\; ,j\!=\!m, i\!=\!j,$$

$$,i\,!\!=\!j, j\!=\!m, \;\Leftrightarrow\; ,j\!=\!m, i\,!\!=\!j,$$

$$,i\,!\!=\!j, j\,!\!=\!m, \;\Leftrightarrow\; ,j\,!\!=\!m, i\,!\!=\!j,$$

9.3 Node null comparison

9.3.1 Operators

$$,i\circledast m, if(i\!=\!\varnothing)\!\!-\!\!\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!\!-\!, \;\Leftrightarrow\; ,if(i\!=\!\varnothing)\!\!-\!\!\left[\begin{array}{l} ,i\circledast m,\copyright c_1, \\ \\ ,i\circledast m,\copyright c_2, \end{array}\right]\!\!-\!,$$

proof:

$$,i\circledast m, if(i\!=\!\varnothing)\!\!-\!\!\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!\!-\!,$$

$$\Leftrightarrow\; ,i\circledast m, \circledcirc j, if(i\!=\!j)\!\!-\!\!\left[\begin{array}{l} ,j\textcircled{\downarrow},\copyright c_1, \\ \\ ,j\textcircled{\downarrow},\copyright c_2, \end{array}\right]\!\!-\!,$$

$$\Leftrightarrow\; ,\circledcirc j, i\circledast m, if(i\!=\!j)\!\!-\!\!\left[\begin{array}{l} ,j\textcircled{\downarrow},\copyright c_1, \\ \\ ,j\textcircled{\downarrow},\copyright c_2, \end{array}\right]\!\!-\!,$$

$$\Leftrightarrow\ ,\circledcirc j, if(i = j)\text{-}\left[\begin{array}{l}, i\text{☮}m, j\text{⏚}, \copyright c_1, \\ \\ , i\text{☮}m, j\text{⏚}, \copyright c_2, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,\circledcirc j, if(i = j)\text{-}\left[\begin{array}{l}, j\text{⏚}, i\text{☮}m, \copyright c_1, \\ \\ , j\text{⏚}, i\text{☮}m, \copyright c_2, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , if(i = \varnothing)\text{-}\left[\begin{array}{l}, i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2, \end{array}\right]\text{-},$$

$$, i\circledast m, if(i=\varnothing)\begin{bmatrix}, \copyright c_1,\\ \\ , \copyright c_2,\end{bmatrix}, \;\Leftrightarrow\; , if(i=\varnothing)\begin{bmatrix}, i\circledast m, \copyright c_1,\\ \\ , i\circledast m, \copyright c_2,\end{bmatrix},$$

$$, i\circleddash m, if(i=\varnothing)\begin{bmatrix}, \copyright c_1,\\ \\ , \copyright c_2,\end{bmatrix}, \;\Leftrightarrow\; , if(i=\varnothing)\begin{bmatrix}, i\circleddash m, \copyright c_1,\\ \\ , i\circleddash m, \copyright c_2,\end{bmatrix},$$

$$, i=\varnothing, i\circledast n, \;\Leftrightarrow\; , i\circledast n, i=\varnothing,$$

$$, i=\varnothing, i\circledcirc n, \;\Leftrightarrow\; , i\circledcirc n, i=\varnothing,$$

$$, i=\varnothing, i\circleddash n, \;\Leftrightarrow\; , i\circleddash n, i=\varnothing,$$

$$, i\,!\!=\varnothing, i\circledast n, \;\Leftrightarrow\; , i\circledast n, i\,!\!=\varnothing,$$

$$, i\,!\!=\varnothing, i\circledcirc n, \;\Leftrightarrow\; , i\circledcirc n, i\,!\!=\varnothing,$$

$$, i\,!\!=\varnothing, i\circleddash n, \;\Leftrightarrow\; , i\circleddash n, i\,!\!=\varnothing,$$

9.3.2 Identical node comparison

$$, if(i\mathring{\bigcirc}j)\begin{bmatrix}, if(i=\varnothing)\begin{bmatrix}, \copyright c_1,\\ \\ , \copyright c_2,\end{bmatrix},\\ , if(i=\varnothing)\begin{bmatrix}, \copyright c_3,\\ \\ , \copyright c_4,\end{bmatrix},\end{bmatrix}, \;\Leftrightarrow\; , if(i=\varnothing)\begin{bmatrix}, if(i\mathring{\bigcirc}j)\begin{bmatrix}, \copyright c_1,\\ \\ , \copyright c_3,\end{bmatrix},\\ , if(i\mathring{\bigcirc}j)\begin{bmatrix}, \copyright c_2,\\ \\ , \copyright c_4,\end{bmatrix},\end{bmatrix},$$

proof:

$$, if(i\mathring{\bigcirc}j)\begin{bmatrix}, if(i=\varnothing)\begin{bmatrix}, \copyright c_1,\\ \\ , \copyright c_2,\end{bmatrix},\\ , if(i=\varnothing)\begin{bmatrix}, \copyright c_3,\\ \\ , \copyright c_4,\end{bmatrix},\end{bmatrix},$$

$$\Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\circledcirc t, if(i=t)\left[\begin{array}{l} ,t\textcircled{$\downarrow$}, \copyright c_1, \\ \\ ,t\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right], \\ ,\circledcirc t, if(i=t)\left[\begin{array}{l} ,t\textcircled{$\downarrow$}, \copyright c_3, \\ \\ ,t\textcircled{$\downarrow$}, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc t, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,if(i=t)\left[\begin{array}{l} ,t\textcircled{$\downarrow$}, \copyright c_1, \\ \\ ,t\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right], \\ ,if(i=t)\left[\begin{array}{l} ,t\textcircled{$\downarrow$}, \copyright c_3, \\ \\ ,t\textcircled{$\downarrow$}, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc t, if(i=t)\left[\begin{array}{l} ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,t\textcircled{$\downarrow$}, \copyright c_1, \\ \\ ,t\textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,t\textcircled{$\downarrow$}, \copyright c_2, \\ \\ ,t\textcircled{$\downarrow$}, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc t, if(i=t)\left[\begin{array}{l} ,t\textcircled{$\downarrow$}, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,t\textcircled{$\downarrow$}, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,i\male j, if(i=\varnothing)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{c} ,i\male j,\,\copyright c_1, \\ \\ ,i\male j,\,\copyright c_2, \end{array}\right],$$

$$,i!\male j, if(i=\varnothing)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{c} ,i!\male j,\,\copyright c_1, \\ \\ ,i!\male j,\,\copyright c_2, \end{array}\right],$$

$$,i=\varnothing, if(i\male j)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\male j)\left[\begin{array}{c} ,i=\varnothing,\,\copyright c_1, \\ \\ ,i=\varnothing,\,\copyright c_2, \end{array}\right],$$

$$,i\,!\!=\varnothing, if(i\male j)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\male j)\left[\begin{array}{c} ,i\,!\!=\varnothing,\,\copyright c_1, \\ \\ ,i\,!\!=\varnothing,\,\copyright c_2, \end{array}\right],$$

$$,i\male j, i=\varnothing, \;\Leftrightarrow\; ,i=\varnothing, i\male j,$$

$$,i\male j, i\,!\!=\varnothing, \;\Leftrightarrow\; ,i\,!\!=\varnothing, i\male j,$$

$$,i!\male j, i=\varnothing, \;\Leftrightarrow\; ,i=\varnothing, i!\male j,$$

$$,i!\male j, i\,!\!=\varnothing, \;\Leftrightarrow\; ,i\,!\!=\varnothing, i!\male j,$$

9.3.3 Node value comparison

$$,if(i=j)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(i=\varnothing)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_3, \end{array}\right], \\ ,if(i=j)\left[\begin{array}{l} ,\copyright c_2, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

proof:

$$,if(i=j)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\iff ,if(i=j)\left[\begin{array}{l} ,\circledcirc t,if(i=t)\left[\begin{array}{l} ,t\text{\textcircled{$\downarrow$}},\copyright c_1, \\ ,t\text{\textcircled{$\downarrow$}},\copyright c_2, \end{array}\right], \\ ,\circledcirc t,if(i=t)\left[\begin{array}{l} ,t\text{\textcircled{$\downarrow$}},\copyright c_3, \\ ,t\text{\textcircled{$\downarrow$}},\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\iff ,\circledcirc t,if(i=j)\left[\begin{array}{l} ,if(i=t)\left[\begin{array}{l} ,t\text{\textcircled{$\downarrow$}},\copyright c_1, \\ ,t\text{\textcircled{$\downarrow$}},\copyright c_2, \end{array}\right], \\ ,if(i=t)\left[\begin{array}{l} ,t\text{\textcircled{$\downarrow$}},\copyright c_3, \\ ,t\text{\textcircled{$\downarrow$}},\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\iff ,\circledcirc t,if(i=t)\left[\begin{array}{l} ,if(i=j)\left[\begin{array}{l} ,t\text{\textcircled{$\downarrow$}},\copyright c_1, \\ ,t\text{\textcircled{$\downarrow$}},\copyright c_3, \end{array}\right], \\ ,if(i=j)\left[\begin{array}{l} ,t\text{\textcircled{$\downarrow$}},\copyright c_2, \\ ,t\text{\textcircled{$\downarrow$}},\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc t, if(i=t)\left[\begin{array}{l} ,t\textcircled{\scriptsize$\downarrow$}, if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,t\textcircled{\scriptsize$\downarrow$}, if(i=j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ , if(i=j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$, i=j, if(i=\varnothing)\left[\begin{array}{c}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{c}, i=j, \copyright c_1, \\ \\ , i=j, \copyright c_2,\end{array}\right],$$

$$, i\,!{=}\,j, if(i=\varnothing)\left[\begin{array}{c}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{c}, i\,!{=}\,j, \copyright c_1, \\ \\ , i\,!{=}\,j, \copyright c_2,\end{array}\right],$$

$$, i=\varnothing, if(i=j)\left[\begin{array}{c}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{array}{c}, i=\varnothing, \copyright c_1, \\ \\ , i=\varnothing, \copyright c_2,\end{array}\right],$$

$$, i\,!{=}\,\varnothing, if(i=j)\left[\begin{array}{c}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{array}{c}, i\,!{=}\,\varnothing, \copyright c_1, \\ \\ , i\,!{=}\,\varnothing, \copyright c_2,\end{array}\right],$$

$$, i=j, i=\varnothing, \;\Leftrightarrow\; , i=\varnothing, i=j,$$

$$, i=j, i\,!{=}\,\varnothing, \;\Leftrightarrow\; , i\,!{=}\,\varnothing, i=j,$$

$$, i\,!{=}\,j, i=\varnothing, \;\Leftrightarrow\; , i=\varnothing, i\,!{=}\,j,$$

$$, i\,!{=}\,j, i\,!{=}\,\varnothing, \;\Leftrightarrow\; , i\,!{=}\,\varnothing, i\,!{=}\,j,$$

10 Theorems of Operators and Relationships

10.1 Identity

$$, \Leftrightarrow \; , i = i,$$

proof:

$,$

$$\Leftrightarrow \; , i \text{☮} j, j \text{⏚}, $$

$$\Leftrightarrow \; , i \text{☮} j, i \bigcirc^{\!\circ} j, j \text{⏚}, $$

$$\Leftrightarrow \; , i \text{☮} j, i \bigcirc^{\!\circ} j, if(i = j) - \left[\begin{array}{l} , \\ \\ , \end{array} \right] -, j \text{⏚}, $$

$$\Leftrightarrow \; , i \text{☮} j, i \bigcirc^{\!\circ} j, if(i = i) - \left[\begin{array}{l} , \\ \\ , \end{array} \right] -, j \text{⏚}, $$

$$\Leftrightarrow \; , i \text{☮} j, if(i = i) - \left[\begin{array}{l} , \\ \\ , \end{array} \right] -, j \text{⏚}, $$

$$\Leftrightarrow \; , i \text{☮} j, i \circledequal i - \left[\begin{array}{l} , \\ \\ , \end{array} \right] -, j \text{⏚}, $$

$$\Leftrightarrow \; , i \text{☮} j, i \circledequal i - \left[\begin{array}{l} , \\ \\ , \otimes , \end{array} \right] -, j \text{⏚}, $$

$$\Leftrightarrow \; , i \text{☮} j, i \circledequal i - \left[\begin{array}{l} , j \text{⏚}, \\ \\ , \otimes , \end{array} \right] -, $$

$$\Leftrightarrow \; , i \text{☮} j, i \circledequal i - \left[\begin{array}{l} , j \text{⏚}, \\ \\ , j \text{⏚}, \otimes , \end{array} \right] -, $$

$$\Leftrightarrow\ ,i\text{☮}j,j\text{⏚},i\text{⊜}i\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{⊜}i\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=i)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,i=i,$$

10.2 Global space operator

$$,\odot i,\odot j,\ \Leftrightarrow\ ,\odot i,\odot j,i\overset{\circ}{\bigcirc}j,$$

$$,\odot i,i\text{☮}j,\ \Leftrightarrow\ ,\odot j,j\text{☮}i,$$

10.3 Temporary space operator

$$,\circledcirc i,\ \Leftrightarrow\ ,\circledcirc i,i=\varnothing,$$

proof:
$,\circledcirc i,$

$$\Leftrightarrow\ ,\circledcirc i,\circledcirc j,j\text{⏚},$$

$$\Leftrightarrow\ ,\circledcirc i,\circledcirc j,i=j,j\text{⏚},$$

$$\Leftrightarrow\ ,\circledcirc i,\circledcirc j,if(i=j)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],j\text{⏚},$$

$$\Leftrightarrow\ ,\circledcirc i,\circledcirc j,if(i=j)\left[\begin{array}{l},j\text{⏚},\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc i,\circledcirc j,if(i=j)\left[\begin{array}{l},j\text{⏚},\\ \\ ,j\text{⏚},\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,\circledcirc i, if(i=\varnothing)-\!\!\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right]\!\!-,$$

$$\Leftrightarrow\ ,\circledcirc i, i=\varnothing,$$

10.4 Id operator

$$,i\text{⊛}t,\ \Leftrightarrow\ ,i\text{⊛}t, i!\text{⦲}t,$$

proof:
$,i\text{⊛}t,$

$$\Leftrightarrow\ ,i\text{☮}j, j\text{⏚}, i\text{⊛}t,$$

$$\Leftrightarrow\ ,i\text{☮}j, i\text{⊛}t, j\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}j, i\text{⊛}t, j!\text{⦲}t, j\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}j, i\text{⦲}j, i\text{⊛}t, j!\text{⦲}t, j\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}j, i\text{⊛}t, i\text{⦲}j, j!\text{⦲}t, j\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}j, i\text{⊛}t, i\text{⦲}j, i!\text{⦲}t, j\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}j, i\text{⦲}j, i\text{⊛}t, i!\text{⦲}t, j\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}j, i\text{⊛}t, i!\text{⦲}t, j\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}j, j\text{⏚}, i\text{⊛}t, i!\text{⦲}t,$$

$$\Leftrightarrow\ ,i\text{⊛}t, i!\text{⦲}t,$$

10.5 Copy operator

$$,j=\varnothing, j\text{☮}j_0,\ \Leftrightarrow\ ,j\text{☮}j_0, j_0=\varnothing,$$

$$,j\ !\!=\varnothing, j\text{☮}j_0,\ \Leftrightarrow\ ,j\text{☮}j_0, j_0\ !\!=\varnothing,$$

$$,i\mathring{\bigcirc}j,j\oslash j_0,\ \Leftrightarrow\ ,j\oslash j_0,i\mathring{\bigcirc}j_0,$$

$$,i!\mathring{\bigcirc}j,j\oslash j_0,\ \Leftrightarrow\ ,j\oslash j_0,i!\mathring{\bigcirc}j_0,$$

$$,i\mathring{\bigcirc}j,i\oslash i_0,j\oslash j_0,\ \Leftrightarrow\ ,i\oslash i_0,j\oslash j_0,i_0\mathring{\bigcirc}j_0,$$

$$,i!\mathring{\bigcirc}j,i\oslash i_0,j\oslash j_0,\ \Leftrightarrow\ ,i\oslash i_0,j\oslash j_0,i_0!\mathring{\bigcirc}j_0,$$

$$,i\mathring{\bigcirc}j,i\oslash i_0,j\oslash j_0,\ \Leftrightarrow\ ,\sim,i_0\mathring{\bigcirc}j_0,$$

10.6 Next node operator

$$,i\mathring{\bigcirc}j,i\oplus,j\oplus,\ \Leftrightarrow\ ,i\oplus,j\oplus,i\mathring{\bigcirc}j,$$

proof:
$,i\mathring{\bigcirc}j,i\oplus,j\oplus,$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , \\ ,\otimes, \end{array}\right],i\oplus,j\oplus,$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\oplus,j\oplus, \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\oplus,j\oplus, \\ ,i\oplus,j\oplus,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus,j\oplus,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} , \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus,j\oplus,i\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j,i\oplus,j\oplus,\ \Leftrightarrow\ ,i\oplus,j\oplus,i!\mathring{\bigcirc}j,$$

10.7 Previous node operator

$$, i\ominus, j\ominus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \iff , if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\ominus, j\ominus, \\ \\ , i\ominus, j\ominus,\end{array}\right.$$

proof:

$$, i\ominus, j\ominus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ \\ ,\end{array}\right.$$

$$\iff , i\ominus, j\ominus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\oplus, i\ominus, \\ \\ , i\oplus, i\ominus,\end{array}\right.$$

$$\iff , i\ominus, j\ominus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\oplus, i\ominus, j\oplus, j\ominus, \\ \\ , i\oplus, i\ominus, j\oplus, j\ominus,\end{array}\right.$$

$$\iff , i\ominus, j\ominus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\oplus, j\oplus, i\ominus, j\ominus, \\ \\ , i\oplus, j\oplus, i\ominus, j\ominus,\end{array}\right.$$

$$\iff , i\ominus, j\ominus, i\oplus, j\oplus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\ominus, j\ominus, \\ \\ , i\ominus, j\ominus,\end{array}\right.$$

$$\iff , i\ominus, i\oplus, j\ominus, j\oplus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\ominus, j\ominus, \\ \\ , i\ominus, j\ominus,\end{array}\right.$$

$$\iff , i\oplus, i\ominus, j\ominus, j\oplus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\ominus, j\ominus, \\ \\ , i\ominus, j\ominus,\end{array}\right.$$

$$\iff , j\ominus, j\oplus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\ominus, j\ominus, \\ \\ , i\ominus, j\ominus,\end{array}\right.$$

$$\iff , j\oplus, j\ominus, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\ominus, j\ominus, \\ \\ , i\ominus, j\ominus,\end{array}\right.$$

$$\iff , if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\ominus, j\ominus, \\ \\ , i\ominus, j\ominus,\end{array}\right.$$

$$, i\mathring{\bigcirc}j, i\ominus, j\ominus, \iff , i\ominus, j\ominus, i\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j,i\ominus,j\ominus,\ \Leftrightarrow\ ,i\ominus,j\ominus,i!\mathring{\bigcirc}j,$$

10.8 Subnode operator

$$,i=\varnothing,i\circledast t,\ \Leftrightarrow\ \sim,i\mathring{\bigcirc}t,$$

$$,i\,!\!=\varnothing,j\,!\!=\varnothing,i\circledast t_1,j\circledast t_2,i\mathring{\bigcirc}j,\ \Leftrightarrow\ ,i\,!\!=\varnothing,j\,!\!=\varnothing,i\circledast t_1,j\circledast t_2,t_1\mathring{\bigcirc}t_2,$$

$$,i\,!\!=\varnothing,j\,!\!=\varnothing,i\circledast t_1,j\circledast t_2,i!\mathring{\bigcirc}j,\ \Leftrightarrow\ ,i\,!\!=\varnothing,j\,!\!=\varnothing,i\circledast t_1,j\circledast t_2,t_1!\mathring{\bigcirc}t_2,$$

$$,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,\ \Leftrightarrow\ ,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,t_1\mathring{\bigcirc}t_2,$$

proof:
$,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, \\ ,\end{array}\right],i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2, \\ ,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},i=\varnothing,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2, \\ ,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},i\mathring{\bigcirc}j,i=\varnothing,i\circledast t_1,j\circledast t_2, \\ ,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},i\mathring{\bigcirc}j,i=\varnothing,i\circledast t_1,j\circledast t_2, \\ ,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},i\mathring{\bigcirc}j,i=\varnothing,i\circledast t_1,i\mathring{\bigcirc}t_1,j\circledast t_2, \\ ,i\mathring{\bigcirc}j,i\circledast t_1,j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\mathring{\bigcirc}j, i=\varnothing, i\circledast t_1, i\mathring{\bigcirc}t_1, j\circledast t_2, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\mathring{\bigcirc}j, i=\varnothing, j\circledast t_2, i\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\mathring{\bigcirc}j, j=\varnothing, j\circledast t_2, i\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\mathring{\bigcirc}j, j=\varnothing, j\circledast t_2, i\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\mathring{\bigcirc}j, j=\varnothing, j\circledast t_2, j\mathring{\bigcirc}t_2, i\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, j=\varnothing, j\circledast t_2, i\mathring{\bigcirc}j, j\mathring{\bigcirc}t_2, i\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, j=\varnothing, j\circledast t_2, i\mathring{\bigcirc}j, i\mathring{\bigcirc}t_2, i\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, j=\varnothing, j\circledast t_2, i\mathring{\bigcirc}j, i\mathring{\bigcirc}t_2, t_2\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, j=\varnothing, j\circledast t_2, i\mathring{\bigcirc}j, j\mathring{\bigcirc}t_2, t_2\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\mathring{\bigcirc}j, j=\varnothing, j\circledast t_2, j\mathring{\bigcirc}t_2, t_2\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\mathring{\bigcirc}j, j=\varnothing, j\circledast t_2, t_2\mathring{\bigcirc}t_1, \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\overset{\circ}{\bigcirc}j, j=\varnothing, j\circledast t_2, t_2\overset{\circ}{\bigcirc}t_1,\\ , i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\circledast t_1, i\overset{\circ}{\bigcirc}j, i=\varnothing, j\circledast t_2, t_2\overset{\circ}{\bigcirc}t_1,\\ , i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i=\varnothing, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_2\overset{\circ}{\bigcirc}t_1,\\ , i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_2\overset{\circ}{\bigcirc}t_1,\\ , i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\overset{\circ}{\bigcirc}t_2,\\ , i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\overset{\circ}{\bigcirc}t_2,\\ , i\,!\!=\varnothing, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\overset{\circ}{\bigcirc}t_2,\\ , i\overset{\circ}{\bigcirc}j, i\,!\!=\varnothing, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\overset{\circ}{\bigcirc}t_2,\\ , i\overset{\circ}{\bigcirc}j, i\,!\!=\varnothing, i\,!\!=\varnothing, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\overset{\circ}{\bigcirc}t_2,\\ , i\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, i\,!\!=\varnothing, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\overset{\circ}{\bigcirc}t_2,\\ , i\overset{\circ}{\bigcirc}j, i\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, i\,!\!=\varnothing, i\circledast t_1, j\circledast t_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\overset{\circ}{\bigcirc}t_2,\\ , i\overset{\circ}{\bigcirc}j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\circledast t_1, j\circledast t_2, i\overset{\circ}{\bigcirc}j,\end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \\ \\ , i\mathring{\bigcirc}j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \\ \\ , i\mathring{\bigcirc}j, j\,!\!=\varnothing, i\,!\!=\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \\ \\ , i\mathring{\bigcirc}j, i\,!\!=\varnothing, i\,!\!=\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \\ \\ , i\mathring{\bigcirc}j, i\,!\!=\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \\ \\ , i\,!\!=\varnothing, i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \\ \\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right], i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2,$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}j, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}t_2,$$

$$, i\circledast t_1, i\circledast t_2, \ \Leftrightarrow\ , i\circledast t_1, i\circledast t_2, t_1\mathring{\bigcirc}t_2,$$

proof:
$, i\circledast t_1, i\circledast t_2,$

$$\Leftrightarrow\ , i\otimes j, j\ominus, i\circledast t_1, i\circledast t_2,$$

$$\Leftrightarrow\ , i\otimes j, i\circledast t_1, i\circledast t_2, j\ominus,$$

$$\Leftrightarrow\ , i\otimes j, i\mathring{\bigcirc}j, i\circledast t_1, i\circledast t_2, j\ominus,$$

$$\Leftrightarrow\ , i\otimes j, i\mathring{\bigcirc}j, j\circledast t_1, i\circledast t_2, j\ominus,$$

$$\Leftrightarrow\ , i\circledast j, j\mathring{\bigcirc} i, j\circledcirc t_1, i\circledcirc t_2, j\circleddash,$$

$$\Leftrightarrow\ , i\circledast j, j\mathring{\bigcirc} i, j\circledcirc t_1, i\circledcirc t_2, t_1\mathring{\bigcirc} t_2, j\circleddash,$$

$$\Leftrightarrow\ , i\circledast j, i\mathring{\bigcirc} j, j\circledcirc t_1, i\circledcirc t_2, t_1\mathring{\bigcirc} t_2, j\circleddash,$$

$$\Leftrightarrow\ , i\circledast j, i\mathring{\bigcirc} j, i\circledcirc t_1, i\circledcirc t_2, t_1\mathring{\bigcirc} t_2, j\circleddash,$$

$$\Leftrightarrow\ , i\circledast j, i\circledcirc t_1, i\circledcirc t_2, t_1\mathring{\bigcirc} t_2, j\circleddash,$$

$$\Leftrightarrow\ , i\circledast j, j\circleddash, i\circledcirc t_1, i\circledcirc t_2, t_1\mathring{\bigcirc} t_2,$$

$$\Leftrightarrow\ , i\circledcirc t_1, i\circledcirc t_2, t_1\mathring{\bigcirc} t_2,$$

$$, i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \ \Leftrightarrow\ , i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, i!\mathring{\bigcirc} j,$$

proof:

$, i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2,$

$$\Leftrightarrow\ , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , \\ , \end{array}\right], i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2,$$

$$\Leftrightarrow\ , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \\ , i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , i\mathring{\bigcirc} j, i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \\ , i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , i\mathring{\bigcirc} j, i\circledcirc t_1, j\circledcirc t_2, t_1\mathring{\bigcirc} t_2, t_1!\mathring{\bigcirc} t_2, \\ , i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , i\mathring{\bigcirc} j, i\circledcirc t_1, j\circledcirc t_2, \otimes, \\ , i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathring{\bigcirc} j)\left[\begin{array}{l} , i\circledcirc t_1, j\circledcirc t_2, \otimes, \\ , i\circledcirc t_1, j\circledcirc t_2, t_1!\mathring{\bigcirc} t_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},\otimes,\\ \\ ,i\oslash t_1,j\oslash t_2,t_1!\mathring{\bigcirc}t_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},\otimes,\\ \\ ,\end{array}\right]\text{-},i\oslash t_1,j\oslash t_2,t_1!\mathring{\bigcirc}t_2,$$

$$\Leftrightarrow\ ,i!\mathring{\bigcirc}j,i\oslash t_1,j\oslash t_2,t_1!\mathring{\bigcirc}t_2,$$

$$\Leftrightarrow\ ,i\oslash t_1,j\oslash t_2,t_1!\mathring{\bigcirc}t_2,i!\mathring{\bigcirc}j,$$

10.9 Other

$$,i\,!{=}\,j,\ \Leftrightarrow\ \sim,i!\mathring{\bigcirc}j,$$

proof:

$,i\,!{=}\,j,$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},\\ \\ ,\end{array}\right]\text{-},i\,!{=}\,j,$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},i\,!{=}\,j,\\ \\ ,i\,!{=}\,j,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},i\mathring{\bigcirc}j,i\,!{=}\,j,\\ \\ ,i\,!{=}\,j,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},i\mathring{\bigcirc}j,i=j,i\,!{=}\,j,\\ \\ ,i\,!{=}\,j,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},i\mathring{\bigcirc}j,\otimes,\\ \\ ,i\,!{=}\,j,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},\otimes,\\ \\ ,i\,!{=}\,j,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l},\otimes,\\ \\ ,\end{array}\right]\text{-},i\,!{=}\,j,$$

$$\Leftrightarrow\ ,i!\mathring{\bigcirc}j,i\,!{=}\,j,$$

$\Leftrightarrow\ ,i\,!{=}\,j, i!\circlearrowright j,$

$$,i=\varnothing, j\,!{=}\,\varnothing,\ \Leftrightarrow\ \sim, i!\circlearrowright j,$$

proof:
$,i=\varnothing, j\,!{=}\,\varnothing,$
$\Leftrightarrow\ ,i=\varnothing, i\,!{=}\,j,$

$\Leftrightarrow\ ,i=\varnothing, i\,!{=}\,j, i!\circlearrowright j,$

$\Leftrightarrow\ ,i=\varnothing, j\,!{=}\,\varnothing, i!\circlearrowright j,$

11 Next Order Induction

11.1 Definition of flag object &SHi with identical node.

11.1.1 Swap definition:

$$, \&SHi\male i, \circledcirc m, \iff , \circledcirc m, \&SHi\male i,$$

$$, \&SHi\male i, \odot m, \iff , \odot m, \&SHi\male i,$$

$$, \&SHi\male i, j\text{☮}n, \iff , j\text{☮}n, \&SHi\male i,$$

$$, \&SHi\male i, j\text{☮}n, \iff , j\text{☮}n, \&SHi\male i,$$

$$, \&SHi\male i, j\text{☮}n, \iff , j\text{☮}n, \&SHi\male i,$$

$$, \&SHi\male i, j\circledbot, \iff , j\circledbot, \&SHi\male i,$$

$$, \&SHi\male i, j\oplus, \iff , j\oplus, \&SHi\male i,$$

$$, \&SHi\male i, \otimes, \iff , \otimes, \&SHi\male i,$$

$$, \&SHi\male i, m\circledequal n\left[\begin{array}{l}, \\ , \end{array}\right. \iff , m\circledequal n\left[\begin{array}{l}, \&SHi\male i, \\ , \&SHi\male i, \end{array}\right.$$

$$, \&SHi\male i, \left[\begin{array}{l}, \\ , \end{array}\right. \iff , \left[\begin{array}{l}, \&SHi\male i, \\ , \&SHi\male i, \end{array}\right.$$

$$\left.\begin{array}{l}, \\ , \end{array}\right], \&SHi\male i, \iff \left.\begin{array}{l}, \&SHi\male i, \\ , \&SHi\male i, \end{array}\right],$$

11.1.2 Substitution definition:

$$, i \mathring{\bigcirc} j,\ \&SHi \mathring{\bigcirc} i, \iff , i \mathring{\bigcirc} j,\ \&SHi \mathring{\bigcirc} j,$$

11.2 Definition of flag object &SHi with next node.

$$, \&SHi \rightarrow i, \iff , i \text{☮} i_0, i_0 \ominus, \&SHi \mathring{\bigcirc} i_0, i_0 \text{⏚},$$

11.3 Definition of flag object &SHi with prev node.

$$, \&SHi \leftarrow i, \iff , i \text{☮} i_0, i_0 \oplus, \&SHi \mathring{\bigcirc} i_0, i_0 \text{⏚},$$

11.4 Theorems of flag object &SHi with identical node.

11.4.1 Swap with previous node operator:

$$, \&SHi \mathring{\bigcirc} i, j \ominus, \iff , j \ominus, \&SHi \mathring{\bigcirc} i,$$

11.4.2 Swap with branch function:

$$, \&SHi \mathring{\bigcirc} i, if(m = n) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \iff , if(m = n) \left[\begin{array}{l} , \&SHi \mathring{\bigcirc} i, \copyright c_1, \\ \\ , \&SHi \mathring{\bigcirc} i, \copyright c_2, \end{array} \right],$$

$$, \&SHi \mathring{\bigcirc} i, if(m = \varnothing) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \iff , if(m = \varnothing) \left[\begin{array}{l} , \&SHi \mathring{\bigcirc} i, \copyright c_1, \\ \\ , \&SHi \mathring{\bigcirc} i, \copyright c_2, \end{array} \right],$$

$$, \&SHi \mathring{\bigcirc} i, if(m \mathring{\bigcirc} n) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \iff , if(m \mathring{\bigcirc} n) \left[\begin{array}{l} , \&SHi \mathring{\bigcirc} i, \copyright c_1, \\ \\ , \&SHi \mathring{\bigcirc} i, \copyright c_2, \end{array} \right],$$

11.4.3 Swap with propositions:

$$, \&SHi\,\mathring{\bigcirc}i, m=n, \;\Leftrightarrow\; , m=n,\ \&SHi\,\mathring{\bigcirc}i,$$

$$, \&SHi\,\mathring{\bigcirc}i, m=\varnothing, \;\Leftrightarrow\; , m=\varnothing,\ \&SHi\,\mathring{\bigcirc}i,$$

$$, \&SHi\,\mathring{\bigcirc}i, m\mathring{\bigcirc}n, \;\Leftrightarrow\; , m\mathring{\bigcirc}n,\ \&SHi\,\mathring{\bigcirc}i,$$

$$, \&SHi\,\mathring{\bigcirc}i, m\,!\!=n, \;\Leftrightarrow\; , m\,!\!=n,\ \&SHi\,\mathring{\bigcirc}i,$$

$$, \&SHi\,\mathring{\bigcirc}i, m\,!\!=\varnothing, \;\Leftrightarrow\; , m\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i,$$

$$, \&SHi\,\mathring{\bigcirc}i, m!\mathring{\bigcirc}n, \;\Leftrightarrow\; , m!\mathring{\bigcirc}n,\ \&SHi\,\mathring{\bigcirc}i,$$

11.4.4 Swap with the same operand's operator:

$$, \&SHi\,\mathring{\bigcirc}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHi\,\mathring{\bigcirc}i,$$

$$, \&SHi\,\mathring{\bigcirc}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHi\,\mathring{\bigcirc}i,$$

$$, \&SHi\,\mathring{\bigcirc}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHi\,\mathring{\bigcirc}i,$$

11.4.5 Swap with the same operand's branch function:

$$, \&SHi\,\mathring{\bigcirc}i, if(i=j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{array}{l}, \&SHi\,\mathring{\bigcirc}i, \copyright c_1, \\ \\ , \&SHi\,\mathring{\bigcirc}i, \copyright c_2,\end{array}\right],$$

proof:

$$, \&SHi\,\mathring{\bigcirc}i, if(i=j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; , i\text{☮}i_0, i_0\text{⏚}, \&SHi\,\mathring{\bigcirc}i, if(i=j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0,\ \&SHi\,\text{♂}i, if(i=j) \left[\begin{array}{l} , i_0\text{⏚},\ \text{©}c_1, \\ \\ , i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, i\text{♂}i_0,\ \&SHi\,\text{♂}i, if(i=j) \left[\begin{array}{l} , i_0\text{⏚},\ \text{©}c_1, \\ \\ , i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, i\text{♂}i_0,\ \&SHi\,\text{♂}i_0, if(i=j) \left[\begin{array}{l} , i_0\text{⏚},\ \text{©}c_1, \\ \\ , i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, i\text{♂}i_0, if(i=j) \left[\begin{array}{l} ,\ \&SHi\,\text{♂}i_0, i_0\text{⏚},\ \text{©}c_1, \\ \\ ,\ \&SHi\,\text{♂}i_0, i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, if(i=j) \left[\begin{array}{l} , i\text{♂}i_0,\ \&SHi\,\text{♂}i_0, i_0\text{⏚},\ \text{©}c_1, \\ \\ , i\text{♂}i_0,\ \&SHi\,\text{♂}i_0, i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, if(i=j) \left[\begin{array}{l} , i\text{♂}i_0,\ \&SHi\,\text{♂}i, i_0\text{⏚},\ \text{©}c_1, \\ \\ , i\text{♂}i_0,\ \&SHi\,\text{♂}i, i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, i\text{♂}i_0, if(i=j) \left[\begin{array}{l} ,\ \&SHi\,\text{♂}i, i_0\text{⏚},\ \text{©}c_1, \\ \\ ,\ \&SHi\,\text{♂}i, i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, if(i=j) \left[\begin{array}{l} ,\ \&SHi\,\text{♂}i, i_0\text{⏚},\ \text{©}c_1, \\ \\ ,\ \&SHi\,\text{♂}i, i_0\text{⏚},\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\text{☮}i_0, i_0\text{⏚}, if(i=j) \left[\begin{array}{l} ,\ \&SHi\,\text{♂}i,\ \text{©}c_1, \\ \\ ,\ \&SHi\,\text{♂}i,\ \text{©}c_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i=j)\left[\begin{array}{l}, \&SHi\mathring{\bigcirc}i, ©c_1, \\ \\ , \&SHi\mathring{\bigcirc}i, ©c_2,\end{array}\right],$$

$$, \&SHi\mathring{\bigcirc}i, if(i=\varnothing)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l}, \&SHi\mathring{\bigcirc}i, ©c_1, \\ \\ , \&SHi\mathring{\bigcirc}i, ©c_2,\end{array}\right],$$

$$, \&SHi\mathring{\bigcirc}i, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ , if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, \&SHi\mathring{\bigcirc}i, ©c_1, \\ \\ , \&SHi\mathring{\bigcirc}i, ©c_2,\end{array}\right],$$

11.4.6 Swap with the same operand's propositions:

$$, \&SHi\mathring{\bigcirc}i, i=j,\ \Leftrightarrow\ , i=j, \&SHi\mathring{\bigcirc}i,$$

$$, \&SHi\mathring{\bigcirc}i, i=\varnothing,\ \Leftrightarrow\ , i=\varnothing, \&SHi\mathring{\bigcirc}i,$$

$$, \&SHi\mathring{\bigcirc}i, i\mathring{\bigcirc}j,\ \Leftrightarrow\ , i\mathring{\bigcirc}j, \&SHi\mathring{\bigcirc}i,$$

$$, \&SHi\mathring{\bigcirc}i, i\,!\!=j,\ \Leftrightarrow\ , i\,!\!=j, \&SHi\mathring{\bigcirc}i,$$

$$, \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing,\ \Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\mathring{\bigcirc}i,$$

$$, \&SHi\mathring{\bigcirc}i, i!\mathring{\bigcirc}j,\ \Leftrightarrow\ , i!\mathring{\bigcirc}j, \&SHi\mathring{\bigcirc}i,$$

11.5 Theorems of flag object &SHi with next node.

11.5.1 Swap with the same operand's next node operator:

$$, \&SHi\,\mathring{\bigcirc}i, i\oplus, \;\Leftrightarrow\; , i\oplus, \&SHi \rightarrow i,$$

proof:

$, \&SHi\,\mathring{\bigcirc}i, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\ominus, \&SHi\,\mathring{\bigcirc}i, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i_0\textcircled{$\downarrow$}, i\ominus, \&SHi\,\mathring{\bigcirc}i, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i_0\ominus, i_0\textcircled{$\downarrow$}, i\ominus, \&SHi\,\mathring{\bigcirc}i, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i\ominus, i_0\ominus, i_0\textcircled{$\downarrow$}, \&SHi\,\mathring{\bigcirc}i, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i\ominus, i_0\ominus, \&SHi\,\mathring{\bigcirc}i, i_0\textcircled{$\downarrow$}, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i\mathring{\bigcirc}i_0, i\ominus, i_0\ominus, \&SHi\,\mathring{\bigcirc}i, i_0\textcircled{$\downarrow$}, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i\ominus, i_0\ominus, i\mathring{\bigcirc}i_0, \&SHi\,\mathring{\bigcirc}i, i_0\textcircled{$\downarrow$}, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i\ominus, i_0\ominus, i\mathring{\bigcirc}i_0, \&SHi\,\mathring{\bigcirc}i_0, i_0\textcircled{$\downarrow$}, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i\ominus, i_0\ominus, \&SHi\,\mathring{\bigcirc}i_0, i_0\textcircled{$\downarrow$}, i\oplus,$

$\Leftrightarrow\; , i\oplus, i\circledwedge i_0, i_0\ominus, \&SHi\,\mathring{\bigcirc}i_0, i_0\textcircled{$\downarrow$}, i\ominus, i\oplus,$

$\Leftrightarrow\; , i\oplus, \&SHi \rightarrow i, i\ominus, i\oplus,$

$\Leftrightarrow\; , i\oplus, \&SHi \rightarrow i, i\oplus, i\ominus,$

$$\Leftrightarrow\ ,i\oplus,\ \&SHi\rightarrow i,$$

11.5.2 Swap with operator:

$$,\ \&SHi\rightarrow i,\circledcirc m,\ \Leftrightarrow\ ,\circledcirc m,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,\odot m,\ \Leftrightarrow\ ,\odot m,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,j\circledast n,\ \Leftrightarrow\ ,j\circledast n,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,j\circledast n,\ \Leftrightarrow\ ,j\circledast n,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,j\circledast n,\ \Leftrightarrow\ ,j\circledast n,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,j\circledS,\ \Leftrightarrow\ ,j\circledS,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,j\oplus,\ \Leftrightarrow\ ,j\oplus,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,j\ominus,\ \Leftrightarrow\ ,j\ominus,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,\otimes,\ \Leftrightarrow\ ,\otimes,\ \&SHi\rightarrow i,$$

$$,\ \&SHi\rightarrow i,m\circeq n\left[\begin{array}{l},\\ ,\end{array}\right.\ \Leftrightarrow\ ,m\circeq n\left[\begin{array}{l},\ \&SHi\rightarrow i,\\ ,\ \&SHi\rightarrow i,\end{array}\right.$$

$$,\ \&SHi\rightarrow i,\left[\begin{array}{l},\\ ,\end{array}\right.\ \Leftrightarrow\ ,\left[\begin{array}{l},\ \&SHi\rightarrow i,\\ ,\ \&SHi\rightarrow i,\end{array}\right.$$

$$\left.\begin{array}{l},\\ ,\end{array}\right],\ \&SHi\rightarrow i,\ \Leftrightarrow\ \left.\begin{array}{l},\ \&SHi\rightarrow i,\\ ,\ \&SHi\rightarrow i,\end{array}\right],$$

11.5.3 Swap with branch function:

$$, \&SHi \rightarrow i, if(m=n)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(m=n)\left[\begin{array}{c} ,\ \&SHi \rightarrow i,\ \copyright c_1, \\ \\ ,\ \&SHi \rightarrow i,\ \copyright c_2, \end{array}\right],$$

$$, \&SHi \rightarrow i, if(m=\varnothing)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(m=\varnothing)\left[\begin{array}{c} ,\ \&SHi \rightarrow i,\ \copyright c_1, \\ \\ ,\ \&SHi \rightarrow i,\ \copyright c_2, \end{array}\right],$$

$$, \&SHi \rightarrow i, if(m\mathring{\bigcirc}n)\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(m\mathring{\bigcirc}n)\left[\begin{array}{c} ,\ \&SHi \rightarrow i,\ \copyright c_1, \\ \\ ,\ \&SHi \rightarrow i,\ \copyright c_2, \end{array}\right],$$

11.5.4 Swap with propositions:

$$, \&SHi \rightarrow i, m=n, \;\Leftrightarrow\; , m=n,\ \&SHi \rightarrow i,$$

$$, \&SHi \rightarrow i, m=\varnothing, \;\Leftrightarrow\; , m=\varnothing,\ \&SHi \rightarrow i,$$

$$, \&SHi \rightarrow i, m\mathring{\bigcirc}n, \;\Leftrightarrow\; , m\mathring{\bigcirc}n,\ \&SHi \rightarrow i,$$

$$, \&SHi \rightarrow i, m\,!\!=n, \;\Leftrightarrow\; , m\,!\!=n,\ \&SHi \rightarrow i,$$

$$, \&SHi \rightarrow i, m\,!\!=\varnothing, \;\Leftrightarrow\; , m\,!\!=\varnothing,\ \&SHi \rightarrow i,$$

$$, \&SHi \rightarrow i, m!\mathring{\bigcirc}n, \;\Leftrightarrow\; , m!\mathring{\bigcirc}n,\ \&SHi \rightarrow i,$$

11.5.5 Swap with the same operand's operator:

$$, \&SHi \rightarrow i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHi \rightarrow i,$$

$$, \&SHi \rightarrow i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHi \rightarrow i,$$

$$, \&SHi \rightarrow i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHi \rightarrow i,$$

11.5.6 Swap with the same operand's branch function:

$$, \&SHi \to i, if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{array}{l} , \&SHi \to i, \copyright c_1, \\ \\ , \&SHi \to i, \copyright c_2, \end{array}\right],$$

$$, \&SHi \to i, if(i=\varnothing)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l} , \&SHi \to i, \copyright c_1, \\ \\ , \&SHi \to i, \copyright c_2, \end{array}\right],$$

$$, \&SHi \to i, if(i♂j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i♂j)\left[\begin{array}{l} , \&SHi \to i, \copyright c_1, \\ \\ , \&SHi \to i, \copyright c_2, \end{array}\right],$$

11.5.7 Swap with the same operand's propositions:

$$, \&SHi \to i, i=j, \;\Leftrightarrow\; , i=j, \&SHi \to i,$$

$$, \&SHi \to i, i=\varnothing, \;\Leftrightarrow\; , i=\varnothing, \&SHi \to i,$$

$$, \&SHi \to i, i♂j, \;\Leftrightarrow\; , i♂j, \&SHi \to i,$$

$$, \&SHi \to i, i\,!\!=j, \;\Leftrightarrow\; , i\,!\!=j, \&SHi \to i,$$

$$, \&SHi \to i, i\,!\!=\varnothing, \;\Leftrightarrow\; , i\,!\!=\varnothing, \&SHi \to i,$$

$$, \&SHi \to i, i!♂j, \;\Leftrightarrow\; , i!♂j, \&SHi \to i,$$

11.6 Axiom of next order induction

11.6.1 axiom of inference:

$$\left\{\begin{array}{l} <premise\ 1> \\ <premise\ 2> \end{array}\right\} \Rightarrow \;<conclusion>$$

11.6.2 premise 1:

$, i = \varnothing, \textcircled{t} c_1, \quad \Leftrightarrow \quad , i = \varnothing, \textcircled{t} c_2,$

11.6.3 premise 2:

$, \ \& SHi \rightarrow i, \textcircled{t} c_1, \quad \Leftrightarrow \quad , \ \& SHi \rightarrow i, \textcircled{t} c_2, \quad =\!\Rightarrow$

$, i \,!\!= \varnothing, \ \& SHi \bigcirc\!\!^{\circ} i, \textcircled{t} c_1, \quad \Leftrightarrow \quad , i \,!\!= \varnothing, \ \& SHi \bigcirc\!\!^{\circ} i, \textcircled{t} c_2,$

11.6.4 conclusion:

$, \textcircled{t} c_1, \quad \Leftrightarrow \quad , \textcircled{t} c_2,$

12 Recursive Function R(i)

12.1 Definition of R(i)

$$,R(i), \;\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, R(i), \end{array}\right],$$

12.2 Theorems of R(i)

12.2.1 Transformation:

$$,i=\varnothing, R(i), \;\Leftrightarrow\; ,i=\varnothing,$$

$$,i\,!{=}\,\varnothing, R(i), \;\Leftrightarrow\; ,i\,!{=}\,\varnothing, i\oplus, R(i),$$

$$,R(i), \;\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, \end{array}\right], R(i),$$

proof:

$,R(i),$

$$\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, R(i), \end{array}\right],$$

$$\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, \\ , i\oplus, R(i), \end{array}\right],$$

$$\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, R(i), \\ , i\oplus, R(i), \end{array}\right],$$

$$\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{l} , R(i), \\ , i\oplus, R(i), \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,i\oplus,\end{array}\right],R(i),$$

12.2.2 Result:

$$,R(i),\ \Leftrightarrow\ ,R(i),i=\varnothing,$$

induction proof:

$premise\ 1:$

$,i=\varnothing,R(i),$

$\Leftrightarrow\ ,i=\varnothing,$

$\Leftrightarrow\ ,i=\varnothing,i=\varnothing,$

$\Leftrightarrow\ ,i=\varnothing,R(i),i=\varnothing,$

$premise\ 2:$

$,\ \&SHi\rightarrow i,R(i),\ \Leftrightarrow\ ,\ \&SHi\rightarrow i,R(i),i=\varnothing,\ \Rightarrow$

$,i\,!\!=\varnothing,\ \&SHi\,\bigcirc^{\circ}i,R(i),$

$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\circ}i,i\,!\!=\varnothing,R(i),$

$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\circ}i,i\,!\!=\varnothing,i\oplus,R(i),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\,\bigcirc^{\circ}i,i\oplus,R(i),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,i\oplus,\ \&SHi\rightarrow i,R(i),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,i\oplus,\ \&SHi\rightarrow i,R(i),i=\varnothing,$

$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\,\bigcirc^{\circ}i,i\oplus,R(i),i=\varnothing,$

$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\circ}i,i\,!\!=\varnothing,i\oplus,R(i),i=\varnothing,$

$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\circ}i,i\,!\!=\varnothing,R(i),i=\varnothing,$

$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\,\bigcirc^{\circ}i,R(i),i=\varnothing,$

$conclusion:$
$, R(i), \;\Leftrightarrow\; , R(i), i=\varnothing,$

12.2.3 Operator:

$$, R(i), i\text{⏚}, \;\Leftrightarrow\; , i\text{⏚},$$

induction proof:
$premise\ 1:$
$, i=\varnothing, R(i), i\text{⏚},$
$\Leftrightarrow\; , i=\varnothing, i\text{⏚},$

$premise\ 2:$
$, \&SHi \rightarrow i, R(i), i\text{⏚}, \;\Leftrightarrow\; , \&SHi \rightarrow i, i\text{⏚}, \;\Longrightarrow$

$, i\,!\!=\varnothing, \&SHi\,\text{♂}i, R(i), i\text{⏚},$

$\Leftrightarrow\; , \&SHi\,\text{♂}i, i\,!\!=\varnothing, R(i), i\text{⏚},$

$\Leftrightarrow\; , \&SHi\,\text{♂}i, i\,!\!=\varnothing, i\oplus, R(i), i\text{⏚},$

$\Leftrightarrow\; , i\,!\!=\varnothing, \&SHi\,\text{♂}i, i\oplus, R(i), i\text{⏚},$

$\Leftrightarrow\; , i\,!\!=\varnothing, i\oplus, \&SHi \rightarrow i, R(i), i\text{⏚},$

$\Leftrightarrow\; , i\,!\!=\varnothing, i\oplus, \&SHi \rightarrow i, i\text{⏚},$

$\Leftrightarrow\; , i\,!\!=\varnothing, \&SHi\,\text{♂}i, i\oplus, i\text{⏚},$

$\Leftrightarrow\; , i\,!\!=\varnothing, \&SHi\,\text{♂}i, i\text{⏚},$

$conclusion:$
$, R(i), i\text{⏚}, \;\Leftrightarrow\; , i\text{⏚},$

$$, R(i), \otimes, \;\Leftrightarrow\; , \otimes,$$

induction proof:
$premise\ 1:$

$, i = \varnothing, R(i), \otimes,$
$\Leftrightarrow \; , i = \varnothing, \otimes,$

$premise\ 2:$
$, \ \&SHi \rightarrow i, R(i), \otimes, \ \ \Leftrightarrow \ , \ \&SHi \rightarrow i, \otimes, \ \ \Rightarrow$

$, i \mathrel{!=} \varnothing, \ \&SHi \bigcirc^{\!\circ} i, R(i), \otimes,$

$\Leftrightarrow \ , \ \&SHi \bigcirc^{\!\circ} i, i \mathrel{!=} \varnothing, R(i), \otimes,$

$\Leftrightarrow \ , \ \&SHi \bigcirc^{\!\circ} i, i \mathrel{!=} \varnothing, i\oplus, R(i), \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHi \bigcirc^{\!\circ} i, i\oplus, R(i), \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i\oplus, \ \&SHi \rightarrow i, R(i), \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i\oplus, \ \&SHi \rightarrow i, \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHi \bigcirc^{\!\circ} i, i\oplus, \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHi \bigcirc^{\!\circ} i, \otimes,$

$conclusion:$
$, R(i), \otimes, \ \ \Leftrightarrow \ , \otimes,$

12.2.4 Swap with operator:

$$, R(i), \circledcirc j, \ \ \Leftrightarrow \ , \circledcirc j, R(i),$$

$$, R(i), \odot j, \ \ \Leftrightarrow \ , \odot j, R(i),$$

$$, R(i), j \circledcirc n, \ \ \Leftrightarrow \ , j \circledcirc n, R(i),$$

$$, R(i), j \circledast n, \ \ \Leftrightarrow \ , j \circledast n, R(i),$$

$$, R(i), j \circleddash n, \ \ \Leftrightarrow \ , j \circleddash n, R(i),$$

$$, R(i), j\oplus, \ \ \Leftrightarrow \ , j\oplus, R(i),$$

induction proof:

$premise\ 1:$

$, i = \varnothing, R(i), j\oplus,$

$\Leftrightarrow\ , i = \varnothing, j\oplus,$

$\Leftrightarrow\ , j\oplus, i = \varnothing,$

$\Leftrightarrow\ , j\oplus, i = \varnothing, R(i),$

$\Leftrightarrow\ , i = \varnothing, j\oplus, R(i),$

$premise\ 2:$

$, \ \&SHi \to i, R(i), j\oplus, \ \ \Leftrightarrow\ \ , \ \&SHi \to i, j\oplus, R(i), \ \ \Longrightarrow$

$, i\,!\!= \varnothing, \ \&SHi\,\mathring{\bigcirc} i, R(i), j\oplus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc} i, i\,!\!= \varnothing, R(i), j\oplus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc} i, i\,!\!= \varnothing, i\oplus, R(i), j\oplus,$

$\Leftrightarrow\ , i\,!\!= \varnothing, \ \&SHi\,\mathring{\bigcirc} i, i\oplus, R(i), j\oplus,$

$\Leftrightarrow\ , i\,!\!= \varnothing, i\oplus, \ \&SHi \to i, R(i), j\oplus,$

$\Leftrightarrow\ , i\,!\!= \varnothing, i\oplus, \ \&SHi \to i, j\oplus, R(i),$

$\Leftrightarrow\ , i\,!\!= \varnothing, \ \&SHi\,\mathring{\bigcirc} i, i\oplus, j\oplus, R(i),$

$\Leftrightarrow\ , i\,!\!= \varnothing, \ \&SHi\,\mathring{\bigcirc} i, j\oplus, i\oplus, R(i),$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc} i, j\oplus, i\,!\!= \varnothing, i\oplus, R(i),$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc} i, j\oplus, i\,!\!= \varnothing, R(i),$

$\Leftrightarrow\ , i\,!\!= \varnothing, \ \&SHi\,\mathring{\bigcirc} i, j\oplus, R(i),$

$conclusion:$

$, R(i), j\oplus, \ \ \Leftrightarrow\ \ , j\oplus, R(i),$

$$, R(i), j\ominus, \ \ \Leftrightarrow\ \ , j\ominus, R(i),$$

$$, R(i), j\textcircled{\underline{\downarrow}}, \ \ \Leftrightarrow\ \ , j\textcircled{\underline{\downarrow}}, R(i),$$

12.2.5 Swap with branch function:

$$,R(i),if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l} ,R(i),\copyright c_1, \\ \\ ,R(i),\copyright c_2, \end{array}\right],$$

induction proof:

$premise\ 1:$

$$,i=\varnothing,R(i),if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,i=\varnothing,if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l} ,i=\varnothing,\copyright c_1, \\ \\ ,i=\varnothing,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l} ,i=\varnothing,R(i),\copyright c_1, \\ \\ ,i=\varnothing,R(i),\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,i=\varnothing,if(m=n)\left[\begin{array}{l} ,R(i),\copyright c_1, \\ \\ ,R(i),\copyright c_2, \end{array}\right],$$

$premise\ 2:$

$$,\ \&SHi\rightarrow i,R(i),if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\ \&SHi\rightarrow i,if(m=n)\left[\begin{array}{l} ,R(i),\copyright c_1, \\ \\ ,R(i),\copyright c_2, \end{array}\right], \;\Longrightarrow$$

$$,i\,!\!=\varnothing,\ \&SHi\,\male i,R(i),if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,\ \&SHi\,\male i,i\,!\!=\varnothing,R(i),if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,\ \&SHi\,\male i,i\,!\!=\varnothing,i\oplus,R(i),if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; ,i\,!\!=\varnothing,\ \&SHi\,\male i,i\oplus,R(i),if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i\oplus,\ \&SHi\rightarrow i,R(i),if(m{=}n)\left[\begin{matrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i\oplus,\ \&SHi\rightarrow i,if(m{=}n)\left[\begin{matrix},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,\ \&SHi\mathring{O}i,i\oplus,if(m{=}n)\left[\begin{matrix},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{O}i,i\,!{=}\varnothing,i\oplus,if(m{=}n)\left[\begin{matrix},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{O}i,i\,!{=}\varnothing,if(m{=}n)\left[\begin{matrix},i\oplus,R(i),\copyright c_1,\\ \\ ,i\oplus,R(i),\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{O}i,if(m{=}n)\left[\begin{matrix},i\,!{=}\varnothing,i\oplus,R(i),\copyright c_1,\\ \\ ,i\,!{=}\varnothing,i\oplus,R(i),\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{O}i,if(m{=}n)\left[\begin{matrix},i\,!{=}\varnothing,R(i),\copyright c_1,\\ \\ ,i\,!{=}\varnothing,R(i),\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{O}i,i\,!{=}\varnothing,if(m{=}n)\left[\begin{matrix},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,\ \&SHi\mathring{O}i,if(m{=}n)\left[\begin{matrix},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{matrix}\right],$$

conclusion :

$$,R(i),if(m{=}n)\left[\begin{matrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{matrix}\right],\ \Leftrightarrow\ ,if(m{=}n)\left[\begin{matrix},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{matrix}\right],$$

$$,R(i),if(m{=}\varnothing)\left[\begin{matrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{matrix}\right],\ \Leftrightarrow\ ,if(m{=}\varnothing)\left[\begin{matrix},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{matrix}\right],$$

$$,R(i),if(m\mars n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(m\mars n)\left[\begin{array}{l},R(i),\copyright c_1,\\ \\ ,R(i),\copyright c_2,\end{array}\right],$$

12.2.6 Swap with propositions:

$$,m\!=\!n,R(i), \;\Leftrightarrow\; ,R(i),m\!=\!n,$$

proof:

$,m\!=\!n,R(i),$

$\Leftrightarrow\; ,if(m\!=\!n)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],R(i),$

$\Leftrightarrow\; ,if(m\!=\!n)\left[\begin{array}{l},R(i),\\ \\ ,\otimes,\end{array}\right],$

$\Leftrightarrow\; ,if(m\!=\!n)\left[\begin{array}{l},R(i),\\ \\ ,R(i),\otimes,\end{array}\right],$

$\Leftrightarrow\; ,R(i),if(m\!=\!n)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],$

$\Leftrightarrow\; ,R(i),m\!=\!n,$

$$,m\,!\!=n,R(i), \;\Leftrightarrow\; ,R(i),m\,!\!=n,$$

$$,m\!=\!\varnothing,R(i), \;\Leftrightarrow\; ,R(i),m\!=\!\varnothing,$$
$$,m\,!\!=\varnothing,R(i), \;\Leftrightarrow\; ,R(i),m\,!\!=\varnothing,$$

$$,m\mars n,R(i), \;\Leftrightarrow\; ,R(i),m\mars n,$$
$$,m!\mars n,R(i), \;\Leftrightarrow\; ,R(i),m!\mars n,$$

12.2.7 Swap with self:

$$, R(i), R(j), \;\Leftrightarrow\; , R(j), R(i),$$

induction proof:
premise 1 :
$, i = \varnothing, R(i), R(j),$
$\Leftrightarrow\;, i = \varnothing, R(j),$
$\Leftrightarrow\;, R(j), i = \varnothing,$
$\Leftrightarrow\;, R(j), i = \varnothing, R(i),$
$\Leftrightarrow\;, i = \varnothing, R(j), R(i),$

premise 2 :
$, \&SHi \rightarrow i, R(i), R(j), \;\Leftrightarrow\; , \&SHi \rightarrow i, R(j), R(i), \;\Longrightarrow$

$, i \,!{=}\, \varnothing, \&SHi \bigcirc\!\!\!^{\circ} i, R(i), R(j),$

$\Leftrightarrow\;, \&SHi \bigcirc\!\!\!^{\circ} i, i \,!{=}\, \varnothing, R(i), R(j),$

$\Leftrightarrow\;, \&SHi \bigcirc\!\!\!^{\circ} i, i \,!{=}\, \varnothing, i\oplus, R(i), R(j),$

$\Leftrightarrow\;, i \,!{=}\, \varnothing, \&SHi \bigcirc\!\!\!^{\circ} i, i\oplus, R(i), R(j),$

$\Leftrightarrow\;, i \,!{=}\, \varnothing, i\oplus, \&SHi \rightarrow i, R(i), R(j),$

$\Leftrightarrow\;, i \,!{=}\, \varnothing, i\oplus, \&SHi \rightarrow i, R(j), R(i),$

$\Leftrightarrow\;, i \,!{=}\, \varnothing, \&SHi \bigcirc\!\!\!^{\circ} i, i\oplus, R(j), R(i),$

$\Leftrightarrow\;, i \,!{=}\, \varnothing, \&SHi \bigcirc\!\!\!^{\circ} i, R(j), i\oplus, R(i),$

$\Leftrightarrow\;, \&SHi \bigcirc\!\!\!^{\circ} i, R(j), i \,!{=}\, \varnothing, i\oplus, R(i),$

$\Leftrightarrow\;, \&SHi \bigcirc\!\!\!^{\circ} i, R(j), i \,!{=}\, \varnothing, R(i),$

$\Leftrightarrow\;, i \,!{=}\, \varnothing, \&SHi \bigcirc\!\!\!^{\circ} i, R(j), R(i),$

conclusion :
$, R(i), R(j), \;\Leftrightarrow\; , R(j), R(i),$

12.2.8 Swap with flag object :

$$,R(i),\ \&SHi\male j,\ \Leftrightarrow\ ,\&SHi\male j,R(i),$$

induction proof:
premise 1 :
$,i=\varnothing,R(i),\ \&SHi\male j,$
$\Leftrightarrow\ ,i=\varnothing,\ \&SHi\male j,$
$\Leftrightarrow\ ,\&SHi\male j,i=\varnothing,$
$\Leftrightarrow\ ,\&SHi\male j,i=\varnothing,R(i),$
$\Leftrightarrow\ ,i=\varnothing,\ \&SHi\male j,R(i),$

premise 2 :
$,\&SHi\rightarrow i,R(i),\ \&SHi\male j,\ \Leftrightarrow\ ,\&SHi\rightarrow i,\ \&SHi\male j,R(i),\ \Rightarrow$

$,i\,!=\varnothing,\ \&SHi\male i,R(i),\ \&SHi\male j,$

$\Leftrightarrow\ ,\&SHi\male i,i\,!=\varnothing,R(i),\ \&SHi\male j,$

$\Leftrightarrow\ ,\&SHi\male i,i\,!=\varnothing,i\oplus,R(i),\ \&SHi\male j,$

$\Leftrightarrow\ ,i\,!=\varnothing,\ \&SHi\male i,i\oplus,R(i),\ \&SHi\male j,$

$\Leftrightarrow\ ,i\,!=\varnothing,i\oplus,\ \&SHi\rightarrow i,R(i),\ \&SHi\male j,$

$\Leftrightarrow\ ,i\,!=\varnothing,i\oplus,\ \&SHi\rightarrow i,\ \&SHi\male j,R(i),$

$\Leftrightarrow\ ,i\,!=\varnothing,\ \&SHi\male i,i\oplus,\ \&SHi\male j,R(i),$

$\Leftrightarrow\ ,i\,!=\varnothing,\ \&SHi\male i,\ \&SHi\male j,i\oplus,R(i),$

$\Leftrightarrow\ ,\&SHi\male i,\ \&SHi\male j,i\,!=\varnothing,i\oplus,R(i),$

$\Leftrightarrow\ ,\&SHi\male i,\ \&SHi\male j,i\,!=\varnothing,R(i),$

$\Leftrightarrow\ ,i\,!=\varnothing,\ \&SHi\male i,\ \&SHi\male j,R(i),$

conclusion :
$,R(i),\ \&SHi\male j,\ \Leftrightarrow\ ,\&SHi\male j,R(i),$

$$,R(i),\ \&SHi\rightarrow j,\ \Leftrightarrow\ ,\&SHi\rightarrow j,R(i),$$

12.2.9 Identical node:

$$, i\overset{\circ}{\bigcirc}j, R(i), R(j), \ \Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, R(i), R(j), i\overset{\circ}{\bigcirc}j,$$

induction proof:

$premise\ 1:$

$, i=\varnothing, i\overset{\circ}{\bigcirc}j, R(i), R(j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i=\varnothing, R(i), R(j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i=\varnothing, R(j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, j=\varnothing, R(j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, j=\varnothing,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i\overset{\circ}{\bigcirc}j, j=\varnothing,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, j=\varnothing, i\overset{\circ}{\bigcirc}j,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, j=\varnothing, R(j), i\overset{\circ}{\bigcirc}j,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i=\varnothing, R(j), i\overset{\circ}{\bigcirc}j,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i=\varnothing, R(i), R(j), i\overset{\circ}{\bigcirc}j,$

$\Leftrightarrow\ , i=\varnothing, i\overset{\circ}{\bigcirc}j, R(i), R(j), i\overset{\circ}{\bigcirc}j,$

$premise\ 2:$

$, \&SHi \rightarrow i, i\overset{\circ}{\bigcirc}j, R(i), R(j), \ \Leftrightarrow\ , \&SHi \rightarrow i, i\overset{\circ}{\bigcirc}j, R(i), R(j), i\overset{\circ}{\bigcirc}j, \ \Rightarrow$

$, i\,!=\varnothing, \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}j, R(i), R(j),$

$\Leftrightarrow\ , \&SHi\overset{\circ}{\bigcirc}i, i\,!=\varnothing, i\overset{\circ}{\bigcirc}j, R(i), R(j),$

$\Leftrightarrow\ , \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}j, i\,!=\varnothing, R(i), R(j),$

$\Leftrightarrow\ , \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}j, i\,!=\varnothing, i\oplus, R(i), R(j),$

$\Leftrightarrow\ , \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}j, j\,!=\varnothing, i\oplus, R(i), R(j),$

$\Leftrightarrow\ , \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}j, i\oplus, j\,!=\varnothing, R(i), R(j),$

$\Leftrightarrow\ , \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}j, i\oplus, R(i), j\,!=\varnothing, R(j),$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, i\oplus, R(i), j \mathrel{!=} \varnothing, j\oplus, R(j),$$

$$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, i\oplus, R(i), j\oplus, R(j),$$

$$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, i\oplus, j\oplus, R(i), R(j),$$

$$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \overset{\circ}{\bigcirc} i, i\oplus, j\oplus, i\overset{\circ}{\bigcirc} j, R(i), R(j),$$

$$\Leftrightarrow\ , j \mathrel{!=} \varnothing, i\oplus,\ \&SHi \rightarrow i, j\oplus, i\overset{\circ}{\bigcirc} j, R(i), R(j),$$

$$\Leftrightarrow\ , j \mathrel{!=} \varnothing, i\oplus, j\oplus,\ \&SHi \rightarrow i, i\overset{\circ}{\bigcirc} j, R(i), R(j),$$

$$\Leftrightarrow\ , j \mathrel{!=} \varnothing, i\oplus, j\oplus,\ \&SHi \rightarrow i, i\overset{\circ}{\bigcirc} j, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, j \mathrel{!=} \varnothing, i\oplus, j\oplus, i\overset{\circ}{\bigcirc} j, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, j \mathrel{!=} \varnothing, i\oplus, j\oplus, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, j \mathrel{!=} \varnothing, i\oplus, R(i), j\oplus, R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, i\oplus, R(i), j \mathrel{!=} \varnothing, j\oplus, R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, i\oplus, R(i), j \mathrel{!=} \varnothing, R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, j \mathrel{!=} \varnothing, i\oplus, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, i \mathrel{!=} \varnothing, i\oplus, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, i \mathrel{!=} \varnothing, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc} j, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

$conclusion:$
$$, i\overset{\circ}{\bigcirc} j, R(i), R(j),\ \Leftrightarrow\ , i\overset{\circ}{\bigcirc} j, R(i), R(j), i\overset{\circ}{\bigcirc} j,$$

13 Previous Order Induction

13.1 Definition of flag object &SHj with identical node.

13.1.1 Swap definition:

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, \circledcirc m,\ \Leftrightarrow\ , \circledcirc m,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, \odot m,\ \Leftrightarrow\ , \odot m,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, j\circledast n,\ \Leftrightarrow\ , j\circledast n,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, j\circledast n,\ \Leftrightarrow\ , j\circledast n,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, j\circledast n,\ \Leftrightarrow\ , j\circledast n,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, j\ominus,\ \Leftrightarrow\ , j\ominus,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, j\oplus,\ \Leftrightarrow\ , j\oplus,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, \otimes,\ \Leftrightarrow\ , \otimes,\ \&SHj\,\bigcirc\!\!\!^{\circ}i,$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, m\circleddash n\!\left[\begin{smallmatrix},\\ ,\end{smallmatrix}\right.\ \Leftrightarrow\ , m\circleddash n\!\left[\begin{smallmatrix}, \&SHj\,\bigcirc\!\!\!^{\circ}i,\\ , \&SHj\,\bigcirc\!\!\!^{\circ}i,\end{smallmatrix}\right.$$

$$,\ \&SHj\,\bigcirc\!\!\!^{\circ}i, \left[\begin{smallmatrix},\\ ,\end{smallmatrix}\right.\ \Leftrightarrow\ , \left[\begin{smallmatrix}, \&SHj\,\bigcirc\!\!\!^{\circ}i,\\ , \&SHj\,\bigcirc\!\!\!^{\circ}i,\end{smallmatrix}\right.$$

$$\left.\begin{smallmatrix},\\ ,\end{smallmatrix}\right], \&SHj\,\bigcirc\!\!\!^{\circ}i,\ \Leftrightarrow\ \left.\begin{smallmatrix}, \&SHj\,\bigcirc\!\!\!^{\circ}i,\\ , \&SHj\,\bigcirc\!\!\!^{\circ}i,\end{smallmatrix}\right],$$

13.1.2 Substitution definition:

$$,i\overset{\circ}{\bigcirc}j,\ \&SHj\,\overset{\circ}{\bigcirc}i,\ \ \Leftrightarrow\ \ ,i\overset{\circ}{\bigcirc}j,\ \&SHj\,\overset{\circ}{\bigcirc}j,$$

13.2 Definition of flag object &SHj with next node.

$$,\ \&SHj\rightarrow i,\ \ \Leftrightarrow\ \ ,i\text{☮}i_0,i_0\ominus,\ \&SHj\,\overset{\circ}{\bigcirc}i_0,i_0\text{⏚},$$

13.3 Definition of flag object &SHj with previous node.

$$,\ \&SHj\leftarrow i,\ \ \Leftrightarrow\ \ ,i\text{☮}i_0,i_0\oplus,\ \&SHj\,\overset{\circ}{\bigcirc}i_0,i_0\text{⏚},$$

13.4 Theorems of flag object &SHj with identical node.

13.4.1 Swap with previous node operator:

$$,\ \&SHj\,\overset{\circ}{\bigcirc}i,j\ominus,\ \ \Leftrightarrow\ \ ,j\ominus,\ \&SHj\,\overset{\circ}{\bigcirc}i,$$

13.4.2 Swap with branch function:

$$,\ \&SHj\,\overset{\circ}{\bigcirc}i,if(m=n)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix},\ \ \Leftrightarrow\ \ ,if(m=n)\begin{bmatrix},\ \&SHj\,\overset{\circ}{\bigcirc}i,\ \copyright c_1,\\ \\ ,\ \&SHj\,\overset{\circ}{\bigcirc}i,\ \copyright c_2,\end{bmatrix},$$

$$,\ \&SHj\,\overset{\circ}{\bigcirc}i,if(m=\varnothing)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix},\ \ \Leftrightarrow\ \ ,if(m=\varnothing)\begin{bmatrix},\ \&SHj\,\overset{\circ}{\bigcirc}i,\ \copyright c_1,\\ \\ ,\ \&SHj\,\overset{\circ}{\bigcirc}i,\ \copyright c_2,\end{bmatrix},$$

$$,\ \&SHj\,\overset{\circ}{\bigcirc}i,if(m\overset{\circ}{\bigcirc}n)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix},\ \ \Leftrightarrow\ \ ,if(m\overset{\circ}{\bigcirc}n)\begin{bmatrix},\ \&SHj\,\overset{\circ}{\bigcirc}i,\ \copyright c_1,\\ \\ ,\ \&SHj\,\overset{\circ}{\bigcirc}i,\ \copyright c_2,\end{bmatrix},$$

13.4.3 Swap with propositions:

$$, \&SHj\,\text{♂}i, m=n, \;\Leftrightarrow\; , m=n,\ \&SHj\,\text{♂}i,$$

$$, \&SHj\,\text{♂}i, m=\varnothing, \;\Leftrightarrow\; , m=\varnothing,\ \&SHj\,\text{♂}i,$$

$$, \&SHj\,\text{♂}i, m\text{♂}n, \;\Leftrightarrow\; , m\text{♂}n,\ \&SHj\,\text{♂}i,$$

$$, \&SHj\,\text{♂}i, m\,!\!=n, \;\Leftrightarrow\; , m\,!\!=n,\ \&SHj\,\text{♂}i,$$

$$, \&SHj\,\text{♂}i, m\,!\!=\varnothing, \;\Leftrightarrow\; , m\,!\!=\varnothing,\ \&SHj\,\text{♂}i,$$

$$, \&SHj\,\text{♂}i, m!\text{♂}n, \;\Leftrightarrow\; , m!\text{♂}n,\ \&SHj\,\text{♂}i,$$

13.4.4 Swap with the same operand's operator:

$$, \&SHj\,\text{♂}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHj\,\text{♂}i,$$

$$, \&SHj\,\text{♂}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHj\,\text{♂}i,$$

$$, \&SHj\,\text{♂}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n,\ \&SHj\,\text{♂}i,$$

13.4.5 Swap with the same operand's branch function:

$$, \&SHj\,\text{♂}i, if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{array}{l} , \&SHj\,\text{♂}i, \copyright c_1, \\ \\ , \&SHj\,\text{♂}i, \copyright c_2, \end{array}\right],$$

proof:

$$, \&SHj\,\text{♂}i, if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , i\text{☮}i_0, i_0\text{⏚},\ \&SHj\,\text{♂}i, if(i=j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0,\ \&SHj\,♂i, if(i=j)\left[\begin{array}{l},i_0⦽,\ ©c_1,\\ \\ ,i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, i♂i_0,\ \&SHj\,♂i, if(i=j)\left[\begin{array}{l},i_0⦽,\ ©c_1,\\ \\ ,i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, i♂i_0,\ \&SHj\,♂i_0, if(i=j)\left[\begin{array}{l},i_0⦽,\ ©c_1,\\ \\ ,i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, i♂i_0, if(i=j)\left[\begin{array}{l},\ \&SHj\,♂i_0, i_0⦽,\ ©c_1,\\ \\ ,\ \&SHj\,♂i_0, i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, if(i=j)\left[\begin{array}{l},i♂i_0,\ \&SHj\,♂i_0, i_0⦽,\ ©c_1,\\ \\ ,i♂i_0,\ \&SHj\,♂i_0, i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, if(i=j)\left[\begin{array}{l},i♂i_0,\ \&SHj\,♂i, i_0⦽,\ ©c_1,\\ \\ ,i♂i_0,\ \&SHj\,♂i, i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, i♂i_0, if(i=j)\left[\begin{array}{l},\ \&SHj\,♂i, i_0⦽,\ ©c_1,\\ \\ ,\ \&SHj\,♂i, i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, if(i=j)\left[\begin{array}{l},\ \&SHj\,♂i, i_0⦽,\ ©c_1,\\ \\ ,\ \&SHj\,♂i, i_0⦽,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i☮i_0, i_0⦽, if(i=j)\left[\begin{array}{l},\ \&SHj\,♂i,\ ©c_1,\\ \\ ,\ \&SHj\,♂i,\ ©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=j)\left[\begin{array}{l}, \&SHj\overset{\circ}{\bigcirc}i,\ ©c_1,\\ \\ , \&SHj\overset{\circ}{\bigcirc}i,\ ©c_2,\end{array}\right],$$

$$, \&SHj\overset{\circ}{\bigcirc}i, if(i=\varnothing)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, \&SHj\overset{\circ}{\bigcirc}i,\ ©c_1,\\ \\ , \&SHj\overset{\circ}{\bigcirc}i,\ ©c_2,\end{array}\right],$$

$$, \&SHj\overset{\circ}{\bigcirc}i, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l}, \&SHj\overset{\circ}{\bigcirc}i,\ ©c_1,\\ \\ , \&SHj\overset{\circ}{\bigcirc}i,\ ©c_2,\end{array}\right],$$

13.4.6 Swap with the same operand's propositions:

$$, \&SHj\overset{\circ}{\bigcirc}i, i=j,\ \Leftrightarrow\ , i=j,\ \&SHj\overset{\circ}{\bigcirc}i,$$

$$, \&SHj\overset{\circ}{\bigcirc}i, i=\varnothing,\ \Leftrightarrow\ , i=\varnothing,\ \&SHj\overset{\circ}{\bigcirc}i,$$

$$, \&SHj\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}j,\ \Leftrightarrow\ , i\overset{\circ}{\bigcirc}j,\ \&SHj\overset{\circ}{\bigcirc}i,$$

$$, \&SHj\overset{\circ}{\bigcirc}i, i\,!\!=j,\ \Leftrightarrow\ , i\,!\!=j,\ \&SHj\overset{\circ}{\bigcirc}i,$$

$$, \&SHj\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing,\ \Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHj\overset{\circ}{\bigcirc}i,$$

$$, \&SHj\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}j,\ \Leftrightarrow\ , i!\overset{\circ}{\bigcirc}j,\ \&SHj\overset{\circ}{\bigcirc}i,$$

13.5 Theorems of flag object &SHj with previous node.

13.5.1 Swap with the same operand's next node operator:

$$, \&SHj \mathring{\bigcirc} i, i\ominus, \;\Leftrightarrow\; , i\ominus, \&SHj \leftarrow i,$$

proof:

$, \&SHj \mathring{\bigcirc} i, i\ominus,$

$\Leftrightarrow\; , i\oplus, i\ominus, \&SHj \mathring{\bigcirc} i, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oplus, \&SHj \mathring{\bigcirc} i, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i_0\circledast, i\oplus, \&SHj \mathring{\bigcirc} i, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i_0\oplus, i_0\circledast, i\oplus, \&SHj \mathring{\bigcirc} i, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i\oplus, i_0\oplus, i_0\circledast, \&SHj \mathring{\bigcirc} i, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i\oplus, i_0\oplus, \&SHj \mathring{\bigcirc} i, i_0\circledast, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i\mathring{\bigcirc} i_0, i\oplus, i_0\oplus, \&SHj \mathring{\bigcirc} i, i_0\circledast, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i\oplus, i_0\oplus, i\mathring{\bigcirc} i_0, \&SHj \mathring{\bigcirc} i, i_0\circledast, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i\oplus, i_0\oplus, i\mathring{\bigcirc} i_0, \&SHj \mathring{\bigcirc} i_0, i_0\circledast, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i\oplus, i_0\oplus, \&SHj \mathring{\bigcirc} i_0, i_0\circledast, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oslash i_0, i_0\oplus, \&SHj \mathring{\bigcirc} i_0, i_0\circledast, i\oplus, i\ominus,$

$\Leftrightarrow\; , i\ominus, \&SHj \leftarrow i, i\oplus, i\ominus,$

$\Leftrightarrow\ , i\ominus,\ \&SHj \leftarrow i,$

13.5.2 Swap with operator:

$$, \&SHj \leftarrow i, \circledcirc m, \ \Leftrightarrow\ , \circledcirc m,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, \odot m, \ \Leftrightarrow\ , \odot m,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, j\text{☮}n, \ \Leftrightarrow\ , j\text{☮}n,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, j\text{☮}n, \ \Leftrightarrow\ , j\text{☮}n,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, j\text{☮}n, \ \Leftrightarrow\ , j\text{☮}n,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, j\text{⏚}, \ \Leftrightarrow\ , j\text{⏚},\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, j\oplus, \ \Leftrightarrow\ , j\oplus,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, j\ominus, \ \Leftrightarrow\ , j\ominus,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, \otimes, \ \Leftrightarrow\ , \otimes,\ \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, m\circledequal n\left[\begin{array}{l} , \\ , \end{array}\right. \ \Leftrightarrow\ , m\circledequal n\left[\begin{array}{l} , \&SHj \leftarrow i, \\ , \&SHj \leftarrow i, \end{array}\right.$$

$$, \&SHj \leftarrow i, \left[\begin{array}{l} , \\ , \end{array}\right. \ \Leftrightarrow\ , \left[\begin{array}{l} , \&SHj \leftarrow i, \\ , \&SHj \leftarrow i, \end{array}\right.$$

$$\left.\begin{array}{l} , \\ , \end{array}\right], \&SHj \leftarrow i, \ \Leftrightarrow\ \left.\begin{array}{l} , \&SHj \leftarrow i, \\ , \&SHj \leftarrow i, \end{array}\right],$$

13.5.3 Swap with branch function:

$$, \&SHj \leftarrow i, if(m=n) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \ \Leftrightarrow \ , if(m=n) \left[\begin{array}{l} , \&SHj \leftarrow i, ©c_1, \\ \\ , \&SHj \leftarrow i, ©c_2, \end{array} \right],$$

$$, \&SHj \leftarrow i, if(m=\varnothing) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \ \Leftrightarrow \ , if(m=\varnothing) \left[\begin{array}{l} , \&SHj \leftarrow i, ©c_1, \\ \\ , \&SHj \leftarrow i, ©c_2, \end{array} \right],$$

$$, \&SHj \leftarrow i, if(m\mathring{\bigcirc}n) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \ \Leftrightarrow \ , if(m\mathring{\bigcirc}n) \left[\begin{array}{l} , \&SHj \leftarrow i, ©c_1, \\ \\ , \&SHj \leftarrow i, ©c_2, \end{array} \right],$$

13.5.4 Swap with propositions:

$$, \&SHj \leftarrow i, m=n, \ \Leftrightarrow \ , m=n, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, m=\varnothing, \ \Leftrightarrow \ , m=\varnothing, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, m\mathring{\bigcirc}n, \ \Leftrightarrow \ , m\mathring{\bigcirc}n, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, m\,!\!=n, \ \Leftrightarrow \ , m\,!\!=n, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, m\,!\!=\varnothing, \ \Leftrightarrow \ , m\,!\!=\varnothing, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, m!\mathring{\bigcirc}n, \ \Leftrightarrow \ , m!\mathring{\bigcirc}n, \&SHj \leftarrow i,$$

13.5.5 Swap with the same operand's operator:

$$, \&SHj \leftarrow i, i\circledast n, \ \Leftrightarrow \ , i\circledast n, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, i\circledcirc n, \ \Leftrightarrow \ , i\circledcirc n, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, i\circleddash n, \ \Leftrightarrow \ , i\circleddash n, \&SHj \leftarrow i,$$

13.5.6 Swap with the same operand's branch function:

$$, \&SHj \leftarrow i, if(i=j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \iff , if(i=j)\left[\begin{array}{l} , \&SHj \leftarrow i, \copyright c_1, \\ \\ , \&SHj \leftarrow i, \copyright c_2, \end{array}\right],$$

$$, \&SHj \leftarrow i, if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \iff , if(i=\varnothing)\left[\begin{array}{l} , \&SHj \leftarrow i, \copyright c_1, \\ \\ , \&SHj \leftarrow i, \copyright c_2, \end{array}\right],$$

$$, \&SHj \leftarrow i, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \iff , if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} , \&SHj \leftarrow i, \copyright c_1, \\ \\ , \&SHj \leftarrow i, \copyright c_2, \end{array}\right],$$

13.5.7 Swap with the same operand's propositions:

$$, \&SHj \leftarrow i, i=j, \iff , i=j, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, i=\varnothing, \iff , i=\varnothing, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, i\overset{\circ}{\bigcirc}j, \iff , i\overset{\circ}{\bigcirc}j, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, i\,!{=}\,j, \iff , i\,!{=}\,j, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, i\,!{=}\,\varnothing, \iff , i\,!{=}\,\varnothing, \&SHj \leftarrow i,$$

$$, \&SHj \leftarrow i, i!\overset{\circ}{\bigcirc}j, \iff , i!\overset{\circ}{\bigcirc}j, \&SHj \leftarrow i,$$

13.6 Axiom of previous order induction

13.6.1 axiom of inference:

$$\left\{\begin{array}{l} <premise\ 1> \\ <premise\ 2> \end{array}\right\} \Longrightarrow \; < conclusion >$$

13.6.2 premise 1:

$, i = \varnothing, \textcircled{t} c_1, \;\; \Leftrightarrow \;\; , i = \varnothing, \textcircled{t} c_2,$

13.6.3 premise 2:

$, \ \& SHj \leftarrow i, \textcircled{t} c_1, \;\; \Leftrightarrow \;\; , \ \& SHj \leftarrow i, \textcircled{t} c_2, \;\; \Rightarrow$

$, i \,!\!= \varnothing, \ \& SHj \bigcirc\!\!^{\circ} i, \textcircled{t} c_1, \;\; \Leftrightarrow \;\; , i \,!\!= \varnothing, \ \& SHj \bigcirc\!\!^{\circ} i, \textcircled{t} c_2,$

13.6.4 conclusion:

$, \textcircled{t} c_1, \;\; \Leftrightarrow \;\; , \textcircled{t} c_2,$

14 Recursive Function R_(i)

14.1 Definition of R_(i)

$$, R_{\_}(i), \;\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , i\ominus, R_{\_}(i), \end{array}\right],$$

14.2 Theorems of R_(i)

14.2.1 Transformation:

$$, i=\varnothing, R_{\_}(i), \;\Leftrightarrow\; , i=\varnothing,$$

$$, i\,!\!=\varnothing, R_{\_}(i), \;\Leftrightarrow\; , i\,!\!=\varnothing, i\ominus, R_{\_}(i),$$

$$, R_{\_}(i), \;\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , i\ominus, \end{array}\right], R_{\_}(i),$$

proof:

$, R_{\_}(i),$

$$\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , i\ominus, R_{\_}(i), \end{array}\right],$$

$$\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, \\ \\ , i\ominus, R_{\_}(i), \end{array}\right],$$

$$\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, R_{\_}(i), \\ \\ , i\ominus, R_{\_}(i), \end{array}\right],$$

$$\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l} , R_{\_}(i), \\ \\ , i\ominus, R_{\_}(i), \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)-\left[\begin{array}{l}, \\ \\ , i\ominus, \end{array}\right]-, R_{\_}(i),$$

14.2.2 Result:

$$, R_{\_}(i),\ \Leftrightarrow\ , R_{\_}(i), i=\varnothing,$$

induction proof:

$premise\ 1:$

$, i=\varnothing, R_{\_}(i),$

$\Leftrightarrow\ , i=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, i=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, R_{\_}(i), i=\varnothing,$

$premise\ 2:$

$, \&SHj\leftarrow i, R_{\_}(i),\ \Leftrightarrow\ , \&SHj\leftarrow i, R_{\_}(i), i=\varnothing,\ \Rightarrow$

$, i\,!\!=\varnothing, \&SHj\overset{\circ}{\bigcirc}i, R_{\_}(i),$

$\Leftrightarrow\ , \&SHj\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, R_{\_}(i),$

$\Leftrightarrow\ , \&SHj\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, i\ominus, R_{\_}(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHj\overset{\circ}{\bigcirc}i, i\ominus, R_{\_}(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing, i\ominus, \&SHj\leftarrow i, R_{\_}(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing, i\ominus, \&SHj\leftarrow i, R_{\_}(i), i=\varnothing,$

$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHj\overset{\circ}{\bigcirc}i, i\ominus, R_{\_}(i), i=\varnothing,$

$\Leftrightarrow\ , \&SHj\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, i\ominus, R_{\_}(i), i=\varnothing,$

$\Leftrightarrow\ , \&SHj\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, R_{\_}(i), i=\varnothing,$

$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHj\overset{\circ}{\bigcirc}i, R_{\_}(i), i=\varnothing,$

$conclusion:$
$, R_{\_}(i), \;\Leftrightarrow\; , R_{\_}(i), i = \varnothing,$

14.2.3 Operator:

$$, R_{\_}(i), i\textcircled{$\downarrow$}, \;\Leftrightarrow\; , i\textcircled{$\downarrow$},$$

induction proof:
$premise\; 1:$
$, i = \varnothing, R_{\_}(i), i\textcircled{$\downarrow$},$
$\Leftrightarrow\; , i = \varnothing, i\textcircled{$\downarrow$},$

$premise\; 2:$
$, \&SHj \leftarrow i, R_{\_}(i), i\textcircled{$\downarrow$}, \;\Leftrightarrow\; , \&SHj \leftarrow i, i\textcircled{$\downarrow$}, \;\Longrightarrow$

$, i \mathrel{!=} \varnothing, \&SHj \mathring{\bigcirc} i, R_{\_}(i), i\textcircled{$\downarrow$},$

$\Leftrightarrow\; , \&SHj \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, R_{\_}(i), i\textcircled{$\downarrow$},$

$\Leftrightarrow\; , \&SHj \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, i\ominus, R_{\_}(i), i\textcircled{$\downarrow$},$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, \&SHj \mathring{\bigcirc} i, i\ominus, R_{\_}(i), i\textcircled{$\downarrow$},$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, i\ominus, \&SHj \leftarrow i, R_{\_}(i), i\textcircled{$\downarrow$},$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, i\ominus, \&SHj \leftarrow i, i\textcircled{$\downarrow$},$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, \&SHj \mathring{\bigcirc} i, i\ominus, i\textcircled{$\downarrow$},$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, \&SHj \mathring{\bigcirc} i, i\textcircled{$\downarrow$},$

$conclusion:$
$, R_{\_}(i), i\textcircled{$\downarrow$}, \;\Leftrightarrow\; , i\textcircled{$\downarrow$},$

$$, R_{\_}(i), \otimes, \;\Leftrightarrow\; , \otimes,$$

induction proof:
$premise\; 1:$

$, i = \varnothing, R\_(i), \otimes,$
$\Leftrightarrow \ , i = \varnothing, \otimes,$

$premise\ 2:$
$, \ \&SHj \leftarrow i, R\_(i), \otimes, \ \ \Leftrightarrow \ , \ \&SHj \leftarrow i, \otimes, \ \ \Rightarrow$

$, i \mathrel{!=} \varnothing, \ \&SHj \mathring{\bigcirc} i, R\_(i), \otimes,$

$\Leftrightarrow \ , \ \&SHj \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, R\_(i), \otimes,$

$\Leftrightarrow \ , \ \&SHj \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, i\ominus, R\_(i), \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHj \mathring{\bigcirc} i, i\ominus, R\_(i), \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i\ominus, \ \&SHj \leftarrow i, R\_(i), \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i\ominus, \ \&SHj \leftarrow i, \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHj \mathring{\bigcirc} i, i\ominus, \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHj \mathring{\bigcirc} i, \otimes,$

$conclusion:$
$, R\_(i), \otimes, \ \ \Leftrightarrow \ , \otimes,$

14.2.4 Swap with operator:

$$, R\_(i), \circledcirc j, \ \ \Leftrightarrow \ , \circledcirc j, R\_(i),$$
$$, R\_(i), \odot j, \ \ \Leftrightarrow \ , \odot j, R\_(i),$$

$$, R\_(i), j \circledast n, \ \ \Leftrightarrow \ , j \circledast n, R\_(i),$$
$$, R\_(i), j \circledast n, \ \ \Leftrightarrow \ , j \circledast n, R\_(i),$$
$$, R\_(i), j \circledast n, \ \ \Leftrightarrow \ , j \circledast n, R\_(i),$$

$$, R\_(i), j\oplus, \ \ \Leftrightarrow \ , j\oplus, R\_(i),$$

induction proof:
$premise\ 1:$
$, i=\varnothing, R_{-}(i), j\oplus,$
$\Leftrightarrow\ , i=\varnothing, j\oplus,$
$\Leftrightarrow\ , j\oplus, i=\varnothing,$
$\Leftrightarrow\ , j\oplus, i=\varnothing, R_{-}(i),$
$\Leftrightarrow\ , i=\varnothing, j\oplus, R_{-}(i),$

$premise\ 2:$
$, \&SHj \leftarrow i, R_{-}(i), j\oplus, \ \Leftrightarrow\ , \&SHj \leftarrow i, j\oplus, R_{-}(i), \ \Rightarrow$

$, i\,!\!=\varnothing, \ \&SHj \bigcirc\!\!^{\circ} i, R_{-}(i), j\oplus,$

$\Leftrightarrow\ , \&SHj \bigcirc\!\!^{\circ} i, i\,!\!=\varnothing, R_{-}(i), j\oplus,$

$\Leftrightarrow\ , \&SHj \bigcirc\!\!^{\circ} i, i\,!\!=\varnothing, i\ominus, R_{-}(i), j\oplus,$

$\Leftrightarrow\ , i\,!\!=\varnothing, \ \&SHj \bigcirc\!\!^{\circ} i, i\ominus, R_{-}(i), j\oplus,$

$\Leftrightarrow\ , i\,!\!=\varnothing, i\ominus, \ \&SHj \leftarrow i, R_{-}(i), j\oplus,$

$\Leftrightarrow\ , i\,!\!=\varnothing, i\ominus, \ \&SHj \leftarrow i, j\oplus, R_{-}(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing, \ \&SHj \bigcirc\!\!^{\circ} i, i\ominus, j\oplus, R_{-}(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing, \ \&SHj \bigcirc\!\!^{\circ} i, j\oplus, i\ominus, R_{-}(i),$

$\Leftrightarrow\ , \&SHj \bigcirc\!\!^{\circ} i, j\oplus, i\,!\!=\varnothing, i\ominus, R_{-}(i),$

$\Leftrightarrow\ , \&SHj \bigcirc\!\!^{\circ} i, j\oplus, i\,!\!=\varnothing, R_{-}(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing, \ \&SHj \bigcirc\!\!^{\circ} i, j\oplus, R_{-}(i),$

$conclusion:$
$, R_{-}(i), j\oplus, \ \Leftrightarrow\ , j\oplus, R_{-}(i),$

$$, R_{-}(i), j\ominus, \ \Leftrightarrow\ , j\ominus, R_{-}(i),$$
$$, R_{-}(i), j\textcircled{$\downarrow$}, \ \Leftrightarrow\ , j\textcircled{$\downarrow$}, R_{-}(i),$$

14.2.5 Swap with branch function:

$$, R\_(i), if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(m\!=\!n) \left[\begin{array}{c} , R\_(i), \copyright c_1, \\ \\ , R\_(i), \copyright c_2, \end{array}\right],$$

induction proof:

premise 1 :

$$, i\!=\!\varnothing, R\_(i), if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , i\!=\!\varnothing, if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , if(m\!=\!n) \left[\begin{array}{c} , i\!=\!\varnothing, \copyright c_1, \\ \\ , i\!=\!\varnothing, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , if(m\!=\!n) \left[\begin{array}{c} , i\!=\!\varnothing, R\_(i), \copyright c_1, \\ \\ , i\!=\!\varnothing, R\_(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , i\!=\!\varnothing, if(m\!=\!n) \left[\begin{array}{c} , R\_(i), \copyright c_1, \\ \\ , R\_(i), \copyright c_2, \end{array}\right],$$

premise 2 :

$$, \&SHj \leftarrow i, R\_(i), if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , \&SHj \leftarrow i, if(m\!=\!n) \left[\begin{array}{c} , R\_(i), \copyright c_1, \\ \\ , R\_(i), \copyright c_2, \end{array}\right], \;\Rightarrow$$

$$, i \,!\!=\varnothing, \&SHj \overset{\circ}{\bigcirc} i, R\_(i), if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , \&SHj \overset{\circ}{\bigcirc} i, i \,!\!=\varnothing, R\_(i), if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , \&SHj \overset{\circ}{\bigcirc} i, i \,!\!=\varnothing, i\ominus, R\_(i), if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\; , i \,!\!=\varnothing, \&SHj \overset{\circ}{\bigcirc} i, i\ominus, R\_(i), if(m\!=\!n) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, i\ominus,\ \&SHj \leftarrow i, R_{-}(i), if(m{=}n) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, i\ominus,\ \&SHj \leftarrow i, if(m{=}n) \left[\begin{array}{l} , R_{-}(i), \copyright c_1, \\ \\ , R_{-}(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHj \bigcirc\!\!\!^{\circ}\, i, i\ominus, if(m{=}n) \left[\begin{array}{l} , R_{-}(i), \copyright c_1, \\ \\ , R_{-}(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHj \bigcirc\!\!\!^{\circ}\, i, i\,!{=}\varnothing, i\ominus, if(m{=}n) \left[\begin{array}{l} , R_{-}(i), \copyright c_1, \\ \\ , R_{-}(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHj \bigcirc\!\!\!^{\circ}\, i, i\,!{=}\varnothing, if(m{=}n) \left[\begin{array}{l} , i\ominus, R_{-}(i), \copyright c_1, \\ \\ , i\ominus, R_{-}(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHj \bigcirc\!\!\!^{\circ}\, i, if(m{=}n) \left[\begin{array}{l} , i\,!{=}\varnothing, i\ominus, R_{-}(i), \copyright c_1, \\ \\ , i\,!{=}\varnothing, i\ominus, R_{-}(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHj \bigcirc\!\!\!^{\circ}\, i, if(m{=}n) \left[\begin{array}{l} , i\,!{=}\varnothing, R_{-}(i), \copyright c_1, \\ \\ , i\,!{=}\varnothing, R_{-}(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHj \bigcirc\!\!\!^{\circ}\, i, i\,!{=}\varnothing, if(m{=}n) \left[\begin{array}{l} , R_{-}(i), \copyright c_1, \\ \\ , R_{-}(i), \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHj \bigcirc\!\!\!^{\circ}\, i, if(m{=}n) \left[\begin{array}{l} , R_{-}(i), \copyright c_1, \\ \\ , R_{-}(i), \copyright c_2, \end{array}\right],$$

conclusion :

$$, R_{-}(i), if(m{=}n) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow\ , if(m{=}n) \left[\begin{array}{l} , R_{-}(i), \copyright c_1, \\ \\ , R_{-}(i), \copyright c_2, \end{array}\right],$$

$$, R_{-}(i), if(m{=}\varnothing) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow\ , if(m{=}\varnothing) \left[\begin{array}{l} , R_{-}(i), \copyright c_1, \\ \\ , R_{-}(i), \copyright c_2, \end{array}\right],$$

$$,R_{\_}(i),if(m♂n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(m♂n)\left[\begin{array}{l},R_{\_}(i),\copyright c_1,\\ \\ ,R_{\_}(i),\copyright c_2,\end{array}\right],$$

14.2.6 Swap with propositions:

$$,m=n,R_{\_}(i), \;\Leftrightarrow\; ,R_{\_}(i),m=n,$$

proof:

$$,m=n,R_{\_}(i),$$

$$\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],R_{\_}(i),$$

$$\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l},R_{\_}(i),\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l},R_{\_}(i),\\ \\ ,R_{\_}(i),\otimes,\end{array}\right],$$

$$\Leftrightarrow\; ,R_{\_}(i),if(m=n)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\; ,R_{\_}(i),m=n,$$

$$,m\,!\!=n,R_{\_}(i), \;\Leftrightarrow\; ,R_{\_}(i),m\,!\!=n,$$

$$,m=\varnothing,R_{\_}(i), \;\Leftrightarrow\; ,R_{\_}(i),m=\varnothing,$$
$$,m\,!\!=\varnothing,R_{\_}(i), \;\Leftrightarrow\; ,R_{\_}(i),m\,!\!=\varnothing,$$

$$,m♂n,R_{\_}(i), \;\Leftrightarrow\; ,R_{\_}(i),m♂n,$$
$$,m!♂n,R_{\_}(i), \;\Leftrightarrow\; ,R_{\_}(i),m!♂n,$$

14.2.7 Swap with self:

$$, R\_(i), R\_(j), \iff , R\_(j), R\_(i),$$

induction proof:
$premise\ 1:$
$, i = \varnothing, R\_(i), R\_(j),$
$\iff , i = \varnothing, R\_(j),$
$\iff , R\_(j), i = \varnothing,$
$\iff , R\_(j), i = \varnothing, R\_(i),$
$\iff , i = \varnothing, R\_(j), R\_(i),$

$premise\ 2:$
$, \&SHj \leftarrow i, R\_(i), R\_(j), \iff , \&SHj \leftarrow i, R\_(j), R\_(i), \implies$

$, i\,!\!= \varnothing, \&SHj \overset{\circ}{\bigcirc} i, R\_(i), R\_(j),$

$\iff , \&SHj \overset{\circ}{\bigcirc} i, i\,!\!= \varnothing, R\_(i), R\_(j),$

$\iff , \&SHj \overset{\circ}{\bigcirc} i, i\,!\!= \varnothing, i\ominus, R\_(i), R\_(j),$

$\iff , i\,!\!= \varnothing, \&SHj \overset{\circ}{\bigcirc} i, i\ominus, R\_(i), R\_(j),$

$\iff , i\,!\!= \varnothing, i\ominus, \&SHj \leftarrow i, R\_(i), R\_(j),$

$\iff , i\,!\!= \varnothing, i\ominus, \&SHj \leftarrow i, R\_(j), R\_(i),$

$\iff , i\,!\!= \varnothing, \&SHj \overset{\circ}{\bigcirc} i, i\ominus, R\_(j), R\_(i),$

$\iff , i\,!\!= \varnothing, \&SHj \overset{\circ}{\bigcirc} i, R\_(j), i\ominus, R\_(i),$

$\iff , \&SHj \overset{\circ}{\bigcirc} i, R\_(j), i\,!\!= \varnothing, i\ominus, R\_(i),$

$\iff , \&SHj \overset{\circ}{\bigcirc} i, R\_(j), i\,!\!= \varnothing, R\_(i),$

$\iff , i\,!\!= \varnothing, \&SHj \overset{\circ}{\bigcirc} i, R\_(j), R\_(i),$

$conclusion:$
$, R\_(i), R\_(j), \iff , R\_(j), R\_(i),$

14.2.8 Swap with R(j):

$$, R_-(i), R(j), \;\Leftrightarrow\; , R(j), R_-(i),$$

14.2.9 Swap with flag object :

$$, R_-(i), \; \&SHj \,♂ j, \;\Leftrightarrow\; , \&SHj \,♂ j, R_-(i),$$

induction proof:
$premise\ 1:$
$, i = \varnothing, R_-(i), \; \&SHj \,♂ j,$
$\Leftrightarrow \; , i = \varnothing, \; \&SHj \,♂ j,$
$\Leftrightarrow \; , \&SHj \,♂ j, i = \varnothing,$
$\Leftrightarrow \; , \&SHj \,♂ j, i = \varnothing, R_-(i),$
$\Leftrightarrow \; , i = \varnothing, \; \&SHj \,♂ j, R_-(i),$

$premise\ 2:$
$, \&SHj \leftarrow i, R_-(i), \; \&SHj \,♂ j, \;\Leftrightarrow\; , \&SHj \leftarrow i, \; \&SHj \,♂ j, R_-(i), \;\Rightarrow$

$, i \mathrel{!=} \varnothing, \; \&SHj \,♂ i, R_-(i), \; \&SHj \,♂ j,$

$\Leftrightarrow \; , \&SHj \,♂ i, i \mathrel{!=} \varnothing, R_-(i), \; \&SHj \,♂ j,$

$\Leftrightarrow \; , \&SHj \,♂ i, i \mathrel{!=} \varnothing, i\ominus, R_-(i), \; \&SHj \,♂ j,$

$\Leftrightarrow \; , i \mathrel{!=} \varnothing, \; \&SHj \,♂ i, i\ominus, R_-(i), \; \&SHj \,♂ j,$

$\Leftrightarrow \; , i \mathrel{!=} \varnothing, i\ominus, \; \&SHj \leftarrow i, R_-(i), \; \&SHj \,♂ j,$

$\Leftrightarrow \; , i \mathrel{!=} \varnothing, i\ominus, \; \&SHj \leftarrow i, \; \&SHj \,♂ j, R_-(i),$

$\Leftrightarrow \; , i \mathrel{!=} \varnothing, \; \&SHj \,♂ i, i\ominus, \; \&SHj \,♂ j, R_-(i),$

$\Leftrightarrow \; , i \mathrel{!=} \varnothing, \; \&SHj \,♂ i, \; \&SHj \,♂ j, i\ominus, R_-(i),$

$\Leftrightarrow \; , \&SHj \,♂ i, \; \&SHj \,♂ j, i \mathrel{!=} \varnothing, i\ominus, R_-(i),$

$\Leftrightarrow \; , \&SHj \,♂ i, \; \&SHj \,♂ j, i \mathrel{!=} \varnothing, R_-(i),$

$\Leftrightarrow \; , i \mathrel{!=} \varnothing, \; \&SHj \,♂ i, \; \&SHj \,♂ j, R_-(i),$

conclusion :
$, R_{\text{-}}(i),\ \&SHj \circlearrowleft j,\ \Leftrightarrow\ , \&SHj \circlearrowleft j, R_{\text{-}}(i),$

$$, R_{\text{-}}(i),\ \&SHi \circlearrowleft j,\ \Leftrightarrow\ , \&SHi \circlearrowleft j, R_{\text{-}}(i),$$

$$, R_{\text{-}}(i),\ \&SHj \leftarrow j,\ \Leftrightarrow\ , \&SHj \leftarrow j, R_{\text{-}}(i),$$

$$, R_{\text{-}}(i),\ \&SHi \rightarrow j,\ \Leftrightarrow\ , \&SHi \rightarrow j, R_{\text{-}}(i),$$

$$, R(i),\ \&SHj \circlearrowleft j,\ \Leftrightarrow\ , \&SHj \circlearrowleft j, R(i),$$

$$, R(i),\ \&SHj \leftarrow j,\ \Leftrightarrow\ , \&SHj \leftarrow j, R(i),$$

14.2.10 Identical node:

$$, i \circlearrowleft j, R_{\text{-}}(i), R_{\text{-}}(j),\ \Leftrightarrow\ , i \circlearrowleft j, R_{\text{-}}(i), R_{\text{-}}(j), i \circlearrowleft j,$$

induction proof:
premise 1 :
$, i = \varnothing, i \circlearrowleft j, R_{\text{-}}(i), R_{\text{-}}(j),$

$\Leftrightarrow\ , i \circlearrowleft j, i = \varnothing, R_{\text{-}}(i), R_{\text{-}}(j),$

$\Leftrightarrow\ , i \circlearrowleft j, i = \varnothing, R_{\text{-}}(j),$

$\Leftrightarrow\ , i \circlearrowleft j, j = \varnothing, R_{\text{-}}(j),$

$\Leftrightarrow\ , i \circlearrowleft j, j = \varnothing,$

$\Leftrightarrow\ , i \circlearrowleft j, i \circlearrowleft j, j = \varnothing,$

$\Leftrightarrow\ , i \circlearrowleft j, j = \varnothing, i \circlearrowleft j,$

$\Leftrightarrow\ , i \circlearrowleft j, j = \varnothing, R_{\text{-}}(j), i \circlearrowleft j,$

$\Leftrightarrow\ , i \circlearrowleft j, i = \varnothing, R_{\text{-}}(j), i \circlearrowleft j,$

$\Leftrightarrow\ , i \circlearrowleft j, i = \varnothing, R_{\text{-}}(i), R_{\text{-}}(j), i \circlearrowleft j,$

$$\Leftrightarrow\ , i = \varnothing, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j), i\bigcirc\!\!^{\circ}j,$$

$premise\ 2:$

$$, \&SHj \leftarrow i, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j), \ \Leftrightarrow\ , \&SHj \leftarrow i, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j), i\bigcirc\!\!^{\circ}j, \ \Longrightarrow$$

$$, i\,!\!= \varnothing, \ \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\,!\!= \varnothing, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\,!\!= \varnothing, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\,!\!= \varnothing, i\ominus, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, j\,!\!= \varnothing, i\ominus, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\ominus, j\,!\!= \varnothing, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\ominus, R_{-}(i), j\,!\!= \varnothing, R_{-}(j),$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\ominus, R_{-}(i), j\,!\!= \varnothing, j\ominus, R_{-}(j),$$

$$\Leftrightarrow\ , j\,!\!= \varnothing, \ \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\ominus, R_{-}(i), j\ominus, R_{-}(j),$$

$$\Leftrightarrow\ , j\,!\!= \varnothing, \ \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\ominus, j\ominus, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , j\,!\!= \varnothing, \ \&SHj\,\bigcirc\!\!^{\circ}i, i\ominus, j\ominus, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , j\,!\!= \varnothing, i\ominus, \ \&SHj \leftarrow i, j\ominus, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , j\,!\!= \varnothing, i\ominus, j\ominus, \ \&SHj \leftarrow i, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j),$$

$$\Leftrightarrow\ , j\,!\!= \varnothing, i\ominus, j\ominus, \ \&SHj \leftarrow i, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j), i\bigcirc\!\!^{\circ}j,$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, j\,!\!= \varnothing, i\ominus, j\ominus, i\bigcirc\!\!^{\circ}j, R_{-}(i), R_{-}(j), i\bigcirc\!\!^{\circ}j,$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, j\,!\!= \varnothing, i\ominus, j\ominus, R_{-}(i), R_{-}(j), i\bigcirc\!\!^{\circ}j,$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, j\,!\!= \varnothing, i\ominus, R_{-}(i), j\ominus, R_{-}(j), i\bigcirc\!\!^{\circ}j,$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\ominus, R_{-}(i), j\,!\!= \varnothing, j\ominus, R_{-}(j), i\bigcirc\!\!^{\circ}j,$$

$$\Leftrightarrow\ , \&SHj\,\bigcirc\!\!^{\circ}i, i\bigcirc\!\!^{\circ}j, i\ominus, R_{-}(i), j\,!\!= \varnothing, R_{-}(j), i\bigcirc\!\!^{\circ}j,$$

$\Leftrightarrow\ ,\ \&SHj\bigcirc^{\circ}i, i\bigcirc^{\circ}j, j \mathrel{!=} \varnothing, i\ominus, R_{\_}(i), R_{\_}(j), i\bigcirc^{\circ}j,$

$\Leftrightarrow\ ,\ \&SHj\bigcirc^{\circ}i, i\bigcirc^{\circ}j, i \mathrel{!=} \varnothing, i\ominus, R_{\_}(i), R_{\_}(j), i\bigcirc^{\circ}j,$

$\Leftrightarrow\ ,\ \&SHj\bigcirc^{\circ}i, i\bigcirc^{\circ}j, i \mathrel{!=} \varnothing, R_{\_}(i), R_{\_}(j), i\bigcirc^{\circ}j,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHj\bigcirc^{\circ}i, i\bigcirc^{\circ}j, R_{\_}(i), R_{\_}(j), i\bigcirc^{\circ}j,$

$conclusion:$
$, i\bigcirc^{\circ}j, R_{\_}(i), R_{\_}(j),\ \Leftrightarrow\ , i\bigcirc^{\circ}j, R_{\_}(i), R_{\_}(j), i\bigcirc^{\circ}j,$

15 Rules of Node Ring

15.1 Axiom of node ring

$$, i\bigcirc^{\!\!\circ} j, i\oplus, R(i), R_{-}(j), \;\Leftrightarrow\; \sim, i\bigcirc^{\!\!\circ} j,$$

15.2 Theorems of node ring

$$, i\bigcirc^{\!\!\circ} j, R(i), j\ominus, R_{-}(j), \;\Leftrightarrow\; \sim, i\bigcirc^{\!\!\circ} j,$$

proof:

$, i\bigcirc^{\!\!\circ} j, R(i), j\ominus, R_{-}(j),$

$\Leftrightarrow\; , i\bigcirc^{\!\!\circ} j, i\oplus, i\ominus, R(i), j\ominus, R_{-}(j),$

$\Leftrightarrow\; , i\bigcirc^{\!\!\circ} j, i\oplus, i\ominus, j\ominus, R(i), R_{-}(j),$

$\Leftrightarrow\; , i\bigcirc^{\!\!\circ} j, i\ominus, j\ominus, i\oplus, R(i), R_{-}(j),$

$\Leftrightarrow\; , i\ominus, j\ominus, i\bigcirc^{\!\!\circ} j, i\oplus, R(i), R_{-}(j),$

$\Leftrightarrow\; , i\ominus, j\ominus, i\bigcirc^{\!\!\circ} j, i\oplus, R(i), R_{-}(j), i\bigcirc^{\!\!\circ} j,$

$\Leftrightarrow\; , i\bigcirc^{\!\!\circ} j, i\ominus, j\ominus, i\oplus, R(i), R_{-}(j), i\bigcirc^{\!\!\circ} j,$

$\Leftrightarrow\; , i\bigcirc^{\!\!\circ} j, i\oplus, i\ominus, j\ominus, R(i), R_{-}(j), i\bigcirc^{\!\!\circ} j,$

$\Leftrightarrow\; , i\bigcirc^{\!\!\circ} j, j\ominus, R(i), R_{-}(j), i\bigcirc^{\!\!\circ} j,$

$\Leftrightarrow\; , i\bigcirc^{\!\!\circ} j, R(i), j\ominus, R_{-}(j), i\bigcirc^{\!\!\circ} j,$

$$, i\bigcirc^{\!\!\circ} j, i\,!\!=\varnothing, R(i), R_{-}(j), \;\Leftrightarrow\; \sim, i\bigcirc^{\!\!\circ} j,$$

proof:

$, i\mathring{\bigcirc}j, i\,!\!=\varnothing, R(i), R_{-}(j),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i\,!\!=\varnothing, i\oplus, R(i), R_{-}(j),$

$\Leftrightarrow\ , i\,!\!=\varnothing, i\mathring{\bigcirc}j, i\oplus, R(i), R_{-}(j),$

$\Leftrightarrow\ , i\,!\!=\varnothing, i\mathring{\bigcirc}j, i\oplus, R(i), R_{-}(j), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i\,!\!=\varnothing, i\oplus, R(i), R_{-}(j), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i\,!\!=\varnothing, R(i), R_{-}(j), i\mathring{\bigcirc}j,$

$$, i\mathring{\bigcirc}j, i=\varnothing, i\oplus, i=\varnothing, \quad\Leftrightarrow\quad \sim, i\mathring{\bigcirc}j,$$

proof:

$, i\mathring{\bigcirc}j, i=\varnothing, i\oplus, i=\varnothing,$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\text{⏚}, i\oplus, i=\varnothing,$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i_0\text{⏚}, i\oplus, i=\varnothing,$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i_0\text{⏚}, i=\varnothing,$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i_0\text{⏚}, i=\varnothing, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i=\varnothing, i_0\text{⏚}, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i\mathring{\bigcirc}i_0, i_0\oplus, i\oplus, i=\varnothing, i_0\text{⏚}, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\mathring{\bigcirc}i_0, i=\varnothing, i_0\text{⏚}, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\mathring{\bigcirc}i_0, i_0=\varnothing, i_0\text{⏚}, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i\mathring{\bigcirc}i_0, i_0\oplus, i\oplus, i_0=\varnothing, i_0\text{⏚}, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i_0=\varnothing, i_0\text{⏚}, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i_0=\varnothing, i_0\text{⏚}, i\oplus, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, j=\varnothing, i\text{☮}i_0, i_0\oplus, i_0=\varnothing, i_0\text{⏚}, i\oplus, R(i),$

$\Leftrightarrow\ , i\mathring{\bigcirc}j, j=\varnothing, R_{-}(j), i\text{☮}i_0, i_0\oplus, i_0=\varnothing, i_0\text{⏚}, i\oplus, R(i),$

$\Leftrightarrow\ ,i\text{♂}j,j=\varnothing,i\text{☮}i_0,i_0\oplus,i_0=\varnothing,i_0\text{⦽},i\oplus,R(i),R_{-}(j),$

$\Leftrightarrow\ ,j=\varnothing,i\text{☮}i_0,i_0\oplus,i_0=\varnothing,i_0\text{⦽},i\text{♂}j,i\oplus,R(i),R_{-}(j),$

$\Leftrightarrow\ ,j=\varnothing,i\text{☮}i_0,i_0\oplus,i_0=\varnothing,i_0\text{⦽},i\text{♂}j,i\oplus,R(i),R_{-}(j),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,j=\varnothing,i\text{☮}i_0,i_0\oplus,i_0=\varnothing,i_0\text{⦽},i\oplus,R(i),R_{-}(j),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,j=\varnothing,R_{-}(j),i\text{☮}i_0,i_0\oplus,i_0=\varnothing,i_0\text{⦽},i\oplus,R(i),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,j=\varnothing,i\text{☮}i_0,i_0\oplus,i_0=\varnothing,i_0\text{⦽},i\oplus,R(i),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\oplus,i\oplus,i_0=\varnothing,i_0\text{⦽},R(i),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i\text{♂}i_0,i_0\oplus,i\oplus,i_0=\varnothing,i_0\text{⦽},R(i),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\oplus,i\oplus,i\text{♂}i_0,i_0=\varnothing,i_0\text{⦽},R(i),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\oplus,i\oplus,i\text{♂}i_0,i=\varnothing,i_0\text{⦽},R(i),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\oplus,i\oplus,i\text{♂}i_0,i_0\text{⦽},i=\varnothing,R(i),i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\oplus,i\oplus,i\text{♂}i_0,i_0\text{⦽},i=\varnothing,i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i\text{♂}i_0,i_0\oplus,i\oplus,i_0\text{⦽},i=\varnothing,i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\oplus,i\oplus,i_0\text{⦽},i=\varnothing,i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\oplus,i_0\text{⦽},i\oplus,i=\varnothing,i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\text{☮}i_0,i_0\text{⦽},i\oplus,i=\varnothing,i\text{♂}j,$

$\Leftrightarrow\ ,i\text{♂}j,i=\varnothing,i\oplus,i=\varnothing,i\text{♂}j,$

$$,i\text{♂}j,i=\varnothing,i\oplus,i\,!\!=\varnothing,R(i),\ \Leftrightarrow\ \sim,i\text{♂}j,$$

proof:

$,i\text{♂}j,i=\varnothing,i\oplus,i\,!\!=\varnothing,R(i),$

$\Leftrightarrow\ ,i\text{♂}j,j=\varnothing,i\oplus,i\,!\!=\varnothing,R(i),$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, j=\varnothing, R_{-}(j), i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\mathring{\bigcirc}j, R_{-}(j), i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\mathring{\bigcirc}j, i\varowedge i_0, i_0\circledbot, R_{-}(j), i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\mathring{\bigcirc}j, i\varowedge i_0, i_0\oplus, t_0\circledbot, R_{-}(j), i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\mathring{\bigcirc}j, i\varowedge i_0, i_0\oplus, R(i_0), i_0\circledbot, R_{-}(j), i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\mathring{\bigcirc}j, i\varowedge i_0, i_0\oplus, R(i_0), R_{-}(j), i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\mathring{\bigcirc}j, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), R_{-}(j), i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}j, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), R_{-}(j), i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}j, j\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), R_{-}(j), i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}j, j\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), R_{-}(j), i_0\mathring{\bigcirc}j, i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , j=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}j, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), R_{-}(j), i_0\mathring{\bigcirc}j, i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, j=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), R_{-}(j), i_0\mathring{\bigcirc}j, i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), j=\varnothing, R_{-}(j), i_0\mathring{\bigcirc}j, i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), j=\varnothing, i_0\mathring{\bigcirc}j, i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, j=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), i_0\mathring{\bigcirc}j, i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, R(i_0), i_0\mathring{\bigcirc}j, i_0\circledbot, i\oplus, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, i\oplus, R(i_0), i_0\mathring{\bigcirc}j, i_0\circledbot, i\,!\!=\varnothing, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, i\oplus, i\,!\!=\varnothing, R(i_0), i_0\mathring{\bigcirc}j, i_0\circledbot, R(i),$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i=\varnothing, i\varowedge i_0, i\mathring{\bigcirc}i_0, i_0\oplus, i\oplus, i\,!\!=\varnothing, R(i_0), R(i), i_0\mathring{\bigcirc}j, i_0\circledbot,$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i=\varnothing, i\varowedge i_0, i_0\oplus, i\oplus, i\mathring{\bigcirc}i_0, i\,!\!=\varnothing, R(i_0), R(i), i_0\mathring{\bigcirc}j, i_0\circledbot,$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i=\varnothing, i\varowedge i_0, i_0\oplus, i\oplus, i\,!\!=\varnothing, i\mathring{\bigcirc}i_0, R(i_0), R(i), i_0\mathring{\bigcirc}j, i_0\circledbot,$$

$$\Leftrightarrow \ , i\mathring{\bigcirc}j, i=\varnothing, i\varowedge i_0, i_0\oplus, i\oplus, i\,!\!=\varnothing, i\mathring{\bigcirc}i_0, R(i_0), R(i), i\mathring{\bigcirc}i_0, i_0\mathring{\bigcirc}j, i_0\circledbot,$$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\,!{=}\varnothing, i\mathring{\bigcirc}i_0, R(i_0), R(i), i\mathring{\bigcirc}i_0, i\mathring{\bigcirc}j, i_0\text{⏚},$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\,!{=}\varnothing, i\mathring{\bigcirc}i_0, R(i_0), R(i), i\mathring{\bigcirc}i_0, i_0\text{⏚}, i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\,!{=}\varnothing, i\mathring{\bigcirc}i_0, R(i_0), R(i), i_0\text{⏚}, i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\mathring{\bigcirc}i_0, i\,!{=}\varnothing, R(i_0), R(i), i_0\text{⏚}, i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i\mathring{\bigcirc}i_0, i_0\oplus, i\oplus, i\,!{=}\varnothing, R(i_0), R(i), i_0\text{⏚}, i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\,!{=}\varnothing, R(i_0), R(i), i_0\text{⏚}, i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, i\,!{=}\varnothing, R(i_0), i_0\text{⏚}, R(i), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i\oplus, R(i_0), i_0\text{⏚}, i\,!{=}\varnothing, R(i), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, R(i_0), i_0\text{⏚}, i\oplus, i\,!{=}\varnothing, R(i), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\oplus, i_0\text{⏚}, i\oplus, i\,!{=}\varnothing, R(i), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\text{☮}i_0, i_0\text{⏚}, i\oplus, i\,!{=}\varnothing, R(i), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i=\varnothing, i\oplus, i\,!{=}\varnothing, R(i), i\mathring{\bigcirc}j,$

$$,i\mathring{\bigcirc}j, i=\varnothing, i\oplus, R(i), \ \Leftrightarrow\ \sim, i\mathring{\bigcirc}j,$$

proof:

$,i\mathring{\bigcirc}j, i=\varnothing, i\oplus, R(i),$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, j=\varnothing, i\oplus, R(i),$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i\oplus, R(i), j=\varnothing,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i\oplus, R(i), j=\varnothing, R_{\text{-}}(j),$

$\Leftrightarrow\ ,j=\varnothing, i\mathring{\bigcirc}j, i\oplus, R(i), R_{\text{-}}(j),$

$\Leftrightarrow\ ,j=\varnothing, i\mathring{\bigcirc}j, i\oplus, R(i), R_{\text{-}}(j), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i\oplus, R(i), j=\varnothing, R_{\text{-}}(j), i\mathring{\bigcirc}j,$

$\Leftrightarrow\ ,i\mathring{\bigcirc}j, i\oplus, R(i), j=\varnothing, i\mathring{\bigcirc}j,$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, j=\varnothing, i\oplus, R(i), i\overset{\circ}{\bigcirc}j,$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i=\varnothing, i\oplus, R(i), i\overset{\circ}{\bigcirc}j,$$

$$, i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \ \Leftrightarrow\ \sim, i\overset{\circ}{\bigcirc}j,$$

proof:

$$, i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , \\ , \end{array}\right], i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\overset{\circ}{\bigcirc}j, i=\varnothing, R(i), R_{-}(j), \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\overset{\circ}{\bigcirc}j, i=\varnothing, R_{-}(j), \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\overset{\circ}{\bigcirc}j, j=\varnothing, R_{-}(j), \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l} , i\overset{\circ}{\bigcirc}j, j=\varnothing, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i\overset{\circ}{\bigcirc}j, j=\varnothing, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, j=\varnothing, i\overset{\circ}{\bigcirc}j, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, j=\varnothing, R_{-}(j), i\overset{\circ}{\bigcirc}j, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i=\varnothing, R_{-}(j), i\overset{\circ}{\bigcirc}j, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, i=\varnothing, R(i), R_{-}(j), i\overset{\circ}{\bigcirc}j, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i=\varnothing, i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), i\overset{\circ}{\bigcirc}j, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), i\overset{\circ}{\bigcirc}j, \\ , i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j), i\overset{\circ}{\bigcirc}j, \\ , i\,!\!=\varnothing, i\overset{\circ}{\bigcirc}j, R(i), R_{-}(j),\end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\mathring{\bigcirc}j, i\,!\!=\varnothing, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\mathring{\bigcirc}j, i\,!\!=\varnothing, i\oplus, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\,!\!=\varnothing, i\mathring{\bigcirc}j, i\oplus, R(i), R_{-}(j), \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\,!\!=\varnothing, i\mathring{\bigcirc}j, i\oplus, R(i), R_{-}(j), i\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\mathring{\bigcirc}j, i\,!\!=\varnothing, i\oplus, R(i), R_{-}(j), i\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\mathring{\bigcirc}j, i\,!\!=\varnothing, R(i), R_{-}(j), i\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\,!\!=\varnothing, i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \\ ,i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow \ ,if(i=\varnothing)\left[\begin{array}{l} , \\ , \end{array}\right], i\mathring{\bigcirc}j, R(i), R_{-}(j), i\mathring{\bigcirc}j,$$

$\Leftrightarrow\ , i \mathring{\bigcirc} j, R(i), R_{-}(j), i \mathring{\bigcirc} j,$

16 Rules of Relationship of Node Connectivity

16.1 Definition of Node Connectivity

$$,if(i\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j)\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \;\Leftrightarrow\; ,i\mathbin{☮}i_1,j\mathbin{☮}j_1,R(i_1),R(j_1),if(i_1\mathbin{\overset{\circ}{\bigcirc}}j_1)\!-\!\left[\begin{array}{l} ,i_1⏚,j_1⏚, \\ \\ ,i_1⏚,j_1⏚, \end{array}\right.$$

$$,i\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j, \;\Leftrightarrow\; ,if(i\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j)\!-\!\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right]\!-,$$

$$,i!\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j, \;\Leftrightarrow\; ,if(i\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j)\!-\!\left[\begin{array}{c} ,\otimes, \\ \\ , \end{array}\right]\!-,$$

16.2 Axiom of node id operator

$$,i\mathbin{☮}m, \;\Leftrightarrow\; \sim,m!\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j,$$

16.3 Theorems of Relationship of Node Connectivity

16.3.1 Node Connectivity propositions to Identical node comparison propositions

$$,i\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j, \;\Leftrightarrow\; ,i\mathbin{☮}i_1,j\mathbin{☮}j_1,R(i_1),R(j_1),i_1\mathbin{\overset{\circ}{\bigcirc}}j_1,i_1⏚,j_1⏚,$$

$$,i!\mathbin{\overset{\circ}{\bigcirc}{}^{\circ}}j, \;\Leftrightarrow\; ,i\mathbin{☮}i_1,j\mathbin{☮}j_1,R(i_1),R(j_1),i_1!\mathbin{\overset{\circ}{\bigcirc}}j_1,i_1⏚,j_1⏚,$$

16.3.2 Branch function to propositions

$$, if(i \bigcirc^{\!\circ\circ} j) \left[\begin{array}{l} , \copyright c, \\ \\ , \otimes, \end{array}\right], \;\Leftrightarrow\; , i \bigcirc^{\!\circ\circ} j, \copyright c,$$

$$, if(i \bigcirc^{\!\circ\circ} j) \left[\begin{array}{l} , \otimes, \\ \\ , \copyright c, \end{array}\right], \;\Leftrightarrow\; , i! \bigcirc^{\!\circ\circ} j, \copyright c,$$

16.3.3 Empty branch function

$$, if(i \bigcirc^{\!\circ\circ} j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \;\Leftrightarrow\; , \left[\begin{array}{l} , i \bigcirc^{\!\circ\circ} j, \\ \\ , i! \bigcirc^{\!\circ\circ} j, \end{array}\right.$$

proof:

$$, if(i \bigcirc^{\!\circ\circ} j) \left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

$$\Leftrightarrow \; , i \oslash i_1, j \oslash j_1, R(i_1), R(j_1), if(i_1 \bigcirc^{\!\circ} j_1) \left[\begin{array}{l} , i_1 \circledast, j_1 \circledast, \\ \\ , i_1 \circledast, j_1 \circledast, \end{array}\right.$$

$$\Leftrightarrow \; , i \oslash i_1, j \oslash j_1, R(i_1), R(j_1), \left[\begin{array}{l} , i_1 \bigcirc^{\!\circ} j_1, i_1 \circledast, j_1 \circledast, \\ \\ , i_1 ! \bigcirc^{\!\circ} j_1, i_1 \circledast, j_1 \circledast, \end{array}\right.$$

$$\Leftrightarrow \; , \left[\begin{array}{l} , i \oslash i_1, j \oslash j_1, R(i_1), R(j_1), i_1 \bigcirc^{\!\circ} j_1, i_1 \circledast, j_1 \circledast, \\ \\ , i \oslash i_1, j \oslash j_1, R(i_1), R(j_1), i_1 ! \bigcirc^{\!\circ} j_1, i_1 \circledast, j_1 \circledast, \end{array}\right.$$

$$\Leftrightarrow \; , \left[\begin{array}{l} , i \bigcirc^{\!\circ\circ} j, \\ \\ , i! \bigcirc^{\!\circ\circ} j, \end{array}\right.$$

16.3.4 Unity

$$, \Leftrightarrow \; , if(i \overset{\circ\circ}{\bigcirc} j) \left[\begin{array}{c} , \\ , \end{array}\right],$$

proof:

$$, if(i \overset{\circ\circ}{\bigcirc} j \left[\begin{array}{c} , \\ \\ , \end{array}\right],$$

$$\Leftrightarrow \; , i \text{☮} i_1, j \text{☮} j_1, R(i_1), R(j_1), if(i_1 \overset{\circ}{\bigcirc} j_1) \left[\begin{array}{c} , i_1 \text{⏚}, j_1 \text{⏚}, \\ \\ , i_1 \text{⏚}, j_1 \text{⏚}, \end{array}\right],$$

$$\Leftrightarrow \; , i \text{☮} i_1, j \text{☮} j_1, R(i_1), R(j_1), if(i_1 \overset{\circ}{\bigcirc} j_1) \left[\begin{array}{c} , \\ \\ , \end{array}\right], i_1 \text{⏚}, j_1 \text{⏚},$$

$$\Leftrightarrow \; , i \text{☮} i_1, j \text{☮} j_1, R(i_1), R(j_1), i_1 \text{⏚}, j_1 \text{⏚},$$

$$\Leftrightarrow \; , i \text{☮} i_1, j \text{☮} j_1, R(i_1), i_1 \text{⏚}, R(j_1), j_1 \text{⏚},$$

$$\Leftrightarrow \; , i \text{☮} i_1, j \text{☮} j_1, i_1 \text{⏚}, R(j_1), j_1 \text{⏚},$$

$$\Leftrightarrow \; , i \text{☮} i_1, j \text{☮} j_1, i_1 \text{⏚}, j_1 \text{⏚},$$

$$\Leftrightarrow \; , i \text{☮} i_1, i_1 \text{⏚}, j \text{☮} j_1, j_1 \text{⏚},$$

$$\Leftrightarrow \; , j \text{☮} j_1, j_1 \text{⏚},$$

$\Leftrightarrow$,

$$, i \text{♂} j, \otimes, \;\Leftrightarrow\; , \otimes,$$
$$, i! \text{♂} j, \otimes, \;\Leftrightarrow\; , \otimes,$$

16.3.5 Symmetry

$$, if(i \text{♂} j) \begin{cases} , \\ , \end{cases} \;\Leftrightarrow\; , if(j \text{♂} i) \begin{cases} , \\ , \end{cases}$$

proof:

$$, if(i \text{♂} j) \begin{cases} , \\ , \end{cases}$$

$$\Leftrightarrow\; , i \text{☮} i_1, j \text{☮} j_1, R(i_1), R(j_1), if(i_1 \text{♂} j_1) \begin{cases} , i_1 \circleddash, j_1 \circleddash, \\ , i_1 \circleddash, j_1 \circleddash, \end{cases}$$

$$\Leftrightarrow\; , j \text{☮} j_1, i \text{☮} i_1, R(i_1), R(j_1), if(i_1 \text{♂} j_1) \begin{cases} , i_1 \circleddash, j_1 \circleddash, \\ , i_1 \circleddash, j_1 \circleddash, \end{cases}$$

$$\Leftrightarrow\; , j \text{☮} j_1, i \text{☮} i_1, R(j_1), R(i_1), if(i_1 \text{♂} j_1) \begin{cases} , i_1 \circleddash, j_1 \circleddash, \\ , i_1 \circleddash, j_1 \circleddash, \end{cases}$$

$$\Leftrightarrow\; , j \text{☮} j_1, i \text{☮} i_1, R(j_1), R(i_1), if(j_1 \text{♂} i_1) \begin{cases} , i_1 \circleddash, j_1 \circleddash, \\ , i_1 \circleddash, j_1 \circleddash, \end{cases}$$

$$\Leftrightarrow\; , j \text{☮} j_1, i \text{☮} i_1, R(j_1), R(i_1), if(j_1 \text{♂} i_1) \begin{cases} , j_1 \circleddash, i_1 \circleddash, \\ , j_1 \circleddash, i_1 \circleddash, \end{cases}$$

$$\Leftrightarrow\ , if(j \mathring{\bigcirc}\!\circ\, i) \begin{cases} , \\ , \end{cases}$$

$$, i \mathring{\bigcirc}\!\circ\, j, \ \Leftrightarrow\ , j \mathring{\bigcirc}\!\circ\, i,$$

$$, i ! \mathring{\bigcirc}\!\circ\, j, \ \Leftrightarrow\ , j ! \mathring{\bigcirc}\!\circ\, i,$$

16.3.6 Swap

Branch function and operator:

$$, \circledcirc m, if(i \mathring{\bigcirc}\!\circ\, j) \begin{cases} , \\ , \end{cases} \Leftrightarrow\ , if(i \mathring{\bigcirc}\!\circ\, j) \begin{cases} , \circledcirc m, \\ , \circledcirc m, \end{cases}$$

proof:

$$, \circledcirc m, if(i \mathring{\bigcirc}\!\circ\, j) \begin{cases} , \\ , \end{cases}$$

$$\Leftrightarrow\ , \circledcirc m, i \circledast i_1, j \circledast j_1, R(i_1), R(j_1), if(i_1 \mathring{\bigcirc} j_1) \begin{cases} , i_1 \ominus, j_1 \ominus, \\ , i_1 \ominus, j_1 \ominus, \end{cases}$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_1, \circledcirc m, R(i_1), R(j_1), if(i_1 \mathring{\bigcirc} j_1) \begin{cases} , i_1 \ominus, j_1 \ominus, \\ , i_1 \ominus, j_1 \ominus, \end{cases}$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_1, R(i_1), R(j_1), \circledcirc m, if(i_1 \mathring{\bigcirc} j_1) \begin{cases} , i_1 \ominus, j_1 \ominus, \\ , i_1 \ominus, j_1 \ominus, \end{cases}$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_1, R(i_1), R(j_1), if(i_1 \mathring{\bigcirc} j_1) \begin{cases} , \circledcirc m, i_1 \ominus, j_1 \ominus, \\ , \circledcirc m, i_1 \ominus, j_1 \ominus, \end{cases}$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_1, R(i_1), R(j_1), if(i_1 \mathring{\bigcirc} j_1) \begin{cases} , i_1 \ominus, j_1 \ominus, \circledcirc m, \\ , i_1 \ominus, j_1 \ominus, \circledcirc m, \end{cases}$$

$$\Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \circledcirc m, \\ \\ , \circledcirc m, \end{array}\right.$$

$$, \odot m, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \odot m, \\ \\ , \odot m, \end{array}\right.$$

$$, m \circledast n, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , m \circledast n, \\ \\ , m \circledast n, \end{array}\right.$$

$$, m \circledast^{\circ} n, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , m \circledast^{\circ} n, \\ \\ , m \circledast^{\circ} n, \end{array}\right.$$

$$, m \circledast^{\lrcorner} n, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , m \circledast^{\lrcorner} n, \\ \\ , m \circledast^{\lrcorner} n, \end{array}\right.$$

$$, m \bigcirc\!\!\!\!\downarrow, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , m \bigcirc\!\!\!\!\downarrow, \\ \\ , m \bigcirc\!\!\!\!\downarrow, \end{array}\right.$$

$$, m \oplus, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , m \oplus, \\ \\ , m \oplus, \end{array}\right.$$

$$, m \ominus, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , m \ominus, \\ \\ , m \ominus, \end{array}\right.$$

Branch function and Branch function:

$$, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , if(m \mathbin{\bigcirc^{\!\!\circ\circ}} n) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right] - , \\ , if(m \mathbin{\bigcirc^{\!\!\circ\circ}} n) - \left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array}\right] - , \end{array}\right] - , \Leftrightarrow\ , if(m \mathbin{\bigcirc^{\!\!\circ\circ}} n) - \left[\begin{array}{l} , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array}\right] - , \\ , if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , \copyright c_2, \\ \\ , \copyright c_4, \end{array}\right] - , \end{array}\right] - ,$$

proof:

$$, if(i \mathbin{\bigcirc^{\!\!\circ\circ}} j) - \left[\begin{array}{l} , if(m \mathbin{\bigcirc^{\!\!\circ\circ}} n) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right] - , \\ , if(m \mathbin{\bigcirc^{\!\!\circ\circ}} n) - \left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array}\right] - , \end{array}\right] - ,$$

$$\Leftrightarrow\ , i\oslash i_1, j\oslash j_1, R(i_1), R(j_1),$$

$$if(i_1 \male j_1)\left[\begin{array}{l} , i_1\oplus, j_1\oplus, if(m \male n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ , i_1\oplus, j_1\oplus, if(m \male n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash i_1, j\oslash j_1, R(i_1), R(j_1),$$

$$if(i_1 \male j_1)\left[\begin{array}{l} , if(m \male n)\left[\begin{array}{l} , i_1\oplus, j_1\oplus, \copyright c_1, \\ \\ , i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right], \\ , if(m \male n)\left[\begin{array}{l} , i_1\oplus, j_1\oplus, \copyright c_3, \\ \\ , i_1\oplus, j_1\oplus, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\oslash i_1, j\oslash j_1, R(i_1), R(j_1),$$

$$if(i_1 \male j_1)\left[\begin{array}{l} , m\oslash m_1, n\oslash n_1, R(m_1), R(n_1), if(m_1 \male n_1)\left[\begin{array}{l} , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_1, \\ \\ , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right], \\ , m\oslash m_1, n\oslash n_1, R(m_1), R(n_1), if(m_1 \male n_1)\left[\begin{array}{l} , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_3, \\ \\ , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , m\oslash m_1, n\oslash n_1, i\oslash i_1, j\oslash j_1, R(i_1), R(j_1),$$

$$if(i_1 \male j_1)\left[\begin{array}{l} , R(m_1), R(n_1), if(m_1 \male n_1)\left[\begin{array}{l} , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_1, \\ \\ , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right], \\ , R(m_1), R(n_1), if(m_1 \male n_1)\left[\begin{array}{l} , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_3, \\ \\ , m_1\oplus, n_1\oplus, i_1\oplus, j_1\oplus, \copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , m\oslash m_1, n\oslash n_1, R(m_1), R(n_1), i\oslash i_1, j\oslash j_1, R(i_1), R(j_1),$$

$$if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,if(m_1\mathring{\bigcirc}n_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_1, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_2, \end{array}\right], \\ ,if(m_1\mathring{\bigcirc}n_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_3, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,m\circledast m_1,n\circledast n_1,R(m_1),R(n_1),i\circledast i_1,j\circledast j_1,R(i_1),R(j_1),$$

$$if(m_1\mathring{\bigcirc}n_1)\left[\begin{array}{l} ,if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_1, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_3, \end{array}\right], \\ ,if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_2, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,m\circledast m_1,n\circledast n_1,R(m_1),R(n_1),i\circledast i_1,j\circledast j_1,$$

$$if(m_1\mathring{\bigcirc}n_1)\left[\begin{array}{l} ,R(i_1),R(j_1),if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_1, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_3, \end{array}\right], \\ ,R(i_1),R(j_1),if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_2, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,m\circledast m_1,n\circledast n_1,R(m_1),R(n_1),$$

$$if(m_1\mathring{\bigcirc}n_1)\left[\begin{array}{l} ,i\circledast i_1,j\circledast j_1,R(i_1),R(j_1),if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_1, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_3, \end{array}\right], \\ ,i\circledast i_1,j\circledast j_1,R(i_1),R(j_1),if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_2, \\ \\ ,m_1\ominus,n_1\ominus,i_1\ominus,j_1\ominus,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,m\circledast m_1,n\circledast n_1,R(m_1),R(n_1),$$

$$if(m_1\mathring{\bigcirc}n_1)\left[\begin{array}{l} ,i\circledast i_1,j\circledast j_1,R(i_1),R(j_1),if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\ominus,j_1\ominus,m_1\ominus,n_1\ominus,\copyright c_1, \\ \\ ,i_1\ominus,j_1\ominus,m_1\ominus,n_1\ominus,\copyright c_3, \end{array}\right], \\ ,i\circledast i_1,j\circledast j_1,R(i_1),R(j_1),if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\ominus,j_1\ominus,m_1\ominus,n_1\ominus,\copyright c_2, \\ \\ ,i_1\ominus,j_1\ominus,m_1\ominus,n_1\ominus,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,m\text{☮}m_1,n\text{☮}n_1,R(m_1),R(n_1),$$

$$if(m_1\overset{\circ}{\bigcirc}n_1)\left[\begin{array}{l} ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,m_1\text{\textcircled{$\downarrow$}},n_1\text{\textcircled{$\downarrow$}},\copyright c_1, \\ \\ ,m_1\text{\textcircled{$\downarrow$}},n_1\text{\textcircled{$\downarrow$}},\copyright c_3, \end{array}\right], \\ ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,m_1\text{\textcircled{$\downarrow$}},n_1\text{\textcircled{$\downarrow$}},\copyright c_2, \\ \\ ,m_1\text{\textcircled{$\downarrow$}},n_1\text{\textcircled{$\downarrow$}},\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,m\text{☮}m_1,n\text{☮}n_1,R(m_1),R(n_1),$$

$$if(m_1\overset{\circ}{\bigcirc}n_1)\left[\begin{array}{l} ,m_1\text{\textcircled{$\downarrow$}},n_1\text{\textcircled{$\downarrow$}},if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,m_1\text{\textcircled{$\downarrow$}},n_1\text{\textcircled{$\downarrow$}},if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ\circ}{\bigcirc}n)\left[\begin{array}{l} ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \ \Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \ \Leftrightarrow\ ,if(m=n)\left[\begin{array}{l} ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$, if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l} , if(m=\varnothing)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \\ , if(m=\varnothing)\left[\begin{array}{l}, \copyright c_3, \\ \\ , \copyright c_4,\end{array}\right], \end{array}\right], \;\Leftrightarrow\; , if(m=\varnothing)\left[\begin{array}{l} , if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_3,\end{array}\right], \\ , if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, \copyright c_2, \\ \\ , \copyright c_4,\end{array}\right], \end{array}\right],$$

Branch function and propositions:

$$, m\overset{\circ}{\bigcirc}\!\!\circ n, if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, m\overset{\circ}{\bigcirc}\!\!\circ n, \copyright c_1, \\ \\ , m\overset{\circ}{\bigcirc}\!\!\circ n, \copyright c_2,\end{array}\right],$$

$$, m!\overset{\circ}{\bigcirc}\!\!\circ n, if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, m!\overset{\circ}{\bigcirc}\!\!\circ n, \copyright c_1, \\ \\ , m!\overset{\circ}{\bigcirc}\!\!\circ n, \copyright c_2,\end{array}\right],$$

$$, m\overset{\circ}{\bigcirc} n, if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, m\overset{\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m\overset{\circ}{\bigcirc} n, \copyright c_2,\end{array}\right],$$

$$, m!\overset{\circ}{\bigcirc} n, if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, m!\overset{\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m!\overset{\circ}{\bigcirc} n, \copyright c_2,\end{array}\right],$$

$$, m=n, if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\overset{\circ}{\bigcirc}\!\!\circ j)\left[\begin{array}{l}, m=n, \copyright c_1, \\ \\ , m=n, \copyright c_2,\end{array}\right],$$

$$,m\,!{=}\,n, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{c},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{c},m\,!{=}\,n,\copyright c_1,\\ \\,m\,!{=}\,n,\copyright c_2,\end{array}\right],$$

$$,m=\varnothing, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{c},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{c},m=\varnothing,\copyright c_1,\\ \\,m=\varnothing,\copyright c_2,\end{array}\right],$$

$$,m\,!{=}\,\varnothing, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{c},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{c},m\,!{=}\,\varnothing,\copyright c_1,\\ \\,m\,!{=}\,\varnothing,\copyright c_2,\end{array}\right],$$

$$,m\overset{\circ}{\bigcirc}{}^{\circ}n, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{c},m\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_1,\\ \\,m\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_2,\end{array}\right],$$

$$,m!\overset{\circ}{\bigcirc}{}^{\circ}n, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{c},m!\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_1,\\ \\,m!\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_2,\end{array}\right],$$

$$,m\overset{\circ}{\bigcirc}{}^{\circ}n, if(i=j)\left[\begin{array}{c},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{c},m\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_1,\\ \\,m\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_2,\end{array}\right],$$

$$,m!\overset{\circ}{\bigcirc}{}^{\circ}n, if(i=j)\left[\begin{array}{c},\copyright c_1,\\ \\,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{c},m!\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_1,\\ \\,m!\overset{\circ}{\bigcirc}{}^{\circ}n,\copyright c_2,\end{array}\right],$$

$$,m\mathring{\bigcirc}n,if(i=\varnothing)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{c},m\mathring{\bigcirc}n,\copyright c_1,\\ \\ ,m\mathring{\bigcirc}n,\copyright c_2,\end{array}\right],$$

$$,m!\mathring{\bigcirc}n,if(i=\varnothing)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=\varnothing)\left[\begin{array}{c},m!\mathring{\bigcirc}n,\copyright c_1,\\ \\ ,m!\mathring{\bigcirc}n,\copyright c_2,\end{array}\right],$$

Branch function and recursive function:

$$,R(m),if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},R(m),\copyright c_1,\\ \\ ,R(m),\copyright c_2,\end{array}\right],$$

$$,R_{-}(m),if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},R_{-}(m),\copyright c_1,\\ \\ ,R_{-}(m),\copyright c_2,\end{array}\right],$$

Branch function and flag object :

$$,\ \&SHi\,\mathring{\bigcirc}m,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\ \&SHi\,\mathring{\bigcirc}m,\copyright c_1,\\ \\ ,\ \&SHi\,\mathring{\bigcirc}m,\copyright c_2,\end{array}\right],$$

$$,\ \&SHi\rightarrow m,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\ \&SHi\rightarrow m,\copyright c_1,\\ \\ ,\ \&SHi\rightarrow m,\copyright c_2,\end{array}\right],$$

$$, \&SHj \mathring{\bigcirc} m, if(i\mathring{\bigcirc}^{\circ} j) \begin{bmatrix} , ©c_1, \\ \\ , ©c_2, \end{bmatrix}, \Leftrightarrow \; , if(i\mathring{\bigcirc}^{\circ} j) \begin{bmatrix} , \&SHj \mathring{\bigcirc} m, ©c_1, \\ \\ , \&SHj \mathring{\bigcirc} m, ©c_2, \end{bmatrix},$$

$$, \&SHj \leftarrow m, if(i\mathring{\bigcirc}^{\circ} j) \begin{bmatrix} , ©c_1, \\ \\ , ©c_2, \end{bmatrix}, \Leftrightarrow \; , if(i\mathring{\bigcirc}^{\circ} j) \begin{bmatrix} , \&SHj \leftarrow m, ©c_1, \\ \\ , \&SHj \leftarrow m, ©c_2, \end{bmatrix},$$

Propositions and operator:

$$, i\mathring{\bigcirc}^{\circ} j, \circledcirc m, \Leftrightarrow \; , \circledcirc m, i\mathring{\bigcirc}^{\circ} j,$$
$$, i\mathring{\bigcirc}^{\circ} j, \odot m, \Leftrightarrow \; , \odot m, i\mathring{\bigcirc}^{\circ} j,$$

$$, i\mathring{\bigcirc}^{\circ} j, m \oslash n, \Leftrightarrow \; , m \oslash n, i\mathring{\bigcirc}^{\circ} j,$$
$$, i\mathring{\bigcirc}^{\circ} j, m \circledast n, \Leftrightarrow \; , m \circledast n, i\mathring{\bigcirc}^{\circ} j,$$
$$, i\mathring{\bigcirc}^{\circ} j, m \circledast n, \Leftrightarrow \; , m \circledast n, i\mathring{\bigcirc}^{\circ} j,$$

$$, i\mathring{\bigcirc}^{\circ} j, m \circledS, \Leftrightarrow \; , m \circledS, i\mathring{\bigcirc}^{\circ} j,$$
$$, i\mathring{\bigcirc}^{\circ} j, m \oplus, \Leftrightarrow \; , m \oplus, i\mathring{\bigcirc}^{\circ} j,$$
$$, i\mathring{\bigcirc}^{\circ} j, m \ominus, \Leftrightarrow \; , m \ominus, i\mathring{\bigcirc}^{\circ} j,$$

$$, i!\mathring{\bigcirc}^{\circ} j, \circledcirc m, \Leftrightarrow \; , \circledcirc m, i!\mathring{\bigcirc}^{\circ} j,$$
$$, i!\mathring{\bigcirc}^{\circ} j, \odot m, \Leftrightarrow \; , \odot m, i!\mathring{\bigcirc}^{\circ} j,$$

$$, i!\mathring{\bigcirc}^{\circ} j, m \oslash n, \Leftrightarrow \; , m \oslash n, i!\mathring{\bigcirc}^{\circ} j,$$
$$, i!\mathring{\bigcirc}^{\circ} j, m \circledast n, \Leftrightarrow \; , m \circledast n, i!\mathring{\bigcirc}^{\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m \oslash n, \;\Leftrightarrow\; , m \oslash n, i!\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m \circledast, \;\Leftrightarrow\; , m \circledast, i!\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m \oplus, \;\Leftrightarrow\; , m \oplus, i!\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m \ominus, \;\Leftrightarrow\; , m \ominus, i!\bigcirc^{\circ\circ} j,$$

Propositions and Propositions:

$$, i\bigcirc^{\circ\circ} j, m\bigcirc^{\circ\circ} n, \;\Leftrightarrow\; , m\bigcirc^{\circ\circ} n, i\bigcirc^{\circ\circ} j,$$

$$, i\bigcirc^{\circ\circ} j, m!\bigcirc^{\circ\circ} n, \;\Leftrightarrow\; , m!\bigcirc^{\circ\circ} n, i\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m!\bigcirc^{\circ\circ} n, \;\Leftrightarrow\; , m!\bigcirc^{\circ\circ} n, i!\bigcirc^{\circ\circ} j,$$

$$, i\bigcirc^{\circ\circ} j, m\bigcirc^{\circ} n, \;\Leftrightarrow\; , m\bigcirc^{\circ} n, i\bigcirc^{\circ\circ} j,$$

$$, i\bigcirc^{\circ\circ} j, m!\bigcirc^{\circ} n, \;\Leftrightarrow\; , m!\bigcirc^{\circ} n, i\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m\bigcirc^{\circ} n, \;\Leftrightarrow\; , m\bigcirc^{\circ} n, i!\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m!\bigcirc^{\circ} n, \;\Leftrightarrow\; , m!\bigcirc^{\circ} n, i!\bigcirc^{\circ\circ} j,$$

$$, i\bigcirc^{\circ\circ} j, m = n, \;\Leftrightarrow\; , m = n, i\bigcirc^{\circ\circ} j,$$

$$, i\bigcirc^{\circ\circ} j, m \mathrel{!=} n, \;\Leftrightarrow\; , m \mathrel{!=} n, i\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m = n, \;\Leftrightarrow\; , m = n, i!\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m \mathrel{!=} n, \;\Leftrightarrow\; , m \mathrel{!=} n, i!\bigcirc^{\circ\circ} j,$$

$$, i\bigcirc^{\circ\circ} j, m = \varnothing, \;\Leftrightarrow\; , m = \varnothing, i\bigcirc^{\circ\circ} j,$$

$$, i\bigcirc^{\circ\circ} j, m \mathrel{!=} \varnothing, \;\Leftrightarrow\; , m \mathrel{!=} \varnothing, i\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m = \varnothing, \;\Leftrightarrow\; , m = \varnothing, i!\bigcirc^{\circ\circ} j,$$

$$, i!\bigcirc^{\circ\circ} j, m \mathrel{!=} \varnothing, \;\Leftrightarrow\; , m \mathrel{!=} \varnothing, i!\bigcirc^{\circ\circ} j,$$

Propositions and recursive function:

$$
\begin{aligned}
, i \mathring{\bigcirc}\!\circ j, R(m), &\iff , R(m), i \mathring{\bigcirc}\!\circ j, \\
, i \mathring{\bigcirc}\!\circ j, R_{-}(m), &\iff , R_{-}(m), i \mathring{\bigcirc}\!\circ j, \\
, i ! \mathring{\bigcirc}\!\circ j, R(m), &\iff , R(m), i ! \mathring{\bigcirc}\!\circ j, \\
, i ! \mathring{\bigcirc}\!\circ j, R_{-}(m), &\iff , R_{-}(m), i ! \mathring{\bigcirc}\!\circ j,
\end{aligned}
$$

Propositions and flag object:

$$
\begin{aligned}
, i \mathring{\bigcirc}\!\circ j,\ \&SHi\, \mathring{\bigcirc} m, &\iff ,\ \&SHi\, \mathring{\bigcirc} m, i \mathring{\bigcirc}\!\circ j, \\
, i \mathring{\bigcirc}\!\circ j,\ \&SHi \rightarrow m, &\iff ,\ \&SHi \rightarrow m, i \mathring{\bigcirc}\!\circ j, \\
, i ! \mathring{\bigcirc}\!\circ j,\ \&SHi\, \mathring{\bigcirc} m, &\iff ,\ \&SHi\, \mathring{\bigcirc} m, i ! \mathring{\bigcirc}\!\circ j, \\
, i ! \mathring{\bigcirc}\!\circ j,\ \&SHi \rightarrow m, &\iff ,\ \&SHi \rightarrow m, i ! \mathring{\bigcirc}\!\circ j,
\end{aligned}
$$

$$
\begin{aligned}
, i \mathring{\bigcirc}\!\circ j,\ \&SHj\, \mathring{\bigcirc} m, &\iff ,\ \&SHj\, \mathring{\bigcirc} m, i \mathring{\bigcirc}\!\circ j, \\
, i \mathring{\bigcirc}\!\circ j,\ \&SHj \leftarrow m, &\iff ,\ \&SHj \leftarrow m, i \mathring{\bigcirc}\!\circ j, \\
, i ! \mathring{\bigcirc}\!\circ j,\ \&SHj\, \mathring{\bigcirc} m, &\iff ,\ \&SHj\, \mathring{\bigcirc} m, i ! \mathring{\bigcirc}\!\circ j, \\
, i ! \mathring{\bigcirc}\!\circ j,\ \&SHj \leftarrow m, &\iff ,\ \&SHj \leftarrow m, i ! \mathring{\bigcirc}\!\circ j,
\end{aligned}
$$

Propositions to Propositions with branch function

$$
, if(i \mathring{\bigcirc}\!\circ j) \left[\begin{array}{l} , m ! \mathring{\bigcirc}\!\circ n, \\ \\ , \end{array} \right], \iff , if(m \mathring{\bigcirc}\!\circ n) \left[\begin{array}{l} , i ! \mathring{\bigcirc}\!\circ j, \\ \\ , \end{array} \right],
$$

$$
, if(i \mathring{\bigcirc}\!\circ j) \left[\begin{array}{l} , \\ \\ , m \mathring{\bigcirc}\!\circ n, \end{array} \right], \iff , if(m \mathring{\bigcirc}\!\circ n) \left[\begin{array}{l} , \\ \\ , i \mathring{\bigcirc}\!\circ j, \end{array} \right],
$$

$$
, if(i \mathring{\bigcirc}\!\circ j) \left[\begin{array}{l} , m ! \mathring{\bigcirc} n, \\ \\ , \end{array} \right], \iff , if(m \mathring{\bigcirc} n) \left[\begin{array}{l} , i ! \mathring{\bigcirc}\!\circ j, \\ \\ , \end{array} \right],
$$

$$, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , \\ , m\overset{\circ}{\bigcirc}n, \end{array}\right], \ \Leftrightarrow \ , if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} , \\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, \end{array}\right],$$

$$, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , m\,!{=}\,n, \\ , \end{array}\right], \ \Leftrightarrow \ , if(m=n)\left[\begin{array}{l} , i!\overset{\circ}{\bigcirc}{}^{\circ}j, \\ , \end{array}\right],$$

$$, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , \\ , m=n, \end{array}\right], \ \Leftrightarrow \ , if(m=n)\left[\begin{array}{l} , \\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, \end{array}\right],$$

$$, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , m\,!{=}\,\varnothing, \\ , \end{array}\right], \ \Leftrightarrow \ , if(m=\varnothing)\left[\begin{array}{l} , i!\overset{\circ}{\bigcirc}{}^{\circ}j, \\ , \end{array}\right],$$

$$, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , \\ , m=\varnothing, \end{array}\right], \ \Leftrightarrow \ , if(m=\varnothing)\left[\begin{array}{l} , \\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, \end{array}\right],$$

16.3.7 Transitivity

Branch function with branch function:

$$, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_3, \end{array}\right], \\ , \copyright c_2, \end{array}\right],$$

proof:

$$, if(i\overset{\circ}{\bigcirc}{}^{\circ}j)\left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, R(i_1), R(j_1), if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, R(i_1), R(j_1), if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3,\end{array}\right], \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, i_2\text{⏚}, j\text{☮}j_1, R(i_1), R(j_1),$$

$$if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3,\end{array}\right], \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, R(i_2), i_2\text{⏚}, j\text{☮}j_1, R(i_1), R(j_1),$$

$$if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3,\end{array}\right], \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_2 a,\end{array}\right]$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, R(i_2), j\text{☮}j_1, R(i_1), R(j_1),$$

$$if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_2\text{⏚}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3,\end{array}\right], \\ , i_2\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, j\text{☮}j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_2\text{⏚}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3,\end{array}\right], \\ , i_2\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, i\mathring{\bigcirc}i_2, j\text{☮}j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_2 \textcircled{$\downarrow$}, if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ , i_2 \textcircled{$\downarrow$}, i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \owedge i_1, i \overset{\circ}{\bigcirc} i_1, i \owedge i_2, i \overset{\circ}{\bigcirc} i_2, j \owedge j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_2 \textcircled{$\downarrow$}, if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ , i_2 \textcircled{$\downarrow$}, i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \owedge i_1, i \owedge i_2, i \overset{\circ}{\bigcirc} i_1, i \overset{\circ}{\bigcirc} i_2, j \owedge j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_2 \textcircled{$\downarrow$}, if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ , i_2 \textcircled{$\downarrow$}, i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \owedge i_1, i \owedge i_2, i \overset{\circ}{\bigcirc} i_1, i_1 \overset{\circ}{\bigcirc} i_2, j \owedge j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_2 \textcircled{$\downarrow$}, if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ , i_2 \textcircled{$\downarrow$}, i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \owedge i_1, i \owedge i_2, i \overset{\circ}{\bigcirc} i_1, j \owedge j_1, i_1 \overset{\circ}{\bigcirc} i_2, R(i_2), R(i_1), R(j_1),$$

$$if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_2 \textcircled{$\downarrow$}, if(i_1 \overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ , i_2 \textcircled{$\downarrow$}, i_1 \textcircled{$\downarrow$}, j_1 \textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \owedge i_1, i \owedge i_2, i \overset{\circ}{\bigcirc} i_1, j \owedge j_1, i_1 \overset{\circ}{\bigcirc} i_2, R(i_2), R(i_1), i_1 \overset{\circ}{\bigcirc} i_2, R(j_1),$$

$$if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_2\textcircled{\scriptsize$\downarrow$}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_1, \\ ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\scriptsize$\downarrow$}, i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,i\circledast i_1, i\circledast i_2, i\mathring{\bigcirc}i_1, j\circledast j_1, i_1\mathring{\bigcirc}i_2, R(i_2), R(i_1), R(j_1), i_1\mathring{\bigcirc}i_2,$$

$$if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_2\textcircled{\scriptsize$\downarrow$}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_1, \\ ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\scriptsize$\downarrow$}, i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,i\circledast i_1, i\circledast i_2, i\mathring{\bigcirc}i_1, j\circledast j_1, i_1\mathring{\bigcirc}i_2, R(i_2), R(i_1), R(j_1), i_1\mathring{\bigcirc}i_2,$$

$$if(i_2\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_2\textcircled{\scriptsize$\downarrow$}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_1, \\ ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\scriptsize$\downarrow$}, i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,i\circledast i_1, i\circledast i_2, i\mathring{\bigcirc}i_1, j\circledast j_1, i_1\mathring{\bigcirc}i_2, R(i_2), R(i_1), i_1\mathring{\bigcirc}i_2, R(j_1),$$

$$if(i_2\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_2\textcircled{\scriptsize$\downarrow$}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_1, \\ ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\scriptsize$\downarrow$}, i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,i\circledast i_1, i\circledast i_2, i\mathring{\bigcirc}i_1, j\circledast j_1, i_1\mathring{\bigcirc}i_2, R(i_2), R(i_1), R(j_1),$$

$$if(i_2\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_2\textcircled{\scriptsize$\downarrow$}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_1, \\ ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\scriptsize$\downarrow$}, i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,i\circledast i_1, i\circledast i_2, i\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j\circledast j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_2\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_2\textcircled{\scriptsize$\downarrow$}, if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_1, \\ ,i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\scriptsize$\downarrow$}, i_1\textcircled{\scriptsize$\downarrow$}, j_1\textcircled{\scriptsize$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\otimes i_1, i\otimes i_2, i\mathring{\bigcirc} i_1, i\mathring{\bigcirc} i_2, j\otimes j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_2\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_2\oplus, if(i_1\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_1\oplus, j_1\oplus, \copyright c_1, \\ ,i_1\oplus, j_1\oplus, \copyright c_3, \end{array}\right], \\ ,i_2\oplus, i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\otimes i_1, i\mathring{\bigcirc} i_1, i\otimes i_2, i\mathring{\bigcirc} i_2, j\otimes j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_2\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_2\oplus, if(i_1\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_1\oplus, j_1\oplus, \copyright c_1, \\ ,i_1\oplus, j_1\oplus, \copyright c_3, \end{array}\right], \\ ,i_2\oplus, i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\otimes i_1, i\otimes i_2, i\mathring{\bigcirc} i_2, j\otimes j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_2\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_2\oplus, if(i_1\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_1\oplus, j_1\oplus, \copyright c_1, \\ ,i_1\oplus, j_1\oplus, \copyright c_3, \end{array}\right], \\ ,i_2\oplus, i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\otimes i_1, i\otimes i_2, j\otimes j_1, R(i_2), R(i_1), R(j_1),$$

$$if(i_2\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_2\oplus, if(i_1\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_1\oplus, j_1\oplus, \copyright c_1, \\ ,i_1\oplus, j_1\oplus, \copyright c_3, \end{array}\right], \\ ,i_2\oplus, i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, R(j_1),$$

$$if(i_2\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_2\oplus, if(i_1\mathring{\bigcirc} j_1)\left[\begin{array}{l} ,i_1\oplus, j_1\oplus, \copyright c_1, \\ ,i_1\oplus, j_1\oplus, \copyright c_3, \end{array}\right], \\ ,i_2\oplus, i_1\oplus, j_1\oplus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, j_2\oplus, R(j_1),$$

$$if(i_2 \text{♂} j_1)\left[\begin{array}{l} ,i_2\text{☮}, if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{☮}, j_1\text{☮}, \copyright c_1, \\ \\ ,i_1\text{☮}, j_1\text{☮}, \copyright c_3, \end{array}\right], \\ ,i_2\text{☮}, i_1\text{☮}, j_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, R(j_2), j_2\text{☮}, R(j_1),$$

$$if(i_2 \text{♂} j_1)\left[\begin{array}{l} ,i_2\text{☮}, if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{☮}, j_1\text{☮}, \copyright c_1, \\ \\ ,i_1\text{☮}, j_1\text{☮}, \copyright c_3, \end{array}\right], \\ ,i_2\text{☮}, i_1\text{☮}, j_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, R(j_2), R(j_1),$$

$$if(i_2 \text{♂} j_1)\left[\begin{array}{l} ,i_2\text{☮}, j_2\text{☮}, if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{☮}, j_1\text{☮}, \copyright c_1, \\ \\ ,i_1\text{☮}, j_1\text{☮}, \copyright c_3, \end{array}\right], \\ ,i_2\text{☮}, j_2\text{☮}, i_1\text{☮}, j_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\text{♂}j_1, j\otimes j_2, R(j_2), R(j_1),$$

$$if(i_2 \text{♂} j_1)\left[\begin{array}{l} ,i_2\text{☮}, j_2\text{☮}, if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{☮}, j_1\text{☮}, \copyright c_1, \\ \\ ,i_1\text{☮}, j_1\text{☮}, \copyright c_3, \end{array}\right], \\ ,i_2\text{☮}, j_2\text{☮}, i_1\text{☮}, j_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\text{♂}j_1, j\otimes j_2, j\text{♂}j_2, R(j_2), R(j_1),$$

$$if(i_2 \text{♂} j_1)\left[\begin{array}{l} ,i_2\text{☮}, j_2\text{☮}, if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{☮}, j_1\text{☮}, \copyright c_1, \\ \\ ,i_1\text{☮}, j_1\text{☮}, \copyright c_3, \end{array}\right], \\ ,i_2\text{☮}, j_2\text{☮}, i_1\text{☮}, j_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, j\text{♂}j_1, j\text{♂}j_2, R(j_2), R(j_1),$$

$$if(i_2 \text{♂} j_1)\left[\begin{array}{l} ,i_2\text{☮}, j_2\text{☮}, if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{☮}, j_1\text{☮}, \copyright c_1, \\ \\ ,i_1\text{☮}, j_1\text{☮}, \copyright c_3, \end{array}\right], \\ ,i_2\text{☮}, j_2\text{☮}, i_1\text{☮}, j_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, j\overset{\circ}{\bigcirc} j_1, j_1\overset{\circ}{\bigcirc} j_2, R(j_2), R(j_1),$$

$$if(i_2\overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_2\ominus, j_2\ominus, if(i_1\overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1\ominus, j_1\ominus, \copyright c_1, \\ , i_1\ominus, j_1\ominus, \copyright c_3, \end{array}\right], \\ , i_2\ominus, j_2\ominus, i_1\ominus, j_1\ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, j\overset{\circ}{\bigcirc} j_1, j_1\overset{\circ}{\bigcirc} j_2, R(j_2), R(j_1), j_1\overset{\circ}{\bigcirc} j_2,$$

$$if(i_2\overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_2\ominus, j_2\ominus, if(i_1\overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1\ominus, j_1\ominus, \copyright c_1, \\ , i_1\ominus, j_1\ominus, \copyright c_3, \end{array}\right], \\ , i_2\ominus, j_2\ominus, i_1\ominus, j_1\ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, j\overset{\circ}{\bigcirc} j_1, j_1\overset{\circ}{\bigcirc} j_2, R(j_2), R(j_1), j_1\overset{\circ}{\bigcirc} j_2,$$

$$if(i_2\overset{\circ}{\bigcirc} j_2)\left[\begin{array}{l} , i_2\ominus, j_2\ominus, if(i_1\overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1\ominus, j_1\ominus, \copyright c_1, \\ , i_1\ominus, j_1\ominus, \copyright c_3, \end{array}\right], \\ , i_2\ominus, j_2\ominus, i_1\ominus, j_1\ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, j\overset{\circ}{\bigcirc} j_1, j_1\overset{\circ}{\bigcirc} j_2, R(j_2), R(j_1),$$

$$if(i_2\overset{\circ}{\bigcirc} j_2)\left[\begin{array}{l} , i_2\ominus, j_2\ominus, if(i_1\overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1\ominus, j_1\ominus, \copyright c_1, \\ , i_1\ominus, j_1\ominus, \copyright c_3, \end{array}\right], \\ , i_2\ominus, j_2\ominus, i_1\ominus, j_1\ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\otimes j_2, j\overset{\circ}{\bigcirc} j_1, j\overset{\circ}{\bigcirc} j_2, R(j_2), R(j_1),$$

$$if(i_2\overset{\circ}{\bigcirc} j_2)\left[\begin{array}{l} , i_2\ominus, j_2\ominus, if(i_1\overset{\circ}{\bigcirc} j_1)\left[\begin{array}{l} , i_1\ominus, j_1\ominus, \copyright c_1, \\ , i_1\ominus, j_1\ominus, \copyright c_3, \end{array}\right], \\ , i_2\ominus, j_2\ominus, i_1\ominus, j_1\ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i\otimes i_1, i\otimes i_2, R(i_2), R(i_1), j\otimes j_1, j\overset{\circ}{\bigcirc} j_1, j\otimes j_2, j\overset{\circ}{\bigcirc} j_2, R(j_2), R(j_1),$$

$$if(i_2\text{♂}j_2)\left[\begin{array}{l} ,i_2\text{⏚},j_2\text{⏚},if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚},\copyright c_1, \\ ,i_1\text{⏚},j_1\text{⏚},\copyright c_3, \end{array}\right], \\ ,i_2\text{⏚},j_2\text{⏚},i_1\text{⏚},j_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,R(i_2),R(i_1),j\text{☮}j_1,j\text{☮}j_2,j\text{♂}j_2,R(j_2),R(j_1),$$

$$if(i_2\text{♂}j_2)\left[\begin{array}{l} ,i_2\text{⏚},j_2\text{⏚},if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚},\copyright c_1, \\ ,i_1\text{⏚},j_1\text{⏚},\copyright c_3, \end{array}\right], \\ ,i_2\text{⏚},j_2\text{⏚},i_1\text{⏚},j_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,R(i_2),R(i_1),j\text{☮}j_1,j\text{☮}j_2,R(j_2),R(j_1),$$

$$if(i_2\text{♂}j_2)\left[\begin{array}{l} ,i_2\text{⏚},j_2\text{⏚},if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚},\copyright c_1, \\ ,i_1\text{⏚},j_1\text{⏚},\copyright c_3, \end{array}\right], \\ ,i_2\text{⏚},j_2\text{⏚},i_1\text{⏚},j_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,R(i_2),R(i_1),j\text{☮}j_1,j\text{☮}j_2,R(j_2),$$

$$if(i_2\text{♂}j_2)\left[\begin{array}{l} ,i_2\text{⏚},j_2\text{⏚},R(j_1),if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚},\copyright c_1, \\ ,i_1\text{⏚},j_1\text{⏚},\copyright c_3, \end{array}\right], \\ ,i_2\text{⏚},j_2\text{⏚},i_1\text{⏚},R(j_1),j_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,R(i_2),R(i_1),j\text{☮}j_2,R(j_2),$$

$$if(i_2\text{♂}j_2)\left[\begin{array}{l} ,i_2\text{⏚},j_2\text{⏚},j\text{☮}j_1,R(j_1),if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚},\copyright c_1, \\ ,i_1\text{⏚},j_1\text{⏚},\copyright c_3, \end{array}\right], \\ ,i_2\text{⏚},j_2\text{⏚},i_1\text{⏚},j\text{☮}j_1,R(j_1),j_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,R(i_2),R(i_1),j\text{☮}j_2,R(j_2),$$

$$if(i_2\text{♂}j_2)\left[\begin{array}{l} ,i_2\text{⏚},j_2\text{⏚},j\text{☮}j_1,R(j_1),if(i_1\text{♂}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚},\copyright c_1, \\ ,i_1\text{⏚},j_1\text{⏚},\copyright c_3, \end{array}\right], \\ ,i_2\text{⏚},j_2\text{⏚},i_1\text{⏚},j\text{☮}j_1,j_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, R(i_2), R(i_1), j\text{☮}j_2, R(j_2),$$

$$if(i_2\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l} , i_2\text{⏚}, j_2\text{⏚}, j\text{☮}j_1, R(j_1), if(i_1\overset{\circ}{\bigcirc}j_1)\left[\begin{array}{l} , i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , i_2\text{⏚}, j_2\text{⏚}, i_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, R(i_2), j\text{☮}j_2, R(j_2),$$

$$if(i_2\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l} , i_2\text{⏚}, j_2\text{⏚}, R(i_1), j\text{☮}j_1, R(j_1), if(i_1\overset{\circ}{\bigcirc}j_1)\left[\begin{array}{l} , i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , i_2\text{⏚}, j_2\text{⏚}, R(i_1), i_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_2, R(i_2), j\text{☮}j_2, R(j_2),$$

$$if(i_2\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l} , i_2\text{⏚}, j_2\text{⏚}, i\text{☮}i_1, R(i_1), j\text{☮}j_1, R(j_1), if(i_1\overset{\circ}{\bigcirc}j_1)\left[\begin{array}{l} , i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , i_2\text{⏚}, j_2\text{⏚}, i\text{☮}i_1, R(i_1), i_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_2, R(i_2), j\text{☮}j_2, R(j_2),$$

$$if(i_2\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l} , i_2\text{⏚}, j_2\text{⏚}, i\text{☮}i_1, R(i_1), j\text{☮}j_1, R(j_1), if(i_1\overset{\circ}{\bigcirc}j_1)\left[\begin{array}{l} , i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , i_2\text{⏚}, j_2\text{⏚}, i\text{☮}i_1, i_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_2, R(i_2), j\text{☮}j_2, R(j_2),$$

$$if(i_2\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l} , i_2\text{⏚}, j_2\text{⏚}, i\text{☮}i_1, R(i_1), j\text{☮}j_1, R(j_1), if(i_1\overset{\circ}{\bigcirc}j_1)\left[\begin{array}{l} , i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ , i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ , i_2\text{⏚}, j_2\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\oslash i_1, R(i_1), j\oslash j_1, R(j_1), if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\ominus, j_1\ominus, \copyright c_1, \\ \\ ,i_1\ominus, j_1\ominus, \copyright c_3, \end{array}\right], \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\oslash i_1, j\oslash j_1, R(i_1), R(j_1), if(i_1\mathring{\bigcirc}j_1)\left[\begin{array}{l} ,i_1\ominus, j_1\ominus, \copyright c_1, \\ \\ ,i_1\ominus, j_1\ominus, \copyright c_3, \end{array}\right], \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

Branch function with propositions:

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\mathring{\bigcirc}j, \copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,i!\mathring{\bigcirc}j, \copyright c_2, \end{array}\right],$$

Propositions with branch function:

$$,i\overset{\circ}{\bigcirc}\!\circ j, if(i\overset{\circ}{\bigcirc}\!\circ j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}\!\circ j,\copyright c_1,$$

$$,i!\overset{\circ}{\bigcirc}\!\circ j, if(i\overset{\circ}{\bigcirc}\!\circ j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \Leftrightarrow\ ,i!\overset{\circ}{\bigcirc}\!\circ j,\copyright c_2,$$

Propositions with propositions:

$$,i\overset{\circ}{\bigcirc}\!\circ j, \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}\!\circ j, i\overset{\circ}{\bigcirc}\!\circ j,$$

$$,i!\overset{\circ}{\bigcirc}\!\circ j, \Leftrightarrow\ ,i!\overset{\circ}{\bigcirc}\!\circ j, i!\overset{\circ}{\bigcirc}\!\circ j,$$

16.3.8 Substitution

Propositions with branch function:

$$,i\overset{\circ}{\bigcirc}\!\circ j, if(j\overset{\circ}{\bigcirc}\!\circ m)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}\!\circ j, if(i\overset{\circ}{\bigcirc}\!\circ m)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

proof:

$$,i\overset{\circ}{\bigcirc}\!\circ j, if(j\overset{\circ}{\bigcirc}\!\circ m)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\circledwedge i_1, j\circledwedge j_1, R(i_1), R(j_1), i_1\overset{\circ}{\bigcirc} j_1, i_1\textcircled{$\downarrow$}, j_1\textcircled{$\downarrow$}, if(j\overset{\circ}{\bigcirc}\!\circ m)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\circledwedge i_1, j\circledwedge j_1, R(i_1), R(j_1), i_1\overset{\circ}{\bigcirc} j_1, i_1\textcircled{$\downarrow$}, j_1\textcircled{$\downarrow$},$$

$$j\circledwedge j_2, m\circledwedge m_2, R(j_2), R(m_2), if(j_2\overset{\circ}{\bigcirc} m_2)\left[\begin{array}{c},j_2\textcircled{$\downarrow$}, m_2\textcircled{$\downarrow$},\copyright c_1,\\ \\ ,j_2\textcircled{$\downarrow$}, m_2\textcircled{$\downarrow$},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, j\otimes j_1, j\otimes j_2, R(j_1), R(j_2), R(i_1), i_1\mathring{\bigcirc} j_1,$$

$$i_1\ominus, j_1\ominus, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc} m_2)\left[\begin{array}{l}, j_2\ominus, m_2\ominus, \copyright c_1,\\ \\ , j_2\ominus, m_2\ominus, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), R(i_1), i_1\mathring{\bigcirc} j_1,$$

$$i_1\ominus, j_1\ominus, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc} m_2)\left[\begin{array}{l}, j_2\ominus, m_2\ominus, \copyright c_1,\\ \\ , j_2\ominus, m_2\ominus, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2, R(i_1), i_1\mathring{\bigcirc} j_1,$$

$$i_1\ominus, j_1\ominus, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc} m_2)\left[\begin{array}{l}, j_2\ominus, m_2\ominus, \copyright c_1,\\ \\ , j_2\ominus, m_2\ominus, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), R(i_1), j_1\mathring{\bigcirc} j_2, i_1\mathring{\bigcirc} j_1,$$

$$i_1\ominus, j_1\ominus, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc} m_2)\left[\begin{array}{l}, j_2\ominus, m_2\ominus, \copyright c_1,\\ \\ , j_2\ominus, m_2\ominus, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\otimes i_1, j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), R(i_1), j_1\mathring{\bigcirc} j_2, i_1\mathring{\bigcirc} j_2,$$

$$i_1\ominus, j_1\ominus, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc} m_2)\left[\begin{array}{l}, j_2\ominus, m_2\ominus, \copyright c_1,\\ \\ , j_2\ominus, m_2\ominus, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2,$$

$$i\otimes i_1, R(i_1), i_1\mathring{\bigcirc} j_2,$$

$$i_1\ominus, j_1\ominus, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc} m_2)\left[\begin{array}{l}, j_2\ominus, m_2\ominus, \copyright c_1,\\ \\ , j_2\ominus, m_2\ominus, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2,$$

$$i\otimes i_1, i\otimes i_2, i_2\ominus, R(i_1), i_1\mathring{\bigcirc} j_2,$$

$$i_1\odot, j_1\odot, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc}m_2)\left[\begin{array}{l}, j_2\odot, m_2\odot, \copyright c_1, \\ \\ , j_2\odot, m_2\odot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc}j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc}j_2,$$

$$i\otimes i_1, i\otimes i_2, R(i_2), i_2\odot, R(i_1), i_1\mathring{\bigcirc}j_2,$$

$$i_1\odot, j_1\odot, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc}m_2)\left[\begin{array}{l}, j_2\odot, m_2\odot, \copyright c_1, \\ \\ , j_2\odot, m_2\odot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc}j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc}j_2,$$

$$i\otimes i_1, i\otimes i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc}j_2,$$

$$i_1\odot, j_1\odot, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc}m_2)\left[\begin{array}{l}, i_2\odot, j_2\odot, m_2\odot, \copyright c_1, \\ \\ , i_2\odot, j_2\odot, m_2\odot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc}j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc}j_2,$$

$$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc}i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc}j_2,$$

$$i_1\odot, j_1\odot, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc}m_2)\left[\begin{array}{l}, i_2\odot, j_2\odot, m_2\odot, \copyright c_1, \\ \\ , i_2\odot, j_2\odot, m_2\odot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc}j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc}j_2,$$

$$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc}i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc}i_2, i_1\mathring{\bigcirc}j_2,$$

$$i_1\odot, j_1\odot, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc}m_2)\left[\begin{array}{l}, i_2\odot, j_2\odot, m_2\odot, \copyright c_1, \\ \\ , i_2\odot, j_2\odot, m_2\odot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc}j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc}j_2,$$

$$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc}i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc}i_2, i_2\mathring{\bigcirc}j_2,$$

$$i_1\odot, j_1\odot, m\otimes m_2, R(m_2), if(j_2\mathring{\bigcirc}m_2)\left[\begin{array}{l}, i_2\odot, j_2\odot, m_2\odot, \copyright c_1, \\ \\ , i_2\odot, j_2\odot, m_2\odot, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc}j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc}j_2,$$

$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc} i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc} i_2, i_1\circleddash, j_1\circleddash,$

$m\otimes m_2, R(m_2), i_2\mathring{\bigcirc} j_2, if(j_2\mathring{\bigcirc} m_2)\left[\begin{array}{l} , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_1, \\ \\ , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_2, \end{array}\right],$

$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2,$

$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc} i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc} i_2, i_1\circleddash, j_1\circleddash,$

$m\otimes m_2, R(m_2), i_2\mathring{\bigcirc} j_2, if(i_2\mathring{\bigcirc} m_2)\left[\begin{array}{l} , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_1, \\ \\ , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_2, \end{array}\right],$

$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2,$

$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc} i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc} i_2, i_2\mathring{\bigcirc} j_2, i_1\circleddash, j_1\circleddash,$

$m\otimes m_2, R(m_2), if(i_2\mathring{\bigcirc} m_2)\left[\begin{array}{l} , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_1, \\ \\ , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_2, \end{array}\right],$

$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2,$

$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc} i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc} i_2, i_1\mathring{\bigcirc} j_2, i_1\circleddash, j_1\circleddash,$

$m\otimes m_2, R(m_2), if(i_2\mathring{\bigcirc} m_2)\left[\begin{array}{l} , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_1, \\ \\ , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_2, \end{array}\right],$

$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2,$

$i\otimes i_1, i\otimes i_2, i_1\mathring{\bigcirc} i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc} j_2, i_1\circleddash, j_1\circleddash,$

$m\otimes m_2, R(m_2), if(i_2\mathring{\bigcirc} m_2)\left[\begin{array}{l} , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_1, \\ \\ , i_2\circleddash, j_2\circleddash, m_2\circleddash, \copyright c_2, \end{array}\right],$

$\Leftrightarrow\ , j\otimes j_1, j\otimes j_2, j_1\mathring{\bigcirc} j_2, R(j_1), R(j_2), j_1\mathring{\bigcirc} j_2,$

$i\otimes i_1, i\otimes i_2, R(i_1), R(i_2), i_1\mathring{\bigcirc} j_2, i_1\circleddash, j_1\circleddash,$

$$m\text{☮}m_2, R(m_2), if(i_2\text{♂}m_2)\left[\begin{array}{l} ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_1, \\ \\ ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,j\text{☮}j_1, j\text{☮}j_2, j_1\text{♂}j_2, R(j_1), R(j_2),$$

$$i\text{☮}i_1, i\text{☮}i_2, R(i_1), R(i_2), j_1\text{♂}j_2, i_1\text{♂}j_2, i_1\text{⏚}, j_1\text{⏚},$$

$$m\text{☮}m_2, R(m_2), if(i_2\text{♂}m_2)\left[\begin{array}{l} ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_1, \\ \\ ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,j\text{☮}j_1, j\text{☮}j_2, j_1\text{♂}j_2, R(j_1), R(j_2),$$

$$i\text{☮}i_1, i\text{☮}i_2, R(i_1), R(i_2), j_1\text{♂}j_2, i_1\text{♂}j_1, i_1\text{⏚}, j_1\text{⏚},$$

$$m\text{☮}m_2, R(m_2), if(i_2\text{♂}m_2)\left[\begin{array}{l} ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_1, \\ \\ ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,j\text{☮}j_1, j\text{☮}j_2, j_1\text{♂}j_2, R(j_1), R(j_2), j_1\text{♂}j_2,$$

$$i\text{☮}i_1, i\text{☮}i_2, R(i_1), R(i_2), i_1\text{♂}j_1, i_1\text{⏚}, j_1\text{⏚},$$

$$m\text{☮}m_2, R(m_2), if(i_2\text{♂}m_2)\left[\begin{array}{l} ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_1, \\ \\ ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,j\text{☮}j_1, j\text{☮}j_2, j_1\text{♂}j_2, R(j_1), R(j_2),$$

$$i\text{☮}i_1, i\text{☮}i_2, R(i_1), R(i_2), i_1\text{♂}j_1, i_1\text{⏚}, j_1\text{⏚},$$

$$m\text{☮}m_2, R(m_2), if(i_2\text{♂}m_2)\left[\begin{array}{l} ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_1, \\ \\ ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,j\text{☮}j_1, j\text{☮}j_2, R(j_1), R(j_2),$$

$$i\text{☮}i_1, i\text{☮}i_2, R(i_1), R(i_2), i_1\text{♂}j_1, i_1\text{⏚}, j_1\text{⏚},$$

$$m\text{☮}m_2, R(m_2), if(i_2\text{♂}m_2)\left[\begin{array}{l} ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_1, \\ \\ ,i_2\text{⏚}, j_2\text{⏚}, m_2\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1 j\text{☮}j_1, R(i_1), R(j_1), i_1\text{♂}j_1, i_1\text{⏚}, j_1\text{⏚},$$

$$, i \text{☮} i_2, R(i_2), j \text{☮} j_2, R(j_2),$$

$$m \text{☮} m_2, R(m_2), if(i_2 \overset{\circ}{\bigcirc} m_2) \left[\begin{array}{l} , i_2 \text{⏚}, j_2 \text{⏚}, m_2 \text{⏚}, \copyright c_1, \\ \\ , i_2 \text{⏚}, j_2 \text{⏚}, m_2 \text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \bigcirc^{\circ\circ} j, i \text{☮} i_2, R(i_2), j \text{☮} j_2, R(j_2), j_2 \text{⏚},$$

$$m \text{☮} m_2, R(m_2), if(i_2 \overset{\circ}{\bigcirc} m_2) \left[\begin{array}{l} , i_2 \text{⏚}, m_2 \text{⏚}, \copyright c_1, \\ \\ , i_2 \text{⏚}, m_2 \text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \bigcirc^{\circ\circ} j, i \text{☮} i_2, R(i_2), m \text{☮} m_2, R(m_2), if(i_2 \overset{\circ}{\bigcirc} m_2) \left[\begin{array}{l} , i_2 \text{⏚}, m_2 \text{⏚}, \copyright c_1, \\ \\ , i_2 \text{⏚}, m_2 \text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \bigcirc^{\circ\circ} j, i \text{☮} i_2, m \text{☮} m_2, R(i_2), R(m_2), if(i_2 \overset{\circ}{\bigcirc} m_2) \left[\begin{array}{l} , i_2 \text{⏚}, m_2 \text{⏚}, \copyright c_1, \\ \\ , i_2 \text{⏚}, m_2 \text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ , i \bigcirc^{\circ\circ} j, if(i \bigcirc^{\circ\circ} m) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

Propositions with propositions:

$$, i \bigcirc^{\circ\circ} j, j \bigcirc^{\circ\circ} m, \ \Leftrightarrow \ , i \bigcirc^{\circ\circ} j, i \bigcirc^{\circ\circ} m,$$

$$, i \bigcirc^{\circ\circ} j, j ! \bigcirc^{\circ\circ} m, \ \Leftrightarrow \ , i \bigcirc^{\circ\circ} j, i ! \bigcirc^{\circ\circ} m,$$

Identical node propositions with branch function:

$$, i \overset{\circ}{\bigcirc} j, if(j \bigcirc^{\circ\circ} m) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , i \overset{\circ}{\bigcirc} j, if(i \bigcirc^{\circ\circ} m) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

Identical node propositions with propositions:

$$,i\mathring{\bigcirc}j,j\mathring{\bigcirc}^{\circ}m,\ \Leftrightarrow\ ,i\mathring{\bigcirc}j,i\mathring{\bigcirc}^{\circ}m,$$

$$,i\mathring{\bigcirc}j,j!\mathring{\bigcirc}^{\circ}m,\ \Leftrightarrow\ ,i\mathring{\bigcirc}j,i!\mathring{\bigcirc}^{\circ}m,$$

16.3.9 Opposition

$$,i\mathring{\bigcirc}^{\circ}j,i!\mathring{\bigcirc}^{\circ}j,\ \Leftrightarrow\ ,\otimes,$$

$$,i!\mathring{\bigcirc}^{\circ}j,i\mathring{\bigcirc}^{\circ}j,\ \Leftrightarrow\ ,\otimes,$$

16.3.10 Swap of the same operand

Operators:

$$,i\circledast_{\circ}m,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},i\circledast_{\circ}m,\copyright c_1,\\ \\ ,i\circledast_{\circ}m,\copyright c_2,\end{array}\right],$$

$$,i\circledast m,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},i\circledast m,\copyright c_1,\\ \\ ,i\circledast m,\copyright c_2,\end{array}\right],$$

$$,i\circledast' m,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},i\circledast' m,\copyright c_1,\\ \\ ,i\circledast' m,\copyright c_2,\end{array}\right],$$

$$,i\oplus,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},i\oplus,\copyright c_1,\\ \\ ,i\oplus,\copyright c_2,\end{array}\right],$$

$$,i\ominus,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},i\ominus,\copyright c_1,\\ \\ ,i\ominus,\copyright c_2,\end{array}\right],$$

$$,i\ddot{\bigcirc}j,i☮n,\ \Leftrightarrow\ ,i☮n,i\ddot{\bigcirc}j,$$

$$,i\ddot{\bigcirc}j,i☮_{\circ}n,\ \Leftrightarrow\ ,i☮_{\circ}n,i\ddot{\bigcirc}j,$$

$$,i\ddot{\bigcirc}j,i☮'n,\ \Leftrightarrow\ ,i☮'n,i\ddot{\bigcirc}j,$$

$$,i\ddot{\bigcirc}j,i\oplus,\ \Leftrightarrow\ ,i\oplus,i\ddot{\bigcirc}j,$$

proof:

$,i\ddot{\bigcirc}j,i\oplus,$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},i\oplus,$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},i\oplus,i\owedge i_2,i_2\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},i\oplus,i\owedge i_2,R(i_2),i_2\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,i\oplus,i\owedge i_2,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,R(i_2),i_2\textcircled{\downarrow},i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\owedge i_1,i\mathring{\bigcirc}i_1,j\owedge j_1,i\oplus,i\owedge i_2,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,R(i_2),i_2\textcircled{\downarrow},i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,i\mathring{\bigcirc}i_1,i\oplus,i\owedge i_2,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,R(i_2),i_2\textcircled{\downarrow},i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,if(i=\varnothing)\begin{bmatrix},\\ ,\end{bmatrix},i\mathring{\bigcirc}i_1,i\oplus,i\owedge i_2,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,R(i_2),i_2\textcircled{\downarrow},i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,$

$$if(i=\varnothing)\begin{bmatrix},i\mathring{\bigcirc}i_1,i\oplus,i\owedge i_2,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,R(i_2),\\ ,i\mathring{\bigcirc}i_1,i\oplus,i\owedge i_2,R(i_1),R(j_1),i_1\mathring{\bigcirc}j_1,R(i_2),\end{bmatrix},i_2\textcircled{\downarrow},i_1\textcircled{\downarrow},j_1\textcircled{\downarrow},$$

$\Leftrightarrow\ ,i\owedge i_1,j\owedge j_1,$

$$if(i=\varnothing)\left[\begin{array}{l} ,i=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,R(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2), \\ ,i\,!\!=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,R(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2), \end{array}\right],i_2\textcircled{\scriptsize$\downarrow$},i_1\textcircled{\scriptsize$\downarrow$},j_1\textcircled{\scriptsize$\downarrow$},$$

$$\Leftrightarrow \quad <1>$$

$$<2> \quad \Leftrightarrow \quad ,i=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,R(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$<3> \quad \Leftrightarrow \quad ,i\,!\!=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,R(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$<2>$$

$$\Leftrightarrow \quad ,i\overset{\circ}{\bigcirc}i_1,i=\varnothing,i\oplus,i\oslash i_2,R(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i\overset{\circ}{\bigcirc}i_1,i_1=\varnothing,i\oplus,i\oslash i_2,R(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,i_1=\varnothing,R(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,i_1=\varnothing,R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,i_1=\varnothing,R_{-}(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i\overset{\circ}{\bigcirc}i_1,i\oplus,i\oslash i_2,i_1=\varnothing,i_1\oplus,i_1\ominus,R_{-}(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i_1=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i_1\oplus,i\oslash i_2,i_1\ominus,R_{-}(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i_1=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i_1\oplus,i\overset{\circ}{\bigcirc}i_1,i\oslash i_2,i_1\ominus,R_{-}(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i_1=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i_1\oplus,i\overset{\circ}{\bigcirc}i_1,i\oslash i_2,i\overset{\circ}{\bigcirc}i_2,i_1\ominus,R_{-}(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$$\Leftrightarrow \quad ,i_1=\varnothing,i\overset{\circ}{\bigcirc}i_1,i\oplus,i_1\oplus,i\oslash i_2,i\overset{\circ}{\bigcirc}i_1,i\overset{\circ}{\bigcirc}i_2,i_1\ominus,R_{-}(i_1),R(j_1),i_1\overset{\circ}{\bigcirc}j_1,R(i_2),$$

$\Leftrightarrow\ , i_1 = \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i_1\oplus, i\text{☮}i_2, i\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1\ominus, R_{\text{-}}(i_1), R(j_1), i_1\mathring{\bigcirc}j_1, R(i_2),$

$\Leftrightarrow\ , i_1 = \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i_1\oplus, i\text{☮}i_2, i\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, R(i_2), i_1\ominus, R_{\text{-}}(i_1), R(j_1), i_1\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i_1 = \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i_1\oplus, i\text{☮}i_2, i\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, R(i_2), i_1\ominus, R_{\text{-}}(i_1), i_1\mathring{\bigcirc}i_2, R(j_1), i_1\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i_1 = \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i_1\oplus, i\text{☮}i_2, i\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, R(i_2), i_1\ominus, R_{\text{-}}(i_1), R(j_1), i_1\mathring{\bigcirc}i_2, i_1\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i_1 = \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i_1\oplus, i\text{☮}i_2, i\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, R(i_2), i_1\ominus, R_{\text{-}}(i_1), R(j_1), i_1\mathring{\bigcirc}i_2, i_2\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i_1 = \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i_1\oplus, i\text{☮}i_2, R(i_2), i_1\ominus, R_{\text{-}}(i_1), R(j_1), i_2\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i\mathring{\bigcirc}i_1, i\oplus, i_1 = \varnothing, i_1\oplus, i_1\ominus, R_{\text{-}}(i_1), i\text{☮}i_2, R(i_2), R(j_1), i_2\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i\mathring{\bigcirc}i_1, i_1 = \varnothing, i\oplus, i\text{☮}i_2, R(i_2), R(j_1), i_2\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i\mathring{\bigcirc}i_1, i = \varnothing, i\oplus, i\text{☮}i_2, R(i_2), R(j_1), i_2\mathring{\bigcirc}j_1,$

$\Leftrightarrow\ , i = \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i\text{☮}i_2, R(i_2), R(j_1), i_2\mathring{\bigcirc}j_1,$

$< 3 >$

$\Leftrightarrow\ , i\mathring{\bigcirc}i_1, i \mathrel{!=} \varnothing, i\oplus, i\text{☮}i_2, R(i_1), R(j_1), i_1\mathring{\bigcirc}j_1, R(i_2),$

$\Leftrightarrow\ , i\mathring{\bigcirc}i_1, i_1 \mathrel{!=} \varnothing, i\oplus, i\text{☮}i_2, R(i_1), R(j_1), i_1\mathring{\bigcirc}j_1, R(i_2),$

$\Leftrightarrow\ , i\mathring{\bigcirc}i_1, i\oplus, i_1 \mathrel{!=} \varnothing, R(i_1), i\text{☮}i_2, R(j_1), i_1\mathring{\bigcirc}j_1, R(i_2),$

$\Leftrightarrow\ , i\mathring{\bigcirc}i_1, i\oplus, i_1 \mathrel{!=} \varnothing, i_1\oplus, R(i_1), i\text{☮}i_2, R(j_1), i_1\mathring{\bigcirc}j_1, R(i_2),$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, R(i_1), i\oslash i_2, R(j_1), i_1 \mathring{\bigcirc} j_1, R(i_2),$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i \mathring{\bigcirc} i_1, R(i_1), i\oslash i_2, R(j_1), i_1 \mathring{\bigcirc} j_1, R(i_2),$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i \mathring{\bigcirc} i_1, i\oslash i_2, R(i_1), R(i_2), R(j_1), i_1 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i \mathring{\bigcirc} i_1, i\oslash i_2, i \mathring{\bigcirc} i_2, R(i_1), R(i_2), R(j_1), i_1 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i\oslash i_2, i \mathring{\bigcirc} i_1, i \mathring{\bigcirc} i_2, R(i_1), R(i_2), R(j_1), i_1 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i\oslash i_2, i \mathring{\bigcirc} i_1, i_1 \mathring{\bigcirc} i_2, R(i_1), R(i_2), R(j_1), i_1 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i\oslash i_2, i \mathring{\bigcirc} i_1, i_1 \mathring{\bigcirc} i_2, R(i_1), R(i_2), i_1 \mathring{\bigcirc} i_2, R(j_1), i_1 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i\oslash i_2, i \mathring{\bigcirc} i_1, i_1 \mathring{\bigcirc} i_2, R(i_1), R(i_2), R(j_1), i_1 \mathring{\bigcirc} i_2, i_1 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i\oslash i_2, i \mathring{\bigcirc} i_1, i_1 \mathring{\bigcirc} i_2, R(i_1), R(i_2), R(j_1), i_1 \mathring{\bigcirc} i_2, i_2 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i\oslash i_2, R(i_1), R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i \mathring{\bigcirc} i_1, i_1 \mathrel{!}= \varnothing, i_1\oplus, R(i_1), i\oplus, i\oslash i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i \mathring{\bigcirc} i_1, i_1 \mathrel{!}= \varnothing, R(i_1), i\oplus, i\oslash i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i \mathring{\bigcirc} i_1, i \mathrel{!}= \varnothing, R(i_1), i\oplus, i\oslash i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1,$$

$$\Leftrightarrow\ , i \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, R(i_1), i\oplus, i\oslash i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1,$$

$< 1 >$

$$\Leftrightarrow\ , i \owedge i_1, j \owedge j_1,$$

$$if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, i \mathring{\bigcirc} i_1, i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \\ \\ , i\ != \varnothing, i \mathring{\bigcirc} i_1, R(i_1), i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \end{array}\right], i_2 \obot, i_1 \obot, j_1 \obot,$$

$$\Leftrightarrow\ , i \owedge i_1, j \owedge j_1,$$

$$if(i=\varnothing)\left[\begin{array}{l} , i \mathring{\bigcirc} i_1, i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \\ \\ , i \mathring{\bigcirc} i_1, R(i_1), i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \end{array}\right], i_2 \obot, i_1 \obot, j_1 \obot,$$

$$\Leftrightarrow\ , i \owedge i_1, i \mathring{\bigcirc} i_1, j \owedge j_1, if(i=\varnothing)\left[\begin{array}{l} , i_1 \obot, i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \\ \\ , R(i_1), i_1 \obot, i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \end{array}\right], i_2 \obot, j_1 \obot,$$

$$\Leftrightarrow\ , i \owedge i_1, j \owedge j_1, if(i=\varnothing)\left[\begin{array}{l} , i_1 \obot, i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \\ \\ , i_1 \obot, i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \end{array}\right], i_2 \obot, j_1 \obot,$$

$$\Leftrightarrow\ , i \owedge i_1, i_1 \obot, j \owedge j_1, if(i=\varnothing)\left[\begin{array}{l} , i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \\ \\ , i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \end{array}\right], i_2 \obot, j_1 \obot,$$

$$\Leftrightarrow\ , j \owedge j_1, if(i=\varnothing)\left[\begin{array}{l} , i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \\ \\ , i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, \end{array}\right], i_2 \obot, j_1 \obot,$$

$$\Leftrightarrow\ , j \owedge j_1, if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right], i\oplus, i \owedge i_2, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, i_2 \obot, j_1 \obot,$$

$$\Leftrightarrow\ , i\oplus, i \owedge i_2, j \owedge j_1, R(i_2), R(j_1), i_2 \mathring{\bigcirc} j_1, i_2 \obot, j_1 \obot,$$

$\Leftrightarrow\ , i\oplus, i\bigcirc^{\circ} j,$

$$, i\bigcirc^{\circ} j, i\ominus, \ \Leftrightarrow\ , i\ominus, i\bigcirc^{\circ} j,$$

$$, i!\bigcirc^{\circ} j, i\circledast n, \ \Leftrightarrow\ , i\circledast n, i!\bigcirc^{\circ} j,$$

$$, i!\bigcirc^{\circ} j, i\circledcirc n, \ \Leftrightarrow\ , i\circledcirc n, i!\bigcirc^{\circ} j,$$

$$, i!\bigcirc^{\circ} j, i\circledS n, \ \Leftrightarrow\ , i\circledS n, i!\bigcirc^{\circ} j,$$

$$, i!\bigcirc^{\circ} j, i\oplus, \ \Leftrightarrow\ , i\oplus, i!\bigcirc^{\circ} j,$$

proof:

$, i!\bigcirc^{\circ} j, i\oplus,$

$\Leftrightarrow\ , i!\bigcirc^{\circ} j, i\oplus, if(i\bigcirc^{\circ} j)\left[\begin{array}{l} , \\ , \end{array}\right],$

$\Leftrightarrow\ , i!\bigcirc^{\circ} j, i\oplus, \left[\begin{array}{l} , i\bigcirc^{\circ} j, \\ , i!\bigcirc^{\circ} j, \end{array}\right],$

$\Leftrightarrow\ , \left[\begin{array}{l} , i!\bigcirc^{\circ} j, i\oplus, i\bigcirc^{\circ} j, \\ , i!\bigcirc^{\circ} j, i\oplus, i!\bigcirc^{\circ} j, \end{array}\right],$

$\Leftrightarrow\ , \left[\begin{array}{l} , i!\bigcirc^{\circ} j, i\bigcirc^{\circ} j, i\oplus, \\ , i!\bigcirc^{\circ} j, i\oplus, i!\bigcirc^{\circ} j, \end{array}\right],$

$\Leftrightarrow\ , \left[\begin{array}{l} , \otimes, i\oplus, \\ , i!\bigcirc^{\circ} j, i\oplus, i!\bigcirc^{\circ} j, \end{array}\right],$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,\otimes, \\ \\ ,i!\overset{\circ}{\bigcirc}^{\!\circ}j,i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,i\oplus,\otimes, \\ \\ ,i!\overset{\circ}{\bigcirc}^{\!\circ}j,i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,i\oplus,i\overset{\circ}{\bigcirc}^{\!\circ}j,i!\overset{\circ}{\bigcirc}^{\!\circ}j, \\ \\ ,i!\overset{\circ}{\bigcirc}^{\!\circ}j,i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,i\overset{\circ}{\bigcirc}^{\!\circ}j,i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j, \\ \\ ,i!\overset{\circ}{\bigcirc}^{\!\circ}j,i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,i\overset{\circ}{\bigcirc}^{\!\circ}j, \\ \\ ,i!\overset{\circ}{\bigcirc}^{\!\circ}j, \end{array}\right],i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j,$$

$$\Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}^{\!\circ}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right],i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j,$$

$$\Leftrightarrow\ ,i\oplus,i!\overset{\circ}{\bigcirc}^{\!\circ}j,$$

$$,i!\overset{\circ}{\bigcirc}^{\!\circ}j,i\ominus,\ \Leftrightarrow\ ,i\ominus,i!\overset{\circ}{\bigcirc}^{\!\circ}j,$$

Identical node comparison:

$$,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,if(j\overset{\circ}{\bigcirc}^{\!\circ}m)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right], \\ ,if(j\overset{\circ}{\bigcirc}^{\!\circ}m)\left[\begin{array}{l} ,©c_3, \\ \\ ,©c_4, \end{array}\right], \end{array}\right],\ \Leftrightarrow\ ,if(j\overset{\circ}{\bigcirc}^{\!\circ}m)\left[\begin{array}{l} ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_3, \end{array}\right], \\ ,if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} ,©c_2, \\ \\ ,©c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\\ ,if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right]-,\end{array}\right]-,\ \Leftrightarrow\ ,if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l},if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right]-,\\ ,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right]-,\end{array}\right]-,$$

$$,i\overset{\circ}{\bigcirc}j,if(j\overset{\circ\circ}{\bigcirc}m)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\ \Leftrightarrow\ ,if(j\overset{\circ\circ}{\bigcirc}m)-\left[\begin{array}{l},i\overset{\circ}{\bigcirc}j,\copyright c_1,\\ \\ ,i\overset{\circ}{\bigcirc}j,\copyright c_2,\end{array}\right]-,$$

$$,i!\overset{\circ}{\bigcirc}j,if(j\overset{\circ\circ}{\bigcirc}m)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\ \Leftrightarrow\ ,if(j\overset{\circ\circ}{\bigcirc}m)-\left[\begin{array}{l},i!\overset{\circ}{\bigcirc}j,\copyright c_1,\\ \\ ,i!\overset{\circ}{\bigcirc}j,\copyright c_2,\end{array}\right]-,$$

$$,i\overset{\circ}{\bigcirc}j,if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\ \Leftrightarrow\ ,if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l},i\overset{\circ}{\bigcirc}j,\copyright c_1,\\ \\ ,i\overset{\circ}{\bigcirc}j,\copyright c_2,\end{array}\right]-,$$

$$,i!\overset{\circ}{\bigcirc}j,if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\ \Leftrightarrow\ ,if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l},i!\overset{\circ}{\bigcirc}j,\copyright c_1,\\ \\ ,i!\overset{\circ}{\bigcirc}j,\copyright c_2,\end{array}\right]-,$$

$$,j\overset{\circ\circ}{\bigcirc}m,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\ \Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},j\overset{\circ\circ}{\bigcirc}m,\copyright c_1,\\ \\ ,j\overset{\circ\circ}{\bigcirc}m,\copyright c_2,\end{array}\right]-,$$

$$,j!\overset{\circ\circ}{\bigcirc}m,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\ \Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},j!\overset{\circ\circ}{\bigcirc}m,\copyright c_1,\\ \\ ,j!\overset{\circ\circ}{\bigcirc}m,\copyright c_2,\end{array}\right]-,$$

$$,i\overset{\circ\circ}{\bigcirc}j,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\ \Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l},i\overset{\circ\circ}{\bigcirc}j,\copyright c_1,\\ \\ ,i\overset{\circ\circ}{\bigcirc}j,\copyright c_2,\end{array}\right]-,$$

$$,i!\mathring{\bigcirc}^{\circ}j, if(i\mathring{\bigcirc}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}j)\left[\begin{array}{c},i!\mathring{\bigcirc}^{\circ}j,\copyright c_1,\\ \\ ,i!\mathring{\bigcirc}^{\circ}j,\copyright c_2,\end{array}\right],$$

$$,i\mathring{\bigcirc}j, i\mathring{\bigcirc}^{\circ}j, \;\Leftrightarrow\; ,i\mathring{\bigcirc}^{\circ}j, i\mathring{\bigcirc}j,$$

$$,i\mathring{\bigcirc}j, i!\mathring{\bigcirc}^{\circ}j, \;\Leftrightarrow\; ,i!\mathring{\bigcirc}^{\circ}j, i\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j, i\mathring{\bigcirc}^{\circ}j, \;\Leftrightarrow\; ,i\mathring{\bigcirc}^{\circ}j, i!\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j, i!\mathring{\bigcirc}^{\circ}j, \;\Leftrightarrow\; ,i!\mathring{\bigcirc}^{\circ}j, i!\mathring{\bigcirc}j,$$

$$,i\mathring{\bigcirc}j, j\mathring{\bigcirc}^{\circ}m, \;\Leftrightarrow\; ,j\mathring{\bigcirc}^{\circ}m, i\mathring{\bigcirc}j,$$

$$,i\mathring{\bigcirc}j, j!\mathring{\bigcirc}^{\circ}m, \;\Leftrightarrow\; ,j!\mathring{\bigcirc}^{\circ}m, i\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j, j\mathring{\bigcirc}^{\circ}m, \;\Leftrightarrow\; ,j\mathring{\bigcirc}^{\circ}m, i!\mathring{\bigcirc}j,$$

$$,i!\mathring{\bigcirc}j, j!\mathring{\bigcirc}^{\circ}m, \;\Leftrightarrow\; ,j!\mathring{\bigcirc}^{\circ}m, i!\mathring{\bigcirc}j,$$

Node value comparison:

$$,if(i=j)\left[\begin{array}{l},if(j\mathring{\bigcirc}^{\circ}m)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(j\mathring{\bigcirc}^{\circ}m)\left[\begin{array}{c},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(j\mathring{\bigcirc}^{\circ}m)\left[\begin{array}{l},if(i=j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i=j)\left[\begin{array}{c},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i=j)\left[\begin{array}{l},if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{c},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(i\mathring{\bigcirc}^{\circ}j)\left[\begin{array}{l},if(i=j)\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i=j)\left[\begin{array}{c},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,i=j,if(j\mathbin{\bigcirc\!\!\!^{\circ}}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(j\mathbin{\bigcirc\!\!\!^{\circ}}m)\left[\begin{array}{l},i=j,\copyright c_1,\\ \\ ,i=j,\copyright c_2,\end{array}\right],$$

$$,i\,!\!=j,if(j\mathbin{\bigcirc\!\!\!^{\circ}}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(j\mathbin{\bigcirc\!\!\!^{\circ}}m)\left[\begin{array}{l},i\,!\!=j,\copyright c_1,\\ \\ ,i\,!\!=j,\copyright c_2,\end{array}\right],$$

$$,i=j,if(i\mathbin{\bigcirc\!\!\!^{\circ}}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathbin{\bigcirc\!\!\!^{\circ}}j)\left[\begin{array}{l},i=j,\copyright c_1,\\ \\ ,i=j,\copyright c_2,\end{array}\right],$$

$$,i\,!\!=j,if(i\mathbin{\bigcirc\!\!\!^{\circ}}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathbin{\bigcirc\!\!\!^{\circ}}j)\left[\begin{array}{l},i\,!\!=j,\copyright c_1,\\ \\ ,i\,!\!=j,\copyright c_2,\end{array}\right],$$

$$,j\mathbin{\bigcirc\!\!\!^{\circ}}m,if(i=j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l},j\mathbin{\bigcirc\!\!\!^{\circ}}m,\copyright c_1,\\ \\ ,j\mathbin{\bigcirc\!\!\!^{\circ}}m,\copyright c_2,\end{array}\right],$$

$$,j!\mathbin{\bigcirc\!\!\!^{\circ}}m,if(i=j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l},j!\mathbin{\bigcirc\!\!\!^{\circ}}m,\copyright c_1,\\ \\ ,j!\mathbin{\bigcirc\!\!\!^{\circ}}m,\copyright c_2,\end{array}\right],$$

$$,i\mathbin{\bigcirc\!\!\!^{\circ}}j,if(i=j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l},i\mathbin{\bigcirc\!\!\!^{\circ}}j,\copyright c_1,\\ \\ ,i\mathbin{\bigcirc\!\!\!^{\circ}}j,\copyright c_2,\end{array}\right],$$

$$,i!\mathbin{\bigcirc\!\!\!^{\circ}}j,if(i=j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l},i!\mathbin{\bigcirc\!\!\!^{\circ}}j,\copyright c_1,\\ \\ ,i!\mathbin{\bigcirc\!\!\!^{\circ}}j,\copyright c_2,\end{array}\right],$$

$$,i=j,i\mathbin{\bigcirc\!\!\!^{\circ}}j, \;\Leftrightarrow\; ,i\mathbin{\bigcirc\!\!\!^{\circ}}j,i=j,$$

$$,i=j,i!\bigcirc\!\!^{\circ}j, \;\Leftrightarrow\; ,i!\bigcirc\!\!^{\circ}j,i=j,$$

$$,i\,!\!=j,i\bigcirc\!\!^{\circ}j, \;\Leftrightarrow\; ,i\bigcirc\!\!^{\circ}j,i\,!\!=j,$$

$$,i\,!\!=j,i!\bigcirc\!\!^{\circ}j, \;\Leftrightarrow\; ,i!\bigcirc\!\!^{\circ}j,i\,!\!=j,$$

$$,i=j,j\bigcirc\!\!^{\circ}m, \;\Leftrightarrow\; ,j\bigcirc\!\!^{\circ}m,i=j,$$

$$,i=j,j!\bigcirc\!\!^{\circ}m, \;\Leftrightarrow\; ,j!\bigcirc\!\!^{\circ}m,i=j,$$

$$,i\,!\!=j,j\bigcirc\!\!^{\circ}m, \;\Leftrightarrow\; ,j\bigcirc\!\!^{\circ}m,i\,!\!=j,$$

$$,i\,!\!=j,j!\bigcirc\!\!^{\circ}m, \;\Leftrightarrow\; ,j!\bigcirc\!\!^{\circ}m,i\,!\!=j,$$

Node null comparison:

$$,if(i=\varnothing)\left[\begin{array}{l} ,if(i\bigcirc\!\!^{\circ}j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(i\bigcirc\!\!^{\circ}j)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(i\bigcirc\!\!^{\circ}j)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_3, \end{array}\right], \\ ,if(i=\varnothing)\left[\begin{array}{l} ,\copyright c_2, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,i=\varnothing,if(i\bigcirc\!\!^{\circ}j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\bigcirc\!\!^{\circ}j)\left[\begin{array}{l} ,i=\varnothing,\copyright c_1, \\ ,i=\varnothing,\copyright c_2, \end{array}\right],$$

$$,i\,!\!=\varnothing,if(i\bigcirc\!\!^{\circ}j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\bigcirc\!\!^{\circ}j)\left[\begin{array}{l} ,i\,!\!=\varnothing,\copyright c_1, \\ ,i\,!\!=\varnothing,\copyright c_2, \end{array}\right],$$

$$, i \circledcirc j, if(i = \varnothing) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \quad \Leftrightarrow \quad , if(i = \varnothing) \left[\begin{array}{l} , i \circledcirc j, \copyright c_1, \\ \\ , i \circledcirc j, \copyright c_2, \end{array} \right],$$

$$, i ! \circledcirc j, if(i = \varnothing) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \quad \Leftrightarrow \quad , if(i = \varnothing) \left[\begin{array}{l} , i ! \circledcirc j, \copyright c_1, \\ \\ , i ! \circledcirc j, \copyright c_2, \end{array} \right],$$

$$, i = \varnothing, i \circledcirc j, \quad \Leftrightarrow \quad , i \circledcirc j, i = \varnothing,$$

$$, i = \varnothing, i ! \circledcirc j, \quad \Leftrightarrow \quad , i ! \circledcirc j, i = \varnothing,$$

$$, i \,!\!= \varnothing, i \circledcirc j, \quad \Leftrightarrow \quad , i \circledcirc j, i \,!\!= \varnothing,$$

$$, i \,!\!= \varnothing, i ! \circledcirc j, \quad \Leftrightarrow \quad , i ! \circledcirc j, i \,!\!= \varnothing,$$

Itself:

$$, if(i \circledcirc j) \left[\begin{array}{l} , if(j \circledcirc m) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \\ , if(j \circledcirc m) \left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_4, \end{array} \right], \end{array} \right], \quad \Leftrightarrow \quad , if(j \circledcirc m) \left[\begin{array}{l} , if(i \circledcirc j) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array} \right], \\ , if(i \circledcirc j) \left[\begin{array}{l} , \copyright c_2, \\ \\ , \copyright c_4, \end{array} \right], \end{array} \right],$$

$$, i \circledcirc j, if(j \circledcirc m) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \quad \Leftrightarrow \quad , if(j \circledcirc m) \left[\begin{array}{l} , i \circledcirc j, \copyright c_1, \\ \\ , i \circledcirc j, \copyright c_2, \end{array} \right],$$

$$, i ! \circledcirc j, if(j \circledcirc m) \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right], \quad \Leftrightarrow \quad , if(j \circledcirc m) \left[\begin{array}{l} , i ! \circledcirc j, \copyright c_1, \\ \\ , i ! \circledcirc j, \copyright c_2, \end{array} \right],$$

$$, i\bigcirc^{\!\circ\circ} j, j\bigcirc^{\!\circ\circ} m, \;\Leftrightarrow\; , j\bigcirc^{\!\circ\circ} m, i\bigcirc^{\!\circ\circ} j,$$

$$, i\bigcirc^{\!\circ\circ} j, j!\bigcirc^{\!\circ\circ} m, \;\Leftrightarrow\; , j!\bigcirc^{\!\circ\circ} m, i\bigcirc^{\!\circ\circ} j,$$

$$, i!\bigcirc^{\!\circ\circ} j, j\bigcirc^{\!\circ\circ} m, \;\Leftrightarrow\; , j\bigcirc^{\!\circ\circ} m, i!\bigcirc^{\!\circ\circ} j,$$

$$, i!\bigcirc^{\!\circ\circ} j, j!\bigcirc^{\!\circ\circ} m, \;\Leftrightarrow\; , j!\bigcirc^{\!\circ\circ} m, i!\bigcirc^{\!\circ\circ} j,$$

flag object:

$$, \ \&SHi\,\mathring{\bigcirc} i, if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \ \&SHi\,\mathring{\bigcirc} i, \copyright c_1, \\ \\ , \ \&SHi\,\mathring{\bigcirc} i, \copyright c_2, \end{array}\right],$$

$$, \ \&SHi \rightarrow i, if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \ \&SHi \rightarrow i, \copyright c_1, \\ \\ , \ \&SHi \rightarrow i, \copyright c_2, \end{array}\right],$$

$$, \ \&SHj\,\mathring{\bigcirc} i, if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \ \&SHj\,\mathring{\bigcirc} i, \copyright c_1, \\ \\ , \ \&SHj\,\mathring{\bigcirc} i, \copyright c_2, \end{array}\right],$$

$$, \ \&SHj \leftarrow i, if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i\bigcirc^{\!\circ\circ} j)\left[\begin{array}{l} , \ \&SHj \leftarrow i, \copyright c_1, \\ \\ , \ \&SHj \leftarrow i, \copyright c_2, \end{array}\right],$$

$$, i\bigcirc^{\!\circ\circ} j, \ \&SHi\,\mathring{\bigcirc} i, \;\Leftrightarrow\; , \ \&SHi\,\mathring{\bigcirc} i, i\bigcirc^{\!\circ\circ} j,$$

$$, i!\bigcirc^{\!\circ\circ} j, \ \&SHi\,\mathring{\bigcirc} i, \;\Leftrightarrow\; , \ \&SHi\,\mathring{\bigcirc} i, i!\bigcirc^{\!\circ\circ} j,$$

$$, i\bigcirc^{\!\circ\circ} j, \ \&SHi \rightarrow i, \;\Leftrightarrow\; , \ \&SHi \rightarrow i, i\bigcirc^{\!\circ\circ} j,$$

$$, i!\overset{\circ}{\bigcirc}\!{\circ} j,\ \&SHi \rightarrow i, \ \Leftrightarrow\ , \&SHi \rightarrow i, i!\overset{\circ}{\bigcirc}\!{\circ} j,$$

$$, i\overset{\circ}{\bigcirc}\!{\circ} j,\ \&SHj \overset{\circ}{\bigcirc} i, \ \Leftrightarrow\ , \&SHj \overset{\circ}{\bigcirc} i, i\overset{\circ}{\bigcirc}\!{\circ} j,$$

$$, i!\overset{\circ}{\bigcirc}\!{\circ} j,\ \&SHj \overset{\circ}{\bigcirc} i, \ \Leftrightarrow\ , \&SHj \overset{\circ}{\bigcirc} i, i!\overset{\circ}{\bigcirc}\!{\circ} j,$$

$$, i\overset{\circ}{\bigcirc}\!{\circ} j,\ \&SHj \leftarrow i, \ \Leftrightarrow\ , \&SHj \leftarrow i, i\overset{\circ}{\bigcirc}\!{\circ} j,$$

$$, i!\overset{\circ}{\bigcirc}\!{\circ} j,\ \&SHj \leftarrow i, \ \Leftrightarrow\ , \&SHj \leftarrow i, i!\overset{\circ}{\bigcirc}\!{\circ} j,$$

16.3.11 Node Connectivity propositions to identical node propositions

$$, i\overset{\circ}{\bigcirc} j, \ \Leftrightarrow\ \sim, i\overset{\circ}{\bigcirc}\!{\circ} j,$$

proof:

$, i\overset{\circ}{\bigcirc} j,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc} j, i\text{☮} i_1, i_1\text{⏚},$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc} j, i\text{☮} i_1, R(i_1), i_1\text{⏚},$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc} j, i\text{☮} i_1, R(i_1), i_1\text{⏚}, j\text{☮} j_1, j_1\text{⏚},$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc} j, i\text{☮} i_1, R(i_1), i_1\text{⏚}, j\text{☮} j_1, R(j_1), j_1\text{⏚},$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc} j, i\text{☮} i_1, j\text{☮} j_1, R(i_1), R(j_1), i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc} j, i\text{☮} i_1, i\overset{\circ}{\bigcirc} i_1, j\text{☮} j_1, R(i_1), R(j_1), i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮} i_1, i\overset{\circ}{\bigcirc} j, i\overset{\circ}{\bigcirc} i_1, j\text{☮} j_1, R(i_1), R(j_1), i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮} i_1, i\overset{\circ}{\bigcirc} j, j\overset{\circ}{\bigcirc} i_1, j\text{☮} j_1, R(i_1), R(j_1), i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮} i_1, i\overset{\circ}{\bigcirc} j, j\overset{\circ}{\bigcirc} i_1, j\text{☮} j_1, j\overset{\circ}{\bigcirc} j_1, R(i_1), R(j_1), i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\circledast i_1, i\male j, j\circledast j_1, j\male i_1, j\male j_1, R(i_1), R(j_1), i_1\circleddash, j_1\circleddash,$

$\Leftrightarrow\ , i\circledast i_1, i\male j, j\circledast j_1, j\male i_1, i_1\male j_1, R(i_1), R(j_1), i_1\circleddash, j_1\circleddash,$

$\Leftrightarrow\ , i\circledast i_1, i\male j, j\circledast j_1, j\male i_1, i_1\male j_1, R(i_1), R(j_1), i_1\male j_1, i_1\circleddash, j_1\circleddash,$

$\Leftrightarrow\ , i\male j, i\circledast i_1, j\circledast j_1, R(i_1), R(j_1), i_1\male j_1, i_1\circleddash, j_1\circleddash,$

$\Leftrightarrow\ , i\male j, i\male^{\circ} j,$

$$, i!\male^{\circ} j, \ \Leftrightarrow\ \sim, i!\male j,$$

proof:

$, i!\male^{\circ} j,$

$\Leftrightarrow\ , i!\male^{\circ} j, if(i\male j)\left[\begin{array}{l}, \\ , \end{array}\right],$

$\Leftrightarrow\ , i!\male^{\circ} j, if(i\male j)\left[\begin{array}{l}, i\male j, \\ , \end{array}\right],$

$\Leftrightarrow\ , i!\male^{\circ} j, if(i\male j)\left[\begin{array}{l}, i\male j, i\male^{\circ} j, \\ , \end{array}\right],$

$\Leftrightarrow\ , if(i\male j)\left[\begin{array}{l}, i!\male^{\circ} j, i\male j, i\male^{\circ} j, \\ , i!\male^{\circ} j, \end{array}\right],$

$\Leftrightarrow\ , if(i\male j)\left[\begin{array}{l}, i!\male^{\circ} j, i\male^{\circ} j, i\male j, \\ , i!\male^{\circ} j, \end{array}\right],$

$\Leftrightarrow\ , if(i\male j)\left[\begin{array}{l}, \otimes, i\male j, \\ , i!\male^{\circ} j, \end{array}\right],$

$$\Leftrightarrow\ ,if(i\bigcirc^{\circ}j)\left[\begin{array}{l} ,\otimes, \\ \\ ,i!\bigcirc^{\circ\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\bigcirc^{\circ}j)\left[\begin{array}{l} ,i!\bigcirc^{\circ\circ}j,\otimes, \\ \\ ,i!\bigcirc^{\circ\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,i!\bigcirc^{\circ\circ}j,if(i\bigcirc^{\circ}j)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right],$$

$$\Leftrightarrow\ ,i!\bigcirc^{\circ\circ}j,i!\bigcirc^{\circ}j,$$

16.3.12 Node null proposition

$$,i=\varnothing,j=\varnothing,if(i\bigcirc^{\circ}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ ,i=\varnothing,j=\varnothing,if(i\bigcirc^{\circ\circ}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

proof:

$$,i=\varnothing,j=\varnothing,if(i\bigcirc^{\circ\circ}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

$$\Leftrightarrow\ ,i=\varnothing,j=\varnothing,i\text{☮}i_1,j\text{☮}j_1,R(i_1),R(j_1),if(i_1\bigcirc^{\circ}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚}, \\ \\ ,i_1\text{⏚},j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\ ,j=\varnothing,i\text{☮}i_1,i=\varnothing,j\text{☮}j_1,R(i_1),R(j_1),if(i_1\bigcirc^{\circ}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚}, \\ \\ ,i_1\text{⏚},j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\ ,j=\varnothing,i\text{☮}i_1,i\bigcirc^{\circ}i_1,i=\varnothing,j\text{☮}j_1,R(i_1),R(j_1),if(i_1\bigcirc^{\circ}j_1)\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚}, \\ \\ ,i_1\text{⏚},j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\ ,j=\varnothing,i\circledast i_1,i\bigcirc^{\circ} i_1,i_1=\varnothing,j\circledast j_1,R(i_1),R(j_1),if(i_1\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,j=\varnothing,i\circledast i_1,i\bigcirc^{\circ} i_1,i_1=\varnothing,R(i_1),j\circledast j_1,R(j_1),if(i_1\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,j=\varnothing,i\circledast i_1,i\bigcirc^{\circ} i_1,i_1=\varnothing,j\circledast j_1,R(j_1),if(i_1\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,j=\varnothing,i\circledast i_1,i\bigcirc^{\circ} i_1,i=\varnothing,j\circledast j_1,R(j_1),if(i_1\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,i=\varnothing,j=\varnothing,i\circledast i_1,j\circledast j_1,R(j_1),i\bigcirc^{\circ} i_1,if(i_1\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,i=\varnothing,j=\varnothing,i\circledast i_1,j\circledast j_1,R(j_1),i\bigcirc^{\circ} i_1,if(i\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,i=\varnothing,j=\varnothing,i\circledast i_1,i\bigcirc^{\circ} i_1,j\circledast j_1,R(j_1),if(i\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,i=\varnothing,j=\varnothing,i\circledast i_1,j\circledast j_1,R(j_1),if(i\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow\ ,i=\varnothing,i\circledast i_1,j\circledast j_1,j=\varnothing,R(j_1),if(i\bigcirc^{\circ} j_1)\begin{cases},i_1\obot,j_1\obot,\\ ,i_1\obot,j_1\obot,\end{cases}$$

$$\Leftrightarrow \ , i = \varnothing, i \circledwedge i_1, j \circledwedge j_1, j \overset{\circ}{\bigcirc} j_1, j = \varnothing, R(j_1), if(i \overset{\circ}{\bigcirc} j_1) - \left[\begin{array}{l} , i_1 \circledvee, j_1 \circledvee, \\ \\ , i_1 \circledvee, j_1 \circledvee, \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, i \circledwedge i_1, j \circledwedge j_1, j \overset{\circ}{\bigcirc} j_1, j_1 = \varnothing, R(j_1), if(i \overset{\circ}{\bigcirc} j_1) - \left[\begin{array}{l} , i_1 \circledvee, j_1 \circledvee, \\ \\ , i_1 \circledvee, j_1 \circledvee, \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, i \circledwedge i_1, j \circledwedge j_1, j \overset{\circ}{\bigcirc} j_1, j_1 = \varnothing, if(i \overset{\circ}{\bigcirc} j_1) - \left[\begin{array}{l} , i_1 \circledvee, j_1 \circledvee, \\ \\ , i_1 \circledvee, j_1 \circledvee, \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, i \circledwedge i_1, j \circledwedge j_1, j \overset{\circ}{\bigcirc} j_1, j = \varnothing, if(i \overset{\circ}{\bigcirc} j_1) - \left[\begin{array}{l} , i_1 \circledvee, j_1 \circledvee, \\ \\ , i_1 \circledvee, j_1 \circledvee, \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, j = \varnothing, i \circledwedge i_1, j \circledwedge j_1, j \overset{\circ}{\bigcirc} j_1, if(i \overset{\circ}{\bigcirc} j_1) - \left[\begin{array}{l} , i_1 \circledvee, j_1 \circledvee, \\ \\ , i_1 \circledvee, j_1 \circledvee, \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, j = \varnothing, i \circledwedge i_1, j \circledwedge j_1, j \overset{\circ}{\bigcirc} j_1, if(i \overset{\circ}{\bigcirc} j) - \left[\begin{array}{l} , i_1 \circledvee, j_1 \circledvee, \\ \\ , i_1 \circledvee, j_1 \circledvee, \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, j = \varnothing, i \circledwedge i_1, j \circledwedge j_1, if(i \overset{\circ}{\bigcirc} j) - \left[\begin{array}{l} , i_1 \circledvee, j_1 \circledvee, \\ \\ , i_1 \circledvee, j_1 \circledvee, \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, j = \varnothing, i \circledwedge i_1, i_1 \circledvee, j \circledwedge j_1, j_1 \circledvee, if(i \overset{\circ}{\bigcirc} j) - \left[\begin{array}{l} , \\ \\ , \end{array} \right.$$

$$\Leftrightarrow \ , i = \varnothing, j = \varnothing, if(i \overset{\circ}{\bigcirc} j) - \left[\begin{array}{l} , \\ \\ , \end{array} \right.$$

$$, i = \varnothing, j = \varnothing, i \overset{\circ}{\bigcirc} j, \;\; \Leftrightarrow \;\; , i = \varnothing, j = \varnothing, i \bigcirc^{\circ} j,$$

$$, i = \varnothing, j = \varnothing, i ! \overset{\circ}{\bigcirc} j, \;\; \Leftrightarrow \;\; , i = \varnothing, j = \varnothing, i ! \bigcirc^{\circ} j,$$

16.3.13 Temporary space operator

$$, \circledcirc i, \;\; \Leftrightarrow \;\; \sim, i ! \bigcirc^{\circ} j,$$

proof:

$, \circledcirc i,$
$\Leftrightarrow \;, j \text{☮} j_1, j_1 \text{\textcircled{$\downarrow$}}, \circledcirc i,$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), j_1 \text{\textcircled{$\downarrow$}}, \circledcirc i,$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i ! \overset{\circ}{\bigcirc} j_1, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i = \varnothing, i ! \overset{\circ}{\bigcirc} j_1, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i = \varnothing, i \text{☮} i_1, i_1 \text{\textcircled{$\downarrow$}}, i ! \overset{\circ}{\bigcirc} j_1, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i = \varnothing, i \text{☮} i_1, i ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i = \varnothing, i \text{☮} i_1, i \overset{\circ}{\bigcirc} i_1, i ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i = \varnothing, i \text{☮} i_1, i \overset{\circ}{\bigcirc} i_1, i_1 ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i \text{☮} i_1, i \overset{\circ}{\bigcirc} i_1, i = \varnothing, i_1 ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i \text{☮} i_1, i \overset{\circ}{\bigcirc} i_1, i_1 = \varnothing, i_1 ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i \text{☮} i_1, i \overset{\circ}{\bigcirc} i_1, i_1 = \varnothing, R(i_1), i_1 ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, j \text{☮} j_1, R(j_1), \circledcirc i, i \text{☮} i_1, R(i_1), i_1 ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc i, i \text{☮} i_1, j \text{☮} j_1, R(i_1), R(j_1), i_1 ! \overset{\circ}{\bigcirc} j_1, i_1 \text{\textcircled{$\downarrow$}}, j_1 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc i, i ! \bigcirc^{\circ} j,$

16.3.14 Node id operator

$$, i\text{☮}m, \;\Leftrightarrow\; \sim, m!\overset{\circ}{\bigcirc}j,$$

16.3.15 Transformation of definition

$$, if(i\overset{\circ}{\bigcirc}\!\circ j)-\!\begin{cases} , \\ , \end{cases} \Leftrightarrow\; , i\text{☮}i_1, j\text{☮}j_1, R(i_1), R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc}j_1)-\!\begin{cases} , i_1\text{⏚}, j_1\text{⏚}, \\ , i_1\text{⏚}, j_1\text{⏚}, \end{cases}$$

proof:

$$, if(i\overset{\circ}{\bigcirc}\!\circ j)-\!\begin{cases} , \\ , \end{cases}$$

$$\Leftrightarrow\; , i\text{☮}i_1, j\text{☮}j_2, R(i_1), R(j_2), if(i_1\overset{\circ}{\bigcirc}j_2)-\!\begin{cases} , i_1\text{⏚}, j_2\text{⏚}, \\ , i_1\text{⏚}, j_2\text{⏚}, \end{cases}$$

$$\Leftrightarrow\; , i\text{☮}i_1, j\text{☮}j_2, j\text{☮}j_1, j_1\text{⏚}, R(i_1), R(j_2), if(i_1\overset{\circ}{\bigcirc}j_2)-\!\begin{cases} , i_1\text{⏚}, j_2\text{⏚}, \\ , i_1\text{⏚}, j_2\text{⏚}, \end{cases}$$

$$\Leftrightarrow\; , i\text{☮}i_1, j\text{☮}j_2, j\text{☮}j_1, R_{-}(j_1), j_1\text{⏚}, R(i_1), R(j_2), if(i_1\overset{\circ}{\bigcirc}j_2)-\!\begin{cases} , i_1\text{⏚}, j_2\text{⏚}, \\ , i_1\text{⏚}, j_2\text{⏚}, \end{cases}$$

$$\Leftrightarrow\; , i\text{☮}i_1, R(i_1), j\text{☮}j_2, j\text{☮}j_1, R(j_2), R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc}j_2)-\!\begin{cases} , i_1\text{⏚}, j_1\text{⏚}, j_2\text{⏚}, \\ , i_1\text{⏚}, j_1\text{⏚}, j_2\text{⏚}, \end{cases}$$

$$\Leftrightarrow\; , i\text{☮}i_1, R(i_1), j\text{☮}j_2, j\text{☮}j_1, j_2\overset{\circ}{\bigcirc}j_1, R(j_2), R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc}j_2)-\!\begin{cases} , i_1\text{⏚}, j_1\text{⏚}, j_2\text{⏚}, \\ , i_1\text{⏚}, j_1\text{⏚}, j_2\text{⏚}, \end{cases}$$

$$\Leftrightarrow\ , i\oslash i_1, R(i_1), j\oslash j_2, j\oslash j_1, j_2\overset{\circ}{\bigcirc} j_1, R(j_2), R_{-}(j_1), j_2\overset{\circ}{\bigcirc} j_1, if(i_1\overset{\circ}{\bigcirc} j_2)\begin{cases}, i_1\circleddash, j_1\circleddash, j_2\circleddash, \\ , i_1\circleddash, j_1\circleddash, j_2\circleddash,\end{cases}$$

$$\Leftrightarrow\ , i\oslash i_1, R(i_1), j\oslash j_2, j\oslash j_1, j_2\overset{\circ}{\bigcirc} j_1, R(j_2), R_{-}(j_1), j_2\overset{\circ}{\bigcirc} j_1, if(i_1\overset{\circ}{\bigcirc} j_1)\begin{cases}, i_1\circleddash, j_1\circleddash, j_2\circleddash, \\ , i_1\circleddash, j_1\circleddash, j_2\circleddash,\end{cases}$$

$$\Leftrightarrow\ , i\oslash i_1, R(i_1), j\oslash j_2, j\oslash j_1, j_2\overset{\circ}{\bigcirc} j_1, R(j_2), R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc} j_1)\begin{cases}, i_1\circleddash, j_1\circleddash, j_2\circleddash, \\ , i_1\circleddash, j_1\circleddash, j_2\circleddash,\end{cases}$$

$$\Leftrightarrow\ , i\oslash i_1, R(i_1), j\oslash j_2, j\oslash j_1, R(j_2), R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc} j_1)\begin{cases}, i_1\circleddash, j_1\circleddash, j_2\circleddash, \\ , i_1\circleddash, j_1\circleddash, j_2\circleddash,\end{cases}$$

$$\Leftrightarrow\ , i\oslash i_1, R(i_1), j\oslash j_2, R(j_2), j_2\circleddash, j\oslash j_1, R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc} j_1)\begin{cases}, i_1\circleddash, j_1\circleddash, \\ , i_1\circleddash, j_1\circleddash,\end{cases}$$

$$\Leftrightarrow\ , i\oslash i_1, R(i_1), j\oslash j_1, R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc} j_1)\begin{cases}, i_1\circleddash, j_1\circleddash, \\ , i_1\circleddash, j_1\circleddash,\end{cases}$$

$$\Leftrightarrow\ , i\oslash i_1, j\oslash j_1, R(i_1), R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc} j_1)\begin{cases}, i_1\circleddash, j_1\circleddash, \\ , i_1\circleddash, j_1\circleddash,\end{cases}$$

$$, if(i\overset{\circ}{\bigcirc}\!\!\circ j)\begin{cases}, \\ ,\end{cases} \Leftrightarrow\ , i\oslash i_1, j\oslash j_1, R_{-}(i_1), R_{-}(j_1), if(i_1\overset{\circ}{\bigcirc} j_1)\begin{cases}, i_1\circleddash, j_1\circleddash, \\ , i_1\circleddash, j_1\circleddash,\end{cases}$$

$$, i\overset{\circ}{\bigcirc}\!\!\circ j, i\oslash n, \ \Leftrightarrow\ , i\oslash n, n\overset{\circ}{\bigcirc}\!\!\circ j,$$

17 Rules of Relationship of Node Continuity

17.1 Definition of Node Continuity

$$,if(i{\to}j)\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \;\Leftrightarrow\; ,i\circledast i_1, i_1\oplus, if(i_1\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l} ,i_1\circledcirc, \\ \\ ,i_1\circledcirc, \end{array}\right.$$

$$,i{\to}j, \;\Leftrightarrow\; ,if(i{\to}j)\!-\!\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right]\!-,$$

$$,i!{\to}j, \;\Leftrightarrow\; ,if(i{\to}j)\!-\!\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right]\!-,$$

$$,i{\to}i, \;\Leftrightarrow\; ,i\circledast i_0, i_0{\to}i, i_0\circledcirc,$$

$$,i!{\to}i, \;\Leftrightarrow\; ,i\circledast i_0, i_0!{\to}i, i_0\circledcirc,$$

$$,if(i{\leftarrow}j)\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right. \;\Leftrightarrow\; ,i\circledast i_1, i_1\ominus, if(i_1\mathring{\bigcirc}j)\!-\!\left[\begin{array}{l} ,i_1\circledcirc, \\ \\ ,i_1\circledcirc, \end{array}\right.$$

$$,i{\leftarrow}j, \;\Leftrightarrow\; ,if(i{\leftarrow}j)\!-\!\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right]\!-,$$

$$,i!{\leftarrow}j, \;\Leftrightarrow\; ,if(i{\leftarrow}j)\text{-}\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right]\text{-},$$

17.2 Theorems of Relationship of Node Continuity

17.2.1 Next node to previous node

$$,if(i{\rightarrow}j)\text{-}\left[\begin{array}{l} , \\ \\ , \end{array}\right. \;\Leftrightarrow\; ,if(j{\leftarrow}i)\text{-}\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

proof:

$$,if(i{\rightarrow}j)\text{-}\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

$$\Leftrightarrow\; ,i\text{☮}i_1,i_1\oplus,if(i_1\bigcirc^{\!\circ}j)\text{-}\left[\begin{array}{l} ,i_1\text{⏚}, \\ \\ ,i_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\; ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,j_1\text{⏚},if(i_1\bigcirc^{\!\circ}j)\text{-}\left[\begin{array}{l} ,i_1\text{⏚}, \\ \\ ,i_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\; ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,if(i_1\bigcirc^{\!\circ}j)\text{-}\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚}, \\ \\ ,i_1\text{⏚},j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\; ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,j\bigcirc^{\!\circ}j_1,if(i_1\bigcirc^{\!\circ}j)\text{-}\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚}, \\ \\ ,i_1\text{⏚},j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\; ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,j\bigcirc^{\!\circ}j_1,if(i_1\bigcirc^{\!\circ}j_1)\text{-}\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚}, \\ \\ ,i_1\text{⏚},j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\; ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,if(i_1\bigcirc^{\!\circ}j_1)\text{-}\left[\begin{array}{l} ,i_1\text{⏚},j_1\text{⏚}, \\ \\ ,i_1\text{⏚},j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,if(i_1\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\ominus,i_1\text{⦽},j_1\ominus,j_1\text{⦽},\\ \\,i_1\ominus,i_1\text{⦽},j_1\ominus,j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,if(i_1\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\ominus,j_1\ominus,i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\ominus,j_1\ominus,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,j\text{☮}j_1,i_1\ominus,j_1\ominus,if(i_1\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\ominus,j\text{☮}j_1,j_1\ominus,if(i_1\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_1,j\text{☮}j_1,j_1\ominus,if(i_1\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_1,j_1\ominus,i\text{☮}i_1,if(i_1\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_1,j_1\ominus,i\text{☮}i_1,i\mathring{\bigcirc}i_1,if(i_1\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_1,j_1\ominus,i\text{☮}i_1,i\mathring{\bigcirc}i_1,if(i\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_1,j_1\ominus,i\text{☮}i_1,if(i\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i_1\text{⦽},j_1\text{⦽},\\ \\,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_1,j_1\ominus,if(i\mathring{\bigcirc}j_1)\left\{\begin{array}{l},i\text{☮}i_1,i_1\text{⦽},j_1\text{⦽},\\ \\,i\text{☮}i_1,i_1\text{⦽},j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_1,j_1\ominus,if(i\mathring{\bigcirc}j_1)\left\{\begin{array}{l},j_1\text{⦽},\\ \\,j_1\text{⦽},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_1, j_1\ominus, if(j_1\text{♂}i) \left[\begin{array}{l} ,j_1\text{⏚}, \\ \\ ,j_1\text{⏚}, \end{array}\right.$$

$$\Leftrightarrow\ ,if(j\leftarrow i) \left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

$$,i\rightarrow j,\ \Leftrightarrow\ ,j\leftarrow i,$$

$$,i!\rightarrow j,\ \Leftrightarrow\ ,j!\leftarrow i,$$

17.2.2 Next node propositions to Identical node comparison propositions

$$,i\rightarrow j,\ \Leftrightarrow\ ,i\text{☮}i_1, i_1\oplus, i_1\text{♂}j, i_1\text{⏚},$$

$$,i!\rightarrow j,\ \Leftrightarrow\ ,i\text{☮}i_1, i_1\oplus, i_1!\text{♂}j, i_1\text{⏚},$$

17.2.3 Branch function to propositions

$$,if(i\rightarrow j) \left[\begin{array}{l} ,\ \copyright c, \\ \\ ,\otimes, \end{array}\right],\ \Leftrightarrow\ ,i\rightarrow j, \copyright c,$$

$$,if(i\rightarrow j) \left[\begin{array}{l} ,\otimes, \\ \\ ,\ \copyright c, \end{array}\right],\ \Leftrightarrow\ ,i!\rightarrow j, \copyright c,$$

17.2.4 Empty branch function

$$,if(i{\to}j)-\left[\begin{array}{l}, \\ \\ ,\end{array}\right. \Leftrightarrow\ ,-\left[\begin{array}{l},i{\to}j, \\ \\ ,i!{\to}j,\end{array}\right.$$

17.2.5 Unity

$$,\ \Leftrightarrow\ ,if(i{\to}j)-\left[\begin{array}{c}, \\ ,\end{array}\right]-,$$

proof:

$$,if(i{\to}j)-\left[\begin{array}{c}, \\ \\ ,\end{array}\right]-,$$

$$\Leftrightarrow\ ,i\owedge i_1,i_1\oplus,if(i_1\mathbin{\text{♂}}j)-\left[\begin{array}{c},i_1\text{⏚}, \\ \\ ,i_1\text{⏚},\end{array}\right]-,$$

$$\Leftrightarrow\ ,i\owedge i_1,i_1\oplus,if(i_1\mathbin{\text{♂}}j)-\left[\begin{array}{c}, \\ \\ ,\end{array}\right]-,i_1\text{⏚},$$

$$\Leftrightarrow\ ,i\owedge i_1,i_1\oplus,i_1\text{⏚},$$

$$\Leftrightarrow\ ,i\owedge i_1,i_1\text{⏚},$$

$$\Leftrightarrow\ ,$$

$$,i{\to}j,\otimes,\ \Leftrightarrow\ ,\otimes,$$

$$,i!{\to}j,\otimes,\ \Leftrightarrow\ ,\otimes,$$

17.2.6 Swap

Branch function and operator:

$$,\circledcirc m, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , \circledcirc m, \\ \\ , \circledcirc m, \end{array}\right.$$

proof:

$$,\circledcirc m, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

$$\Leftrightarrow \ ,\circledcirc m, i \varowedge i_1, i_1 \oplus, if(i_1 \mathring{\bigcirc} j) \left[\begin{array}{l} , i_1 \ovee, \\ \\ , i_1 \ovee, \end{array}\right.$$

$$\Leftrightarrow \ , i \varowedge i_1, i_1 \oplus, if(i_1 \mathring{\bigcirc} j) \left[\begin{array}{l} , i_1 \ovee, \circledcirc m, \\ \\ , i_1 \ovee, \circledcirc m, \end{array}\right.$$

$$\Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , \circledcirc m, \\ \\ , \circledcirc m, \end{array}\right.$$

$$,\odot m, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , \odot m, \\ \\ , \odot m, \end{array}\right.$$

$$, m \varowedge n, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , m \varowedge n, \\ \\ , m \varowedge n, \end{array}\right.$$

$$, m \varocircle n, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , m \varocircle n, \\ \\ , m \varocircle n, \end{array}\right.$$

$$, m \varobslash n, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , m \varobslash n, \\ \\ , m \varobslash n, \end{array}\right.$$

$$, m \ovee, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , m \ovee, \\ \\ , m \ovee, \end{array}\right.$$

$$, m \oplus, if(i \to j) \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i \to j) \left[\begin{array}{l} , m \oplus, \\ \\ , m \oplus, \end{array}\right.$$

$$, m\ominus, if(i\to j)\begin{cases} , \\ , \end{cases} \Leftrightarrow\ , if(i\to j)\begin{cases} , m\ominus, \\ , m\ominus, \end{cases}$$

Branch function and Branch function:

$$, if(i\to j)\left[\begin{array}{l} , if(m\to n)\left[\begin{array}{l} , ©c_1, \\ , ©c_2, \end{array}\right], \\ , if(m\to n)\left[\begin{array}{l} , ©c_3, \\ , ©c_4, \end{array}\right], \end{array}\right], \Leftrightarrow\ , if(m\to n)\left[\begin{array}{l} , if(i\to j)\left[\begin{array}{l} , ©c_1, \\ , ©c_3, \end{array}\right], \\ , if(i\to j)\left[\begin{array}{l} , ©c_2, \\ , ©c_4, \end{array}\right], \end{array}\right],$$

proof:

$$, if(i\to j)\left[\begin{array}{l} , if(m\to n)\left[\begin{array}{l} , ©c_1, \\ , ©c_2, \end{array}\right], \\ , if(m\to n)\left[\begin{array}{l} , ©c_3, \\ , ©c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i_1\oplus, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} , i_1\textcircled{$\downarrow$}, if(m\to n)\left[\begin{array}{l} , ©c_1, \\ , ©c_2, \end{array}\right], \\ , i_1\textcircled{$\downarrow$}, if(m\to n)\left[\begin{array}{l} , ©c_3, \\ , ©c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i_1\oplus, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} , i_1\textcircled{$\downarrow$}, m\text{☮}m_1, m_1\oplus, if(m_1\mathring{\bigcirc}n)\left[\begin{array}{l} , m_1\textcircled{$\downarrow$}, ©c_1, \\ , m_1\textcircled{$\downarrow$}, ©c_2, \end{array}\right], \\ , i_1\textcircled{$\downarrow$}, m\text{☮}m_1, m_1\oplus, if(m_1\mathring{\bigcirc}n)\left[\begin{array}{l} , m_1\textcircled{$\downarrow$}, ©c_3, \\ , m_1\textcircled{$\downarrow$}, ©c_4, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , m\text{☮}m_1, m_1\oplus, i\text{☮}i_1, i_1\oplus,$$

$$if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} , if(m_1\mathring{\bigcirc}n)\left[\begin{array}{l}, m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_1,\\ \\ , m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_2,\end{array}\right], \\ , if(m_1\mathring{\bigcirc}n)\left[\begin{array}{l}, m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_3,\\ \\ , m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_4,\end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , m\oslash m_1, m_1\oplus, i\oslash i_1, i_1\oplus,$$

$$if(m_1\mathring{\bigcirc}n)\left[\begin{array}{l} , if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l}, m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_1,\\ \\ , m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_3,\end{array}\right], \\ , if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l}, m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_2,\\ \\ , m_1\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, \copyright c_4,\end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , m\oslash m_1, m_1\oplus, if(m_1\mathring{\bigcirc}n)\left[\begin{array}{l} , m_1\textcircled{$\downarrow$}, i\oslash i_1, i_1\oplus, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l}, i_1\textcircled{$\downarrow$}, \copyright c_1,\\ \\ , i_1\textcircled{$\downarrow$}, \copyright c_3,\end{array}\right], \\ , m_1\textcircled{$\downarrow$}, i\oslash i_1, i_1\oplus, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l}, i_1\textcircled{$\downarrow$}, \copyright c_2,\\ \\ , i_1\textcircled{$\downarrow$}, \copyright c_4,\end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , if(m\to n)\left[\begin{array}{l} , if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_3,\end{array}\right], \\ , if(i\to j)\left[\begin{array}{l}, \copyright c_2,\\ \\ , \copyright c_4,\end{array}\right], \end{array}\right],$$

$$, if(i\to j)\left[\begin{array}{l} , if(m\mathring{\bigcirc}\!\circ n)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \\ , if(m\mathring{\bigcirc}\!\circ n)\left[\begin{array}{l}, \copyright c_3,\\ \\ , \copyright c_4,\end{array}\right], \end{array}\right],\ \Leftrightarrow\ , if(m\mathring{\bigcirc}\!\circ n)\left[\begin{array}{l} , if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_3,\end{array}\right], \\ , if(i\to j)\left[\begin{array}{l}, \copyright c_2,\\ \\ , \copyright c_4,\end{array}\right], \end{array}\right],$$

$$,if(i{\rightarrow}j)\left[\begin{array}{l} ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i{\rightarrow}j)\left[\begin{array}{l} ,if(m{=}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m{=}n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(m{=}n)\left[\begin{array}{l} ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i{\rightarrow}j)\left[\begin{array}{l} ,if(m{=}\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m{=}\varnothing)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(m{=}\varnothing)\left[\begin{array}{l} ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

Branch function and propositions:

$$,m{\rightarrow}n,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{\rightarrow}j)\left[\begin{array}{l} ,m{\rightarrow}n,\copyright c_1, \\ \\ ,m{\rightarrow}n,\copyright c_2, \end{array}\right],$$

$$,m!{\rightarrow}n,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{\rightarrow}j)\left[\begin{array}{l} ,m!{\rightarrow}n,\copyright c_1, \\ \\ ,m!{\rightarrow}n,\copyright c_2, \end{array}\right],$$

$$, m \overset{\circ\circ}{\bigcirc} n, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m \overset{\circ\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m \overset{\circ\circ}{\bigcirc} n, \copyright c_2, \end{bmatrix},$$

$$, m ! \overset{\circ\circ}{\bigcirc} n, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m ! \overset{\circ\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m ! \overset{\circ\circ}{\bigcirc} n, \copyright c_2, \end{bmatrix},$$

$$, m \overset{\circ}{\bigcirc} n, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m \overset{\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m \overset{\circ}{\bigcirc} n, \copyright c_2, \end{bmatrix},$$

$$, m ! \overset{\circ}{\bigcirc} n, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m ! \overset{\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m ! \overset{\circ}{\bigcirc} n, \copyright c_2, \end{bmatrix},$$

$$, m = n, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m = n, \copyright c_1, \\ \\ , m = n, \copyright c_2, \end{bmatrix},$$

$$, m \mathrel{!=} n, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m \mathrel{!=} n, \copyright c_1, \\ \\ , m \mathrel{!=} n, \copyright c_2, \end{bmatrix},$$

$$, m = \varnothing, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m = \varnothing, \copyright c_1, \\ \\ , m = \varnothing, \copyright c_2, \end{bmatrix},$$

$$, m \mathrel{!=} \varnothing, if(i \to j) \begin{bmatrix} , \copyright c_1, \\ \\ , \copyright c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i \to j) \begin{bmatrix} , m \mathrel{!=} \varnothing, \copyright c_1, \\ \\ , m \mathrel{!=} \varnothing, \copyright c_2, \end{bmatrix},$$

$$,m{\to}n, if(i\bigcirc^{\circ\circ}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-, \quad\Leftrightarrow\quad ,if(i\bigcirc^{\circ\circ}j)-\left[\begin{array}{l},m{\to}n,\copyright c_1,\\ \\ ,m{\to}n,\copyright c_2,\end{array}\right]-,$$

$$,m!{\to}n, if(i\bigcirc^{\circ\circ}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-, \quad\Leftrightarrow\quad ,if(i\bigcirc^{\circ\circ}j)-\left[\begin{array}{l},m!{\to}n,\copyright c_1,\\ \\ ,m!{\to}n,\copyright c_2,\end{array}\right]-,$$

$$,m{\to}n, if(i\bigcirc^{\circ}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-, \quad\Leftrightarrow\quad ,if(i\bigcirc^{\circ}j)-\left[\begin{array}{l},m{\to}n,\copyright c_1,\\ \\ ,m{\to}n,\copyright c_2,\end{array}\right]-,$$

$$,m!{\to}n, if(i\bigcirc^{\circ}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-, \quad\Leftrightarrow\quad ,if(i\bigcirc^{\circ}j)-\left[\begin{array}{l},m!{\to}n,\copyright c_1,\\ \\ ,m!{\to}n,\copyright c_2,\end{array}\right]-,$$

$$,m{\to}n, if(i{=}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-, \quad\Leftrightarrow\quad ,if(i{=}j)-\left[\begin{array}{l},m{\to}n,\copyright c_1,\\ \\ ,m{\to}n,\copyright c_2,\end{array}\right]-,$$

$$,m!{\to}n, if(i{=}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-, \quad\Leftrightarrow\quad ,if(i{=}j)-\left[\begin{array}{l},m!{\to}n,\copyright c_1,\\ \\ ,m!{\to}n,\copyright c_2,\end{array}\right]-,$$

$$,m{\to}n, if(i{=}\varnothing)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-, \quad\Leftrightarrow\quad ,if(i{=}\varnothing)-\left[\begin{array}{l},m{\to}n,\copyright c_1,\\ \\ ,m{\to}n,\copyright c_2,\end{array}\right]-,$$

$$, m!\to n, if(i=\varnothing)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i=\varnothing)\left[\begin{array}{l}, m!\to n, \copyright c_1,\\ \\ , m!\to n, \copyright c_2,\end{array}\right],$$

Branch function and recursive function:

$$, R(m), if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\to j)\left[\begin{array}{l}, R(m), \copyright c_1,\\ \\ , R(m), \copyright c_2,\end{array}\right],$$

$$, R_{-}(m), if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\to j)\left[\begin{array}{l}, R_{-}(m), \copyright c_1,\\ \\ , R_{-}(m), \copyright c_2,\end{array}\right],$$

Branch function and flag object :

$$, \&SHi\,\text{♂}m, if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\to j)\left[\begin{array}{l}, \&SHi\,\text{♂}m, \copyright c_1,\\ \\ , \&SHi\,\text{♂}m, \copyright c_2,\end{array}\right],$$

$$, \&SHi\to m, if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\to j)\left[\begin{array}{l}, \&SHi\to m, \copyright c_1,\\ \\ , \&SHi\to m, \copyright c_2,\end{array}\right],$$

$$, \&SHj\,\text{♂}m, if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(i\to j)\left[\begin{array}{l}, \&SHj\,\text{♂}m, \copyright c_1,\\ \\ , \&SHj\,\text{♂}m, \copyright c_2,\end{array}\right],$$

$$, \&SHj \leftarrow m, if(i \rightarrow j) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \;\Leftrightarrow\; , if(i \rightarrow j) \left[\begin{array}{l} , \&SHj \leftarrow m, ©c_1, \\ \\ , \&SHj \leftarrow m, ©c_2, \end{array} \right],$$

Propositions and operator:

$$, i \rightarrow j, \circledcirc m, \;\Leftrightarrow\; , \circledcirc m, i \rightarrow j,$$
$$, i \rightarrow j, \odot m, \;\Leftrightarrow\; , \odot m, i \rightarrow j,$$

$$, i \rightarrow j, m \text{☮} n, \;\Leftrightarrow\; , m \text{☮} n, i \rightarrow j,$$
$$, i \rightarrow j, m \text{☮} n, \;\Leftrightarrow\; , m \text{☮} n, i \rightarrow j,$$
$$, i \rightarrow j, m \text{☮} n, \;\Leftrightarrow\; , m \text{☮} n, i \rightarrow j,$$

$$, i \rightarrow j, m \text{⦹}, \;\Leftrightarrow\; , m \text{⦹}, i \rightarrow j,$$
$$, i \rightarrow j, m \oplus, \;\Leftrightarrow\; , m \oplus, i \rightarrow j,$$
$$, i \rightarrow j, m \ominus, \;\Leftrightarrow\; , m \ominus, i \rightarrow j,$$

$$, i! \rightarrow j, \circledcirc m, \;\Leftrightarrow\; , \circledcirc m, i! \rightarrow j,$$
$$, i! \rightarrow j, \odot m, \;\Leftrightarrow\; , \odot m, i! \rightarrow j,$$

$$, i! \rightarrow j, m \text{☮} n, \;\Leftrightarrow\; , m \text{☮} n, i! \rightarrow j,$$
$$, i! \rightarrow j, m \text{☮} n, \;\Leftrightarrow\; , m \text{☮} n, i! \rightarrow j,$$
$$, i! \rightarrow j, m \text{☮} n, \;\Leftrightarrow\; , m \text{☮} n, i! \rightarrow j,$$

$$, i! \rightarrow j, m \text{⦹}, \;\Leftrightarrow\; , m \text{⦹}, i! \rightarrow j,$$
$$, i! \rightarrow j, m \oplus, \;\Leftrightarrow\; , m \oplus, i! \rightarrow j,$$
$$, i! \rightarrow j, m \ominus, \;\Leftrightarrow\; , m \ominus, i! \rightarrow j,$$

Propositions and Propositions:

$$
\begin{aligned}
,i{\rightarrow}j,m{\rightarrow}n, \quad &\Leftrightarrow \quad ,m{\rightarrow}n,i{\rightarrow}j,\\
,i{\rightarrow}j,m!{\rightarrow}n, \quad &\Leftrightarrow \quad ,m!{\rightarrow}n,i{\rightarrow}j,\\
,i!{\rightarrow}j,m!{\rightarrow}n, \quad &\Leftrightarrow \quad ,m!{\rightarrow}n,i!{\rightarrow}j,
\end{aligned}
$$

$$
\begin{aligned}
,i{\rightarrow}j,m{\bigcirc^{\circ\circ}}n, \quad &\Leftrightarrow \quad ,m{\bigcirc^{\circ\circ}}n,i{\rightarrow}j,\\
,i{\rightarrow}j,m!{\bigcirc^{\circ\circ}}n, \quad &\Leftrightarrow \quad ,m!{\bigcirc^{\circ\circ}}n,i{\rightarrow}j,\\
,i!{\rightarrow}j,m{\bigcirc^{\circ\circ}}n, \quad &\Leftrightarrow \quad ,m{\bigcirc^{\circ\circ}}n,i!{\rightarrow}j,\\
,i!{\rightarrow}j,m!{\bigcirc^{\circ\circ}}n, \quad &\Leftrightarrow \quad ,m!{\bigcirc^{\circ\circ}}n,i!{\rightarrow}j,
\end{aligned}
$$

$$
\begin{aligned}
,i{\rightarrow}j,m{\bigcirc^{\circ}}n, \quad &\Leftrightarrow \quad ,m{\bigcirc^{\circ}}n,i{\rightarrow}j,\\
,i{\rightarrow}j,m!{\bigcirc^{\circ}}n, \quad &\Leftrightarrow \quad ,m!{\bigcirc^{\circ}}n,i{\rightarrow}j,\\
,i!{\rightarrow}j,m{\bigcirc^{\circ}}n, \quad &\Leftrightarrow \quad ,m{\bigcirc^{\circ}}n,i!{\rightarrow}j,\\
,i!{\rightarrow}j,m!{\bigcirc^{\circ}}n, \quad &\Leftrightarrow \quad ,m!{\bigcirc^{\circ}}n,i!{\rightarrow}j,
\end{aligned}
$$

$$
\begin{aligned}
,i{\rightarrow}j,m=n, \quad &\Leftrightarrow \quad ,m=n,i{\rightarrow}j,\\
,i{\rightarrow}j,m\,!{=}\,n, \quad &\Leftrightarrow \quad ,m\,!{=}\,n,i{\rightarrow}j,\\
,i!{\rightarrow}j,m=n, \quad &\Leftrightarrow \quad ,m=n,i!{\rightarrow}j,\\
,i!{\rightarrow}j,m\,!{=}\,n, \quad &\Leftrightarrow \quad ,m\,!{=}\,n,i!{\rightarrow}j,
\end{aligned}
$$

$$
\begin{aligned}
,i{\rightarrow}j,m=\varnothing, \quad &\Leftrightarrow \quad ,m=\varnothing,i{\rightarrow}j,\\
,i{\rightarrow}j,m\,!{=}\,\varnothing, \quad &\Leftrightarrow \quad ,m\,!{=}\,\varnothing,i{\rightarrow}j,\\
,i!{\rightarrow}j,m=\varnothing, \quad &\Leftrightarrow \quad ,m=\varnothing,i!{\rightarrow}j,\\
,i!{\rightarrow}j,m\,!{=}\,\varnothing, \quad &\Leftrightarrow \quad ,m\,!{=}\,\varnothing,i!{\rightarrow}j,
\end{aligned}
$$

Propositions and recursive function:

$$
\begin{aligned}
,i\rightarrow j, R(m), &\Leftrightarrow ,R(m), i\rightarrow j,\\
,i\rightarrow j, R_{-}(m), &\Leftrightarrow ,R_{-}(m), i\rightarrow j,\\
,i!\rightarrow j, R(m), &\Leftrightarrow ,R(m), i!\rightarrow j,\\
,i!\rightarrow j, R_{-}(m), &\Leftrightarrow ,R_{-}(m), i!\rightarrow j,
\end{aligned}
$$

Propositions and flag object:

$$
\begin{aligned}
,i\rightarrow j,\ \&SHi\,\mathring{\bigcirc}m, &\Leftrightarrow ,\ \&SHi\,\mathring{\bigcirc}m, i\rightarrow j,\\
,i\rightarrow j,\ \&SHi\rightarrow m, &\Leftrightarrow ,\ \&SHi\rightarrow m, i\rightarrow j,\\
,i!\rightarrow j,\ \&SHi\,\mathring{\bigcirc}m, &\Leftrightarrow ,\ \&SHi\,\mathring{\bigcirc}m, i!\rightarrow j,\\
,i!\rightarrow j,\ \&SHi\rightarrow m, &\Leftrightarrow ,\ \&SHi\rightarrow m, i!\rightarrow j,
\end{aligned}
$$

$$
\begin{aligned}
,i\rightarrow j,\ \&SHj\,\mathring{\bigcirc}m, &\Leftrightarrow ,\ \&SHj\,\mathring{\bigcirc}m, i\rightarrow j,\\
,i\rightarrow j,\ \&SHj\leftarrow m, &\Leftrightarrow ,\ \&SHj\leftarrow m, i\rightarrow j,\\
,i!\rightarrow j,\ \&SHj\,\mathring{\bigcirc}m, &\Leftrightarrow ,\ \&SHj\,\mathring{\bigcirc}m, i!\rightarrow j,\\
,i!\rightarrow j,\ \&SHj\leftarrow m, &\Leftrightarrow ,\ \&SHj\leftarrow m, i!\rightarrow j,
\end{aligned}
$$

Propositions to Propositions with branch function

$$
,if(i\rightarrow j)-\left[\begin{array}{l} ,m!\rightarrow n, \\ \\ , \end{array}\right]-, \Leftrightarrow ,if(m\rightarrow n)-\left[\begin{array}{l} ,i!\rightarrow j, \\ \\ , \end{array}\right]-,
$$

$$
,if(i\rightarrow j)-\left[\begin{array}{l} , \\ \\ ,m\rightarrow n, \end{array}\right]-, \Leftrightarrow ,if(m\rightarrow n)-\left[\begin{array}{l} , \\ \\ ,i\rightarrow j, \end{array}\right]-,
$$

$$
,if(i\rightarrow j)-\left[\begin{array}{l} ,m!\mathring{\bigcirc}\!\circ n, \\ \\ , \end{array}\right]-, \Leftrightarrow ,if(m\mathring{\bigcirc}\!\circ n)-\left[\begin{array}{l} ,i!\rightarrow j, \\ \\ , \end{array}\right]-,
$$

$$,if(i\rightarrow j)\left[\begin{array}{l} , \\ \\ , m\bigcirc\!\!\!{}^{\circ\circ} n, \end{array}\right], \;\Leftrightarrow\; ,if(m\bigcirc\!\!\!{}^{\circ\circ} n)\left[\begin{array}{l} , \\ \\ , i\rightarrow j, \end{array}\right],$$

$$,if(i\rightarrow j)\left[\begin{array}{l} , m!\bigcirc\!\!\!{}^{\circ} n, \\ \\ , \end{array}\right], \;\Leftrightarrow\; ,if(m\bigcirc\!\!\!{}^{\circ} n)\left[\begin{array}{l} , i!\rightarrow j, \\ \\ , \end{array}\right],$$

$$,if(i\rightarrow j)\left[\begin{array}{l} , \\ \\ , m\bigcirc\!\!\!{}^{\circ} n, \end{array}\right], \;\Leftrightarrow\; ,if(m\bigcirc\!\!\!{}^{\circ} n)\left[\begin{array}{l} , \\ \\ , i\rightarrow j, \end{array}\right],$$

$$,if(i\rightarrow j)\left[\begin{array}{l} , m\,!\!=n, \\ \\ , \end{array}\right], \;\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l} , i!\rightarrow j, \\ \\ , \end{array}\right],$$

$$,if(i\rightarrow j)\left[\begin{array}{l} , \\ \\ , m=n, \end{array}\right], \;\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l} , \\ \\ , i\rightarrow j, \end{array}\right],$$

$$,if(i\rightarrow j)\left[\begin{array}{l} , m\,!\!=\varnothing, \\ \\ , \end{array}\right], \;\Leftrightarrow\; ,if(m=\varnothing)\left[\begin{array}{l} , i!\rightarrow j, \\ \\ , \end{array}\right],$$

$$,if(i\rightarrow j)\left[\begin{array}{l} , \\ \\ , m=\varnothing, \end{array}\right], \;\Leftrightarrow\; ,if(m=\varnothing)\left[\begin{array}{l} , \\ \\ , i\rightarrow j, \end{array}\right],$$

17.2.7 Transitivity

Branch function with branch function:

$$,if(i\rightarrow j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\rightarrow j)\left[\begin{array}{l},if(i\rightarrow j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,\copyright c_2,\end{array}\right],$$

proof:

$$,if(i\rightarrow j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,i\circledast i_1,i_1\oplus,if(i_1\mathbin{\text{♂}}j)\left[\begin{array}{l},i_1\textcircled{$\downarrow$},\copyright c_1,\\ \\ ,i_1\textcircled{$\downarrow$},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,i\circledast i_1,i_1\oplus,if(i_1\mathbin{\text{♂}}j)\left[\begin{array}{l},if(i_1\mathbin{\text{♂}}j)\left[\begin{array}{l},i_1\textcircled{$\downarrow$},\copyright c_1,\\ \\ ,i_1\textcircled{$\downarrow$},\copyright c_3,\end{array}\right],\\ ,i_1\textcircled{$\downarrow$},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,i\circledast i_1,i\circledast i_2,i_2\textcircled{$\downarrow$},i_1\oplus,$$

$$if(i_1\mathbin{\text{♂}}j)\left[\begin{array}{l},if(i_1\mathbin{\text{♂}}j)\left[\begin{array}{l},i_1\textcircled{$\downarrow$},\copyright c_1,\\ \\ ,i_1\textcircled{$\downarrow$},\copyright c_3,\end{array}\right],\\ ,i_1\textcircled{$\downarrow$},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,i\circledast i_1,i\circledast i_2,i_2\oplus,i_2\textcircled{$\downarrow$},i_1\oplus,$$

$$if(i_1\mathbin{\text{♂}}j)\left[\begin{array}{l},if(i_1\mathbin{\text{♂}}j)\left[\begin{array}{l},i_1\textcircled{$\downarrow$},\copyright c_1,\\ \\ ,i_1\textcircled{$\downarrow$},\copyright c_3,\end{array}\right],\\ ,i_1\textcircled{$\downarrow$},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\; ,i\circledast i_1,i\circledast i_2,i_1\oplus,i_2\oplus,$$

$$if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_2\textcircled{\downarrow}, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_1\textcircled{\downarrow}, \copyright c_1, \\ ,i_1\textcircled{\downarrow}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\downarrow}, i_1\textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\circledast i_1, i\circledast i_2, i_1\mathring{\bigcirc}i_2, i_1\oplus, i_2\oplus,$$

$$if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_2\textcircled{\downarrow}, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_1\textcircled{\downarrow}, \copyright c_1, \\ ,i_1\textcircled{\downarrow}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\downarrow}, i_1\textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\circledast i_1, i\circledast i_2, i_1\oplus, i_2\oplus, i_1\mathring{\bigcirc}i_2,$$

$$if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_2\textcircled{\downarrow}, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_1\textcircled{\downarrow}, \copyright c_1, \\ ,i_1\textcircled{\downarrow}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\downarrow}, i_1\textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\circledast i_1, i\circledast i_2, i_1\oplus, i_2\oplus, i_1\mathring{\bigcirc}i_2,$$

$$if(i_2\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_2\textcircled{\downarrow}, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_1\textcircled{\downarrow}, \copyright c_1, \\ ,i_1\textcircled{\downarrow}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\downarrow}, i_1\textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\circledast i_1, i\circledast i_2, i_1\oplus, i_2\oplus,$$

$$if(i_2\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_2\textcircled{\downarrow}, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_1\textcircled{\downarrow}, \copyright c_1, \\ ,i_1\textcircled{\downarrow}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\downarrow}, i_1\textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\circledast i_2, i_2\oplus,$$

$$if(i_2\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_2\textcircled{\downarrow}, i\circledast i_1, i_1\oplus, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} ,i_1\textcircled{\downarrow}, \copyright c_1, \\ ,i_1\textcircled{\downarrow}, \copyright c_3, \end{array}\right], \\ ,i_2\textcircled{\downarrow}, i\circledast i_1, i_1\oplus, i_1\textcircled{\downarrow}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \;, if(i{\rightarrow}j)\left[\begin{array}{l} , i\,\text{☮}\,i_1, i_1\oplus, if(i_1 \overset{\circ}{\bigcirc} j)\left[\begin{array}{l} , i_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1\textcircled{$\downarrow$}, \copyright c_3, \end{array}\right], \\ \\ , i\,\text{☮}\,i_1, i_1\oplus, i_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \;, if(i{\rightarrow}j)\left[\begin{array}{l} , if(i{\rightarrow}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_3, \end{array}\right], \\ \\ , \copyright c_2, \end{array}\right],$$

$$, if(i{\rightarrow}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i{\rightarrow}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , if(i{\rightarrow}j)\left[\begin{array}{l} , \copyright c_3, \\ \\ , \copyright c_2, \end{array}\right], \end{array}\right],$$

Branch function with propositions:

$$, if(i{\rightarrow}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i{\rightarrow}j)\left[\begin{array}{l} , i{\rightarrow}j, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$, if(i{\rightarrow}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , if(i{\rightarrow}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , i!{\rightarrow}j, \copyright c_2, \end{array}\right],$$

Propositions with branch function:

$$,i{\to}j, if(i{\to}j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-}, \iff ,i{\to}j, \copyright c_1,$$

$$,i!{\to}j, if(i{\to}j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-}, \iff ,i!{\to}j, \copyright c_2,$$

Propositions with propositions:

$$,i{\to}j, \iff ,i{\to}j, i{\to}j,$$

$$,i!{\to}j, \iff ,i!{\to}j, i!{\to}j,$$

17.2.8 Substitution

Propositions with branch function:

$$,i{\to}j, if(m{\to}j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-}, \iff ,i{\to}j, if(m\mathring{\bigcirc}i)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},$$

proof:

$$,i{\to}j, if(m{\to}j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},$$

$$\iff ,i\text{☮}i_1, i_1\oplus, i_1\mathring{\bigcirc}j, i_1\text{⏚}, if(m{\to}j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},$$

$$\iff ,i\text{☮}i_1, i_1\oplus, i_1\mathring{\bigcirc}j, i_1\text{⏚}, m\text{☮}m_1, m_1\oplus, if(m_1\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l} ,m_1\text{⏚}, \copyright c_1, \\ \\ ,m_1\text{⏚}, \copyright c_2, \end{array}\right]\text{-},$$

$$\iff ,i\text{☮}i_1, i_1\oplus, i_1\mathring{\bigcirc}j, m\text{☮}m_1, m_1\oplus, if(m_1\mathring{\bigcirc}j)\text{-}\left[\begin{array}{l} ,m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ ,m_1\text{⏚}, i_1\text{⏚}, \copyright c_2, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,m\text{☮}m_1,m_1\oplus,i_1\text{☌}j,if(m_1\text{☌}j)\left[\begin{array}{l},m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,m\text{☮}m_1,m_1\oplus,i_1\text{☌}j,if(m_1\text{☌}i_1)\left[\begin{array}{l},m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i\ominus,i_1\oplus,m\text{☮}m_1,m\oplus,m\ominus,m_1\oplus,i_1\text{☌}j,$$

$$if(m_1\text{☌}i_1)\left[\begin{array}{l},m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i_1\oplus,m\text{☮}m_1,m\oplus,m_1\oplus,i_1\text{☌}j,$$

$$if(m_1\text{☌}i_1)\left[\begin{array}{l},m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☌}i_1,i\oplus,i_1\oplus,m\text{☮}m_1,m\oplus,m_1\oplus,i_1\text{☌}j,$$

$$if(m_1\text{☌}i_1)\left[\begin{array}{l},m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i_1\oplus,i\text{☌}i_1,m\text{☮}m_1,m\oplus,m_1\oplus,i_1\text{☌}j,$$

$$if(m_1\text{☌}i_1)\left[\begin{array}{l},m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i_1\oplus,m\text{☮}m_1,m\oplus,m_1\oplus,i_1\text{☌}j,i\text{☌}i_1,$$

$$if(m_1\text{☌}i_1)\left[\begin{array}{l},m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i_1\oplus,m\text{☮}m_1,m\oplus,m_1\oplus,i_1\text{☌}j,i\text{☌}i_1,$$

$$if(m_1\text{☌}i)\left[\begin{array}{l},m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_1,\\ \\ ,m\ominus,i\ominus,m_1\text{⏚},i_1\text{⏚},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i_1\oplus, m\text{☮}m_1, m\oplus, m_1\oplus, i_1\text{♂}j,$$

$$if(m_1\text{♂}i)\text{-}\left[\begin{array}{l}, m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ , m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i_1\oplus, m\text{☮}m_1, m\text{♂}m_1, m\oplus, m_1\oplus, i_1\text{♂}j,$$

$$if(m_1\text{♂}i)\text{-}\left[\begin{array}{l}, m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ , m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i_1\oplus, m\text{☮}m_1, m\oplus, m_1\oplus, m\text{♂}m_1, i_1\text{♂}j,$$

$$if(m_1\text{♂}i)\text{-}\left[\begin{array}{l}, m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ , m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i_1\oplus, m\text{☮}m_1, m\oplus, m_1\oplus, i_1\text{♂}j, m\text{♂}m_1,$$

$$if(m_1\text{♂}i)\text{-}\left[\begin{array}{l}, m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ , m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i_1\oplus, m\text{☮}m_1, m\oplus, m_1\oplus, i_1\text{♂}j, m\text{♂}m_1,$$

$$if(m\text{♂}i)\text{-}\left[\begin{array}{l}, m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ , m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i_1\oplus, m\text{☮}m_1, m\oplus, m_1\oplus, i_1\text{♂}j,$$

$$if(m\text{♂}i)\text{-}\left[\begin{array}{l}, m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ , m\ominus, i\ominus, m_1\text{⏚}, i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i_1\oplus, m\text{☮}m_1, m\oplus, m_1\oplus, i_1\text{♂}j, m\ominus, i\ominus,$$

$$if(m\text{♂}i)\text{-}\left[\begin{array}{l}, m_1\text{⏚}, i_1\text{⏚}, \copyright c_1, \\ \\ , m_1\text{⏚}, i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i\oplus, i\ominus, i_1\oplus, m\text{☮}m_1, m\oplus, m\ominus, m_1\oplus, i_1\text{♂}j,$$

$$if(m\text{♂}i)\left[\begin{array}{c} ,m_1\text{⏚},i_1\text{⏚},©c_1, \\ \\ ,m_1\text{⏚},i_1\text{⏚},©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,m\text{☮}m_1,m_1\oplus,i_1\text{♂}j,if(m\text{♂}i)\left[\begin{array}{c} ,m_1\text{⏚},i_1\text{⏚},©c_1, \\ \\ ,m_1\text{⏚},i_1\text{⏚},©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\text{♂}j,i_1\text{⏚},m\text{☮}m_1,m_1\oplus,m_1\text{⏚},if(m\text{♂}i)\left[\begin{array}{c} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\text{♂}j,i_1\text{⏚},if(m\text{♂}i)\left[\begin{array}{c} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\rightarrow j,if(m\text{♂}i)\left[\begin{array}{c} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$,i\rightarrow j,if(i\rightarrow m)\left[\begin{array}{c} ,©c_1, \\ \\ ,©c_2, \end{array}\right],\ \Leftrightarrow\ ,i\rightarrow j,if(m\text{♂}j)\left[\begin{array}{c} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

Propositions with propositions:

$$,i\rightarrow j,m\rightarrow j,\ \Leftrightarrow\ ,i\rightarrow j,m\text{♂}i,$$
$$,i\rightarrow j,m!\rightarrow j,\ \Leftrightarrow\ ,i\rightarrow j,m!\text{♂}i,$$

$$,i\rightarrow j,i\rightarrow m,\ \Leftrightarrow\ ,i\rightarrow j,m\text{♂}j,$$
$$,i\rightarrow j,i!\rightarrow m,\ \Leftrightarrow\ ,i\rightarrow j,m!\text{♂}j,$$

Identical node propositions with branch function:

$$,i\overset{\circ}{\bigcirc}j,if(j{\rightarrow}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \iff ,i\overset{\circ}{\bigcirc}j,if(i{\rightarrow}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

proof:

$$,i\overset{\circ}{\bigcirc}j,if(j{\rightarrow}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\iff ,i\overset{\circ}{\bigcirc}j,j\owedge j_1,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,\copyright c_1,\\ \\ ,j_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\overset{\circ}{\bigcirc}j,i\owedge i_1,i_1\obot,j\owedge j_1,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,\copyright c_1,\\ \\ ,j_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\overset{\circ}{\bigcirc}j,i\owedge i_1,i_1\oplus,i_1\obot,j\owedge j_1,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,\copyright c_1,\\ \\ ,j_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\overset{\circ}{\bigcirc}j,i\owedge i_1,i_1\oplus,j\owedge j_1,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,i_1\obot,\copyright c_1,\\ \\ ,j_1\obot,i_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\overset{\circ}{\bigcirc}j,i\owedge i_1,j\owedge j_1,i_1\oplus,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,i_1\obot,\copyright c_1,\\ \\ ,j_1\obot,i_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\overset{\circ}{\bigcirc}j,i\owedge i_1,i\overset{\circ}{\bigcirc}i_1,j\owedge j_1,i_1\oplus,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,i_1\obot,\copyright c_1,\\ \\ ,j_1\obot,i_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\owedge i_1,i\overset{\circ}{\bigcirc}j,i\overset{\circ}{\bigcirc}i_1,j\owedge j_1,i_1\oplus,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,i_1\obot,\copyright c_1,\\ \\ ,j_1\obot,i_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\owedge i_1,i\overset{\circ}{\bigcirc}j,j\overset{\circ}{\bigcirc}i_1,j\owedge j_1,i_1\oplus,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,i_1\obot,\copyright c_1,\\ \\ ,j_1\obot,i_1\obot,\copyright c_2,\end{array}\right],$$

$$\iff ,i\owedge i_1,i\overset{\circ}{\bigcirc}j,j\overset{\circ}{\bigcirc}i_1,j\owedge j_1,j\overset{\circ}{\bigcirc}j_1,i_1\oplus,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\obot,i_1\obot,\copyright c_1,\\ \\ ,j_1\obot,i_1\obot,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\owedge i_1,i\overset{\circ}{\bigcirc}j,j\owedge j_1,j\overset{\circ}{\bigcirc}i_1,j\overset{\circ}{\bigcirc}j_1,i_1\oplus,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\ovee,i_1\ovee,\copyright c_1,\\ \\ ,j_1\ovee,i_1\ovee,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\owedge i_1,i\overset{\circ}{\bigcirc}j,j\owedge j_1,j\overset{\circ}{\bigcirc}i_1,i_1\overset{\circ}{\bigcirc}j_1,i_1\oplus,j_1\oplus,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\ovee,i_1\ovee,\copyright c_1,\\ \\ ,j_1\ovee,i_1\ovee,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\owedge i_1,i\overset{\circ}{\bigcirc}j,j\owedge j_1,j\overset{\circ}{\bigcirc}i_1,i_1\oplus,j_1\oplus,i_1\overset{\circ}{\bigcirc}j_1,if(j_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\ovee,i_1\ovee,\copyright c_1,\\ \\ ,j_1\ovee,i_1\ovee,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\owedge i_1,i\overset{\circ}{\bigcirc}j,j\owedge j_1,j\overset{\circ}{\bigcirc}i_1,i_1\oplus,j_1\oplus,i_1\overset{\circ}{\bigcirc}j_1,if(i_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\ovee,i_1\ovee,\copyright c_1,\\ \\ ,j_1\ovee,i_1\ovee,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i\owedge i_1,j\owedge j_1,i_1\oplus,j_1\oplus,if(i_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},j_1\ovee,i_1\ovee,\copyright c_1,\\ \\ ,j_1\ovee,i_1\ovee,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,j\owedge j_1,j_1\oplus,j_1\ovee,i\owedge i_1,i_1\oplus,if(i_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},i_1\ovee,\copyright c_1,\\ \\ ,i_1\ovee,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i\owedge i_1,i_1\oplus,if(i_1\overset{\circ}{\bigcirc}m)\left[\begin{array}{l},i_1\ovee,\copyright c_1,\\ \\ ,i_1\ovee,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(i\rightarrow m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$,i\overset{\circ}{\bigcirc}j,if(m\rightarrow j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,if(m\rightarrow i)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

Identical node propositions with propositions:

$$,i\overset{\circ}{\bigcirc}j,j\rightarrow m,\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,i\rightarrow m,$$
$$,i\overset{\circ}{\bigcirc}j,m\rightarrow j,\ \Leftrightarrow\ ,i\overset{\circ}{\bigcirc}j,m\rightarrow i,$$

$$, i\overset{\circ}{\bigcirc}j, j!{\to}m, \;\Leftrightarrow\; , i\overset{\circ}{\bigcirc}j, i!{\to}m,$$

$$, i\overset{\circ}{\bigcirc}j, m!{\to}j, \;\Leftrightarrow\; , i\overset{\circ}{\bigcirc}j, m!{\to}i,$$

17.2.9 Opposition

$$, i{\to}j, i!{\to}j, \;\Leftrightarrow\; , \otimes,$$

$$, i!{\to}j, i{\to}j, \;\Leftrightarrow\; , \otimes,$$

17.2.10 Swap of the same operand

Operators:

$$, i\text{☮}m, if(i{\to}j)\text{-}\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right]\text{-}, \;\Leftrightarrow\; , if(i{\to}j)\text{-}\left[\begin{array}{l}, i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2,\end{array}\right]\text{-},$$

$$, i\text{☮}m, if(i{\to}j)\text{-}\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right]\text{-}, \;\Leftrightarrow\; , if(i{\to}j)\text{-}\left[\begin{array}{l}, i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2,\end{array}\right]\text{-},$$

proof:

$$, i\text{☮}m, if(i{\to}j)\text{-}\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\; , i\text{☮}m, i\text{☮}i_1, i_1\oplus, if(i_1\overset{\circ}{\bigcirc}j)\text{-}\left[\begin{array}{l}, i_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\; , i\text{☮}i_1, i\text{☮}m, i_1\oplus, if(i_1\overset{\circ}{\bigcirc}j)\text{-}\left[\begin{array}{l}, i_1\text{⏚}, \copyright c_1, \\ \\ , i_1\text{⏚}, \copyright c_2,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\text{☮}i_1, i_1\oplus, if(i_1\mathring{\bigcirc}j)\left[\begin{array}{l} , i_1\text{☮}, i\text{☮}m, \copyright c_1, \\ \\ , i_1\text{☮}, i\text{☮}m, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\rightarrow j)\left[\begin{array}{l} , i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2, \end{array}\right],$$

$$, i\text{☮}m, if(i\rightarrow j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i\rightarrow j)\left[\begin{array}{l} , i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2, \end{array}\right],$$

$$, i\text{☮}m, if(j\rightarrow i)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(j\rightarrow i)\left[\begin{array}{l} , i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2, \end{array}\right],$$

$$, i\text{☮}m, if(j\rightarrow i)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(j\rightarrow i)\left[\begin{array}{l} , i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2, \end{array}\right],$$

$$, i\text{☮}m, if(j\rightarrow i)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(j\rightarrow i)\left[\begin{array}{l} , i\text{☮}m, \copyright c_1, \\ \\ , i\text{☮}m, \copyright c_2, \end{array}\right],$$

$$, i\rightarrow j, i\text{☮}n, \quad \Leftrightarrow \quad , i\text{☮}n, i\rightarrow j,$$

$$, i\rightarrow j, i\text{☮}n, \quad \Leftrightarrow \quad , i\text{☮}n, i\rightarrow j,$$

$$, i\rightarrow j, i\text{☮}n, \quad \Leftrightarrow \quad , i\text{☮}n, i\rightarrow j,$$

$$, i!\rightarrow j, i\text{☮}n, \quad \Leftrightarrow \quad , i\text{☮}n, i!\rightarrow j,$$

$$, i!\rightarrow j, i\text{☮}n, \quad \Leftrightarrow \quad , i\text{☮}n, i!\rightarrow j,$$

$$, i!\rightarrow j, i\text{☮}n, \quad \Leftrightarrow \quad , i\text{☮}n, i!\rightarrow j,$$

$$,j{\to}i,i☮n, \iff ,i☮n,j{\to}i,$$

$$,j{\to}i,i☮n, \iff ,i☮n,j{\to}i,$$

$$,j{\to}i,i☮n, \iff ,i☮n,j{\to}i,$$

$$,j!{\to}i,i☮n, \iff ,i☮n,j!{\to}i,$$

$$,j!{\to}i,i☮n, \iff ,i☮n,j!{\to}i,$$

$$,j!{\to}i,i☮n, \iff ,i☮n,j!{\to}i,$$

Node connectivity:

$$,if(i\bigcirc j)\left[\begin{array}{l} ,if(j{\to}m)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(j{\to}m)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(j{\to}m)\left[\begin{array}{l} ,if(i\bigcirc j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\bigcirc j)\left[\begin{array}{l} ,\copyright c_2, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\bigcirc j)\left[\begin{array}{l} ,if(m{\to}j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(m{\to}j)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(m{\to}j)\left[\begin{array}{l} ,if(i\bigcirc j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\bigcirc j)\left[\begin{array}{l} ,\copyright c_2, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\bigcirc j)\left[\begin{array}{l} ,if(i{\to}j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_2, \end{array}\right], \\ ,if(i{\to}j)\left[\begin{array}{l} ,\copyright c_3, \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(i{\to}j)\left[\begin{array}{l} ,if(i\bigcirc j)\left[\begin{array}{l} ,\copyright c_1, \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\bigcirc j)\left[\begin{array}{l} ,\copyright c_2, \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$, i\bigcirc\!\!\!\!^{\circ}\, j, if(j\to m)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(j\to m)\left[\begin{array}{l}, i\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_1,\\ \\ , i\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_2,\end{array}\right],$$

$$, i!\bigcirc\!\!\!\!^{\circ}\, j, if(j\to m)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(j\to m)\left[\begin{array}{l}, i!\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_1,\\ \\ , i!\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_2,\end{array}\right],$$

$$, i\bigcirc\!\!\!\!^{\circ}\, j, if(m\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(m\to j)\left[\begin{array}{l}, i\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_1,\\ \\ , i\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_2,\end{array}\right],$$

$$, i!\bigcirc\!\!\!\!^{\circ}\, j, if(m\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(m\to j)\left[\begin{array}{l}, i!\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_1,\\ \\ , i!\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_2,\end{array}\right],$$

$$, i\bigcirc\!\!\!\!^{\circ}\, j, if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(i\to j)\left[\begin{array}{l}, i\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_1,\\ \\ , i\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_2,\end{array}\right],$$

$$, i!\bigcirc\!\!\!\!^{\circ}\, j, if(i\to j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(i\to j)\left[\begin{array}{l}, i!\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_1,\\ \\ , i!\bigcirc\!\!\!\!^{\circ}\, j, \copyright c_2,\end{array}\right],$$

$$, j\to m, if(i\bigcirc\!\!\!\!^{\circ}\, j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(i\bigcirc\!\!\!\!^{\circ}\, j)\left[\begin{array}{l}, j\to m, \copyright c_1,\\ \\ , j\to m, \copyright c_2,\end{array}\right],$$

$$, j!\to m, if(i\bigcirc\!\!\!\!^{\circ}\, j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(i\bigcirc\!\!\!\!^{\circ}\, j)\left[\begin{array}{l}, j!\to m, \copyright c_1,\\ \\ , j!\to m, \copyright c_2,\end{array}\right],$$

$$, m\to j, if(i\bigcirc\!\!\!\!^{\circ}\, j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \iff , if(i\bigcirc\!\!\!\!^{\circ}\, j)\left[\begin{array}{l}, m\to j, \copyright c_1,\\ \\ , m\to j, \copyright c_2,\end{array}\right],$$

$$, m!{\rightarrow}j, if(i{\circledcirc}j)\begin{bmatrix} , ©c_1, \\ \\ , ©c_2, \end{bmatrix}, \quad\Leftrightarrow\quad , if(i{\circledcirc}j)\begin{bmatrix} , m!{\rightarrow}j, ©c_1, \\ \\ , m!{\rightarrow}j, ©c_2, \end{bmatrix},$$

$$, i{\rightarrow}j, if(i{\circledcirc}j)\begin{bmatrix} , ©c_1, \\ \\ , ©c_2, \end{bmatrix}, \quad\Leftrightarrow\quad , if(i{\circledcirc}j)\begin{bmatrix} , i{\rightarrow}j, ©c_1, \\ \\ , i{\rightarrow}j, ©c_2, \end{bmatrix},$$

$$, i!{\rightarrow}j, if(i{\circledcirc}j)\begin{bmatrix} , ©c_1, \\ \\ , ©c_2, \end{bmatrix}, \quad\Leftrightarrow\quad , if(i{\circledcirc}j)\begin{bmatrix} , i!{\rightarrow}j, ©c_1, \\ \\ , i!{\rightarrow}j, ©c_2, \end{bmatrix},$$

$$, i{\circledcirc}j, i{\rightarrow}j, \quad\Leftrightarrow\quad , i{\rightarrow}j, i{\circledcirc}j,$$

$$, i{\circledcirc}j, i!{\rightarrow}j, \quad\Leftrightarrow\quad , i!{\rightarrow}j, i{\circledcirc}j,$$

$$, i!{\circledcirc}j, i{\rightarrow}j, \quad\Leftrightarrow\quad , i{\rightarrow}j, i!{\circledcirc}j,$$

$$, i!{\circledcirc}j, i!{\rightarrow}j, \quad\Leftrightarrow\quad , i!{\rightarrow}j, i!{\circledcirc}j,$$

$$, i{\circledcirc}j, j{\rightarrow}m, \quad\Leftrightarrow\quad , j{\rightarrow}m, i{\circledcirc}j,$$

$$, i{\circledcirc}j, j!{\rightarrow}m, \quad\Leftrightarrow\quad , j!{\rightarrow}m, i{\circledcirc}j,$$

$$, i!{\circledcirc}j, j{\rightarrow}m, \quad\Leftrightarrow\quad , j{\rightarrow}m, i!{\circledcirc}j,$$

$$, i!{\circledcirc}j, j!{\rightarrow}m, \quad\Leftrightarrow\quad , j!{\rightarrow}m, i!{\circledcirc}j,$$

$$, i{\circledcirc}j, m{\rightarrow}j, \quad\Leftrightarrow\quad , m{\rightarrow}j, i{\circledcirc}j,$$

$$, i{\circledcirc}j, m!{\rightarrow}j, \quad\Leftrightarrow\quad , m!{\rightarrow}j, i{\circledcirc}j,$$

$$, i!{\circledcirc}j, m{\rightarrow}j, \quad\Leftrightarrow\quad , m{\rightarrow}j, i!{\circledcirc}j,$$

$$, i!{\circledcirc}j, m!{\rightarrow}j, \quad\Leftrightarrow\quad , m!{\rightarrow}j, i!{\circledcirc}j,$$

Identical node comparison:

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,if(j{\rightarrow}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(j{\rightarrow}m)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(j{\rightarrow}m)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,if(m{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(m{\rightarrow}j)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(i{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(i{\rightarrow}j)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,i\mathring{\bigcirc}j,if(j{\rightarrow}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(j{\rightarrow}m)\left[\begin{array}{l} ,i\mathring{\bigcirc}j,\copyright c_1, \\ \\ ,i\mathring{\bigcirc}j,\copyright c_2, \end{array}\right],$$

$$,i!\mathring{\bigcirc}j,if(j{\rightarrow}m)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(j{\rightarrow}m)\left[\begin{array}{l} ,i!\mathring{\bigcirc}j,\copyright c_1, \\ \\ ,i!\mathring{\bigcirc}j,\copyright c_2, \end{array}\right],$$

$$,i\mathring{\bigcirc}j,if(m{\rightarrow}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(m{\rightarrow}j)\left[\begin{array}{l} ,i\mathring{\bigcirc}j,\copyright c_1, \\ \\ ,i\mathring{\bigcirc}j,\copyright c_2, \end{array}\right],$$

$$,\, i!\overset{\circ}{\bigcirc}j,\, if(m{\rightarrow}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(m{\rightarrow}j) \left[\begin{array}{c} ,\, i!\overset{\circ}{\bigcirc}j,\, \copyright c_1, \\ \\ ,\, i!\overset{\circ}{\bigcirc}j,\, \copyright c_2, \end{array}\right],$$

$$,\, i\overset{\circ}{\bigcirc}j,\, if(i{\rightarrow}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i{\rightarrow}j) \left[\begin{array}{c} ,\, i\overset{\circ}{\bigcirc}j,\, \copyright c_1, \\ \\ ,\, i\overset{\circ}{\bigcirc}j,\, \copyright c_2, \end{array}\right],$$

$$,\, i!\overset{\circ}{\bigcirc}j,\, if(i{\rightarrow}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i{\rightarrow}j) \left[\begin{array}{c} ,\, i!\overset{\circ}{\bigcirc}j,\, \copyright c_1, \\ \\ ,\, i!\overset{\circ}{\bigcirc}j,\, \copyright c_2, \end{array}\right],$$

$$,\, j{\rightarrow}m,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, j{\rightarrow}m,\, \copyright c_1, \\ \\ ,\, j{\rightarrow}m,\, \copyright c_2, \end{array}\right],$$

$$,\, j!{\rightarrow}m,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, j!{\rightarrow}m,\, \copyright c_1, \\ \\ ,\, j!{\rightarrow}m,\, \copyright c_2, \end{array}\right],$$

$$,\, m{\rightarrow}j,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, m{\rightarrow}j,\, \copyright c_1, \\ \\ ,\, m{\rightarrow}j,\, \copyright c_2, \end{array}\right],$$

$$,\, m!{\rightarrow}j,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, m!{\rightarrow}j,\, \copyright c_1, \\ \\ ,\, m!{\rightarrow}j,\, \copyright c_2, \end{array}\right],$$

$$,\, i{\rightarrow}j,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, i{\rightarrow}j,\, \copyright c_1, \\ \\ ,\, i{\rightarrow}j,\, \copyright c_2, \end{array}\right],$$

$$,\, i!{\rightarrow}j,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, \copyright c_1, \\ \\ ,\, \copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,\, if(i\overset{\circ}{\bigcirc}j) \left[\begin{array}{c} ,\, i!{\rightarrow}j,\, \copyright c_1, \\ \\ ,\, i!{\rightarrow}j,\, \copyright c_2, \end{array}\right],$$

$$, i\mars j, i{\rightarrow}j, \;\Leftrightarrow\; , i{\rightarrow}j, i\mars j,$$

$$, i\mars j, i!{\rightarrow}j, \;\Leftrightarrow\; , i!{\rightarrow}j, i\mars j,$$

$$, i!\mars j, i{\rightarrow}j, \;\Leftrightarrow\; , i{\rightarrow}j, i!\mars j,$$

$$, i!\mars j, i!{\rightarrow}j, \;\Leftrightarrow\; , i!{\rightarrow}j, i!\mars j,$$

$$, i\mars j, j{\rightarrow}m, \;\Leftrightarrow\; , j{\rightarrow}m, i\mars j,$$

$$, i\mars j, j!{\rightarrow}m, \;\Leftrightarrow\; , j!{\rightarrow}m, i\mars j,$$

$$, i!\mars j, j{\rightarrow}m, \;\Leftrightarrow\; , j{\rightarrow}m, i!\mars j,$$

$$, i!\mars j, j!{\rightarrow}m, \;\Leftrightarrow\; , j!{\rightarrow}m, i!\mars j,$$

$$, i\mars j, m{\rightarrow}j, \;\Leftrightarrow\; , m{\rightarrow}j, i\mars j,$$

$$, i\mars j, m!{\rightarrow}j, \;\Leftrightarrow\; , m!{\rightarrow}j, i\mars j,$$

$$, i!\mars j, m{\rightarrow}j, \;\Leftrightarrow\; , m{\rightarrow}j, i!\mars j,$$

$$, i!\mars j, m!{\rightarrow}j, \;\Leftrightarrow\; , m!{\rightarrow}j, i!\mars j,$$

Node value comparison:

$$,if(i{=}j)\left[\begin{array}{l},if(j{\to}m)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_2,\end{array}\right],\\ ,if(j{\to}m)\left[\begin{array}{l},\copyright c_3,\\ ,\copyright c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(j{\to}m)\left[\begin{array}{l},if(i{=}j)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_3,\end{array}\right],\\ ,if(i{=}j)\left[\begin{array}{l},\copyright c_2,\\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{=}j)\left[\begin{array}{l},if(m{\to}j)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_2,\end{array}\right],\\ ,if(m{\to}j)\left[\begin{array}{l},\copyright c_3,\\ ,\copyright c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(m{\to}j)\left[\begin{array}{l},if(i{=}j)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_3,\end{array}\right],\\ ,if(i{=}j)\left[\begin{array}{l},\copyright c_2,\\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{=}j)\left[\begin{array}{l},if(i{\to}j)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_2,\end{array}\right],\\ ,if(i{\to}j)\left[\begin{array}{l},\copyright c_3,\\ ,\copyright c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(i{\to}j)\left[\begin{array}{l},if(i{=}j)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_3,\end{array}\right],\\ ,if(i{=}j)\left[\begin{array}{l},\copyright c_2,\\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,i{=}j,if(j{\to}m)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(j{\to}m)\left[\begin{array}{l},i{=}j,\copyright c_1,\\ ,i{=}j,\copyright c_2,\end{array}\right],$$

$$,i\,!{=}j,if(j{\to}m)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(j{\to}m)\left[\begin{array}{l},i\,!{=}j,\copyright c_1,\\ ,i\,!{=}j,\copyright c_2,\end{array}\right],$$

$$,i{=}j,if(m{\to}j)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(m{\to}j)\left[\begin{array}{l},i{=}j,\copyright c_1,\\ ,i{=}j,\copyright c_2,\end{array}\right],$$

$$, i\,!\!=j, if(m\to j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(m\to j)\left[\begin{matrix}, i\,!\!=j, \copyright c_1, \\ \\ , i\,!\!=j, \copyright c_2,\end{matrix}\right],$$

$$, i=j, if(i\to j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i\to j)\left[\begin{matrix}, i=j, \copyright c_1, \\ \\ , i=j, \copyright c_2,\end{matrix}\right],$$

$$, i\,!\!=j, if(i\to j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i\to j)\left[\begin{matrix}, i\,!\!=j, \copyright c_1, \\ \\ , i\,!\!=j, \copyright c_2,\end{matrix}\right],$$

$$, j\to m, if(i=j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{matrix}, j\to m, \copyright c_1, \\ \\ , j\to m, \copyright c_2,\end{matrix}\right],$$

$$, j!\!\to m, if(i=j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{matrix}, j!\!\to m, \copyright c_1, \\ \\ , j!\!\to m, \copyright c_2,\end{matrix}\right],$$

$$, m\to j, if(i=j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{matrix}, m\to j, \copyright c_1, \\ \\ , m\to j, \copyright c_2,\end{matrix}\right],$$

$$, m!\!\to j, if(i=j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{matrix}, m!\!\to j, \copyright c_1, \\ \\ , m!\!\to j, \copyright c_2,\end{matrix}\right],$$

$$, i\to j, if(i=j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{matrix}, i\to j, \copyright c_1, \\ \\ , i\to j, \copyright c_2,\end{matrix}\right],$$

$$, i!\!\to j, if(i=j)\left[\begin{matrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{matrix}\right], \;\Leftrightarrow\; , if(i=j)\left[\begin{matrix}, i!\!\to j, \copyright c_1, \\ \\ , i!\!\to j, \copyright c_2,\end{matrix}\right],$$

$$, i = j, i \rightarrow j, \;\Leftrightarrow\; , i \rightarrow j, i = j,$$

$$, i = j, i ! \rightarrow j, \;\Leftrightarrow\; , i ! \rightarrow j, i = j,$$

$$, i \,!\!= j, i \rightarrow j, \;\Leftrightarrow\; , i \rightarrow j, i \,!\!= j,$$

$$, i \,!\!= j, i ! \rightarrow j, \;\Leftrightarrow\; , i ! \rightarrow j, i \,!\!= j,$$

$$, i = j, j \rightarrow m, \;\Leftrightarrow\; , j \rightarrow m, i = j,$$

$$, i = j, j ! \rightarrow m, \;\Leftrightarrow\; , j ! \rightarrow m, i = j,$$

$$, i \,!\!= j, j \rightarrow m, \;\Leftrightarrow\; , j \rightarrow m, i \,!\!= j,$$

$$, i \,!\!= j, j ! \rightarrow m, \;\Leftrightarrow\; , j ! \rightarrow m, i \,!\!= j,$$

$$, i = j, m \rightarrow j, \;\Leftrightarrow\; , m \rightarrow j, i = j,$$

$$, i = j, m ! \rightarrow j, \;\Leftrightarrow\; , m ! \rightarrow j, i = j,$$

$$, i \,!\!= j, m \rightarrow j, \;\Leftrightarrow\; , m \rightarrow j, i \,!\!= j,$$

$$, i \,!\!= j, m ! \rightarrow j, \;\Leftrightarrow\; , m ! \rightarrow j, i \,!\!= j,$$

Node null comparison:

$$,if(i\!=\!\varnothing)\left[\begin{array}{l} ,if(i\!\to\! j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(i\!\to\! j)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(i\!\to\! j)\left[\begin{array}{l} ,if(i\!=\!\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\!=\!\varnothing)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i\!=\!\varnothing)\left[\begin{array}{l} ,if(j\!\to\! i)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(j\!\to\! i)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(j\!\to\! i)\left[\begin{array}{l} ,if(i\!=\!\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\!=\!\varnothing)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,i\!=\!\varnothing, if(i\!\to\! j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\!\to\! j)\left[\begin{array}{l} ,i\!=\!\varnothing, \copyright c_1, \\ \\ ,i\!=\!\varnothing, \copyright c_2, \end{array}\right],$$

$$,i\,!\!=\!\varnothing, if(i\!\to\! j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\!\to\! j)\left[\begin{array}{l} ,i\,!\!=\!\varnothing, \copyright c_1, \\ \\ ,i\,!\!=\!\varnothing, \copyright c_2, \end{array}\right],$$

$$,i\!\to\! j, if(i\!=\!\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\!=\!\varnothing)\left[\begin{array}{l} ,i\!\to\! j, \copyright c_1, \\ \\ ,i\!\to\! j, \copyright c_2, \end{array}\right],$$

$$,i!\!\to\! j, if(i\!=\!\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i\!=\!\varnothing)\left[\begin{array}{l} ,i!\!\to\! j, \copyright c_1, \\ \\ ,i!\!\to\! j, \copyright c_2, \end{array}\right],$$

$$,i\!=\!\varnothing,if(j\!\to\! i)\!\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!, \quad\Leftrightarrow\quad ,if(j\!\to\! i)\!\left[\begin{array}{l},i\!=\!\varnothing,\,\copyright c_1,\\ \\ ,i\!=\!\varnothing,\,\copyright c_2,\end{array}\right]\!,$$

$$,i\,!\!=\!\varnothing,if(j\!\to\! i)\!\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!, \quad\Leftrightarrow\quad ,if(j\!\to\! i)\!\left[\begin{array}{l},i\,!\!=\!\varnothing,\,\copyright c_1,\\ \\ ,i\,!\!=\!\varnothing,\,\copyright c_2,\end{array}\right]\!,$$

$$,j\!\to\! i,if(i\!=\!\varnothing)\!\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!, \quad\Leftrightarrow\quad ,if(i\!=\!\varnothing)\!\left[\begin{array}{l},j\!\to\! i,\,\copyright c_1,\\ \\ ,j\!\to\! i,\,\copyright c_2,\end{array}\right]\!,$$

$$,j!\!\to\! i,if(i\!=\!\varnothing)\!\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!, \quad\Leftrightarrow\quad ,if(i\!=\!\varnothing)\!\left[\begin{array}{l},j!\!\to\! i,\,\copyright c_1,\\ \\ ,j!\!\to\! i,\,\copyright c_2,\end{array}\right]\!,$$

$$,i\!=\!\varnothing,i\!\to\! j, \quad\Leftrightarrow\quad ,i\!\to\! j,i\!=\!\varnothing,$$

$$,i\!=\!\varnothing,i!\!\to\! j, \quad\Leftrightarrow\quad ,i!\!\to\! j,i\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing,i\!\to\! j, \quad\Leftrightarrow\quad ,i\!\to\! j,i\,!\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing,i!\!\to\! j, \quad\Leftrightarrow\quad ,i!\!\to\! j,i\,!\!=\!\varnothing,$$

$$,i\!=\!\varnothing,j\!\to\! i, \quad\Leftrightarrow\quad ,j\!\to\! i,i\!=\!\varnothing,$$

$$,i\!=\!\varnothing,j!\!\to\! i, \quad\Leftrightarrow\quad ,j!\!\to\! i,i\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing,j\!\to\! i, \quad\Leftrightarrow\quad ,j\!\to\! i,i\,!\!=\!\varnothing,$$

$$,i\,!\!=\!\varnothing,j!\!\to\! i, \quad\Leftrightarrow\quad ,j!\!\to\! i,i\,!\!=\!\varnothing,$$

Itself:

$$,if(i{\to}j)\left[\begin{array}{l},if(j{\to}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(j{\to}m)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \iff ,if(j{\to}m)\left[\begin{array}{l},if(i{\to}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i{\to}j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{\to}j)\left[\begin{array}{l},if(m{\to}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(m{\to}j)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \iff ,if(m{\to}j)\left[\begin{array}{l},if(i{\to}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i{\to}j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{\to}j)\left[\begin{array}{l},if(i{\to}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(i{\to}m)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \iff ,if(i{\to}m)\left[\begin{array}{l},if(i{\to}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i{\to}j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{\to}j)\left[\begin{array}{l},if(m{\to}i)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\ ,if(m{\to}i)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \iff ,if(m{\to}i)\left[\begin{array}{l},if(i{\to}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ ,if(i{\to}j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,i{\to}j,if(j{\to}m)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \iff ,if(j{\to}m)\left[\begin{array}{l},i{\to}j,\copyright c_1,\\ \\ ,i{\to}j,\copyright c_2,\end{array}\right],$$

$$, i!{\to}j, if(j{\to}m)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(j{\to}m)\left[\begin{array}{l} , i!{\to}j, \copyright c_1, \\ \\ , i!{\to}j, \copyright c_2, \end{array}\right],$$

$$, i{\to}j, if(m{\to}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(m{\to}j)\left[\begin{array}{l} , i{\to}j, \copyright c_1, \\ \\ , i{\to}j, \copyright c_2, \end{array}\right],$$

$$, i!{\to}j, if(m{\to}j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(m{\to}j)\left[\begin{array}{l} , i!{\to}j, \copyright c_1, \\ \\ , i!{\to}j, \copyright c_2, \end{array}\right],$$

$$, i{\to}j, if(i{\to}m)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{\to}m)\left[\begin{array}{l} , i{\to}j, \copyright c_1, \\ \\ , i{\to}j, \copyright c_2, \end{array}\right],$$

$$, i!{\to}j, if(i{\to}m)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{\to}m)\left[\begin{array}{l} , i!{\to}j, \copyright c_1, \\ \\ , i!{\to}j, \copyright c_2, \end{array}\right],$$

$$, i{\to}j, if(m{\to}i)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(m{\to}i)\left[\begin{array}{l} , i{\to}j, \copyright c_1, \\ \\ , i{\to}j, \copyright c_2, \end{array}\right],$$

$$, i!{\to}j, if(m{\to}i)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(m{\to}i)\left[\begin{array}{l} , i!{\to}j, \copyright c_1, \\ \\ , i!{\to}j, \copyright c_2, \end{array}\right],$$

$$, i{\to}j, j{\to}m, \quad \Leftrightarrow \quad , j{\to}m, i{\to}j,$$

$$, i{\to}j, j!{\to}m, \quad \Leftrightarrow \quad , j!{\to}m, i{\to}j,$$

$$, i!{\to}j, j{\to}m, \quad \Leftrightarrow \quad , j{\to}m, i!{\to}j,$$

$$, i!{\to}j, j!{\to}m, \quad \Leftrightarrow \quad , j!{\to}m, i!{\to}j,$$

$$, i{\rightarrow}j, m{\rightarrow}j, \;\Leftrightarrow\; , m{\rightarrow}j, i{\rightarrow}j,$$

$$, i{\rightarrow}j, m!{\rightarrow}j, \;\Leftrightarrow\; , m!{\rightarrow}j, i{\rightarrow}j,$$

$$, i!{\rightarrow}j, m{\rightarrow}j, \;\Leftrightarrow\; , m{\rightarrow}j, i!{\rightarrow}j,$$

$$, i!{\rightarrow}j, m!{\rightarrow}j, \;\Leftrightarrow\; , m!{\rightarrow}j, i!{\rightarrow}j,$$

$$, i{\rightarrow}j, i{\rightarrow}m, \;\Leftrightarrow\; , i{\rightarrow}m, i{\rightarrow}j,$$

$$, i{\rightarrow}j, i!{\rightarrow}m, \;\Leftrightarrow\; , i!{\rightarrow}m, i{\rightarrow}j,$$

$$, i!{\rightarrow}j, i{\rightarrow}m, \;\Leftrightarrow\; , i{\rightarrow}m, i!{\rightarrow}j,$$

$$, i!{\rightarrow}j, i!{\rightarrow}m, \;\Leftrightarrow\; , i!{\rightarrow}m, i!{\rightarrow}j,$$

$$, i{\rightarrow}j, m{\rightarrow}i, \;\Leftrightarrow\; , m{\rightarrow}i, i{\rightarrow}j,$$

$$, i{\rightarrow}j, m!{\rightarrow}i, \;\Leftrightarrow\; , m!{\rightarrow}i, i{\rightarrow}j,$$

$$, i!{\rightarrow}j, m{\rightarrow}i, \;\Leftrightarrow\; , m{\rightarrow}i, i!{\rightarrow}j,$$

$$, i!{\rightarrow}j, m!{\rightarrow}i, \;\Leftrightarrow\; , m!{\rightarrow}i, i!{\rightarrow}j,$$

flag object:

$$,\ \&SHi\,\overset{\circ}{\bigcirc}i, if(i\rightarrow j)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(i\rightarrow j)-\left[\begin{array}{c} ,\ \&SHi\,\overset{\circ}{\bigcirc}i, \copyright c_1, \\ \\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, \copyright c_2, \end{array}\right]-,$$

$$,\ \&SHi \rightarrow i, if(i\rightarrow j)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(i\rightarrow j)-\left[\begin{array}{c} ,\ \&SHi \rightarrow i, \copyright c_1, \\ \\ ,\ \&SHi \rightarrow i, \copyright c_2, \end{array}\right]-,$$

$$,\ \&SHj\,\overset{\circ}{\bigcirc}i, if(i\rightarrow j)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(i\rightarrow j)-\left[\begin{array}{c} ,\ \&SHj\,\overset{\circ}{\bigcirc}i, \copyright c_1, \\ \\ ,\ \&SHj\,\overset{\circ}{\bigcirc}i, \copyright c_2, \end{array}\right]-,$$

$$,\ \&SHj \leftarrow i, if(i\rightarrow j)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(i\rightarrow j)-\left[\begin{array}{c} ,\ \&SHj \leftarrow i, \copyright c_1, \\ \\ ,\ \&SHj \leftarrow i, \copyright c_2, \end{array}\right]-,$$

$$,\ \&SHi\,\overset{\circ}{\bigcirc}i, if(j\rightarrow i)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(j\rightarrow i)-\left[\begin{array}{c} ,\ \&SHi\,\overset{\circ}{\bigcirc}i, \copyright c_1, \\ \\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, \copyright c_2, \end{array}\right]-,$$

$$,\ \&SHi \rightarrow i, if(j\rightarrow i)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(j\rightarrow i)-\left[\begin{array}{c} ,\ \&SHi \rightarrow i, \copyright c_1, \\ \\ ,\ \&SHi \rightarrow i, \copyright c_2, \end{array}\right]-,$$

$$,\ \&SHj\,\overset{\circ}{\bigcirc}i, if(j\rightarrow i)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(j\rightarrow i)-\left[\begin{array}{c} ,\ \&SHj\,\overset{\circ}{\bigcirc}i, \copyright c_1, \\ \\ ,\ \&SHj\,\overset{\circ}{\bigcirc}i, \copyright c_2, \end{array}\right]-,$$

$$,\ \&SHj \leftarrow i, if(j\rightarrow i)-\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-,\ \Leftrightarrow\ , if(j\rightarrow i)-\left[\begin{array}{c} ,\ \&SHj \leftarrow i, \copyright c_1, \\ \\ ,\ \&SHj \leftarrow i, \copyright c_2, \end{array}\right]-,$$

$$, i\rightarrow j,\ \&SHi\,\overset{\circ}{\bigcirc}i,\ \Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, i\rightarrow j,$$

$$, i! {\to} j,\ \&SHi \mathring{\bigcirc} i,\ \Leftrightarrow\ ,\ \&SHi \mathring{\bigcirc} i, i! {\to} j,$$

$$, i {\to} j,\ \&SHi {\to} i,\ \Leftrightarrow\ ,\ \&SHi {\to} i, i {\to} j,$$

$$, i! {\to} j,\ \&SHi {\to} i,\ \Leftrightarrow\ ,\ \&SHi {\to} i, i! {\to} j,$$

$$, i {\to} j,\ \&SHj \mathring{\bigcirc} i,\ \Leftrightarrow\ ,\ \&SHj \mathring{\bigcirc} i, i {\to} j,$$

$$, i! {\to} j,\ \&SHj \mathring{\bigcirc} i,\ \Leftrightarrow\ ,\ \&SHj \mathring{\bigcirc} i, i! {\to} j,$$

$$, i {\to} j,\ \&SHj {\leftarrow} i,\ \Leftrightarrow\ ,\ \&SHj {\leftarrow} i, i {\to} j,$$

$$, i! {\to} j,\ \&SHj {\leftarrow} i,\ \Leftrightarrow\ ,\ \&SHj {\leftarrow} i, i! {\to} j,$$

$$, j {\to} i,\ \&SHi \mathring{\bigcirc} i,\ \Leftrightarrow\ ,\ \&SHi \mathring{\bigcirc} i, j {\to} i,$$

$$, j! {\to} i,\ \&SHi \mathring{\bigcirc} i,\ \Leftrightarrow\ ,\ \&SHi \mathring{\bigcirc} i, j! {\to} i,$$

$$, j {\to} i,\ \&SHi {\to} i,\ \Leftrightarrow\ ,\ \&SHi {\to} i, j {\to} i,$$

$$, j! {\to} i,\ \&SHi {\to} i,\ \Leftrightarrow\ ,\ \&SHi {\to} i, j! {\to} i,$$

$$, j {\to} i,\ \&SHj \mathring{\bigcirc} i,\ \Leftrightarrow\ ,\ \&SHj \mathring{\bigcirc} i, j {\to} i,$$

$$, j! {\to} i,\ \&SHj \mathring{\bigcirc} i,\ \Leftrightarrow\ ,\ \&SHj \mathring{\bigcirc} i, j! {\to} i,$$

$$, j {\to} i,\ \&SHj {\leftarrow} i,\ \Leftrightarrow\ ,\ \&SHj {\leftarrow} i, j {\to} i,$$

$$, j! {\to} i,\ \&SHj {\leftarrow} i,\ \Leftrightarrow\ ,\ \&SHj {\leftarrow} i, j! {\to} i,$$

17.2.11 Node Continuity propositions to node Connectivity propositions

$$,i{\rightarrow}j,\ \Leftrightarrow\ \sim,i\overset{\circ}{\bigcirc}^{\circ}j,$$

proof:

$,i{\rightarrow}j,$
$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i\overset{\circ}{\bigcirc}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i\overset{\circ}{\bigcirc}i_1,i\overset{\circ}{\bigcirc}^{\circ}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i\overset{\circ}{\bigcirc}^{\circ}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i\overset{\circ}{\bigcirc}^{\circ}i_1,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i\overset{\circ}{\bigcirc}^{\circ}i_1,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i\overset{\circ}{\bigcirc}^{\circ}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},i\overset{\circ}{\bigcirc}^{\circ}j,$

$\Leftrightarrow\ ,i{\rightarrow}j,i\overset{\circ}{\bigcirc}^{\circ}j,$

17.2.12 Node Continuity propositions to identical node propositions

$$,i{\rightarrow}j,i\oplus,\ \Leftrightarrow\ ,i\oplus,i\overset{\circ}{\bigcirc}j,$$

proof:

$,i{\rightarrow}j,i\oplus,$
$\Leftrightarrow\ ,i\text{☮}i_1,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},i\oplus,$

$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i\overset{\circ}{\bigcirc}i_1,i\oplus,i_1\oplus,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i_1\oplus,i\overset{\circ}{\bigcirc}i_1,i_1\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ ,i\text{☮}i_1,i\oplus,i_1\oplus,i\overset{\circ}{\bigcirc}i_1,i\overset{\circ}{\bigcirc}j,i_1\text{⏚},$

$\Leftrightarrow\ , i\circledast i_1, i\oplus, i_1\oplus, i\mathring{\bigcirc}j, i_1\circledcirc,$

$\Leftrightarrow\ , i\circledast i_1, i\oplus, i_1\oplus, i_1\circledcirc, i\mathring{\bigcirc}j,$

$\Leftrightarrow\ , i\circledast i_1, i_1\oplus, i_1\circledcirc, i\oplus, i\mathring{\bigcirc}j,$

$\Leftrightarrow\ , i\oplus, i\mathring{\bigcirc}j,$

$$, i!{\rightarrow}j, i\oplus, \ \Leftrightarrow\ , i\oplus, i!\mathring{\bigcirc}j,$$

$$, i{\rightarrow}j, j\ominus, \ \Leftrightarrow\ , j\ominus, i\mathring{\bigcirc}j,$$
$$, i!{\rightarrow}j, j\ominus, \ \Leftrightarrow\ , j\ominus, i!\mathring{\bigcirc}j,$$

17.2.13 Empty node ring

$$, i{\rightarrow}i, \ \Leftrightarrow\ , i\circledast i_0, i{\rightarrow}i_0, i_0\circledcirc,$$
$$, i{\rightarrow}i, \ \Leftrightarrow\ , i\circledast i_0, i_0\oplus, i_0\mathring{\bigcirc}i, i_0\circledcirc,$$

$$, i{\rightarrow}i, \ \Leftrightarrow\ , i{\rightarrow}i, i{\rightarrow}i,$$

$$, i\mathring{\bigcirc}j, i\oplus, i\mathring{\bigcirc}j, j\oplus, \ \Leftrightarrow\ , i\oplus, i\mathring{\bigcirc}j, j\oplus, i\mathring{\bigcirc}j,$$

proof:

$, i\mathring{\bigcirc}j, i\oplus, i\mathring{\bigcirc}j, j\oplus,$
$\Leftrightarrow\ , i\circledast i_0, i_0\circledcirc, i\mathring{\bigcirc}j, i\oplus, i\mathring{\bigcirc}j, j\oplus,$

$\Leftrightarrow\ , i\circledast i_0, i_0\oplus, i_0\circledcirc, i\mathring{\bigcirc}j, i\oplus, i\mathring{\bigcirc}j, j\oplus,$

$\Leftrightarrow\ , i\circledast i_0, i\mathring{\bigcirc}j, i_0\oplus, i\oplus, i\mathring{\bigcirc}j, j\oplus, i_0\circledcirc,$

$\Leftrightarrow\ , i\circledast i_0, i\mathring{\bigcirc}i_0, i\mathring{\bigcirc}j, i_0\oplus, i\oplus, i\mathring{\bigcirc}j, j\oplus, i_0\circledcirc,$

$\Leftrightarrow\ , i\circledast i_0, i\mathring{\bigcirc}j, i\mathring{\bigcirc}i_0, i_0\oplus, i\oplus, i\mathring{\bigcirc}j, j\oplus, i_0\circledcirc,$

$\Leftrightarrow\ , i\circledast i_0, i\mathring{\bigcirc}j, i_0\oplus, i\oplus, i\mathring{\bigcirc}i_0, i\mathring{\bigcirc}j, j\oplus, i_0\circledcirc,$

$\Leftrightarrow\ , i\text{☮}i_0, i\overset{\circ}{\bigcirc}j, i_0\oplus, i\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}i_0, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\overset{\circ}{\bigcirc}i_0, i\overset{\circ}{\bigcirc}j, i_0\oplus, i\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}i_0, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\overset{\circ}{\bigcirc}i_0, i_0\overset{\circ}{\bigcirc}j, i_0\oplus, i\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}i_0, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i_0\overset{\circ}{\bigcirc}j, i\oplus, i\overset{\circ}{\bigcirc}j, i_0\oplus, j\oplus, i\overset{\circ}{\bigcirc}i_0, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\oplus, i\overset{\circ}{\bigcirc}j, i_0\overset{\circ}{\bigcirc}j, i_0\oplus, j\oplus, i\overset{\circ}{\bigcirc}i_0, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\oplus, i\overset{\circ}{\bigcirc}j, i_0\oplus, j\oplus, i_0\overset{\circ}{\bigcirc}j, i\overset{\circ}{\bigcirc}i_0, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\oplus, i\overset{\circ}{\bigcirc}j, i_0\oplus, j\oplus, i\overset{\circ}{\bigcirc}i_0, i_0\overset{\circ}{\bigcirc}j, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\oplus, i\overset{\circ}{\bigcirc}j, i_0\oplus, j\oplus, i\overset{\circ}{\bigcirc}i_0, i\overset{\circ}{\bigcirc}j, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\oplus, i_0\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}i_0, i\overset{\circ}{\bigcirc}j, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\oplus, i_0\oplus, i\overset{\circ}{\bigcirc}i_0, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}j, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\overset{\circ}{\bigcirc}i_0, i\oplus, i_0\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}j, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\oplus, i_0\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}j, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i_0\oplus, i_0\text{⏚}, i\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}j,$

$\Leftrightarrow\ , i\oplus, i\overset{\circ}{\bigcirc}j, j\oplus, i\overset{\circ}{\bigcirc}j,$

$$, i{\rightarrow}i, i\overset{\circ}{\bigcirc}i_1, i\oplus, \ \Leftrightarrow\ , i{\rightarrow}i, i\oplus, i\overset{\circ}{\bigcirc}i_1,$$

proof:

$, i{\rightarrow}i, i\overset{\circ}{\bigcirc}i_1, i\oplus,$
$\Leftrightarrow\ , i\text{☮}i_2, i_2\oplus, i_2\overset{\circ}{\bigcirc}i, i_2\text{⏚}, i\overset{\circ}{\bigcirc}i_1, i\oplus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}i_1, i\text{☮}i_2, i_2\oplus, i_2\overset{\circ}{\bigcirc}i, i_2\text{⏚}, i\oplus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}i_1, i\text{☮}i_2, i\overset{\circ}{\bigcirc}i_2, i_2\oplus, i_2\overset{\circ}{\bigcirc}i, i_2\text{⏚}, i\oplus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}i_1, i\text{☮}i_2, i_2\overset{\circ}{\bigcirc}i, i_2\oplus, i_2\overset{\circ}{\bigcirc}i, i_2\text{⏚}, i\oplus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}i_1, i\text{☮}i_2, i_2\overset{\circ}{\bigcirc}i, i_2\oplus, i_2\overset{\circ}{\bigcirc}i, i\oplus, i_2\text{⏚},$

$\Leftrightarrow\ , i\mathring{\bigcirc} i_1, i\circledast i_2, i_2\oplus, i_2\mathring{\bigcirc} i, i\oplus, i_2\mathring{\bigcirc} i, i_2\odot,$

$\Leftrightarrow\ , i\circledast i_2, i_2\oplus, i_2\mathring{\bigcirc} i, i\mathring{\bigcirc} i_1, i\oplus, i_2\mathring{\bigcirc} i, i_2\odot,$

$\Leftrightarrow\ , i\circledast i_2, i_2\oplus, i_2\mathring{\bigcirc} i, i_2\mathring{\bigcirc} i_1, i\oplus, i_2\mathring{\bigcirc} i, i_2\odot,$

$\Leftrightarrow\ , i\circledast i_2, i_2\oplus, i_2\mathring{\bigcirc} i, i\oplus, i_2\mathring{\bigcirc} i, i_2\mathring{\bigcirc} i_1, i_2\odot,$

$\Leftrightarrow\ , i\circledast i_2, i_2\oplus, i_2\mathring{\bigcirc} i, i\oplus, i_2\mathring{\bigcirc} i, i\mathring{\bigcirc} i_1, i_2\odot,$

$\Leftrightarrow\ , i\circledast i_2, i_2\mathring{\bigcirc} i, i_2\oplus, i_2\mathring{\bigcirc} i, i\oplus, i\mathring{\bigcirc} i_1, i_2\odot,$

$\Leftrightarrow\ , i\circledast i_2, i_2\oplus, i_2\mathring{\bigcirc} i, i\oplus, i\mathring{\bigcirc} i_1, i_2\odot,$

$\Leftrightarrow\ , i\circledast i_2, i_2\oplus, i_2\mathring{\bigcirc} i, i_2\odot, i\oplus, i\mathring{\bigcirc} i_1,$

$\Leftrightarrow\ , i{\rightarrow}i, i\oplus, i\mathring{\bigcirc} i_1,$

$$, i{\rightarrow}i, i\oplus, \ \Leftrightarrow\ , i\oplus, i{\rightarrow}i,$$

proof:

$, i{\rightarrow}i, i\oplus,$

$\Leftrightarrow\ , i\circledast i_1, i_1\oplus, i_1\mathring{\bigcirc} i, i_1\odot, i\oplus,$

$\Leftrightarrow\ , i\circledast i_1, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\odot,$

$\Leftrightarrow\ , i\circledast i_1, i\mathring{\bigcirc} i_1, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\odot,$

$\Leftrightarrow\ , i\circledast i_1, i_1\mathring{\bigcirc} i, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\odot,$

$\Leftrightarrow\ , i\circledast i_1, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\mathring{\bigcirc} i, i_1\odot,$

$\Leftrightarrow\ , i\circledast i_1, i\circledast i_2, i_2\odot, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\mathring{\bigcirc} i, i_1\odot,$

$\Leftrightarrow\ , i\circledast i_1, i\circledast i_2, i_2\oplus, i_2\odot, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\mathring{\bigcirc} i, i_1\odot,$

$\Leftrightarrow\ , i\circledast i_2, i\circledast i_1, i_2\oplus, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\mathring{\bigcirc} i, i_2\odot, i_1\odot,$

$\Leftrightarrow\ , i\circledast i_2, i\circledast i_1, i_2\mathring{\bigcirc} i_1, i_2\oplus, i_1\oplus, i_1\mathring{\bigcirc} i, i\oplus, i_1\mathring{\bigcirc} i, i_2\odot, i_1\odot,$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i_1\oplus, i_2\bigcirc^{\circ} i_1, i_1\bigcirc^{\circ} i, i\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i_1\oplus, i_2\bigcirc^{\circ} i_1, i_2\bigcirc^{\circ} i, i\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i_1\oplus, i_2\bigcirc^{\circ} i, i\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i_2\bigcirc^{\circ} i, i\oplus, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\bigcirc^{\circ} i_2, i\owedge i_1, i_2\oplus, i_2\bigcirc^{\circ} i, i\oplus, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i_2\bigcirc^{\circ} i, i\owedge i_1, i_2\oplus, i_2\bigcirc^{\circ} i, i\oplus, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\bigcirc^{\circ} i, i_2\oplus, i_2\bigcirc^{\circ} i, i\oplus, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i_2\bigcirc^{\circ} i, i\oplus, i_2\bigcirc^{\circ} i, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i\bigcirc^{\circ} i_1, i_2\oplus, i_2\bigcirc^{\circ} i, i\oplus, i_2\bigcirc^{\circ} i, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i\bigcirc^{\circ} i_1, i_2\bigcirc^{\circ} i, i\oplus, i_2\bigcirc^{\circ} i, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i\bigcirc^{\circ} i_1, i_2\bigcirc^{\circ} i_1, i\oplus, i_2\bigcirc^{\circ} i, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i_2\bigcirc^{\circ} i_1, i\oplus, i_2\bigcirc^{\circ} i, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i\oplus, i_2\bigcirc^{\circ} i, i_2\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i\oplus, i_2\bigcirc^{\circ} i, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\bigcirc^{\circ} i, i_2\oplus, i\oplus, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i_2\bigcirc^{\circ} i, i\owedge i_1, i_2\oplus, i\oplus, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i\owedge i_1, i_2\oplus, i\oplus, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_2\obot, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_2, i_2\oplus, i_2\obot, i\owedge i_1, i\oplus, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_1, i\oplus, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_1, i\oplus, i\owedge i_3, i_3\obot, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_1, i\oplus, i\owedge i_3, i_3\oplus, i_3\obot, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_1\obot,$$

$$\Leftrightarrow\ , i\owedge i_1, i\oplus, i\owedge i_3, i_3\oplus, i\bigcirc^{\circ} i_1, i_1\oplus, i_1\bigcirc^{\circ} i, i_1\obot, i_3\obot,$$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i\oslash i_3, i\mathring{\bigcirc} i_1, i_1\oplus, i_3\oplus, i_1\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i\oslash i_3, i\mathring{\bigcirc} i_3, i\mathring{\bigcirc} i_1, i_1\oplus, i_3\oplus, i_1\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i\oslash i_3, i\mathring{\bigcirc} i_3, i_3\mathring{\bigcirc} i_1, i_1\oplus, i_3\oplus, i_1\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i\oslash i_3, i_3\mathring{\bigcirc} i_1, i_1\oplus, i_3\oplus, i_1\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i\oslash i_3, i_1\oplus, i_3\oplus, i_3\mathring{\bigcirc} i_1, i_1\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i\oslash i_3, i_1\oplus, i_3\oplus, i_1\mathring{\bigcirc} i, i_3\mathring{\bigcirc} i_1, i_1\textcircled{$\downarrow$}, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i\oslash i_3, i_1\oplus, i_3\oplus, i_1\mathring{\bigcirc} i, i_3\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i_1\oplus, i_1\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i\oslash i_3, i_3\oplus, i_3\mathring{\bigcirc} i, i_3\textcircled{$\downarrow$},$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i_1\oplus, i_1\mathring{\bigcirc} i, i_1\textcircled{$\downarrow$}, i\rightarrow i,$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i_1\oplus, i\mathring{\bigcirc} i_1, i_1\textcircled{$\downarrow$}, i\rightarrow i,$

$\Leftrightarrow\ , i\oslash i_1, i\mathring{\bigcirc} i_1, i\oplus, i_1\oplus, i_1\textcircled{$\downarrow$}, i\rightarrow i,$

$\Leftrightarrow\ , i\oslash i_1, i\oplus, i_1\oplus, i_1\textcircled{$\downarrow$}, i\rightarrow i,$

$\Leftrightarrow\ , i\oplus, i\oslash i_1, i_1\oplus, i_1\textcircled{$\downarrow$}, i\rightarrow i,$

$\Leftrightarrow\ , i\oplus, i\oslash i_1, i_1\textcircled{$\downarrow$}, i\rightarrow i,$

$\Leftrightarrow\ , i\oplus, i\rightarrow i,$

$$, i\rightarrow i, i\ominus, \ \Leftrightarrow\ , i\ominus, i\rightarrow i,$$

proof:

$, i\rightarrow i, i\ominus,$
$\Leftrightarrow\ , i\oplus, i\ominus, i\rightarrow i, i\ominus,$

$\Leftrightarrow\ , i\ominus, i\oplus, i\rightarrow i, i\ominus,$

$\Leftrightarrow\ , i\ominus, i\rightarrow i, i\oplus, i\ominus,$

$\Leftrightarrow\ , i\ominus, i\rightarrow i,$

$$, i{\to}i, i\mathring{\bigcirc}i_1, i\ominus, \;\Leftrightarrow\; , i{\to}i, i\ominus, i\mathring{\bigcirc}i_1,$$

proof:

$, i{\to}i, i\mathring{\bigcirc}i_1, i\ominus,$
$\Leftrightarrow\; , i\oplus, i\ominus, i{\to}i, i\mathring{\bigcirc}i_1, i\ominus,$

$\Leftrightarrow\; , i\ominus, i\oplus, i{\to}i, i\mathring{\bigcirc}i_1, i\ominus,$

$\Leftrightarrow\; , i\ominus, i{\to}i, i\oplus, i\mathring{\bigcirc}i_1, i\ominus,$

$\Leftrightarrow\; , i\ominus, i{\to}i, i\mathring{\bigcirc}i_1, i\oplus, i\ominus,$

$\Leftrightarrow\; , i\ominus, i{\to}i, i\mathring{\bigcirc}i_1,$

$\Leftrightarrow\; , i{\to}i, i\ominus, i\mathring{\bigcirc}i_1,$

$$, i{\to}i, i=\varnothing, i\ominus, \;\Leftrightarrow\; , i{\to}i, i\ominus, i=\varnothing,$$

proof:

$, i{\to}i, i=\varnothing, i\ominus,$
$\Leftrightarrow\; , i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i=\varnothing, i\ominus,$

$\Leftrightarrow\; , i=\varnothing, i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i\ominus,$

$\Leftrightarrow\; , i=\varnothing, i\text{☮}i_1, i_1\text{⏚}, i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i\ominus,$

$\Leftrightarrow\; , i=\varnothing, i\text{☮}i_1, i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i\ominus, i_1\text{⏚},$

$\Leftrightarrow\; , i=\varnothing, i\text{☮}i_1, i\mathring{\bigcirc}i_1, i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i\ominus, i_1\text{⏚},$

$\Leftrightarrow\; , i=\varnothing, i\text{☮}i_1, i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i\mathring{\bigcirc}i_1, i\ominus, i_1\text{⏚},$

$\Leftrightarrow\; , i=\varnothing, i\text{☮}i_1, i{\to}i, i\mathring{\bigcirc}i_1, i\ominus, i_1\text{⏚},$

$\Leftrightarrow\; , i=\varnothing, i\text{☮}i_1, i{\to}i, i\ominus, i\mathring{\bigcirc}i_1, i_1\text{⏚},$

$\Leftrightarrow\; , i\text{☮}i_1, i=\varnothing, i{\to}i, i\ominus, i\mathring{\bigcirc}i_1, i_1\text{⏚},$

$\Leftrightarrow\; , i\text{☮}i_1, i\mathring{\bigcirc}i_1, i=\varnothing, i{\to}i, i\ominus, i\mathring{\bigcirc}i_1, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i\mathring{\bigcirc}i_1, i_1=\varnothing, i{\to}i, i\ominus, i\mathring{\bigcirc}i_1, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1=\varnothing, i{\to}i, i\ominus, i\mathring{\bigcirc}i_1, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1=\varnothing, i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i\ominus, i\mathring{\bigcirc}i_1, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i\text{☮}i_2, i{\to}i_2, i_2\text{⏚}, i\ominus, i\mathring{\bigcirc}i_1, i_1=\varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i{\to}i, i\ominus, i\mathring{\bigcirc}i_1, i_1=\varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i{\to}i, i\ominus, i\mathring{\bigcirc}i_1, i=\varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i{\to}i, i\mathring{\bigcirc}i_1, i\ominus, i=\varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i\mathring{\bigcirc}i_1, i{\to}i, i\ominus, i=\varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i{\to}i, i\ominus, i=\varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1\text{⏚}, i{\to}i, i\ominus, i=\varnothing,$

$\Leftrightarrow\ , i{\to}i, i\ominus, i=\varnothing,$

$$, i{\to}i, i=\varnothing, i\oplus, \ \Leftrightarrow\ , i{\to}i, i\oplus, i=\varnothing,$$

proof:

$, i{\to}i, i=\varnothing, i\oplus,$
$\Leftrightarrow\ , i{\to}i, i\oplus, i\ominus, i=\varnothing, i\oplus,$

$\Leftrightarrow\ , i\oplus, i{\to}i, i\ominus, i=\varnothing, i\oplus,$

$\Leftrightarrow\ , i\oplus, i{\to}i, i=\varnothing, i\ominus, i\oplus,$

$\Leftrightarrow\ , i\oplus, i{\to}i, i=\varnothing, i\oplus, i\ominus,$

$\Leftrightarrow\ , i\oplus, i{\to}i, i=\varnothing,$

$\Leftrightarrow\ , i{\to}i, i\oplus, i=\varnothing,$

$$, i=\varnothing, i\oplus, i=\varnothing, \ \Leftrightarrow\ \sim, i{\to}i,$$

proof:

$, i = \varnothing, i\oplus, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\text{⏚}, i\oplus, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i_1\text{⏚}, i\oplus, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\oplus, i = \varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i\bigcirc^{\!\circ}i_1, i_1\oplus, i\oplus, i = \varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\oplus, i\bigcirc^{\!\circ}i_1, i = \varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\oplus, i\bigcirc^{\!\circ}i_1, i = \varnothing, i = \varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\oplus, i\bigcirc^{\!\circ}i_1, i_1 = \varnothing, i = \varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\oplus, i_1 = \varnothing, i = \varnothing, i_1\text{⏚},$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i_1 = \varnothing, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i\text{☮}i_1, i = \varnothing, i_1\oplus, i_1 = \varnothing, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i\text{☮}i_1, i\bigcirc^{\!\circ}i_1, i = \varnothing, i_1\oplus, i_1 = \varnothing, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i\text{☮}i_1, i\bigcirc^{\!\circ}i_1, i_1 = \varnothing, i_1\oplus, i_1 = \varnothing, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i\text{☮}i_1, i\bigcirc^{\!\circ}i_1, i_1 = \varnothing, i_1\oplus, i_1 = \varnothing, i\bigcirc^{\!\circ}i_1, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i\text{☮}i_1, i\bigcirc^{\!\circ}i_1, i = \varnothing, i_1\oplus, i_1 = \varnothing, i\bigcirc^{\!\circ}i_1, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i\text{☮}i_1, i = \varnothing, i_1\oplus, i_1 = \varnothing, i\bigcirc^{\!\circ}i_1, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i_1 = \varnothing, i\bigcirc^{\!\circ}i_1, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\bigcirc^{\!\circ}i_1, i_1 = \varnothing, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\bigcirc^{\!\circ}i_1, i = \varnothing, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i = \varnothing, i\text{☮}i_1, i_1\oplus, i\bigcirc^{\!\circ}i_1, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i = \varnothing, i\text{☮}i_1, i_1\oplus, i\bigcirc^{\!\circ}i_1, i\oplus, i_1\text{⏚}, i = \varnothing,$

$\Leftrightarrow\ , i=\varnothing, i\text{☮}i_1, i_1\oplus, i\text{♂}i_1, i_1\text{⏚}, i\oplus, i=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, i\to i, i\oplus, i=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, i\oplus, i\to i, i=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, i\oplus, i=\varnothing, i\to i,$

$$, \circledcirc i, \quad \Leftrightarrow \quad \sim, i\to i,$$

proof:

$, \circledcirc i,$
$\Leftrightarrow\ , \circledcirc i, i\oplus, i\ominus,$

$\Leftrightarrow\ , \circledcirc i, i\oplus, i=\varnothing, i\ominus,$

$\Leftrightarrow\ , \circledcirc i, i=\varnothing, i\oplus, i=\varnothing, i\ominus,$

$\Leftrightarrow\ , \circledcirc i, i=\varnothing, i\oplus, i=\varnothing, i\to i, i\ominus,$

$\Leftrightarrow\ , \circledcirc i, i=\varnothing, i\oplus, i=\varnothing, i\ominus, i\to i,$

$\Leftrightarrow\ , \circledcirc i, i\oplus, i\ominus, i\to i,$

$\Leftrightarrow\ , \circledcirc i, i\to i,$

$$, i\to i, \quad \Leftrightarrow \quad \sim, i=\varnothing,$$

induction proof:
$premise\ 1:$
$, i=\varnothing, i\to i,$
$\Leftrightarrow\ , i=\varnothing, i\to i, i=\varnothing,$

$premise\ 2:$
$, \&SHi\to i, i\to i, \quad \Leftrightarrow \quad , \&SHi\to i, i\to i, i=\varnothing, \quad \Rightarrow$

$, i\, != \varnothing, \&SHi\,\text{♂}i, i\to i,$

$\Leftrightarrow \;, i \,!\!=\varnothing,\ \&SHi\,\bigcirc\!\!\!^{\circ} i, i\!\rightarrow\! i, i\oplus, i\ominus,$

$\Leftrightarrow \;, i \,!\!=\varnothing,\ \&SHi\,\bigcirc\!\!\!^{\circ} i, i\oplus, i\!\rightarrow\! i, i\ominus,$

$\Leftrightarrow \;, i \,!\!=\varnothing, i\oplus,\ \&SHi\rightarrow i, i\!\rightarrow\! i, i\ominus,$

$\Leftrightarrow \;, i \,!\!=\varnothing, i\oplus,\ \&SHi\rightarrow i, i\!\rightarrow\! i, i=\varnothing, i\ominus,$

$\Leftrightarrow \;, i \,!\!=\varnothing,\ \&SHi\,\bigcirc\!\!\!^{\circ} i, i\oplus, i\!\rightarrow\! i, i=\varnothing, i\ominus,$

$\Leftrightarrow \;, i \,!\!=\varnothing,\ \&SHi\,\bigcirc\!\!\!^{\circ} i, i\oplus, i\!\rightarrow\! i, i\ominus, i=\varnothing,$

$\Leftrightarrow \;, i \,!\!=\varnothing,\ \&SHi\,\bigcirc\!\!\!^{\circ} i, i\!\rightarrow\! i, i\oplus, i\ominus, i=\varnothing,$

$\Leftrightarrow \;, i \,!\!=\varnothing,\ \&SHi\,\bigcirc\!\!\!^{\circ} i, i\!\rightarrow\! i, i=\varnothing,$

$conclusion:$
$, i\!\rightarrow\! i, \;\Leftrightarrow\; , i\!\rightarrow\! i, i=\varnothing,$

17.2.14 Other

$$, j\text{☮}k, k\oplus, \;\Leftrightarrow\; \sim, j\!\rightarrow\! k,$$

18 Rules of Relationship of Subnode

18.1 Definition of Node Subnode

$$,if(i\obot j)\left[\begin{array}{l},\\ \\ ,\end{array}\right. \Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l},i\owedge n,\\ \\ ,i\ovee t,if(t\bigcirc j)\left[\begin{array}{l},\circledcirc n,\\ \\ ,i\owedge n,\end{array}\right],t\oslash,\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\oslash,\\ \\ ,n\oslash,\end{array}\right.$$

$$,i\obot j,\ \Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l},\\ \\ ,\otimes,\end{array}\right],$$

$$,i!\obot j,\ \Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l},\otimes,\\ \\ ,\end{array}\right],$$

$$,i\obot i,\ \Leftrightarrow\ ,i\owedge i_0,i_0\obot i,i_0\oslash,$$
$$,i!\obot i,\ \Leftrightarrow\ ,i\owedge i_0,i_0!\obot i,i_0\oslash,$$

18.2 Theorems of Relationship of Subnode

18.2.1 Subnode propositions to Node Connectivity propositions

$$,i\obot j,\ \Leftrightarrow\ ,i\,!\!=\varnothing,i\ovee t,t\bigcirc j,t\oslash,$$

proof:
$,i\obot j,$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\text{☮}n,\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚},\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\text{⏚},\\ , n\text{⏚}, \otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\text{☮}n,\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚},\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\text{⏚},\\ , \otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\text{☮}n,\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚},\end{array}\right], n=\varnothing, n\text{⏚},$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\text{☮}n, n=\varnothing, n\text{⏚},\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚}, n=\varnothing, n\text{⏚},\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\text{☮}n, n=\varnothing, n\text{⏚},\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚}, n=\varnothing, n\text{⏚},\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\text{☮}n, n\,!\!=\varnothing, n=\varnothing, n\text{⏚},\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚}, n=\varnothing, n\text{⏚},\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, i\text{☮}n, \otimes, n\text{⏚},\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚}, n=\varnothing, n\text{⏚},\end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l}, \otimes,\\ , i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚}, n=\varnothing, n\text{⏚},\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n,\\ , i\text{☮}n,\end{array}\right], t\text{⏚}, n=\varnothing, n\text{⏚},$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash t, if(t\bigcirc j)\left[\begin{array}{l}, \circledcirc n, n=\varnothing, n\text{⏚},\\ , i\text{☮}n, n=\varnothing, n\text{⏚},\end{array}\right], t\text{⏚},$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oslash t, if(t\bigcirc^{\circ}j)\left[\begin{array}{l}, \circledcirc n, n=\varnothing, n\textcircled{\downarrow}, \\ \\ , i\obslash n, n=\varnothing, n\textcircled{\downarrow},\end{array}\right], t\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oslash t, if(t\bigcirc^{\circ}j)\left[\begin{array}{l}, \circledcirc n, n\textcircled{\downarrow}, \\ \\ , i\obslash n, n=\varnothing, n\textcircled{\downarrow},\end{array}\right], t\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oslash t, if(t\bigcirc^{\circ}j)\left[\begin{array}{l}, \\ \\ , i\obslash n, n=\varnothing, n\textcircled{\downarrow},\end{array}\right], t\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oslash t, if(t\bigcirc^{\circ}j)\left[\begin{array}{l}, \\ \\ , i\obslash n, n\,!{=}\,\varnothing, n=\varnothing, n\textcircled{\downarrow},\end{array}\right], t\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oslash t, if(t\bigcirc^{\circ}j)\left[\begin{array}{l}, \\ \\ , i\obslash n, \otimes, n\textcircled{\downarrow},\end{array}\right], t\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oslash t, if(t\bigcirc^{\circ}j)\left[\begin{array}{l}, \\ \\ , \otimes,\end{array}\right], t\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oslash t, t\bigcirc^{\circ}j, t\textcircled{\downarrow},$$

$$, i\obar j, \ \Leftrightarrow\ \sim, i\,!{=}\,\varnothing,$$

$$, i!\obar j, \ \Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l}, \\ \\ , i\oslash t, t!\bigcirc^{\circ}j, t\textcircled{\downarrow},\end{array}\right],$$

18.2.2 Branch function to propositions

$$, if(i\obar j)\left[\begin{array}{l}, \text{\copyright}c, \\ \\ , \otimes,\end{array}\right], \ \Leftrightarrow\ , i\obar j, \text{\copyright}c,$$

$$, if(i\obar j)\left[\begin{array}{l}, \otimes, \\ \\ , \text{\copyright}c,\end{array}\right], \ \Leftrightarrow\ , i!\obar j, \text{\copyright}c,$$

18.2.3 Empty branch function

$$, if(i \obot j) \!-\!\! \left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \; , \!-\!\! \left[\begin{array}{l} , i \obot j, \\ \\ , i ! \obot j, \end{array}\right.$$

18.2.4 Unity

$$, \;\Leftrightarrow\; , if(i \obot j) \!-\!\! \left[\begin{array}{l} , \\ , \end{array}\right] \!-, $$

proof:

$$, if(i \obot j) \!-\!\! \left[\begin{array}{l} , \\ , \end{array}\right] \!-,$$

$$\Leftrightarrow \; , if(i = \varnothing) \!-\!\! \left[\begin{array}{l} , i \owedge n, \\ \\ , i \ovee t, if(t \bigcirc^{\circ\circ} j) \!-\!\! \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \owedge n, \end{array}\right] \!-, t \odownarrow, \end{array}\right] \!-, if(n = \varnothing) \!-\!\! \left[\begin{array}{l} , n \odownarrow, \\ \\ , n \odownarrow, \end{array}\right] \!-,$$

$$\Leftrightarrow \; , if(i = \varnothing) \!-\!\! \left[\begin{array}{l} , i \owedge n, \\ \\ , i \ovee t, if(t \bigcirc^{\circ\circ} j) \!-\!\! \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \owedge n, \end{array}\right] \!-, t \odownarrow, \end{array}\right] \!-, if(n = \varnothing) \!-\!\! \left[\begin{array}{l} , \\ \\ , \end{array}\right] \!-, n \odownarrow,$$

$$\Leftrightarrow \; , if(i = \varnothing) \!-\!\! \left[\begin{array}{l} , i \owedge n, \\ \\ , i \ovee t, if(t \bigcirc^{\circ\circ} j) \!-\!\! \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \owedge n, \end{array}\right] \!-, t \odownarrow, \end{array}\right] \!-, n \odownarrow,$$

$$\Leftrightarrow \; , if(i = \varnothing) \!-\!\! \left[\begin{array}{l} , i \owedge n, n \odownarrow, \\ \\ , i \ovee t, if(t \bigcirc^{\circ\circ} j) \!-\!\! \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \owedge n, \end{array}\right] \!-, t \odownarrow, n \odownarrow, \end{array}\right] \!-,$$

$$\Leftrightarrow \; , if(i = \varnothing) \!-\!\! \left[\begin{array}{l} , \\ \\ , i \ovee t, if(t \bigcirc^{\circ\circ} j) \!-\!\! \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \owedge n, \end{array}\right] \!-, t \odownarrow, n \odownarrow, \end{array}\right] \!-,$$

$$\Leftrightarrow \; , if(i = \varnothing) \!-\!\! \left[\begin{array}{l} , \\ \\ , i \ovee t, if(t \bigcirc^{\circ\circ} j) \!-\!\! \left[\begin{array}{l} , \circledcirc n, n \odownarrow, \\ \\ , i \owedge n, n \odownarrow, \end{array}\right] \!-, t \odownarrow, \end{array}\right] \!-,$$

$$\Leftrightarrow\ , if(i=\varnothing)\begin{cases} , \\ , i\oslash t, if(t\circlearrowleft j)\begin{cases} , \\ , \end{cases}, t\circleddash, \end{cases},$$

$$\Leftrightarrow\ , if(i=\varnothing)\begin{cases} , \\ , i\oslash t, t\circleddash, \end{cases},$$

$$\Leftrightarrow\ , if(i=\varnothing)\begin{cases} , \\ , \end{cases},$$

$$\Leftrightarrow\ ,$$

$$, i\oplus j, \otimes, \ \Leftrightarrow\ , \otimes,$$

$$, i!\oplus j, \otimes, \ \Leftrightarrow\ , \otimes,$$

18.2.5 Swap

Branch function and operator:

$$, \circledcirc m, if(i\oplus j)\begin{cases} , \\ , \end{cases} \Leftrightarrow\ , if(i\oplus j)\begin{cases} , \circledcirc m, \\ , \circledcirc m, \end{cases}$$

proof:

$$, \circledcirc m, if(i\oplus j)\begin{cases} , \\ , \end{cases}$$

$$\Leftrightarrow\ , \circledcirc m, if(i=\varnothing)\begin{cases} , i\oslash n, \\ , i\oslash t, if(t\circlearrowleft j)\begin{cases} , \circledcirc n, \\ , i\oslash n, \end{cases}, t\circleddash, \end{cases}, if(n=\varnothing)\begin{cases} , n\circleddash, \\ , n\circleddash, \end{cases}$$

$$\Leftrightarrow\ , if(i=\varnothing)\begin{cases} , \circledcirc m, i\oslash n, \\ , \circledcirc m, i\oslash t, if(t\circlearrowleft j)\begin{cases} , \circledcirc n, \\ , i\oslash n, \end{cases}, t\circleddash, \end{cases}, if(n=\varnothing)\begin{cases} , n\circleddash, \\ , n\circleddash, \end{cases}$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, \circledcirc m, i\circledast n,\\ , i\otimes t, \circledcirc m, if(t\between j)\left[\begin{array}{l}, \circledcirc n,\\ , i\circledast n,\end{array}\right], t\oplus,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\oplus,\\ , n\oplus,\end{array}\right.$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, \circledcirc m, i\circledast n,\\ , i\otimes t, if(t\between j)\left[\begin{array}{l}, \circledcirc m, \circledcirc n,\\ , \circledcirc m, i\circledast n,\end{array}\right], t\oplus,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\oplus,\\ , n\oplus,\end{array}\right.$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, \circledcirc m, i\circledast n,\\ , i\otimes t, if(t\between j)\left[\begin{array}{l}, \circledcirc n, \circledcirc m,\\ , i\circledast n, \circledcirc m,\end{array}\right], t\oplus,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\oplus,\\ , n\oplus,\end{array}\right.$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, \circledcirc m, i\circledast n,\\ , i\otimes t, if(t\between j)\left[\begin{array}{l}, \circledcirc n,\\ , i\circledast n,\end{array}\right], \circledcirc m, t\oplus,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\oplus,\\ , n\oplus,\end{array}\right.$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, i\circledast n, \circledcirc m,\\ , i\otimes t, if(t\between j)\left[\begin{array}{l}, \circledcirc n,\\ , i\circledast n,\end{array}\right], t\oplus, \circledcirc m,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\oplus,\\ , n\oplus,\end{array}\right.$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, i\circledast n,\\ , i\otimes t, if(t\between j)\left[\begin{array}{l}, \circledcirc n,\\ , i\circledast n,\end{array}\right], t\oplus,\end{array}\right], \circledcirc m, if(n=\varnothing)\left[\begin{array}{l}, n\oplus,\\ , n\oplus,\end{array}\right.$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, i\circledast n,\\ , i\otimes t, if(t\between j)\left[\begin{array}{l}, \circledcirc n,\\ , i\circledast n,\end{array}\right], t\oplus,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, \circledcirc m, n\oplus,\\ , \circledcirc m, n\oplus,\end{array}\right.$$

$$\Leftrightarrow \;, if(i=\varnothing)\left[\begin{array}{l}, i\circledast n,\\ , i\otimes t, if(t\between j)\left[\begin{array}{l}, \circledcirc n,\\ , i\circledast n,\end{array}\right], t\oplus,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\oplus, \circledcirc m,\\ , n\oplus, \circledcirc m,\end{array}\right.$$

$$\Leftrightarrow \;, if(i\ominus j)\left[\begin{array}{l}, \circledcirc m,\\ , \circledcirc m,\end{array}\right.$$

$$,\odot m, if(i\text{⦹}j)\begin{cases} , \\ , \end{cases} \Leftrightarrow \; , if(i\text{⦹}j)\begin{cases} ,\odot m, \\ ,\odot m, \end{cases}$$

$$,m\text{☮}n, if(i\text{⦹}j)\begin{cases} , \\ , \end{cases} \Leftrightarrow \; , if(i\text{⦹}j)\begin{cases} ,m\text{☮}n, \\ ,m\text{☮}n, \end{cases}$$

$$,m\circledast n, if(i\text{⦹}j)\begin{cases} , \\ , \end{cases} \Leftrightarrow \; , if(i\text{⦹}j)\begin{cases} ,m\circledast n, \\ ,m\circledast n, \end{cases}$$

$$,m\oslash n, if(i\text{⦹}j)\begin{cases} , \\ , \end{cases} \Leftrightarrow \; , if(i\text{⦹}j)\begin{cases} ,m\oslash n, \\ ,m\oslash n, \end{cases}$$

$$,m\text{⏚}, if(i\text{⦹}j)\begin{cases} , \\ , \end{cases} \Leftrightarrow \; , if(i\text{⦹}j)\begin{cases} ,m\text{⏚}, \\ ,m\text{⏚}, \end{cases}$$

$$,m\oplus, if(i\text{⦹}j)\begin{cases} , \\ , \end{cases} \Leftrightarrow \; , if(i\text{⦹}j)\begin{cases} ,m\oplus, \\ ,m\oplus, \end{cases}$$

$$,m\ominus, if(i\text{⦹}j)\begin{cases} , \\ , \end{cases} \Leftrightarrow \; , if(i\text{⦹}j)\begin{cases} ,m\ominus, \\ ,m\ominus, \end{cases}$$

Branch function and Branch function:

$$, if(i\text{⦹}j)\begin{cases} , if(m\text{⦹}n)\begin{cases} ,©c_1, \\ ,©c_2, \end{cases}, \\ , if(m\text{⦹}n)\begin{cases} ,©c_3, \\ ,©c_4, \end{cases}, \end{cases}, \Leftrightarrow \; , if(m\text{⦹}n)\begin{cases} , if(i\text{⦹}j)\begin{cases} ,©c_1, \\ ,©c_3, \end{cases}, \\ , if(i\text{⦹}j)\begin{cases} ,©c_2, \\ ,©c_4, \end{cases}, \end{cases},$$

proof:

$$<1> \;\Leftrightarrow\; if(i=\varnothing)\begin{cases} ,i\circledast s_1, \\ ,i\oslash t_1, if(t_1\mathring{\bigcirc}j)\begin{cases} ,\circledcirc s_1, \\ ,i\circledast s_1, \end{cases}, t_1\text{⏚}, \end{cases},$$

$$< 2 > \;\Leftrightarrow\; if(m=\varnothing)\left[\begin{array}{l}, m\circledast s_2, \\ , m\circledast t_2, if(t_2 \mathring{\bigcirc} n)\left[\begin{array}{l}, \circledcirc s_2, \\ , m\circledast s_2,\end{array}\right], t_2\circledast,\end{array}\right],$$

$$, if(i\ominus j)\left[\begin{array}{l}, if(m\ominus n)\left[\begin{array}{l}, \copyright c_1, \\ , \copyright c_2,\end{array}\right], \\ , if(m\ominus n)\left[\begin{array}{l}, \copyright c_3, \\ , \copyright c_4,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\; , < 1 >, if(s_1=\varnothing)\left[\begin{array}{l}, s_1\circledast, < 2 >, if(s_2=\varnothing)\left[\begin{array}{l}, s_2\circledast, \copyright c_1, \\ , s_2\circledast, \copyright c_2,\end{array}\right], \\ , s_1\circledast, < 2 >, if(s_2=\varnothing)\left[\begin{array}{l}, s_2\circledast, \copyright c_3, \\ , s_2\circledast, \copyright c_4,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\; , < 1 >, < 2 >, if(s_1=\varnothing)\left[\begin{array}{l}, if(s_2=\varnothing)\left[\begin{array}{l}, s_1\circledast, s_2\circledast, \copyright c_1, \\ , s_1\circledast, s_2\circledast, \copyright c_2,\end{array}\right], \\ , if(s_2=\varnothing)\left[\begin{array}{l}, s_1\circledast, s_2\circledast, \copyright c_3, \\ , s_1\circledast, s_2\circledast, \copyright c_4,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\; , < 1 >, < 2 >, if(s_2=\varnothing)\left[\begin{array}{l}, if(s_1=\varnothing)\left[\begin{array}{l}, s_1\circledast, s_2\circledast, \copyright c_1, \\ , s_1\circledast, s_2\circledast, \copyright c_3,\end{array}\right], \\ , if(s_1=\varnothing)\left[\begin{array}{l}, s_1\circledast, s_2\circledast, \copyright c_2, \\ , s_1\circledast, s_2\circledast, \copyright c_4,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\; , < 2 >, if(s_2=\varnothing)\left[\begin{array}{l}, s_2\circledast, < 1 >, if(s_1=\varnothing)\left[\begin{array}{l}, s_1\circledast, \copyright c_1, \\ , s_1\circledast, \copyright c_3,\end{array}\right], \\ , s_2\circledast, < 1 >, if(s_1=\varnothing)\left[\begin{array}{l}, s_1\circledast, \copyright c_2, \\ , s_1\circledast, \copyright c_4,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\oplus n)\left[\begin{array}{l} ,if(i\oplus j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right], \\ ,if(i\oplus j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\oplus j)\left[\begin{array}{l} ,if(m\rightarrow n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \\ ,if(m\rightarrow n)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m\rightarrow n)\left[\begin{array}{l} ,if(i\oplus j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right], \\ ,if(i\oplus j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\oplus j)\left[\begin{array}{l} ,if(m\overset{\circ}{\bigcirc}{}^{\circ} n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \\ ,if(m\overset{\circ}{\bigcirc}{}^{\circ} n)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}{}^{\circ} n)\left[\begin{array}{l} ,if(i\oplus j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right], \\ ,if(i\oplus j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\oplus j)\left[\begin{array}{l} ,if(m\overset{\circ}{\bigcirc} n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \\ ,if(m\overset{\circ}{\bigcirc} n)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc} n)\left[\begin{array}{l} ,if(i\oplus j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right], \\ ,if(i\oplus j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\oplus j)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \\ ,if(m=n)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m=n)\left[\begin{array}{l} ,if(i\oplus j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right], \\ ,if(i\oplus j)\left[\begin{array}{l},\copyright c_2,\\ \\ ,\copyright c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\oplus j)\left[\begin{array}{l} ,if(m=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=\varnothing)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \quad \Leftrightarrow \quad ,if(m=\varnothing)\left[\begin{array}{l} ,if(i\oplus j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i\oplus j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

Branch function and propositions:

$$,m\oplus n,if(i\oplus j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad ,if(i\oplus j)\left[\begin{array}{l} ,m\oplus n,\copyright c_1, \\ \\ ,m\oplus n,\copyright c_2, \end{array}\right],$$

$$,m!\oplus n,if(i\oplus j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad ,if(i\oplus j)\left[\begin{array}{l} ,m!\oplus n,\copyright c_1, \\ \\ ,m!\oplus n,\copyright c_2, \end{array}\right],$$

$$,m\rightarrow n,if(i\oplus j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad ,if(i\oplus j)\left[\begin{array}{l} ,m\rightarrow n,\copyright c_1, \\ \\ ,m\rightarrow n,\copyright c_2, \end{array}\right],$$

$$,m!\rightarrow n,if(i\oplus j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad ,if(i\oplus j)\left[\begin{array}{l} ,m!\rightarrow n,\copyright c_1, \\ \\ ,m!\rightarrow n,\copyright c_2, \end{array}\right],$$

$$,m\circledcirc n,if(i\oplus j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad ,if(i\oplus j)\left[\begin{array}{l} ,m\circledcirc n,\copyright c_1, \\ \\ ,m\circledcirc n,\copyright c_2, \end{array}\right],$$

$$, m!\bigcirc^{\!\circ\circ} n, if(i \obot j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \obot j)\!-\!\left[\begin{array}{c}, m!\bigcirc^{\!\circ\circ} n, ©c_1, \\ \\ , m!\bigcirc^{\!\circ\circ} n, ©c_2,\end{array}\right]\!-,$$

$$, m\bigcirc^{\!\circ} n, if(i \obot j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \obot j)\!-\!\left[\begin{array}{c}, m\bigcirc^{\!\circ} n, ©c_1, \\ \\ , m\bigcirc^{\!\circ} n, ©c_2,\end{array}\right]\!-,$$

$$, m!\bigcirc^{\!\circ} n, if(i \obot j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \obot j)\!-\!\left[\begin{array}{c}, m!\bigcirc^{\!\circ} n, ©c_1, \\ \\ , m!\bigcirc^{\!\circ} n, ©c_2,\end{array}\right]\!-,$$

$$, m = n, if(i \obot j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \obot j)\!-\!\left[\begin{array}{c}, m = n, ©c_1, \\ \\ , m = n, ©c_2,\end{array}\right]\!-,$$

$$, m \mathrel{!=} n, if(i \obot j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \obot j)\!-\!\left[\begin{array}{c}, m \mathrel{!=} n, ©c_1, \\ \\ , m \mathrel{!=} n, ©c_2,\end{array}\right]\!-,$$

$$, m = \varnothing, if(i \obot j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \obot j)\!-\!\left[\begin{array}{c}, m = \varnothing, ©c_1, \\ \\ , m = \varnothing, ©c_2,\end{array}\right]\!-,$$

$$, m \mathrel{!=} \varnothing, if(i \obot j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \obot j)\!-\!\left[\begin{array}{c}, m \mathrel{!=} \varnothing, ©c_1, \\ \\ , m \mathrel{!=} \varnothing, ©c_2,\end{array}\right]\!-,$$

$$, m \obot n, if(i \bigcirc^{\!\circ\circ} j)\!-\!\left[\begin{array}{c}, ©c_1, \\ \\ , ©c_2,\end{array}\right]\!-, \quad \Leftrightarrow \quad , if(i \bigcirc^{\!\circ\circ} j)\!-\!\left[\begin{array}{c}, m \obot n, ©c_1, \\ \\ , m \obot n, ©c_2,\end{array}\right]\!-,$$

$$, m!\circledperp n, if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l}, \copyright c_1,\\ \\, \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\overset{\circ\circ}{\bigcirc}j)\left[\begin{array}{l}, m!\circledperp n, \copyright c_1,\\ \\, m!\circledperp n, \copyright c_2,\end{array}\right],$$

$$, m\circledperp n, if(i\rightarrow j)\left[\begin{array}{l}, \copyright c_1,\\ \\, \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\rightarrow j)\left[\begin{array}{l}, m\circledperp n, \copyright c_1,\\ \\, m\circledperp n, \copyright c_2,\end{array}\right],$$

$$, m!\circledperp n, if(i\rightarrow j)\left[\begin{array}{l}, \copyright c_1,\\ \\, \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\rightarrow j)\left[\begin{array}{l}, m!\circledperp n, \copyright c_1,\\ \\, m!\circledperp n, \copyright c_2,\end{array}\right],$$

$$, m\circledperp n, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l}, \copyright c_1,\\ \\, \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l}, m\circledperp n, \copyright c_1,\\ \\, m\circledperp n, \copyright c_2,\end{array}\right],$$

$$, m!\circledperp n, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l}, \copyright c_1,\\ \\, \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l}, m!\circledperp n, \copyright c_1,\\ \\, m!\circledperp n, \copyright c_2,\end{array}\right],$$

$$, m\circledperp n, if(i=j)\left[\begin{array}{l}, \copyright c_1,\\ \\, \copyright c_2,\end{array}\right], \Leftrightarrow , if(i=j)\left[\begin{array}{l}, m\circledperp n, \copyright c_1,\\ \\, m\circledperp n, \copyright c_2,\end{array}\right],$$

$$, m!\circledperp n, if(i=j)\left[\begin{array}{l}, \copyright c_1,\\ \\, \copyright c_2,\end{array}\right], \Leftrightarrow , if(i=j)\left[\begin{array}{l}, m!\circledperp n, \copyright c_1,\\ \\, m!\circledperp n, \copyright c_2,\end{array}\right],$$

$$, m\oplus n, if(i=\varnothing)-\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-, \iff , if(i=\varnothing)-\left[\begin{array}{l} , m\oplus n, \copyright c_1, \\ \\ , m\oplus n, \copyright c_2, \end{array}\right]-,$$

$$, m!\oplus n, if(i=\varnothing)-\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-, \iff , if(i=\varnothing)-\left[\begin{array}{l} , m!\oplus n, \copyright c_1, \\ \\ , m!\oplus n, \copyright c_2, \end{array}\right]-,$$

Branch function and recursive function:

$$, R(m), if(i\oplus j)-\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-, \iff , if(i\oplus j)-\left[\begin{array}{l} , R(m), \copyright c_1, \\ \\ , R(m), \copyright c_2, \end{array}\right]-,$$

$$, R_{-}(m), if(i\oplus j)-\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-, \iff , if(i\oplus j)-\left[\begin{array}{l} , R_{-}(m), \copyright c_1, \\ \\ , R_{-}(m), \copyright c_2, \end{array}\right]-,$$

Branch function and flag object :

$$, \&SHi\mathring{\bigcirc} m, if(i\oplus j)-\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-, \iff , if(i\oplus j)-\left[\begin{array}{l} , \&SHi\mathring{\bigcirc} m, \copyright c_1, \\ \\ , \&SHi\mathring{\bigcirc} m, \copyright c_2, \end{array}\right]-,$$

$$, \&SHi\rightarrow m, if(i\oplus j)-\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]-, \iff , if(i\oplus j)-\left[\begin{array}{l} , \&SHi\rightarrow m, \copyright c_1, \\ \\ , \&SHi\rightarrow m, \copyright c_2, \end{array}\right]-,$$

$$, \&SHj\,\mathring{\bigcirc}m, if(i⦹j)-\left[\begin{array}{c} , ©c_1, \\ \\ , ©c_2, \end{array}\right]-, \Leftrightarrow ,if(i⦹j)-\left[\begin{array}{c} , \&SHj\,\mathring{\bigcirc}m, ©c_1, \\ \\ , \&SHj\,\mathring{\bigcirc}m, ©c_2, \end{array}\right]-,$$

$$, \&SHj \leftarrow m, if(i⦹j)-\left[\begin{array}{c} , ©c_1, \\ \\ , ©c_2, \end{array}\right]-, \Leftrightarrow ,if(i⦹j)-\left[\begin{array}{c} , \&SHj \leftarrow m, ©c_1, \\ \\ , \&SHj \leftarrow m, ©c_2, \end{array}\right]-,$$

Propositions and operator:

$$, i⦹j, \circledcirc m, \Leftrightarrow , \circledcirc m, i⦹j,$$
$$, i⦹j, \odot m, \Leftrightarrow , \odot m, i⦹j,$$

$$, i⦹j, m☮n, \Leftrightarrow , m☮n, i⦹j,$$
$$, i⦹j, m☮n, \Leftrightarrow , m☮n, i⦹j,$$
$$, i⦹j, m☮n, \Leftrightarrow , m☮n, i⦹j,$$

$$, i⦹j, m⏚, \Leftrightarrow , m⏚, i⦹j,$$
$$, i⦹j, m\oplus, \Leftrightarrow , m\oplus, i⦹j,$$
$$, i⦹j, m\ominus, \Leftrightarrow , m\ominus, i⦹j,$$

$$, i!⦹j, \circledcirc m, \Leftrightarrow , \circledcirc m, i!⦹j,$$
$$, i!⦹j, \odot m, \Leftrightarrow , \odot m, i!⦹j,$$

$$, i!⦹j, m☮n, \Leftrightarrow , m☮n, i!⦹j,$$
$$, i!⦹j, m☮n, \Leftrightarrow , m☮n, i!⦹j,$$

$$, i!\operp j, m\mathbin{☮}n, \iff , m\mathbin{☮}n, i!\operp j,$$

$$, i!\operp j, m\mathbin{⏚}, \iff , m\mathbin{⏚}, i!\operp j,$$
$$, i!\operp j, m\oplus, \iff , m\oplus, i!\operp j,$$
$$, i!\operp j, m\ominus, \iff , m\ominus, i!\operp j,$$

Propositions and Propositions:

$$, i\operp j, m\operp n, \iff , m\operp n, i\operp j,$$
$$, i\operp j, m!\operp n, \iff , m!\operp n, i\operp j,$$
$$, i!\operp j, m!\operp n, \iff , m!\operp n, i!\operp j,$$

$$, i\operp j, m{\rightarrow}n, \iff , m{\rightarrow}n, i\operp j,$$
$$, i\operp j, m!{\rightarrow}n, \iff , m!{\rightarrow}n, i\operp j,$$
$$, i!\operp j, m{\rightarrow}n, \iff , m{\rightarrow}n, i!\operp j,$$
$$, i!\operp j, m!{\rightarrow}n, \iff , m!{\rightarrow}n, i!\operp j,$$

$$, i\operp j, m\mathbin{⚮}n, \iff , m\mathbin{⚮}n, i\operp j,$$
$$, i\operp j, m!\mathbin{⚮}n, \iff , m!\mathbin{⚮}n, i\operp j,$$
$$, i!\operp j, m\mathbin{⚮}n, \iff , m\mathbin{⚮}n, i!\operp j,$$
$$, i!\operp j, m!\mathbin{⚮}n, \iff , m!\mathbin{⚮}n, i!\operp j,$$

$$, i\operp j, m\mathbin{♂}n, \iff , m\mathbin{♂}n, i\operp j,$$
$$, i\operp j, m!\mathbin{♂}n, \iff , m!\mathbin{♂}n, i\operp j,$$
$$, i!\operp j, m\mathbin{♂}n, \iff , m\mathbin{♂}n, i!\operp j,$$
$$, i!\operp j, m!\mathbin{♂}n, \iff , m!\mathbin{♂}n, i!\operp j,$$

$$, i\operp j, m{=}n, \iff , m{=}n, i\operp j,$$
$$, i\operp j, m\,!{=}n, \iff , m\,!{=}n, i\operp j,$$
$$, i!\operp j, m{=}n, \iff , m{=}n, i!\operp j,$$
$$, i!\operp j, m\,!{=}n, \iff , m\,!{=}n, i!\operp j,$$

$$
\begin{aligned}
,i\obot j, m = \varnothing, &\iff ,m = \varnothing, i\obot j,\\
,i\obot j, m \mathrel{!=} \varnothing, &\iff ,m \mathrel{!=} \varnothing, i\obot j,\\
,i!\obot j, m = \varnothing, &\iff ,m = \varnothing, i!\obot j,\\
,i!\obot j, m \mathrel{!=} \varnothing, &\iff ,m \mathrel{!=} \varnothing, i!\obot j,
\end{aligned}
$$

Propositions and recursive function:

$$
\begin{aligned}
,i\obot j, R(m), &\iff ,R(m), i\obot j,\\
,i\obot j, R_{-}(m), &\iff ,R_{-}(m), i\obot j,\\
,i!\obot j, R(m), &\iff ,R(m), i!\obot j,\\
,i!\obot j, R_{-}(m), &\iff ,R_{-}(m), i!\obot j,
\end{aligned}
$$

Propositions and flag object:

$$
\begin{aligned}
,i\obot j,\ \&SHi\male m, &\iff ,\ \&SHi\male m, i\obot j,\\
,i\obot j,\ \&SHi\rightarrow m, &\iff ,\ \&SHi\rightarrow m, i\obot j,\\
,i!\obot j,\ \&SHi\male m, &\iff ,\ \&SHi\male m, i!\obot j,\\
,i!\obot j,\ \&SHi\rightarrow m, &\iff ,\ \&SHi\rightarrow m, i!\obot j,
\end{aligned}
$$

$$
\begin{aligned}
,i\obot j,\ \&SHj\male m, &\iff ,\ \&SHj\male m, i\obot j,\\
,i\obot j,\ \&SHj\leftarrow m, &\iff ,\ \&SHj\leftarrow m, i\obot j,\\
,i!\obot j,\ \&SHj\male m, &\iff ,\ \&SHj\male m, i!\obot j,\\
,i!\obot j,\ \&SHj\leftarrow m, &\iff ,\ \&SHj\leftarrow m, i!\obot j,
\end{aligned}
$$

Propositions to Propositions with branch function

$$, if(i \obar j) \left[\begin{array}{l} , m ! \obar n, \\ \\ , \end{array} \right], \iff , if(m \obar n) \left[\begin{array}{l} , i ! \obar j, \\ \\ , \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , \\ \\ , m \obar n, \end{array} \right], \iff , if(m \obar n) \left[\begin{array}{l} , \\ \\ , i \obar j, \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , m ! \rightarrow n, \\ \\ , \end{array} \right], \iff , if(m \rightarrow n) \left[\begin{array}{l} , i ! \obar j, \\ \\ , \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , \\ \\ , m \rightarrow n, \end{array} \right], \iff , if(m \rightarrow n) \left[\begin{array}{l} , \\ \\ , i \obar j, \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , m ! \mathring{\bigcirc}\!{\circ} n, \\ \\ , \end{array} \right], \iff , if(m \mathring{\bigcirc}\!{\circ} n) \left[\begin{array}{l} , i ! \obar j, \\ \\ , \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , \\ \\ , m \mathring{\bigcirc}\!{\circ} n, \end{array} \right], \iff , if(m \mathring{\bigcirc}\!{\circ} n) \left[\begin{array}{l} , \\ \\ , i \obar j, \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , m ! \mathring{\bigcirc} n, \\ \\ , \end{array} \right], \iff , if(m \mathring{\bigcirc} n) \left[\begin{array}{l} , i ! \obar j, \\ \\ , \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , \\ \\ , m \mathring{\bigcirc} n, \end{array} \right], \iff , if(m \mathring{\bigcirc} n) \left[\begin{array}{l} , \\ \\ , i \obar j, \end{array} \right],$$

$$, if(i \obar j) \left[\begin{array}{l} , m \,!\!= n, \\ \\ , \end{array} \right], \iff , if(m = n) \left[\begin{array}{l} , i ! \obar j, \\ \\ , \end{array} \right],$$

$$,if(i\oplus j)\text{-}\left[\begin{array}{l},\\ ,m=n,\end{array}\right]\text{-}, \;\Leftrightarrow\; ,if(m=n)\text{-}\left[\begin{array}{l},\\ ,i\oplus j,\end{array}\right]\text{-},$$

$$,if(i\oplus j)\text{-}\left[\begin{array}{l},m\,!=\varnothing,\\ ,\end{array}\right]\text{-}, \;\Leftrightarrow\; ,if(m=\varnothing)\text{-}\left[\begin{array}{l},i!\oplus j,\\ ,\end{array}\right]\text{-},$$

$$,if(i\oplus j)\text{-}\left[\begin{array}{l},\\ ,m=\varnothing,\end{array}\right]\text{-}, \;\Leftrightarrow\; ,if(m=\varnothing)\text{-}\left[\begin{array}{l},\\ ,i\oplus j,\end{array}\right]\text{-},$$

18.2.6 Swap of the same operand

(skip......)

18.2.7 Transitivity

Branch function with branch function:

$$,if(i\oplus j)\text{-}\left[\begin{array}{l},©c_1,\\ ,©c_2,\end{array}\right]\text{-}, \;\Leftrightarrow\; ,if(i\oplus j)\text{-}\left[\begin{array}{l},if(i\oplus j)\text{-}\left[\begin{array}{l},©c_1,\\ ,©c_3,\end{array}\right]\text{-},\\ ,©c_2,\end{array}\right]\text{-},$$

$$,if(i\oplus j)\text{-}\left[\begin{array}{l},©c_1,\\ ,©c_2,\end{array}\right]\text{-}, \;\Leftrightarrow\; ,if(i\oplus j)\text{-}\left[\begin{array}{l},©c_1,\\ ,if(i\oplus j)\text{-}\left[\begin{array}{l},©c_3,\\ ,©c_2,\end{array}\right]\text{-},\end{array}\right]\text{-},$$

proof:

$$<1> \;\Leftrightarrow\; ,i\circledast t_1, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, \\ \\ ,i\odot s_1, \end{array}\right]-,t_1\ominus,$$

$$<2> \;\Leftrightarrow\; ,i\circledast t_2, if(t_2\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-,t_2\ominus,$$

$<1>, <2>$

$$\Leftrightarrow\; ,i\circledast t_1, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, \\ \\ ,i\odot s_1, \end{array}\right]-,t_1\ominus, i\circledast t_2, if(t_2\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-,t_2\ominus,$$

$$\Leftrightarrow\; ,i\circledast t_1, i\circledast t_2, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, \\ \\ ,i\odot s_1, \end{array}\right]-, if(t_2\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-,t_1\ominus, t_2\ominus,$$

$$\Leftrightarrow\; ,i\circledast t_1, i\circledast t_2, t_1\mathring{\bigcirc} t_2, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, \\ \\ ,i\odot s_1, \end{array}\right]-, if(t_2\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-,t_1\ominus, t_2\ominus,$$

$$\Leftrightarrow\; ,i\circledast t_1, i\circledast t_2, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, \\ \\ ,i\odot s_1, \end{array}\right]-, t_1\mathring{\bigcirc} t_2, if(t_2\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-,t_1\ominus, t_2\ominus,$$

$$\Leftrightarrow\; ,i\circledast t_1, i\circledast t_2, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, \\ \\ ,i\odot s_1, \end{array}\right]-, t_1\mathring{\bigcirc} t_2, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-,t_1\ominus, t_2\ominus,$$

$$\Leftrightarrow\; ,i\circledast t_1, i\circledast t_2, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, \\ \\ ,i\odot s_1, \end{array}\right]-, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-,t_1\ominus, t_2\ominus,$$

$$\Leftrightarrow\; ,i\circledast t_1, i\circledast t_2, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_1, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-, \\ \\ ,i\odot s_1, if(t_1\bigcirc^{\circ\circ} j)-\left[\begin{array}{l} ,\circledcirc s_2, \\ \\ ,i\odot s_2, \end{array}\right]-, \end{array}\right]-,t_1\ominus, t_2\ominus,$$

$$\Leftrightarrow\ , i\circledast t_1, i\circledast t_2, if(t_1 \bigcirc j)\left[\begin{array}{l} , if(t_1 \bigcirc j)\left[\begin{array}{l} , \circledcirc s_1, \circledcirc s_2, \\ \\ , \circledcirc s_1, i\otimes s_2, \end{array}\right], \\ , if(t_1 \bigcirc j)\left[\begin{array}{l} , i\otimes s_1, \circledcirc s_2, \\ \\ , i\otimes s_1, i\otimes s_2, \end{array}\right], \end{array}\right], t_1\oslash, t_2\oslash,$$

$$\Leftrightarrow\ , i\circledast t_1, i\circledast t_2, if(t_1 \bigcirc j)\left[\begin{array}{l} , \circledcirc s_1, \circledcirc s_2, \\ \\ , i\otimes s_1, i\otimes s_2, \end{array}\right], t_1\oslash, t_2\oslash,$$

$$\Leftrightarrow\ , i\circledast t_1, i\circledast t_2, if(t_1 \bigcirc j)\left[\begin{array}{l} , \circledcirc s_1, \circledcirc s_2, s_1 = s_2, \\ \\ , i\otimes s_1, i\otimes s_2, s_1 = s_2, \end{array}\right], t_1\oslash, t_2\oslash,$$

$$\Leftrightarrow\ , i\circledast t_1, i\circledast t_2, if(t_1 \bigcirc j)\left[\begin{array}{l} , \circledcirc s_1, \circledcirc s_2, \\ \\ , i\otimes s_1, i\otimes s_2, \end{array}\right], s_1 = s_2, t_1\oslash, t_2\oslash,$$

$$\Leftrightarrow\ , <1>, <2>, s_1 = s_2,$$

$<2>, s_2\oslash,$

$$\Leftrightarrow\ , i\circledast t_2, if(t_2 \bigcirc j)\left[\begin{array}{l} , \circledcirc s_2, \\ \\ , i\otimes s_2, \end{array}\right], t_2\oslash, s_2\oslash,$$

$$\Leftrightarrow\ , i\circledast t_2, if(t_2 \bigcirc j)\left[\begin{array}{l} , \circledcirc s_2, s_2\oslash \\ \\ , i\otimes s_2, s_2\oslash \end{array}\right], t_2\oslash, ,$$

$$\Leftrightarrow\ , i\circledast t_2, if(t_2 \bigcirc j)\left[\begin{array}{l} , \\ \\ , \end{array}\right], t_2\oslash, ,$$

$$\Leftrightarrow\ , i\circledast t_2, t_2\oslash,$$

$$\Leftrightarrow\ ,$$

$$, if(i\ominus j)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1, \\ ,i\text{☮}t_1,if(t_1\text{☌}j)\left[\begin{array}{l} ,\circledcirc s_1, \\ ,i\text{☮}s_1, \end{array}\right],t_1\text{⏚}, \end{array}\right],if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_1, \\ ,s_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1, \\ ,<1>, \end{array}\right],if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_1, \\ ,s_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1,i\text{☮}s_2,s_2\text{⏚}, \\ ,<1>,<2>,s_2\text{⏚}, \end{array}\right],if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_1, \\ ,s_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1,i\text{☮}s_2,s_1=s_2,s_2\text{⏚}, \\ ,<1>,<2>,s_1=s_2,s_2\text{⏚}, \end{array}\right],if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_1, \\ ,s_1\text{⏚},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1,i\text{☮}s_2,s_1=s_2,s_2\text{⏚}, \\ ,<1>,<2>,s_1=s_2,s_2\text{⏚}, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_1, \\ ,if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_3, \\ ,s_1\text{⏚},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1,i\text{☮}s_2, \\ ,<1>,<2>, \end{array}\right],s_1=s_2,s_2\text{⏚},$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_1, \\ ,if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\text{⏚},\copyright c_3, \\ ,s_1\text{⏚},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1,i\text{☮}s_2, \\ ,<1>,<2>, \end{array}\right],s_1=s_2,$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_2\text{⏚},s_1\text{⏚},\copyright c_1, \\ ,if(s_1=\varnothing)\left[\begin{array}{l} ,s_2\text{⏚},s_1\text{⏚},\copyright c_3, \\ ,s_2\text{⏚},s_1\text{⏚},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}s_1,i\text{☮}s_2, \\ ,<1>,<2>, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_1=s_2,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_1, \\ ,s_1=s_2,if(s_1=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_3, \\ \\ ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\wedge}s_1,i\textcircled{\wedge}s_2, \\ \\ ,<1>,<2>, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_1=s_2,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_1, \\ ,s_1=s_2,if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_3, \\ \\ ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\wedge}s_1,i\textcircled{\wedge}s_2, \\ \\ ,<1>,<2>, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_1, \\ ,if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_3, \\ \\ ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\wedge}s_1,i\textcircled{\wedge}s_2, \\ \\ ,i\textcircled{\wedge}s_1,<2>, \end{array}\right], \\ ,if(i=\varnothing)\left[\begin{array}{l} ,<1>,i\textcircled{\wedge}s_2, \\ \\ ,<1>,<2>, \end{array}\right], \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_1, \\ ,if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_3, \\ \\ ,s_2\textcircled{\downarrow},s_1\textcircled{\downarrow},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\wedge}s_1,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\wedge}s_2, \\ \\ ,<2>, \end{array}\right], \\ ,<1>,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\wedge}s_2, \\ \\ ,<2>, \end{array}\right], \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_1, \\ ,if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_3, \\ ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_1, \\ ,<1>, \end{array}\right],if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_2, \\ ,<2>, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_1, \\ ,if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_3, \\ ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_1, \\ ,<1>, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_2, \\ ,<2>, \end{array}\right],s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_1, \\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_2, \\ ,<2>, \end{array}\right],if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_3, \\ ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_1, \\ ,<1>, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_2,s_2\textcircled{\scriptsize$\downarrow$}, \\ ,<2>,s_2\textcircled{\scriptsize$\downarrow$}, \end{array}\right],s_1\textcircled{\scriptsize$\downarrow$},\copyright c_1, \\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_2, \\ ,<2>, \end{array}\right],if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_3, \\ ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_1, \\ ,<1>, \end{array}\right],$$

$$if(s_1=\varnothing)\left[\begin{array}{l} ,s_1\textcircled{\scriptsize$\downarrow$},\copyright c_1, \\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\textcircled{\scriptsize$\uparrow$}s_2, \\ ,<2>, \end{array}\right],if(s_2=\varnothing)\left[\begin{array}{l} ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_3, \\ ,s_2\textcircled{\scriptsize$\downarrow$},s_1\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\text{-}\left[\begin{array}{l} ,i\circledcirc s_1, \\ \\ ,<1>, \end{array}\right]\text{-},$$

$$if(s_1=\varnothing)\text{-}\left[\begin{array}{l} ,s_1\textcircled{\downarrow},\ \copyright c_1, \\ ,s_1\textcircled{\downarrow},if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_2, \end{array}\right]\text{-}, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ ,if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_2, \end{array}\right]\text{-}, \end{array}\right]\text{-},$$

Branch function with propositions:

$$,if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},\ \Leftrightarrow\ ,if(i\obot j)\text{-}\left[\begin{array}{l} ,i\obot j,\ \copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},$$

$$,if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},\ \Leftrightarrow\ ,if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,i!\obot j,\ \copyright c_2, \end{array}\right]\text{-},$$

Propositions with branch function:

$$,i\obot j,if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},\ \Leftrightarrow\ ,i\obot j,\ \copyright c_1,$$

$$,i!\obot j,if(i\obot j)\text{-}\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},\ \Leftrightarrow\ ,i!\obot j,\ \copyright c_2,$$

Propositions with propositions:

$$,i\oplus j, \;\Leftrightarrow\; ,i\oplus j, i\oplus j,$$

$$,i!\oplus j, \;\Leftrightarrow\; ,i!\oplus j, i!\oplus j,$$

18.2.8 Substitution

Identical node Propositions with branch function:

$$,i\overset{\circ}{\bigcirc}j, if(i\oplus t)\!-\!\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!-\!, \;\Leftrightarrow\; ,i\overset{\circ}{\bigcirc}j, if(j\oplus t)\!-\!\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!-\!,$$

proof:

$$,i\overset{\circ}{\bigcirc}j, if(i\oplus t)\!-\!\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\!-\!,$$

$$\Leftrightarrow\; ,i\overset{\circ}{\bigcirc}j, if(i=\varnothing)\!-\!\left[\begin{array}{l} ,i\otimes n, \\ ,i\otimes m, if(m\overset{\circ}{\bigcirc}\!\circ t)\!-\!\left[\begin{array}{l} ,\circledcirc n, \\ \\ ,i\otimes n, \end{array}\right]\!-\!, m\oslash, \end{array}\right]\!-\!, if(n=\varnothing)\!-\!\left[\begin{array}{l} ,n\oslash, \copyright c_1, \\ \\ ,n\oslash, \copyright c_2, \end{array}\right]\!-\!,$$

$$\Leftrightarrow\; ,i\overset{\circ}{\bigcirc}j, if(j=\varnothing)\!-\!\left[\begin{array}{l} ,i\otimes n, \\ ,i\otimes m, if(m\overset{\circ}{\bigcirc}\!\circ t)\!-\!\left[\begin{array}{l} ,\circledcirc n, \\ \\ ,i\otimes n, \end{array}\right]\!-\!, m\oslash, \end{array}\right]\!-\!, if(n=\varnothing)\!-\!\left[\begin{array}{l} ,n\oslash, \copyright c_1, \\ \\ ,n\oslash, \copyright c_2, \end{array}\right]\!-\!,$$

$$\Leftrightarrow\; ,if(j=\varnothing)\!-\!\left[\begin{array}{l} ,i\overset{\circ}{\bigcirc}j, i\otimes n, \\ ,i\overset{\circ}{\bigcirc}j, i\otimes m, if(m\overset{\circ}{\bigcirc}\!\circ t)\!-\!\left[\begin{array}{l} ,\circledcirc n, \\ \\ ,i\otimes n, \end{array}\right]\!-\!, m\oslash, \end{array}\right]\!-\!, if(n=\varnothing)\!-\!\left[\begin{array}{l} ,n\oslash, \copyright c_1, \\ \\ ,n\oslash, \copyright c_2, \end{array}\right]\!-\!,$$

$$\Leftrightarrow\; ,if(j=\varnothing)\!-\!\left[\begin{array}{l} ,i\overset{\circ}{\bigcirc}j, j\otimes n, \\ ,i\overset{\circ}{\bigcirc}j, j\otimes m, if(m\overset{\circ}{\bigcirc}\!\circ t)\!-\!\left[\begin{array}{l} ,\circledcirc n, \\ \\ ,i\otimes n, \end{array}\right]\!-\!, m\oslash, \end{array}\right]\!-\!, if(n=\varnothing)\!-\!\left[\begin{array}{l} ,n\oslash, \copyright c_1, \\ \\ ,n\oslash, \copyright c_2, \end{array}\right]\!-\!,$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i\mathring{\bigcirc}j, j\circledast n,\\ , j\circledast m, if(m\bigcirc^{\!\circ}t)\left[\begin{array}{l}, i\mathring{\bigcirc}j, \circledcirc n,\\ , i\mathring{\bigcirc}j, i\circledast n,\end{array}\right], m\ovee,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\ovee, \copyright c_1,\\ , n\ovee, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i\mathring{\bigcirc}j, j\circledast n,\\ , j\circledast m, if(m\bigcirc^{\!\circ}t)\left[\begin{array}{l}, i\mathring{\bigcirc}j, \circledcirc n,\\ , i\mathring{\bigcirc}j, j\circledast n,\end{array}\right], m\ovee,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\ovee, \copyright c_1,\\ , n\ovee, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}j, if(j=\varnothing)\left[\begin{array}{l}, j\circledast n,\\ , j\circledast m, if(m\bigcirc^{\!\circ}t)\left[\begin{array}{l}, \circledcirc n,\\ , j\circledast n,\end{array}\right], m\ovee,\end{array}\right], if(n=\varnothing)\left[\begin{array}{l}, n\ovee, \copyright c_1,\\ , n\ovee, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}j, if(j\obot t)\left[\begin{array}{l}, \copyright c_1,\\ , \copyright c_2,\end{array}\right],$$

Propositions with propositions:

$$, i\mathring{\bigcirc}j, i\obot t, \ \Leftrightarrow\ , i\mathring{\bigcirc}j, j\obot t,$$
$$, i\mathring{\bigcirc}j, i!\obot t, \ \Leftrightarrow\ , i\mathring{\bigcirc}j, j!\obot t,$$

Node connectivity Propositions with branch function:

$$, i\bigcirc^{\!\circ}j, if(t\obot i)\left[\begin{array}{l}, \copyright c_1,\\ , \copyright c_2,\end{array}\right], \ \Leftrightarrow\ , i\bigcirc^{\!\circ}j, if(t\obot j)\left[\begin{array}{l}, \copyright c_1,\\ , \copyright c_2,\end{array}\right],$$

proof:

$$, i\bigcirc^{\!\circ}j, if(t\obot i)\left[\begin{array}{l}, \copyright c_1,\\ , \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\mathring{\bigcirc}j,if(t{=}\varnothing)\left[\begin{array}{l},t\circledast n,\\ ,t\otimes m,if(m\mathring{\bigcirc}i)\left[\begin{array}{l},\circledcirc n,\\ ,t\circledast n,\end{array}\right],m\ominus,\end{array}\right],if(n{=}\varnothing)\left[\begin{array}{l},n\ominus,\copyright c_1,\\ ,n\ominus,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(t{=}\varnothing)\left[\begin{array}{l},i\mathring{\bigcirc}j,t\circledast n,\\ ,i\mathring{\bigcirc}j,t\otimes m,if(m\mathring{\bigcirc}i)\left[\begin{array}{l},\circledcirc n,\\ ,t\circledast n,\end{array}\right],m\ominus,\end{array}\right],if(n{=}\varnothing)\left[\begin{array}{l},n\ominus,\copyright c_1,\\ ,n\ominus,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(t{=}\varnothing)\left[\begin{array}{l},i\mathring{\bigcirc}j,t\circledast n,\\ ,t\otimes m,i\mathring{\bigcirc}j,if(m\mathring{\bigcirc}i)\left[\begin{array}{l},\circledcirc n,\\ ,t\circledast n,\end{array}\right],m\ominus,\end{array}\right],if(n{=}\varnothing)\left[\begin{array}{l},n\ominus,\copyright c_1,\\ ,n\ominus,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(t{=}\varnothing)\left[\begin{array}{l},i\mathring{\bigcirc}j,t\circledast n,\\ ,t\otimes m,i\mathring{\bigcirc}j,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},\circledcirc n,\\ ,t\circledast n,\end{array}\right],m\ominus,\end{array}\right],if(n{=}\varnothing)\left[\begin{array}{l},n\ominus,\copyright c_1,\\ ,n\ominus,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\mathring{\bigcirc}j,if(t{=}\varnothing)\left[\begin{array}{l},t\circledast n,\\ ,t\otimes m,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},\circledcirc n,\\ ,t\circledast n,\end{array}\right],m\ominus,\end{array}\right],if(n{=}\varnothing)\left[\begin{array}{l},n\ominus,\copyright c_1,\\ ,n\ominus,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\mathring{\bigcirc}j,if(t\oplus j)\left[\begin{array}{l},\copyright c_1,\\ ,\copyright c_2,\end{array}\right],$$

Propositions with propositions:

$$,i\mathring{\bigcirc}j,t\oplus i,\ \Leftrightarrow\ ,i\mathring{\bigcirc}j,t\oplus j,$$
$$,i\mathring{\bigcirc}j,t!\oplus i,\ \Leftrightarrow\ ,i\mathring{\bigcirc}j,t!\oplus j,$$

Subnode Propositions with branch function:

$$,i\text{⦹}j,if(i\text{⦹}t)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \iff ,i\text{⦹}j,if(j\overset{\circ\circ}{\bigcirc}t)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right],$$

proof:

$$,i\text{⦹}j,if(i\text{⦹}t)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right],$$

$$\iff ,i\overset{\circ}{\bigcirc}j,if(i=\varnothing)\left[\begin{array}{l},i\text{☮}n,\\ ,i\text{☮}m,if(m\overset{\circ\circ}{\bigcirc}t)\left[\begin{array}{l},\circledcirc n,\\ \\ ,i\text{☮}n,\end{array}\right],m\text{⏚},\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\text{⏚},\text{©}c_1,\\ \\ ,n\text{⏚},\text{©}c_2,\end{array}\right],$$

$$\iff ,i\,!=\varnothing,i\text{☮}m_1,m_1\overset{\circ\circ}{\bigcirc}j,m_1\text{⏚},$$

$$if(i=\varnothing)\left[\begin{array}{l},i\text{☮}n,\\ ,i\text{☮}m,if(m\overset{\circ\circ}{\bigcirc}t)\left[\begin{array}{l},\circledcirc n,\\ \\ ,i\text{☮}n,\end{array}\right],m\text{⏚},\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\text{⏚},\text{©}c_1,\\ \\ ,n\text{⏚},\text{©}c_2,\end{array}\right],$$

$$\iff ,i\text{☮}m_1,m_1\overset{\circ\circ}{\bigcirc}j,m_1\text{⏚},i\,!=\varnothing,$$

$$if(i=\varnothing)\left[\begin{array}{l},i\text{☮}n,\\ ,i\text{☮}m,if(m\overset{\circ\circ}{\bigcirc}t)\left[\begin{array}{l},\circledcirc n,\\ \\ ,i\text{☮}n,\end{array}\right],m\text{⏚},\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\text{⏚},\text{©}c_1,\\ \\ ,n\text{⏚},\text{©}c_2,\end{array}\right],$$

$$\iff ,i\text{☮}m_1,m_1\overset{\circ\circ}{\bigcirc}j,m_1\text{⏚},i\,!=\varnothing,i\text{☮}m,if(m\overset{\circ\circ}{\bigcirc}t)\left[\begin{array}{l},\circledcirc n,\\ \\ ,i\text{☮}n,\end{array}\right],$$

$$m\text{⏚},if(n=\varnothing)\left[\begin{array}{l},n\text{⏚},\text{©}c_1,\\ \\ ,n\text{⏚},\text{©}c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i \circledast m_1, i \circledast m, m_1 \overset{\circ}{\bigcirc}{}^{\circ} j, if(m \overset{\circ}{\bigcirc}{}^{\circ} t) \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \odot n, \end{array}\right],$$

$$m_1 \ominus, m \ominus, if(n = \varnothing) \left[\begin{array}{l} , n \ominus, \copyright c_1, \\ \\ , n \ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i \circledast m_1, i \circledast m, m_1 \overset{\circ}{\bigcirc} m, m_1 \overset{\circ}{\bigcirc}{}^{\circ} j, if(m \overset{\circ}{\bigcirc}{}^{\circ} t) \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \odot n, \end{array}\right],$$

$$m_1 \ominus, m \ominus, if(n = \varnothing) \left[\begin{array}{l} , n \ominus, \copyright c_1, \\ \\ , n \ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i \circledast m_1, i \circledast m, m_1 \overset{\circ}{\bigcirc} m, m \overset{\circ}{\bigcirc}{}^{\circ} j, if(m \overset{\circ}{\bigcirc}{}^{\circ} t) \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \odot n, \end{array}\right],$$

$$m_1 \ominus, m \ominus, if(n = \varnothing) \left[\begin{array}{l} , n \ominus, \copyright c_1, \\ \\ , n \ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i \circledast m_1, i \circledast m, m_1 \overset{\circ}{\bigcirc} m, m \overset{\circ}{\bigcirc}{}^{\circ} j, if(j \overset{\circ}{\bigcirc}{}^{\circ} t) \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \odot n, \end{array}\right],$$

$$m_1 \ominus, m \ominus, if(n = \varnothing) \left[\begin{array}{l} , n \ominus, \copyright c_1, \\ \\ , n \ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i \circledast m_1, i \circledast m, m_1 \overset{\circ}{\bigcirc}{}^{\circ} j, if(j \overset{\circ}{\bigcirc}{}^{\circ} t) \left[\begin{array}{l} , \circledcirc n, \\ \\ , i \odot n, \end{array}\right],$$

$$m_1 \ominus, m \ominus, if(n = \varnothing) \left[\begin{array}{l} , n \ominus, \copyright c_1, \\ \\ , n \ominus, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash m_1, m_1\male j, m_1\textcircled{$\downarrow$}, i\oslash m, m\textcircled{$\downarrow$}, if(j\male t)\begin{bmatrix}, \circledcirc n, \\ \\ , i\circledast n,\end{bmatrix},$$

$$if(n=\varnothing)\begin{bmatrix}, n\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , n\textcircled{$\downarrow$}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ ,i\obot j, if(j\male t)\begin{bmatrix}, \circledcirc n, \\ \\ , i\circledast n,\end{bmatrix}, if(n=\varnothing)\begin{bmatrix}, n\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , n\textcircled{$\downarrow$}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ ,i\obot j, if(j\male t)\begin{bmatrix}, \circledcirc n, n=\varnothing, \\ \\ , i\circledast n, n\,!\!=\varnothing,\end{bmatrix}, if(n=\varnothing)\begin{bmatrix}, n\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , n\textcircled{$\downarrow$}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ ,i\obot j, if(j\male t)\begin{bmatrix}, \circledcirc n, n=\varnothing, if(n=\varnothing)\begin{bmatrix}, n\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , n\textcircled{$\downarrow$}, \copyright c_2,\end{bmatrix}, \\ , i\circledast n, n\,!\!=\varnothing, if(n=\varnothing)\begin{bmatrix}, n\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , n\textcircled{$\downarrow$}, \copyright c_2,\end{bmatrix},\end{bmatrix},$$

$$\Leftrightarrow\ ,i\obot j, if(j\male t)\begin{bmatrix}, \circledcirc n, n=\varnothing, n\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i\circledast n, n\,!\!=\varnothing, n\textcircled{$\downarrow$}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ ,i\obot j, if(j\male t)\begin{bmatrix}, \circledcirc n, n\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i\circledast n, n\textcircled{$\downarrow$}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ ,i\obot j, if(j\male t)\begin{bmatrix}, \copyright c_1, \\ \\ , \copyright c_2,\end{bmatrix},$$

Propositions with propositions:

$$,i\obot j,i\obot t,\ \Leftrightarrow\ ,i\obot j,j\bigcirc^{\circ} t,$$

$$,i\obot j,i!\obot t,\ \Leftrightarrow\ ,i\obot j,j!\bigcirc^{\circ} t,$$

Subnode Propositions with branch function:

$$,i\obot j,if(t\obot j)\left[\begin{matrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{matrix}\right],\ \Leftrightarrow\ ,i\obot j,if(i\bigcirc^{\circ} t)\left[\begin{matrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{matrix}\right],$$

proof:

$$,i\obot j,if(t\obot j)\left[\begin{matrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,i\obot j,if(t=\varnothing)\left[\begin{matrix},t\otimes n,\\ \\ ,t\otimes m,if(m\bigcirc^{\circ} j)\left[\begin{matrix},\circledcirc n,\\ \\ ,t\otimes n,\end{matrix}\right],m\ovee,\end{matrix}\right],if(n=\varnothing)\left[\begin{matrix},n\ovee,\copyright c_1,\\ \\ ,n\ovee,\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,i\otimes m_1,m_1\bigcirc^{\circ} j,m_1\ovee,$$

$$if(t=\varnothing)\left[\begin{matrix},t\otimes n,\\ \\ ,t\otimes m,if(m\bigcirc^{\circ} j)\left[\begin{matrix},\circledcirc n,\\ \\ ,t\otimes n,\end{matrix}\right],m\ovee,\end{matrix}\right],if(n=\varnothing)\left[\begin{matrix},n\ovee,\copyright c_1,\\ \\ ,n\ovee,\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow\ ,if(t=\varnothing)\left[\begin{matrix},i\,!\!=\varnothing,i\otimes m_1,m_1\bigcirc^{\circ} j,m_1\ovee,t\otimes n,\\ \\ ,i\,!\!=\varnothing,i\otimes m_1,m_1\bigcirc^{\circ} j,m_1\ovee,t\otimes m,if(m\bigcirc^{\circ} j)\left[\begin{matrix},\circledcirc n,\\ \\ ,t\otimes n,\end{matrix}\right],m\ovee,\end{matrix}\right],$$

$$if(n=\varnothing)\left[\begin{matrix},n\ovee,\copyright c_1,\\ \\ ,n\ovee,\copyright c_2,\end{matrix}\right],$$

$$\Leftrightarrow \ ,if(t=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, i\text{☮}m_1, m_1\text{⚨}j, m_1\text{⬇}, t\text{☮}n, \\ ,i\,!\!=\varnothing, i\text{☮}m_1, t\text{☮}m, m_1\text{⚨}j, if(m\text{⚨}j)\left[\begin{array}{l} ,\circledcirc n, \\ ,t\text{☮}n, \end{array}\right], m\text{⬇}, m_1\text{⬇}, \end{array}\right],$$

$$if(n=\varnothing)\left[\begin{array}{l} ,n\text{⬇}, \copyright c_1, \\ ,n\text{⬇}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,if(t=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, i\text{☮}m_1, m_1\text{⚨}j, t\text{☮}n, \\ ,i\,!\!=\varnothing, i\text{☮}m_1, t\text{☮}m, m_1\text{⚨}j, if(m\text{⚨}m_1)\left[\begin{array}{l} ,\circledcirc n, \\ ,t\text{☮}n, \end{array}\right], m\text{⬇}, \end{array}\right],$$

$$m_1\text{⬇}, if(n=\varnothing)\left[\begin{array}{l} ,n\text{⬇}, \copyright c_1, \\ ,n\text{⬇}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,if(t=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, i\text{☮}m_1, m_1\text{⚨}j, t\text{☮}n, \\ ,i\,!\!=\varnothing, i\text{☮}m_1, m_1=\varnothing, t\text{☮}m, m=\varnothing, m_1\text{⚨}j, if(m\text{⚨}m_1)\left[\begin{array}{l} ,\circledcirc n, \\ ,t\text{☮}n, \end{array}\right], m\text{⬇}, \end{array}\right],$$

$$m_1\text{⬇}, if(n=\varnothing)\left[\begin{array}{l} ,n\text{⬇}, \copyright c_1, \\ ,n\text{⬇}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,if(t=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, i\text{☮}m_1, m_1\text{⚨}j, t\text{☮}n, \\ ,i\,!\!=\varnothing, i\text{☮}m_1, t\text{☮}m, m_1\text{⚨}j, m_1=\varnothing, m=\varnothing, if(m\text{⚨}m_1)\left[\begin{array}{l} ,\circledcirc n, \\ ,t\text{☮}n, \end{array}\right], m\text{⬇}, \end{array}\right],$$

$$m_1\text{⬇}, if(n=\varnothing)\left[\begin{array}{l} ,n\text{⬇}, \copyright c_1, \\ ,n\text{⬇}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow \ ,if(t=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, i\text{☮}m_1, m_1\text{⚨}j, t\text{☮}n, \\ ,i\,!\!=\varnothing, i\text{☮}m_1, t\text{☮}m, m_1\text{⚨}j, m_1=\varnothing, m=\varnothing, if(m\bigcirc m_1)\left[\begin{array}{l} ,\circledcirc n, \\ ,t\text{☮}n, \end{array}\right], m\text{⬇}, \end{array}\right],$$

$$m_1\textcircled{\downarrow}, if(n=\varnothing)\begin{bmatrix}, n\textcircled{\downarrow}, \copyright c_1, \\ , n\textcircled{\downarrow}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ , if(t=\varnothing)\begin{bmatrix}, i\,!\!=\varnothing, i\varovee m_1, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, t\varowedge n, \\ , i\,!\!=\varnothing, i\varovee m_1, t\varovee m, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, if(m\overset{\circ}{\bigcirc}m_1)\begin{bmatrix}, \circledcirc n, \\ , t\varowedge n,\end{bmatrix}, m\textcircled{\downarrow},\end{bmatrix},$$

$$m_1\textcircled{\downarrow}, if(n=\varnothing)\begin{bmatrix}, n\textcircled{\downarrow}, \copyright c_1, \\ , n\textcircled{\downarrow}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ , if(t=\varnothing)\begin{bmatrix}, i\,!\!=\varnothing, i\varovee m_1, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, t\varowedge n, \\ , i\,!\!=\varnothing, i\varovee m_1, t\varovee m, if(m\overset{\circ}{\bigcirc}m_1)\begin{bmatrix}, \circledcirc n, \\ , t\varowedge n,\end{bmatrix}, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, m\textcircled{\downarrow},\end{bmatrix},$$

$$m_1\textcircled{\downarrow}, if(n=\varnothing)\begin{bmatrix}, n\textcircled{\downarrow}, \copyright c_1, \\ , n\textcircled{\downarrow}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ , if(t=\varnothing)\begin{bmatrix}, i\,!\!=\varnothing, i\varovee m_1, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, t\varowedge n, \\ , t\,!\!=\varnothing, i\,!\!=\varnothing, i\varovee m_1, t\varovee m, if(m\overset{\circ}{\bigcirc}m_1)\begin{bmatrix}, \circledcirc n, \\ , t\varowedge n,\end{bmatrix}, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, m\textcircled{\downarrow},\end{bmatrix},$$

$$m_1\textcircled{\downarrow}, if(n=\varnothing)\begin{bmatrix}, n\textcircled{\downarrow}, \copyright c_1, \\ , n\textcircled{\downarrow}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ , if(t=\varnothing)\begin{bmatrix}, i\,!\!=\varnothing, i\varovee m_1, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, t\varowedge n, \\ , t\,!\!=\varnothing, i\,!\!=\varnothing, i\varovee m_1, t\varovee m, if(i\overset{\circ}{\bigcirc}t)\begin{bmatrix}, \circledcirc n, \\ , t\varowedge n,\end{bmatrix}, m_1\overset{\circ}{\bigcirc}{}^{\circ}j, m\textcircled{\downarrow},\end{bmatrix},$$

$$m_1\textcircled{\downarrow}, if(n=\varnothing)\begin{bmatrix}, n\textcircled{\downarrow}, \copyright c_1, \\ , n\textcircled{\downarrow}, \copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ ,if(t=\varnothing)\left[\begin{array}{l},i\,!=\varnothing,i\text{☮}m_1,m_1\text{♂}j,t\text{☮}n,\\ ,i\,!=\varnothing,i\text{☮}m_1,t\text{☮}m,if(i\text{♂}t)\left[\begin{array}{l},\circledcirc n,\\ ,t\text{☮}n,\end{array}\right],m_1\text{♂}j,m\text{⊍},\end{array}\right],$$

$$m_1\text{⊍},if(n=\varnothing)\left[\begin{array}{l},n\text{⊍},\copyright c_1,\\ ,n\text{⊍},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,if(t=\varnothing)\left[\begin{array}{l},i\,!=\varnothing,i\text{☮}m_1,m_1\text{♂}j,m_1\text{⊍},t\text{☮}n,\\ ,i\,!=\varnothing,i\text{☮}m_1,m_1\text{♂}j,m_1\text{⊍},t\text{☮}m,m\text{⊍},if(i\text{♂}t)\left[\begin{array}{l},\circledcirc n,\\ ,t\text{☮}n,\end{array}\right],\end{array}\right],$$

$$if(n=\varnothing)\left[\begin{array}{l},n\text{⊍},\copyright c_1,\\ ,n\text{⊍},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!=\varnothing,i\text{☮}m_1,m_1\text{♂}j,m_1\text{⊍},if(t=\varnothing)\left[\begin{array}{l},t\text{☮}n,\\ ,if(i\text{♂}t)\left[\begin{array}{l},\circledcirc n,\\ ,t\text{☮}n,\end{array}\right],\end{array}\right],$$

$$if(n=\varnothing)\left[\begin{array}{l},n\text{⊍},\copyright c_1,\\ ,n\text{⊍},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{⦶}j,if(t=\varnothing)\left[\begin{array}{l},t\text{☮}n,\\ ,if(i\text{♂}t)\left[\begin{array}{l},\circledcirc n,\\ ,t\text{☮}n,\end{array}\right],\end{array}\right],$$

$$if(n=\varnothing)\left[\begin{array}{l},n\text{⊍},\copyright c_1,\\ ,n\text{⊍},\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{⦶}j,if(t=\varnothing)\left[\begin{array}{l},t\text{☮}n,n\,!=\varnothing,\\ ,if(i\text{♂}t)\left[\begin{array}{l},\circledcirc n,n=\varnothing,\\ ,t\text{☮}n,n\,!=\varnothing,\end{array}\right],\end{array}\right],$$

$$if(n=\varnothing)\left[\begin{array}{l}, n\textcircled{\downarrow}, \copyright c_1,\\ \\ , n\textcircled{\downarrow}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\obot j, if(t=\varnothing)\left[\begin{array}{l}, t\textcircled{\wedge}n, n\,!\!=\varnothing, if(n=\varnothing)\left[\begin{array}{l}, n\textcircled{\downarrow}, \copyright c_1,\\ \\ , n\textcircled{\downarrow}, \copyright c_2,\end{array}\right],\\ \\ , if(i\mathring{\bigcirc}t)\left[\begin{array}{l}, \circledcirc n, n=\varnothing, if(n=\varnothing)\left[\begin{array}{l}, n\textcircled{\downarrow}, \copyright c_1,\\ \\ , n\textcircled{\downarrow}, \copyright c_2,\end{array}\right],\\ \\ , t\textcircled{\wedge}n, n\,!\!=\varnothing, if(n=\varnothing)\left[\begin{array}{l}, n\textcircled{\downarrow}, \copyright c_1,\\ \\ , n\textcircled{\downarrow}, \copyright c_2,\end{array}\right],\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ , i\obot j, if(t=\varnothing)\left[\begin{array}{l}, t\textcircled{\wedge}n, n\,!\!=\varnothing, n\textcircled{\downarrow}, \copyright c_2,\\ , if(i\mathring{\bigcirc}t)\left[\begin{array}{l}, \circledcirc n, n=\varnothing, n\textcircled{\downarrow}, \copyright c_1,\\ \\ , t\textcircled{\wedge}n, n\,!\!=\varnothing, n\textcircled{\downarrow}, \copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ , i\obot j, if(t=\varnothing)\left[\begin{array}{l}, t\textcircled{\wedge}n, n\textcircled{\downarrow}, \copyright c_2,\\ , if(i\mathring{\bigcirc}t)\left[\begin{array}{l}, \circledcirc n, n\textcircled{\downarrow}, \copyright c_1,\\ \\ , t\textcircled{\wedge}n, n\textcircled{\downarrow}, \copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ , i\obot j, if(t=\varnothing)\left[\begin{array}{l}, \copyright c_2,\\ , if(i\mathring{\bigcirc}t)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ , i\obot j, i\,!\!=\varnothing, if(t=\varnothing)\left[\begin{array}{l}, \copyright c_2,\\ , if(i\mathring{\bigcirc}t)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ , i\obot j, if(t=\varnothing)\left[\begin{array}{l}, i\,!\!=\varnothing, \copyright c_2,\\ , i\,!\!=\varnothing, if(i\mathring{\bigcirc}t)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus j, if(t=\varnothing)\left[\begin{array}{l} ,t=\varnothing, i\,!\!=\varnothing, \copyright c_2, \\ ,i\,!\!=\varnothing, if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus j, if(t=\varnothing)\left[\begin{array}{l} ,t=\varnothing, i\,!\!=\varnothing, i!\mathring{\bigcirc}t, \copyright c_2, \\ ,i\,!\!=\varnothing, if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus j, if(t=\varnothing)\left[\begin{array}{l} ,t=\varnothing, i\,!\!=\varnothing, i!\mathring{\bigcirc}t, if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,i\,!\!=\varnothing, if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus j, if(t=\varnothing)\left[\begin{array}{l} ,if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus j, if(t=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right], if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\oplus j, if(i\mathring{\bigcirc}t)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

Propositions with propositions:

$$,i\oplus j, t\oplus j, \ \Leftrightarrow\ ,i\oplus j, i\mathring{\bigcirc}t,$$

$$,i\oplus j, t!\oplus j, \ \Leftrightarrow\ ,i\oplus j, i!\mathring{\bigcirc}t,$$

18.2.9 Opposition

$$,i⦹j,i!⦹j,\ \Leftrightarrow\ ,\otimes,$$

$$,i!⦹j,i⦹j,\ \Leftrightarrow\ ,\otimes,$$

18.2.10 Other

$$,i\,!{=}\varnothing,i☮t,\ \Leftrightarrow\ \sim,i⦹t,$$

proof:

$,i\,!{=}\varnothing,i☮t,$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i☮t,if(i⦹t)\!-\!\left[\begin{array}{l},\\ \\ ,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i☮t,-\!\left[\begin{array}{l},i⦹t,\\ \\ ,i!⦹t,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i☮t,-\!\left[\begin{array}{l},i⦹t,\\ \\ ,if(i{=}\varnothing)\!-\!\left[\begin{array}{l},\\ \\ ,i☮t_0,t_0!⚨t,t_0⏚,\end{array}\right]\!-,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,-\!\left[\begin{array}{l},i\,!{=}\varnothing,i☮t,i⦹t,\\ \\ ,i\,!{=}\varnothing,i☮t,if(i{=}\varnothing)\!-\!\left[\begin{array}{l},\\ \\ ,i☮t_0,t_0!⚨t,t_0⏚,\end{array}\right]\!-,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,-\!\left[\begin{array}{l},i\,!{=}\varnothing,i☮t,i⦹t,\\ \\ ,i☮t,i\,!{=}\varnothing,if(i{=}\varnothing)\!-\!\left[\begin{array}{l},\\ \\ ,i☮t_0,t_0!⚨t,t_0⏚,\end{array}\right]\!-,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,-\!\left[\begin{array}{l},i\,!{=}\varnothing,i☮t,i⦹t,\\ \\ ,i☮t,i\,!{=}\varnothing,i☮t_0,t_0!⚨t,t_0⏚,\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,\left[\begin{array}{l},i\,!{=}\varnothing,i\circledast t,i\ominus t,\\ \\ ,i\,!{=}\varnothing,i\circledast t,i\circledast t_0,t_0!\overset{\circ}{\bigcirc}{}^{\circ}t,t_0\circleddash,\end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l},i\,!{=}\varnothing,i\circledast t,i\ominus t,\\ \\ ,i\,!{=}\varnothing,i\circledast t,i\circledast t_0,t\overset{\circ}{\bigcirc}t_0,t_0!\overset{\circ}{\bigcirc}{}^{\circ}t,t_0\circleddash,\end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l},i\,!{=}\varnothing,i\circledast t,i\ominus t,\\ \\ ,i\,!{=}\varnothing,i\circledast t,i\circledast t_0,t\overset{\circ}{\bigcirc}t_0,t\overset{\circ}{\bigcirc}{}^{\circ}t_0,t_0!\overset{\circ}{\bigcirc}{}^{\circ}t,t_0\circleddash,\end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l},i\,!{=}\varnothing,i\circledast t,i\ominus t,\\ \\ ,i\,!{=}\varnothing,i\circledast t,i\circledast t_0,t\overset{\circ}{\bigcirc}t_0,\otimes,t_0\circleddash,\end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l},i\,!{=}\varnothing,i\circledast t,i\ominus t,\\ \\ ,i\,!{=}\varnothing,i\circledast t,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i\circledast t,\left[\begin{array}{l},i\ominus t,\\ \\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i\circledast t,\left[\begin{array}{l},i\ominus t,\\ \\ ,i!\ominus t,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i\circledast t,if(i\ominus t)\left[\begin{array}{l},\\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i\circledast t,i\ominus t,$$

$$,i=\varnothing,\ \Leftrightarrow\ \sim,i!\ominus t,$$

proof:

$,i=\varnothing,$

$\Leftrightarrow\ ,i=\varnothing,i\circledast n,n\circleddash,$

$\Leftrightarrow\ ,i=\varnothing,i\circledast n,n\,!{=}\varnothing,n\circleddash,$

$$\Leftrightarrow\ ,i=\varnothing,i\oslash n,if(n=\varnothing)\begin{cases},\otimes,\\,\end{cases},n\ovee,$$

$$\Leftrightarrow\ ,i=\varnothing,i\oslash n,if(n=\varnothing)\begin{cases},\otimes,\\,n\ovee,\end{cases},$$

$$\Leftrightarrow\ ,i=\varnothing,i\oslash n,if(n=\varnothing)\begin{cases},n\ovee,\otimes,\\,n\ovee,\end{cases},$$

$$\Leftrightarrow\ ,i=\varnothing,if(i=\varnothing)\begin{cases},i\oslash n,if(n=\varnothing)\begin{cases},n\ovee,\otimes,\\,n\ovee,\end{cases},\\,i\oslash m,if(m\mathring{\bigcirc}^{\circ}t)\begin{cases},\circledcirc n,\\,i\oslash n,\end{cases},m\ovee,if(n=\varnothing)\begin{cases},n\ovee,\otimes,\\,n\ovee,\end{cases},\end{cases},$$

$$\Leftrightarrow\ ,i=\varnothing,if(i=\varnothing)\begin{cases},i\oslash n,\\,i\oslash m,if(m\mathring{\bigcirc}^{\circ}t)\begin{cases},\circledcirc n,\\,i\oslash n,\end{cases},m\ovee,\end{cases},if(n=\varnothing)\begin{cases},n\ovee,\otimes,\\,n\ovee,\end{cases},$$

$$\Leftrightarrow\ ,i=\varnothing,i!\obar t,$$

$$,i\obar m,j\obar n,if(i\mathring{\bigcirc}j)\begin{cases},\\,\end{cases}\Leftrightarrow\ ,i\obar m,j\obar n,if(m\mathring{\bigcirc}^{\circ}n)\begin{cases},\\,\end{cases}$$

proof:

$$,i\obar m,j\obar n,if(i\mathring{\bigcirc}j)\begin{cases},\\,\end{cases}$$

$$\Leftrightarrow\ ,i\,!=\varnothing,i\oslash t_1,t_1\mathring{\bigcirc}^{\circ}m,t_1\ovee,j\,!=\varnothing,j\oslash t_2,t_2\mathring{\bigcirc}^{\circ}n,t_2\ovee,if(i\mathring{\bigcirc}j)\begin{cases},\\,\end{cases}$$

$$\Leftrightarrow\ ,i\,!=\varnothing,j\,!=\varnothing,i\oslash t_1,j\oslash t_2,if(i\mathring{\bigcirc}j)\begin{cases},t_1\mathring{\bigcirc}^{\circ}m,t_2\mathring{\bigcirc}^{\circ}n,t_1\ovee,t_2\ovee,\\,t_1\mathring{\bigcirc}^{\circ}m,t_2\mathring{\bigcirc}^{\circ}n,t_1\ovee,t_2\ovee,\end{cases}$$

$$\Leftrightarrow\ ,i\,!=\varnothing,j\,!=\varnothing,i\oslash t_1,j\oslash t_2,if(t_1\mathring{\bigcirc}t_2)\begin{cases},t_1\mathring{\bigcirc}^{\circ}m,t_2\mathring{\bigcirc}^{\circ}n,t_1\ovee,t_2\ovee,\\,t_1\mathring{\bigcirc}^{\circ}m,t_2\mathring{\bigcirc}^{\circ}n,t_1\ovee,t_2\ovee,\end{cases}$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, t_1=\varnothing, j\circledast t_2, t_2=\varnothing, if(t_1\mathring{\bigcirc}t_2)\left[\begin{array}{l}, t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\\ \\ , t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, j\circledast t_2, t_1=\varnothing, t_2=\varnothing, if(t_1\mathring{\bigcirc}t_2)\left[\begin{array}{l}, t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\\ \\ , t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, j\circledast t_2, t_1=\varnothing, t_2=\varnothing, if(t_1\mathring{\bigcirc}\!\circ t_2)\left[\begin{array}{l}, t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\\ \\ , t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, j\circledast t_2, if(t_1\mathring{\bigcirc}\!\circ t_2)\left[\begin{array}{l}, t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\\ \\ , t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}\!\circ m, if(t_1\mathring{\bigcirc}\!\circ t_2)\left[\begin{array}{l}, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\\ \\ , t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}\!\circ m, if(m\mathring{\bigcirc}\!\circ t_2)\left[\begin{array}{l}, t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\\ \\ , t_2\mathring{\bigcirc}\!\circ n, t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, if(m\mathring{\bigcirc}\!\circ t_2)\left[\begin{array}{l}, t_1\odot, t_2\odot,\\ \\ , t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, j\,!{=}\varnothing, i\circledast t_1, j\circledast t_2, t_1\mathring{\bigcirc}\!\circ m, t_2\mathring{\bigcirc}\!\circ n, if(m\mathring{\bigcirc}\!\circ n)\left[\begin{array}{l}, t_1\odot, t_2\odot,\\ \\ , t_1\odot, t_2\odot,\end{array}\right.$$

$$\Leftrightarrow\ , i\,!{=}\varnothing, i\circledast t_1, t_1\mathring{\bigcirc}\!\circ m, t_1\odot, j\,!{=}\varnothing, j\circledast t_2, t_2\mathring{\bigcirc}\!\circ n, t_2\odot, if(m\mathring{\bigcirc}\!\circ n)\left[\begin{array}{l},\\ \\ ,\end{array}\right.$$

$$\Leftrightarrow\ , i\ominus m, j\ominus n, if(m\mathring{\bigcirc}\!\circ n)\left[\begin{array}{l},\\ \\ ,\end{array}\right.$$

$$, i\ominus m, j\ominus n, i\mathring{\bigcirc}j, \ \Leftrightarrow\ i\ominus m, j\ominus n, m\mathring{\bigcirc}\!\circ n,$$

$$, i\ominus m, j\ominus n, i!\mathring{\bigcirc}j, \ \Leftrightarrow\ i\ominus m, j\ominus n, m!\mathring{\bigcirc}\!\circ n,$$

$$, i \obot j, j \oplus, \iff , j \oplus, i \obot j,$$

proof:

$, i \obot j, j \oplus,$

$\iff , i \mathrel{!}= \varnothing, i \owyebent m, m \occirc j, m \oground, j \oplus,$

$\iff , i \mathrel{!}= \varnothing, i \owyebent m, m \occirc j, j \oplus, m \oground,$

$\iff , i \mathrel{!}= \varnothing, i \owyebent m, j \oplus, m \occirc j, m \oground,$

$\iff , j \oplus, i \mathrel{!}= \varnothing, i \owyebent m, m \occirc j, m \oground,$

$\iff , j \oplus, i \obot j,$

$$, i \obot j, j \ominus, \iff , j \ominus, i \obot j,$$

$$, i \owye i_0, i \obot i_0, i_0 \oground, \iff , i \owye i_0, i_0 \obot i, i_0 \oground,$$

proof:

$, i \owye i_0, i \obot i_0, i_0 \oground,$

$\iff , i \owye i_0, i \owye i_1, i_1 \oground, i \obot i_0, i_0 \oground,$

$\iff , i \owye i_0, i \owye i_1, i \obot i_0, i_1 \oground, i_0 \oground,$

$\iff , i \owye i_0, i \owye i_1, i \ocirc i_1, i \obot i_0, i_1 \oground, i_0 \oground,$

$\iff , i \owye i_0, i \owye i_1, i \ocirc i_1, i_1 \obot i_0, i_1 \oground, i_0 \oground,$

$\iff , i \owye i_0, i \owye i_1, i_1 \obot i_0, i_1 \oground, i_0 \oground,$

$\iff , i \owye i_0, i \ocirc i_0, i \owye i_1, i_1 \obot i_0, i_1 \oground, i_0 \oground,$

$\iff , i \owye i_0, i \ocirc i_0, i \occirc i_0, i \owye i_1, i_1 \obot i_0, i_1 \oground, i_0 \oground,$

$\iff , i \owye i_0, i \ocirc i_0, i \owye i_1, i \occirc i_0, i_1 \obot i_0, i_1 \oground, i_0 \oground,$

$\iff , i \owye i_0, i \ocirc i_0, i \owye i_1, i \occirc i_0, i_1 \obot i, i_1 \oground, i_0 \oground,$

$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, i_1\text{⦹}i, i_1\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, i_0\text{⧂}i_1, i_1\text{⦹}i, i_1\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, i_0\text{⧂}i_1, i_0\text{⦹}i, i_1\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, i_0\text{⦹}i, i_1\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, i_1\text{⏚}, i_0\text{⦹}i, i_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i_0\text{⦹}i, i_0\text{⏚},$

$$, i\text{⦹}i, \quad \Leftrightarrow \quad , i\text{☮}i_0, i\text{⦹}i_0, i_0\text{⏚},$$

19 Tree Order Induction

19.1 Definition of flag object &SVi with identical node.

19.1.1 Swap definition:

$$, \&SVi\,\bigcirc^{\!\circ}i, \circledcirc m, \;\Leftrightarrow\; , \circledcirc m, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, \odot m, \;\Leftrightarrow\; , \odot m, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, j\text{☮}n, \;\Leftrightarrow\; , j\text{☮}n, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, j\text{☮}n, \;\Leftrightarrow\; , j\text{☮}n, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, j\text{☮}n, \;\Leftrightarrow\; , j\text{☮}n, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, j\text{⏚}, \;\Leftrightarrow\; , j\text{⏚}, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, j\oplus, \;\Leftrightarrow\; , j\oplus, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, \otimes, \;\Leftrightarrow\; , \otimes, \&SVi\,\bigcirc^{\!\circ}i,$$

$$, \&SVi\,\bigcirc^{\!\circ}i, m\circeq n\!\left[\begin{matrix}, \\ ,\end{matrix}\right. \;\Leftrightarrow\; , m\circeq n\!\left[\begin{matrix}, \&SVi\,\bigcirc^{\!\circ}i, \\ , \&SVi\,\bigcirc^{\!\circ}i,\end{matrix}\right.$$

$$, \&SVi\,\bigcirc^{\!\circ}i, \left[\begin{matrix}, \\ ,\end{matrix}\right. \;\Leftrightarrow\; , \left[\begin{matrix}, \&SVi\,\bigcirc^{\!\circ}i, \\ , \&SVi\,\bigcirc^{\!\circ}i,\end{matrix}\right.$$

$$\left.\begin{matrix}, \\ ,\end{matrix}\right], \&SVi\,\bigcirc^{\!\circ}i, \;\Leftrightarrow\; \left.\begin{matrix}, \&SVi\,\bigcirc^{\!\circ}i, \\ , \&SVi\,\bigcirc^{\!\circ}i,\end{matrix}\right],$$

19.1.2 Substitution definition:

$$, i\overset{\circ}{\bigcirc}j,\ \&SVi\,\overset{\circ}{\bigcirc}i,\ \Leftrightarrow\ , i\overset{\circ}{\bigcirc}j,\ \&SVi\,\overset{\circ}{\bigcirc}j,$$

19.2 Definition of flag object &SVi with subnode.

$$, i\oplus j,\ \&SVi\oplus j,\ \Leftrightarrow\ , i\oplus j,\ \&SVi\,\overset{\circ}{\bigcirc}i,$$

19.3 Theorems of flag object &SVi with identical node.

19.3.1 Swap with previous node operator:

$$,\ \&SVi\,\overset{\circ}{\bigcirc}i, j\ominus,\ \Leftrightarrow\ , j\ominus,\ \&SVi\,\overset{\circ}{\bigcirc}i,$$

19.3.2 Swap with branch function:

$$,\ \&SVi\,\overset{\circ}{\bigcirc}i, if(m{=}n)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ , if(m{=}n)\left[\begin{array}{l}, \&SVi\,\overset{\circ}{\bigcirc}i, ©c_1,\\ \\ , \&SVi\,\overset{\circ}{\bigcirc}i, ©c_2,\end{array}\right],$$

$$,\ \&SVi\,\overset{\circ}{\bigcirc}i, if(m{=}\varnothing)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ , if(m{=}\varnothing)\left[\begin{array}{l}, \&SVi\,\overset{\circ}{\bigcirc}i, ©c_1,\\ \\ , \&SVi\,\overset{\circ}{\bigcirc}i, ©c_2,\end{array}\right],$$

$$,\ \&SVi\,\overset{\circ}{\bigcirc}i, if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ , if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l}, \&SVi\,\overset{\circ}{\bigcirc}i, ©c_1,\\ \\ , \&SVi\,\overset{\circ}{\bigcirc}i, ©c_2,\end{array}\right],$$

$$,\ \&SVi\,\overset{\circ}{\bigcirc}i, if(m{\rightarrow}n)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right],\ \Leftrightarrow\ , if(m{\rightarrow}n)\left[\begin{array}{l}, \&SVi\,\overset{\circ}{\bigcirc}i, ©c_1,\\ \\ , \&SVi\,\overset{\circ}{\bigcirc}i, ©c_2,\end{array}\right],$$

$$, \&SVi\male i, if(m\male\!\circ n)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(m\male\!\circ n)\left[\begin{array}{l}, \&SVi\male i, \copyright c_1,\\ \\ , \&SVi\male i, \copyright c_2,\end{array}\right],$$

$$, \&SVi\male i, if(m\obot n)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \;\Leftrightarrow\; , if(m\obot n)\left[\begin{array}{l}, \&SVi\male i, \copyright c_1,\\ \\ , \&SVi\male i, \copyright c_2,\end{array}\right],$$

19.3.3 Swap with propositions:

$$, \&SVi\male i, m=n, \;\Leftrightarrow\; , m=n, \&SVi\male i,$$

$$, \&SVi\male i, m=\varnothing, \;\Leftrightarrow\; , m=\varnothing, \&SVi\male i,$$

$$, \&SVi\male i, m\male n, \;\Leftrightarrow\; , m\male n, \&SVi\male i,$$

$$, \&SVi\male i, m\rightarrow n, \;\Leftrightarrow\; , m\rightarrow n, \&SVi\male i,$$

$$, \&SVi\male i, m\male\!\circ n, \;\Leftrightarrow\; , m\male\!\circ n, \&SVi\male i,$$

$$, \&SVi\male i, m\obot n, \;\Leftrightarrow\; , m\obot n, \&SVi\male i,$$

$$, \&SVi\male i, m\,!\!=n, \;\Leftrightarrow\; , m\,!\!=n, \&SVi\male i,$$

$$, \&SVi\male i, m\,!\!=\varnothing, \;\Leftrightarrow\; , m\,!\!=\varnothing, \&SVi\male i,$$

$$, \&SVi\male i, m!\male n, \;\Leftrightarrow\; , m!\male n, \&SVi\male i,$$

$$, \&SVi\male i, m!\rightarrow n, \;\Leftrightarrow\; , m!\rightarrow n, \&SVi\male i,$$

$$, \&SVi\male i, m!\male\!\circ n, \;\Leftrightarrow\; , m!\male\!\circ n, \&SVi\male i,$$

$$, \&SVi\male i, m!\obot n, \;\Leftrightarrow\; , m!\obot n, \&SVi\male i,$$

19.3.4 Propositions and recursive function:

$$, \&SVi\text{♂}i, R(m), \;\Leftrightarrow\; , R(m), \&SVi\text{♂}i,$$

$$, \&SVi\text{♂}i, R_{\text{-}}(m), \;\Leftrightarrow\; , R_{\text{-}}(m), \&SVi\text{♂}i,$$

19.3.5 Swap with the same operand's operator:

$$, \&SVi\text{♂}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n, \&SVi\text{♂}i,$$

proof:

$, \&SVi\text{♂}i, i\text{☮}n,$

$\Leftrightarrow \; , i\text{☮}i_0, i_0\text{⏚}, \&SVi\text{♂}i, i\text{☮}n,$

$\Leftrightarrow \; , i\text{☮}i_0, \&SVi\text{♂}i, i\text{☮}n, i_0\text{⏚},$

$\Leftrightarrow \; , i\text{☮}i_0, i\text{♂}i_0, \&SVi\text{♂}i, i\text{☮}n, i_0\text{⏚},$

$\Leftrightarrow \; , i\text{☮}i_0, i\text{♂}i_0, \&SVi\text{♂}i_0, i\text{☮}n, i_0\text{⏚},$

$\Leftrightarrow \; , i\text{☮}i_0, i\text{♂}i_0, i\text{☮}n, \&SVi\text{♂}i_0, i_0\text{⏚},$

$\Leftrightarrow \; , i\text{☮}i_0, i\text{☮}n, i\text{♂}i_0, \&SVi\text{♂}i_0, i_0\text{⏚},$

$\Leftrightarrow \; , i\text{☮}i_0, i\text{☮}n, i\text{♂}i_0, \&SVi\text{♂}i, i_0\text{⏚},$

$\Leftrightarrow \; , i\text{☮}i_0, i\text{☮}n, \&SVi\text{♂}i, i_0\text{⏚},$

$\Leftrightarrow \; , i\text{☮}i_0, i_0\text{⏚}, i\text{☮}n, \&SVi\text{♂}i,$

$\Leftrightarrow \; , i\text{☮}n, \&SVi\text{♂}i,$

$$, \&SVi\text{♂}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n, \&SVi\text{♂}i,$$

$$, \&SVi\text{♂}i, i\text{☮}n, \;\Leftrightarrow\; , i\text{☮}n, \&SVi\text{♂}i,$$

19.3.6 Swap with the same operand's branch function:

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(i=j)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(i=j)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(i=\varnothing)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(i=\varnothing)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(i\overset{\circ}{\bigcirc}j)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(i\overset{\circ\circ}{\bigcirc}j)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(i\rightarrow j)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(i\rightarrow j)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(j\rightarrow i)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(j\rightarrow i)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(i\obot j)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(i\obot j)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

$$,\ \&SVi\overset{\circ}{\bigcirc}i, if(j\obot i)-\left[\begin{array}{l} ,\ \copyright c_1, \\ \\ ,\ \copyright c_2, \end{array}\right]-, \quad\Leftrightarrow\quad , if(j\obot i)-\left[\begin{array}{l} ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_1, \\ \\ ,\ \&SVi\overset{\circ}{\bigcirc}i,\ \copyright c_2, \end{array}\right]-,$$

19.3.7 Swap with the same operand's propositions:

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i = j, \iff , i = j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i = \varnothing, \iff , i = \varnothing, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i \mathbin{\bigcirc\mkern-8mu^{\circ}} j, \iff , i \mathbin{\bigcirc\mkern-8mu^{\circ}} j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}\mkern-4mu\circ} i, i \mathbin{\bigcirc\mkern-8mu^{\circ}} j, \iff , i \mathbin{\bigcirc\mkern-8mu^{\circ}\mkern-4mu\circ} j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i \rightarrow j, \iff , i \rightarrow j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, j \rightarrow i, \iff , j \rightarrow i, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i \mathbin{\bigcirc\mkern-15mu\perp} j, \iff , i \mathbin{\bigcirc\mkern-15mu\perp} j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, j \mathbin{\bigcirc\mkern-15mu\perp} i, \iff , j \mathbin{\bigcirc\mkern-15mu\perp} i, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i \mathrel{!=} j, \iff , i \mathrel{!=} j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i \mathrel{!=} \varnothing, \iff , i \mathrel{!=} \varnothing, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i ! \mathbin{\bigcirc\mkern-8mu^{\circ}} j, \iff , i ! \mathbin{\bigcirc\mkern-8mu^{\circ}} j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i ! \mathbin{\bigcirc\mkern-8mu^{\circ}\mkern-4mu\circ} j, \iff , i ! \mathbin{\bigcirc\mkern-8mu^{\circ}\mkern-4mu\circ} j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i ! \rightarrow j, \iff , i ! \rightarrow j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, j ! \rightarrow i, \iff , j ! \rightarrow i, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, i ! \mathbin{\bigcirc\mkern-15mu\perp} j, \iff , i ! \mathbin{\bigcirc\mkern-15mu\perp} j, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

$$, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i, j ! \mathbin{\bigcirc\mkern-15mu\perp} i, \iff , j ! \mathbin{\bigcirc\mkern-15mu\perp} i, \&SVi \mathbin{\bigcirc\mkern-8mu^{\circ}} i,$$

19.4 Axiom of tree order induction

19.4.1 axiom of inference:

$\left\{ {}^{<premise\ 1>}_{<premise\ 2>} \right\} \Longrightarrow \ < conclusion >$

19.4.2 premise 1:

$, i = \varnothing, \textcircled{t} c_1, \ \Leftrightarrow \ , i = \varnothing, \textcircled{t} c_2,$

19.4.3 premise 2:

$, \ \&\mathit{SV}i \,⦹ i, \textcircled{t} c_1, \ \Leftrightarrow \ , \ \&\mathit{SV}i \,⦹ i, \textcircled{t} c_2, \ \Longrightarrow$

$, i \,!\!= \varnothing, \ \&\mathit{SV}i \,\overset{\circ}{\bigcirc} i, \textcircled{t} c_1, \ \Leftrightarrow \ , i \,!\!= \varnothing, \ \&\mathit{SV}i \,\overset{\circ}{\bigcirc} i, \textcircled{t} c_2,$

19.4.4 conclusion:

$, \textcircled{t} c_1, \ \Leftrightarrow \ , \textcircled{t} c_2,$

19.5 Theorems of tree order induction

$$, i ⦹ j, j ⦹ i, \ \Leftrightarrow \ , \otimes,$$

induction proof:
$premise\ 1:$
$, i = \varnothing, i ⦹ j, j ⦹ i,$
$\Leftrightarrow \ , i = \varnothing, i ! ⦹ j, i ⦹ j, j ⦹ i,$
$\Leftrightarrow \ , i = \varnothing, \otimes, j ⦹ i,$
$\Leftrightarrow \ , i = \varnothing, \otimes,$

$premise\ 2:$
$, \ \&\mathit{SV}i \,⦹ i, i ⦹ j, j ⦹ i, \ \Leftrightarrow \ , \ \&\mathit{SV}i \,⦹ i, \otimes, \ \Longrightarrow$

$, i \,!\!= \varnothing, \ \&\mathit{SV}i \,\overset{\circ}{\bigcirc} i, i ⦹ j, j ⦹ i,$

$\Leftrightarrow \ , i \,!\!= \varnothing, \ \&\mathit{SV}i \,\overset{\circ}{\bigcirc} i, i ⦹ j, i ⦹ j, j ⦹ i,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i \obot j, \ \&SVi \mathring{\bigcirc} i, i \obot j, j \obot i,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i \obot j, \ \&SVi \obot j, i \obot j, j \obot i,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i \obot j, \ \&SVi \obot j, j \obot i, i \obot j,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i \obot j, \ \&SVi \obot j, \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, i \obot j, \ \&SVi \mathring{\bigcirc} i, \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SVi \mathring{\bigcirc} i, i \obot j, \otimes,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SVi \mathring{\bigcirc} i, \otimes,$

$conclusion:$
$, i \obot j, j \obot i, \ \Leftrightarrow \ , \otimes,$

$$, i \obot i, \ \Leftrightarrow \ , \otimes,$$

proof:
$, i \obot i,$
$\Leftrightarrow \ , i \owedge i_1, i_1 \obot i, i_1 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i_1 \obot i, i_1 \obot i, i_1 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i \owedge i_2, i_2 \odownarrow, i_1 \obot i, i_1 \obot i, i_1 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i \owedge i_2, i_1 \obot i, i_1 \obot i, i_1 \odownarrow, i_2 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i \owedge i_2, i_1 \mathring{\bigcirc} i_2, i_1 \obot i, i_1 \obot i, i_1 \odownarrow, i_2 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i \owedge i_2, i_1 \mathring{\bigcirc} i_2, i_2 \obot i, i_1 \obot i, i_1 \odownarrow, i_2 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i \owedge i_2, i_2 \obot i, i_1 \obot i, i_1 \odownarrow, i_2 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i \mathring{\bigcirc} i_1, i \owedge i_2, i_2 \obot i, i_1 \obot i, i_1 \odownarrow, i_2 \odownarrow,$

$\Leftrightarrow \ , i \owedge i_1, i \owedge i_2, i \mathring{\bigcirc} i_1, i_2 \obot i, i_1 \obot i, i_1 \odownarrow, i_2 \odownarrow,$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,i\overset{\circ}{\bigcirc}i_1,i\overset{\circ\circ}{\bigcirc}i_1,i_2\text{⦹}i,i_1\text{⦹}i,i_1\text{⦽},i_2\text{⦽},$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,i\overset{\circ}{\bigcirc}i_1,i\overset{\circ\circ}{\bigcirc}i_1,i_2\text{⦹}i_1,i_1\text{⦹}i,i_1\text{⦽},i_2\text{⦽},$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,i_2\text{⦹}i_1,i_1\text{⦹}i,i_1\text{⦽},i_2\text{⦽},$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,i\overset{\circ}{\bigcirc}i_2,i_2\text{⦹}i_1,i_1\text{⦹}i,i_1\text{⦽},i_2\text{⦽},$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,i\overset{\circ}{\bigcirc}i_2,i\text{⦹}i_1,i_1\text{⦹}i,i_1\text{⦽},i_2\text{⦽},$$

$$\Leftrightarrow\ ,i\text{☮}i_1,i\text{☮}i_2,i\overset{\circ}{\bigcirc}i_2,\otimes,i_1\text{⦽},i_2\text{⦽},$$

$$\Leftrightarrow\ ,\otimes,$$

$$,i\overset{\circ}{\bigcirc}j,i\text{⦹}j,\ \Leftrightarrow\ ,\otimes,$$

$$,i\overset{\circ\circ}{\bigcirc}j,\ \Leftrightarrow\ ,\sim,i!\text{⦹}j,$$

proof:

$,i\overset{\circ\circ}{\bigcirc}j,$

$$\Leftrightarrow\ ,i\overset{\circ\circ}{\bigcirc}j,if(i\text{⦹}j)\left[\begin{array}{l},\\ ,\end{array}\right],$$

$$\Leftrightarrow\ ,i\overset{\circ\circ}{\bigcirc}j,if(i\text{⦹}j)\left[\begin{array}{l},i\text{⦹}j,\\ ,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i\text{⦹}j)\left[\begin{array}{l},i\overset{\circ\circ}{\bigcirc}j,i\text{⦹}j,\\ ,i\overset{\circ\circ}{\bigcirc}j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i\text{⦹}j)\left[\begin{array}{l},i\overset{\circ\circ}{\bigcirc}j,i\text{☮}i_0,i_0\text{⦽},i\text{⦹}j,\\ ,i\overset{\circ\circ}{\bigcirc}j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i\text{⦹}j)\left[\begin{array}{l},i\overset{\circ\circ}{\bigcirc}j,i\text{☮}i_0,i\text{⦹}j,i_0\text{⦽},\\ ,i\overset{\circ\circ}{\bigcirc}j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l}, i\owedge i_0, i\overset{\circ\circ}{\bigcirc}j, i\obot j, i_0\textcircled{$\downarrow$}, \\ \\ , i\overset{\circ\circ}{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l}, i\owedge i_0, i\overset{\circ}{\bigcirc}i_0, i\overset{\circ\circ}{\bigcirc}j, i\obot j, i_0\textcircled{$\downarrow$}, \\ \\ , i\overset{\circ\circ}{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l}, i\owedge i_0, i\overset{\circ}{\bigcirc}i_0, i_0\overset{\circ\circ}{\bigcirc}j, i\obot j, i_0\textcircled{$\downarrow$}, \\ \\ , i\overset{\circ\circ}{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l}, i\owedge i_0, i\overset{\circ}{\bigcirc}i_0, i_0\overset{\circ\circ}{\bigcirc}j, i\obot i_0, i_0\textcircled{$\downarrow$}, \\ \\ , i\overset{\circ\circ}{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l}, i\overset{\circ\circ}{\bigcirc}j, i\owedge i_0, i\obot i_0, i_0\textcircled{$\downarrow$}, \\ \\ , i\overset{\circ\circ}{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l}, i\overset{\circ\circ}{\bigcirc}j, i\obot i, \\ \\ , i\overset{\circ\circ}{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\obot j)\left[\begin{array}{l}, i\overset{\circ\circ}{\bigcirc}j, \otimes, \\ \\ , i\overset{\circ\circ}{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ , i\overset{\circ\circ}{\bigcirc}j, if(i\obot j)\left[\begin{array}{l}, \otimes, \\ \\ , \end{array}\right],$$

$$\Leftrightarrow\ , i\overset{\circ\circ}{\bigcirc}j, i!\obot j,$$

$$, i\overset{\circ}{\bigcirc}j, \ \Leftrightarrow\ , \sim, i!\obot j,$$

19.6 Definition of Rd(i):r

$$, Rd(i) : r, \ \Leftrightarrow\ , if(i=\varnothing)\left[\begin{array}{l}, i\owedge r, \\ \\ , i\oslash j, j\oplus, Rd(j) : r, j\textcircled{$\downarrow$}, \end{array}\right],$$

19.7 Theorems of Rd(i):r

$$,i=\varnothing, Rd(i):r, \;\Leftrightarrow\; ,i=\varnothing, i\oslash r,$$

$$,i\,!\!=\varnothing, Rd(i):r, \;\Leftrightarrow\; ,i\,!\!=\varnothing, i\oslash j, j\oplus, Rd(j):r, j\ovee,$$

$$,Rd(i):r, \;\Leftrightarrow\; \sim, r=\varnothing,$$

induction proof:
$premise\ 1:$
$,i=\varnothing, Rd(i):r,$
$\Leftrightarrow\ ,i=\varnothing, i\oslash r,$
$\Leftrightarrow\ ,i=\varnothing, i\oslash r, i\mathring{\bigcirc}r,$
$\Leftrightarrow\ ,i=\varnothing, i=\varnothing, i\oslash r, i\mathring{\bigcirc}r,$
$\Leftrightarrow\ ,i=\varnothing, i\oslash r, i\mathring{\bigcirc}r, i=\varnothing,$
$\Leftrightarrow\ ,i=\varnothing, i\oslash r, i\mathring{\bigcirc}r, r=\varnothing,$
$\Leftrightarrow\ ,i=\varnothing, i\oslash r, r=\varnothing,$
$\Leftrightarrow\ ,i=\varnothing, Rd(i):r, r=\varnothing,$

$premise\ 2:$
$, \&SVi\obot i, Rd(i):r, \;\Leftrightarrow\; , \&SVi\obot i, Rd(i):r, r=\varnothing, \;\Rightarrow$

$,i\,!\!=\varnothing, \&SVi\mathring{\bigcirc}i, Rd(i):r,$

$\Leftrightarrow\ , \&SVi\mathring{\bigcirc}i, i\,!\!=\varnothing, Rd(i):r,$

$\Leftrightarrow\ , \&SVi\mathring{\bigcirc}i, i\,!\!=\varnothing, i\oslash j, j\oplus, Rd(j):r, j\ovee,$

$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash j, j\oplus, \&SVi\mathring{\bigcirc}i, Rd(j):r, j\ovee,$

$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash j, i\obot j, j\oplus, \&SVi\mathring{\bigcirc}i, Rd(j):r, j\ovee,$

$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash j, j\oplus, i\obot j, \&SVi\mathring{\bigcirc}i, Rd(j):r, j\ovee,$

$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash j, j\oplus, i\obot j, \&SVi\obot j, Rd(j):r, j\ovee,$

$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash j, j\oplus, i\obot j, \&SVi\obot j, Rd(j):r, r=\varnothing, j\ovee,$

$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash j, j\oplus, i\obot j, \&SVi\mathring{\bigcirc}i, Rd(j):r, r=\varnothing, j\ovee,$

$\Leftrightarrow\ ,i\,!\!=\varnothing, i\oslash j, j\oplus, \&SVi\mathring{\bigcirc}i, Rd(j):r, r=\varnothing, j\ovee,$

$\Leftrightarrow\ , \&SVi\mathring{\bigcirc}i, i\,!\!=\varnothing, i\oslash j, j\oplus, Rd(j):r, j\ovee, r=\varnothing,$

$$\Leftrightarrow\ , \&SVi \bigcirc^{\!\circ} i, i \mathrel{!}= \varnothing, Rd(i) : r, r = \varnothing,$$

$$\Leftrightarrow\ , i \mathrel{!}= \varnothing,\ \&SVi \bigcirc^{\!\circ} i, Rd(i) : r, r = \varnothing,$$

conclusion :
$, Rd(i) : r,\ \Leftrightarrow\ , Rd(i) : r, r = \varnothing,$

$$, Rd(i) : r, \otimes,\ \Leftrightarrow\ , \otimes,$$

20 Recursive Function Rc(i;j)

20.1 Definition of Rc(i;j)

$$,Rc(i;j), \iff ,if(i=\varnothing)\left[\begin{array}{l} , \\ ,if(j=\varnothing)\left[\begin{array}{l} , \\ ,i\oplus,j\oplus,Rc(i;j), \end{array}\right], \end{array}\right],$$

20.2 Theorems of Rc(i;j)

20.2.1 Transformation:

$$,i=\varnothing,Rc(i;j), \iff ,i=\varnothing,$$

$$,j=\varnothing,Rc(i;j), \iff ,j=\varnothing,$$

$$,i\,!\!=\varnothing,Rc(i;j), \iff ,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l} , \\ ,i\oplus,j\oplus,Rc(i;j), \end{array}\right],$$

$$,j\,!\!=\varnothing,Rc(i;j), \iff ,j\,!\!=\varnothing,if(i=\varnothing)\left[\begin{array}{l} , \\ ,i\oplus,j\oplus,Rc(i;j), \end{array}\right],$$

$$,i\,!\!=\varnothing,j\,!\!=\varnothing,Rc(i;j), \iff ,i\,!\!=\varnothing,j\,!\!=\varnothing,i\oplus,j\oplus,Rc(i;j),$$

$$,Rc(i;j), \iff ,if(i=\varnothing)\left[\begin{array}{l} , \\ ,if(j=\varnothing)\left[\begin{array}{l} , \\ ,i\oplus,j\oplus, \end{array}\right], \end{array}\right],Rc(i;j),$$

20.2.2 Result:

$$,Rc(i;j),\ \Leftrightarrow\ ,Rc(i;j),if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,j=\varnothing,\end{array}\right],$$

induction proof:
premise 1 :

$,i=\varnothing,Rc(i;j),$

$\Leftrightarrow\ ,i=\varnothing,$

$$\Leftrightarrow\ ,i=\varnothing,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,j=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,i=\varnothing,Rc(i;j),if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,j=\varnothing,\end{array}\right],$$

premise 2 :

$$,\ \&SHi\rightarrow i,Rc(i;j),\ \Leftrightarrow\ ,\ \&SHi\rightarrow i,Rc(i;j),if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,j=\varnothing,\end{array}\right],\ \Rightarrow$$

$,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,Rc(i;j),$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,Rc(i;j),$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},\\ \\ ,i\oplus,j\oplus,Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,\\ \\ ,i\oplus,j\oplus,Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,if(j=\varnothing)\left[\begin{array}{l},\\ \\ ,i=\varnothing,\end{array}\right],\\ ,i\oplus,j\oplus,Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,j=\varnothing,\end{array}\right],\\ ,i\oplus,j\oplus,Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\text{-}\left[\begin{array}{l} , if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-}, \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\text{-}\left[\begin{array}{l} ,\ \&SHi\overset{\circ}{\bigcirc}i, if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-}, \\ ,\ \&SHi\overset{\circ}{\bigcirc}i, i\oplus, j\oplus, Rc(i;j), \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\text{-}\left[\begin{array}{l} ,\ \&SHi\overset{\circ}{\bigcirc}i, if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-}, \\ , i\oplus, j\oplus,\ \&SHi\rightarrow i, Rc(i;j), \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\text{-}\left[\begin{array}{l} ,\ \&SHi\overset{\circ}{\bigcirc}i, if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-}, \\ , i\oplus, j\oplus,\ \&SHi\rightarrow i, Rc(i;j), if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-}, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\text{-}\left[\begin{array}{l} ,\ \&SHi\overset{\circ}{\bigcirc}i, \\ , i\oplus, j\oplus,\ \&SHi\rightarrow i, Rc(i;j), \end{array}\right]\text{-}, if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\text{-}\left[\begin{array}{l} , \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right]\text{-}, if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, Rc(i;j), if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-},$$

$conclusion:$

$$, Rc(i;j),\ \Leftrightarrow\ , Rc(i;j), if(i=\varnothing)\text{-}\left[\begin{array}{l} , \\ , j=\varnothing, \end{array}\right]\text{-},$$

20.2.3 With R(i):

$$, Rc(i;j), R(i), R(j), \ \Leftrightarrow \ , R(i), R(j),$$

induction proof:

$premise\ 1:$

$, i = \varnothing, Rc(i;j), R(i), R(j),$

$\Leftrightarrow \ , i = \varnothing, R(i), R(j),$

$premise\ 2:$

$, \ \&SHi \rightarrow i, Rc(i;j), R(i), R(j), \ \Leftrightarrow \ , \ \&SHi \rightarrow i, R(i), R(j), \ \Rightarrow$

$, i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, Rc(i;j), R(i), R(j),$

$\Leftrightarrow \ , \ \&SHi \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, Rc(i;j), R(i), R(j),$

$$\Leftrightarrow \ , \ \&SHi \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \\ \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right], R(i), R(j),$$

$$\Leftrightarrow \ , \ \&SHi \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , R(i), R(j), \\ \\ , i\oplus, j\oplus, Rc(i;j), R(i), R(j), \end{array}\right],$$

$$\Leftrightarrow \ , i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \ \&SHi \mathring{\bigcirc} i, R(i), R(j), \\ \\ , \ \&SHi \mathring{\bigcirc} i, i\oplus, j\oplus, Rc(i;j), R(i), R(j), \end{array}\right],$$

$$\Leftrightarrow \ , i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \ \&SHi \mathring{\bigcirc} i, R(i), R(j), \\ \\ , i\oplus, j\oplus, \ \&SHi \rightarrow i, Rc(i;j), R(i), R(j), \end{array}\right],$$

$$\Leftrightarrow \ , i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \ \&SHi \mathring{\bigcirc} i, R(i), R(j), \\ \\ , i\oplus, j\oplus, \ \&SHi \rightarrow i, R(i), R(j), \end{array}\right],$$

$$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, if(j = \varnothing) \left[\begin{array}{l} , R(i), R(j), \\ \\ , i\oplus, j\oplus, R(i), R(j), \end{array}\right],$$

$$\Leftrightarrow \ , i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, if(j = \varnothing) \left[\begin{array}{l} , R(i), R(j), \\ \\ , j \mathrel{!=} \varnothing, i\oplus, j\oplus, R(i), R(j), \end{array}\right],$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc} i, if(j=\varnothing)\text{-}\left[\begin{array}{l}, R(i), R(j),\\ \\ , i\oplus, R(i), j \mathrel{!=} \varnothing, j\oplus, R(j),\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc} i, if(j=\varnothing)\text{-}\left[\begin{array}{l}, R(i), R(j),\\ \\ , i\oplus, R(i), j \mathrel{!=} \varnothing, R(j),\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc} i, if(j=\varnothing)\text{-}\left[\begin{array}{l}, R(i), R(j),\\ \\ , i\oplus, R(i), R(j),\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc} i, if(j=\varnothing)\text{-}\left[\begin{array}{l}, i \mathrel{!=} \varnothing, R(i), R(j),\\ \\ , i \mathrel{!=} \varnothing, i\oplus, R(i), R(j),\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc} i, if(j=\varnothing)\text{-}\left[\begin{array}{l}, i \mathrel{!=} \varnothing, R(i), R(j),\\ \\ , i \mathrel{!=} \varnothing, R(i), R(j),\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc} i, if(j=\varnothing)\text{-}\left[\begin{array}{l}, R(i), R(j),\\ \\ , R(i), R(j),\end{array}\right]\text{-},$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc} i, if(j=\varnothing)\text{-}\left[\begin{array}{l},\\ \\ ,\end{array}\right]\text{-}, R(i), R(j),$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc} i, R(i), R(j),$$

conclusion :

$, Rc(i;j), R(i), R(j),\ \Leftrightarrow\ , R(i), R(j),$

20.2.4 With operator:

$$, Rc(i;j), i\textcircled{\scriptsize$\downarrow$}, j\textcircled{\scriptsize$\downarrow$},\ \Leftrightarrow\ , i\textcircled{\scriptsize$\downarrow$}, j\textcircled{\scriptsize$\downarrow$},$$

proof:
$, Rc(i;j), i\textcircled{\scriptsize$\downarrow$}, j\textcircled{\scriptsize$\downarrow$},$

$\Leftrightarrow\ ,Rc(i;j),R(i),i\textcircled{\downarrow},j\textcircled{\downarrow},$

$\Leftrightarrow\ ,Rc(i;j),R(i),i\textcircled{\downarrow},R(j),j\textcircled{\downarrow},$

$\Leftrightarrow\ ,Rc(i;j),R(i),R(j),i\textcircled{\downarrow},j\textcircled{\downarrow},$

$\Leftrightarrow\ ,R(i),R(j),i\textcircled{\downarrow},j\textcircled{\downarrow},$

$\Leftrightarrow\ ,R(i),i\textcircled{\downarrow},R(j),j\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\textcircled{\downarrow},R(j),j\textcircled{\downarrow},$

$\Leftrightarrow\ ,i\textcircled{\downarrow},j\textcircled{\downarrow},$

$$,\quad\Leftrightarrow\ ,i\textcircled{\curlywedge}i_0,j\textcircled{\curlywedge}j_0,Rc(i_0;j_0),i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$,Rc(i;j),\otimes,\quad\Leftrightarrow\ ,\otimes,$$

induction proof:

$premise\ 1:$

$,i=\varnothing,Rc(i;j),\otimes,$

$\Leftrightarrow\ ,i=\varnothing,\otimes,$

$premise\ 2:$

$,\ \&SHi\rightarrow i,Rc(i;j),\otimes,\quad\Leftrightarrow\ ,\ \&SHi\rightarrow i,\otimes,\quad\Rightarrow$

$,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,Rc(i;j),\otimes,$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,Rc(i;j),\otimes,$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)-\left[\begin{array}{l},\\ \\ ,i\oplus,j\oplus,Rc(i;j),\end{array}\right]-,\otimes,$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)-\left[\begin{array}{l},\otimes,\\ \\ ,i\oplus,j\oplus,Rc(i;j),\otimes,\end{array}\right]-,$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,if(j=\varnothing)-\left[\begin{array}{l},\ \&SHi\mathring{\bigcirc}i,\otimes,\\ \\ ,\ \&SHi\mathring{\bigcirc}i,i\oplus,j\oplus,Rc(i;j),\otimes,\end{array}\right]-,$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l} ,\ \&SHi\mathring{\bigcirc}i,\otimes, \\ \\ ,i\oplus,j\oplus,\ \&SHi\rightarrow i,Rc(i;j),\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l} ,\ \&SHi\mathring{\bigcirc}i,\otimes, \\ \\ ,i\oplus,j\oplus,\ \&SHi\rightarrow i,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,if(j=\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ ,i\oplus,j\oplus,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,if(j=\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ ,i\oplus,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,if(j=\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,if(j=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right],\otimes,$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,\otimes,$$

$conclusion:$

$$,Rc(i;j),\otimes,\ \Leftrightarrow\ ,\otimes,$$

20.2.5 Symmetry:

$$,Rc(i;j),\ \Leftrightarrow\ ,Rc(j;i),$$

induction proof:
$premise\ 1:$
$,i=\varnothing,Rc(i;j),$

$$\Leftrightarrow\ ,i=\varnothing,$$

$$\Leftrightarrow \; ,i = \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \\ , \end{array}\right],$$

$$\Leftrightarrow \; ,if(j = \varnothing) \left[\begin{array}{l} ,i = \varnothing, \\ ,i = \varnothing, \end{array}\right],$$

$$\Leftrightarrow \; ,if(j = \varnothing) \left[\begin{array}{l} ,i = \varnothing, \\ ,i = \varnothing, if(i = \varnothing) \left[\begin{array}{l} , \\ ,j\oplus, i\oplus, Rc(j;i), \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \; ,i = \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \\ ,if(i = \varnothing) \left[\begin{array}{l} , \\ ,j\oplus, i\oplus, Rc(j;i), \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \; ,i = \varnothing, Rc(j;i),$$

$premise\ 2:$
$$, \&SHi \rightarrow i, Rc(i;j), \;\; \Leftrightarrow \;\; , \&SHi \rightarrow i, Rc(j;i), \;\; \Rightarrow$$

$$,i \mathrel{!=} \varnothing, \; \&SHi \mathring{\bigcirc} i, Rc(i;j),$$

$$\Leftrightarrow \; , \&SHi \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, Rc(i;j),$$

$$\Leftrightarrow \; , \&SHi \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \\ ,i\oplus, j\oplus, Rc(i;j), \end{array}\right],$$

$$\Leftrightarrow \; ,i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \; \&SHi \mathring{\bigcirc} i, \\ ,i\oplus, j\oplus, \; \&SHi \rightarrow i, Rc(i;j), \end{array}\right],$$

$$\Leftrightarrow \; ,i \mathrel{!=} \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \; \&SHi \mathring{\bigcirc} i, \\ ,i\oplus, j\oplus, \; \&SHi \rightarrow i, Rc(j;i), \end{array}\right],$$

$$\Leftrightarrow \; ,i \mathrel{!=} \varnothing, \; \&SHi \mathring{\bigcirc} i, if(j = \varnothing) \left[\begin{array}{l} , \\ ,i\oplus, j\oplus, Rc(j;i), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i\,!\!=\varnothing,\\ \\ , i\,!\!=\varnothing, i\oplus, j\oplus, Rc(j;i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i\,!\!=\varnothing,\\ \\ , i\,!\!=\varnothing, if(i=\varnothing)\left[\begin{array}{l},\\ \\ , i\oplus, j\oplus, Rc(j;i),\end{array}\right]\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l},\\ \\ , if(i=\varnothing)\left[\begin{array}{l},\\ \\ , i\oplus, j\oplus, Rc(j;i),\end{array}\right]\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l},\\ \\ , if(i=\varnothing)\left[\begin{array}{l},\\ \\ , j\oplus, i\oplus, Rc(j;i),\end{array}\right]\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, Rc(j;i),$$

$conclusion$:

$$, Rc(i;j),\ \Leftrightarrow\ , Rc(j;i),$$

20.2.6 Swap with operator:

$$, Rc(i;j), \circledcirc m,\ \Leftrightarrow\ , \circledcirc m, Rc(i;j),$$
$$, Rc(i;j), \odot m,\ \Leftrightarrow\ , \odot m, Rc(i;j),$$

$$, Rc(i;j), m\circledast n,\ \Leftrightarrow\ , m\circledast n, Rc(i;j),$$
$$, Rc(i;j), m\circledast n,\ \Leftrightarrow\ , m\circledast n, Rc(i;j),$$
$$, Rc(i;j), m\circledast n,\ \Leftrightarrow\ , m\circledast n, Rc(i;j),$$

$$, Rc(i;j), m\oplus,\ \Leftrightarrow\ , m\oplus, Rc(i;j),$$

induction proof:
$premise$ 1 :
$, i=\varnothing, Rc(i;j), m\oplus,$

$\Leftrightarrow\ , i=\varnothing, m\oplus,$

$\Leftrightarrow\ , m\oplus, i=\varnothing,$

$\Leftrightarrow\ , m\oplus, i=\varnothing, Rc(i;j),$

$\Leftrightarrow\ , i=\varnothing, m\oplus, Rc(i;j),$

$premise\ 2:$

$, \&SHi\rightarrow i, Rc(i;j), m\oplus,\ \Leftrightarrow\ , \&SHi\rightarrow i, m\oplus, Rc(i;j),\ \Rightarrow$

$, i\,!\!=\varnothing, \&SHi\,♂i, Rc(i;j), m\oplus,$

$\Leftrightarrow\ , \&SHi\,♂i, i\,!\!=\varnothing, Rc(i;j), m\oplus,$

$$\Leftrightarrow\ , \&SHi\,♂i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, \\ , i\oplus, j\oplus, Rc(i;j),\end{array}\right], m\oplus,$$

$$\Leftrightarrow\ , \&SHi\,♂i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, m\oplus, \\ , i\oplus, j\oplus, Rc(i;j), m\oplus,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, \&SHi\,♂i, m\oplus, \\ , i\oplus, j\oplus, \&SHi\rightarrow i, Rc(i;j), m\oplus,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, \&SHi\,♂i, m\oplus, \\ , i\oplus, j\oplus, \&SHi\rightarrow i, m\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\,♂i, if(j=\varnothing)\left[\begin{array}{l}, m\oplus, \\ , i\oplus, j\oplus, m\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\,♂i, if(j=\varnothing)\left[\begin{array}{l}, m\oplus, \\ , m\oplus, i\oplus, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\,♂i, m\oplus, if(j=\varnothing)\left[\begin{array}{l}, \\ , i\oplus, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, m\oplus, i \,!\!= \varnothing, if(j = \varnothing) \left[\begin{array}{l} , \\ \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, m\oplus, i \,!\!= \varnothing, Rc(i;j),$$

$$\Leftrightarrow\ , i \,!\!= \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}i, m\oplus, Rc(i;j),$$

$conclusion:$

$$, Rc(i;j), m\oplus, \ \Leftrightarrow\ , m\oplus, Rc(i;j),$$

$$, Rc(i;j), m\ominus, \ \Leftrightarrow\ , m\ominus, Rc(i;j),$$
$$, Rc(i;j), m\textcircled{$\downarrow$}, \ \Leftrightarrow\ , m\textcircled{$\downarrow$}, Rc(i;j),$$

20.2.7 Swap with branch function:

$$, Rc(i;j), if(m = n) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array}\right], \ \Leftrightarrow\ , if(m = n) \left[\begin{array}{l} , Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2, \end{array}\right],$$

$$, Rc(i;j), if(m = \varnothing) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array}\right], \ \Leftrightarrow\ , if(m = \varnothing) \left[\begin{array}{l} , Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2, \end{array}\right],$$

$$, Rc(i;j), if(m\overset{\circ}{\bigcirc}n) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array}\right], \ \Leftrightarrow\ , if(m\overset{\circ}{\bigcirc}n) \left[\begin{array}{l} , Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2, \end{array}\right],$$

$$, Rc(i;j), if(m\overset{\circ}{\bigcirc}{\circ}n) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array}\right], \ \Leftrightarrow\ , if(m\overset{\circ}{\bigcirc}{\circ}n) \left[\begin{array}{l} , Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2, \end{array}\right],$$

induction proof:

premise 1 :

$$,i=\varnothing, Rc(i;j), if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i=\varnothing, if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,i=\varnothing,\ ©c_1, \\ \\ ,i=\varnothing,\ ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,i=\varnothing, Rc(i;j),\ ©c_1, \\ \\ ,i=\varnothing, Rc(i;j),\ ©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i=\varnothing, if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,Rc(i;j),\ ©c_1, \\ \\ ,Rc(i;j),\ ©c_2, \end{array}\right],$$

premise 2 :

$$,\ \&SHi\rightarrow i, Rc(i;j), if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],\ \Leftrightarrow\ ,\ \&SHi\rightarrow i, if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,Rc(i;j),\ ©c_1, \\ \\ ,Rc(i;j),\ ©c_2, \end{array}\right],\ \Rightarrow$$

$$,i\,!\!=\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, Rc(i;j), if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, Rc(i;j), if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \\ \\ ,i\oplus, j\oplus, Rc(i;j), \end{array}\right], if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \\ , i\oplus, j\oplus, Rc(i;j), if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \&SHi\,\mathring{\bigcirc}i, if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \\ , i\oplus, j\oplus,\ \&SHi\rightarrow i, Rc(i;j), if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \&SHi\,\mathring{\bigcirc}i, if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \\ , i\oplus, j\oplus,\ \&SHi\rightarrow i, if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \\ , i\oplus, j\oplus, if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \\ , j\,!\!=\varnothing, i\oplus, j\oplus, if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} , i\,!\!=\varnothing, if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \\ , i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, if(m\mathring{\bigcirc}n)\left[\begin{array}{l}, Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), \copyright c_1, \\ \\ ,i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), \copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), \copyright c_1, \\ \\ ,i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), \copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,i\,!\!=\varnothing, if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,i\,!\!=\varnothing, j\,!\!=\varnothing, if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,Rc(i;j), \copyright c_1, \\ \\ ,Rc(i;j), \copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,Rc(i;j), \copyright c_1, \\ \\ ,Rc(i;j), \copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} ,j=\varnothing, if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l} ,Rc(i;j), \copyright c_1, \\ \\ ,Rc(i;j), \copyright c_2, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l},j=\varnothing,\copyright c_1,\\ \\ ,j=\varnothing,\copyright c_2,\end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l},j=\varnothing,Rc(i;j),\copyright c_1,\\ \\ ,j=\varnothing,Rc(i;j),\copyright c_2,\end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right], \\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right],\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right], if(m\mathring{\bigcirc}n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i, if(m\mathring{\bigcirc}n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right],$$

conclusion :

$$,Rc(i;j),if(m\mathring{\bigcirc}n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,if(m\mathring{\bigcirc}n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right],$$

$$,Rc(i;j),if(m\to n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\ \Leftrightarrow\ ,if(m\to n)\left[\begin{array}{l},Rc(i;j),\copyright c_1,\\ \\ ,Rc(i;j),\copyright c_2,\end{array}\right],$$

$$, Rc(i;j), if(m \obot n) \left[\begin{array}{c} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \quad \Leftrightarrow \quad , if(m \obot n) \left[\begin{array}{c} , Rc(i;j), ©c_1, \\ \\ , Rc(i;j), ©c_2, \end{array} \right],$$

20.2.8 Swap with propositions:

$$, m = n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m = n,$$

$$, m \mathrel{!=} n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m \mathrel{!=} n,$$

$$, m = \varnothing, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m = \varnothing,$$

$$, m \mathrel{!=} \varnothing, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m \mathrel{!=} \varnothing,$$

$$, m \mathring{\bigcirc} n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m \mathring{\bigcirc} n,$$

$$, m! \mathring{\bigcirc} n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m! \mathring{\bigcirc} n,$$

$$, m \mathring{\bigcirc}\!{\circ} n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m \mathring{\bigcirc}\!{\circ} n,$$

$$, m! \mathring{\bigcirc}\!{\circ} n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m! \mathring{\bigcirc}\!{\circ} n,$$

$$, m \rightarrow n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m \rightarrow n,$$

$$, m! \rightarrow n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m! \rightarrow n,$$

$$, m \obot n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m \obot n,$$

$$, m! \obot n, Rc(i;j), \quad \Leftrightarrow \quad , Rc(i;j), m! \obot n,$$

20.2.9 Swap with recursive function:

$$,Rc(i;j),R(m),\ \Leftrightarrow\ ,R(m),Rc(i;j),$$

$$,Rc(i;j),R\_(m),\ \Leftrightarrow\ ,R\_(m),Rc(i;j),$$

$$,Rc(i;j),Rc(m;n),\ \Leftrightarrow\ ,Rc(m;n),Rc(i;j),$$

induction proof:

$premise\ 1:$

$,i=\varnothing,Rc(i;j),Rc(m;n),$

$\Leftrightarrow\ ,i=\varnothing,Rc(m;n),$

$\Leftrightarrow\ ,Rc(m;n),i=\varnothing,$

$\Leftrightarrow\ ,Rc(m;n),i=\varnothing,Rc(i;j),$

$\Leftrightarrow\ ,i=\varnothing,Rc(m;n),Rc(i;j),$

$premise\ 2:$

$,\ \&SHi\rightarrow i,Rc(i;j),Rc(m;n),\ \Leftrightarrow\ ,\ \&SHi\rightarrow i,Rc(m;n),Rc(i;j),\ \Rightarrow$

$,i\,!\!=\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i,Rc(i;j),Rc(m;n),$

$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i,i\,!\!=\varnothing,Rc(i;j),Rc(m;n),$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},\\ \\ ,i\oplus,j\oplus,Rc(i;j),\end{array}\right],Rc(m;n),$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},Rc(m;n),\\ \\ ,i\oplus,j\oplus,Rc(i;j),Rc(m;n),\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},\ \&SHi\overset{\circ}{\bigcirc}i,Rc(m;n),\\ \\ ,i\oplus,j\oplus,\ \&SHi\rightarrow i,Rc(i;j),Rc(m;n),\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},\ \&SHi\overset{\circ}{\bigcirc}i,Rc(m;n),\\ \\ ,i\oplus,j\oplus,\ \&SHi\rightarrow i,Rc(m;n),Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, Rc(m;n),\\ \\ , i\oplus, j\oplus, Rc(m;n), Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, Rc(m;n),\\ \\ , Rc(m;n), i\oplus, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, Rc(m;n), if(j=\varnothing)\left[\begin{array}{l},\\ \\ , i\oplus, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, Rc(m;n), i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l},\\ \\ , i\oplus, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, Rc(m;n), i\,!\!=\varnothing, Rc(i;j),$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, Rc(m;n), Rc(i;j),$$

conclusion :

$$, Rc(i;j), Rc(m;n),\ \Leftrightarrow\ , Rc(m;n), Rc(i;j),$$

20.2.10 Swap with flag object :

$$, Rc(i;j),\ \&SHj\,\mathring{\bigcirc}m,\ \Leftrightarrow\ ,\ \&SHj\,\mathring{\bigcirc}m, Rc(i;j),$$

$$, Rc(i;j),\ \&SHi\,\mathring{\bigcirc}m,\ \Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}m, Rc(i;j),$$

$$, Rc(i;j),\ \&SHj\leftarrow m,\ \Leftrightarrow\ ,\ \&SHj\leftarrow m, Rc(i;j),$$

$$, Rc(i;j),\ \&SHi\rightarrow m,\ \Leftrightarrow\ ,\ \&SHi\rightarrow m, Rc(i;j),$$

20.2.11 Fundamental properties:

$$,Rc(i;j),i\,!{=}\,\varnothing, \;\Leftrightarrow\; ,i\,!{=}\,\varnothing,Rc(i;j),i\,!{=}\,\varnothing,$$

proof:

$,Rc(i;j),i\,!{=}\,\varnothing,$

$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right],Rc(i;j),i\,!{=}\,\varnothing,$

$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,Rc(i;j),i\,!{=}\,\varnothing, \\ \\ ,Rc(i;j),i\,!{=}\,\varnothing, \end{array}\right],$

$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,i{=}\varnothing,Rc(i;j),i\,!{=}\,\varnothing, \\ \\ ,Rc(i;j),i\,!{=}\,\varnothing, \end{array}\right],$

$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,i{=}\varnothing,i\,!{=}\,\varnothing, \\ \\ ,Rc(i;j),i\,!{=}\,\varnothing, \end{array}\right],$

$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ ,Rc(i;j),i\,!{=}\,\varnothing, \end{array}\right],$

$\Leftrightarrow\; ,if(i{=}\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right],Rc(i;j),i\,!{=}\,\varnothing,$

$\Leftrightarrow\; ,i\,!{=}\,\varnothing,Rc(i;j),i\,!{=}\,\varnothing,$

$$,i\,!{=}\,\varnothing,Rc(i;j),i{=}\varnothing, \;\Leftrightarrow\; ,i\,!{=}\,\varnothing,j\,!{=}\,\varnothing,Rc(i;j),i{=}\varnothing,$$

proof:

$,i\,!{=}\,\varnothing,Rc(i;j),i{=}\varnothing,$

$$\Leftrightarrow\ ,i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \\ \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right], i=\varnothing,$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , i=\varnothing, \\ \\ , i\oplus, j\oplus, Rc(i;j), i=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l} , i=\varnothing, \\ \\ , j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), i=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , if(j=\varnothing)\left[\begin{array}{l} , i\,!\!=\varnothing, i=\varnothing, \\ \\ , i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), i=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , if(j=\varnothing)\left[\begin{array}{l} , \otimes, \\ \\ , i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), i=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , if(j=\varnothing)\left[\begin{array}{l} , \otimes, \\ \\ , i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), i=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , if(j=\varnothing)\left[\begin{array}{l} , \otimes, \\ \\ , \end{array}\right], i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), i=\varnothing,$$

$$\Leftrightarrow\ , j\,!\!=\varnothing, i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), i=\varnothing,$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, j\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), i=\varnothing,$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), i=\varnothing,$$

$$, i\,!\!=\varnothing, Rc(i;j), i\overset{\circ}{\bigcirc}j, \quad \Leftrightarrow \quad , i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), i\overset{\circ}{\bigcirc}j,$$

proof:
$, i\,!\!=\varnothing, Rc(i;j), i\overset{\circ}{\bigcirc}j,$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),if(i{=}\varnothing)\left[\begin{array}{l},\\ \\ ,j{=}\varnothing,\end{array}\right],i\bigcirc^{\!\circ}j,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),if(i{=}\varnothing)\left[\begin{array}{l},i{=}\varnothing,\\ \\ ,j{=}\varnothing,\end{array}\right],i\bigcirc^{\!\circ}j,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),if(i{=}\varnothing)\left[\begin{array}{l},i{=}\varnothing,i\bigcirc^{\!\circ}j,\\ \\ ,j{=}\varnothing,i\bigcirc^{\!\circ}j,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),if(i{=}\varnothing)\left[\begin{array}{l},i{=}\varnothing,i\bigcirc^{\!\circ}j,\\ \\ ,i\bigcirc^{\!\circ}j,j{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),if(i{=}\varnothing)\left[\begin{array}{l},i{=}\varnothing,i\bigcirc^{\!\circ}j,\\ \\ ,i\bigcirc^{\!\circ}j,i{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),if(i{=}\varnothing)\left[\begin{array}{l},i{=}\varnothing,\\ \\ ,i{=}\varnothing,\end{array}\right],i\bigcirc^{\!\circ}j,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),if(i{=}\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right],i{=}\varnothing,i\bigcirc^{\!\circ}j,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,Rc(i;j),i{=}\varnothing,i\bigcirc^{\!\circ}j,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,j\,!{=}\varnothing,Rc(i;j),i{=}\varnothing,i\bigcirc^{\!\circ}j,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,j\,!{=}\varnothing,Rc(i;j),i\bigcirc^{\!\circ}j,$$

$$,Rc(i;j),i{=}\varnothing,j{=}\varnothing,\ \Leftrightarrow\ ,if(i{=}\varnothing)\left[\begin{array}{l},j{=}\varnothing,\\ \\ ,j\,!{=}\varnothing,\end{array}\right],Rc(i;j),i{=}\varnothing,j{=}\varnothing,$$

$$,i\bigcirc^{\!\circ}j,Rc(i;j),\ \Leftrightarrow\ ,Rc(i;j),i\bigcirc^{\!\circ}j,$$

induction proof:
premise 1 :
$, i = \varnothing, i♂j, Rc(i; j),$

$\Leftrightarrow \ , i♂j, i = \varnothing, Rc(i; j),$

$\Leftrightarrow \ , i♂j, i = \varnothing,$

$\Leftrightarrow \ , i = \varnothing, i♂j,$

$\Leftrightarrow \ , i = \varnothing, Rc(i; j), i♂j,$

premise 2 :
$, \ \&SHi \to i, i♂j, Rc(i; j), \ \Leftrightarrow \ , \&SHi \to i, Rc(i; j), i♂j, \ \Rightarrow$

$, i \mathrel{!}= \varnothing, \ \&SHi ♂i, i♂j, Rc(i; j),$

$\Leftrightarrow \ , \ \&SHi ♂i, i♂j, i \mathrel{!}= \varnothing, Rc(i; j),$

$\Leftrightarrow \ , \ \&SHi ♂i, i♂j, i \mathrel{!}= \varnothing, i \mathrel{!}= \varnothing, Rc(i; j),$

$\Leftrightarrow \ , \ \&SHi ♂i, i♂j, j \mathrel{!}= \varnothing, i \mathrel{!}= \varnothing, Rc(i; j),$

$\Leftrightarrow \ , \ \&SHi ♂i, i♂j, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, Rc(i; j),$

$\Leftrightarrow \ , \ \&SHi ♂i, i♂j, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, Rc(i; j),$

$\Leftrightarrow \ , \ \&SHi ♂i, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i♂j, i\oplus, j\oplus, Rc(i; j),$

$\Leftrightarrow \ , \ \&SHi ♂i, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, i♂j, Rc(i; j),$

$\Leftrightarrow \ , i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, \ \&SHi \to i, i♂j, Rc(i; j),$

$\Leftrightarrow \ , i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, \ \&SHi \to i, Rc(i; j), i♂j,$

$\Leftrightarrow \ , \ \&SHi ♂i, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, Rc(i; j), i♂j,$

$\Leftrightarrow \ , \ \&SHi ♂i, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, Rc(i; j), i♂j,$

$\Leftrightarrow \ , \ \&SHi ♂i, i \mathrel{!}= \varnothing, Rc(i; j), i♂j,$

$\Leftrightarrow \ , i \mathrel{!}= \varnothing, \ \&SHi ♂i, Rc(i; j), i♂j,$

conclusion :

$, i\mathring{\bigcirc}j, Rc(i;j), \ \Leftrightarrow \ , Rc(i;j), i\mathring{\bigcirc}j,$

$$, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), \ \Leftrightarrow \ \sim, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$$

induction proof:

premise 1 :

$, i_1 = \varnothing, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2),$

$\Leftrightarrow \ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 = \varnothing, Rc(i_1;j_1), Rc(i_2;j_2),$

$\Leftrightarrow \ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 = \varnothing, Rc(i_2;j_2),$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_1 = \varnothing, Rc(i_2;j_2),$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_2 = \varnothing, Rc(i_2;j_2),$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_2 = \varnothing,$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_1\mathring{\bigcirc}i_2, i_2 = \varnothing,$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_2 = \varnothing, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_2 = \varnothing, Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_1 = \varnothing, Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$

$\Leftrightarrow \ , j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, i_1 = \varnothing, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$

$\Leftrightarrow \ , i_1 = \varnothing, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$

premise 2 :

$, \ \&SHi \rightarrow i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), \ \Leftrightarrow$

$, \ \&SHi \rightarrow i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \ \Rightarrow$

$, i_1 \mathrel{!}= \varnothing, \ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2),$

$\Leftrightarrow \ , \ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 \mathrel{!}= \varnothing, Rc(i_1;j_1), Rc(i_2;j_2),$

$$\Leftrightarrow \ , \ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 \mathrel{!}= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} , \\ \\ , i_1\oplus, j_1\oplus, Rc(i_1;j_1), \end{array}\right], Rc(i_2;j_2),$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,Rc(i_2;j_2), \\ \\ ,i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1 = \varnothing, Rc(i_2;j_2), \\ \\ ,j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1\mathring{\bigcirc}j_2, j_1 = \varnothing, Rc(i_2;j_2), \\ \\ ,j_1\mathring{\bigcirc}j_2, j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1\mathring{\bigcirc}j_2, j_2 = \varnothing, Rc(i_2;j_2), \\ \\ ,j_1\mathring{\bigcirc}j_2, j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1\mathring{\bigcirc}j_2, j_2 = \varnothing, \\ \\ ,j_1\mathring{\bigcirc}j_2, j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1\mathring{\bigcirc}j_2, j_1 = \varnothing, \\ \\ ,j_1\mathring{\bigcirc}j_2, j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1 = \varnothing, j_1\mathring{\bigcirc}j_2, \\ \\ ,j_1\mathring{\bigcirc}j_2, j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1\mathring{\bigcirc}j_2, \\ \\ ,j_1\mathring{\bigcirc}j_2, j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, if(j_1 = \varnothing)\left[\begin{array}{l} ,j_1\mathring{\bigcirc}j_2, \\ \\ ,j_1\mathring{\bigcirc}j_2, j_2 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, if(j_1 = \varnothing)\left[\begin{array}{l} ,i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, j_1\mathring{\bigcirc}j_2, \\ \\ ,i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, j_1\mathring{\bigcirc}j_2, j_2 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, if(j_1 = \varnothing)\left[\begin{array}{l} ,i_1\mathring{\bigcirc}i_2, i_1 \,!\!= \varnothing, j_1\mathring{\bigcirc}j_2, \\ \\ ,i_1\mathring{\bigcirc}i_2, i_2 \,!\!= \varnothing, j_1\mathring{\bigcirc}j_2, j_2 \,!\!= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, i_1 \mathrel{!=}\varnothing, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), i_2 \mathrel{!=}\varnothing, j_2 \mathrel{!=}\varnothing, Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, i_1 \mathrel{!=}\varnothing, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), i_2 \mathrel{!=}\varnothing, j_2 \mathrel{!=}\varnothing, i_2\oplus, j_2\oplus, Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, i_1 \mathrel{!=}\varnothing, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, i_1\oplus, j_1\oplus, i_2 \mathrel{!=}\varnothing, j_2 \mathrel{!=}\varnothing, i_2\oplus, j_2\oplus, Rc(i_1;j_1), Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, i_1 \mathrel{!=}\varnothing, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_2 \mathrel{!=}\varnothing, j_2 \mathrel{!=}\varnothing, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, i_1\oplus, j_1\oplus, i_2\oplus, j_2\oplus, Rc(i_1;j_1), Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, i_1 \mathrel{!=}\varnothing, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_1 \mathrel{!=}\varnothing, j_1 \mathrel{!=}\varnothing, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, i_1\oplus, j_1\oplus, i_2\oplus, j_2\oplus, Rc(i_1;j_1), Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ , i_1 \mathrel{!=}\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2,\\ , , i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, i_1\oplus, j_1\oplus, i_2\oplus, j_2\oplus, Rc(i_1;j_1), Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ , i_1 \mathrel{!=}\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2,\\ , j_1\overset{\circ}{\bigcirc}j_2, i_1\oplus, i_2\oplus, j_1\oplus, j_2\oplus, i_1\overset{\circ}{\bigcirc}i_2, Rc(i_1;j_1), Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ , i_1 \mathrel{!=}\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i_1, if(j_1=\varnothing)\left[\begin{array}{l}, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_1\oplus, i_2\oplus, j_1\oplus, j_2\oplus, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ , i_1 \mathrel{!=}\varnothing, if(j_1=\varnothing)\left[\begin{array}{l}, \ \&SHi\overset{\circ}{\bigcirc}i_1, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_1\oplus, i_2\oplus, j_1\oplus, j_2\oplus,\ \&SHi\rightarrow i_1, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2),\end{array}\right],$$

$$\Leftrightarrow\ , i_1 \mathrel{!=}\varnothing, if(j_1=\varnothing)\left[\begin{array}{l}, \ \&SHi\overset{\circ}{\bigcirc}i_1, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2,\\ , i_1\oplus, i_2\oplus, j_1\oplus, j_2\oplus,\ \&SHi\rightarrow i_1, i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), i_1\overset{\circ}{\bigcirc}i_2, j_1\overset{\circ}{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 \mathrel{!=}\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\\ , i_1\oplus, i_2\oplus, j_1\oplus, j_2\oplus, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, i_2\oplus, j_1\oplus, j_2\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, i_1 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2,\\ , i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, i_2\oplus, j_1\oplus, j_2\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, i_1 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2,\\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_2\oplus, j_2\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, i_1 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2,\\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_2\oplus, j_2\oplus, Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, i_1 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2,\\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, i_1 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2,\\ , i_1\mathring{\bigcirc}i_2, i_2 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2, j_2 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, i_1 != \varnothing, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, i_1 != \varnothing, j_1\mathring{\bigcirc}j_2, j_1 != \varnothing, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, j_1=\varnothing, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, j_1=\varnothing, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, j_2=\varnothing, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, j_2=\varnothing, Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,\end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \end{array}\right],$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\mathring{\bigcirc}i_1,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, \\ \\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1\oplus, j_1\oplus, Rc(i_1;j_1), \end{array}\right], Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 != \varnothing,$$

$$if(j_1=\varnothing)\left[\begin{array}{l}, \\ \\ , i_1\oplus, j_1\oplus, Rc(i_1;j_1), \end{array}\right], Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, i_1 != \varnothing, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$$

$$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$$

$conclusion:$

$$, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), \ \Leftrightarrow\ , i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, Rc(i_1;j_1), Rc(i_2;j_2), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2,$$

$$, Rc(i;j), i != \varnothing, i\oplus, \ \Leftrightarrow\ , i != \varnothing, i\oplus, Rc(i;j), j=\varnothing,$$

induction proof 1:
$premise\ 1:$
$, i=\varnothing, Rc(i;j), i != \varnothing, i\oplus,$

$$\Leftrightarrow\ , i=\varnothing, i != \varnothing, i\oplus,$$

$$\Leftrightarrow\ , \otimes, i\oplus,$$

$$\Leftrightarrow\ , \otimes,$$

$$\Leftrightarrow\ , \otimes, i\oplus, Rc(i;j), j=\varnothing,$$

$$\Leftrightarrow\ ,i=\varnothing,i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,$$

premise 2 :

$$,\ \&SHi\rightarrow i,Rc(i;j),i\,!\!=\varnothing,i\oplus,\ \ \Leftrightarrow\ ,\ \&SHi\rightarrow i,i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,\ \ \Rightarrow$$

$$,i\,!\!=\varnothing,\ \&SHi\,\mathring{O}i,Rc(i;j),i\,!\!=\varnothing,i\oplus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,Rc(i;j),i\,!\!=\varnothing,i\oplus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},\\ ,i\oplus,j\oplus,Rc(i;j),\end{array}\right],i\,!\!=\varnothing,i\oplus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},i\,!\!=\varnothing,i\oplus,\\ ,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,i\,!\!=\varnothing,i\oplus,\\ ,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},i\,!\!=\varnothing,i\oplus,j=\varnothing,\\ ,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},i\,!\!=\varnothing,i\oplus,j=\varnothing,j=\varnothing,\\ ,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},i\,!\!=\varnothing,i\oplus,j=\varnothing,Rc(i;j),j=\varnothing,\\ ,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,\\ ,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,if(j=\varnothing)\left[\begin{array}{l},\ \&SHi\,\mathring{O}i,i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,\\ ,i\oplus,j\oplus,\ \&SHi\rightarrow i,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing, if(j{=}\varnothing)\left[\begin{array}{l}, \ \&SHi\,{\bigcirc}\!\!^{\circ}i, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , i\oplus, j\oplus, \ \&SHi\rightarrow i, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, if(j{=}\varnothing)\left[\begin{array}{l}, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , i\oplus, j\oplus, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, if(j{=}\varnothing)\left[\begin{array}{l}, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , j\,!{=}\varnothing, i\oplus, j\oplus, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, if(j{=}\varnothing)\left[\begin{array}{l}, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , i\oplus, i\,!{=}\varnothing, j\,!{=}\varnothing, i\oplus, j\oplus, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, if(j{=}\varnothing)\left[\begin{array}{l}, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , i\oplus, i\,!{=}\varnothing, j\,!{=}\varnothing, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, if(j{=}\varnothing)\left[\begin{array}{l}, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , i\oplus, j\,!{=}\varnothing, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, if(j{=}\varnothing)\left[\begin{array}{l}, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , i\oplus, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, if(j{=}\varnothing)\left[\begin{array}{l}, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \\ \\ , i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, if(j{=}\varnothing)\left[\begin{array}{l}, \\ \\ , \end{array}\right], i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing,$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing,$$

$$\Leftrightarrow\ , \&SHi\rightarrow i, i\,!{=}\varnothing, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing,$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing, \ \&SHi\,{\bigcirc}\!\!^{\circ}i, i\,!{=}\varnothing, i\oplus, Rc(i;j), j{=}\varnothing,$$

$conclusion:$
$$, Rc(i;j), i\,!\!=\varnothing, i\oplus, \;\Leftrightarrow\; , i\,!\!=\varnothing, i\oplus, Rc(i;j), j=\varnothing,$$

induction proof 2:

$premise\ 1:$
$$, j=\varnothing, Rc(i;j), i\,!\!=\varnothing, i\oplus,$$

$$\Leftrightarrow\; , j=\varnothing, i\,!\!=\varnothing, i\oplus,$$

$$\Leftrightarrow\; , i\,!\!=\varnothing, i\oplus, j=\varnothing,$$

$$\Leftrightarrow\; , i\,!\!=\varnothing, i\oplus, j=\varnothing, j=\varnothing,$$

$$\Leftrightarrow\; , i\,!\!=\varnothing, i\oplus, j=\varnothing, Rc(i;j), j=\varnothing,$$

$$\Leftrightarrow\; , j=\varnothing, i\,!\!=\varnothing, i\oplus, Rc(i;j), j=\varnothing,$$

$premise\ 2:$
$$, \&SHi\rightarrow j, Rc(i;j), i\,!\!=\varnothing, i\oplus, \;\Leftrightarrow\; , \&SHi\rightarrow j, i\,!\!=\varnothing, i\oplus, Rc(i;j), j=\varnothing, \;\Rightarrow$$

$$, j\,!\!=\varnothing, \&SHi\overset{\circ}{\bigcirc}j, Rc(i;j), i\,!\!=\varnothing, i\oplus,$$

$$\Leftrightarrow\; , \&SHi\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, Rc(i;j), i\,!\!=\varnothing, i\oplus,$$

$$\Leftrightarrow\; , \&SHi\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, if(i=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right], i\,!\!=\varnothing, i\oplus,$$

$$\Leftrightarrow\; , \&SHi\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, if(i=\varnothing)\left[\begin{array}{l} , i\,!\!=\varnothing, i\oplus, \\ , i\oplus, j\oplus, Rc(i;j), i\,!\!=\varnothing, i\oplus, \end{array}\right],$$

$$\Leftrightarrow\; , \&SHi\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, i\,!\!=\varnothing, i\oplus, \\ , i\oplus, j\oplus, Rc(i;j), i\,!\!=\varnothing, i\oplus, \end{array}\right],$$

$$\Leftrightarrow\; , \&SHi\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, if(i=\varnothing)\left[\begin{array}{l} , \otimes, i\oplus, \\ , i\oplus, j\oplus, Rc(i;j), i\,!\!=\varnothing, i\oplus, \end{array}\right],$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,j\,!\!=\varnothing,if(i=\varnothing)\left[\begin{array}{l},\otimes,\\ \\ ,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,\end{array}\right],$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,j\,!\!=\varnothing,if(i=\varnothing)\left[\begin{array}{l},\otimes,\\ \\ ,\end{array}\right],i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,j\,!\!=\varnothing,i\,!\!=\varnothing,i\oplus,j\oplus,Rc(i;j),i\,!\!=\varnothing,i\oplus,$$

$$\Leftrightarrow\ ,j\,!\!=\varnothing,i\,!\!=\varnothing,i\oplus,j\oplus,\ \&SHi\rightarrow j,Rc(i;j),i\,!\!=\varnothing,i\oplus,$$

$$\Leftrightarrow\ ,j\,!\!=\varnothing,i\,!\!=\varnothing,i\oplus,j\oplus,\ \&SHi\rightarrow j,i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,j\,!\!=\varnothing,i\,!\!=\varnothing,i\oplus,j\oplus,i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,j\,!\!=\varnothing,i\,!\!=\varnothing,i\oplus,i\,!\!=\varnothing,i\oplus,j\oplus,Rc(i;j),j=\varnothing,$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,i\,!\!=\varnothing,i\oplus,i\,!\!=\varnothing,j\,!\!=\varnothing,i\oplus,j\oplus,Rc(i;j),j=\varnothing,$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,i\,!\!=\varnothing,i\oplus,i\,!\!=\varnothing,j\,!\!=\varnothing,Rc(i;j),j=\varnothing,$$

$$\Leftrightarrow\ ,\&SHi\mathring{\bigcirc}j,i\,!\!=\varnothing,i\oplus,j\,!\!=\varnothing,Rc(i;j),j=\varnothing,$$

$$\Leftrightarrow\ ,j\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}j,i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,$$

conclusion :
$,Rc(i;j),i\,!\!=\varnothing,i\oplus,\ \Leftrightarrow\ ,i\,!\!=\varnothing,i\oplus,Rc(i;j),j=\varnothing,$

$$,Rc(i;j),Rc(i;k),j=\varnothing,k=\varnothing,\ \Leftrightarrow\ ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing,$$

induction proof 1:
premise 1 :
$,j=\varnothing,Rc(i;j),Rc(i;k),j=\varnothing,k=\varnothing,$

$$\Leftrightarrow\ , j=\varnothing, Rc(i;k), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow\ , Rc(i;k), j=\varnothing, j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow\ , Rc(i;k), j=\varnothing, Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow\ , j=\varnothing, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,$$

$premise\ 2:$
$, \&SHi \rightarrow j, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$
$\Leftrightarrow\ , \&SHi \rightarrow j, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\ \ \Rightarrow$

$$, j \mathrel{!}= \varnothing, \&SHi\,\text{♂}j, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow\ , \&SHi\,\text{♂}j, j \mathrel{!}= \varnothing, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow\ , \&SHi\,\text{♂}j, j \mathrel{!}= \varnothing, if(i=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right], Rc(i;k), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow\ , \&SHi\,\text{♂}j, j \mathrel{!}= \varnothing, if(i=\varnothing)\left[\begin{array}{l} , Rc(i;k), j=\varnothing, k=\varnothing, \\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\text{♂}j, j \mathrel{!}= \varnothing, if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, Rc(i;k), j=\varnothing, k=\varnothing, \\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\text{♂}j, j \mathrel{!}= \varnothing, if(i=\varnothing)\left[\begin{array}{l} , i=\varnothing, j=\varnothing, k=\varnothing, \\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\text{♂}j, j \mathrel{!}= \varnothing, if(i=\varnothing)\left[\begin{array}{l} , j=\varnothing, k=\varnothing, \\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\text{♂}j, if(i=\varnothing)\left[\begin{array}{l} , j \mathrel{!}= \varnothing, j=\varnothing, k=\varnothing, \\ , j \mathrel{!}= \varnothing, i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, if(i=\varnothing) - \left[\begin{array}{l} ,\otimes, k=\varnothing, \\ \\ , j\,!\!= \varnothing, i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right] - ,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, if(i=\varnothing) - \left[\begin{array}{l} ,\otimes, \\ \\ , j\,!\!= \varnothing, i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right] - ,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, \ \&SHi \to j, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, \ \&SHi \to j, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, Rc(i;k), j\oplus, Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, i\,!\!=\varnothing, i\oplus, Rc(i;k), j\,!\!=\varnothing, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, i\,!\!=\varnothing, i\oplus, Rc(i;k), j\,!\!=\varnothing, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, i\,!\!=\varnothing, i\oplus, Rc(i;k), k=\varnothing, j\,!\!=\varnothing, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, Rc(i;k), i\,!\!=\varnothing, i\oplus, j\,!\!=\varnothing, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, Rc(i;k), i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, Rc(i;k), i\,!\!=\varnothing, j\,!\!=\varnothing, Rc(i;j), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow \ , \&SHi \bigcirc\!\!\!^{\circ} j, Rc(i;k), j\,!\!=\varnothing, Rc(i;j), j=\varnothing, k=\varnothing,$$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \bigcirc\!\!^{\circ} j, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,$

conclusion :
$, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,\ \ \Leftrightarrow\ \ , Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,$

induction proof 2:
premise 1 :
$, i=\varnothing, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, Rc(i;k), j=\varnothing, k=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, j=\varnothing, k=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, Rc(i;j), j=\varnothing, k=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,$

premise 2 :
$,\ \&SHi \rightarrow i, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$
$\Leftrightarrow\ ,\ \&SHi \rightarrow i, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\ \ \Rightarrow$

$, i \mathrel{!=} \varnothing,\ \&SHi \bigcirc\!\!^{\circ} i, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$

$\Leftrightarrow\ ,\ \&SHi \bigcirc\!\!^{\circ} i, i \mathrel{!=} \varnothing, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc\!\!^{\circ} i, i \mathrel{!=} \varnothing, if(j=\varnothing) \left[\begin{array}{l} , \\ \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right], Rc(i;k), j=\varnothing, k=\varnothing,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc\!\!^{\circ} i, i \mathrel{!=} \varnothing, if(j=\varnothing) \left[\begin{array}{l} , Rc(i;k), j=\varnothing, k=\varnothing, \\ \\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc\!\!^{\circ} i, i \mathrel{!=} \varnothing, if(j=\varnothing) \left[\begin{array}{l} , Rc(i;k), j=\varnothing, j=\varnothing, k=\varnothing, \\ \\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc\!\!^{\circ} i, i \mathrel{!=} \varnothing, if(j=\varnothing) \left[\begin{array}{l} , Rc(i;k), j=\varnothing, Rc(i;j), j=\varnothing, k=\varnothing, \\ \\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, j=\varnothing, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\oplus, j\oplus, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, \&SHi\mathring{\bigcirc}i, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\oplus, j\oplus, \&SHi\to i, Rc(i;j), Rc(i;k), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, if(j=\varnothing)\left[\begin{array}{l}, \&SHi\mathring{\bigcirc}i, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\oplus, j\oplus, \&SHi\to i, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\oplus, j\oplus, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i\,!\!=\varnothing, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i\,!\!=\varnothing, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\,!\!=\varnothing, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\,!\!=\varnothing, i\oplus, Rc(i;k), j\,!\!=\varnothing, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\,!\!=\varnothing, i\oplus, Rc(i;k), j\,!\!=\varnothing, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, Rc(i;k), Rc(i;j), j=\varnothing, k=\varnothing,\\ , i\,!\!=\varnothing, i\oplus, Rc(i;k), k=\varnothing, j\,!\!=\varnothing, j\oplus, Rc(i;j), j=\varnothing, k=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \\ ,Rc(i;k),i \mathrel{!=} \varnothing,i\oplus,j \mathrel{!=} \varnothing,j\oplus,Rc(i;j),j=\varnothing,k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \\ ,Rc(i;k),i \mathrel{!=} \varnothing,j \mathrel{!=} \varnothing,i\oplus,j\oplus,Rc(i;j),j=\varnothing,k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \\ ,Rc(i;k),i \mathrel{!=} \varnothing,j \mathrel{!=} \varnothing,Rc(i;j),j=\varnothing,k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \\ ,Rc(i;k),j \mathrel{!=} \varnothing,Rc(i;j),j=\varnothing,k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \\ ,j \mathrel{!=} \varnothing,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \\ ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l} , \\ , \end{array}\right],Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing,$$

$$\Leftrightarrow\ ,i \mathrel{!=} \varnothing,\ \&SHi\mathring{\bigcirc}i,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing,$$

conclusion :
$,Rc(i;j),Rc(i;k),j=\varnothing,k=\varnothing,\ \Leftrightarrow\ ,Rc(i;k),Rc(i;j),j=\varnothing,k=\varnothing,$

$$,Rc(i;j),Rc(i;k),i=\varnothing,j=\varnothing,k=\varnothing,\ \Leftrightarrow\ ,Rc(i;k),Rc(i;j),i=\varnothing,j=\varnothing,k=\varnothing,$$

$$,Rc(i;j),Rc(i;k),i \mathrel{!=} \varnothing,j=\varnothing,k=\varnothing,\ \Leftrightarrow\ ,Rc(i;k),Rc(i;j),i \mathrel{!=} \varnothing,j=\varnothing,k=\varnothing,$$

$$,i\mathring{\bigcirc}j,Rc(i;j),\ \Leftrightarrow\ ,Rc(i;j),i\mathring{\bigcirc}j,$$

induction proof:
premise 1 :
$,i=\varnothing,i\mathring{\bigcirc}j,Rc(i;j),$

$\Leftrightarrow\ , i\bigcirc^{\circ}j, i=\varnothing, Rc(i;j),$

$\Leftrightarrow\ , i\bigcirc^{\circ}j, i=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, i\bigcirc^{\circ}j,$

$\Leftrightarrow\ , i=\varnothing, Rc(i;j), i\bigcirc^{\circ}j,$

$premise\ 2:$
$, \&SHi\to i, i\bigcirc^{\circ}j, Rc(i;j),\ \Leftrightarrow\ , \&SHi\to i, Rc(i;j), i\bigcirc^{\circ}j,\ \Rightarrow$

$, i\,!{=}\,\varnothing, \&SHi\,\mathring{\bigcirc}i, i\bigcirc^{\circ}j, Rc(i;j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\bigcirc^{\circ}j, i\,!{=}\,\varnothing, Rc(i;j),$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\bigcirc^{\circ}j, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , i\bigcirc^{\circ}j, \\ , i\bigcirc^{\circ}j, i\oplus, j\oplus, Rc(i;j), \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , i\bigcirc^{\circ}j, \\ , i\oplus, j\oplus, i\bigcirc^{\circ}j, Rc(i;j), \end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \&SHi\,\mathring{\bigcirc}i, i\bigcirc^{\circ}j, \\ , i\oplus, j\oplus, \&SHi\to i, i\bigcirc^{\circ}j, Rc(i;j), \end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \&SHi\,\mathring{\bigcirc}i, i\bigcirc^{\circ}j, \\ , i\oplus, j\oplus, \&SHi\to i, Rc(i;j), i\bigcirc^{\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , i\bigcirc^{\circ}j, \\ , i\oplus, j\oplus, Rc(i;j), i\bigcirc^{\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right], i\bigcirc^{\circ}j,$$

$$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i, i \,!\!= \varnothing, Rc(i;j), i \mathring{\bigcirc}\!\!\circ j,$$

$$\Leftrightarrow\ , i \,!\!= \varnothing,\ \&SHi \mathring{\bigcirc} i, Rc(i;j), i \mathring{\bigcirc}\!\!\circ j,$$

$conclusion:$
$, i \mathring{\bigcirc}\!\!\circ j, Rc(i;j), \ \Leftrightarrow\ , Rc(i;j), i \mathring{\bigcirc}\!\!\circ j,$

$$, i! \mathring{\bigcirc}\!\!\circ j, Rc(i;j), \ \Leftrightarrow\ , Rc(i;j), i! \mathring{\bigcirc}\!\!\circ j,$$

$$, i \mathring{\bigcirc}\!\!\circ m, Rc(i;j), \ \Leftrightarrow\ , Rc(i;j), i \mathring{\bigcirc}\!\!\circ m,$$

$$, i! \mathring{\bigcirc}\!\!\circ m, Rc(i;j), \ \Leftrightarrow\ , Rc(i;j), i! \mathring{\bigcirc}\!\!\circ m,$$

21 Rules of Number Equal Relationship

21.1 Definition of Number Equal

$$,if(i\mathrel{\text{⌶}}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),if(i_0=j_0)\left[\begin{array}{l} ,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i_0\text{⏚},j_0\text{⏚}, \end{array}\right.$$

$$,i\mathrel{\text{⌶}}j,\ \Leftrightarrow\ ,if(i\mathrel{\text{⌶}}j)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],$$

$$,i!\mathrel{\text{⌶}}j,\ \Leftrightarrow\ ,if(i\mathrel{\text{⌶}}j)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right],$$

21.2 Theorems of Number Equal Relationship

21.2.1 Number Equal propositions to definition

$$,i\mathrel{\text{⌶}}j,\ \Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),i_0=j_0,i_0\text{⏚},j_0\text{⏚},$$

$$,i\mathrel{\text{⌶}}j,\ \Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),i_0=\varnothing,j_0=\varnothing,i_0\text{⏚},j_0\text{⏚},$$

proof:

$,i\mathrel{\text{⌶}}j,$

$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),i_0=j_0,i_0\text{⏚},j_0\text{⏚},$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),if(i_0=\varnothing)\left[\begin{array}{l} , \\ \\ ,j_0=\varnothing, \end{array}\right],i_0=j_0,i_0\text{⏚},j_0\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),if(i_0=\varnothing)\left[\begin{array}{l} ,i_0=j_0, \\ \\ ,j_0=\varnothing,i_0=j_0, \end{array}\right],i_0\text{⏚},j_0\text{⏚},$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 = j_0, \\ \\ , j_0 = \varnothing, i_0 = \varnothing, \end{array}\right], i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 = j_0, \\ \\ , i_0 = \varnothing, j_0 = \varnothing, \end{array}\right], i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 = j_0, \\ \\ , i_0 != \varnothing, i_0 = \varnothing, j_0 = \varnothing, \end{array}\right], i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 = j_0, \\ \\ , \otimes, j_0 = \varnothing, \end{array}\right], i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 = j_0, \\ \\ , \otimes, \end{array}\right], i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), i_0 = \varnothing, i_0 = j_0, i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$$

$$, i != j, \ \Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), i_0 != j_0, i_0 \obot, j_0 \obot,$$

$$, i != j, \ \Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , j_0 != \varnothing, \\ \\ , j_0 = \varnothing, \end{array}\right], i_0 \obot, j_0 \obot,$$

proof:

$, i != j,$

$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), i_0 != j_0, i_0 \obot, j_0 \obot,$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , \\ \\ , j_0 = \varnothing, \end{array}\right], i_0 != j_0, i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ , i \owedge i_0, j \owedge j_0, Rc(i_0, j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 != j_0, \\ \\ , j_0 = \varnothing, i_0 != j_0, \end{array}\right], i_0 \obot, j_0 \obot,$$

$$\Leftrightarrow\ ,i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=\varnothing)-\left[\begin{array}{l} ,i_0\,!{=}\,j_0, \\ \\ ,j_0=\varnothing, i_0\,!{=}\,\varnothing, \end{array}\right]-, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=\varnothing)-\left[\begin{array}{l} ,i_0\,!{=}\,j_0, \\ \\ ,i_0\,!{=}\,\varnothing, j_0=\varnothing, \end{array}\right]-, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=\varnothing)-\left[\begin{array}{l} ,i_0\,!{=}\,j_0, \\ \\ ,j_0=\varnothing, \end{array}\right]-, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=\varnothing)-\left[\begin{array}{l} ,i_0=\varnothing, i_0\,!{=}\,j_0, \\ \\ ,j_0=\varnothing, \end{array}\right]-, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=\varnothing)-\left[\begin{array}{l} ,i_0=\varnothing, j_0\,!{=}\,\varnothing, \\ \\ ,j_0=\varnothing, \end{array}\right]-, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ ,i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=\varnothing)-\left[\begin{array}{l} ,j_0\,!{=}\,\varnothing, \\ \\ ,j_0=\varnothing, \end{array}\right]-, i_0\text{⏚}, j_0\text{⏚},$$

21.2.2 Branch function to propositions

$$,if(i\,\text{⌶}\,j)-\left[\begin{array}{l} ,©c, \\ \\ ,\otimes, \end{array}\right]-, \quad \Leftrightarrow\ ,i\,\text{⌶}\,j, ©c,$$

$$,if(i\,\text{⌶}\,j)-\left[\begin{array}{l} ,\otimes, \\ \\ ,©c, \end{array}\right]-, \quad \Leftrightarrow\ ,i!\text{⌶}\,j, ©c,$$

21.2.3 Empty branch function

$$,if(i\,\text{⌶}\,j)-\left[\begin{array}{l} , \\ \\ , \end{array}\right. \quad \Leftrightarrow\ ,-\left[\begin{array}{l} ,i\,\text{⌶}\,j, \\ \\ ,i!\text{⌶}\,j, \end{array}\right.$$

21.2.4 Unity

$$, \quad \Leftrightarrow \quad , if(i{=}j)\left[\begin{array}{l} , \\ , \end{array}\right],$$

proof:

$$
\begin{aligned}
&,\\
\Leftrightarrow\ &, i\oslash i_0, i_0\obar, j\oslash j_0, j_0\obar,\\
\Leftrightarrow\ &, i\oslash i_0, j\oslash j_0, i_0\obar, j_0\obar,\\
\Leftrightarrow\ &, i\oslash i_0, j\oslash j_0, Rc(i_0,j_0), i_0\obar, j_0\obar,\\
\Leftrightarrow\ &, i\oslash i_0, j\oslash j_0, Rc(i_0,j_0), if(i_0{=}j_0)\left[\begin{array}{l} , \\ \\ , \end{array}\right], i_0\obar, j_0\obar,\\
\Leftrightarrow\ &, i\oslash i_0, j\oslash j_0, Rc(i_0,j_0), if(i_0{=}j_0)\left[\begin{array}{l} , i_0\obar, j_0\obar, \\ \\ , i_0\obar, j_0\obar, \end{array}\right],\\
\Leftrightarrow\ &, if(i{=}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right],
\end{aligned}
$$

$$
\begin{aligned}
, i{=}j, \otimes, \quad &\Leftrightarrow \quad , \otimes,\\
, i!{=}j, \otimes, \quad &\Leftrightarrow \quad , \otimes,
\end{aligned}
$$

21.2.5 Symmetry

$$, if(i{=}j)\left[\begin{array}{l} , \\ , \end{array}\right. \quad \Leftrightarrow \quad , if(j{=}i)\left[\begin{array}{l} , \\ , \end{array}\right.$$

proof:

$$
\begin{aligned}
&, if(i{=}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right.\\
\Leftrightarrow\ &, i\oslash i_0, j\oslash j_0, Rc(i_0,j_0), if(i_0{=}j_0)\left[\begin{array}{l} , i_0\obar, j_0\obar, \\ \\ , i_0\obar, j_0\obar, \end{array}\right.\\
\Leftrightarrow\ &, j\oslash j_0, i\oslash i_0, Rc(i_0,j_0), if(i_0{=}j_0)\left[\begin{array}{l} , i_0\obar, j_0\obar, \\ \\ , i_0\obar, j_0\obar, \end{array}\right.\\
\Leftrightarrow\ &, j\oslash j_0, i\oslash i_0, Rc(j_0,i_0), if(i_0{=}j_0)\left[\begin{array}{l} , i_0\obar, j_0\obar, \\ \\ , i_0\obar, j_0\obar, \end{array}\right.
\end{aligned}
$$

$$\Leftrightarrow\ ,j\text{☮}j_0,i\text{☮}i_0,Rc(j_0,i_0),if(j_0=i_0)\left[\begin{array}{l},i_0\text{⏚},j_0\text{⏚},\\ \\,i_0\text{⏚},j_0\text{⏚},\end{array}\right.$$

$$\Leftrightarrow\ ,j\text{☮}j_0,i\text{☮}i_0,Rc(j_0,i_0),if(j_0=i_0)\left[\begin{array}{l},j_0\text{⏚},i_0\text{⏚},\\ \\,j_0\text{⏚},i_0\text{⏚},\end{array}\right.$$

$$\Leftrightarrow\ ,if(j\text{⌶}i)\left[\begin{array}{l},\\ \\,\end{array}\right.$$

$$,i\text{⌶}j,\ \Leftrightarrow\ ,j\text{⌶}i,$$
$$,i!\text{⌶}j,\ \Leftrightarrow\ ,j!\text{⌶}i,$$

21.2.6 Swap

Branch function and operator:

$$,\circledcirc m,if(i\text{⌶}j)\left[\begin{array}{l},\\ \\,\end{array}\right.\ \Leftrightarrow\ ,if(i\text{⌶}j)\left[\begin{array}{l},\circledcirc m,\\ \\,\circledcirc m,\end{array}\right.$$

proof:

$$,if(i\text{⌶}j)\left[\begin{array}{l},\\ \\,\end{array}\right.$$

$$\Leftrightarrow\ ,\circledcirc m,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),if(i_0=j_0)\left[\begin{array}{l},i_0\text{⏚},j_0\text{⏚},\\ \\,i_0\text{⏚},j_0\text{⏚},\end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,\circledcirc m,Rc(i_0,j_0),if(i_0=j_0)\left[\begin{array}{l},i_0\text{⏚},j_0\text{⏚},\\ \\,i_0\text{⏚},j_0\text{⏚},\end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),\circledcirc m,if(i_0=j_0)\left[\begin{array}{l},i_0\text{⏚},j_0\text{⏚},\\ \\,i_0\text{⏚},j_0\text{⏚},\end{array}\right.$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),if(i_0=j_0)\left[\begin{array}{l},i_0\text{⏚},j_0\text{⏚},\circledcirc m,\\ \\,i_0\text{⏚},j_0\text{⏚},\circledcirc m,\end{array}\right.$$

$$\Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,\circledcirc m, \\ \\ ,\circledcirc m, \end{array}\right.$$

$$, \odot m, if(i\text{⌶}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,\odot m, \\ \\ ,\odot m, \end{array}\right.$$

$$, m\text{☮}n, if(i\text{⌶}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,m\text{☮}n, \\ \\ ,m\text{☮}n, \end{array}\right.$$

$$, m\text{☮}n, if(i\text{⌶}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,m\text{☮}n, \\ \\ ,m\text{☮}n, \end{array}\right.$$

$$, m\text{☮}n, if(i\text{⌶}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,m\text{☮}n, \\ \\ ,m\text{☮}n, \end{array}\right.$$

$$, m\text{⏚}, if(i\text{⌶}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,m\text{⏚}, \\ \\ ,m\text{⏚}, \end{array}\right.$$

$$, m\oplus, if(i\text{⌶}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,m\oplus, \\ \\ ,m\oplus, \end{array}\right.$$

$$, m\ominus, if(i\text{⌶}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \ , if(i\text{⌶}j)\left[\begin{array}{l} ,m\ominus, \\ \\ ,m\ominus, \end{array}\right.$$

Branch function and Branch function:

$$, if(i\text{⌶}j)\left[\begin{array}{l} , if(m\text{⌶}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ , if(m\text{⌶}n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \ \Leftrightarrow \ , if(m\text{⌶}n)\left[\begin{array}{l} , if(i\text{⌶}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ , if(i\text{⌶}j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

proof:

$$
,if(i{=}j)\left[\begin{array}{l}
,if(m{=}n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\
,if(m{=}n)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$
\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),
$$

$$
if(i_0=j_0)\left[\begin{array}{l}
,i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},if(m{=}n)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],\\
,i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},if(m{=}n)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$
\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),
$$

$$
if(i_0=j_0)\left[\begin{array}{l}
,i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},m\text{☮}m_0,n\text{☮}n_0,Rc(m_0;n_0),if(m_0=n_0)\left[\begin{array}{l},m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},\copyright c_1,\\ \\ ,m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},\copyright c_2,\end{array}\right],\\
,i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},m\text{☮}m_0,n\text{☮}n_0,Rc(m_0;n_0),if(m_0=n_0)\left[\begin{array}{l},m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},\copyright c_3,\\ \\ ,m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},\copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$
\Leftrightarrow\ ,m\text{☮}m_0,n\text{☮}n_0,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),
$$

$$
if(i_0=j_0)\left[\begin{array}{l}
,Rc(m_0;n_0),if(m_0=n_0)\left[\begin{array}{l},m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},\copyright c_1,\\ \\ ,m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},\copyright c_2,\end{array}\right],\\
,Rc(m_0;n_0),if(m_0=n_0)\left[\begin{array}{l},m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},\copyright c_3,\\ \\ ,m_0\text{\textcircled{$\downarrow$}},n_0\text{\textcircled{$\downarrow$}},i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},\copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$
\Leftrightarrow\ ,m\text{☮}m_0,n\text{☮}n_0,Rc(m_0;n_0),i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),
$$

$$
if(i_0=j_0)\left[\begin{array}{l}
, if(m_0=n_0)\left[\begin{array}{l}, m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_1, \\ \\ , m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_2,\end{array}\right], \\
, if(m_0=n_0)\left[\begin{array}{l}, m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_3, \\ \\ , m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$
\Leftrightarrow \ , m\text{☮}m_0, n\text{☮}n_0, Rc(m_0;n_0), i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0),
$$

$$
if(m_0=n_0)\left[\begin{array}{l}
, if(i_0=j_0)\left[\begin{array}{l}, m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_1, \\ \\ , m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_3,\end{array}\right], \\
, if(i_0=j_0)\left[\begin{array}{l}, m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_2, \\ \\ , m_0\text{⏚}, n_0\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$
\Leftrightarrow \ , m\text{☮}m_0, n\text{☮}n_0, Rc(m_0;n_0),
$$

$$
if(m_0=n_0)\left[\begin{array}{l}
, m_0\text{⏚}, n_0\text{⏚}, i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=j_0)\left[\begin{array}{l}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_1, \\ \\ , i_0\text{⏚}, j_0\text{⏚}, \copyright c_3,\end{array}\right], \\
, m_0\text{⏚}, n_0\text{⏚}, i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), if(i_0=j_0)\left[\begin{array}{l}, i_0\text{⏚}, j_0\text{⏚}, \copyright c_2, \\ \\ , i_0\text{⏚}, j_0\text{⏚}, \copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$
\Leftrightarrow \ , if(m\mp n)\left[\begin{array}{l}
, if(i\mp j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_3,\end{array}\right], \\
, if(i\mp j)\left[\begin{array}{l}, \copyright c_2, \\ \\ , \copyright c_4,\end{array}\right],
\end{array}\right],
$$

$$,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},if(m\mathbin{\bigcirc\mkern-15mu\perp}n)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right],\\ ,if(m\mathbin{\bigcirc\mkern-15mu\perp}n)\left[\begin{array}{l},\text{©}c_3,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(m\mathbin{\bigcirc\mkern-15mu\perp}n)\left[\begin{array}{l},if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_3,\end{array}\right],\\ ,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_2,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},if(m\rightarrow n)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right],\\ ,if(m\rightarrow n)\left[\begin{array}{l},\text{©}c_3,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(m\rightarrow n)\left[\begin{array}{l},if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_3,\end{array}\right],\\ ,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_2,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},if(m\mathbin{\bigcirc^{\circ}}n)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right],\\ ,if(m\mathbin{\bigcirc^{\circ}}n)\left[\begin{array}{l},\text{©}c_3,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(m\mathbin{\bigcirc^{\circ}}n)\left[\begin{array}{l},if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_3,\end{array}\right],\\ ,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_2,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},if(m\mathbin{\overset{\circ}{\bigcirc}}n)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right],\\ ,if(m\mathbin{\overset{\circ}{\bigcirc}}n)\left[\begin{array}{l},\text{©}c_3,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(m\mathbin{\overset{\circ}{\bigcirc}}n)\left[\begin{array}{l},if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_3,\end{array}\right],\\ ,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_2,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right],$$

$$,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},if(m=n)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right],\\ ,if(m=n)\left[\begin{array}{l},\text{©}c_3,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right], \;\Leftrightarrow\; ,if(m=n)\left[\begin{array}{l},if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_3,\end{array}\right],\\ ,if(i\mathrel{\mathrm{I}}j)\left[\begin{array}{l},\text{©}c_2,\\ \\ ,\text{©}c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{=}j)\left[\begin{array}{l} ,if(m{=}\varnothing)\left[\begin{array}{l} ,\textcircled{c}c_1, \\ \\ ,\textcircled{c}c_2, \end{array}\right], \\ ,if(m{=}\varnothing)\left[\begin{array}{l} ,\textcircled{c}c_3, \\ \\ ,\textcircled{c}c_4, \end{array}\right], \end{array}\right], \;\Leftrightarrow\; ,if(m{=}\varnothing)\left[\begin{array}{l} ,if(i{=}j)\left[\begin{array}{l} ,\textcircled{c}c_1, \\ \\ ,\textcircled{c}c_3, \end{array}\right], \\ ,if(i{=}j)\left[\begin{array}{l} ,\textcircled{c}c_2, \\ \\ ,\textcircled{c}c_4, \end{array}\right], \end{array}\right],$$

Branch function and propositions:

$$,m{=}n,if(i{=}j)\left[\begin{array}{l} ,\textcircled{c}c_1, \\ \\ ,\textcircled{c}c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l} ,m{=}n,\textcircled{c}c_1, \\ \\ ,m{=}n,\textcircled{c}c_2, \end{array}\right],$$

$$,m!{=}n,if(i{=}j)\left[\begin{array}{l} ,\textcircled{c}c_1, \\ \\ ,\textcircled{c}c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l} ,m!{=}n,\textcircled{c}c_1, \\ \\ ,m!{=}n,\textcircled{c}c_2, \end{array}\right],$$

$$,m\oplus n,if(i{=}j)\left[\begin{array}{l} ,\textcircled{c}c_1, \\ \\ ,\textcircled{c}c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l} ,m\oplus n,\textcircled{c}c_1, \\ \\ ,m\oplus n,\textcircled{c}c_2, \end{array}\right],$$

$$,m!\oplus n,if(i{=}j)\left[\begin{array}{l} ,\textcircled{c}c_1, \\ \\ ,\textcircled{c}c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l} ,m!\oplus n,\textcircled{c}c_1, \\ \\ ,m!\oplus n,\textcircled{c}c_2, \end{array}\right],$$

$$,m{\rightarrow}n,if(i{=}j)\left[\begin{array}{l} ,\textcircled{c}c_1, \\ \\ ,\textcircled{c}c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l} ,m{\rightarrow}n,\textcircled{c}c_1, \\ \\ ,m{\rightarrow}n,\textcircled{c}c_2, \end{array}\right],$$

$$,m!\to n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m!\to n, \copyright c_1,\\ \\ , m!\to n, \copyright c_2,\end{array}\right],$$

$$,m\overset{\circ}{\bigcirc}\!\circ n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m\overset{\circ}{\bigcirc}\!\circ n, \copyright c_1,\\ \\ , m\overset{\circ}{\bigcirc}\!\circ n, \copyright c_2,\end{array}\right],$$

$$,m!\overset{\circ}{\bigcirc}\!\circ n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m!\overset{\circ}{\bigcirc}\!\circ n, \copyright c_1,\\ \\ , m!\overset{\circ}{\bigcirc}\!\circ n, \copyright c_2,\end{array}\right],$$

$$,m\overset{\circ}{\bigcirc} n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m\overset{\circ}{\bigcirc} n, \copyright c_1,\\ \\ , m\overset{\circ}{\bigcirc} n, \copyright c_2,\end{array}\right],$$

$$,m!\overset{\circ}{\bigcirc} n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m!\overset{\circ}{\bigcirc} n, \copyright c_1,\\ \\ , m!\overset{\circ}{\bigcirc} n, \copyright c_2,\end{array}\right],$$

$$,m=n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m=n, \copyright c_1,\\ \\ , m=n, \copyright c_2,\end{array}\right],$$

$$,m\,!{=}\,n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m\,!{=}\,n, \copyright c_1,\\ \\ , m\,!{=}\,n, \copyright c_2,\end{array}\right],$$

$$,m=\varnothing, if(i{=}j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i{=}j)\left[\begin{array}{l}, m=\varnothing, \copyright c_1,\\ \\ , m=\varnothing, \copyright c_2,\end{array}\right],$$

$$,m\,!{=}\,\varnothing,if(i{\vdash\!\dashv}j)\!-\!\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\quad\Leftrightarrow\quad ,if(i{\vdash\!\dashv}j)\!-\!\left[\begin{array}{c},m\,!{=}\,\varnothing,\copyright c_1,\\ \\ ,m\,!{=}\,\varnothing,\copyright c_2,\end{array}\right]\!-,$$

$$,m{\vdash\!\dashv}n,if(i\,\textcircled{$\scriptstyle\bot$}\,j)\!-\!\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\quad\Leftrightarrow\quad ,if(i\,\textcircled{$\scriptstyle\bot$}\,j)\!-\!\left[\begin{array}{c},m{\vdash\!\dashv}n,\copyright c_1,\\ \\ ,m{\vdash\!\dashv}n,\copyright c_2,\end{array}\right]\!-,$$

$$,m!{\vdash\!\dashv}n,if(i\,\textcircled{$\scriptstyle\bot$}\,j)\!-\!\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\quad\Leftrightarrow\quad ,if(i\,\textcircled{$\scriptstyle\bot$}\,j)\!-\!\left[\begin{array}{c},m!{\vdash\!\dashv}n,\copyright c_1,\\ \\ ,m!{\vdash\!\dashv}n,\copyright c_2,\end{array}\right]\!-,$$

$$,m{\vdash\!\dashv}n,if(i\overset{\circ}{\bigcirc}j)\!-\!\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\quad\Leftrightarrow\quad ,if(i\overset{\circ}{\bigcirc}j)\!-\!\left[\begin{array}{c},m{\vdash\!\dashv}n,\copyright c_1,\\ \\ ,m{\vdash\!\dashv}n,\copyright c_2,\end{array}\right]\!-,$$

$$,m!{\vdash\!\dashv}n,if(i\overset{\circ}{\bigcirc}j)\!-\!\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\quad\Leftrightarrow\quad ,if(i\overset{\circ}{\bigcirc}j)\!-\!\left[\begin{array}{c},m!{\vdash\!\dashv}n,\copyright c_1,\\ \\ ,m!{\vdash\!\dashv}n,\copyright c_2,\end{array}\right]\!-,$$

$$,m{\vdash\!\dashv}n,if(i{\to}j)\!-\!\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\quad\Leftrightarrow\quad ,if(i{\to}j)\!-\!\left[\begin{array}{c},m{\vdash\!\dashv}n,\copyright c_1,\\ \\ ,m{\vdash\!\dashv}n,\copyright c_2,\end{array}\right]\!-,$$

$$,m!{\vdash\!\dashv}n,if(i{\to}j)\!-\!\left[\begin{array}{c},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]\!-,\quad\Leftrightarrow\quad ,if(i{\to}j)\!-\!\left[\begin{array}{c},m!{\vdash\!\dashv}n,\copyright c_1,\\ \\ ,m!{\vdash\!\dashv}n,\copyright c_2,\end{array}\right]\!-,$$

$$,m\mathrel{\mathrm{I}}n,if(i\bigcirc\!\!^{\circ}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\quad\Leftrightarrow\quad,if(i\bigcirc\!\!^{\circ}j)-\left[\begin{array}{l},m\mathrel{\mathrm{I}}n,\copyright c_1,\\ \\ ,m\mathrel{\mathrm{I}}n,\copyright c_2,\end{array}\right]-,$$

$$,m!\mathrel{\mathrm{I}}n,if(i\bigcirc\!\!^{\circ}j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\quad\Leftrightarrow\quad,if(i\bigcirc\!\!^{\circ}j)-\left[\begin{array}{l},m!\mathrel{\mathrm{I}}n,\copyright c_1,\\ \\ ,m!\mathrel{\mathrm{I}}n,\copyright c_2,\end{array}\right]-,$$

$$,m\mathrel{\mathrm{I}}n,if(i=j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\quad\Leftrightarrow\quad,if(i=j)-\left[\begin{array}{l},m\mathrel{\mathrm{I}}n,\copyright c_1,\\ \\ ,m\mathrel{\mathrm{I}}n,\copyright c_2,\end{array}\right]-,$$

$$,m!\mathrel{\mathrm{I}}n,if(i=j)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\quad\Leftrightarrow\quad,if(i=j)-\left[\begin{array}{l},m!\mathrel{\mathrm{I}}n,\copyright c_1,\\ \\ ,m!\mathrel{\mathrm{I}}n,\copyright c_2,\end{array}\right]-,$$

$$,m\mathrel{\mathrm{I}}n,if(i=\varnothing)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\quad\Leftrightarrow\quad,if(i=\varnothing)-\left[\begin{array}{l},m\mathrel{\mathrm{I}}n,\copyright c_1,\\ \\ ,m\mathrel{\mathrm{I}}n,\copyright c_2,\end{array}\right]-,$$

$$,m!\mathrel{\mathrm{I}}n,if(i=\varnothing)-\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right]-,\quad\Leftrightarrow\quad,if(i=\varnothing)-\left[\begin{array}{l},m!\mathrel{\mathrm{I}}n,\copyright c_1,\\ \\ ,m!\mathrel{\mathrm{I}}n,\copyright c_2,\end{array}\right]-,$$

Branch function and recursive function:

$$, R(m), if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , R(m), \copyright c_1, \\ \\ , R(m), \copyright c_2, \end{array}\right],$$

$$, R\_(m), if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , R\_(m), \copyright c_1, \\ \\ , R\_(m), \copyright c_2, \end{array}\right],$$

$$, Rc(m;n), if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , Rc(m;n), \copyright c_1, \\ \\ , Rc(m;n), \copyright c_2, \end{array}\right],$$

Branch function and flag object :

$$, \ \&SHi \text{♂} m, if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \ \&SHi \text{♂} m, \copyright c_1, \\ \\ , \ \&SHi \text{♂} m, \copyright c_2, \end{array}\right],$$

$$, \ \&SHi \rightarrow m, if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \ \&SHi \rightarrow m, \copyright c_1, \\ \\ , \ \&SHi \rightarrow m, \copyright c_2, \end{array}\right],$$

$$, \ \&SHj \text{♂} m, if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \ \&SHj \text{♂} m, \copyright c_1, \\ \\ , \ \&SHj \text{♂} m, \copyright c_2, \end{array}\right],$$

$$, \ \&SHj \leftarrow m, if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \ \Leftrightarrow \ , if(i \mathrel{\top\!\!\!\!\bot} j) \left[\begin{array}{c} , \ \&SHj \leftarrow m, \copyright c_1, \\ \\ , \ \&SHj \leftarrow m, \copyright c_2, \end{array}\right],$$

$$, \&SVi\overset{\circ}{\bigcirc}m, if(i\text{工}j)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i\text{工}j)\left[\begin{array}{l}, \&SVi\overset{\circ}{\bigcirc}m, ©c_1, \\ \\ , \&SVi\overset{\circ}{\bigcirc}m, ©c_2,\end{array}\right],$$

$$, \&SVi\obot m, if(i\text{工}j)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \quad\Leftrightarrow\quad , if(i\text{工}j)\left[\begin{array}{l}, \&SVi\obot m, ©c_1, \\ \\ , \&SVi\obot m, ©c_2,\end{array}\right],$$

Propositions and operator:

$$, i\text{工}j, \circledcirc m, \quad\Leftrightarrow\quad , \circledcirc m, i\text{工}j,$$
$$, i\text{工}j, \odot m, \quad\Leftrightarrow\quad , \odot m, i\text{工}j,$$

$$, i\text{工}j, m\text{☮}n, \quad\Leftrightarrow\quad , m\text{☮}n, i\text{工}j,$$
$$, i\text{工}j, m\text{☮}^{\circ}n, \quad\Leftrightarrow\quad , m\text{☮}^{\circ}n, i\text{工}j,$$
$$, i\text{工}j, m\text{☮}^{\prime}n, \quad\Leftrightarrow\quad , m\text{☮}^{\prime}n, i\text{工}j,$$

$$, i\text{工}j, m\text{⏚}, \quad\Leftrightarrow\quad , m\text{⏚}, i\text{工}j,$$
$$, i\text{工}j, m\oplus, \quad\Leftrightarrow\quad , m\oplus, i\text{工}j,$$
$$, i\text{工}j, m\ominus, \quad\Leftrightarrow\quad , m\ominus, i\text{工}j,$$

$$, i!\text{工}j, \circledcirc m, \quad\Leftrightarrow\quad , \circledcirc m, i!\text{工}j,$$
$$, i!\text{工}j, \odot m, \quad\Leftrightarrow\quad , \odot m, i!\text{工}j,$$

$$, i!\text{工}j, m\text{☮}n, \quad\Leftrightarrow\quad , m\text{☮}n, i!\text{工}j,$$
$$, i!\text{工}j, m\text{☮}^{\circ}n, \quad\Leftrightarrow\quad , m\text{☮}^{\circ}n, i!\text{工}j,$$

$$, i!\vdash\!\dashv j, m\oslash n, \ \Leftrightarrow\ , m\oslash n, i!\vdash\!\dashv j,$$

$$, i!\vdash\!\dashv j, m\owedge, \ \Leftrightarrow\ , m\owedge, i!\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m\oplus, \ \Leftrightarrow\ , m\oplus, i!\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m\ominus, \ \Leftrightarrow\ , m\ominus, i!\vdash\!\dashv j,$$

Propositions and Propositions:

$$, i\vdash\!\dashv j, m\vdash\!\dashv n, \ \Leftrightarrow\ , m\vdash\!\dashv n, i\vdash\!\dashv j,$$
$$, i\vdash\!\dashv j, m!\vdash\!\dashv n, \ \Leftrightarrow\ , m!\vdash\!\dashv n, i\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m!\vdash\!\dashv n, \ \Leftrightarrow\ , m!\vdash\!\dashv n, i!\vdash\!\dashv j,$$

$$, i\vdash\!\dashv j, m\obot n, \ \Leftrightarrow\ , m\obot n, i\vdash\!\dashv j,$$
$$, i\vdash\!\dashv j, m!\obot n, \ \Leftrightarrow\ , m!\obot n, i\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m\obot n, \ \Leftrightarrow\ , m\obot n, i!\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m!\obot n, \ \Leftrightarrow\ , m!\obot n, i!\vdash\!\dashv j,$$

$$, i\vdash\!\dashv j, m\rightarrow n, \ \Leftrightarrow\ , m\rightarrow n, i\vdash\!\dashv j,$$
$$, i\vdash\!\dashv j, m!\rightarrow n, \ \Leftrightarrow\ , m!\rightarrow n, i\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m\rightarrow n, \ \Leftrightarrow\ , m\rightarrow n, i!\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m!\rightarrow n, \ \Leftrightarrow\ , m!\rightarrow n, i!\vdash\!\dashv j,$$

$$, i\vdash\!\dashv j, m\bigcirc^{\circ\circ} n, \ \Leftrightarrow\ , m\bigcirc^{\circ\circ} n, i\vdash\!\dashv j,$$
$$, i\vdash\!\dashv j, m!\bigcirc^{\circ\circ} n, \ \Leftrightarrow\ , m!\bigcirc^{\circ\circ} n, i\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m\bigcirc^{\circ\circ} n, \ \Leftrightarrow\ , m\bigcirc^{\circ\circ} n, i!\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m!\bigcirc^{\circ\circ} n, \ \Leftrightarrow\ , m!\bigcirc^{\circ\circ} n, i!\vdash\!\dashv j,$$

$$, i\vdash\!\dashv j, m\bigcirc^{\circ} n, \ \Leftrightarrow\ , m\bigcirc^{\circ} n, i\vdash\!\dashv j,$$
$$, i\vdash\!\dashv j, m!\bigcirc^{\circ} n, \ \Leftrightarrow\ , m!\bigcirc^{\circ} n, i\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m\bigcirc^{\circ} n, \ \Leftrightarrow\ , m\bigcirc^{\circ} n, i!\vdash\!\dashv j,$$
$$, i!\vdash\!\dashv j, m!\bigcirc^{\circ} n, \ \Leftrightarrow\ , m!\bigcirc^{\circ} n, i!\vdash\!\dashv j,$$

$$, i \mathrel{⌶} j, m = n, \iff , m = n, i \mathrel{⌶} j,$$

$$, i \mathrel{⌶} j, m \mathrel{!=} n, \iff , m \mathrel{!=} n, i \mathrel{⌶} j,$$

$$, i \mathrel{!⌶} j, m = n, \iff , m = n, i \mathrel{!⌶} j,$$

$$, i \mathrel{!⌶} j, m \mathrel{!=} n, \iff , m \mathrel{!=} n, i \mathrel{!⌶} j,$$

$$, i \mathrel{⌶} j, m = \varnothing, \iff , m = \varnothing, i \mathrel{⌶} j,$$

$$, i \mathrel{⌶} j, m \mathrel{!=} \varnothing, \iff , m \mathrel{!=} \varnothing, i \mathrel{⌶} j,$$

$$, i \mathrel{!⌶} j, m = \varnothing, \iff , m = \varnothing, i \mathrel{!⌶} j,$$

$$, i \mathrel{!⌶} j, m \mathrel{!=} \varnothing, \iff , m \mathrel{!=} \varnothing, i \mathrel{!⌶} j,$$

Propositions and recursive function:

$$, i \mathrel{⌶} j, R(m), \iff , R(m), i \mathrel{⌶} j,$$

$$, i \mathrel{!⌶} j, R(m), \iff , R(m), i \mathrel{!⌶} j,$$

$$, i \mathrel{⌶} j, R_{-}(m), \iff , R_{-}(m), i \mathrel{⌶} j,$$

$$, i \mathrel{!⌶} j, R_{-}(m), \iff , R_{-}(m), i \mathrel{!⌶} j,$$

$$, i \mathrel{⌶} j, Rc(m; n), \iff , Rc(m; n), i \mathrel{⌶} j,$$

$$, i \mathrel{!⌶} j, Rc(m; n), \iff , Rc(m; n), i \mathrel{!⌶} j,$$

Propositions and flag object:

$$, i \text{⌶} j,\ \&SHi \text{☌} m, \iff , \&SHi \text{☌} m, i \text{⌶} j,$$

$$, i \text{⌶} j,\ \&SHi \rightarrow m, \iff , \&SHi \rightarrow m, i \text{⌶} j,$$

$$, i! \text{⌶} j,\ \&SHi \text{☌} m, \iff , \&SHi \text{☌} m, i! \text{⌶} j,$$

$$, i! \text{⌶} j,\ \&SHi \rightarrow m, \iff , \&SHi \rightarrow m, i! \text{⌶} j,$$

$$, i \text{⌶} j,\ \&SHj \text{☌} m, \iff , \&SHj \text{☌} m, i \text{⌶} j,$$

$$, i \text{⌶} j,\ \&SHj \leftarrow m, \iff , \&SHj \leftarrow m, i \text{⌶} j,$$

$$, i! \text{⌶} j,\ \&SHj \text{☌} m, \iff , \&SHj \text{☌} m, i! \text{⌶} j,$$

$$, i! \text{⌶} j,\ \&SHj \leftarrow m, \iff , \&SHj \leftarrow m, i! \text{⌶} j,$$

$$, i \text{⌶} j,\ \&SVi \text{☌} m, \iff , \&SVi \text{☌} m, i \text{⌶} j,$$

$$, i \text{⌶} j,\ \&SVi \oplus m, \iff , \&SVi \oplus m, i \text{⌶} j,$$

$$, i! \text{⌶} j,\ \&SVi \text{☌} m, \iff , \&SVi \text{☌} m, i! \text{⌶} j,$$

$$, i! \text{⌶} j,\ \&SVi \oplus m, \iff , \&SVi \oplus m, i! \text{⌶} j,$$

Propositions to Propositions with branch function

(Skip......)

21.2.7 Swap of the same operand

(skip......)

21.2.8 Transitivity

Branch function with branch function:

$$,if(i{=}j)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix}, \;\Leftrightarrow\; ,if(i{=}j)\begin{bmatrix},if(i{=}j)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_3,\end{bmatrix},\\ ,\copyright c_2,\end{bmatrix},$$

proof:

$$,if(i{=}j)\begin{bmatrix},\copyright c_1,\\ \\ ,\copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\; ,i\oslash i_0,j\oslash j_0,Rc(i_0,j_0),if(i_0=j_0)\begin{bmatrix},i_0\circledcirc,j_0\circledcirc,\copyright c_1,\\ \\ ,i_0\circledcirc,j_0\circledcirc,\copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\; ,i\oslash i_0,j\oslash j_0,Rc(i_0,j_0),i\oslash i_1,i_1\circledcirc,j\oslash j_1,j_1\circledcirc,$$

$$if(i_0=j_0)\begin{bmatrix},i_0\circledcirc,j_0\circledcirc,\copyright c_1,\\ \\ ,i_0\circledcirc,j_0\circledcirc,\copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\; ,i\oslash i_0,i\oslash i_1,j\oslash j_0,j\oslash j_1,Rc(i_0,j_0),i_1\circledcirc,j_1\circledcirc,$$

$$if(i_0=j_0)\begin{bmatrix},i_0\circledcirc,j_0\circledcirc,\copyright c_1,\\ \\ ,i_0\circledcirc,j_0\circledcirc,\copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\; ,i\oslash i_0,i\oslash i_1,j\oslash j_0,j\oslash j_1,Rc(i_0,j_0),Rc(i_1,j_1),i_1\circledcirc,j_1\circledcirc,$$

$$if(i_0=j_0)\begin{bmatrix},i_0\circledcirc,j_0\circledcirc,\copyright c_1,\\ \\ ,i_0\circledcirc,j_0\circledcirc,\copyright c_2,\end{bmatrix},$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l}, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, i_0\text{♂}i_1, j\text{☮}j_0, j\text{☮}j_1, j_0\text{♂}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l}, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\text{♂}i_1, j_0\text{♂}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l}, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\text{♂}i_1, j_0\text{♂}j_1, Rc(i_0,j_0), Rc(i_1,j_1), i_0\text{♂}i_1, j_0\text{♂}j_1,$$

$$if(i_0=j_0)\left[\begin{array}{l}, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\text{♂}i_1, j_0\text{♂}j_1, Rc(i_0,j_0), Rc(i_1,j_1), i_0\text{♂}i_1, j_0\text{♂}j_1,$$

$$if(i_0=j_0)\left[\begin{array}{l}, if(i_0=j_0)\left[\begin{array}{l}, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ , i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_3,\end{array}\right], \\ , i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\text{♂}i_1, j_0\text{♂}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, if(i_0=j_0)\left[\begin{array}{l} ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ ,i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, if(i_0=j_1)\left[\begin{array}{l} ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ ,i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,j_0\mathring{\bigcirc}j_1, i_0\mathring{\bigcirc}i_1, if(i_0=j_1)\left[\begin{array}{l} ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ ,i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,j_0\mathring{\bigcirc}j_1, i_0\mathring{\bigcirc}i_1, if(i_1=j_1)\left[\begin{array}{l} ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ ,i_0\mathring{\bigcirc}i_1, j_0\mathring{\bigcirc}j_1, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, Rc(i_0,j_0), Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,if(i_1=j_1)\left[\begin{array}{l} ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_1, \\ \\ ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_3, \end{array}\right], \\ ,i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\text{☮}i_0, j\text{☮}j_0, Rc(i_0,j_0), i\text{☮}i_1, j\text{☮}j_1, Rc(i_1,j_1),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,i_0\textcircled{\scriptsize$\downarrow$},j_0\textcircled{\scriptsize$\downarrow$},if(i_1=j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$},j_1\textcircled{\scriptsize$\downarrow$},\copyright c_1, \\ \\ ,i_1\textcircled{\scriptsize$\downarrow$},j_1\textcircled{\scriptsize$\downarrow$},\copyright c_3, \end{array}\right], \\ ,i_0\textcircled{\scriptsize$\downarrow$},j_0\textcircled{\scriptsize$\downarrow$},i_1\textcircled{\scriptsize$\downarrow$},j_1\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\textcircled{\scriptsize$\curlywedge$}i_0,j\textcircled{\scriptsize$\curlywedge$}j_0,Rc(i_0,j_0),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,i_0\textcircled{\scriptsize$\downarrow$},j_0\textcircled{\scriptsize$\downarrow$},i\textcircled{\scriptsize$\curlywedge$}i_1,j\textcircled{\scriptsize$\curlywedge$}j_1,Rc(i_1,j_1),if(i_1=j_1)\left[\begin{array}{l} ,i_1\textcircled{\scriptsize$\downarrow$},j_1\textcircled{\scriptsize$\downarrow$},\copyright c_1, \\ \\ ,i_1\textcircled{\scriptsize$\downarrow$},j_1\textcircled{\scriptsize$\downarrow$},\copyright c_3, \end{array}\right], \\ ,i_0\textcircled{\scriptsize$\downarrow$},j_0\textcircled{\scriptsize$\downarrow$},i\textcircled{\scriptsize$\curlywedge$}i_1,j\textcircled{\scriptsize$\curlywedge$}j_1,Rc(i_1,j_1),i_1\textcircled{\scriptsize$\downarrow$},j_1\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i\textcircled{\scriptsize$\curlywedge$}i_0,j\textcircled{\scriptsize$\curlywedge$}j_0,Rc(i_0,j_0),$$

$$if(i_0=j_0)\left[\begin{array}{l} ,i_0\textcircled{\scriptsize$\downarrow$},j_0\textcircled{\scriptsize$\downarrow$},if(i\mathrel{\pm}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,i_0\textcircled{\scriptsize$\downarrow$},j_0\textcircled{\scriptsize$\downarrow$},\copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathrel{\pm}j)\left[\begin{array}{l} ,if(i\mathrel{\pm}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i\mathrel{\pm}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],\quad\Leftrightarrow\quad ,if(i\mathrel{\pm}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,if(i\mathrel{\pm}j)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_2, \end{array}\right], \end{array}\right],$$

Branch function with propositions:

$$,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l} ,i{=}j,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right],$$

$$,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,i!{=}j,\copyright c_2, \end{array}\right],$$

Propositions with branch function:

$$,i{=}j,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,i{=}j,\copyright c_1,$$

$$,i!{=}j,if(i{=}j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,i!{=}j,\copyright c_2,$$

Propositions with propositions:

$$,i{=}j, \;\Leftrightarrow\; ,i{=}j,i{=}j,$$

$$,i!{=}j, \;\Leftrightarrow\; ,i!{=}j,i!{=}j,$$

21.2.9 With node null propositions

$$, i = \varnothing, j = \varnothing, \ \Leftrightarrow \ \sim, i \text{⌶} j,$$
$$, i = \varnothing, j \mathrel{!=} \varnothing, \ \Leftrightarrow \ \sim, i ! \text{⌶} j,$$

$$, i \text{⌶} j, i = \varnothing, \ \Leftrightarrow \ , i \text{⌶} j, j = \varnothing,$$

proof:

$, i \text{⌶} j, i = \varnothing,$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚}, i = \varnothing,$

$\Leftrightarrow \ , i \text{☮} i_0, i = \varnothing, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, i \text{⧂} i_0, i = \varnothing, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, i \text{⧂} i_0, i_0 = \varnothing, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, i_0 = \varnothing, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, i_0 = \varnothing, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, i_0 = \varnothing, i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, i_0 = j_0, i_0 = \varnothing, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, i_0 = j_0, j_0 = \varnothing, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, j_0 = \varnothing, i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, j_0 = \varnothing, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, j \text{⧂} j_0, j_0 = \varnothing, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, j \text{⧂} j_0, j = \varnothing, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, j = \varnothing, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow \ , i \text{☮} i_0, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚}, j = \varnothing,$

$\Leftrightarrow \ , i \text{⌶} j, j = \varnothing,$

$$, i \text{⌶} j, i \mathrel{!=} \varnothing, \ \Leftrightarrow \ , i \text{⌶} j, j \mathrel{!=} \varnothing,$$

proof 1:

$, i \text{⌶} j, i \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚}, i \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \text{☮} i_0, i \mathrel{!=} \varnothing, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow\ , i \text{☮} i_0, i \mathring{\bigcirc} i_0, i \mathrel{!=} \varnothing, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow\ , i \text{☮} i_0, i \mathring{\bigcirc} i_0, i_0 \mathrel{!=} \varnothing, j \text{☮} j_0, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, i_0 \mathrel{!=} \varnothing, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, i_0 \mathrel{!=} \varnothing, if(j_0 = \varnothing) \left[\begin{array}{l} , \\ , i_0 \oplus, j_0 \oplus, Rc(i_0, j_0), \end{array} \right], i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$$

$$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, i_0 \mathrel{!=} \varnothing, if(j_0 = \varnothing) \left[\begin{array}{l} , i_0 = j_0, \\ , i_0 \oplus, j_0 \oplus, Rc(i_0, j_0), i_0 = j_0, \end{array} \right], i_0 \text{⏚}, j_0 \text{⏚},$$

$$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, if(j_0 = \varnothing) \left[\begin{array}{l} , i_0 \mathrel{!=} \varnothing, i_0 = j_0, \\ , i_0 \mathrel{!=} \varnothing, i_0 \oplus, j_0 \oplus, Rc(i_0, j_0), i_0 = j_0, \end{array} \right], i_0 \text{⏚}, j_0 \text{⏚},$$

$$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, if(j_0 = \varnothing) \left[\begin{array}{l} , j_0 \mathrel{!=} \varnothing, i_0 = j_0, \\ , i_0 \mathrel{!=} \varnothing, i_0 \oplus, j_0 \oplus, Rc(i_0, j_0), i_0 = j_0, \end{array} \right], i_0 \text{⏚}, j_0 \text{⏚},$$

$$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, if(j_0 = \varnothing) \left[\begin{array}{l} , j_0 = \varnothing, j_0 \mathrel{!=} \varnothing, i_0 = j_0, \\ , i_0 \mathrel{!=} \varnothing, i_0 \oplus, j_0 \oplus, Rc(i_0, j_0), i_0 = j_0, \end{array} \right], i_0 \text{⏚}, j_0 \text{⏚},$$

$$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, if(j_0 = \varnothing) \left[\begin{array}{l} , \otimes, \\ , i_0 \mathrel{!=} \varnothing, i_0 \oplus, j_0 \oplus, Rc(i_0, j_0), i_0 = j_0, \end{array} \right], i_0 \text{⏚}, j_0 \text{⏚},$$

$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, j_0 \mathrel{!=} \varnothing, i_0 \mathrel{!=} \varnothing, i_0 \oplus, j_0 \oplus, Rc(i_0, j_0), i_0 = j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,i_0 \mathrel{!=}\varnothing,j_0 \mathrel{!=}\varnothing,i_0\oplus,j_0\oplus,Rc(i_0,j_0),i_0=j_0,i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,if(i_0=\varnothing)\left[\begin{array}{l},\otimes,\\ \\ ,j_0 \mathrel{!=}\varnothing,i_0\oplus,j_0\oplus,Rc(i_0,j_0),i_0=j_0,\end{array}\right],i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,if(i_0=\varnothing)\left[\begin{array}{l},i_0=\varnothing,i_0 \mathrel{!=}\varnothing,i_0=j_0,\\ \\ ,j_0 \mathrel{!=}\varnothing,i_0\oplus,j_0\oplus,Rc(i_0,j_0),i_0=j_0,\end{array}\right],i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,if(i_0=\varnothing)\left[\begin{array}{l},i_0=\varnothing,j_0 \mathrel{!=}\varnothing,i_0=j_0,\\ \\ ,j_0 \mathrel{!=}\varnothing,i_0\oplus,j_0\oplus,Rc(i_0,j_0),i_0=j_0,\end{array}\right],i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,j_0 \mathrel{!=}\varnothing,if(i_0=\varnothing)\left[\begin{array}{l},i_0=\varnothing,\\ \\ ,i_0\oplus,j_0\oplus,Rc(i_0,j_0),\end{array}\right],i_0=j_0,i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,j_0 \mathrel{!=}\varnothing,if(i_0=\varnothing)\left[\begin{array}{l},\\ \\ ,i_0\oplus,j_0\oplus,Rc(i_0,j_0),\end{array}\right],i_0=j_0,i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,j_0 \mathrel{!=}\varnothing,Rc(i_0,j_0),i_0=j_0,i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,j\overset{\circ}{\bigcirc}j_0,j_0 \mathrel{!=}\varnothing,Rc(i_0,j_0),i_0=j_0,i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,j\overset{\circ}{\bigcirc}j_0,j \mathrel{!=}\varnothing,Rc(i_0,j_0),i_0=j_0,i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0,j_0),i_0=j_0,i_0\textcircled{\downarrow},j_0\textcircled{\downarrow},j \mathrel{!=}\varnothing,$$

$$\Leftrightarrow \;, i \mathrel{\mathtt{I}} j, j \,!{=}\, \varnothing,$$

proof 2:

$$, i \mathrel{\mathtt{I}} j, i \,!{=}\, \varnothing,$$

$$\Leftrightarrow \;, i \mathrel{\mathtt{I}} j, if(i=\varnothing) \left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right],$$

$$\Leftrightarrow \;, if(i=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, \otimes, \\ \\ , i \mathrel{\mathtt{I}} j, \end{array}\right],$$

$$\Leftrightarrow \;, if(i=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, \otimes, \\ \\ , i \mathrel{\mathtt{I}} j, if(j=\varnothing) \left[\begin{array}{l} , \\ \\ , \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \;, if(i=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, \otimes, \\ \\ , i \,!{=}\, \varnothing, i \mathrel{\mathtt{I}} j, if(j=\varnothing) \left[\begin{array}{l} , \\ \\ , \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \;, if(i=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, \otimes, \\ \\ , if(j=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, i \,!{=}\, \varnothing, \\ \\ , i \mathrel{\mathtt{I}} j, i \,!{=}\, \varnothing, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \;, if(i=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, \otimes, \\ \\ , if(j=\varnothing) \left[\begin{array}{l} , j=\varnothing, i \mathrel{\mathtt{I}} j, i \,!{=}\, \varnothing, \\ \\ , i \mathrel{\mathtt{I}} j, i \,!{=}\, \varnothing, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow \;, if(i=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, \otimes, \\ \\ , if(j=\varnothing) \left[\begin{array}{l} , i \mathrel{\mathtt{I}} j, j=\varnothing, i \,!{=}\, \varnothing, \\ \\ , i \mathrel{\mathtt{I}} j, i \,!{=}\, \varnothing, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,\otimes, \\ ,if(j=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,i=\varnothing,i\,!\!=\varnothing, \\ ,i\mathrel{\vdash\!\!\dashv}j,i\,!\!=\varnothing, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,\otimes, \\ ,if(j=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,\otimes, \\ ,i\mathrel{\vdash\!\!\dashv}j,i\,!\!=\varnothing, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,\otimes, \\ ,if(j=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,i\,!\!=\varnothing,\otimes, \\ ,i\mathrel{\vdash\!\!\dashv}j,i\,!\!=\varnothing, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,\otimes, \\ ,i\,!\!=\varnothing,i\mathrel{\vdash\!\!\dashv}j,if(j=\varnothing)\left[\begin{array}{l} ,\otimes, \\ , \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,\otimes, \\ ,i\mathrel{\vdash\!\!\dashv}j,if(j=\varnothing)\left[\begin{array}{l} ,\otimes, \\ , \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,\otimes, \\ ,i\mathrel{\vdash\!\!\dashv}j,j\,!\!=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,j=\varnothing,j\,!\!=\varnothing, \\ ,i\mathrel{\vdash\!\!\dashv}j,j\,!\!=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\mathrel{\vdash\!\!\dashv}j,i=\varnothing,j\,!\!=\varnothing, \\ ,i\mathrel{\vdash\!\!\dashv}j,j\,!\!=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,i\mathrel{=\!\!\!\mid}j,if(i=\varnothing)\left[\begin{array}{l},i=\varnothing,j\,!{=}\varnothing,\\ \\ ,j\,!{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,i\mathrel{=\!\!\!\mid}j,if(i=\varnothing)\left[\begin{array}{l},i=\varnothing,\\ \\ ,\end{array}\right],j\,!{=}\varnothing,$$

$$\Leftrightarrow\ ,i\mathrel{=\!\!\!\mid}j,if(i=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right],j\,!{=}\varnothing,$$

$$\Leftrightarrow\ ,i\mathrel{=\!\!\!\mid}j,j\,!{=}\varnothing,$$

$$,i_1\mathrel{=\!\!\!\mid}i_2,j_1\mathrel{=\!\!\!\mid}j_2,if(i_1=\varnothing)\left[\begin{array}{l},\\ \\ ,j_1=\varnothing,\end{array}\right],if(i_1=j_1)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1\mathrel{=\!\!\!\mid}i_2,j_1\mathrel{=\!\!\!\mid}j_2,if(i_1=\varnothing)\left[\begin{array}{l},\\ \\ ,j_1=\varnothing,\end{array}\right],if(i_2=j_2)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],$$

proof:

$$,i_1\mathrel{=\!\!\!\mid}i_2,j_1\mathrel{=\!\!\!\mid}j_2,if(i_1=\varnothing)\left[\begin{array}{l},\\ \\ ,j_1=\varnothing,\end{array}\right],if(i_1=j_1)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1\mathrel{=\!\!\!\mid}i_2,j_1\mathrel{=\!\!\!\mid}j_2,if(i_1=\varnothing)\left[\begin{array}{l},i_1=\varnothing,\\ \\ ,i_1\,!{=}\varnothing,j_1=\varnothing,\end{array}\right],if(i_1=j_1)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1\mathrel{=\!\!\!\mid}i_2,j_1\mathrel{=\!\!\!\mid}j_2,if(i_1=\varnothing)\left[\begin{array}{l},i_1=\varnothing,if(j_1=\varnothing)\left[\begin{array}{l},\\ ,\end{array}\right],\\ \\ ,i_1\,!{=}\varnothing,j_1=\varnothing,\end{array}\right],if(i_1=j_1)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1{=}i_2,j_1{=}j_2,if(i_1=\varnothing)\left[\begin{array}{l},i_1=\varnothing,if(j_1=\varnothing)\left[\begin{array}{l},j_1=\varnothing,\\ \\ ,j_1\,!{=}\,\varnothing,\end{array}\right],\\ ,i_1\,!{=}\,\varnothing,j_1=\varnothing,\end{array}\right],if(i_1=j_1)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1{=}i_2,j_1{=}j_2,if(i_1=\varnothing)\left[\begin{array}{l},if(j_1=\varnothing)\left[\begin{array}{l},i_1=\varnothing,j_1=\varnothing,\\ \\ ,i_1=\varnothing,j_1\,!{=}\,\varnothing,\end{array}\right],\\ ,i_1\,!{=}\,\varnothing,j_1=\varnothing,\end{array}\right],if(i_1=j_1)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1{=}i_2,j_1{=}j_2,if(i_1=\varnothing)\left[\begin{array}{l},if(j_1=\varnothing)\left[\begin{array}{l},i_1=\varnothing,j_1=\varnothing,i_1=j_1,\\ \\ ,i_1=\varnothing,j_1\,!{=}\,\varnothing,i_1\,!{=}\,j_1,\end{array}\right],\\ ,i_1\,!{=}\,\varnothing,j_1=\varnothing,i_1=j_1,\end{array}\right],$$

$$if(i_1=j_1)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1{=}i_2,j_1{=}j_2,if(i_1=\varnothing)\left[\begin{array}{l},if(j_1=\varnothing)\left[\begin{array}{l},i_1=\varnothing,j_1=\varnothing,i_1=j_1,\copyright c_1,\\ \\ ,i_1=\varnothing,j_1\,!{=}\,\varnothing,i_1\,!{=}\,j_1,\copyright c_2,\end{array}\right],\\ ,i_1\,!{=}\,\varnothing,j_1=\varnothing,i_1=j_1,\copyright c_1,\end{array}\right],$$

$$\Leftrightarrow\ ,i_1{=}i_2,j_1{=}j_2,if(i_1=\varnothing)\left[\begin{array}{l},if(j_1=\varnothing)\left[\begin{array}{l},i_1=\varnothing,j_1=\varnothing,\copyright c_1,\\ \\ ,i_1=\varnothing,j_1\,!{=}\,\varnothing,\copyright c_2,\end{array}\right],\\ ,i_1\,!{=}\,\varnothing,j_1=\varnothing,\copyright c_1,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i_1=\varnothing)\left[\begin{array}{l},if(j_1=\varnothing)\left[\begin{array}{l},i_1{=}i_2,j_1{=}j_2,i_1=\varnothing,j_1=\varnothing,\copyright c_1,\\ \\ ,i_1{=}i_2,j_1{=}j_2,i_1=\varnothing,j_1\,!{=}\,\varnothing,\copyright c_2,\end{array}\right],\\ ,i_1{=}i_2,j_1{=}j_2,i_1\,!{=}\,\varnothing,j_1=\varnothing,\copyright c_1,\end{array}\right],$$

$$\Leftrightarrow\ ,if(i_1=\varnothing)\left[\begin{array}{l} ,if(j_1=\varnothing)\left[\begin{array}{l} ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2=\varnothing,j_2=\varnothing,©c_1, \\ \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2=\varnothing,j_2\,!\!=\varnothing,©c_2, \end{array}\right], \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2\,!\!=\varnothing,j_2=\varnothing,©c_1, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i_1=\varnothing)\left[\begin{array}{l} ,if(j_1=\varnothing)\left[\begin{array}{l} ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2=\varnothing,j_2=\varnothing,i_2=j_2,©c_1, \\ \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2=\varnothing,j_2\,!\!=\varnothing,i_2\,!\!=j_2,©c_2, \end{array}\right], \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2\,!\!=\varnothing,j_2=\varnothing,i_2=j_2,©c_1, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i_1=\varnothing)\left[\begin{array}{l} ,if(j_1=\varnothing)\left[\begin{array}{l} ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2=\varnothing,j_2=\varnothing,i_2=j_2, \\ \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2=\varnothing,j_2\,!\!=\varnothing,i_2\,!\!=j_2, \end{array}\right], \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_2\,!\!=\varnothing,j_2=\varnothing,i_2=j_2, \end{array}\right],$$

$$if(i_2=j_2)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i_1=\varnothing)\left[\begin{array}{l} ,if(j_1=\varnothing)\left[\begin{array}{l} ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_1=\varnothing,j_1=\varnothing, \\ \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_1=\varnothing,j_1\,!\!=\varnothing, \end{array}\right], \\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,i_1\,!\!=\varnothing,j_1=\varnothing, \end{array}\right],$$

$$if(i_2=j_2)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,if(i_1=\varnothing)\left[\begin{array}{l} ,if(j_1=\varnothing)\left[\begin{array}{l} ,i_1=\varnothing,j_1=\varnothing, \\ \\ ,i_1=\varnothing,j_1\,!\!=\varnothing, \end{array}\right], \\ ,i_1\,!\!=\varnothing,j_1=\varnothing, \end{array}\right],if(i_2=j_2)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ ,i_1\mathrel{\overline{\underline{\mid}}}i_2,j_1\mathrel{\overline{\underline{\mid}}}j_2,if(i_1=\varnothing)\left[\begin{array}{l} ,i_1=\varnothing,if(j_1=\varnothing)\left[\begin{array}{l} ,j_1=\varnothing, \\ \\ ,j_1\,!\!=\varnothing, \end{array}\right], \\ ,j_1=\varnothing, \end{array}\right],if(i_2=j_2)\left[\begin{array}{l} ,©c_1, \\ \\ ,©c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i_1 \doteq i_2, j_1 \doteq j_2, if(i_1 = \varnothing) \left[\begin{array}{l} , \\ , j_1 = \varnothing, \end{array}\right], if(i_2 = j_2) \left[\begin{array}{l} , \copyright c_1, \\ , \copyright c_2, \end{array}\right],$$

21.2.10 With node continuity

$$, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i \doteq j, \ \Leftrightarrow\ , i \doteq j, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus,$$

proof:

$, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i \doteq j,$

$\Leftrightarrow\ , i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i\text{☮}i_0, j\text{☮}j_0, Rc(i_0, j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i \,!\!= \varnothing, j \,!\!= \varnothing, i\text{☮}i_1, i_1\text{⏚}, j\text{☮}j_1, j_1\text{⏚}, i\oplus, j\oplus,$

$i\text{☮}i_0, j\text{☮}j_0, Rc(i_0, j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i \,!\!= \varnothing, j \,!\!= \varnothing, i\text{☮}i_1, j\text{☮}j_1, Rc(i_1, j_1), i_1\text{⏚}, j_1\text{⏚}, i\oplus, j\oplus,$

$i\text{☮}i_0, j\text{☮}j_0, Rc(i_0, j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i \,!\!= \varnothing, j \,!\!= \varnothing, i\text{☮}i_1, j\text{☮}j_1, i\oplus, j\oplus, i\text{☮}i_0, j\text{☮}j_0, Rc(i_1, j_1),$

$Rc(i_0, j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i \,!\!= \varnothing, j\text{☮}j_1, j \,!\!= \varnothing, i\oplus, j\oplus, i\text{☮}i_0, j\text{☮}j_0, Rc(i_1, j_1),$

$Rc(i_0, j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i\overset{\circ}{\bigcirc}i_1, i \,!\!= \varnothing, j\text{☮}j_1, j\overset{\circ}{\bigcirc}j_1, j \,!\!= \varnothing, i\oplus, j\oplus, i\text{☮}i_0, j\text{☮}j_0,$

$Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\oslash i_1, i\mathring{\bigcirc} i_1, i_1 \mathrel{!=} \varnothing, j\oslash j_1, j\mathring{\bigcirc} j_1, j_1 \mathrel{!=} \varnothing, i\oplus, j\oplus, i\oslash i_0, j\oslash j_0,$

$Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i\mathring{\bigcirc} i_1, j\oslash j_1, j\mathring{\bigcirc} j_1, i\oplus, j\oplus, i\oslash i_0, j\oslash j_0,$

$i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i\mathring{\bigcirc} i_1, j\oslash j_1, j\mathring{\bigcirc} j_1, i\oplus, j\oplus, i\oslash i_0, j\oslash j_0,$

$i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i_1 \mathrel{!=} \varnothing, i\mathring{\bigcirc} i_1, j\oslash j_1, j_1 \mathrel{!=} \varnothing, j\mathring{\bigcirc} j_1, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\oslash i_0, j\oslash j_0,$

$Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i_1 \mathrel{!=} \varnothing, j\oslash j_1, j_1 \mathrel{!=} \varnothing, i\mathring{\bigcirc} i_1, i\oplus, i_1\oplus, j\mathring{\bigcirc} j_1, j\oplus, j_1\oplus, i\oslash i_0, j\oslash j_0,$

$Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i_1 \mathrel{!=} \varnothing, j\oslash j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, i\mathring{\bigcirc} i_1, j\oplus, j_1\oplus, j\mathring{\bigcirc} j_1, i\oslash i_0, j\oslash j_0,$

$Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i_1 \mathrel{!=} \varnothing, j\oslash j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, i\mathring{\bigcirc} i_1, j\oplus, j_1\oplus, j\mathring{\bigcirc} j_1, i\oslash i_0, i\mathring{\bigcirc} i_0, j\oslash j_0,$

$j\mathring{\bigcirc} j_0, Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i_1 \mathrel{!=} \varnothing, j\oslash j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\oslash i_0, j\oslash j_0,$

$i\mathring{\bigcirc} i_1, i\mathring{\bigcirc} i_0, j\mathring{\bigcirc} j_1, j\mathring{\bigcirc} j_0, Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\oslash i_1, i_1 \mathrel{!=} \varnothing, j\oslash j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\oslash i_0, j\oslash j_0,$

$i\mathring{\bigcirc} i_1, i_1\mathring{\bigcirc} i_0, j\mathring{\bigcirc} j_1, j_1\mathring{\bigcirc} j_0, Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\circledcirc, j_0\circledcirc, i_1\circledcirc, j_1\circledcirc,$

$\Leftrightarrow\ , i\text{☮}i_1, i_1 \mathrel{!=} \varnothing, j\text{☮}j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\text{☮}i_0, j\text{☮}j_0,$

$i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, Rc(i_1, j_1), Rc(i_0, j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1 \mathrel{!=} \varnothing, j\text{☮}j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\text{☮}i_0, j\text{☮}j_0,$

$i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, Rc(i_1, j_1), Rc(i_0, j_0), i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, i_0 = j_0, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1 \mathrel{!=} \varnothing, j\text{☮}j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\text{☮}i_0, j\text{☮}j_0,$

$i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, Rc(i_1, j_1), Rc(i_0, j_0), i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, i_1 = j_1, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1 \mathrel{!=} \varnothing, j\text{☮}j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i\text{☮}i_0, j\text{☮}j_0,$

$Rc(i_1, j_1), Rc(i_0, j_0), i_1 = j_1, i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1 \mathrel{!=} \varnothing, j\text{☮}j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1,$

$Rc(i_1, j_1), i_1 = j_1, i\text{☮}i_0, j\text{☮}j_0, Rc(i_0, j_0), i_0\text{⏚}, j_0\text{⏚}, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1 \mathrel{!=} \varnothing, j\text{☮}j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1,$

$Rc(i_1, j_1), i_1 = j_1, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i\mathring{\bigcirc}i_1, i_1 \mathrel{!=} \varnothing, j\text{☮}j_1, j\mathring{\bigcirc}j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus,$

$Rc(i_1, j_1), i_1 = j_1, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i\mathring{\bigcirc}i_1, j\text{☮}j_1, j\mathring{\bigcirc}j_1, i\oplus, j\oplus, i_1 \mathrel{!=} \varnothing, i_1\oplus, j_1 \mathrel{!=} \varnothing, j_1\oplus,$

$Rc(i_1, j_1), i_1 = j_1, i_1\text{⏚}, j_1\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_1, i\mathring{\bigcirc}i_1, j\text{☮}j_1, j\mathring{\bigcirc}j_1, i\oplus, j\oplus, i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing,$

$Rc(i_1, j_1), i_1 = j_1, i_1⏚, j_1⏚,$

$\Leftrightarrow\ , i☮i_1, i\bigcirc^{\circ}i_1, i_1 \,!\!= \varnothing, j☮j_1, j\bigcirc^{\circ}j_1, j_1 \,!\!= \varnothing, i\oplus, j\oplus,$

$Rc(i_1, j_1), i_1 = j_1, i_1⏚, j_1⏚,$

$\Leftrightarrow\ , i☮i_1, i\bigcirc^{\circ}i_1, i \,!\!= \varnothing, j☮j_1, j\bigcirc^{\circ}j_1, j \,!\!= \varnothing, i\oplus, j\oplus,$

$Rc(i_1, j_1), i_1 = j_1, i_1⏚, j_1⏚,$

$\Leftrightarrow\ , i☮i_1, i \,!\!= \varnothing, j☮j_1, j \,!\!= \varnothing, i\oplus, j\oplus,$

$Rc(i_1, j_1), i_1 = j_1, i_1⏚, j_1⏚,$

$\Leftrightarrow\ , i☮i_1, j☮j_1, Rc(i_1, j_1), i_1 = j_1, i_1⏚, j_1⏚, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus,$

$\Leftrightarrow\ , i⌶j, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus,$

$$, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i⌶j, \quad \Leftrightarrow \quad , i⌶j, i \,!\!= \varnothing, i\oplus, j\oplus,$$

$$, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i!⌶j, \quad \Leftrightarrow \quad , i!⌶j, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus,$$

$$, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, if(i⌶j) \begin{bmatrix} , ©c_1, \\ \\ , ©c_2, \end{bmatrix}, \quad \Leftrightarrow \quad , if(i⌶j) \begin{bmatrix} , i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, ©c_1, \\ \\ , i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, ©c_2, \end{bmatrix},$$

$$, i\ominus, j\ominus, i \,!\!= \varnothing, j \,!\!= \varnothing, i⌶j, \quad \Leftrightarrow \quad , i⌶j, i\ominus, j\ominus, i \,!\!= \varnothing, j \,!\!= \varnothing,$$

proof:
$, i\ominus, j\ominus, i \,!\!= \varnothing, j \,!\!= \varnothing, i⌶j,$

$\Leftrightarrow\ , i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i\text{⌶}j, i\oplus, i\ominus, j\oplus, j\ominus,$

$\Leftrightarrow\ , i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i\text{⌶}j, i\oplus, j\oplus, i\ominus, j\ominus,$

$\Leftrightarrow\ , i\ominus, j\ominus, i\text{⌶}j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, i\ominus, j\ominus,$

$\Leftrightarrow\ , i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus, i\text{⌶}j, i\ominus, j\ominus,$

$\Leftrightarrow\ , i\text{☮}i_1, i_1\text{\textcircled{$\downarrow$}}, j\text{☮}j_1, j_1\text{\textcircled{$\downarrow$}}, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus,$
$i\text{⌶}j, i\ominus, j\ominus,$

$\Leftrightarrow\ , i\text{☮}i_1, i_1\ominus, i_1\text{\textcircled{$\downarrow$}}, j\text{☮}j_1, j_1\ominus, j_1\text{\textcircled{$\downarrow$}}, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus,$
$i\text{⌶}j, i\ominus, j\ominus,$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i_1\ominus, i\ominus, j_1\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus,$
$i\text{⌶}j, i\ominus, j\ominus, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, i\overset{\circ}{\bigcirc}i_1, j\text{☮}j_1, j\overset{\circ}{\bigcirc}j_1, i_1\ominus, i\ominus, j_1\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus,$
$i\text{⌶}j, i\ominus, j\ominus, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i_1\ominus, i\ominus, j_1\ominus, j\ominus, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i\,!\!=\varnothing, j\,!\!=\varnothing, i\oplus, j\oplus,$
$i\text{⌶}j, i\ominus, j\ominus, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i_1\ominus, i\ominus, j_1\ominus, j\ominus, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, i\oplus, j\oplus,$
$i\text{⌶}j, i\ominus, j\ominus, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i_1\ominus, j_1\ominus, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, i\ominus, j\ominus, i\oplus, j\oplus,$
$i\text{⌶}j, i\ominus, j\ominus, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i_1\ominus, j_1\ominus, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, i\oplus, i\ominus, j\oplus, j\ominus,$
$i\text{⌶}j, i\ominus, j\ominus, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i_1\ominus, j_1\ominus, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing,$
$i\text{⌶}j, i\ominus, j\ominus, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i\text{⌶}j, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i_1\ominus, j_1\ominus, i\ominus, j\ominus, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i\text{⌶}j, i_1\ominus, j_1\ominus, i\ominus, j\ominus, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i\text{⌶}j, i_1\ominus, j_1\ominus, i\ominus, j\ominus, i\overset{\circ}{\bigcirc}i_1, j\overset{\circ}{\bigcirc}j_1, i\,!\!=\varnothing, j\,!\!=\varnothing, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, j\text{☮}j_1, i\text{⌶}j, i_1\ominus, j_1\ominus, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i_1\text{\textcircled{$\downarrow$}}, j_1\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i\text{☮}i_1, i_1\ominus, i_1\text{\textcircled{$\downarrow$}}, j\text{☮}j_1, j_1\ominus, j_1\text{\textcircled{$\downarrow$}}, i\text{⌶}j, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing,$

$$\Leftrightarrow\ , i⌶j, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing,$$

$$, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i!⌶j, \ \Leftrightarrow\ , i!⌶j, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing,$$

$$, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, if(i⌶j)\left[\begin{array}{l}, ©c_1, \\ \\ , ©c_2,\end{array}\right], \ \Leftrightarrow\ , if(i⌶j)\left[\begin{array}{l}, i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, ©c_1, \\ \\ , i\ominus, j\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, ©c_2,\end{array}\right],$$

$$, i_1⌶i_2, j_1⌶j_2, Rc(i_1, j_1), Rc(i_2, j_2), \ \Leftrightarrow\ \sim, i_1⌶i_2, j_1⌶j_2,$$

induction proof:

premise 1 :

$, i_1=\varnothing, i_1⌶i_2, j_1⌶j_2, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_1=\varnothing, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_1=\varnothing, Rc(i_2, j_2),$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_2=\varnothing, Rc(i_2, j_2),$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_2=\varnothing,$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_2=\varnothing, i_1⌶i_2, j_1⌶j_2,$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_2=\varnothing, Rc(i_2, j_2), i_1⌶i_2, j_1⌶j_2,$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_1=\varnothing, Rc(i_2, j_2), i_1⌶i_2, j_1⌶j_2,$

$\Leftrightarrow\ , j_1⌶j_2, i_1⌶i_2, i_1=\varnothing, Rc(i_1, j_1), Rc(i_2, j_2), i_1⌶i_2, j_1⌶j_2,$

$\Leftrightarrow\ , i_1=\varnothing, i_1⌶i_2, j_1⌶j_2, Rc(i_1, j_1), Rc(i_2, j_2), i_1⌶i_2, j_1⌶j_2,$

$premise\ 2:$

$, \&SHi \rightarrow i_1, i_1{=}i_2, j_1{=}j_2, Rc(i_1,j_1), Rc(i_2,j_2), \ \Leftrightarrow$

$, \&SHi \rightarrow i_1, i_1{=}i_2, j_1{=}j_2, Rc(i_1,j_1), Rc(i_2,j_2), \ \Rightarrow$

$, i_1 \mathrel{!}= \varnothing, \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, Rc(i_1,j_1), Rc(i_2,j_2),$

$\Leftrightarrow \ , \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, Rc(i_1,j_1), Rc(i_2,j_2),$

$\Leftrightarrow \ , \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing,$

$$if(j_1=\varnothing)\left[\begin{array}{l} , \\ , i_1\oplus, j_1\oplus, Rc(i_1,j_1), \end{array}\right], Rc(i_2,j_2),$$

$$\Leftrightarrow \ , \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, if(j_1=\varnothing)\left[\begin{array}{l} , j_1=\varnothing, \\ , i_1\oplus, j_1\oplus, Rc(i_1,j_1), \end{array}\right], Rc(i_2,j_2),$$

$$\Leftrightarrow \ , \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, if(j_1=\varnothing)\left[\begin{array}{l} , j_1=\varnothing, Rc(i_2,j_2), \\ , i_1\oplus, j_1\oplus, Rc(i_1,j_1), Rc(i_2,j_2), \end{array}\right],$$

$$\Leftrightarrow \ , if(j_1=\varnothing)\left[\begin{array}{l} , \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, j_1=\varnothing, Rc(i_2,j_2), \\ , \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1,j_1), Rc(i_2,j_2), \end{array}\right],$$

$$\Leftrightarrow \ , if(j_1=\varnothing)\left[\begin{array}{l} , i_1 \mathrel{!}= \varnothing, \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, j_1=\varnothing, Rc(i_2,j_2), \\ , \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1,j_1), Rc(i_2,j_2), \end{array}\right],$$

$$\Leftrightarrow \ , if(j_1=\varnothing)\left[\begin{array}{l} , i_1 \mathrel{!}= \varnothing, \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, j_2=\varnothing, Rc(i_2,j_2), \\ , \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1,j_1), Rc(i_2,j_2), \end{array}\right],$$

$$\Leftrightarrow \ , if(j_1=\varnothing)\left[\begin{array}{l} , i_1 \mathrel{!}= \varnothing, \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, j_2=\varnothing, \\ , \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1,j_1), Rc(i_2,j_2), \end{array}\right],$$

$$\Leftrightarrow \ , if(j_1=\varnothing)\left[\begin{array}{l} , i_1 \mathrel{!}= \varnothing, \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, j_2=\varnothing, i_1{=}i_2, j_1{=}j_2, \\ , \ \&SHi \circlearrowright i_1, i_1{=}i_2, j_1{=}j_2, i_1 \mathrel{!}= \varnothing, i_1\oplus, j_1\oplus, Rc(i_1,j_1), Rc(i_2,j_2), \end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1=\varnothing)\!-\!\left[\begin{array}{l},i_1\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,j_2=\varnothing,Rc(i_2,j_2),i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,\\ \\,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,i_1\,!\!=\varnothing,i_1\oplus,j_1\oplus,Rc(i_1,j_1),Rc(i_2,j_2),\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,if(j_1=\varnothing)\!-\!\left[\begin{array}{l},i_1\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,j_1=\varnothing,Rc(i_2,j_2),i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,\\ \\,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,i_1\,!\!=\varnothing,i_1\oplus,j_1\oplus,Rc(i_1,j_1),Rc(i_2,j_2),\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,$$

$$if(j_1=\varnothing)\!-\!\left[\begin{array}{l},i_1\,!\!=\varnothing,j_1=\varnothing,Rc(i_2,j_2),i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,\\ \\,i_1\,!\!=\varnothing,i_1\oplus,j_1\oplus,Rc(i_1,j_1),Rc(i_2,j_2),\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,$$

$$if(j_1=\varnothing)\!-\!\left[\begin{array}{l},i_1\,!\!=\varnothing,j_1=\varnothing,Rc(i_1,j_1),Rc(i_2,j_2),i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,\\ \\,i_1\,!\!=\varnothing,i_1\oplus,j_1\oplus,Rc(i_1,j_1),Rc(i_2,j_2),\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,$$

$$if(j_1=\varnothing)\!-\!\left[\begin{array}{l},j_1=\varnothing,i_1\,!\!=\varnothing,Rc(i_1,j_1),Rc(i_2,j_2),i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,\\ \\,i_1\,!\!=\varnothing,i_1\oplus,j_1\oplus,Rc(i_1,j_1),Rc(i_2,j_2),\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,$$

$$if(j_1=\varnothing)\!-\!\left[\begin{array}{l},i_1\,!\!=\varnothing,Rc(i_1,j_1),Rc(i_2,j_2),i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,\\ \\,i_1\,!\!=\varnothing,i_1\oplus,j_1\oplus,Rc(i_1,j_1),Rc(i_2,j_2),\end{array}\right]\!-,$$

$$\Leftrightarrow\ ,$$

$$if(j_1=\varnothing)\!-\!\left[\begin{array}{l},i_1\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,Rc(i_1,j_1),Rc(i_2,j_2),i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,\\ \\,j_1\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i_1,i_1\mathrel{\mathrm{I}}i_2,j_1\mathrel{\mathrm{I}}j_2,i_1\,!\!=\varnothing,i_1\oplus,j_1\oplus,Rc(i_1,j_1),Rc(i_2,j_2),\end{array}\right]\!-,$$

$$\Leftrightarrow\ <1>$$

$, j_1 \mathrel{!=} \varnothing,\ \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2, i_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2, i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1, j_1), i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, Rc(i_2, j_2),$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, Rc(i_1, j_1), i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_2\oplus, j_2\oplus, Rc(i_2, j_2),$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus, j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1\oplus, j_2\oplus, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus, j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1\oplus, j_2\oplus,$

$i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus, j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1\oplus, j_2\oplus,$

$\&SHi \rightarrow i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2, Rc(i_1, j_1), Rc(i_2, j_2),$

$\Leftrightarrow\ , i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus, j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1\oplus, j_2\oplus,$

$\&SHi \rightarrow i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2, Rc(i_1, j_1), Rc(i_2, j_2), i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus, j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1\oplus, j_2\oplus, Rc(i_1, j_1), Rc(i_2, j_2), i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$\Leftrightarrow\ , \&SHi \bigcirc^{\!\circ} i_1, i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing, i_1\oplus, j_1\oplus, i_2 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_2\oplus, j_2\oplus, Rc(i_1, j_1), Rc(i_2, j_2), i_1 \mathrel{\text{工}} i_2, j_1 \mathrel{\text{工}} j_2,$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, i_1\oplus, j_1\oplus, Rc(i_1,j_1), i_2\,!\!=\varnothing, j_2\,!\!=\varnothing, i_2\oplus, j_2\oplus, Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, Rc(i_1,j_1), i_2\,!\!=\varnothing, j_2\,!\!=\varnothing, Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$i_2\,!\!=\varnothing, j_2\,!\!=\varnothing, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, Rc(i_1,j_1), Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, Rc(i_1,j_1), Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2, i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, Rc(i_1,j_1), Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$< 1 >\ \ \Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$$if(j_1=\varnothing)\left[\begin{array}{l} , i_1\,!\!=\varnothing, Rc(i_1,j_1), Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2, \\ \\ , i_1\,!\!=\varnothing, j_1\,!\!=\varnothing, Rc(i_1,j_1), Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2, \end{array}\right],$$

$\Leftrightarrow\ , i_1\,!\!=\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

$$if(j_1=\varnothing)\left[\begin{array}{l} , \\ \\ , j_1\,!\!=\varnothing, \end{array}\right], Rc(i_1,j_1), Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$$

$\Leftrightarrow\ , i_1\,!\!=\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc} i_1, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2, Rc(i_1,j_1), Rc(i_2,j_2), i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

conclusion :

$, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2, Rc(i_1,j_1), Rc(i_2,j_2),\ \ \Leftrightarrow\ \ \sim, i_1 \mathrel{\overline{\perp}} i_2, j_1 \mathrel{\overline{\perp}} j_2,$

21.2.11 With identical node propositions

$$,i\overset{\circ}{\bigcirc}j, \;\; \Leftrightarrow \;\; \sim, i\doteq j,$$

proof:

$, i\overset{\circ}{\bigcirc}j,$

$\Leftrightarrow \;\; , i\overset{\circ}{\bigcirc}j, i\circledast i_0, j\circledast j_0, Rc(i_0; j_0), i_0\circleddash, j_0\circleddash,$

$\Leftrightarrow \;\; , i\overset{\circ}{\bigcirc}j, i\circledast i_0, j\circledast j_0, i_0\overset{\circ}{\bigcirc}j_0, Rc(i_0; j_0), i_0\circleddash, j_0\circleddash,$

$\Leftrightarrow \;\; , i\overset{\circ}{\bigcirc}j, i\circledast i_0, j\circledast j_0, i_0\overset{\circ}{\bigcirc}j_0, Rc(i_0; j_0), i_0\overset{\circ}{\bigcirc}j_0, i_0\circleddash, j_0\circleddash,$

$\Leftrightarrow \;\; , i\overset{\circ}{\bigcirc}j, i\circledast i_0, j\circledast j_0, i_0\overset{\circ}{\bigcirc}j_0, Rc(i_0; j_0), i_0\overset{\circ}{\bigcirc}j_0, i_0 = j_0, i_0\circleddash, j_0\circleddash,$

$\Leftrightarrow \;\; , i\overset{\circ}{\bigcirc}j, i\circledast i_0, j\circledast j_0, Rc(i_0; j_0), i_0 = j_0, i_0\circleddash, j_0\circleddash,$

$\Leftrightarrow \;\; , i\overset{\circ}{\bigcirc}j, i\doteq j,$

21.2.12 Substitution

Propositions with null node branch function:

$$, i\doteq j, if(j=\varnothing)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\; \Leftrightarrow \;\; , i\doteq j, if(i=\varnothing)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right],$$

Identical node propositions with branch function:

$$, i\overset{\circ}{\bigcirc}j, if(j\doteq m)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \;\; \Leftrightarrow \;\; , i\overset{\circ}{\bigcirc}j, if(i\doteq m)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right],$$

proof 1:

$, i\overset{\circ}{\bigcirc}j, if(j\doteq m)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right],$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, j\oslash j_0, m\oslash m_0, Rc(j_0, m_0), if(j_0=m_0)\left[\begin{array}{l} , j_0\ominus, m_0\ominus, \copyright c_1, \\ \\ , j_0\ominus, m_0\ominus, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i\oslash j_0, m\oslash m_0, Rc(j_0, m_0), if(j_0=m_0)\left[\begin{array}{l} , j_0\ominus, m_0\ominus, \copyright c_1, \\ \\ , j_0\ominus, m_0\ominus, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, if(i\mathrel{\underline{\overline{|}}}m)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

proof 2:

$$, i\overset{\circ}{\bigcirc}j, if(j\mathrel{\underline{\overline{|}}}m)\left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, j\oslash j_0, m\oslash m_0, Rc(j_0, m_0), if(j_0=m_0)\left[\begin{array}{l} , j_0\ominus, m_0\ominus, \copyright c_1, \\ \\ , j_0\ominus, m_0\ominus, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i\oslash i_1, m\oslash m_1, i_1\ominus, m_1\ominus, j\oslash j_0, m\oslash m_0, Rc(j_0, m_0),$$

$$if(j_0=m_0)\left[\begin{array}{l} , j_0\ominus, m_0\ominus, \copyright c_1, \\ \\ , j_0\ominus, m_0\ominus, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i\oslash i_1, m\oslash m_1, Rc(i_1; m_1), i_1\ominus, m_1\ominus, j\oslash j_0, m\oslash m_0, Rc(j_0, m_0),$$

$$if(j_0=m_0)\left[\begin{array}{l} , j_0\ominus, m_0\ominus, \copyright c_1, \\ \\ , j_0\ominus, m_0\ominus, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i\oslash i_1, m\oslash m_1, Rc(i_1; m_1), j\oslash j_0, m\oslash m_0, Rc(j_0, m_0),$$

$$if(j_0=m_0)\left[\begin{array}{l} , j_0\ominus, m_0\ominus, i_1\ominus, m_1\ominus, \copyright c_1, \\ \\ , j_0\ominus, m_0\ominus, i_1\ominus, m_1\ominus, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}j, i\oslash i_1, j\oslash j_0, m\oslash m_1, m\oslash m_0, Rc(i_1; m_1), Rc(j_0, m_0),$$

$$if(j_0=m_0)\left[\begin{array}{l} , j_0\ominus, m_0\ominus, i_1\ominus, m_1\ominus, \copyright c_1, \\ \\ , j_0\ominus, m_0\ominus, i_1\ominus, m_1\ominus, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}j, i\circledast i_1, i\mathring{\bigcirc}i_1, j\circledast j_0, j\mathring{\bigcirc}j_0, m\circledast m_1, m\circledast m_0, m_1\mathring{\bigcirc}m_0, Rc(i_1;m_1), Rc(j_0,m_0),$$

$$if(j_0=m_0)\left[\begin{array}{l} , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\circledast i_1, j\circledast j_0, m\circledast m_1, m\circledast m_0, i\mathring{\bigcirc}j, i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_0, m_1\mathring{\bigcirc}m_0, Rc(i_1;m_1), Rc(j_0,m_0),$$

$$if(j_0=m_0)\left[\begin{array}{l} , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\circledast i_1, j\circledast j_0, m\circledast m_1, m\circledast m_0, i\mathring{\bigcirc}j, j\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}j_0, m_1\mathring{\bigcirc}m_0, Rc(i_1;m_1), Rc(j_0,m_0),$$

$$if(j_0=m_0)\left[\begin{array}{l} , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\circledast i_1, j\circledast j_0, m\circledast m_1, m\circledast m_0, i\mathring{\bigcirc}j, j\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}j_0, m_1\mathring{\bigcirc}m_0, Rc(i_1;m_1), Rc(j_0,m_0),$$

$$i_1\mathring{\bigcirc}j_0, m_1\mathring{\bigcirc}m_0, if(j_0=m_0)\left[\begin{array}{l} , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\circledast i_1, j\circledast j_0, m\circledast m_1, m\circledast m_0, i\mathring{\bigcirc}j, j\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}j_0, m_1\mathring{\bigcirc}m_0, Rc(i_1;m_1), Rc(j_0,m_0),$$

$$i_1\mathring{\bigcirc}j_0, m_1\mathring{\bigcirc}m_0, if(i_1=m_1)\left[\begin{array}{l} , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_1, \\ \\ , j_0\circleddash, m_0\circleddash, i_1\circleddash, m_1\circleddash, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}j, i\circledast i_1, m\circledast m_1, Rc(i_1;m_1), j\circledast j_0, m\circledast m_0, Rc(j_0,m_0), j_0\circleddash, m_0\circleddash,$$

$$if(i_1=m_1)\left[\begin{array}{l} , i_1\circleddash, m_1\circleddash, \copyright c_1, \\ \\ , i_1\circleddash, m_1\circleddash, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}j, i\circledast i_1, m\circledast m_1, Rc(i_1;m_1),$$

$$if(i_1=m_1)\left[\begin{array}{l} , i_1\circleddash, m_1\circleddash, \copyright c_1, \\ \\ , i_1\circleddash, m_1\circleddash, \copyright c_2, \end{array}\right]$$

$$\Leftrightarrow\ ,i\mathring{\bigcirc}j, if(i\mathrel{⌶}m)\text{-}\begin{bmatrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{bmatrix}\text{-},$$

Identical node propositions with propositions:

$$,i\mathring{\bigcirc}j, j\mathrel{⌶}m,\ \Leftrightarrow\ ,i\mathring{\bigcirc}j, i\mathrel{⌶}m,$$

$$,i\mathring{\bigcirc}j, j!\mathrel{⌶}m,\ \Leftrightarrow\ ,i\mathring{\bigcirc}j, i!\mathrel{⌶}m,$$

Propositions with branch function:

$$,i\mathrel{⌶}j, if(j\mathrel{⌶}m)\text{-}\begin{bmatrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{bmatrix}\text{-},\ \Leftrightarrow\ ,i\mathrel{⌶}j, if(i\mathrel{⌶}m)\text{-}\begin{bmatrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{bmatrix}\text{-},$$

proof:

$$,i\mathrel{⌶}j, if(j\mathrel{⌶}m)\text{-}\begin{bmatrix} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{bmatrix}\text{-},$$

$$\Leftrightarrow\ ,i\mathrel{⌶}j, j\mathbin{☮}j_0, m\mathbin{☮}m_0, Rc(j_0;m_0), if(j_0=m_0)\text{-}\begin{bmatrix} ,j_0⏚, m_0⏚, \copyright c_1, \\ \\ j_0⏚, m_0⏚, \copyright c_2, \end{bmatrix}\text{-},$$

$$\Leftrightarrow\ ,i\mathrel{⌶}j, i\mathbin{☮}i_1, m\mathbin{☮}m_1, Rc(i_1;m_1), i_1⏚, m_1⏚, j\mathbin{☮}j_0, m\mathbin{☮}m_0, Rc(j_0;m_0),$$

$$if(j_0=m_0)\text{-}\begin{bmatrix} ,j_0⏚, m_0⏚, \copyright c_1, \\ \\ j_0⏚, m_0⏚, \copyright c_2, \end{bmatrix}\text{-},$$

$$\Leftrightarrow\ ,i\mathrel{⌶}j, i\mathbin{☮}i_1, m\mathbin{☮}m_1, Rc(i_1;m_1), j\mathbin{☮}j_0, m\mathbin{☮}m_0, Rc(j_0;m_0),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, i\circledast i_1, m\circledast m_1, j\circledast j_0, m\circledast m_0, Rc(j_0;m_0), Rc(i_1;m_1),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, i\circledast i_1, j\circledast j_0, m\circledast m_0, m\circledast m_1, Rc(j_0;m_0), Rc(i_1;m_1),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, i\circledast i_1, j\circledast j_0, m\circledast m_0, m\circledast m_1, m_0\text{♂}m_1, Rc(j_0;m_0), Rc(i_1;m_1),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, i\circledast i_1, j\circledast j_0, m\circledast m_0, m\circledast m_1, m_0\text{♂}m_1, m_0\text{⌶}m_1, Rc(j_0;m_0), Rc(i_1;m_1),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, i\circledast i_1, i\text{♂}i_1, j\circledast j_0, m\circledast m_0, m\circledast m_1, m_0\text{⌶}m_1, Rc(j_0;m_0), Rc(i_1;m_1),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast i_1, i\text{♂}i_1, i\text{⌶}j, j\circledast j_0, m\circledast m_0, m\circledast m_1, m_0\text{⌶}m_1, Rc(j_0;m_0), Rc(i_1;m_1),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i\circledast i_1, i\text{♂}i_1, i_1\text{⌶}j, j\circledast j_0, m\circledast m_0, m\circledast m_1, m_0\text{⌶}m_1, Rc(j_0;m_0), Rc(i_1;m_1),$$

$$if(j_0{=}m_0)\left[\begin{array}{l}, j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_1, \\ j_0\text{☮}, m_0\text{☮}, i_1\text{☮}, m_1\text{☮}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_0, j \mathring{\bigcirc} j_0, i_1 \mathrel{\overline{\underline{\mid}}} j, m \circledast m_0, m \circledast m_1, m_0 \mathrel{\overline{\underline{\mid}}} m_1, Rc(j_0; m_0), Rc(i_1; m_1),$$

$$if(j_0 = m_0)\left[\begin{array}{l} , j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_0, j \mathring{\bigcirc} j_0, i_1 \mathrel{\overline{\underline{\mid}}} j_0, m \circledast m_0, m \circledast m_1, m_0 \mathrel{\overline{\underline{\mid}}} m_1, Rc(j_0; m_0), Rc(i_1; m_1),$$

$$if(j_0 = m_0)\left[\begin{array}{l} , j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_0, m \circledast m_0, m \circledast m_1, i_1 \mathrel{\overline{\underline{\mid}}} j_0, m_0 \mathrel{\overline{\underline{\mid}}} m_1, Rc(j_0; m_0), Rc(i_1; m_1),$$

$$if(j_0 = m_0)\left[\begin{array}{l} , j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_0, m \circledast m_0, m \circledast m_1, i_1 \mathrel{\overline{\underline{\mid}}} j_0, m_0 \mathrel{\overline{\underline{\mid}}} m_1, Rc(j_0; m_0), Rc(i_1; m_1),$$

$$if(i_1 = \varnothing)\left[\begin{array}{l} , \\ \\ , m_1 = \varnothing, \end{array}\right], if(j_0 = m_0)\left[\begin{array}{l} , j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_0, m \circledast m_0, m \circledast m_1, i_1 \mathrel{\overline{\underline{\mid}}} j_0, m_0 \mathrel{\overline{\underline{\mid}}} m_1, Rc(j_0; m_0), Rc(i_1; m_1),$$

$$i_1 \mathrel{\overline{\underline{\mid}}} j_0, m_0 \mathrel{\overline{\underline{\mid}}} m_1, if(i_1 = \varnothing)\left[\begin{array}{l} , \\ \\ , m_1 = \varnothing, \end{array}\right], if(j_0 = m_0)\left[\begin{array}{l} , j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_0, m \circledast m_0, m \circledast m_1, i_1 \mathrel{\overline{\underline{\mid}}} j_0, m_0 \mathrel{\overline{\underline{\mid}}} m_1, Rc(j_0; m_0), Rc(i_1; m_1),$$

$$i_1 \mathrel{\overline{\underline{\mid}}} j_0, m_0 \mathrel{\overline{\underline{\mid}}} m_1, if(i_1 = \varnothing)\left[\begin{array}{l} , \\ \\ , m_1 = \varnothing, \end{array}\right], if(i_1 = m_1)\left[\begin{array}{l} , j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \circledast i_1, j \circledast j_0, m \circledast m_0, m \circledast m_1, i_1 \mathrel{\overline{\underline{\mid}}} j_0, m_0 \mathrel{\overline{\underline{\mid}}} m_1, Rc(j_0; m_0), Rc(i_1; m_1),$$

$$if(i_1 = m_1)\left[\begin{array}{l} , j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2, \end{array}\right],$$

$$\Leftrightarrow\ , i \mathrel{\overline{\underline{\mid}}} j, i \circledast i_1, j \circledast j_0, m \circledast m_0, , Rc(j_0; m_0), Rc(i_1; m_1),$$

$$if(i_1{=}m_1)\left[\begin{array}{l}, j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, j\textcircled{$\curlywedge$}j_0, m\textcircled{$\curlywedge$}m_0, Rc(j_0;m_0), j_0\textcircled{$\downarrow$}, m_0\textcircled{$\downarrow$}, i\textcircled{$\curlywedge$}i_1, m\textcircled{$\curlywedge$}m_1, Rc(i_1;m_1),$$

$$if(i_1{=}m_1)\left[\begin{array}{l}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, i\textcircled{$\curlywedge$}i_1, m\textcircled{$\curlywedge$}m_1, Rc(i_1;m_1), if(i_1{=}m_1)\left[\begin{array}{l}, i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_1, \\ \\ , i_1\textcircled{$\downarrow$}, m_1\textcircled{$\downarrow$}, \copyright c_2,\end{array}\right],$$

$$\Leftrightarrow\ , i\text{⌶}j, if(i\text{⌶}m)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right],$$

Propositions with propositions:

$$, i\text{⌶}j, j\text{⌶}m, \ \Leftrightarrow\ , i\text{⌶}j, i\text{⌶}m,$$

$$, i\text{⌶}j, j!\text{⌶}m, \ \Leftrightarrow\ , i\text{⌶}j, i!\text{⌶}m,$$

21.2.13 Opposition

$$, i\text{⌶}j, i!\text{⌶}j, \ \Leftrightarrow\ , \otimes,$$

$$, i!\text{⌶}j, i\text{⌶}j, \ \Leftrightarrow\ , \otimes,$$

21.2.14 With identical node connectivity

$$,i \mathrel{=\!\!\!\mid} j, i \mathrel{\mathring{\bigcirc}^{\circ}} j, \quad \Leftrightarrow \quad , i \mathrel{\mathring{\bigcirc}} j,$$

proof:

$, i \mathrel{=\!\!\!\mid} j, i \mathrel{\mathring{\bigcirc}^{\circ}} j,$

$\Leftrightarrow \; , i \mathrel{\mathring{\bigcirc}^{\circ}} j, i \mathrel{=\!\!\!\mid} j,$

$\Leftrightarrow \; , i \mathrel{\mathring{\bigcirc}^{\circ}} j, i \owedge i_0, j \owedge j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \mathrel{\mathring{\bigcirc}^{\circ}} j, i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, i \mathrel{\mathring{\bigcirc}^{\circ}} j, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, i_0 \mathrel{\mathring{\bigcirc}^{\circ}} j, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, i_0 \mathrel{\mathring{\bigcirc}^{\circ}} j, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, i_0 \mathrel{\mathring{\bigcirc}^{\circ}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 \mathrel{\mathring{\bigcirc}^{\circ}} j_0, i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 \mathrel{\mathring{\bigcirc}} j_0, i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, i_0 \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, i_0 \mathrel{\mathring{\bigcirc}} j, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, i_0 \mathrel{\mathring{\bigcirc}} j, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, i \mathrel{\mathring{\bigcirc}} j, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \mathrel{\mathring{\bigcirc}} j, i \owedge i_0, i \mathrel{\mathring{\bigcirc}} i_0, j \owedge j_0, j \mathrel{\mathring{\bigcirc}} j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \mathrel{\mathring{\bigcirc}} j, i \owedge i_0, j \owedge j_0, Rc(i_0; j_0), i_0 = \varnothing, j_0 = \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow \; , i \mathrel{\mathring{\bigcirc}} j, i \mathrel{=\!\!\!\mid} j,$

$\Leftrightarrow \; , i \mathrel{\mathring{\bigcirc}} j,$

$$, i \mathrel{=\!\!\!\mid} j, i \mathrel{\mathring{\bigcirc}^{\circ}} j, \quad \Leftrightarrow \quad , i \mathrel{=\!\!\!\mid} j, i \mathrel{\mathring{\bigcirc}} j,$$

21.2.15 With recursive function

$$, i \mathrel{\text{⌶}} j, R(i), R(j), \;\Leftrightarrow\; , i \mathrel{\text{⌶}} j, Rc(i;j),$$

induction proof:

premise 1 :

$, i=\varnothing, i \mathrel{\text{⌶}} j, R(i), R(j),$

$\Leftrightarrow\; , i \mathrel{\text{⌶}} j, i=\varnothing, R(i), R(j),$

$\Leftrightarrow\; , i \mathrel{\text{⌶}} j, i=\varnothing, R(j),$

$\Leftrightarrow\; , i \mathrel{\text{⌶}} j, j=\varnothing, R(j),$

$\Leftrightarrow\; , i \mathrel{\text{⌶}} j, j=\varnothing,$

$\Leftrightarrow\; , i \mathrel{\text{⌶}} j, i=\varnothing,$

$\Leftrightarrow\; , i \mathrel{\text{⌶}} j, i=\varnothing, Rc(i;j),$

$\Leftrightarrow\; , i=\varnothing, i \mathrel{\text{⌶}} j, Rc(i;j),$

premise 2 :

$, \&SHi \rightarrow i, i \mathrel{\text{⌶}} j, R(i), R(j), \;\Leftrightarrow\; , \&SHi \rightarrow i, i \mathrel{\text{⌶}} j, Rc(i;j), \;\Rightarrow$

$, i \mathrel{!}= \varnothing, \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, R(i), R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, i \mathrel{!}= \varnothing, R(i), R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, i \mathrel{!}= \varnothing, i\oplus, R(i), R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, i \mathrel{!}= \varnothing, i \mathrel{!}= \varnothing, i\oplus, R(i), R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, j \mathrel{!}= \varnothing, i \mathrel{!}= \varnothing, i\oplus, R(i), R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, i \mathrel{!}= \varnothing, i\oplus, R(i), j \mathrel{!}= \varnothing, R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, i \mathrel{!}= \varnothing, i\oplus, R(i), j \mathrel{!}= \varnothing, j\oplus, R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{\text{⌶}} j, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, R(i), R(j),$

$\Leftrightarrow\; , \&SHi \bigcirc^{\circ} i, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, i \mathrel{\text{⌶}} j, R(i), R(j),$

$\Leftrightarrow\; , i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i\oplus, j\oplus, \&SHi \rightarrow i, i \mathrel{\text{⌶}} j, R(i), R(j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus, \ \& SHi \rightarrow i, i \mathrel{\text{⌶}} j, Rc(i; j),$

$\Leftrightarrow\ , \ \& SHi \mathring{\bigcirc} i, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus, i \mathrel{\text{⌶}} j, Rc(i; j),$

$\Leftrightarrow\ , \ \& SHi \mathring{\bigcirc} i, i \mathrel{\text{⌶}} j, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus, Rc(i; j),$

$\Leftrightarrow\ , \ \& SHi \mathring{\bigcirc} i, i \mathrel{\text{⌶}} j, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, Rc(i; j),$

$\Leftrightarrow\ , \ \& SHi \mathring{\bigcirc} i, i \mathrel{\text{⌶}} j, j \mathrel{!=} \varnothing, i \mathrel{!=} \varnothing, Rc(i; j),$

$\Leftrightarrow\ , \ \& SHi \mathring{\bigcirc} i, i \mathrel{\text{⌶}} j, i \mathrel{!=} \varnothing, i \mathrel{!=} \varnothing, Rc(i; j),$

$\Leftrightarrow\ , \ \& SHi \mathring{\bigcirc} i, i \mathrel{\text{⌶}} j, i \mathrel{!=} \varnothing, Rc(i; j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \ \& SHi \mathring{\bigcirc} i, i \mathrel{\text{⌶}} j, Rc(i; j),$

$conclusion:$

$, i \mathrel{\text{⌶}} j, R(i), R(j), \ \Leftrightarrow\ , i \mathrel{\text{⌶}} j, Rc(i; j),$

21.2.16 With release operator

$$, i \mathrel{\text{⌶}} j, Rc(j; m), i\text{⏚}, j\text{⏚}, \ \Leftrightarrow\ , i \mathrel{\text{⌶}} j, Rc(i; m), i\text{⏚}, j\text{⏚},$$

induction proof:

$premise\ 1:$

$, i = \varnothing, i \mathrel{\text{⌶}} j, Rc(j; m), i\text{⏚}, j\text{⏚},$

$\Leftrightarrow\ , i \mathrel{\text{⌶}} j, i = \varnothing, Rc(j; m), i\text{⏚}, j\text{⏚},$

$\Leftrightarrow\ , i \mathrel{\text{⌶}} j, j = \varnothing, Rc(j; m), i\text{⏚}, j\text{⏚},$

$\Leftrightarrow\ , i \mathrel{\text{⌶}} j, j = \varnothing, i\text{⏚}, j\text{⏚},$

$\Leftrightarrow\ , i \mathrel{\text{⌶}} j, i = \varnothing, i\text{⏚}, j\text{⏚},$

$\Leftrightarrow\ , i \mathrel{\text{⌶}} j, i = \varnothing, Rc(i; m), i\text{⏚}, j\text{⏚},$

$$\Leftrightarrow\ , i=\varnothing, i\mathrel{\vdash\!\dashv}j, Rc(i;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},$$

premise 2 :

$$, \&SHi\rightarrow i, i\mathrel{\vdash\!\dashv}j, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\ \Leftrightarrow\ , \&SHi\rightarrow i, i\mathrel{\vdash\!\dashv}j, Rc(i;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\ \Rightarrow$$

$$, i\,!\!=\varnothing, \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, i\,!\!=\varnothing, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, j\,!\!=\varnothing, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, j\,!\!=\varnothing, if(m=\varnothing)\left[\begin{array}{l}, \\ , j\oplus, m\oplus, Rc(j;m),\end{array}\right], i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, j\,!\!=\varnothing, if(m=\varnothing)\left[\begin{array}{l}, \\ , j\oplus, m\oplus, Rc(j;m),\end{array}\right], i\oplus, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, j\,!\!=\varnothing, if(m=\varnothing)\left[\begin{array}{l}, i\oplus, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}}, \\ , j\oplus, m\oplus, Rc(j;m), i\oplus, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, j\,!\!=\varnothing, if(m=\varnothing)\left[\begin{array}{l}, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}}, \\ , j\oplus, i\oplus, m\oplus, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, j\,!\!=\varnothing, j\,!\!=\varnothing, if(m=\varnothing)\left[\begin{array}{l}, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}}, \\ , j\oplus, i\oplus, m\oplus, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, i\,!\!=\varnothing, j\,!\!=\varnothing, if(m=\varnothing)\left[\begin{array}{l}, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}}, \\ , j\oplus, i\oplus, m\oplus, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\end{array}\right],$$

$$\Leftrightarrow\ , if(m=\varnothing)\left[\begin{array}{l}, \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}}, \\ , \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, i\,!\!=\varnothing, j\,!\!=\varnothing, j\oplus, i\oplus, m\oplus, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\end{array}\right],$$

$$\Leftrightarrow\ , if(m=\varnothing)\left[\begin{array}{l}, \&SHi\text{♂}i, i\mathrel{\vdash\!\dashv}j, i\,!\!=\varnothing, j\,!\!=\varnothing, i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}}, \\ , \&SHi\text{♂}i, i\,!\!=\varnothing, j\,!\!=\varnothing, j\oplus, i\oplus, m\oplus, i\mathrel{\vdash\!\dashv}j, Rc(j;m), i\text{\textcircled{$\downarrow$}}, j\text{\textcircled{$\downarrow$}},\end{array}\right],$$

$$\Leftrightarrow\ , if(m=\varnothing)\left[\begin{array}{l} ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, j\ !=\varnothing, i⏚, j⏚, \\ \\ , i\ !=\varnothing, j\ !=\varnothing, j\oplus, i\oplus, m\oplus,\ \&SHi\rightarrow i, i\underline{\top}j, Rc(j;m), i⏚, j⏚, \end{array}\right],$$

$$\Leftrightarrow\ , if(m=\varnothing)\left[\begin{array}{l} ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, j\ !=\varnothing, i⏚, j⏚, \\ \\ , i\ !=\varnothing, j\ !=\varnothing, j\oplus, i\oplus, m\oplus,\ \&SHi\rightarrow i, i\underline{\top}j, Rc(i;m), i⏚, j⏚, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, j\ !=\varnothing, if(m=\varnothing)\left[\begin{array}{l} , i⏚, j⏚, \\ \\ j\oplus, i\oplus, m\oplus, Rc(i;m), i⏚, j⏚, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, j\ !=\varnothing, if(m=\varnothing)\left[\begin{array}{l} , i⏚, j⏚, \\ \\ i\oplus, m\oplus, Rc(i;m), i⏚, j\oplus, j⏚, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, j\ !=\varnothing, if(m=\varnothing)\left[\begin{array}{l} , i⏚, j⏚, \\ \\ i\oplus, m\oplus, Rc(i;m), i⏚, j⏚, \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, j\ !=\varnothing, if(m=\varnothing)\left[\begin{array}{l} , \\ \\ i\oplus, m\oplus, Rc(i;m), \end{array}\right], i⏚, j⏚,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, j\ !=\varnothing, i\ !=\varnothing, if(m=\varnothing)\left[\begin{array}{l} , \\ \\ i\oplus, m\oplus, Rc(i;m), \end{array}\right], i⏚, j⏚,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, i\ !=\varnothing, Rc(i;m), i⏚, j⏚,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, i\ !=\varnothing, Rc(i;m), i⏚, j⏚,$$

$$\Leftrightarrow\ , i\ !=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\underline{\top}j, Rc(i;m), i⏚, j⏚,$$

conclusion :

$$, i\underline{\top}j, Rc(j;m), i⏚, j⏚,\ \Leftrightarrow\ , i\underline{\top}j, Rc(i;m), i⏚, j⏚,$$

22 Rules of Number More Than and Less Than Relationship

22.1 Definition of Number more than

$$,if(i\gtrdot j)\left[\begin{array}{l},\\ \\,\end{array}\right. \Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),if(i_0=\varnothing)\left[\begin{array}{l},i_0\text{☮}n,\\ \\,\circledcirc n,\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\text{⏚},i_0\text{⏚},j_0\text{⏚},\\ \\,n\text{⏚},i_0\text{⏚},j_0\text{⏚},\end{array}\right.$$

$$,i\gtrdot j,\ \Leftrightarrow\ ,if(i\gtrdot j)\left[\begin{array}{l},\\ \\,\otimes,\end{array}\right],$$

$$,i!\gtrdot j,\ \Leftrightarrow\ ,if(i\gtrdot j)\left[\begin{array}{l},\otimes,\\ \\,\end{array}\right],$$

22.2 Definition of Number less than

$$,if(i\lessdot j)\left[\begin{array}{l},\\ \\,\end{array}\right. \Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),if(j_0=\varnothing)\left[\begin{array}{l},j_0\text{☮}n,\\ \\,\circledcirc n,\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\text{⏚},i_0\text{⏚},j_0\text{⏚},\\ \\,n\text{⏚},i_0\text{⏚},j_0\text{⏚},\end{array}\right.$$

$$,i\lessdot j,\ \Leftrightarrow\ ,if(i\lessdot j)\left[\begin{array}{l},\\ \\,\otimes,\end{array}\right],$$

$$,i!\lessdot j,\ \Leftrightarrow\ ,if(i\lessdot j)\left[\begin{array}{l},\otimes,\\ \\,\end{array}\right],$$

22.3 Theorems of Relationship of more than and less than

$$,if(i>j)\left[\begin{array}{l} , \\ \\ , \end{array}\right. \Leftrightarrow \; ,if(i<j)\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

$$,i>j, \;\Leftrightarrow\; ,j<i,$$

$$,i!>j, \;\Leftrightarrow\; ,j!<i,$$

22.4 Theorems of Number more than Relationship

22.4.1 Number more than branch function to definition

$$,if(i>j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \;\Leftrightarrow\; ,i\owedge i_0,j\owedge j_0,Rc(i_0;j_0),if(i_0=\varnothing)\left[\begin{array}{l} ,i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},\copyright c_2, \\ \\ ,i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},\copyright c_1, \end{array}\right],$$

22.4.2 Number more than propositions to definition

$$,i>j, \;\Leftrightarrow\; ,i\owedge i_0,j\owedge j_0,Rc(i_0;j_0),i_0\,!=\varnothing,i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$,i!>j, \;\Leftrightarrow\; ,i\owedge i_0,j\owedge j_0,Rc(i_0;j_0),i_0=\varnothing,i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

22.4.3 Branch function to propositions

$$,if(i>j)\left[\begin{array}{l} ,\copyright c, \\ \\ ,\otimes, \end{array}\right], \;\Leftrightarrow\; ,i>j,\copyright c,$$

$$,if(i>j)\left[\begin{array}{l} ,\otimes, \\ \\ ,\copyright c, \end{array}\right], \;\Leftrightarrow\; ,i!>j,\copyright c,$$

22.4.4 Empty branch function

$$, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , \left[\begin{array}{l} , i \succ j, \\ \\ , i ! \succ j, \end{array} \right.$$

22.4.5 Unity

$$, \;\Leftrightarrow\; , if(i \succ j) \left[\begin{array}{l} , \\ , \end{array} \right],$$

$$, i \succ j, \otimes, \;\Leftrightarrow\; , \otimes,$$

$$, i ! \succ j, \otimes, \;\Leftrightarrow\; , \otimes,$$

22.4.6 Swap

Branch function and operator:

$$, \circledcirc m, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , if(i \succ j) \left[\begin{array}{l} , \circledcirc m, \\ \\ , \circledcirc m, \end{array} \right.$$

$$, \odot m, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , if(i \succ j) \left[\begin{array}{l} , \odot m, \\ \\ , \odot m, \end{array} \right.$$

$$, m \circledvee n, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , if(i \succ j) \left[\begin{array}{l} , m \circledvee n, \\ \\ , m \circledvee n, \end{array} \right.$$

$$, m \circledvee_{\circ} n, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , if(i \succ j) \left[\begin{array}{l} , m \circledvee_{\circ} n, \\ \\ , m \circledvee_{\circ} n, \end{array} \right.$$

$$, m \circledvee_{\angle} n, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , if(i \succ j) \left[\begin{array}{l} , m \circledvee_{\angle} n, \\ \\ , m \circledvee_{\angle} n, \end{array} \right.$$

$$, m \circledDownarrow, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , if(i \succ j) \left[\begin{array}{l} , m \circledDownarrow, \\ \\ , m \circledDownarrow, \end{array} \right.$$

$$, m \oplus, if(i \succ j) \left[\begin{array}{l} , \\ \\ , \end{array} \right. \Leftrightarrow \; , if(i \succ j) \left[\begin{array}{l} , m \oplus, \\ \\ , m \oplus, \end{array} \right.$$

$$,m\ominus,if(i{>}j)\left[\begin{array}{l},\\ \\ ,\end{array}\right. \Leftrightarrow\ ,if(i{>}j)\left[\begin{array}{l},m\ominus,\\ \\ ,m\ominus,\end{array}\right.$$

Branch function and Branch function:

$$,if(i{>}j)\left[\begin{array}{l},if(m{>}n)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],\\ ,if(m{>}n)\left[\begin{array}{l},©c_3,\\ \\ ,©c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m{>}n)\left[\begin{array}{l},if(i{>}j)\left[\begin{array}{l},©c_1,\\ \\ ,©c_3,\end{array}\right],\\ ,if(i{>}j)\left[\begin{array}{l},©c_2,\\ \\ ,©c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{>}j)\left[\begin{array}{l},if(m\mathrel{\text{⌶}}n)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],\\ ,if(m\mathrel{\text{⌶}}n)\left[\begin{array}{l},©c_3,\\ \\ ,©c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m\mathrel{\text{⌶}}n)\left[\begin{array}{l},if(i{>}j)\left[\begin{array}{l},©c_1,\\ \\ ,©c_3,\end{array}\right],\\ ,if(i{>}j)\left[\begin{array}{l},©c_2,\\ \\ ,©c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{>}j)\left[\begin{array}{l},if(m\circledperp n)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],\\ ,if(m\circledperp n)\left[\begin{array}{l},©c_3,\\ \\ ,©c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m\circledperp n)\left[\begin{array}{l},if(i{>}j)\left[\begin{array}{l},©c_1,\\ \\ ,©c_3,\end{array}\right],\\ ,if(i{>}j)\left[\begin{array}{l},©c_2,\\ \\ ,©c_4,\end{array}\right],\end{array}\right],$$

$$,if(i{>}j)\left[\begin{array}{l},if(m{\rightarrow}n)\left[\begin{array}{l},©c_1,\\ \\ ,©c_2,\end{array}\right],\\ ,if(m{\rightarrow}n)\left[\begin{array}{l},©c_3,\\ \\ ,©c_4,\end{array}\right],\end{array}\right], \Leftrightarrow\ ,if(m{\rightarrow}n)\left[\begin{array}{l},if(i{>}j)\left[\begin{array}{l},©c_1,\\ \\ ,©c_3,\end{array}\right],\\ ,if(i{>}j)\left[\begin{array}{l},©c_2,\\ \\ ,©c_4,\end{array}\right],\end{array}\right],$$

$$,if(i \succ j)\left[\begin{array}{l} ,if(m\overset{\circ}{\bigcirc}{}^{\circ}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\overset{\circ}{\bigcirc}{}^{\circ}n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(m\overset{\circ}{\bigcirc}{}^{\circ}n)\left[\begin{array}{l} ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i \succ j)\left[\begin{array}{l} ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(m\overset{\circ}{\bigcirc}n)\left[\begin{array}{l} ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i \succ j)\left[\begin{array}{l} ,if(m=n)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=n)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(m=n)\left[\begin{array}{l} ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

$$,if(i \succ j)\left[\begin{array}{l} ,if(m=\varnothing)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right], \\ ,if(m=\varnothing)\left[\begin{array}{l} ,\copyright c_3, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right], \iff ,if(m=\varnothing)\left[\begin{array}{l} ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_1, \\ \\ ,\copyright c_3, \end{array}\right], \\ ,if(i \succ j)\left[\begin{array}{l} ,\copyright c_2, \\ \\ ,\copyright c_4, \end{array}\right], \end{array}\right],$$

Branch function and propositions:

$$,m\succ n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m\succ n, \copyright c_1,\\ \\ , m\succ n, \copyright c_2,\end{array}\right],$$

$$,m!\succ n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m!\succ n, \copyright c_1,\\ \\ , m!\succ n, \copyright c_2,\end{array}\right],$$

$$,m\mathrel{\text{⌶}}n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m\mathrel{\text{⌶}}n, \copyright c_1,\\ \\ , m\mathrel{\text{⌶}}n, \copyright c_2,\end{array}\right],$$

$$,m!\mathrel{\text{⌶}}n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m!\mathrel{\text{⌶}}n, \copyright c_1,\\ \\ , m!\mathrel{\text{⌶}}n, \copyright c_2,\end{array}\right],$$

$$,m\mathrel{\text{⦹}}n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m\mathrel{\text{⦹}}n, \copyright c_1,\\ \\ , m\mathrel{\text{⦹}}n, \copyright c_2,\end{array}\right],$$

$$,m!\mathrel{\text{⦹}}n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m!\mathrel{\text{⦹}}n, \copyright c_1,\\ \\ , m!\mathrel{\text{⦹}}n, \copyright c_2,\end{array}\right],$$

$$,m\rightarrow n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m\rightarrow n, \copyright c_1,\\ \\ , m\rightarrow n, \copyright c_2,\end{array}\right],$$

$$,m!\rightarrow n, if(i\succ j)\left[\begin{array}{l}, \copyright c_1,\\ \\ , \copyright c_2,\end{array}\right], \Leftrightarrow , if(i\succ j)\left[\begin{array}{l}, m!\rightarrow n, \copyright c_1,\\ \\ , m!\rightarrow n, \copyright c_2,\end{array}\right],$$

$$, m \overset{\circ\circ}{\bigcirc} n, if(i \succ j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right] -, \quad \Leftrightarrow \quad , if(i \succ j) - \left[\begin{array}{l} , m \overset{\circ\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m \overset{\circ\circ}{\bigcirc} n, \copyright c_2, \end{array} \right] -,$$

$$, m! \overset{\circ\circ}{\bigcirc} n, if(i \succ j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right] -, \quad \Leftrightarrow \quad , if(i \succ j) - \left[\begin{array}{l} , m! \overset{\circ\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m! \overset{\circ\circ}{\bigcirc} n, \copyright c_2, \end{array} \right] -,$$

$$, m \overset{\circ}{\bigcirc} n, if(i \succ j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right] -, \quad \Leftrightarrow \quad , if(i \succ j) - \left[\begin{array}{l} , m \overset{\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m \overset{\circ}{\bigcirc} n, \copyright c_2, \end{array} \right] -,$$

$$, m! \overset{\circ}{\bigcirc} n, if(i \succ j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right] -, \quad \Leftrightarrow \quad , if(i \succ j) - \left[\begin{array}{l} , m! \overset{\circ}{\bigcirc} n, \copyright c_1, \\ \\ , m! \overset{\circ}{\bigcirc} n, \copyright c_2, \end{array} \right] -,$$

$$, m = n, if(i \succ j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right] -, \quad \Leftrightarrow \quad , if(i \succ j) - \left[\begin{array}{l} , m = n, \copyright c_1, \\ \\ , m = n, \copyright c_2, \end{array} \right] -,$$

$$, m \mathrel{!=} n, if(i \succ j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right] -, \quad \Leftrightarrow \quad , if(i \succ j) - \left[\begin{array}{l} , m \mathrel{!=} n, \copyright c_1, \\ \\ , m \mathrel{!=} n, \copyright c_2, \end{array} \right] -,$$

$$, m = \varnothing, if(i \succ j) - \left[\begin{array}{l} , \copyright c_1, \\ \\ , \copyright c_2, \end{array} \right] -, \quad \Leftrightarrow \quad , if(i \succ j) - \left[\begin{array}{l} , m = \varnothing, \copyright c_1, \\ \\ , m = \varnothing, \copyright c_2, \end{array} \right] -,$$

$$,m\,!{=}\varnothing,if(i{\succ}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{\succ}j)\left[\begin{array}{l},m\,!{=}\varnothing,\text{©}c_1,\\ \\ ,m\,!{=}\varnothing,\text{©}c_2,\end{array}\right],$$

$$,m{\succ}n,if(i\mathrel{\text{⌶}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathrel{\text{⌶}}j)\left[\begin{array}{l},m{\succ}n,\text{©}c_1,\\ \\ ,m{\succ}n,\text{©}c_2,\end{array}\right],$$

$$,m!{\succ}n,if(i\mathrel{\text{⌶}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathrel{\text{⌶}}j)\left[\begin{array}{l},m!{\succ}n,\text{©}c_1,\\ \\ ,m!{\succ}n,\text{©}c_2,\end{array}\right],$$

$$,m{\succ}n,if(i\mathrel{\text{⦹}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathrel{\text{⦹}}j)\left[\begin{array}{l},m{\succ}n,\text{©}c_1,\\ \\ ,m{\succ}n,\text{©}c_2,\end{array}\right],$$

$$,m!{\succ}n,if(i\mathrel{\text{⦹}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathrel{\text{⦹}}j)\left[\begin{array}{l},m!{\succ}n,\text{©}c_1,\\ \\ ,m!{\succ}n,\text{©}c_2,\end{array}\right],$$

$$,m{\succ}n,if(i\mathrel{\text{⧂}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathrel{\text{⧂}}j)\left[\begin{array}{l},m{\succ}n,\text{©}c_1,\\ \\ ,m{\succ}n,\text{©}c_2,\end{array}\right],$$

$$,m!{\succ}n,if(i\mathrel{\text{⧂}}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i\mathrel{\text{⧂}}j)\left[\begin{array}{l},m!{\succ}n,\text{©}c_1,\\ \\ ,m!{\succ}n,\text{©}c_2,\end{array}\right],$$

$$,m{\succ}n,if(i{\rightarrow}j)\left[\begin{array}{l},\text{©}c_1,\\ \\ ,\text{©}c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{\rightarrow}j)\left[\begin{array}{l},m{\succ}n,\text{©}c_1,\\ \\ ,m{\succ}n,\text{©}c_2,\end{array}\right],$$

$$, m!{\vartriangleright}n, if(i{\to}j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{\to}j)\left[\begin{array}{l}, m!{\vartriangleright}n, \copyright c_1, \\ \\ , m!{\vartriangleright}n, \copyright c_2, \end{array}\right],$$

$$, m{\vartriangleright}n, if(i{\bigcirc\!\!^{\circ}}j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{\bigcirc\!\!^{\circ}}j)\left[\begin{array}{l}, m{\vartriangleright}n, \copyright c_1, \\ \\ , m{\vartriangleright}n, \copyright c_2, \end{array}\right],$$

$$, m!{\vartriangleright}n, if(i{\bigcirc\!\!^{\circ}}j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{\bigcirc\!\!^{\circ}}j)\left[\begin{array}{l}, m!{\vartriangleright}n, \copyright c_1, \\ \\ , m!{\vartriangleright}n, \copyright c_2, \end{array}\right],$$

$$, m{\vartriangleright}n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{=}j)\left[\begin{array}{l}, m{\vartriangleright}n, \copyright c_1, \\ \\ , m{\vartriangleright}n, \copyright c_2, \end{array}\right],$$

$$, m!{\vartriangleright}n, if(i{=}j)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{=}j)\left[\begin{array}{l}, m!{\vartriangleright}n, \copyright c_1, \\ \\ , m!{\vartriangleright}n, \copyright c_2, \end{array}\right],$$

$$, m{\vartriangleright}n, if(i{=}\varnothing)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{=}\varnothing)\left[\begin{array}{l}, m{\vartriangleright}n, \copyright c_1, \\ \\ , m{\vartriangleright}n, \copyright c_2, \end{array}\right],$$

$$, m!{\vartriangleright}n, if(i{=}\varnothing)\left[\begin{array}{l}, \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \quad \Leftrightarrow \quad , if(i{=}\varnothing)\left[\begin{array}{l}, m!{\vartriangleright}n, \copyright c_1, \\ \\ , m!{\vartriangleright}n, \copyright c_2, \end{array}\right],$$

Branch function and recursive function:

$$,R(m),if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},R(m),\copyright c_1,\\ \\ ,R(m),\copyright c_2,\end{array}\right],$$

$$,R_{-}(m),if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},R_{-}(m),\copyright c_1,\\ \\ ,R_{-}(m),\copyright c_2,\end{array}\right],$$

$$,Rc(m;n),if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},Rc(m;n),\copyright c_1,\\ \\ ,Rc(m;n),\copyright c_2,\end{array}\right],$$

Branch function and flag object :

$$,\ \&SHi\text{♂}m,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},\ \&SHi\text{♂}m,\copyright c_1,\\ \\ ,\ \&SHi\text{♂}m,\copyright c_2,\end{array}\right],$$

$$,\ \&SHi\rightarrow m,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},\ \&SHi\rightarrow m,\copyright c_1,\\ \\ ,\ \&SHi\rightarrow m,\copyright c_2,\end{array}\right],$$

$$,\ \&SHj\text{♂}m,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},\ \&SHj\text{♂}m,\copyright c_1,\\ \\ ,\ \&SHj\text{♂}m,\copyright c_2,\end{array}\right],$$

$$,\ \&SHj\leftarrow m,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},\ \&SHj\leftarrow m,\copyright c_1,\\ \\ ,\ \&SHj\leftarrow m,\copyright c_2,\end{array}\right],$$

$$, \ \&SVi \mathring{\bigcirc} m, if(i \succ j) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \quad \Leftrightarrow \quad , if(i \succ j) \left[\begin{array}{l} , \ \&SVi \mathring{\bigcirc} m, ©c_1, \\ \\ , \ \&SVi \mathring{\bigcirc} m, ©c_2, \end{array} \right],$$

$$, \ \&SVi \obot m, if(i \succ j) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \quad \Leftrightarrow \quad , if(i \succ j) \left[\begin{array}{l} , \ \&SVi \obot m, ©c_1, \\ \\ , \ \&SVi \obot m, ©c_2, \end{array} \right],$$

Propositions and operator:

$$, i \succ j, \circledcirc m, \quad \Leftrightarrow \quad , \circledcirc m, i \succ j,$$
$$, i \succ j, \odot m, \quad \Leftrightarrow \quad , \odot m, i \succ j,$$

$$, i \succ j, m \varowedge n, \quad \Leftrightarrow \quad , m \varowedge n, i \succ j,$$
$$, i \succ j, m \varowedge_{\circ} n, \quad \Leftrightarrow \quad , m \varowedge_{\circ} n, i \succ j,$$
$$, i \succ j, m \varowedge_{\lrcorner} n, \quad \Leftrightarrow \quad , m \varowedge_{\lrcorner} n, i \succ j,$$

$$, i \succ j, m \circledbot, \quad \Leftrightarrow \quad , m \circledbot, i \succ j,$$
$$, i \succ j, m \oplus, \quad \Leftrightarrow \quad , m \oplus, i \succ j,$$
$$, i \succ j, m \ominus, \quad \Leftrightarrow \quad , m \ominus, i \succ j,$$

$$, i! \succ j, \circledcirc m, \quad \Leftrightarrow \quad , \circledcirc m, i! \succ j,$$
$$, i! \succ j, \odot m, \quad \Leftrightarrow \quad , \odot m, i! \succ j,$$

$$, i! \succ j, m \varowedge n, \quad \Leftrightarrow \quad , m \varowedge n, i! \succ j,$$
$$, i! \succ j, m \varowedge_{\circ} n, \quad \Leftrightarrow \quad , m \varowedge_{\circ} n, i! \succ j,$$

$$, i!{>}j, m\text{☮}n, \;\Leftrightarrow\; , m\text{☮}n, i!{>}j,$$

$$, i!{>}j, m\text{⏚}, \;\Leftrightarrow\; , m\text{⏚}, i!{>}j,$$

$$, i!{>}j, m\oplus, \;\Leftrightarrow\; , m\oplus, i!{>}j,$$

$$, i!{>}j, m\ominus, \;\Leftrightarrow\; , m\ominus, i!{>}j,$$

Propositions and Propositions:

$$, i{>}j, m{>}n, \;\Leftrightarrow\; , m{>}n, i{>}j,$$

$$, i{>}j, m!{>}n, \;\Leftrightarrow\; , m!{>}n, i{>}j,$$

$$, i!{>}j, m!{>}n, \;\Leftrightarrow\; , m!{>}n, i!{>}j,$$

$$, i{>}j, m{=}n, \;\Leftrightarrow\; , m{=}n, i{>}j,$$

$$, i{>}j, m!{=}n, \;\Leftrightarrow\; , m!{=}n, i{>}j,$$

$$, i!{>}j, m{=}n, \;\Leftrightarrow\; , m{=}n, i!{>}j,$$

$$, i!{>}j, m!{=}n, \;\Leftrightarrow\; , m!{=}n, i!{>}j,$$

$$, i{>}j, m\text{⦹}n, \;\Leftrightarrow\; , m\text{⦹}n, i{>}j,$$

$$, i{>}j, m!\text{⦹}n, \;\Leftrightarrow\; , m!\text{⦹}n, i{>}j,$$

$$, i!{>}j, m\text{⦹}n, \;\Leftrightarrow\; , m\text{⦹}n, i!{>}j,$$

$$, i!{>}j, m!\text{⦹}n, \;\Leftrightarrow\; , m!\text{⦹}n, i!{>}j,$$

$$, i{>}j, m{\rightarrow}n, \;\Leftrightarrow\; , m{\rightarrow}n, i{>}j,$$

$$, i{>}j, m!{\rightarrow}n, \;\Leftrightarrow\; , m!{\rightarrow}n, i{>}j,$$

$$, i!{>}j, m{\rightarrow}n, \;\Leftrightarrow\; , m{\rightarrow}n, i!{>}j,$$

$$, i!{>}j, m!{\rightarrow}n, \;\Leftrightarrow\; , m!{\rightarrow}n, i!{>}j,$$

$$, i{>}j, m\text{⚲}n, \;\Leftrightarrow\; , m\text{⚲}n, i{>}j,$$

$$, i{>}j, m!\text{⚲}n, \;\Leftrightarrow\; , m!\text{⚲}n, i{>}j,$$

$$, i!{>}j, m\text{⚲}n, \;\Leftrightarrow\; , m\text{⚲}n, i!{>}j,$$

$$, i!{>}j, m!\text{⚲}n, \;\Leftrightarrow\; , m!\text{⚲}n, i!{>}j,$$

$$, i \gtrdot j, m \mathbin{\bigcirc^{\!\circ}} n, \iff , m \mathbin{\bigcirc^{\!\circ}} n, i \gtrdot j,$$

$$, i \gtrdot j, m ! \mathbin{\bigcirc^{\!\circ}} n, \iff , m ! \mathbin{\bigcirc^{\!\circ}} n, i \gtrdot j,$$

$$, i ! \gtrdot j, m \mathbin{\bigcirc^{\!\circ}} n, \iff , m \mathbin{\bigcirc^{\!\circ}} n, i ! \gtrdot j,$$

$$, i ! \gtrdot j, m ! \mathbin{\bigcirc^{\!\circ}} n, \iff , m ! \mathbin{\bigcirc^{\!\circ}} n, i ! \gtrdot j,$$

$$, i \gtrdot j, m = n, \iff , m = n, i \gtrdot j,$$

$$, i \gtrdot j, m \,!{=}\, n, \iff , m \,!{=}\, n, i \gtrdot j,$$

$$, i ! \gtrdot j, m = n, \iff , m = n, i ! \gtrdot j,$$

$$, i ! \gtrdot j, m \,!{=}\, n, \iff , m \,!{=}\, n, i ! \gtrdot j,$$

$$, i \gtrdot j, m = \varnothing, \iff , m = \varnothing, i \gtrdot j,$$

$$, i \gtrdot j, m \,!{=}\, \varnothing, \iff , m \,!{=}\, \varnothing, i \gtrdot j,$$

$$, i ! \gtrdot j, m = \varnothing, \iff , m = \varnothing, i ! \gtrdot j,$$

$$, i ! \gtrdot j, m \,!{=}\, \varnothing, \iff , m \,!{=}\, \varnothing, i ! \gtrdot j,$$

Propositions and recursive function:

$$, i \gtrdot j, R(m), \iff , R(m), i \gtrdot j,$$

$$, i ! \gtrdot j, R(m), \iff , R(m), i ! \gtrdot j,$$

$$, i \gtrdot j, R_{\_}(m), \iff , R_{\_}(m), i \gtrdot j,$$

$$, i ! \gtrdot j, R_{\_}(m), \iff , R_{\_}(m), i ! \gtrdot j,$$

$$, i \gtrdot j, Rc(m; n), \iff , Rc(m; n), i \gtrdot j,$$

$$, i ! \gtrdot j, Rc(m; n), \iff , Rc(m; n), i ! \gtrdot j,$$

Propositions and flag object:

$$
\begin{aligned}
, i\rhd j,\ \&SHi\,\mathring{\bigcirc} m, &\iff , \&SHi\,\mathring{\bigcirc} m, i\rhd j,\\
, i\rhd j,\ \&SHi \rightarrow m, &\iff , \&SHi \rightarrow m, i\rhd j,\\
, i!\rhd j,\ \&SHi\,\mathring{\bigcirc} m, &\iff , \&SHi\,\mathring{\bigcirc} m, i!\rhd j,\\
, i!\rhd j,\ \&SHi \rightarrow m, &\iff , \&SHi \rightarrow m, i!\rhd j,
\end{aligned}
$$

$$
\begin{aligned}
, i\rhd j,\ \&SHj\,\mathring{\bigcirc} m, &\iff , \&SHj\,\mathring{\bigcirc} m, i\rhd j,\\
, i\rhd j,\ \&SHj \leftarrow m, &\iff , \&SHj \leftarrow m, i\rhd j,\\
, i!\rhd j,\ \&SHj\,\mathring{\bigcirc} m, &\iff , \&SHj\,\mathring{\bigcirc} m, i!\rhd j,\\
, i!\rhd j,\ \&SHj \leftarrow m, &\iff , \&SHj \leftarrow m, i!\rhd j,
\end{aligned}
$$

$$
\begin{aligned}
, i\rhd j,\ \&SVi\,\mathring{\bigcirc} m, &\iff , \&SVi\,\mathring{\bigcirc} m, i\rhd j,\\
, i\rhd j,\ \&SVi\,\obot m, &\iff , \&SVi\,\obot m, i\rhd j,\\
, i!\rhd j,\ \&SVi\,\mathring{\bigcirc} m, &\iff , \&SVi\,\mathring{\bigcirc} m, i!\rhd j,\\
, i!\rhd j,\ \&SVi\,\obot m, &\iff , \&SVi\,\obot m, i!\rhd j,
\end{aligned}
$$

Propositions to Propositions with branch function

(Skip......)

22.4.7 Swap of the same operand

(skip......)

22.4.8 Transitivity

Branch function with branch function:

$$,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_3,\end{array}\right],\\ \\ ,\copyright c_2,\end{array}\right],$$

$$,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,if(i{>}j)\left[\begin{array}{l},\copyright c_3,\\ \\ ,\copyright c_2,\end{array}\right],\end{array}\right],$$

Branch function with propositions:

$$,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},i{>}j,\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right],$$

$$,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,\copyright c_2,\end{array}\right], \;\Leftrightarrow\; ,if(i{>}j)\left[\begin{array}{l},\copyright c_1,\\ \\ ,i!{>}j,\copyright c_2,\end{array}\right],$$

Propositions with branch function:

$$, i \succ j, if(i \succ j) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \ \Leftrightarrow \ , i \succ j, ©c_1,$$

$$, i ! \succ j, if(i \succ j) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \ \Leftrightarrow \ , i ! \succ j, ©c_2,$$

Propositions with propositions:

$$, i \succ j, \ \Leftrightarrow \ , i \succ j, i \succ j,$$

$$, i ! \succ j, \ \Leftrightarrow \ , i ! \succ j, i ! \succ j,$$

22.4.9 Substitution

Identical node propositions with branch function:

$$, i \mathring{\bigcirc} j, if(j \succ m) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \ \Leftrightarrow \ , i \mathring{\bigcirc} j, if(i \succ m) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right],$$

$$, i \mathring{\bigcirc} j, if(m \succ j) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right], \ \Leftrightarrow \ , i \mathring{\bigcirc} j, if(m \succ i) \left[\begin{array}{l} , ©c_1, \\ \\ , ©c_2, \end{array} \right],$$

Identical node propositions with propositions:

$$, i \overset{\circ}{\bigcirc} j, j \mathrel{>\!\!\!\mid} m, \;\Leftrightarrow\; , i \overset{\circ}{\bigcirc} j, i \mathrel{>\!\!\!\mid} m,$$

$$, i \overset{\circ}{\bigcirc} j, m \mathrel{>\!\!\!\mid} j, \;\Leftrightarrow\; , i \overset{\circ}{\bigcirc} j, m \mathrel{>\!\!\!\mid} i,$$

$$, i \overset{\circ}{\bigcirc} j, j ! \mathrel{>\!\!\!\mid} m, \;\Leftrightarrow\; , i \overset{\circ}{\bigcirc} j, i ! \mathrel{>\!\!\!\mid} m,$$

$$, i \overset{\circ}{\bigcirc} j, m ! \mathrel{>\!\!\!\mid} j, \;\Leftrightarrow\; , i \overset{\circ}{\bigcirc} j, m ! \mathrel{>\!\!\!\mid} j,$$

Propositions with branch function:

$$, i \mathrel{\overline{\underline{\mid}}} j, if(j \mathrel{>\!\!\!\mid} m) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , i \mathrel{\overline{\underline{\mid}}} j, if(i \mathrel{>\!\!\!\mid} m) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

$$, i \mathrel{\overline{\underline{\mid}}} j, if(m \mathrel{>\!\!\!\mid} j) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right], \;\Leftrightarrow\; , i \mathrel{\overline{\underline{\mid}}} j, if(m \mathrel{>\!\!\!\mid} i) \left[\begin{array}{c} , \copyright c_1, \\ \\ , \copyright c_2, \end{array}\right],$$

Propositions with propositions:

$$, i \mathrel{\overline{\underline{\mid}}} j, j \mathrel{>\!\!\!\mid} m, \;\Leftrightarrow\; , i \mathrel{\overline{\underline{\mid}}} j, i \mathrel{>\!\!\!\mid} m,$$

$$, i \mathrel{\overline{\underline{\mid}}} j, m \mathrel{>\!\!\!\mid} j, \;\Leftrightarrow\; , i \mathrel{\overline{\underline{\mid}}} j, m \mathrel{>\!\!\!\mid} i,$$

$$, i \mathrel{\overline{\underline{\mid}}} j, j ! \mathrel{>\!\!\!\mid} m, \;\Leftrightarrow\; , i \mathrel{\overline{\underline{\mid}}} j, i ! \mathrel{>\!\!\!\mid} m,$$

$$, i \mathrel{\overline{\underline{\mid}}} j, m ! \mathrel{>\!\!\!\mid} j, \;\Leftrightarrow\; , i \mathrel{\overline{\underline{\mid}}} j, m ! \mathrel{>\!\!\!\mid} i,$$

22.4.10 Opposition

$$, i \triangleright j, i ! \triangleright j, \;\Leftrightarrow\; , \otimes,$$

$$, i ! \triangleright j, i \triangleright j, \;\Leftrightarrow\; , \otimes,$$

22.4.11 With identical node propositions

$$, i \triangleright j, \;\Leftrightarrow\; \sim, i ! \overset{\circ}{\bigcirc} j,$$

proof:

$, i \triangleright j,$

$\Leftrightarrow\ , if(i \overset{\circ}{\bigcirc} j) \left[\begin{array}{l} , \\ , \end{array}\right], i \triangleright j,$

$\Leftrightarrow\ , if(i \overset{\circ}{\bigcirc} j) \left[\begin{array}{l} , i \triangleright j, \\ , i \triangleright j, \end{array}\right],$

$\Leftrightarrow\ , if(i \overset{\circ}{\bigcirc} j) \left[\begin{array}{l} , i \varowedge i_0, j \varowedge j_0, Rc(i_0; j_0), i_0 \mathrel{!=} \varnothing, i_0 \textcircled{\downarrow}, j_0 \textcircled{\downarrow}, \\ , i \triangleright j, \end{array}\right],$

$\Leftrightarrow\ , if(i \overset{\circ}{\bigcirc} j) \left[\begin{array}{l} , i \varowedge i_0, j \varowedge j_0, Rc(i_0; j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , \\ , j_0 = \varnothing, \end{array}\right], i_0 \mathrel{!=} \varnothing, i_0 \textcircled{\downarrow}, j_0 \textcircled{\downarrow}, \\ , i \triangleright j, \end{array}\right],$

$\Leftrightarrow\ , if(i \overset{\circ}{\bigcirc} j) \left[\begin{array}{l} , i \varowedge i_0, j \varowedge j_0, Rc(i_0; j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 = \varnothing, \\ , j_0 = \varnothing, \end{array}\right], i_0 \mathrel{!=} \varnothing, i_0 \textcircled{\downarrow}, j_0 \textcircled{\downarrow}, \\ , i \triangleright j, \end{array}\right],$

$\Leftrightarrow\ , if(i \overset{\circ}{\bigcirc} j) \left[\begin{array}{l} , i \varowedge i_0, j \varowedge j_0, Rc(i_0; j_0), if(i_0 = \varnothing) \left[\begin{array}{l} , i_0 = \varnothing, i_0 \mathrel{!=} \varnothing, \\ , i_0 \mathrel{!=} \varnothing, j_0 = \varnothing, \end{array}\right], i_0 \textcircled{\downarrow}, j_0 \textcircled{\downarrow}, \\ , i \triangleright j, \end{array}\right],$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\circledast i_0,j\circledast j_0,Rc(i_0;j_0),if(i_0=\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ j_0=\varnothing, \end{array}\right],i_0\circleddash,j_0\circleddash, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\circledast i_0,j\circledast j_0,Rc(i_0;j_0),i_0\,!\!=\varnothing,j_0=\varnothing,i_0\circleddash,j_0\circleddash, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\mathring{\bigcirc}j,i\circledast i_0,j\circledast j_0,Rc(i_0;j_0),i_0\,!\!=\varnothing,j_0=\varnothing,i_0\circleddash,j_0\circleddash, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\mathring{\bigcirc}j,i\circledast i_0,j\circledast j_0,i_0\mathring{\bigcirc}j_0,Rc(i_0;j_0),i_0\,!\!=\varnothing,j_0=\varnothing,i_0\circleddash,j_0\circleddash, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\mathring{\bigcirc}j,i\circledast i_0,j\circledast j_0,Rc(i_0;j_0),i_0\mathring{\bigcirc}j_0,i_0\,!\!=\varnothing,j_0=\varnothing,i_0\circleddash,j_0\circleddash, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\mathring{\bigcirc}j,i\circledast i_0,j\circledast j_0,Rc(i_0;j_0),i_0\mathring{\bigcirc}j_0,j_0\,!\!=\varnothing,j_0=\varnothing,i_0\circleddash,j_0\circleddash, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,i\mathring{\bigcirc}j,i\circledast i_0,j\circledast j_0,Rc(i_0;j_0),i_0\mathring{\bigcirc}j_0,\otimes,i_0\circleddash,j_0\circleddash, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}j)\left[\begin{array}{l} ,\otimes, \\ \\ ,i\gtrdot j, \end{array}\right],$$

$$\Leftrightarrow\ ,i!\mathring{\bigcirc}j,i\gtrdot j,$$

$$\Leftrightarrow\ ,i\gtrdot j,i!\mathring{\bigcirc}j,$$

22.4.12 With node null propositions

$$,i{\succ}j,\ \Leftrightarrow\ ,\sim,i\,!{=}\varnothing,$$

proof:

$,i{\succ}j,$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right],i{\succ}j,$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i{\succ}j, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),i_0\,!{=}\varnothing,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i=\varnothing,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),i_0\,!{=}\varnothing,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}i_0,i\mathring{\bigcirc}i_0,i=\varnothing,j\text{☮}j_0,Rc(i_0;j_0),i_0\,!{=}\varnothing,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}i_0,i\mathring{\bigcirc}i_0,i_0=\varnothing,j\text{☮}j_0,Rc(i_0;j_0),i_0\,!{=}\varnothing,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}i_0,j\text{☮}j_0,i_0=\varnothing,Rc(i_0;j_0),i_0\,!{=}\varnothing,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}i_0,j\text{☮}j_0,i_0=\varnothing,i_0\,!{=}\varnothing,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,i\text{☮}i_0,j\text{☮}j_0,\otimes,i_0\text{⏚},j_0\text{⏚}, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,\otimes, \\ \\ ,i{\succ}j, \end{array}\right],$$

$$\Leftrightarrow\ ,i\,!{=}\varnothing,i{\succ}j,$$

$\Leftrightarrow\ , i \gtrdot j, i\,!\!= \varnothing,$

$$, i\,!\!= \varnothing, j = \varnothing, \ \Leftrightarrow\ \sim, i \gtrdot j,$$

proof:

$, i\,!\!= \varnothing, j = \varnothing,$
$\Leftrightarrow\ , i\,!\!= \varnothing, j = \varnothing, i\text{☮}i_0, i_0\text{⏚}, j\text{☮}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , i\,!\!= \varnothing, j = \varnothing, i\text{☮}i_0, j\text{☮}j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i\,!\!= \varnothing, j = \varnothing, i\text{☮}i_0, i\mathring{\bigcirc}i_0, j\text{☮}j_0, j\mathring{\bigcirc}j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\mathring{\bigcirc}i_0, i\,!\!= \varnothing, j\text{☮}j_0, j\mathring{\bigcirc}j_0, j = \varnothing, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, i\mathring{\bigcirc}i_0, i_0\,!\!= \varnothing, j\text{☮}j_0, j\mathring{\bigcirc}j_0, j_0 = \varnothing, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, j\text{☮}j_0, j\mathring{\bigcirc}j_0, j_0 = \varnothing, i_0\,!\!= \varnothing, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i\text{☮}i_0, j\text{☮}j_0, j\mathring{\bigcirc}j_0, j_0 = \varnothing, Rc(i_0; j_0), i_0\,!\!= \varnothing, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , j = \varnothing, i\text{☮}i_0, j\text{☮}j_0, Rc(i_0; j_0), i_0\,!\!= \varnothing, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , j = \varnothing, i \gtrdot j,$

$\Leftrightarrow\ , j = \varnothing, i \gtrdot j, i\,!\!= \varnothing,$

$\Leftrightarrow\ , i\,!\!= \varnothing, j = \varnothing, i \gtrdot j,$

22.4.13 With node continuity

$$, i\,!\!= \varnothing, j\,!\!= \varnothing, i\oplus, j\oplus, i \gtrdot j, \ \Leftrightarrow\ , i \gtrdot j, i\,!\!= \varnothing, j\,!\!= \varnothing, i\oplus, j\oplus,$$

proof:

$, i\,!\!= \varnothing, j\,!\!= \varnothing, i\oplus, j\oplus, i \gtrdot j,$
$\Leftrightarrow\ , i\,!\!= \varnothing, j\,!\!= \varnothing, i\oplus, j\oplus, i\text{☮}i_0, j\text{☮}j_0, Rc(i_0; j_0), i_0\,!\!= \varnothing, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i \owedge i_1, i_1 \obot, j \owedge j_1, j_1 \obot, i \oplus, j \oplus,$

$i \owedge i_0, j \owedge j_0, Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\ , i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i \owedge i_1, j \owedge j_1, Rc(i_1; j_1), i_1 \obot, j_1 \obot, i \oplus, j \oplus,$

$i \owedge i_0, j \owedge j_0, Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\ , i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, i \owedge i_1, j \owedge j_1, i \oplus, j \oplus, i \owedge i_0, j \owedge j_0, Rc(i_1; j_1),$

$Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot, i_1 \obot, j_1 \obot,$

$\Leftrightarrow\ , i \owedge i_1, i \mathrel{!}= \varnothing, j \owedge j_1, j \mathrel{!}= \varnothing, i \oplus, j \oplus, i \owedge i_0, j \owedge j_0, Rc(i_1; j_1),$

$Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot, i_1 \obot, j_1 \obot,$

$\Leftrightarrow\ , i \owedge i_1, i \mathring{\bigcirc} i_1, i \mathrel{!}= \varnothing, j \owedge j_1, j \mathring{\bigcirc} j_1, j \mathrel{!}= \varnothing, i \oplus, j \oplus, i \owedge i_0, j \owedge j_0,$

$Rc(i_1; j_1), Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot, i_1 \obot, j_1 \obot,$

$\Leftrightarrow\ , i \owedge i_1, i \mathring{\bigcirc} i_1, i_1 \mathrel{!}= \varnothing, j \owedge j_1, j \mathring{\bigcirc} j_1, j_1 \mathrel{!}= \varnothing, i \oplus, j \oplus, i \owedge i_0, j \owedge j_0,$

$Rc(i_1; j_1), Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot, i_1 \obot, j_1 \obot,$

$\Leftrightarrow\ , i \owedge i_1, i \mathring{\bigcirc} i_1, j \owedge j_1, j \mathring{\bigcirc} j_1, i \oplus, j \oplus, i \owedge i_0, j \owedge j_0,$

$i_1 \mathrel{!}= \varnothing, j_1 \mathrel{!}= \varnothing, Rc(i_1; j_1), Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot, i_1 \obot, j_1 \obot,$

$\Leftrightarrow\ , i \owedge i_1, i \mathring{\bigcirc} i_1, j \owedge j_1, j \mathring{\bigcirc} j_1, i \oplus, j \oplus, i \owedge i_0, j \owedge j_0,$

$i_1 \mathrel{!}= \varnothing, j_1 \mathrel{!}= \varnothing, i_1 \oplus, j_1 \oplus, Rc(i_1; j_1), Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot, i_1 \obot, j_1 \obot,$

$\Leftrightarrow\ , i \owedge i_1, i_1 \mathrel{!}= \varnothing, i \mathring{\bigcirc} i_1, j \owedge j_1, j_1 \mathrel{!}= \varnothing, j \mathring{\bigcirc} j_1, i \oplus, i_1 \oplus, j \oplus, j_1 \oplus, i \owedge i_0, j \owedge j_0,$

$Rc(i_1; j_1), Rc(i_0; j_0), i_0 \mathrel{!}= \varnothing, i_0 \obot, j_0 \obot, i_1 \obot, j_1 \obot,$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\mathring{\bigcirc}i_1, i\oplus, i_1\oplus, j\mathring{\bigcirc}j_1, j\oplus, j_1\oplus, i☮i_0, j☮j_0,$$

$$Rc(i_1;j_1), Rc(i_0;j_0), i_0 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, i\mathring{\bigcirc}i_1, j\oplus, j_1\oplus, j\mathring{\bigcirc}j_1, i☮i_0, j☮j_0,$$

$$Rc(i_1;j_1), Rc(i_0;j_0), i_0 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, i\mathring{\bigcirc}i_1, j\oplus, j_1\oplus, j\mathring{\bigcirc}j_1, i☮i_0, i\mathring{\bigcirc}i_0, j☮j_0,$$

$$j\mathring{\bigcirc}j_0, Rc(i_1;j_1), Rc(i_0;j_0), i_0 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i☮i_0, j☮j_0,$$

$$i\mathring{\bigcirc}i_1, i\mathring{\bigcirc}i_0, j\mathring{\bigcirc}j_1, j\mathring{\bigcirc}j_0, Rc(i_1;j_1), Rc(i_0;j_0), i_0 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i☮i_0, j☮j_0,$$

$$i\mathring{\bigcirc}i_1, i_1\mathring{\bigcirc}i_0, j\mathring{\bigcirc}j_1, j_1\mathring{\bigcirc}j_0, Rc(i_1;j_1), Rc(i_0;j_0), i_0 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i☮i_0, j☮j_0,$$

$$i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, Rc(i_1;j_1), Rc(i_0;j_0), i_0 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i☮i_0, j☮j_0,$$

$$i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, Rc(i_1;j_1), Rc(i_0;j_0), i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, i_0 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i☮i_0, j☮j_0,$$

$$i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, Rc(i_1;j_1), Rc(i_0;j_0), i_1\mathring{\bigcirc}i_0, j_1\mathring{\bigcirc}j_0, i_1 \mathrel{!=} \varnothing, i_0⏚, j_0⏚, i_1⏚, j_1⏚,$$

$$\Leftrightarrow\ , i☮i_1, i_1 \mathrel{!=} \varnothing, j☮j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\mathring{\bigcirc}i_1, j\mathring{\bigcirc}j_1, i☮i_0, j☮j_0,$$

$Rc(i_1;j_1), Rc(i_0;j_0), i_1 \mathrel{!=} \varnothing, i_0\obot, j_0\obot, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i_1 \mathrel{!=} \varnothing, j\owedge j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\mathring{\bigcirc} i_1, j\mathring{\bigcirc} j_1,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), i_0\obot, j_0\obot, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i_1 \mathrel{!=} \varnothing, j\owedge j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus, i\mathring{\bigcirc} i_1, j\mathring{\bigcirc} j_1,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i\mathring{\bigcirc} i_1, i_1 \mathrel{!=} \varnothing, j\owedge j_1, j\mathring{\bigcirc} j_1, j_1 \mathrel{!=} \varnothing, i\oplus, i_1\oplus, j\oplus, j_1\oplus,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i\mathring{\bigcirc} i_1, j\owedge j_1, j\mathring{\bigcirc} j_1, i\oplus, j\oplus, i_1 \mathrel{!=} \varnothing, i_1\oplus, j_1 \mathrel{!=} \varnothing, j_1\oplus,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i\mathring{\bigcirc} i_1, j\owedge j_1, j\mathring{\bigcirc} j_1, i\oplus, j\oplus, i_1 \mathrel{!=} \varnothing, j_1 \mathrel{!=} \varnothing,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i\mathring{\bigcirc} i_1, i_1 \mathrel{!=} \varnothing, j\owedge j_1, j\mathring{\bigcirc} j_1, j_1 \mathrel{!=} \varnothing, i\oplus, j\oplus,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i\mathring{\bigcirc} i_1, i \mathrel{!=} \varnothing, j\owedge j_1, j\mathring{\bigcirc} j_1, j \mathrel{!=} \varnothing, i\oplus, j\oplus,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, i \mathrel{!=} \varnothing, j\owedge j_1, j \mathrel{!=} \varnothing, i\oplus, j\oplus,$

$Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot,$

$\Leftrightarrow\ , i\owedge i_1, j\owedge j_1, Rc(i_1;j_1), i_1 \mathrel{!=} \varnothing, i_1\obot, j_1\obot, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus,$

$$\Leftrightarrow \ , i{>}j, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i\oplus, j\oplus,$$

$$, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i\oplus, j\oplus, i!{>}j, \ \Leftrightarrow \ , i!{>}j, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i\oplus, j\oplus,$$

$$, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i\oplus, j\oplus, if(i{>}j)\left[\begin{array}{c}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \ \Leftrightarrow \ , if(i{>}j)\left[\begin{array}{c}, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i\oplus, j\oplus, \copyright c_1, \\ \\ , i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i\oplus, j\oplus, \copyright c_2,\end{array}\right],$$

$$, i\ominus, j\ominus, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i{>}j, \ \Leftrightarrow \ , i{>}j, i\ominus, j\ominus, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing,$$

$$, i\ominus, j\ominus, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i!{>}j, \ \Leftrightarrow \ , i!{>}j, i\ominus, j\ominus, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing,$$

$$, i\ominus, j\ominus, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, if(i{>}j)\left[\begin{array}{c}, \copyright c_1, \\ \\ , \copyright c_2,\end{array}\right], \ \Leftrightarrow \ , if(i{>}j)\left[\begin{array}{c}, i\ominus, j\ominus, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, \copyright c_1, \\ \\ , i\ominus, j\ominus, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, \copyright c_2,\end{array}\right],$$

22.4.14 With self propositions

$$, i \succ j, j \succ k, \ \Leftrightarrow \ \sim, i \succ k,$$

induction proof:

premise 1 :

$, k = \varnothing, i \succ j, j \succ k,$

$\Leftrightarrow \ , k = \varnothing, i \succ j, j \succ k,$

$\Leftrightarrow \ , k = \varnothing, i \succ j, i \mathrel{!=} \varnothing, j \succ k,$

$\Leftrightarrow \ , i \succ j, j \succ k, i \mathrel{!=} \varnothing, k = \varnothing,$

$\Leftrightarrow \ , i \succ j, j \succ k, i \mathrel{!=} \varnothing, k = \varnothing, i \succ k,$

$\Leftrightarrow \ , k = \varnothing, i \succ j, j \succ k, i \succ k,$

premise 2 :

$, \&SHi \rightarrow k, i \succ j, j \succ k, \ \Leftrightarrow \ , \&SHi \rightarrow k, i \succ j, j \succ k, i \succ k, \ \Rightarrow$

$, k \mathrel{!=} \varnothing, \ \&SHi \overset{\circ}{\bigcirc} k, i \succ j, j \succ k,$

$\Leftrightarrow \ , k \mathrel{!=} \varnothing, \ \&SHi \overset{\circ}{\bigcirc} k, i \succ j, i \mathrel{!=} \varnothing, j \succ k, j \mathrel{!=} \varnothing,$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} k, i \succ j, j \succ k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} k, i \succ j, j \succ k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, i\ominus, k\oplus, k\ominus, j\oplus, j\ominus,$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} k, i \succ j, j \succ k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, k\oplus, j\oplus, i\ominus, k\ominus, j\ominus,$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, k\oplus, j\oplus, i \succ j, j \succ k, i\ominus, k\ominus, j\ominus,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, k\oplus, j\oplus, \ \&SHi \rightarrow k, i \succ j, j \succ k, i\ominus, k\ominus, j\ominus,$

$\Leftrightarrow \ , i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, k\oplus, j\oplus, \ \&SHi \rightarrow k, i \succ j, j \succ k, i \succ k, i\ominus, k\ominus, j\ominus,$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, k\oplus, j\oplus, i \succ j, j \succ k, i \succ k, i\ominus, k\ominus, j\ominus,$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} k, i \succ j, j \succ k, i \succ k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, k\oplus, j\oplus, i\ominus, k\ominus, j\ominus,$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} k, i \succ j, j \succ k, i \succ k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , k\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}k, i\gtrdot j, j\gtrdot k, i\gtrdot k,$

conclusion :

$, i\gtrdot j, j\gtrdot k,\ \ \Leftrightarrow\ \ , i\gtrdot j, j\gtrdot k, i\gtrdot k,$

22.4.15 With next and previous node operator

$$, i\vdash\!\dashv j, j\,!\!=\varnothing, j\oplus,\ \ \Leftrightarrow\ \ \sim, i\gtrdot j,$$

induction proof:
premise 1 :
$, i=\varnothing, i\vdash\!\dashv j, j\,!\!=\varnothing, j\oplus,$

$\Leftrightarrow\ , i\vdash\!\dashv j, i=\varnothing, j\,!\!=\varnothing, j\oplus,$

$\Leftrightarrow\ , i\vdash\!\dashv j, j=\varnothing, j\,!\!=\varnothing, j\oplus,$

$\Leftrightarrow\ , i\vdash\!\dashv j, \otimes, j\oplus,$

$\Leftrightarrow\ , i\vdash\!\dashv j, \otimes, j\oplus, i\gtrdot j,$

$\Leftrightarrow\ , i\vdash\!\dashv j, j=\varnothing, j\,!\!=\varnothing, j\oplus, i\gtrdot j,$

$\Leftrightarrow\ , i\vdash\!\dashv j, i=\varnothing, j\,!\!=\varnothing, j\oplus, i\gtrdot j,$

$\Leftrightarrow\ , i=\varnothing, i\vdash\!\dashv j, j\,!\!=\varnothing, j\oplus, i\gtrdot j,$

premise 2 :
$, \&SHi\rightarrow i, i\vdash\!\dashv j, j\,!\!=\varnothing, j\oplus,\ \ \Leftrightarrow\ \ , \&SHi\rightarrow i, i\vdash\!\dashv j, j\,!\!=\varnothing, j\oplus, i\gtrdot j,\ \ \Rightarrow$

$, i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\vdash\!\dashv j, j\,!\!=\varnothing, j\oplus,$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\vdash\!\dashv j, j\,!\!=\varnothing, j\oplus, if\,(j=\varnothing)\!-\!\left[\begin{array}{c} , \\ \\ , \end{array}\right]\!-,$

$$\Leftrightarrow\ ,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus, \begin{bmatrix} ,j=\varnothing, \\ \\ ,j\,!{=}\,\varnothing, \end{bmatrix},$$

$$\Leftrightarrow\ , \begin{bmatrix} ,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, j=\varnothing, \\ \\ ,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, j\,!{=}\,\varnothing, \end{bmatrix},$$

$$\Leftrightarrow\ , \begin{bmatrix} ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\,!{=}\,\varnothing,\, j=\varnothing, \\ \\ ,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, j\,!{=}\,\varnothing, \end{bmatrix},$$

$$\Leftrightarrow\ , \begin{bmatrix} ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\,!{=}\,\varnothing,\, j=\varnothing,\, i>j, \\ \\ ,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, j\,!{=}\,\varnothing, \end{bmatrix},$$

$$\Leftrightarrow\ , \begin{bmatrix} ,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, j=\varnothing,\, i>j, \\ \\ ,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, j\,!{=}\,\varnothing, \end{bmatrix},$$

$$\Leftrightarrow\ \ <1>$$

$$,i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, j\,!{=}\,\varnothing,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, i\ominus,\, j\oplus,\, j\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, j\oplus,\, i\ominus,\, j\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, j\oplus,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\ominus,\, j\ominus,$$

$$\Leftrightarrow\ ,i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, j\oplus,\ \&SHi\rightarrow i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\ominus,\, j\ominus,$$

$$\Leftrightarrow\ ,i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, j\oplus,\ \&SHi\rightarrow i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i>j,\, i\ominus,\, j\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, j\oplus,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i>j,\, i\ominus,\, j\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, j\oplus,\, i>j,\, i\ominus,\, j\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i>j,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,\, i\oplus,\, j\oplus,\, i\ominus,\, j\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i,\, i⌶j,\, j\,!{=}\,\varnothing,\, j\oplus,\, i>j,\, i\,!{=}\,\varnothing,\, j\,!{=}\,\varnothing,$$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, j\,!\!=\varnothing, j\oplus, j\,!\!=\varnothing, i\succ j,$

$<1>$

$\Leftrightarrow\ ,\left[\begin{array}{l}, i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, j\,!\!=\varnothing, j\oplus, j=\varnothing, i\succ j, \\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, j\,!\!=\varnothing, j\oplus, j\,!\!=\varnothing, i\succ j,\end{array}\right],$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, j\,!\!=\varnothing, j\oplus, \left[\begin{array}{l}, j=\varnothing, \\ , j\,!\!=\varnothing,\end{array}\right], i\succ j,$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, j\,!\!=\varnothing, j\oplus, i\succ j,$

$conclusion:$

$, i\asymp j, j\,!\!=\varnothing, j\oplus,\ \Leftrightarrow\ , i\asymp j, j\,!\!=\varnothing, j\oplus, i\succ j,$

$$, i\asymp j, j\ominus, j\,!\!=\varnothing,\ \Leftrightarrow\ \sim, j\succ i,$$

induction proof:

$premise\ 1:$

$, i=\varnothing, i\asymp j, j\ominus, j\,!\!=\varnothing,$

$\Leftrightarrow\ , i\asymp j, j\ominus, j\,!\!=\varnothing, i=\varnothing, j\succ i,$

$\Leftrightarrow\ , i=\varnothing, i\asymp j, j\ominus, j\,!\!=\varnothing, j\succ i,$

$premise\ 2:$

$,\ \&SHi\rightarrow i, i\asymp j, j\ominus, j\,!\!=\varnothing,\ \Leftrightarrow\ ,\ \&SHi\rightarrow i, i\asymp j, j\ominus, j\,!\!=\varnothing, j\succ i,\ \Rightarrow$

$, i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, j\ominus, j\,!\!=\varnothing,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, i\,!\!=\varnothing, j\ominus, j\,!\!=\varnothing,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, i\asymp j, i\,!\!=\varnothing, i\,!\!=\varnothing, j\ominus, j\,!\!=\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, i\,!{=}\varnothing, j\,!{=}\varnothing, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, j\ominus, i\oplus, i\ominus, j\,!{=}\varnothing, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, j\ominus, i\ominus, j\,!{=}\varnothing, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, i\text{⌶}j, j\ominus, i\ominus, j\,!{=}\varnothing, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, i\text{⌶}j, j\ominus, j\,!{=}\varnothing, i\ominus, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, \&SHi\rightarrow i, i\text{⌶}j, j\ominus, j\,!{=}\varnothing, i\ominus, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, \&SHi\rightarrow i, i\text{⌶}j, j\ominus, j\,!{=}\varnothing, j\succ i, i\ominus, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, i\text{⌶}j, j\ominus, j\,!{=}\varnothing, j\succ i, i\ominus, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, i\text{⌶}j, j\ominus, j\,!{=}\varnothing, j\succ i, i\ominus, j\ominus, i\,!{=}\varnothing, j\,!{=}\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, i\text{⌶}j, j\ominus, j\,!{=}\varnothing, i\ominus, j\ominus, i\,!{=}\varnothing, j\,!{=}\varnothing, j\succ i,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, i\text{⌶}j, i\ominus, j\ominus, i\,!{=}\varnothing, j\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing, j\succ i,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, i\,!{=}\varnothing, j\oplus, i\oplus, i\ominus, j\ominus, i\,!{=}\varnothing, j\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing, j\succ i,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, i\,!{=}\varnothing, i\,!{=}\varnothing, j\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing, j\succ i,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing, j\succ i,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, i\,!{=}\varnothing, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing, j\succ i,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\text{⌶}j, i\,!{=}\varnothing, j\ominus, j\,!{=}\varnothing, j\succ i,$

$\Leftrightarrow\ , i\,!{=}\varnothing, \&SHi\mathring{\bigcirc}i, i\text{⌶}j, j\,!{=}\varnothing, j\oplus, j\succ i,$

conclusion :

$, i\text{⌶}j, j\ominus, j\,!{=}\varnothing,\ \Leftrightarrow\ , i\text{⌶}j, j\ominus, j\,!{=}\varnothing, j\succ i,$

$$, i \mathrel{!=} \varnothing, i \rightarrow j, \iff \sim, i \succ j,$$

proof:

$, i \mathrel{!=} \varnothing, i \rightarrow j,$

$\iff , i \mathrel{!=} \varnothing, i \text{☮} i_0, i_0 \oplus, i_0 \mathring{\bigcirc} j, i_0 \text{⏚},$

$\iff , i \mathrel{!=} \varnothing, i \text{☮} i_0, i \mathring{\bigcirc} i_0, i_0 \oplus, i_0 \mathring{\bigcirc} j, i_0 \text{⏚},$

$\iff , i \mathrel{!=} \varnothing, i \text{☮} i_0, i \mathring{\bigcirc} i_0, i \text{⌶} i_0, i_0 \oplus, i_0 \mathring{\bigcirc} j, i_0 \text{⏚},$

$\iff , i \text{☮} i_0, i \mathring{\bigcirc} i_0, i \text{⌶} i_0, i \mathrel{!=} \varnothing, i_0 \oplus, i_0 \mathring{\bigcirc} j, i_0 \text{⏚},$

$\iff , i \text{☮} i_0, i \mathring{\bigcirc} i_0, i \text{⌶} i_0, i \mathrel{!=} \varnothing, i_0 \oplus, i \succ i_0, i_0 \mathring{\bigcirc} j, i_0 \text{⏚},$

$\iff , i \text{☮} i_0, i \mathring{\bigcirc} i_0, i \text{⌶} i_0, i \mathrel{!=} \varnothing, i_0 \oplus, i_0 \mathring{\bigcirc} j, i \succ i_0, i_0 \text{⏚},$

$\iff , i \text{☮} i_0, i \mathring{\bigcirc} i_0, i \text{⌶} i_0, i \mathrel{!=} \varnothing, i_0 \oplus, i_0 \mathring{\bigcirc} j, i \succ j, i_0 \text{⏚},$

$\iff , i \text{☮} i_0, i \mathrel{!=} \varnothing, i_0 \oplus, i_0 \mathring{\bigcirc} j, i \succ j, i_0 \text{⏚},$

$\iff , i \mathrel{!=} \varnothing, i \text{☮} i_0, i_0 \oplus, i_0 \mathring{\bigcirc} j, i_0 \text{⏚}, i \succ j,$

$\iff , i \mathrel{!=} \varnothing, i \rightarrow j, i \succ j,$

$$, i \mathrel{!=} \varnothing, i \oplus, i \succ k, \iff , i \succ k, \sim$$

induction proof:

premise 1 :

$, k = \varnothing, i \mathrel{!=} \varnothing, i \oplus, i \succ k,$

$\iff , i \mathrel{!=} \varnothing, k = \varnothing, i \oplus, i \succ k,$

$\iff , i \mathrel{!=} \varnothing, k = \varnothing, i \succ k, i \oplus, i \succ k,$

$\iff , k = \varnothing, i \succ k, i \mathrel{!=} \varnothing, i \oplus, i \succ k,$

premise 2 :

$, \&SHi \rightarrow k, i \mathrel{!=} \varnothing, i \oplus, i \succ k, \iff , \&SHi \rightarrow k, i \succ k, i \mathrel{!=} \varnothing, i \oplus, i \succ k, \Rightarrow$

$, k \mathrel{!=} \varnothing, \&SHi \mathring{\bigcirc} k, i \mathrel{!=} \varnothing, i \oplus, i \succ k,$

$$\Leftrightarrow\ , k \mathrel{!=} \varnothing,\ \&SHi \bigcirc^{\!\circ} k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i \mathrel{!=} \varnothing,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, i\ominus, k\oplus, k\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus, i\ominus, k\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i \mathrel{!=} \varnothing, i\oplus, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus, i{>}k, i\ominus, k\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i\ominus, k\ominus,$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus,\ \&SHi \rightarrow k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i\ominus, k\ominus,$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus,\ \&SHi \rightarrow k, i{>}k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i\ominus, k\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus, i{>}k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i\ominus, k\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i{>}k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i\ominus, k\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i{>}k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus, i\ominus, k\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i{>}k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing,$$

$$\Leftrightarrow\ ,\ \&SHi \bigcirc^{\!\circ} k, i{>}k, i \mathrel{!=} \varnothing, i\oplus, i{>}k, k \mathrel{!=} \varnothing,$$

$$\Leftrightarrow\ , k \mathrel{!=} \varnothing,\ \&SHi \bigcirc^{\!\circ} k, i{>}k, i \mathrel{!=} \varnothing, i\oplus, i{>}k,$$

$conclusion:$

$$, i \mathrel{!=} \varnothing, i\oplus, i{>}k,\ \Leftrightarrow\ , i{>}k, i \mathrel{!=} \varnothing, i\oplus, i{>}k,$$

$$, i{>}j, j \mathrel{!=} \varnothing, j\oplus,\ \Leftrightarrow\ \sim, i{>}j,$$

induction proof:

$premise\ 1:$

$$, j = \varnothing, i{>}j, j \mathrel{!=} \varnothing, j\oplus,$$

$$\Leftrightarrow\ , i{>}j, j = \varnothing, j \mathrel{!=} \varnothing, j\oplus,$$

$$\Leftrightarrow\ , i{>}j, \otimes, j\oplus,$$

$$\Leftrightarrow\ , i \succ j, \otimes, j\oplus, i \succ j,$$

$$\Leftrightarrow\ , i \succ j, j = \varnothing, j \mathrel{!}= \varnothing, j\oplus, i \succ j,$$

$$\Leftrightarrow\ , j = \varnothing, i \succ j, j \mathrel{!}= \varnothing, j\oplus, i \succ j,$$

$premise\ 2:$
$$, \ \&SHi \rightarrow j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, \ \Leftrightarrow\ , \ \&SHi \rightarrow j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, i \succ j, \ \Rightarrow$$

$$, j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus,$$

$$\Leftrightarrow\ , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, if(j = \varnothing) - \left[\begin{array}{l} , \\ \\ , \end{array}\right] -,$$

$$\Leftrightarrow\ , i \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i, i \succ j, j \mathrel{!}= \varnothing, j\oplus, - \left[\begin{array}{l} , j = \varnothing, \\ \\ , j \mathrel{!}= \varnothing, \end{array}\right] -,$$

$$\Leftrightarrow\ , - \left[\begin{array}{l} , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, j = \varnothing, \\ \\ , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, j \mathrel{!}= \varnothing, \end{array}\right] -,$$

$$\Leftrightarrow\ , - \left[\begin{array}{l} , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, i \mathrel{!}= \varnothing, j \mathrel{!}= \varnothing, j\oplus, j = \varnothing, \\ \\ , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, j \mathrel{!}= \varnothing, \end{array}\right] -,$$

$$\Leftrightarrow\ , - \left[\begin{array}{l} , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, i \mathrel{!}= \varnothing, j = \varnothing, \\ \\ , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, j \mathrel{!}= \varnothing, \end{array}\right] -,$$

$$\Leftrightarrow\ , - \left[\begin{array}{l} , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, i \mathrel{!}= \varnothing, j = \varnothing, i \succ j, \\ \\ , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, j \mathrel{!}= \varnothing, \end{array}\right] -,$$

$$\Leftrightarrow\ , - \left[\begin{array}{l} , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, j = \varnothing, i \succ j, \\ \\ , j \mathrel{!}= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} j, i \succ j, j \mathrel{!}= \varnothing, j\oplus, j \mathrel{!}= \varnothing, \end{array}\right] -,$$

$$\Leftrightarrow\ < 1 >$$

$$, j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, j \,!\!= \varnothing,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, i \,!\!= \varnothing, j \,!\!= \varnothing, j\oplus, j \,!\!= \varnothing,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, i \,!\!= \varnothing, j \,!\!= \varnothing,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, i\ominus, j\oplus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i > j, j \,!\!= \varnothing, j\oplus, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus,\ \& SHi \rightarrow j, i > j, j \,!\!= \varnothing, j\oplus, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus,\ \& SHi \rightarrow j, i > j, j \,!\!= \varnothing, j\oplus, i > j, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i > j, j \,!\!= \varnothing, j\oplus, i > j, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, j \,!\!= \varnothing, j\oplus, i > j, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i > j, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, i > j, i \,!\!= \varnothing, j \,!\!= \varnothing, i\oplus, j\oplus, i\ominus, j\ominus,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, i > j, i \,!\!= \varnothing, j \,!\!= \varnothing,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, i > j, j \,!\!= \varnothing,$$

$$\Leftrightarrow\ , j \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} j, i > j, j \,!\!= \varnothing, j\oplus, j \,!\!= \varnothing, i > j,$$

$< 1 >$

$$\Leftrightarrow\ , \left[\begin{array}{l} , i \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} i, i > j, j \,!\!= \varnothing, j\oplus, j = \varnothing, i > j, \\ , i \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} i, i > j, j \,!\!= \varnothing, j\oplus, j \,!\!= \varnothing, i > j, \end{array}\right],$$

$$\Leftrightarrow\ , i \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} i, i > j, j \,!\!= \varnothing, j\oplus, \left[\begin{array}{l} , j = \varnothing, \\ , j \,!\!= \varnothing, \end{array}\right], i > j,$$

$$\Leftrightarrow\ , i \,!\!= \varnothing,\ \& SHi \mathring{\bigcirc} i, i > j, j \,!\!= \varnothing, j\oplus, i > j,$$

$conclusion:$

$$,i\gtrdot j,j\,!\!=\varnothing,j\oplus,\ \Leftrightarrow\ ,i\gtrdot j,j\,!\!=\varnothing,j\oplus,i\gtrdot j,$$

$$,i\gtrdot j,i\ominus,i\,!\!=\varnothing,\ \Leftrightarrow\ \sim,i\gtrdot j,$$

proof:

$,i\gtrdot j,i\ominus,i\,!\!=\varnothing,$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},\\ ,\end{array}\right],i\gtrdot j,i\ominus,i\,!\!=\varnothing,$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,\\ ,j\,!\!=\varnothing,\end{array}\right],i\gtrdot j,i\ominus,i\,!\!=\varnothing,$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,i\gtrdot j,i\ominus,i\,!\!=\varnothing,\\ ,j\,!\!=\varnothing,i\gtrdot j,i\ominus,i\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},i\gtrdot j,i\ominus,i\,!\!=\varnothing,j=\varnothing,\\ ,i\gtrdot j,i\ominus,i\,!\!=\varnothing,j\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},i\gtrdot j,i\ominus,i\,!\!=\varnothing,j=\varnothing,i\gtrdot j,\\ ,i\gtrdot j,i\ominus,i\,!\!=\varnothing,j\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,i\gtrdot j,i\ominus,i\,!\!=\varnothing,i\gtrdot j,\\ ,i\gtrdot j,i\ominus,i\,!\!=\varnothing,j\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},i\gtrdot j,i\ominus,i\,!\!=\varnothing,i\gtrdot j,\\ ,i\gtrdot j,i\ominus,i\,!\!=\varnothing,j\,!\!=\varnothing,j\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},i\gtrdot j,i\ominus,i\,!\!=\varnothing,i\gtrdot j,\\ ,i\gtrdot j,i\ominus,i\,!\!=\varnothing,j\,!\!=\varnothing,j\oplus,j\ominus,j\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},i\gtrdot j,i\ominus,i\,!\!=\varnothing,i\gtrdot j,\\ ,i\gtrdot j,j\,!\!=\varnothing,j\oplus,i\ominus,i\,!\!=\varnothing,j\ominus,j\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i>j, i\ominus, i\,!\!=\varnothing, i>j,\\ \\ , i>j, j\,!\!=\varnothing, j\oplus, i>j, i\ominus, i\,!\!=\varnothing, j\ominus, j\,!\!=\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i>j, i\ominus, i\,!\!=\varnothing, i>j,\\ \\ , i>j, j\,!\!=\varnothing, j\oplus, i\ominus, i\,!\!=\varnothing, j\ominus, j\,!\!=\varnothing, i>j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i>j, i\ominus, i\,!\!=\varnothing, i>j,\\ \\ , i>j, i\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, j\oplus, j\ominus, j\,!\!=\varnothing, i>j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i>j, i\ominus, i\,!\!=\varnothing, i>j,\\ \\ , i>j, i\ominus, i\,!\!=\varnothing, j\,!\!=\varnothing, i>j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i>j, i\ominus, i\,!\!=\varnothing, i>j,\\ \\ , j\,!\!=\varnothing, i>j, i\ominus, i\,!\!=\varnothing, i>j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l}, i>j, i\ominus, i\,!\!=\varnothing, i>j,\\ \\ , i>j, i\ominus, i\,!\!=\varnothing, i>j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j=\varnothing)\left[\begin{array}{l},\\ \\ ,\end{array}\right], i>j, i\ominus, i\,!\!=\varnothing, i>j,$$

$$\Leftrightarrow\ , i>j, i\ominus, i\,!\!=\varnothing, i>j,$$

$$, j\ominus, j\,!\!=\varnothing, i>j, \ \Leftrightarrow\ , i>j, \sim$$

proof:

$, j\ominus, j\,!\!=\varnothing, i>j,$

$$\Leftrightarrow\ , j\ominus, j\,!\!=\varnothing, i>j, i\,!\!=\varnothing,$$

$$\Leftrightarrow\ , j\ominus, j\,!\!=\varnothing, i\,!\!=\varnothing, i>j,$$

$$\Leftrightarrow\ , j\ominus, j\,!\!=\varnothing, i\,!\!=\varnothing, i\,!\!=\varnothing, i>j,$$

$$\Leftrightarrow\ , j\ominus, j \mathrel{!=} \varnothing, i \mathrel{!=} \varnothing, i\oplus, i\ominus, i \mathrel{!=} \varnothing, i > j,$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i\oplus, i\ominus, j\ominus, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i > j,$$

$$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i\oplus, i > j, i\ominus, j\ominus, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,$$

$$\Leftrightarrow\ , i > j, i \mathrel{!=} \varnothing, i\oplus, i > j, i\ominus, j\ominus, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,$$

$$\Leftrightarrow\ , i > j, i \mathrel{!=} \varnothing, i\oplus, i\ominus, j\ominus, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i > j,$$

$$\Leftrightarrow\ , i > j, j\ominus, j \mathrel{!=} \varnothing, i \mathrel{!=} \varnothing, i\oplus, i\ominus, i \mathrel{!=} \varnothing, i > j,$$

$$\Leftrightarrow\ , i > j, j\ominus, j \mathrel{!=} \varnothing, i \mathrel{!=} \varnothing, i > j,$$

$$\Leftrightarrow\ , i > j, j\ominus, j \mathrel{!=} \varnothing, i > j, i \mathrel{!=} \varnothing,$$

$$\Leftrightarrow\ , i > j, j\ominus, j \mathrel{!=} \varnothing, i > j,$$

$$, i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, \quad \Leftrightarrow \quad \sim, k = \varnothing,$$

proof:

$, i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing,$

$$\Leftrightarrow\ , i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, if(k = \varnothing) \left[\begin{array}{l} , \\ , \end{array}\right],$$

$$\Leftrightarrow\ , i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, if(k = \varnothing) \left[\begin{array}{l} , \\ , k \mathrel{!=} \varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , if(k = \varnothing) \left[\begin{array}{l} , i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, \\ , i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, k \mathrel{!=} \varnothing, \end{array}\right],$$

$$\Leftrightarrow\ , if(k = \varnothing) \left[\begin{array}{l} , i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, \\ , i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, k \mathrel{!=} \varnothing, k\oplus, k\ominus, \end{array}\right],$$

$$\Leftrightarrow\ , if(k = \varnothing) \left[\begin{array}{l} , i > k, i \mathrel{!=} \varnothing, i\oplus, i = \varnothing, \\ , i > k, i \mathrel{!=} \varnothing, k \mathrel{!=} \varnothing, i\oplus, k\oplus, i = \varnothing, k\ominus, \end{array}\right],$$

$$\Leftrightarrow\ ,if(k=\varnothing)\left[\begin{array}{l} ,i\gtrdot k,i\,!\!=\varnothing,i\oplus,i=\varnothing, \\ \\ ,i\,!\!=\varnothing,k\,!\!=\varnothing,i\oplus,k\oplus,i\gtrdot k,i=\varnothing,k\ominus, \end{array}\right],$$

$$\Leftrightarrow\ ,if(k=\varnothing)\left[\begin{array}{l} ,i\gtrdot k,i\,!\!=\varnothing,i\oplus,i=\varnothing, \\ \\ ,i\,!\!=\varnothing,k\,!\!=\varnothing,i\oplus,k\oplus,i\gtrdot k,i\,!\!=\varnothing,i=\varnothing,k\ominus, \end{array}\right],$$

$$\Leftrightarrow\ ,if(k=\varnothing)\left[\begin{array}{l} ,i\gtrdot k,i\,!\!=\varnothing,i\oplus,i=\varnothing, \\ \\ ,i\,!\!=\varnothing,k\,!\!=\varnothing,i\oplus,k\oplus,i\gtrdot k,\otimes,k\ominus, \end{array}\right],$$

$$\Leftrightarrow\ ,if(k=\varnothing)\left[\begin{array}{l} ,i\gtrdot k,i\,!\!=\varnothing,i\oplus,i=\varnothing, \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,if(k=\varnothing)\left[\begin{array}{l} , \\ \\ ,\otimes, \end{array}\right],i\gtrdot k,i\,!\!=\varnothing,i\oplus,i=\varnothing,$$

$$\Leftrightarrow\ ,i\gtrdot k,i\,!\!=\varnothing,i\oplus,i=\varnothing,k=\varnothing,$$

$$,k\,!\!=\varnothing,i\gtrdot k,i\,!\!=\varnothing,i\oplus,\ \Leftrightarrow\ \sim,i\,!\!=\varnothing,$$

$$,k=\varnothing,\ \Leftrightarrow\ \sim,if(i\gtrdot k)\left[\begin{array}{l} , \\ \\ ,i\mathrel{=\!\!\!\!\mid}k, \end{array}\right],$$

proof:

$,k=\varnothing,$

$$\Leftrightarrow\ ,k=\varnothing,if(i=\varnothing)\left[\begin{array}{l} , \\ \\ , \end{array}\right],$$

$$\Leftrightarrow\ ,k=\varnothing,if(i=\varnothing)\left[\begin{array}{l} ,i=\varnothing, \\ \\ ,i\,!\!=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\left[\begin{array}{l} ,k=\varnothing,i=\varnothing, \\ \\ ,k=\varnothing,i\,!\!=\varnothing, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i=\varnothing)\text{-}\left[\begin{array}{l},k=\varnothing,i=\varnothing,i\mathrel{=\!\!\!\!\mid\;}k,\\ \\ ,k=\varnothing,i\,!\!=\varnothing,i\rhd k,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i=\varnothing)\text{-}\left[\begin{array}{l},k=\varnothing,i=\varnothing,i\mathrel{=\!\!\!\!\mid\;}k,if(i\mathrel{=\!\!\!\!\mid\;}k)\text{-}\left[\begin{array}{l},\\ \\ ,i\rhd k,\end{array}\right]\text{-},\\ ,k=\varnothing,i\,!\!=\varnothing,i\rhd k,if(i\rhd k)\text{-}\left[\begin{array}{l},\\ \\ ,i\mathrel{=\!\!\!\!\mid\;}k,\end{array}\right]\text{-},\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i=\varnothing)\text{-}\left[\begin{array}{l},k=\varnothing,i=\varnothing,i\mathrel{=\!\!\!\!\mid\;}k,if(i\rhd k)\text{-}\left[\begin{array}{l},\\ \\ ,i\mathrel{=\!\!\!\!\mid\;}k,\end{array}\right]\text{-},\\ ,k=\varnothing,i\,!\!=\varnothing,i\rhd k,if(i\rhd k)\text{-}\left[\begin{array}{l},\\ \\ ,i\mathrel{=\!\!\!\!\mid\;}k,\end{array}\right]\text{-},\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i=\varnothing)\text{-}\left[\begin{array}{l},k=\varnothing,i=\varnothing,i\mathrel{=\!\!\!\!\mid\;}k,\\ \\ ,k=\varnothing,i\,!\!=\varnothing,i\rhd k,\end{array}\right]\text{-},if(i\rhd k)\text{-}\left[\begin{array}{l},\\ \\ ,i\mathrel{=\!\!\!\!\mid\;}k,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i=\varnothing)\text{-}\left[\begin{array}{l},k=\varnothing,i=\varnothing,\\ \\ ,k=\varnothing,i\,!\!=\varnothing,\end{array}\right]\text{-},if(i\rhd k)\text{-}\left[\begin{array}{l},\\ \\ ,i\mathrel{=\!\!\!\!\mid\;}k,\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,k=\varnothing,if(i\rhd k)\text{-}\left[\begin{array}{l},\\ \\ ,i\mathrel{=\!\!\!\!\mid\;}k,\end{array}\right]\text{-},$$

$$,i\rhd k,i\,!\!=\varnothing,i\oplus,\ \Leftrightarrow\ \sim,if(i\rhd k)\text{-}\left[\begin{array}{l},\\ \\ ,i\mathrel{=\!\!\!\!\mid\;}k,\end{array}\right]\text{-},$$

induction proof:

premise 1 :

$,k=\varnothing,i\rhd k,i\,!\!=\varnothing,i\oplus,$

$\Leftrightarrow\ ,i\rhd k,i\,!\!=\varnothing,i\oplus,k=\varnothing,$

$$\Leftrightarrow\ , i \gtrdot k, i \,!\!= \varnothing, i\oplus, k = \varnothing, if(i \gtrdot k) \text{-} \left[\begin{array}{l} , \\ \\ , i \asymp k, \end{array}\right] \text{-},$$

$$\Leftrightarrow\ , k = \varnothing, i \gtrdot k, i \,!\!= \varnothing, i\oplus, if(i \gtrdot k) \text{-} \left[\begin{array}{l} , \\ \\ , i \asymp k, \end{array}\right] \text{-},$$

premise 2 :

$$, \ \&SHi \rightarrow k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, \ \Leftrightarrow\ , \ \&SHi \rightarrow k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, if(i \gtrdot k) \text{-} \left[\begin{array}{l} , \\ \\ , i \asymp k, \end{array}\right] \text{-}, \ \Rightarrow$$

$$, k \,!\!= \varnothing, \ \&SHi \mathring{\bigcirc} k, i \gtrdot k, i \,!\!= \varnothing, i\oplus,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, k \,!\!= \varnothing,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, k \,!\!= \varnothing, i \,!\!= \varnothing,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, k\ominus, i\oplus, i\ominus,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, k\ominus, i\ominus,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, i \gtrdot k, i \,!\!= \varnothing, i\oplus, k\ominus, i\ominus,$$

$$\Leftrightarrow\ , k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, \ \&SHi \rightarrow k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, k\ominus, i\ominus,$$

$$\Leftrightarrow\ , k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, \ \&SHi \rightarrow k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, if(i \gtrdot k) \text{-} \left[\begin{array}{l} , \\ \\ , i \asymp k, \end{array}\right] \text{-}, k\ominus, i\ominus,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, i \gtrdot k, i \,!\!= \varnothing, i\oplus, if(i \gtrdot k) \text{-} \left[\begin{array}{l} , \\ \\ , i \asymp k, \end{array}\right] \text{-}, k\ominus, i\ominus,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, if(i \gtrdot k) \text{-} \left[\begin{array}{l} , \\ \\ , i \asymp k, \end{array}\right] \text{-}, k\ominus, i\ominus,$$

$$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} k, i \gtrdot k, i \,!\!= \varnothing, i\oplus, if(i \gtrdot k) \text{-} \left[\begin{array}{l} , k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, \\ \\ , k \,!\!= \varnothing, i \,!\!= \varnothing, k\oplus, i\oplus, i \asymp k, \end{array}\right] \text{-}, k\ominus, i\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}k, i{>}k, i \mathrel{!{=}} \varnothing, i\oplus, if(i{>}k)\text{-}\left[\begin{array}{l} , \\ , i\doteq k, \end{array}\right]\text{-}, k \mathrel{!{=}} \varnothing, i \mathrel{!{=}} \varnothing, k\oplus, i\oplus, k\ominus, i\ominus,$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}k, i{>}k, i \mathrel{!{=}} \varnothing, i\oplus, if(i{>}k)\text{-}\left[\begin{array}{l} , \\ , i\doteq k, \end{array}\right]\text{-}, k \mathrel{!{=}} \varnothing, i \mathrel{!{=}} \varnothing,$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}k, i{>}k, i \mathrel{!{=}} \varnothing, i\oplus, k \mathrel{!{=}} \varnothing, i \mathrel{!{=}} \varnothing, if(i{>}k)\text{-}\left[\begin{array}{l} , \\ , i\doteq k, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}k, i{>}k, i \mathrel{!{=}} \varnothing, i\oplus, k \mathrel{!{=}} \varnothing, if(i{>}k)\text{-}\left[\begin{array}{l} , \\ , i\doteq k, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , k \mathrel{!{=}} \varnothing,\ \&SHi\mathring{\bigcirc}k, i{>}k, i \mathrel{!{=}} \varnothing, i\oplus, if(i{>}k)\text{-}\left[\begin{array}{l} , \\ , i\doteq k, \end{array}\right]\text{-},$$

$conclusion:$

$$, i{>}k, i \mathrel{!{=}} \varnothing, i\oplus,\ \Leftrightarrow\ , i{>}k, i \mathrel{!{=}} \varnothing, i\oplus, if(i{>}k)\text{-}\left[\begin{array}{l} , \\ , i\doteq k, \end{array}\right]\text{-},$$

$$, i{>}k, k\ominus, k \mathrel{!{=}} \varnothing,\ \Leftrightarrow\ \sim, if(i{>}k)\text{-}\left[\begin{array}{l} , \\ , i\doteq k, \end{array}\right]\text{-},$$

22.4.16 relationship of number equal and more than and less than

$$, i!\doteq j,\ \Leftrightarrow\ , if(i{>}j)\text{-}\left[\begin{array}{l} , \\ , i{<}j, \end{array}\right]\text{-},$$

proof:
$, i!\doteq j,$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), i_0 \,!\!= j_0, i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), if(i_0=\varnothing)\left[\begin{array}{l} , \\ , j_0=\varnothing, \end{array}\right], i_0 \,!\!= j_0, i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), \left[\begin{array}{l} , i_0=\varnothing, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, \end{array}\right], i_0 \,!\!= j_0, i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), \left[\begin{array}{l} , i_0=\varnothing, i_0 \,!\!= j_0, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, i_0 \,!\!= j_0, \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), \left[\begin{array}{l} , i_0=\varnothing, j_0 \,!\!=\varnothing, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), \left[\begin{array}{l} , j_0 \,!\!=\varnothing, i_0=\varnothing, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), \left[\begin{array}{l} , j_0 \,!\!=\varnothing, if(j_0=\varnothing)\left[\begin{array}{l} , \\ , i_0=\varnothing, \end{array}\right], \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, if(j_0=\varnothing)\left[\begin{array}{l} , \\ , i_0=\varnothing, \end{array}\right], \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), if(j_0=\varnothing)\left[\begin{array}{l} , \\ , i_0=\varnothing, \end{array}\right], \left[\begin{array}{l} , j_0 \,!\!=\varnothing, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), if(i_0=\varnothing)\left[\begin{array}{l} , \\ , j_0=\varnothing, \end{array}\right], \left[\begin{array}{l} , j_0 \,!\!=\varnothing, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), \left[\begin{array}{l} , j_0 \,!\!=\varnothing, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ , i\owedge i_0, j\owedge j_0, Rc(i_0;j_0), i\owedge i_1, i_1\obot, j\owedge j_1, j_1\obot, \left[\begin{array}{l} , j_0 \,!\!=\varnothing, \\ , i_0 \,!\!=\varnothing, j_0=\varnothing, \end{array}\right], i_0\obot, j_0\obot,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, j\oslash j_0, j\oslash j_1, Rc(i_0;j_0), i_1\circleddash, j_1\circleddash, -\left[\begin{array}{l} ,j_0 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_0 = \varnothing, \end{array}\right]-, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, j\oslash j_0, j\oslash j_1, Rc(i_0;j_0), Rc(i_1;j_1), i_1\circleddash, j_1\circleddash, -\left[\begin{array}{l} ,j_0 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_0 = \varnothing, \end{array}\right]-, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, j\oslash j_0, j\oslash j_1, Rc(i_0;j_0), Rc(i_1;j_1),$$

$$-\left[\begin{array}{l} ,j_0 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_0 = \varnothing, \end{array}\right]-, i_1\circleddash, j_1\circleddash, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, i_0\mathring{\bigcirc} i_1, j\oslash j_0, j\oslash j_1, j_0\mathring{\bigcirc} j_1, Rc(i_0;j_0), Rc(i_1;j_1),$$

$$-\left[\begin{array}{l} ,j_0 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_0 = \varnothing, \end{array}\right]-, i_1\circleddash, j_1\circleddash, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, j\oslash j_0, j\oslash j_1, i_0\mathring{\bigcirc} i_1, j_0\mathring{\bigcirc} j_1, Rc(i_0;j_0), Rc(i_1;j_1),$$

$$i_0\mathring{\bigcirc} i_1, j_0\mathring{\bigcirc} j_1, -\left[\begin{array}{l} ,j_0 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_0 = \varnothing, \end{array}\right]-, i_1\circleddash, j_1\circleddash, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, j\oslash j_0, j\oslash j_1, i_0\mathring{\bigcirc} i_1, j_0\mathring{\bigcirc} j_1, Rc(i_0;j_0), Rc(i_1;j_1),$$

$$i_0\mathring{\bigcirc} i_1, -\left[\begin{array}{l} ,j_0\mathring{\bigcirc} j_1, j_0 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_0\mathring{\bigcirc} j_1, j_0 = \varnothing, \end{array}\right]-, i_1\circleddash, j_1\circleddash, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, j\oslash j_0, j\oslash j_1, i_0\mathring{\bigcirc} i_1, j_0\mathring{\bigcirc} j_1, Rc(i_0;j_0), Rc(i_1;j_1),$$

$$i_0\mathring{\bigcirc} i_1, -\left[\begin{array}{l} ,j_0\mathring{\bigcirc} j_1, j_1 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_0\mathring{\bigcirc} j_1, j_1 = \varnothing, \end{array}\right]-, i_1\circleddash, j_1\circleddash, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,i\oslash i_0, i\oslash i_1, j\oslash j_0, j\oslash j_1, Rc(i_0;j_0), Rc(i_1;j_1),$$

$$-\left[\begin{array}{l} ,j_1 \mathrel{!=} \varnothing, \\ \\ ,i_0 \mathrel{!=} \varnothing, j_1 = \varnothing, \end{array}\right]-, i_1\circleddash, j_1\circleddash, i_0\circleddash, j_0\circleddash,$$

$$\Leftrightarrow\ ,j\oslash j_1, i\oslash i_1, Rc(i_1;j_1),$$

$$\left[\begin{array}{l} ,j_1 \,!\!= \varnothing, i\otimes i_0, j\otimes j_0, Rc(i_0;j_0), i_0\oplus, j_0\oplus, \\ \\ ,i\otimes i_0, j\otimes j_0, Rc(i_0;j_0), i_0 \,!\!= \varnothing, i_0\oplus, j_0\oplus, j_1 = \varnothing, \end{array}\right], i_1\oplus, j_1\oplus,$$

$$\Leftrightarrow\ ,j\otimes j_1, i\otimes i_1, Rc(i_1;j_1), \left[\begin{array}{l} ,j_1 \,!\!= \varnothing, \\ \\ ,i\succ j, j_1 = \varnothing, \end{array}\right], i_1\oplus, j_1\oplus,$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,j\otimes j_1, i\otimes i_1, Rc(i_1;j_1), j_1 \,!\!= \varnothing, i_1\oplus, j_1\oplus, \\ \\ ,j\otimes j_1, i\otimes i_1, Rc(i_1;j_1), j_1 = \varnothing, i_1\oplus, j_1\oplus, i\succ j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,j\otimes j_1, i\otimes i_1, Rc(j_1;i_1), j_1 \,!\!= \varnothing, j_1\oplus, i_1\oplus, \\ \\ ,j\otimes j_1, i\otimes i_1, Rc(j_1;i_1), j_1 = \varnothing, j_1\oplus, i_1\oplus, i\succ j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,j\succ i, \\ \\ ,j!\succ i, i\succ j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j\succ i)\left[\begin{array}{l} , \\ \\ ,i\succ j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\succ j)\left[\begin{array}{l} , \\ \\ ,j\succ i, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\succ j)\left[\begin{array}{l} , \\ \\ ,i\prec j, \end{array}\right],$$

$$,i!\succ j, \ \Leftrightarrow\ ,if(i\asymp j)\left[\begin{array}{l} , \\ \\ ,i\prec j, \end{array}\right],$$

proof:

$,i!\succ j,$

$$\Leftrightarrow\ ,i\otimes i_0, j\otimes j_0, Rc(i_0;j_0), i_0 = \varnothing, i_0\oplus, j_0\oplus,$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),if(i_0=\varnothing)\left[\begin{array}{l},\\ \\ ,j_0=\varnothing,\end{array}\right],i_0=\varnothing,i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),if(j_0=\varnothing)\left[\begin{array}{l},\\ \\ ,i_0=\varnothing,\end{array}\right],i_0=\varnothing,i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),\left[\begin{array}{l},j_0=\varnothing,\\ \\ ,j_0\,!\!=\varnothing,i_0=\varnothing,\end{array}\right],i_0=\varnothing,i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),\left[\begin{array}{l},j_0=\varnothing,i_0=\varnothing,\\ \\ ,j_0\,!\!=\varnothing,i_0=\varnothing,\end{array}\right],i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),\left[\begin{array}{l},i_0=\varnothing,i_0=j_0,\\ \\ ,j_0\,!\!=\varnothing,i_0=\varnothing,i_0\,!\!=j_0,\end{array}\right],i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),\left[\begin{array}{l},i_0=j_0,i_0=\varnothing,\\ \\ ,i_0\,!\!=j_0,j_0\,!\!=\varnothing,i_0=\varnothing,\end{array}\right],i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),\left[\begin{array}{l},i_0=j_0,if(i_0=\varnothing)\left[\begin{array}{l},\\ ,\otimes,\end{array}\right],\\ \\ ,i_0\,!\!=j_0,j_0\,!\!=\varnothing,if(j_0=\varnothing)\left[\begin{array}{l},\\ \\ ,i_0=\varnothing,\end{array}\right],\end{array}\right],i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),\left[\begin{array}{l},if(i_0=\varnothing)\left[\begin{array}{l},i_0=j_0,\\ \\ ,i_0=j_0,j_0\,!\!=\varnothing,j_0=\varnothing,\end{array}\right],\\ ,i_0\,!\!=j_0,j_0\,!\!=\varnothing,if(j_0=\varnothing)\left[\begin{array}{l},\\ \\ ,i_0=\varnothing,\end{array}\right],\end{array}\right],i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\circledwedge i_0,j\circledwedge j_0,Rc(i_0;j_0),\left[\begin{array}{l},if(i_0=\varnothing)\left[\begin{array}{l},i_0=j_0,\\ \\ ,i_0=j_0,i_0\,!\!=\varnothing,j_0=\varnothing,\end{array}\right],\\ ,i_0\,!\!=j_0,j_0\,!\!=\varnothing,if(j_0=\varnothing)\left[\begin{array}{l},\\ \\ ,i_0=\varnothing,\end{array}\right],\end{array}\right],i_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),\left[\begin{array}{l} ,i_0=j_0,if(i_0=\varnothing)\left[\begin{array}{l} , \\ ,i_0\ !=\varnothing,j_0=\varnothing, \end{array}\right], \\ ,i_0\ !=j_0,j_0\ !=\varnothing,if(j_0=\varnothing)\left[\begin{array}{l} , \\ ,i_0=\varnothing, \end{array}\right], \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),\left[\begin{array}{l} ,i_0=j_0,if(i_0=\varnothing)\left[\begin{array}{l} , \\ ,j_0=\varnothing, \end{array}\right], \\ ,i_0\ !=j_0,j_0\ !=\varnothing,if(j_0=\varnothing)\left[\begin{array}{l} , \\ ,i_0=\varnothing, \end{array}\right], \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),\left[\begin{array}{l} ,i_0=j_0,if(i_0=\varnothing)\left[\begin{array}{l} , \\ ,j_0=\varnothing, \end{array}\right], \\ ,i_0\ !=j_0,j_0\ !=\varnothing,if(i_0=\varnothing)\left[\begin{array}{l} , \\ ,j_0=\varnothing, \end{array}\right], \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),if(i_0=\varnothing)\left[\begin{array}{l} , \\ ,j_0=\varnothing, \end{array}\right],\left[\begin{array}{l} ,i_0=j_0, \\ ,i_0\ !=j_0,j_0\ !=\varnothing, \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),\left[\begin{array}{l} ,i_0=j_0, \\ ,i_0\ !=j_0,j_0\ !=\varnothing, \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,j\text{☮}j_0,Rc(i_0;j_0),i\text{☮}i_1,i_1\text{\textcircled{$\downarrow$}},j\text{☮}j_1,j_1\text{\textcircled{$\downarrow$}},\left[\begin{array}{l} ,i_0=j_0, \\ ,i_0\ !=j_0,j_0\ !=\varnothing, \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,i\text{☮}i_1,j\text{☮}j_0,j\text{☮}j_1,Rc(i_0;j_0),i_1\text{\textcircled{$\downarrow$}},j_1\text{\textcircled{$\downarrow$}},\left[\begin{array}{l} ,i_0=j_0, \\ ,i_0\ !=j_0,j_0\ !=\varnothing, \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,i\text{☮}i_1,j\text{☮}j_0,j\text{☮}j_1,Rc(i_0;j_0),Rc(i_1;j_1),i_1\text{\textcircled{$\downarrow$}},j_1\text{\textcircled{$\downarrow$}},\left[\begin{array}{l} ,i_0=j_0, \\ ,i_0\ !=j_0,j_0\ !=\varnothing, \end{array}\right],i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,i\text{☮}i_1,j\text{☮}j_0,j\text{☮}j_1,Rc(i_0;j_0),Rc(i_1;j_1),$$

$$\left[\begin{array}{l} ,i_0=j_0, \\ ,i_0\ !=j_0,j_0\ !=\varnothing, \end{array}\right],i_1\text{\textcircled{$\downarrow$}},j_1\text{\textcircled{$\downarrow$}},i_0\text{\textcircled{$\downarrow$}},j_0\text{\textcircled{$\downarrow$}},$$

$$\Leftrightarrow\ ,i\text{☮}i_0,i\text{☮}i_1,i_0\text{♂}i_1,j\text{☮}j_0,j\text{☮}j_1,j_0\text{♂}j_1,Rc(i_0;j_0),Rc(i_1;j_1),$$

$$\left[\begin{array}{l}, i_0 = j_0, \\ \\ , i_0 \mathrel{!=} j_0, j_0 \mathrel{!=} \varnothing, \end{array}\right], i_1\text{⏚}, j_1\text{⏚}, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\text{♂}i_1, j_0\text{♂}j_1, Rc(i_0; j_0), Rc(i_1; j_1),$$

$$i_0\text{♂}i_1, j_0\text{♂}j_1, \left[\begin{array}{l}, i_0 = j_0, \\ \\ , i_0 \mathrel{!=} j_0, j_0 \mathrel{!=} \varnothing, \end{array}\right], i_1\text{⏚}, j_1\text{⏚}, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\text{♂}i_1, j_0\text{♂}j_1, Rc(i_0; j_0), Rc(i_1; j_1),$$

$$i_0\text{♂}i_1, \left[\begin{array}{l}, j_0\text{♂}j_1, i_0 = j_0, \\ \\ , i_0 \mathrel{!=} j_0, j_0\text{♂}j_1, j_0 \mathrel{!=} \varnothing, \end{array}\right], i_1\text{⏚}, j_1\text{⏚}, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, i_0\text{♂}i_1, j_0\text{♂}j_1, Rc(i_0; j_0), Rc(i_1; j_1),$$

$$i_0\text{♂}i_1, \left[\begin{array}{l}, j_0\text{♂}j_1, i_0 = j_0, \\ \\ , i_0 \mathrel{!=} j_0, j_0\text{♂}j_1, j_1 \mathrel{!=} \varnothing, \end{array}\right], i_1\text{⏚}, j_1\text{⏚}, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ , i\text{☮}i_0, i\text{☮}i_1, j\text{☮}j_0, j\text{☮}j_1, Rc(i_0; j_0), Rc(i_1; j_1),$$

$$\left[\begin{array}{l}, i_0 = j_0, \\ \\ , i_0 \mathrel{!=} j_0, j_1 \mathrel{!=} \varnothing, \end{array}\right], i_1\text{⏚}, j_1\text{⏚}, i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ , i\text{☮}i_0, j\text{☮}j_0, Rc(i_0; j_0),$$

$$\left[\begin{array}{l}, i_0 = j_0, i\text{☮}i_1, j\text{☮}j_1, Rc(i_1; j_1), i_1\text{⏚}, j_1\text{⏚}, \\ \\ , i_0 \mathrel{!=} j_0, i\text{☮}i_1, j\text{☮}j_1, Rc(i_1; j_1), j_1 \mathrel{!=} \varnothing, i_1\text{⏚}, j_1\text{⏚}, \end{array}\right], i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ , i\text{☮}i_0, j\text{☮}j_0, Rc(i_0; j_0), \left[\begin{array}{l}, i_0 = j_0, \\ \\ , i_0 \mathrel{!=} j_0, j \rhd i, \end{array}\right], i_0\text{⏚}, j_0\text{⏚},$$

$$\Leftrightarrow\ , \left[\begin{array}{l}, i\text{☮}i_0, j\text{☮}j_0, Rc(i_0; j_0), i_0 = j_0, i_0\text{⏚}, j_0\text{⏚}, \\ \\ , i\text{☮}i_0, j\text{☮}j_0, Rc(i_0; j_0), i_0 \mathrel{!=} j_0, i_0\text{⏚}, j_0\text{⏚}, j \rhd i, \end{array}\right],$$

$$\Leftrightarrow\ , \left[\begin{array}{l}, i \text{⌶} j \\ \\ , i \mathrel{!}\text{⌶} j, , j \rhd i, \end{array}\right],$$

$$\Leftrightarrow\ , if(i{=}j) \text{-}\left[\begin{array}{l} , \\ , j{>}i, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , if(i{=}j) \text{-}\left[\begin{array}{l} , \\ , i{<}j, \end{array}\right]\text{-},$$

$$,\ \Leftrightarrow\ , if(i{=}j) \text{-}\left[\begin{array}{l} , \\ , if(i{>}j), \text{-}\left[\begin{array}{l} , \\ , i{<}j, \end{array}\right]\text{-}, \end{array}\right]\text{-},$$

proof:

$,$

$$\Leftrightarrow\ , if(i{=}j) \text{-}\left[\begin{array}{l} , \\ , \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , if(i{=}j) \text{-}\left[\begin{array}{l} , \\ , i!{=}j, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ , if(i{=}j) \text{-}\left[\begin{array}{l} , \\ , if(i{>}j), \text{-}\left[\begin{array}{l} , \\ , i{<}j, \end{array}\right]\text{-}, \end{array}\right]\text{-},$$

$$, i!{<}j,\ \Leftrightarrow\ , if(i{=}j) \text{-}\left[\begin{array}{l} , \\ , i{>}j, \end{array}\right]\text{-},$$

proof:

$, i!{<}j,$

$$\Leftrightarrow\ , j!{>}i,$$

$$\Leftrightarrow\ , if(j{=}i) \text{-}\left[\begin{array}{l} , \\ , j{<}i, \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i{=}j)-\left[\begin{array}{l},\\ \\ ,j{<}i,\end{array}\right]-,$$

$$\Leftrightarrow\ ,if(i{=}j)-\left[\begin{array}{l},\\ \\ ,i{>}j,\end{array}\right]-,$$

$$,i{=}j,i{>}j,\ \Leftrightarrow\ ,\otimes,$$

proof:

$,i{=}j,i{>}j,$

$$\Leftrightarrow\ ,i{=}j,i{>}j,if(i{>}j)-\left[\begin{array}{l},\\ \\ ,i{<}j,\end{array}\right]-,$$

$$\Leftrightarrow\ ,i{=}j,i{>}j,i!{=}j,$$

$$\Leftrightarrow\ ,i{=}j,i!{=}j,i{>}j,$$

$$\Leftrightarrow\ ,\otimes,i{>}j,$$

$$\Leftrightarrow\ ,\otimes,$$

$$,i{=}j,i{<}j,\ \Leftrightarrow\ ,\otimes,$$

$$,i{>}j,i{<}j,\ \Leftrightarrow\ ,\otimes,$$

$$,i{=}j,\ \Leftrightarrow\ \sim,i!{>}j,$$

proof:

$,i{=}j,$

$$\Leftrightarrow\ ,i{=}j,if(i{>}j)-\left[\begin{array}{l},\\ \\ ,\end{array}\right]-,$$

$$\Leftrightarrow\ , i{=}j, if(i{>}j) - \left[\begin{array}{l} , i{>}j, \\ \\ , \end{array}\right] - ,$$

$$\Leftrightarrow\ , if(i{>}j) - \left[\begin{array}{l} , i{=}j, i{>}j, \\ \\ , i{=}j, \end{array}\right] - ,$$

$$\Leftrightarrow\ , if(i{>}j) - \left[\begin{array}{l} , \otimes, \\ \\ , i{=}j, \end{array}\right] - ,$$

$$\Leftrightarrow\ , i!{>}j, i{=}j,$$

$$\Leftrightarrow\ , i{=}j, i!{>}j,$$

$$, i{=}j, \quad \Leftrightarrow \quad \sim, i!{<}j,$$

$$, i{>}j, \quad \Leftrightarrow \quad \sim, i!{=}j,$$
$$, i{>}j, \quad \Leftrightarrow \quad \sim, i!{<}j,$$

$$, i{<}j, \quad \Leftrightarrow \quad \sim, i!{=}j,$$
$$, i{<}j, \quad \Leftrightarrow \quad \sim, i!{>}j,$$

23 Rules of assign operator in temporary space

23.1 Definition of Flag object Tm

$$, m\oplus, \&Tm(m), \;\Leftrightarrow\; , \&Tm(m),$$

$$, t\textcircled{$\Rightarrow$}m, \&Tm(m), \;\Leftrightarrow\; , \&Tm(m),$$

$$, t\textcircled{$\Rightarrow$}m, t\textcircled{$\underline{\downarrow}$}, \&Tm(m), \;\Leftrightarrow\; , t\textcircled{$\underline{\downarrow}$}, \&Tm(m),$$

23.2 Definition of Flag object Fam

23.2.1 Transformation

$$, \&Fam(i), i\textcircled{$\curlywedge$}j, \;\Leftrightarrow\; , i\textcircled{$\curlywedge$}j, \&Fam(i), \&Fam(j),$$

$$, \&Fam(i), i\textcircled{$\lrcorner$}j, \;\Leftrightarrow\; , i\textcircled{$\lrcorner$}j, \&Fam(i), \&Fam(j),$$

$$, \&Fam(i), \&Fam(i), \;\Leftrightarrow\; , \&Fam(i),$$

23.2.2 Swap with self

$$, \&Fam(i), \&Fam(j), \;\Leftrightarrow\; , \&Fam(j), \&Fam(i),$$

23.2.3 Swap with operators

$$
\begin{aligned}
,\&Fam(i), j\text{☮}n, &\Leftrightarrow\ , j\text{☮}n, \&Fam(i),\\
,\&Fam(i), i\text{☮}n, &\Leftrightarrow\ , i\text{☮}n, \&Fam(i),
\end{aligned}
$$

$$
\begin{aligned}
,\&Fam(i), \odot j, &\Leftrightarrow\ , \odot j, \&Fam(i),\\
,\&Fam(i), \circledcirc j, &\Leftrightarrow\ , \circledcirc j, \&Fam(i),
\end{aligned}
$$

$$
\begin{aligned}
, i!\mathring{\bigcirc}j, \&Fam(i), j\text{☮}n, &\Leftrightarrow\ , i!\mathring{\bigcirc}j, j\text{☮}n, \&Fam(i),\\
, i!\mathring{\bigcirc}j, \&Fam(i), j\text{☮}n, &\Leftrightarrow\ , i!\mathring{\bigcirc}j, j\text{☮}n, \&Fam(i),
\end{aligned}
$$

$$
,\&Fam(i), j\text{\textcircled{$\downarrow$}}, \Leftrightarrow\ , j\text{\textcircled{$\downarrow$}}, \&Fam(i),
$$

$$
\begin{aligned}
,\&Fam(i), j\oplus, &\Leftrightarrow\ , j\oplus, \&Fam(i),\\
,\&Fam(i), i\oplus, &\Leftrightarrow\ , i\oplus, \&Fam(i),
\end{aligned}
$$

$$
,\&Fam(i), j\text{\textcircled{$\Rightarrow$}}n, \Leftrightarrow\ , j\text{\textcircled{$\Rightarrow$}}n, \&Fam(i),
$$

$$
,\&Fam(i), m\text{⊜}n\left[\begin{smallmatrix} , \\ , \end{smallmatrix}\right. \Leftrightarrow\ , m\text{⊜}n\left[\begin{smallmatrix} ,\&Fam(i), \\ ,\&Fam(i), \end{smallmatrix}\right.
$$

$$
,\&Fam(i), \left[\begin{smallmatrix} , \\ , \end{smallmatrix}\right. \Leftrightarrow\ , \left[\begin{smallmatrix} ,\&Fam(i), \\ ,\&Fam(i), \end{smallmatrix}\right.
$$

$$
\left.\begin{smallmatrix} , \\ , \end{smallmatrix}\right], \&Fam(i), \Leftrightarrow\ \left.\begin{smallmatrix} ,\&Fam(i), \\ ,\&Fam(i), \end{smallmatrix}\right],
$$

$$
,\&Fam(i), \otimes, \Leftrightarrow\ , \otimes, \&Fam(i),
$$

23.2.4 Clear Fam

$$
\begin{aligned}
,\&Fam(i), i\text{☮}m, \copyright c, m\text{\textcircled{$\downarrow$}}, \Leftrightarrow\ , i\text{☮}m, \text{ⓣ}ct, \&Fam(m), m\text{\textcircled{$\downarrow$}}, \ \Rightarrow\\
,\&Fam(i), i\text{☮}m, \copyright c, m\text{\textcircled{$\downarrow$}}, \Leftrightarrow\ , i\text{☮}m, \text{ⓣ}ct, m\text{\textcircled{$\downarrow$}},
\end{aligned}
$$

$$
\begin{aligned}
,\&Fam(i), i\text{☮}m, \copyright c, m\text{\textcircled{$\downarrow$}}, \Leftrightarrow\ , i\text{☮}m, \text{ⓣ}ct, \&Fam(m), m\text{\textcircled{$\downarrow$}}, \ \Rightarrow\\
,\&Fam(i), i\text{☮}m, \copyright c, m\text{\textcircled{$\downarrow$}}, \Leftrightarrow\ , i\text{☮}m, \text{ⓣ}ct, m\text{\textcircled{$\downarrow$}},
\end{aligned}
$$

23.3 Theorems of Flag object Fam

$$,\&Fam(i),i\text{☮}m,\copyright c,m\text{⏚}, \iff ,i\text{☮}m,\text{ⓣ}ct,\&Fam(m),m\text{⏚}, \implies$$
$$,i\text{☮}m,\text{ⓣ}ct,\&Fam(m),m\text{⏚}, \iff ,i\text{☮}m,\text{ⓣ}ct,m\text{⏚},$$

$$,\&Fam(i),i\text{☮}'m,\copyright c,m\text{⏚}, \iff ,i\text{☮}'m,\text{ⓣ}ct,\&Fam(m),m\text{⏚}, \implies$$
$$,i\text{☮}'m,\text{ⓣ}ct,\&Fam(m),m\text{⏚}, \iff ,i\text{☮}'m,\text{ⓣ}ct,m\text{⏚},$$

$$,\&Fam(i),i\ominus, \iff ,i\ominus,\&Fam(i),$$

proof:
$,\&Fam(i),i\ominus,$

$\iff ,i\oplus,i\ominus,\&Fam(i),i\ominus,$

$\iff ,i\ominus,i\oplus,\&Fam(i),i\ominus,$

$\iff ,i\ominus,\&Fam(i),i\oplus,i\ominus,$

$\iff ,i\ominus,\&Fam(i),$

$$,\&Fam(i),m\ominus, \iff ,m\ominus,\&Fam(i),$$

$$,i\overset{\circ}{\bigcirc}j,\&Fam(i), \iff ,\&Fam(i),i\overset{\circ}{\bigcirc}j,$$

proof:
$,i\overset{\circ}{\bigcirc}j,\&Fam(i),$

$\iff ,i\text{☮}m,j\text{☮}n,m=n,m\text{⏚},n\text{⏚},\&Fam(i),$

$\iff ,i\text{☮}m,j\text{☮}n,m\text{⊜}n\left[\begin{array}{l} , \\ ,\otimes, \end{array}\right],m\text{⏚},n\text{⏚},\&Fam(i),$

$\iff ,i\text{☮}m,j\text{☮}n,m\text{⊜}n\left[\begin{array}{l} , \\ ,\otimes, \end{array}\right],\&Fam(i),m\text{⏚},n\text{⏚},$

$\iff ,i\text{☮}m,j\text{☮}n,m\text{⊜}n\left[\begin{array}{l} ,\&Fam(i), \\ ,\otimes,\&Fam(i), \end{array}\right],m\text{⏚},n\text{⏚},$

$\iff ,i\text{☮}m,j\text{☮}n,m\text{⊜}n\left[\begin{array}{l} ,\&Fam(i), \\ ,\&Fam(i),\otimes, \end{array}\right],m\text{⏚},n\text{⏚},$

$$\Leftrightarrow\ ,i\circledast m, j\circledast n, \&Fam(i), m\circeq n\text{-}\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right]\text{-}, m\operp, n\operp,$$

$$\Leftrightarrow\ ,i\circledast m, \&Fam(i), j\circledast n, m\circeq n\text{-}\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right]\text{-}, m\operp, n\operp,$$

$$\Leftrightarrow\ ,\&Fam(i), i\circledast m, j\circledast n, m\circeq n\text{-}\left[\begin{array}{c} , \\ \\ ,\otimes, \end{array}\right]\text{-}, m\operp, n\operp,$$

$$\Leftrightarrow\ ,\&Fam(i), i\text{♂}j,$$

$$,i\text{♂}j, \&Fam(i), j\circledast n,\ \Leftrightarrow\ ,i\text{♂}j, j\circledast n, \&Fam(i), \&Fam(n),$$

proof:
$,i\text{♂}j, \&Fam(i), j\circledast n,$

$\Leftrightarrow\ ,\&Fam(i), i\text{♂}j, j\circledast n,$

$\Leftrightarrow\ ,\&Fam(i), i\text{♂}j, i\circledast n,$

$\Leftrightarrow\ ,i\text{♂}j, \&Fam(i), i\circledast n,$

$\Leftrightarrow\ ,i\text{♂}j, i\circledast n, \&Fam(i), \&Fam(n),$

$\Leftrightarrow\ ,i\text{♂}j, j\circledast n, \&Fam(i), \&Fam(n),$

$$,i\text{♂}j, \&Fam(i), j\circledast n,\ \Leftrightarrow\ ,i\text{♂}j, j\circledast n, \&Fam(i), \&Fam(j),$$

23.3.1 Swap with branch function:

$$,\&Fam(i), if(m=n)\text{-}\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},\ \Leftrightarrow\ ,if(m=n)\text{-}\left[\begin{array}{c} ,\&Fam(i),\copyright c_1, \\ \\ ,\&Fam(i),\copyright c_2, \end{array}\right]\text{-},$$

$$,\&Fam(i), if(m=\varnothing)\text{-}\left[\begin{array}{c} ,\copyright c_1, \\ \\ ,\copyright c_2, \end{array}\right]\text{-},\ \Leftrightarrow\ ,if(m=\varnothing)\text{-}\left[\begin{array}{c} ,\&Fam(i),\copyright c_1, \\ \\ ,\&Fam(i),\copyright c_2, \end{array}\right]\text{-},$$

$$,\&Fam(i), if(m♂n)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right], \;\Leftrightarrow\; , if(m♂n)\left[\begin{array}{l}, \&Fam(i), ©c_1,\\ \\ , \&Fam(i), ©c_2,\end{array}\right],$$

$$,\&Fam(i), if(m{\rightarrow}n)\left[\begin{array}{l}, ©c_1,\\ \\ , ©c_2,\end{array}\right], \;\Leftrightarrow\; , if(m{\rightarrow}n)\left[\begin{array}{l}, \&Fam(i), ©c_1,\\ \\ , \&Fam(i), ©c_2,\end{array}\right],$$

23.3.2 Swap with propositions:

$$,\&Fam(i), m{=}n, \;\Leftrightarrow\; , m{=}n, \&Fam(i),$$

$$,\&Fam(i), m{=}\varnothing, \;\Leftrightarrow\; , m{=}\varnothing, \&Fam(i),$$

$$,\&Fam(i), m♂n, \;\Leftrightarrow\; , m♂n, \&Fam(i),$$

$$,\&Fam(i), m{\rightarrow}n, \;\Leftrightarrow\; , m{\rightarrow}n, \&Fam(i),$$

$$,\&Fam(i), m\,!{=}\,n, \;\Leftrightarrow\; , m\,!{=}\,n, \&Fam(i),$$

$$,\&Fam(i), m\,!{=}\,\varnothing, \;\Leftrightarrow\; , m\,!{=}\,\varnothing, \&Fam(i),$$

$$,\&Fam(i), m!♂n, \;\Leftrightarrow\; , m!♂n, \&Fam(i),$$

$$,\&Fam(i), m!{\rightarrow}n, \;\Leftrightarrow\; , m!{\rightarrow}n, \&Fam(i),$$

23.3.3 Swap with recursive function:

$$,\&Fam(i),R(m),\ \Leftrightarrow\ ,R(m),\&Fam(i),$$

induction proof:
$premise\ 1:$
$,m=\varnothing,\&Fam(i),R(m),$
$\Leftrightarrow\ ,\&Fam(i),m=\varnothing,R(m),$
$\Leftrightarrow\ ,\&Fam(i),m=\varnothing,$
$\Leftrightarrow\ ,m=\varnothing,\&Fam(i),$
$\Leftrightarrow\ ,m=\varnothing,R(m),\&Fam(i),$

$premise\ 2:$
$,\ \&SHi\rightarrow m,\&Fam(i),R(m),\ \Leftrightarrow\ ,\ \&SHi\rightarrow m,R(m),\&Fam(i),\ \Rightarrow$

$,m\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc} m,\&Fam(i),R(m),,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} m,\&Fam(i),m\,!\!=\varnothing,R(m),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} m,\&Fam(i),m\,!\!=\varnothing,m\oplus,R(m),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} m,m\,!\!=\varnothing,m\oplus,\&Fam(i),R(m),$

$\Leftrightarrow\ ,m\,!\!=\varnothing,m\oplus,\ \&SHi\rightarrow m,\&Fam(i),R(m),$

$\Leftrightarrow\ ,m\,!\!=\varnothing,m\oplus,\ \&SHi\rightarrow m,R(m),\&Fam(i),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} m,m\,!\!=\varnothing,m\oplus,R(m),\&Fam(i),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} m,m\,!\!=\varnothing,R(m),\&Fam(i),$

$\Leftrightarrow\ ,m\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc} m,R(m),\&Fam(i),$

$conclusion:$
$,\&Fam(i),R(m),\ \Leftrightarrow\ ,R(m),\&Fam(i),$

$$,\&Fam(i),R\_(m),\ \Leftrightarrow\ ,R\_(m),\&Fam(i),$$

$$,\&Fam(i),Rc(m;n),\ \Leftrightarrow\ ,Rc(m;n),\&Fam(i),$$

$$,\&Fam(i),R(i),\ \Leftrightarrow\ ,R(i),\&Fam(i),$$

induction proof:

$premise\ 1:$

$, i=\varnothing, \&Fam(i), R(i),$

$\Leftrightarrow\ , \&Fam(i), i=\varnothing, R(i),$

$\Leftrightarrow\ , \&Fam(i), i=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, \&Fam(i),$

$\Leftrightarrow\ , i=\varnothing, R(i), \&Fam(i),$

$premise\ 2:$

$, \&SHi \rightarrow i, \&Fam(i), R(i),\ \Leftrightarrow\ , \&SHi \rightarrow i, R(i), \&Fam(i),\ \Rightarrow$

$, i \,!\!= \varnothing,\ \&SHi \text{♂} i, \&Fam(i), R(i), ,$

$\Leftrightarrow\ , \&SHi \text{♂} i, \&Fam(i), i \,!\!= \varnothing, R(i),$

$\Leftrightarrow\ , \&SHi \text{♂} i, \&Fam(i), i \,!\!= \varnothing, i\oplus, R(i),$

$\Leftrightarrow\ , \&SHi \text{♂} i, i \,!\!= \varnothing, i\oplus, \&Fam(i), R(i),$

$\Leftrightarrow\ , i \,!\!= \varnothing, i\oplus,\ \&SHi \rightarrow i, \&Fam(i), R(i),$

$\Leftrightarrow\ , i \,!\!= \varnothing, i\oplus,\ \&SHi \rightarrow i, R(i), \&Fam(i),$

$\Leftrightarrow\ , \&SHi \text{♂} i, i \,!\!= \varnothing, i\oplus, R(i), \&Fam(i),$

$\Leftrightarrow\ , \&SHi \text{♂} i, i \,!\!= \varnothing, R(i), \&Fam(i),$

$\Leftrightarrow\ , i \,!\!= \varnothing,\ \&SHi \text{♂} i, R(i), \&Fam(i),$

$conclusion:$

$, \&Fam(i), R(i),\ \Leftrightarrow\ , R(i), \&Fam(i),$

$$, \&Fam(i), R\_(i),\ \Leftrightarrow\ , R\_(i), \&Fam(i),$$

$$, \&Fam(i), Rc(i;n),\ \Leftrightarrow\ , Rc(i;n), \&Fam(i),$$

$$, \&Fam(i), i\text{☮}m, R(m), R(n), m\text{♂}n, m\text{⏚},\ \Leftrightarrow\ , i\text{☮}m, R(m), R(n), m\text{♂}n, m\text{⏚}, \&Fam(i),$$

proof:

$, \&Fam(i), i\text{☮}m, R(m), R(n), m\text{♂}n, m\text{⏚},$

$$\Leftrightarrow\ , i☮m, \&Fam(i), \&Fam(m), R(m), R(n), m♂n, m\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i☮m, R(m), R(n), m♂n, \&Fam(i), \&Fam(m), m\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i☮m, R(m), R(n), m♂n, \&Fam(i), m\textcircled{\downarrow},$$

$$\Leftrightarrow\ , i☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \&Fam(i),$$

$$, \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow},\ \Leftrightarrow\ , j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \&Fam(i),$$

proof:
$, \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow},$

$$\Leftrightarrow\ , if(i♂j)\left[\begin{array}{l} , \\ , \end{array}\right], \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow},$$

$$\Leftrightarrow\ , if(i♂j)\left[\begin{array}{l} , i♂j, \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \\ , i!♂j, \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \end{array}\right],$$

$$\Leftrightarrow\ , if(i♂j)\left[\begin{array}{l} , i♂j, \&Fam(i), i☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \\ , i!♂j, \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \end{array}\right],$$

$$\Leftrightarrow\ , if(i♂j)\left[\begin{array}{l} , i♂j, i☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \&Fam(i), \\ , i!♂j, \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \end{array}\right],$$

$$\Leftrightarrow\ , if(i♂j)\left[\begin{array}{l} , i♂j, j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \&Fam(i), \\ , i!♂j, \&Fam(i), j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \end{array}\right],$$

$$\Leftrightarrow\ , if(i♂j)\left[\begin{array}{l} , i♂j, j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \&Fam(i), \\ , i!♂j, j☮m, \&Fam(i), R(m), R(n), m♂n, m\textcircled{\downarrow}, \end{array}\right],$$

$$\Leftrightarrow\ , if(i♂j)\left[\begin{array}{l} , i♂j, j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \&Fam(i), \\ , i!♂j, j☮m, R(m), R(n), m♂n, m\textcircled{\downarrow}, \&Fam(i), \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)-\begin{bmatrix} ,j\text{☮}m,R(m),R(n),m\overset{\circ}{\bigcirc}n,m\text{⏚},\&Fam(i), \\ \\ ,j\text{☮}m,R(m),R(n),m\overset{\circ}{\bigcirc}n,m\text{⏚},\&Fam(i), \end{bmatrix}-,$$

$$\Leftrightarrow\ ,if(i\overset{\circ}{\bigcirc}j)-\begin{bmatrix} , \\ \\ , \end{bmatrix}-,j\text{☮}m,R(m),R(n),m\overset{\circ}{\bigcirc}n,m\text{⏚},\&Fam(i),$$

$$\Leftrightarrow\ ,j\text{☮}m,R(m),R(n),m\overset{\circ}{\bigcirc}n,m\text{⏚},\&Fam(i),$$

23.3.4 Swap with branch function:

$$,\&Fam(i),if(m\overset{\circ\circ}{\bigcirc}n)-\begin{bmatrix} ,©c_1, \\ \\ ,©c_2, \end{bmatrix}-,\ \Leftrightarrow\ ,if(m\overset{\circ\circ}{\bigcirc}n)-\begin{bmatrix} ,\&Fam(i),©c_1, \\ \\ ,\&Fam(i),©c_2, \end{bmatrix}-,$$

$$,\&Fam(i),if(m\text{⦹}n)-\begin{bmatrix} ,©c_1, \\ \\ ,©c_2, \end{bmatrix}-,\ \Leftrightarrow\ ,if(m\text{⦹}n)-\begin{bmatrix} ,\&Fam(i),©c_1, \\ \\ ,\&Fam(i),©c_2, \end{bmatrix}-,$$

$$,\&Fam(i),if(m\text{⌶}n)-\begin{bmatrix} ,©c_1, \\ \\ ,©c_2, \end{bmatrix}-,\ \Leftrightarrow\ ,if(m\text{⌶}n)-\begin{bmatrix} ,\&Fam(i),©c_1, \\ \\ ,\&Fam(i),©c_2, \end{bmatrix}-,$$

$$,\&Fam(i),if(m{>\!\!-}n)-\begin{bmatrix} ,©c_1, \\ \\ ,©c_2, \end{bmatrix}-,\ \Leftrightarrow\ ,if(m{>\!\!-}n)-\begin{bmatrix} ,\&Fam(i),©c_1, \\ \\ ,\&Fam(i),©c_2, \end{bmatrix}-,$$

23.3.5 Swap with propositions:

$$,\&Fam(i), i\bigcirc^{\circ}j, \;\Leftrightarrow\; ,i\bigcirc^{\circ}j, \&Fam(i),$$

proof:
$,\&Fam(i), i\bigcirc^{\circ}j,$

$\Leftrightarrow\;,\&Fam(i), i\otimes m, j\otimes n, R(m), R(n), m\mathring{\bigcirc}n, n\odot, m\odot,$

$\Leftrightarrow\;,i\otimes m, \&Fam(i), \&Fam(m), j\otimes n, R(m), R(n), m\mathring{\bigcirc}n, n\odot, m\odot,$

$\Leftrightarrow\;,i\otimes m, j\otimes n, R(m), R(n), m\mathring{\bigcirc}n, n\odot, \&Fam(i), \&Fam(m), m\odot,$

$\Leftrightarrow\;,i\otimes m, j\otimes n, R(m), R(n), m\mathring{\bigcirc}n, n\odot, \&Fam(i), m\odot,$

$\Leftrightarrow\;,i\otimes m, j\otimes n, R(m), R(n), m\mathring{\bigcirc}n, m\odot, n\odot, \&Fam(i),$

$$,\&Fam(i), m\bigcirc^{\circ}n, \;\Leftrightarrow\; ,m\bigcirc^{\circ}n, \&Fam(i),$$

proof:
$,\&Fam(i), m\bigcirc^{\circ}n,$

$\Leftrightarrow\;,\&Fam(i), m\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, m_0\odot, n_0\odot,$

$\Leftrightarrow\;,if(i\mathring{\bigcirc}m)\left[\begin{array}{l}, \\ ,\end{array}\right], \&Fam(i), m\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot,$

$\Leftrightarrow\;,if(i\mathring{\bigcirc}m)\left[\begin{array}{l}, i\mathring{\bigcirc}m, \&Fam(i), m\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot, \\ , i!\mathring{\bigcirc}m, \&Fam(i), m\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot,\end{array}\right],$

$\Leftrightarrow\;,if(i\mathring{\bigcirc}m)\left[\begin{array}{l}, i\mathring{\bigcirc}m, \&Fam(i), i\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot, \\ , i!\mathring{\bigcirc}m, \&Fam(i), m\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot,\end{array}\right],$

$\Leftrightarrow\;,if(i\mathring{\bigcirc}m)\left[\begin{array}{l}, i\mathring{\bigcirc}m, i\otimes m_0, \&Fam(i), \&Fam(m_0), n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot, \\ , i!\mathring{\bigcirc}m, \&Fam(i), m\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot,\end{array}\right],$

$\Leftrightarrow\;,if(i\mathring{\bigcirc}m)\left[\begin{array}{l}, i\mathring{\bigcirc}m, i\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, \&Fam(i), \&Fam(m_0), m_0\odot, \\ , i!\mathring{\bigcirc}m, \&Fam(i), m\otimes m_0, n\otimes n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc}n_0, n_0\odot, m_0\odot,\end{array}\right],$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}m)\text{-}\left[\begin{array}{l},i\mathring{\bigcirc}m,i\owedge m_0,n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,n_0\mathbin{\bigcirc\mkern-14mu\downarrow},\&Fam(i),m_0\mathbin{\bigcirc\mkern-14mu\downarrow},\\ \\ ,i!\mathring{\bigcirc}m,\&Fam(i),m\owedge m_0,n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,n_0\mathbin{\bigcirc\mkern-14mu\downarrow},m_0\mathbin{\bigcirc\mkern-14mu\downarrow},\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}m)\text{-}\left[\begin{array}{l},i\mathring{\bigcirc}m,i\owedge m_0,n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,n_0\mathbin{\bigcirc\mkern-14mu\downarrow},\&Fam(i),m_0\mathbin{\bigcirc\mkern-14mu\downarrow},\\ \\ ,i!\mathring{\bigcirc}m,m\owedge m_0,\&Fam(i),n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,n_0\mathbin{\bigcirc\mkern-14mu\downarrow},m_0\mathbin{\bigcirc\mkern-14mu\downarrow},\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}m)\text{-}\left[\begin{array}{l},i\mathring{\bigcirc}m,i\owedge m_0,n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,n_0\mathbin{\bigcirc\mkern-14mu\downarrow},\&Fam(i),m_0\mathbin{\bigcirc\mkern-14mu\downarrow},\\ \\ ,i!\mathring{\bigcirc}m,m\owedge m_0,n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,n_0\mathbin{\bigcirc\mkern-14mu\downarrow},\&Fam(i),m_0\mathbin{\bigcirc\mkern-14mu\downarrow},\end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(i\mathring{\bigcirc}m)\text{-}\left[\begin{array}{l},i\mathring{\bigcirc}m,\\ \\ ,i!\mathring{\bigcirc}m,\end{array}\right]\text{-},m\owedge m_0,n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,n_0\mathbin{\bigcirc\mkern-14mu\downarrow},\&Fam(i),m_0\mathbin{\bigcirc\mkern-14mu\downarrow},$$

$$\Leftrightarrow\ ,m\owedge m_0,n\owedge n_0,R(m_0),R(n_0),m_0\mathring{\bigcirc}n_0,m_0\mathbin{\bigcirc\mkern-14mu\downarrow},n_0\mathbin{\bigcirc\mkern-14mu\downarrow},\&Fam(i),$$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}\!^{\circ}n,\&Fam(i),$$

$$,\&Fam(i),m\obot n,\ \Leftrightarrow\ ,m\obot n,\&Fam(i),$$

$$,\&Fam(i),m\mathbin{\top\mkern-14mu\bot}n,\ \Leftrightarrow\ ,m\mathbin{\top\mkern-14mu\bot}n,\&Fam(i),$$

$$,\&Fam(i),m\succ n,\ \Leftrightarrow\ ,m\succ n,\&Fam(i),$$

$$,\&Fam(i),m!\mathring{\bigcirc}\!^{\circ}n,\ \Leftrightarrow\ ,m!\mathring{\bigcirc}\!^{\circ}n,\&Fam(i),$$

$$,\&Fam(i),m!\obot n,\ \Leftrightarrow\ ,m!\obot n,\&Fam(i),$$

$$,\&Fam(i),m!\mathbin{\top\mkern-14mu\bot}n,\ \Leftrightarrow\ ,m!\mathbin{\top\mkern-14mu\bot}n,\&Fam(i),$$

$$,\&Fam(i),m!\succ n,\ \Leftrightarrow\ ,m!\succ n,\&Fam(i),$$

23.3.6 Swap of the same operand

(skip......)

23.4 Axiom of Flag object Tm and Fam

23.4.1 axiom of inference 1:

$, \&Fam(i), ©c, \iff , ©c, \&Fam(i), \Longrightarrow$

$, \circledcirc i, ©c, i\textcircled{\downarrow}, \iff , \circledcirc i, ©c, \&Tm(i),$

23.4.2 axiom of inference 2:

$, \&Fam(i), ©c, \iff , ©c, \&Fam(i), \&Fam(m), \Longrightarrow$

$, \circledcirc i, ©c, i\textcircled{\downarrow}, m\textcircled{\downarrow}, \iff , \circledcirc i, ©c, i\textcircled{\downarrow}, \&Tm(m),$

23.5 Theorems of Tm

$$, \&Tm(m), \iff , m\ominus, \&Tm(m),$$

$$, \&Tm(m), \iff , R(m), \&Tm(m),$$

23.6 Theorems of temporary space

$$, \circledcirc m, m\textcircled{\downarrow}, \iff , \circledcirc m, \&Tm(m),$$

$$, \circledcirc m, t\textcircled{\Rightarrow}m, m\textcircled{\downarrow}, \iff , \circledcirc m, t\textcircled{\Rightarrow}m, \&Tm(m),$$

$$, \circledcirc m, \&Tm(m), \iff , \circledcirc m, t\textcircled{\Rightarrow}m, \&Tm(m),$$

$$, \circledcirc m, m\textcircled{\downarrow}, \iff , \circledcirc m, t\textcircled{\Rightarrow}m, m\textcircled{\downarrow},$$

24 Axioms of assign operator

24.1 General axioms

24.1.1 Substitution

$$,t\!=\!j, t\,\text{⇒⃝}\,j, \;\Leftrightarrow\; ,t\!=\!j, j\,\text{⇒⃝}\,j,$$

$$,t_1\!=\!t_2, t_1\,\text{⇒⃝}\,j, \;\Leftrightarrow\; ,t_1\!=\!t_2, t_2\,\text{⇒⃝}\,j,$$

$$,j_1\mathring{\bigcirc}j_2, t\,\text{⇒⃝}\,j_1, \;\Leftrightarrow\; ,j_1\mathring{\bigcirc}j_2, t\,\text{⇒⃝}\,j_2,$$

24.1.2 Unity

$$,i\!=\!\varnothing, \;\Leftrightarrow\; ,i\!=\!\varnothing, i\,\text{⇒⃝}\,i,$$

24.1.3 Swap

$$,g\,\text{↓⃝}, t\,\text{⇒⃝}\,j, \;\Leftrightarrow\; ,t\,\text{⇒⃝}\,j, g\,\text{↓⃝},$$

$$,g\,\text{↓⃝}, j\,\text{⇒⃝}\,j, \;\Leftrightarrow\; ,j\,\text{⇒⃝}\,j, g\,\text{↓⃝},$$

$$,\circledcirc g, t\,\text{⇒⃝}\,j, \;\Leftrightarrow\; ,t\,\text{⇒⃝}\,j, \circledcirc g,$$

$$,m\,\text{☮}\,n, t\,\text{⇒⃝}\,j, \;\Leftrightarrow\; ,t\,\text{⇒⃝}\,j, m\,\text{☮}\,n,$$

$$,m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, m\circeq n\!-\!\left[\begin{array}{l} ,t\,\text{⇒⃝}\,j, \\ \\ ,t\,\text{⇒⃝}\,j, \end{array}\right. \;\Leftrightarrow\; ,m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, t\,\text{⇒⃝}\,j, m\circeq n\!-\!\left[\begin{array}{l} , \\ \\ , \end{array}\right.$$

24.2 Definition of Del(j)

$$,Del(j), \;\Leftrightarrow\; ,j\,!\!=\varnothing, \circledcirc t, t\text{\textcircled{$\scriptstyle\Rightarrow$}}j, t\text{\textcircled{$\scriptstyle\downarrow$}},$$

24.3 Axioms of Del(j)

24.3.1 Mutation

$$,m\overset{\circ}{\bigcirc}j, m\oplus, Del(j), \;\Leftrightarrow\; ,m\overset{\circ}{\bigcirc}j, Del(j),$$

24.3.2 Swap

Id operator

$$,m!\overset{\circ}{\bigcirc}j, m\text{\textcircled{$\scriptstyle\wedge$}}n, Del(j), \;\Leftrightarrow\; ,m!\overset{\circ}{\bigcirc}j, Del(j), m\text{\textcircled{$\scriptstyle\wedge$}}n,$$

Next Node operator

$$,m!\overset{\circ}{\bigcirc}j, m\oplus, Del(j), \;\Leftrightarrow\; ,m!\overset{\circ}{\bigcirc}j, Del(j), m\oplus,$$

Global space operator

$$,\odot g, Del(j), \;\Leftrightarrow\; ,Del(j), \odot g,$$

Subnode operator

$$,m!\overset{\circ}{\bigcirc}j, m\text{\textcircled{$\scriptstyle\vee$}}n, Del(j), \;\Leftrightarrow\; ,m!\overset{\circ}{\bigcirc}j, Del(j), m\text{\textcircled{$\scriptstyle\vee$}}n,$$

24.4 Definition of Ins(t;j)

$$,Ins(t;j), \;\Leftrightarrow\; ,t\,!\!=\varnothing, t\text{\textcircled{$\scriptstyle\Rightarrow$}}j,$$

24.5 Axioms of Ins(t;j)

24.5.1 Mutation

$$, Ins(t;j), \;\Leftrightarrow\; \sim, t=j,$$

$$, Ins(t;j), j\circledast n, \;\Leftrightarrow\; \sim, n!\text{♂}i, n\rightarrow n,$$

24.5.2 Swap

Id operator

$$, j\circledast n, Ins(t;j), \;\Leftrightarrow\; , Ins(t;j), j\otimes j_0, j_0\oplus, j_0\circledast n, j_0\obot,$$

$$, m!\text{♂}j, m\circledast n, Ins(t;j), \;\Leftrightarrow\; , m!\text{♂}j, Ins(t;j), m\circledast n,$$

Next Node operator

$$, m!\text{♂}j, m\oplus, Ins(t;j), \;\Leftrightarrow\; , m!\text{♂}j, Ins(t;j), m\oplus,$$

$$, m\text{♂}j, m\oplus, Ins(t;j), \;\Leftrightarrow\; , m\text{♂}j, Ins(t;j), m\oplus, m\oplus,$$

Global space operator

$$, \odot g, g!\text{♂}j, Ins(t;j), \;\Leftrightarrow\; , Ins(t;j), \odot g, j!\rightarrow g,$$

$$, \odot g, g\text{♂}j, Ins(t;j), \;\Leftrightarrow\; , Ins(t;j), \odot g, g\ominus, g\text{♂}j,$$

Subnode operator

$$, j\,!\!=\varnothing, j\circledast n, Ins(t;j), \;\Leftrightarrow\; , j\,!\!=\varnothing, Ins(t;j), j\otimes j_0, j_0\oplus, j_0\circledast n, j_0\obot,$$

$$, m!\text{♂}j, m\circledast n, n!\text{♂}j, Ins(t;j), \;\Leftrightarrow\; , m!\text{♂}j, Ins(t;j), m\circledast n, j!\rightarrow n,$$

$$, m!\text{♂}j, m\circledast n, n\text{♂}j, Ins(t;j), \;\Leftrightarrow\; , m!\text{♂}j, Ins(t;j), m\circledast n, n\ominus, n\text{♂}j,$$

Node value

$$, m!\text{♂}j, j=m, Ins(t;j), \;\Leftrightarrow\; , m!\text{♂}j, Ins(t;j), j\otimes j_0, j_0\oplus, j_0=m, j_0\obot,$$

24.6 Swap definition of &SHi

24.6.1 Ins(t;j)

$$
\begin{aligned}
,i!\overset{\circ}{\bigcirc}{\circ}j,\ \&SHi\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i!\overset{\circ}{\bigcirc}{\circ}j, Ins(t;j),\ \&SHi\,\overset{\circ}{\bigcirc}i,\\
,i\overset{\circ}{\bigcirc}{\circ}j, j\gtrdot i,\ \&SHi\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i\overset{\circ}{\bigcirc}{\circ}j, j\gtrdot i, Ins(t;j),\ \&SHi\,\overset{\circ}{\bigcirc}i,\\
,i\overset{\circ}{\bigcirc}j,\ \&SHi\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i\overset{\circ}{\bigcirc}j, Ins(t;j),\ \&SHi\leftarrow i,\\
,i\overset{\circ}{\bigcirc}{\circ}j, j\lessdot i,\ \&SHi\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i\overset{\circ}{\bigcirc}{\circ}j, j\lessdot i, Ins(t;j),\ \&SHi\leftarrow i,
\end{aligned}
$$

24.6.2 Del(j)

$$
\begin{aligned}
,i!\overset{\circ}{\bigcirc}{\circ}j,\ \&SHi\,\overset{\circ}{\bigcirc}i, Del(j), &\iff ,i!\overset{\circ}{\bigcirc}{\circ}j, Del(j),\ \&SHi\,\overset{\circ}{\bigcirc}i,\\
,i\overset{\circ}{\bigcirc}{\circ}j, j\gtrdot i,\ \&SHi\,\overset{\circ}{\bigcirc}i, Del(j), &\iff ,i\overset{\circ}{\bigcirc}{\circ}j, j\gtrdot i, Del(j),\ \&SHi\,\overset{\circ}{\bigcirc}i,\\
,i\overset{\circ}{\bigcirc}j,\ \&SHi\,\overset{\circ}{\bigcirc}i, Del(j), &\iff ,i\overset{\circ}{\bigcirc}j, Del(j),\ \&SHi\rightarrow i,\\
,i\overset{\circ}{\bigcirc}{\circ}j, j\lessdot i,\ \&SHi\,\overset{\circ}{\bigcirc}i, Del(j), &\iff ,i\overset{\circ}{\bigcirc}{\circ}j, j\lessdot i, Del(j),\ \&SHi\rightarrow i,
\end{aligned}
$$

24.7 Swap definition of &SHj

24.7.1 Ins(t;j)

$$
\begin{aligned}
,i!\overset{\circ}{\bigcirc}{\circ}j,\ \&SHj\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i!\overset{\circ}{\bigcirc}{\circ}j, Ins(t;j),\ \&SHj\,\overset{\circ}{\bigcirc}i,\\
,i\overset{\circ}{\bigcirc}{\circ}j, j\lessdot i,\ \&SHj\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i\overset{\circ}{\bigcirc}{\circ}j, j\lessdot i, Ins(t;j),\ \&SHj\,\overset{\circ}{\bigcirc}i,\\
,i\overset{\circ}{\bigcirc}j,\ \&SHj\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i\overset{\circ}{\bigcirc}j, Ins(t;j),\ \&SHj\,\overset{\circ}{\bigcirc}i,\\
,i\overset{\circ}{\bigcirc}{\circ}j, j\gtrdot i,\ \&SHj\,\overset{\circ}{\bigcirc}i, Ins(t;j), &\iff ,i\overset{\circ}{\bigcirc}{\circ}j, j\gtrdot i, Ins(t;j),\ \&SHj\rightarrow i,
\end{aligned}
$$

24.7.2 Del(j)

$$, i!\mathring{\bigcirc}\!\circ j,\ \&SHj\,\mathring{\bigcirc}i, Del(j),\ \Leftrightarrow\ , i!\mathring{\bigcirc}\!\circ j, Del(j),\ \&SHj\,\mathring{\bigcirc}i,$$

$$, i\mathring{\bigcirc}\!\circ j, j\lessdot i,\ \&SHj\,\mathring{\bigcirc}i, Del(j),\ \Leftrightarrow\ , i\mathring{\bigcirc}\!\circ j, j\lessdot i, Del(j),\ \&SHj\,\mathring{\bigcirc}i,$$

$$, i\mathring{\bigcirc}j,\ \&SHj\,\mathring{\bigcirc}i, Del(j),\ \Leftrightarrow\ , i\mathring{\bigcirc}j, Del(j),\ \&SHj\,\mathring{\bigcirc}i,$$

$$, i\mathring{\bigcirc}\!\circ j, j\gtrdot i,\ \&SHj\,\mathring{\bigcirc}i, Del(j),\ \Leftrightarrow\ , i\mathring{\bigcirc}\!\circ j, j\gtrdot i, Del(j),\ \&SHj\leftarrow i,$$

24.8 Axioms of swap with self

24.8.1 Ins;Ins

$i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing$:
$i_1 = i_2$:

$, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1 = i_2, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1, i_2 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_2,\ \Leftrightarrow$

$, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1 = i_2, i_2 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_2, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1,$

$i_1 \mathrel{!=} i_2$:

$, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1 \mathrel{!=} i_2, j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_2!\mathring{\bigcirc}j_1, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1, i_2 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_2,\ \Leftrightarrow$

$, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1 \mathrel{!=} i_2, j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_2!\mathring{\bigcirc}j_1, i_2 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_2, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1,$

24.8.2 Del;Del

$i_1 = \varnothing, i_2 = \varnothing$:

$, i_1 = \varnothing, i_2 = \varnothing, j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1, i_2 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_2,\ \Leftrightarrow$

$, i_1 = \varnothing, i_2 = \varnothing, j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, i_2 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_2, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1,$

24.8.3 Ins;Del

$i_1 \mathrel{!{=}} \varnothing, i_2 = \varnothing :$

$j_1 \mathring{\bigcirc} j_2 :$
$, i_1 \mathrel{!{=}} \varnothing, i_2 = \varnothing, j_1 \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, \quad \Leftrightarrow$

$, i_1 \mathrel{!{=}} \varnothing, i_2 = \varnothing, j_1 \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, i_1 \text{\textcircled{$\Rightarrow$}} j_1, i_2 \text{\textcircled{$\Rightarrow$}} j_2,$

$j_1 ! \mathring{\bigcirc} j_2 :$
$, i_1 \mathrel{!{=}} \varnothing, i_2 = \varnothing, j_2 \mathrel{!{=}} \varnothing, j_1 ! \mathring{\bigcirc} j_2, i_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, i_1 \text{\textcircled{$\Rightarrow$}} j_1, i_2 \text{\textcircled{$\Rightarrow$}} j_2, \quad \Leftrightarrow$

$, i_1 \mathrel{!{=}} \varnothing, i_2 = \varnothing, j_2 \mathrel{!{=}} \varnothing, j_1 ! \mathring{\bigcirc} j_2, i_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, i_2 \text{\textcircled{$\Rightarrow$}} j_2, i_1 \text{\textcircled{$\Rightarrow$}} j_1,$

25 Theorems of Insert Node Function Ins(t;j)

25.1 General theorems

25.1.1 Property

$$, Insert(t;j), \ \Leftrightarrow \ , t \mathrel{!=} \varnothing, Insert(t;j),$$

$$, Ins(t;j), j \text{☮} n, \ \Leftrightarrow \ \sim, n ! \mathring{\bigcirc} i,$$

$$, Ins(t;j), j \text{☮} n, \ \Leftrightarrow \ \sim, n \rightarrow n,$$

25.1.2 Substitution

$$, t_1 = t_2, Ins(t_1;j), \ \Leftrightarrow \ , t_1 = t_2, Ins(t_2;j),$$

$$, t_1 \mathring{\bigcirc} t_2, Ins(t_1;j), \ \Leftrightarrow \ , t_1 \mathring{\bigcirc} t_2, Ins(t_2;j),$$

$$, j_1 \mathring{\bigcirc} j_2, Ins(t;j_1), \ \Leftrightarrow \ , j_1 \mathring{\bigcirc} j_2, Ins(t;j_2),$$

25.1.3 Swap with operator

$$, g \text{⏚}, Ins(t;j), \ \Leftrightarrow \ , Ins(t;j), g \text{⏚},$$

$$, \circledcirc g, Ins(t;j), \ \Leftrightarrow \ , Ins(t;j), \circledcirc g,$$

$$, m \text{☮} n, Ins(t;j), \ \Leftrightarrow \ , Ins(t;j), m \text{☮} n,$$

$$, j \text{☮} n, Ins(t;j), \ \Leftrightarrow \ , Ins(t;j), j \text{☮} n,$$

proof:
$, j \text{☮} n, Ins(t;j),$

$\Leftrightarrow \ , j \text{☮} n, j \text{☮} j_0, j_0 \text{⏚}, Ins(t;j),$

$\Leftrightarrow\ , j\circledvee n, j\circledvee j_0, Ins(t;j), j_0\circleddash,$

$\Leftrightarrow\ , j\circledvee n, j\circledvee j_0, j\mathring{\bigcirc} j_0, Ins(t;j), j_0\circleddash,$

$\Leftrightarrow\ , j\circledvee n, j\circledvee j_0, j\mathring{\bigcirc} j_0, Ins(t;j_0), j_0\circleddash,$

$\Leftrightarrow\ , j\circledvee j_0, j\mathring{\bigcirc} j_0, j\circledvee n, Ins(t;j_0), j_0\circleddash,$

$\Leftrightarrow\ , j\circledvee j_0, j\mathring{\bigcirc} j_0, Ins(t;j_0), j_0\circleddash, j\circledvee n,$

$\Leftrightarrow\ , j\circledvee j_0, j\mathring{\bigcirc} j_0, Ins(t;j), j_0\circleddash, j\circledvee n,$

$\Leftrightarrow\ , j\circledvee j_0, Ins(t;j), j_0\circleddash, j\circledvee n,$

$\Leftrightarrow\ , j\circledvee j_0, j_0\circleddash, Ins(t;j), j\circledvee n,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee n,$

$$, m\mathring{\bigcirc} j, m\circledcirc dm, Ins(t;j),\ \Leftrightarrow\ , m\mathring{\bigcirc} j, Ins(t;j), m\circledvee m_0, m_0\oplus, m_0\circledcirc dm, m_0\circleddash,$$

$$, j\circledcirc n, Ins(t;j), j\circledvee j_0, j_0\oplus,\ \Leftrightarrow\ , Ins(t;j), j\circledvee j_0, j_0\oplus, j_0\circledcirc n,$$

proof:
$, j\circledcirc n, Ins(t;j), j\circledvee j_0, j_0\oplus,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j_1\oplus, j_1\circledcirc n, j_1\circleddash, j\circledvee j_0, j_0\oplus,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j\circledvee j_0, j_1\oplus, j_0\oplus, j_1\circledcirc n, j_1\circleddash,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j\circledvee j_0, j_1\mathring{\bigcirc} j_0, j_1\oplus, j_0\oplus, j_1\circledcirc n, j_1\circleddash,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j\circledvee j_0, j_1\oplus, j_0\oplus, j_1\mathring{\bigcirc} j_0, j_1\circledcirc n, j_1\circleddash,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j\circledvee j_0, j_1\oplus, j_0\oplus, j_1\mathring{\bigcirc} j_0, j_0\circledcirc n, j_1\circleddash,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j\circledvee j_0, j_1\oplus, j_0\oplus, j_0\circledcirc n, j_1\circleddash,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j_1\oplus, j_1\circleddash, j\circledvee j_0, j_0\oplus, j_0\circledcirc n,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_1, j_1\circleddash, j\circledvee j_0, j_0\oplus, j_0\circledcirc n,$

$\Leftrightarrow\ , Ins(t;j), j\circledvee j_0, j_0\oplus, j_0\circledcirc n,$

$$, j \mathrel{!=} \varnothing, j \circledcirc n, Ins(t;j), j \text{☮} j_0, j_0\oplus, \iff , j \mathrel{!=} \varnothing, Ins(t;j), j \text{☮} j_0, j_0\oplus, j_0 \circledcirc n,$$

$$, j!\!\rightarrow\! m, m\ominus, Ins(t;j), \iff , j!\!\rightarrow\! m, Ins(t;j), m\ominus,$$

proof:
$, j!\!\rightarrow\! m, m\ominus, Ins(t;j),$

$\iff , m\ominus, m!\text{♂}j, Ins(t;j),$

$\iff , m\ominus, m!\text{♂}j, Ins(t;j), m\oplus, m\ominus,$

$\iff , m\ominus, m!\text{♂}j, m\oplus, Ins(t;j), m\ominus,$

$\iff , j!\!\rightarrow\! m, m\ominus, m\oplus, Ins(t;j), m\ominus,$

$\iff , j!\!\rightarrow\! m, m\oplus, m\ominus, Ins(t;j), m\ominus,$

$\iff , j!\!\rightarrow\! m, Ins(t;j), m\ominus,$

$$, j\!\rightarrow\! m, m\ominus, Ins(t;j), \iff , j\!\rightarrow\! m, Ins(t;j), m\ominus, m\ominus,$$

proof:
$, j\!\rightarrow\! m, m\ominus, Ins(t;j),$

$\iff , m\ominus, m\text{♂}j, Ins(t;j),$

$\iff , m\ominus, m\text{♂}j, Ins(t;j), m\oplus, m\ominus, m\oplus, m\ominus,$

$\iff , m\ominus, m\text{♂}j, Ins(t;j), m\oplus, m\oplus, m\ominus, m\ominus,$

$\iff , m\ominus, m\text{♂}j, m\oplus, Ins(t;j), m\ominus, m\ominus,$

$\iff , j\!\rightarrow\! m, m\ominus, m\oplus, Ins(t;j), m\ominus, m\ominus,$

$\iff , j\!\rightarrow\! m, m\oplus, m\ominus, Ins(t;j), m\ominus, m\ominus,$

$\iff , j\!\rightarrow\! m, Ins(t;j), m\ominus, m\ominus,$

$$, m!\text{♂}j, j = m, Ins(t;j), j \text{☮} j_0, j_0\oplus, \iff , m!\text{♂}j, Ins(t;j), j \text{☮} j_0, j_0\oplus, j_0 = m,$$

25.2 Propositions property

$$,t\,!{=}\,\varnothing,\;\;\Leftrightarrow\;\;,\circledcirc m, Ins(t;m), m\text{⏚},$$

$$,m!\text{♂}j, n!\text{♂}j, m{=}n, Ins(t;j),\;\;\Leftrightarrow\;\;,m!\text{♂}j, n!\text{♂}j, Ins(t;j), m{=}n,$$
$$,m!\text{♂}j, n!\text{♂}j, m\,!{=}\,n, Ins(t;j),\;\;\Leftrightarrow\;\;,m!\text{♂}j, n!\text{♂}j, Ins(t;j), m\,!{=}\,n,$$

$$,m!\text{♂}j, m\,!{=}\,\varnothing, Ins(t;j),\;\;\Leftrightarrow\;\;,m!\text{♂}j, Ins(t;j), m\,!{=}\,\varnothing,$$

proof:
$,m!\text{♂}j, m\,!{=}\,\varnothing, Ins(t;j),$

$\Leftrightarrow\;\;,m!\text{♂}j, \circledcirc n, m\,!{=}\,n, n\text{⏚}, Ins(t;j),$

$\Leftrightarrow\;\;,m!\text{♂}j, \circledcirc n, m\,!{=}\,n, Ins(t;j), n\text{⏚},$

$\Leftrightarrow\;\;,m!\text{♂}j, \circledcirc n, n!\text{♂}j, m\,!{=}\,n, Ins(t;j), n\text{⏚},$

$\Leftrightarrow\;\;,\circledcirc n, m!\text{♂}j, n!\text{♂}j, m\,!{=}\,n, Ins(t;j), n\text{⏚},$

$\Leftrightarrow\;\;,\circledcirc n, m!\text{♂}j, n!\text{♂}j, Ins(t;j), m\,!{=}\,n, n\text{⏚},$

$\Leftrightarrow\;\;,m!\text{♂}j, \circledcirc n, n!\text{♂}j, Ins(t;j), m\,!{=}\,n, n\text{⏚},$

$\Leftrightarrow\;\;,m!\text{♂}j, \circledcirc n, Ins(t;j), m\,!{=}\,n, n\text{⏚},$

$\Leftrightarrow\;\;,m!\text{♂}j, Ins(t;j), \circledcirc n, m\,!{=}\,n, n\text{⏚},$

$\Leftrightarrow\;\;,m!\text{♂}j, Ins(t;j), m\,!{=}\,\varnothing,$

$$,m!\text{♂}j, m{=}\varnothing, Ins(t;j),\;\;\Leftrightarrow\;\;,m!\text{♂}j, Ins(t;j), m{=}\varnothing,$$

$$,t!\text{♂}j, Ins(t;j),\;\;\Leftrightarrow\;\;\sim, j\,!{=}\,\varnothing,$$

proof:
$,t!\text{♂}j, Ins(t;j),$

$\Leftrightarrow\;\;,t!\text{♂}j, t\,!{=}\,\varnothing, Ins(t;j),$

$\Leftrightarrow\;\;,t!\text{♂}j, t\,!{=}\,\varnothing, t\text{☮}t_0, t_0\text{⏚}, Ins(t;j),$

$\Leftrightarrow\ , t!♂j, t\,!{=}\,\varnothing, t☮t_0, Ins(t;j), t_0⊕,$

$\Leftrightarrow\ , t!♂j, t\,!{=}\,\varnothing, t☮t_0, t♂t_0, Ins(t;j), t_0⊕,$

$\Leftrightarrow\ , t!♂j, t\,!{=}\,\varnothing, t☮t_0, t♂t_0, Ins(t_0;j), t_0⊕,$

$\Leftrightarrow\ , t☮t_0, t♂t_0, t!♂j, t\,!{=}\,\varnothing, Ins(t_0;j), t_0⊕,$

$\Leftrightarrow\ , t☮t_0, t♂t_0, t!♂j, Ins(t_0;j), t\,!{=}\,\varnothing, t_0⊕,$

$\Leftrightarrow\ , t!♂j, t☮t_0, t♂t_0, Ins(t_0;j), t\,!{=}\,\varnothing, t_0⊕,$

$\Leftrightarrow\ , t!♂j, t☮t_0, t♂t_0, Ins(t;j), t\,!{=}\,\varnothing, t_0⊕,$

$\Leftrightarrow\ , t!♂j, t☮t_0, t♂t_0, t_0⊕, Ins(t;j), t\,!{=}\,\varnothing,$

$\Leftrightarrow\ , t!♂j, Ins(t;j), t\,!{=}\,\varnothing,$

$\Leftrightarrow\ , t!♂j, Ins(t;j), t=j, t\,!{=}\,\varnothing,$

$\Leftrightarrow\ , t!♂j, Ins(t;j), t=j, j\,!{=}\,\varnothing,$

$\Leftrightarrow\ , t!♂j, Ins(t;j), j\,!{=}\,\varnothing,$

$$, t!♂j, m!♂j, Ins(t;j), m♂j, \ \Leftrightarrow\ , \otimes,$$

proof:
$, t!♂j, m!♂j, Ins(t;j), m♂j,$

$\Leftrightarrow\ , t!♂j, m!♂j, m☮n_0, n_0⊕, Ins(t;j), m♂j,$

$\Leftrightarrow\ , t!♂j, m!♂j, m☮n_0, Ins(t;j), m♂j, n_0⊕,$

$\Leftrightarrow\ , t!♂j, m!♂j, m☮n_0, if(n_0♂j)\left[\begin{array}{l}, \\ \\ , \end{array}\right], Ins(t;j), m♂j, n_0⊕,$

$\Leftrightarrow\ , t!♂j, m!♂j, m☮n_0, \left[\begin{array}{l}, n_0♂j, \\ \\ , n_0!♂j, \end{array}\right], Ins(t;j), m♂j, n_0⊕,$

$\Leftrightarrow\ , \left[\begin{array}{l}, t!♂j, m!♂j, m☮n_0, n_0♂j, Ins(t;j), m♂j, n_0⊕, \\ , t!♂j, m!♂j, m☮n_0, n_0!♂j, Ins(t;j), m♂j, n_0⊕, \end{array}\right],$

$\Leftrightarrow \;\; < 1 >$

$, t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, m\ovee n_0, n_0\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, Ins(t;j), m\ovee n_0, n_0\ominus, n_0\mathring{\bigcirc}j, m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m\ovee n_0, n_0\ominus, n_0\mathring{\bigcirc}j, m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), j \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, n_0\mathring{\bigcirc}j, m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, j \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, n_0\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, m \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, n_0\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, m \mathrel{!=} \varnothing, m\ovee n_0, m\obot n_0, n_0\ominus, n_0\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, m \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, m\obot n_0, n_0\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, m\mathring{\bigcirc}j, m\obot n_0, n_0\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, m\mathring{\bigcirc}j, j\obot n_0, n_0\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, m\mathring{\bigcirc}j, n_0\mathring{\bigcirc}j, j\obot n_0, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , m!\mathring{\bigcirc}j, t!\mathring{\bigcirc}j, Ins(t;j), m \mathrel{!=} \varnothing, m\ovee n_0, n_0\ominus, m\mathring{\bigcirc}j, \otimes, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , \otimes,$

$, t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, m\ovee n_0, n_0!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, m\ovee n_0, n_0!\mathring{\bigcirc}j, Ins(t;j), j\ovee n_1, n_1\textcircled{$\downarrow$}, m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, m\ovee n_0, n_0!\mathring{\bigcirc}j, Ins(t;j), j\ovee n_1, n_1!\bigcirc n_0, n_1\textcircled{$\downarrow$}, m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, Ins(t;j), m\ovee n_0, j!{\rightarrow} n_0, j\ovee n_1, n_1!\bigcirc n_0, n_1\textcircled{$\downarrow$}, m\mathring{\bigcirc}j, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, m\ovee n_0, j\ovee n_1, n_1!\bigcirc n_0, j!{\rightarrow} n_0, n_1\textcircled{$\downarrow$}, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, m\ovee n_0, j\ovee n_1, n_0\bigcirc n_1, n_1!\bigcirc n_0, j!{\rightarrow} n_0, n_1\textcircled{$\downarrow$}, n_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;\; , t!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, m\ovee n_0, j\ovee n_1, \otimes, j!{\rightarrow} n_0, n_1\textcircled{$\downarrow$}, n_0\textcircled{$\downarrow$},$

$$\Leftrightarrow\ ,\otimes,$$

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$$\Leftrightarrow\ ,\left[\begin{array}{l} ,\otimes, \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,m=j,\otimes, \\ \\ ,m\,!=j,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,m=j, \\ \\ ,m\,!=j, \end{array}\right],\otimes,$$

$$\Leftrightarrow\ ,if(m=j)\left[\begin{array}{l} , \\ \\ , \end{array}\right],\otimes,$$

$$\Leftrightarrow\ ,\otimes,$$

$$,t!♂j,m!♂j,Ins(t;j),\ \Leftrightarrow\ \sim,m!♂j,$$

proof:
$,t!♂j,m!♂j,Ins(t;j),$

$$\Leftrightarrow\ ,t!♂j,m!♂j,Ins(t;j),if(m♂j)\left[\begin{array}{l} , \\ \\ , \end{array}\right],$$

$$\Leftrightarrow\ ,t!♂j,m!♂j,Ins(t;j),\left[\begin{array}{l} ,m♂j, \\ \\ ,m!♂j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,t!♂j,m!♂j,Ins(t;j),m♂j, \\ \\ ,t!♂j,m!♂j,Ins(t;j),m!♂j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,\otimes, \\ \\ ,t!♂j,m!♂j,Ins(t;j),m!♂j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,t!♂j,m!♂j,Ins(t;j),m♂j,\otimes, \\ \\ ,t!♂j,m!♂j,Ins(t;j),m!♂j, \end{array}\right],$$

$$\Leftrightarrow\ ,t!\male j, m!\male j, Ins(t;j), \left[\begin{array}{l} ,m\male j,\otimes, \\ \\ ,m!\male j, \end{array}\right],$$

$$\Leftrightarrow\ ,t!\male j, m!\male j, Ins(t;j), if(m\male j)\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right],$$

$$\Leftrightarrow\ ,t!\male j, m!\male j, Ins(t;j), m!\male j,$$

$$,m!\male j, m=t, Ins(t;j),\ \ \Leftrightarrow\ \ \sim, j\,!\!=\varnothing,$$

proof:
$,m!\male j, m=t, Ins(t;j),$

$$\Leftrightarrow\ ,m!\male j, m=t, Ins(t;j),$$

$$\Leftrightarrow\ ,m!\male j, m=t, Ins(m;j),$$

$$\Leftrightarrow\ ,m=t, m!\male j, Ins(m;j),$$

$$\Leftrightarrow\ ,m=t, m!\male j, Ins(m;j), j\,!\!=\varnothing,$$

$$\Leftrightarrow\ ,m!\male j, m=t, Ins(m;j), j\,!\!=\varnothing,$$

$$\Leftrightarrow\ ,m!\male j, m=t, Ins(t;j), j\,!\!=\varnothing,$$

$$,Ins(t;j),\ \ \Leftrightarrow\ \ \sim, j\,!\!=\varnothing,$$

proof:
$,Ins(t;j),$

$$\Leftrightarrow\ ,t\,!\!=\varnothing, Ins(t;j),$$

$$\Leftrightarrow\ ,\circledcirc m, Ins(t;m), m\textcircled{$\downarrow$}, Ins(t;j),$$

$$\Leftrightarrow\ ,\circledcirc m, Ins(t;m), Ins(t;j), m\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc m, t!\male m, j!\male m, Ins(t;m), Ins(t;j), m\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc m, t!\male m, j!\male m, Ins(t;m), j!\male m, Ins(t;j), m\textcircled{$\downarrow$},$$

$\Leftrightarrow\ ,\circledcirc m, t!\male m, j!\male m, Ins(t;m), m = t, j!\male m, Ins(t;j), m\circledast,$

$\Leftrightarrow\ ,\circledcirc m, t!\male m, j!\male m, Ins(t;m), m!\male j, m = t, Ins(t;j), m\circledast,$

$\Leftrightarrow\ ,\circledcirc m, t!\male m, j!\male m, Ins(t;m), m!\male j, m = t, Ins(t;j), j\,!\!=\varnothing, m\circledast,$

$\Leftrightarrow\ ,\circledcirc m, t!\male m, j!\male m, Ins(t;m), Ins(t;j), j\,!\!=\varnothing, m\circledast,$

$\Leftrightarrow\ ,\circledcirc m, Ins(t;m), Ins(t;j), j\,!\!=\varnothing, m\circledast,$

$\Leftrightarrow\ ,\circledcirc m, Ins(t;m), m\circledast, Ins(t;j), j\,!\!=\varnothing,$

$\Leftrightarrow\ , t\,!\!=\varnothing, Ins(t;j), j\,!\!=\varnothing,$

$\Leftrightarrow\ , Ins(t;j), j\,!\!=\varnothing,$

$$, Ins(t;j),\ \Leftrightarrow\ \sim, t\,!\!=\varnothing,$$

$$, m!\male j, Ins(t;j),\ \Leftrightarrow\ \sim, m!\male j,$$

proof:
$, m!\male j, Ins(t;j),$

$\Leftrightarrow\ , m!\male j, t\,!\!=\varnothing, Ins(t;j),$

$\Leftrightarrow\ , t\,!\!=\varnothing, m!\male j, Ins(t;j),$

$\Leftrightarrow\ ,\circledcirc n, Ins(t;n), n\circledast, m!\male j, Ins(t;j),$

$\Leftrightarrow\ ,\circledcirc n, Ins(t;n), m!\male j, Ins(t;j), n\circledast,$

$\Leftrightarrow\ ,\circledcirc n, t!\male n, j!\male n, Ins(t;n), m!\male j, Ins(t;j), n\circledast,$

$\Leftrightarrow\ ,\circledcirc n, t!\male n, j!\male n, Ins(t;n), j!\male n, m!\male j, Ins(t;j), n\circledast,$

$\Leftrightarrow\ ,\circledcirc n, t!\male n, j!\male n, Ins(t;n), t = n, j!\male n, m!\male j, Ins(t;j), n\circledast,$

$\Leftrightarrow\ ,\circledcirc n, t!\male n, j!\male n, Ins(t;n), n!\male j, m!\male j, t = n, Ins(t;j), n\circledast,$

$\Leftrightarrow\ ,\circledcirc n, t!\male n, j!\male n, Ins(t;n), n!\male j, m!\male j, t = n, Ins(n;j), n\circledast,$

$\Leftrightarrow\ ,\circledcirc n, t!\male n, j!\male n, Ins(t;n), t = n, n!\male j, m!\male j, Ins(n;j), n\circledast,$

$$\Leftrightarrow\ ,\circledcirc n, t!\mathring{\bigcirc} n, j!\mathring{\bigcirc} n, Ins(t;n), t=n, n!\mathring{\bigcirc} j, m!\mathring{\bigcirc} j, Ins(n;j), m!\mathring{\bigcirc} j, n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc n, t!\mathring{\bigcirc} n, j!\mathring{\bigcirc} n, Ins(t;n), n!\mathring{\bigcirc} j, m!\mathring{\bigcirc} j, t=n, Ins(n;j), m!\mathring{\bigcirc} j, n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc n, t!\mathring{\bigcirc} n, j!\mathring{\bigcirc} n, Ins(t;n), n!\mathring{\bigcirc} j, m!\mathring{\bigcirc} j, t=n, Ins(t;j), m!\mathring{\bigcirc} j, n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc n, t!\mathring{\bigcirc} n, j!\mathring{\bigcirc} n, Ins(t;n), m!\mathring{\bigcirc} j, t=n, Ins(t;j), m!\mathring{\bigcirc} j, n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc n, Ins(t;n), m!\mathring{\bigcirc} j, t=n, Ins(t;j), m!\mathring{\bigcirc} j, n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc n, Ins(t;n), t=n, m!\mathring{\bigcirc} j, Ins(t;j), m!\mathring{\bigcirc} j, n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc n, Ins(t;n), m!\mathring{\bigcirc} j, Ins(t;j), m!\mathring{\bigcirc} j, n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,\circledcirc n, Ins(t;n), n\textcircled{$\downarrow$}, m!\mathring{\bigcirc} j, Ins(t;j), m!\mathring{\bigcirc} j,$$

$$\Leftrightarrow\ ,t\,!\!=\varnothing, m!\mathring{\bigcirc} j, Ins(t;j), m!\mathring{\bigcirc} j,$$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc} j, t\,!\!=\varnothing, Ins(t;j), m!\mathring{\bigcirc} j,$$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc} j, Ins(t;j), m!\mathring{\bigcirc} j,$$

25.3 Swap with identical node propositions

$$,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Ins(t;j), \ \Leftrightarrow\ ,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, Ins(t;j), m\mathring{\bigcirc} n,$$

proof:
$,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Ins(t;j),$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\textcircled{$\curlywedge$} dm, n\textcircled{$\curlywedge$} dn, dm=dn, dm\textcircled{$\downarrow$}, dn\textcircled{$\downarrow$}, Ins(t;j),$$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\textcircled{$\curlywedge$} dm, n\textcircled{$\curlywedge$} dn, dm=dn, Ins(t;j), dm\textcircled{$\downarrow$}, dn\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\textcircled{$\curlywedge$} dm, dm!\mathring{\bigcirc} j, n\textcircled{$\curlywedge$} dn, dn!\mathring{\bigcirc} j, dm=dn, Ins(t;j), dm\textcircled{$\downarrow$}, dn\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\textcircled{$\curlywedge$} dm, n\textcircled{$\curlywedge$} dn, dm!\mathring{\bigcirc} j, dn!\mathring{\bigcirc} j, dm=dn, Ins(t;j), dm\textcircled{$\downarrow$}, dn\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\textcircled{$\curlywedge$} dm, n\textcircled{$\curlywedge$} dn, dm!\mathring{\bigcirc} j, dn!\mathring{\bigcirc} j, Ins(t;j), dm=dn, dm\textcircled{$\downarrow$}, dn\textcircled{$\downarrow$},$$

$\Leftrightarrow\ , m!⦲j, n!⦲j, m⧂dm, n⧂dn, Ins(t;j), dm=dn, dm⏚, dn⏚,$

$\Leftrightarrow\ , m!⦲j, m⧂dm, n!⦲j, n⧂dn, Ins(t;j), dm=dn, dm⏚, dn⏚,$

$\Leftrightarrow\ , m!⦲j, m⧂dm, n!⦲j, Ins(t;j), n⧂dn, dm=dn, dm⏚, dn⏚,$

$\Leftrightarrow\ , n!⦲j, m!⦲j, m⧂dm, Ins(t;j), n⧂dn, dm=dn, dm⏚, dn⏚,$

$\Leftrightarrow\ , n!⦲j, m!⦲j, Ins(t;j), m⧂dm, n⧂dn, dm=dn, dm⏚, dn⏚,$

$\Leftrightarrow\ , m!⦲j, n!⦲j, Ins(t;j), m⦲n,$

$$, m!⦲j, n!⦲j, m!⦲n, Ins(t;j), \ \Leftrightarrow\ , m!⦲j, n!⦲j, Ins(t;j), m!⦲n,$$

$$, m⦲j, Ins(t;j), \ \Leftrightarrow\ \sim, m⦲j,$$

proof:
$, m⦲j, Ins(t;j),$

$\Leftrightarrow\ , m⦲j, m⦲j, Ins(t;j),$

$\Leftrightarrow\ , m⦲j, m⧂dm, j⧂dj, dm=dj, dm⏚, dj⏚, Ins(t;j),$

$\Leftrightarrow\ , m⦲j, m⧂dm, j⧂dj, dm=dj, Ins(t;j), dm⏚, dj⏚,$

$\Leftrightarrow\ , m⦲j, m⧂dm, dm!⦲j, j⧂dj, dj!⦲j, dm=dj, Ins(t;j), dm⏚, dj⏚,$

$\Leftrightarrow\ , m⦲j, m⧂dm, j⧂dj, dm!⦲j, dj!⦲j, dm=dj, Ins(t;j), dm⏚, dj⏚,$

$\Leftrightarrow\ , m⦲j, m⧂dm, j⧂dj, dm!⦲j, dj!⦲j, Ins(t;j), dm=dj, dm⏚, dj⏚,$

$\Leftrightarrow\ , m⦲j, m⧂dm, j⧂dj, Ins(t;j), dm=dj, dm⏚, dj⏚,$

$\Leftrightarrow\ , m⦲j, m⧂dm, Ins(t;j), j☮j_0, j_0\oplus, j_0⧂dj, j_0⏚, dm=dj, dm⏚, dj⏚,$

$\Leftrightarrow\ , m⦲j, Ins(t;j), m☮m_0, m_0\oplus, m_0⧂dm, m_0⏚, j☮j_0, j_0\oplus, j_0⧂dj, j_0⏚, dm=dj, dm⏚, dj⏚,$

$\Leftrightarrow\ , m⦲j, Ins(t;j), m☮m_0, m_0\oplus, j☮j_0, j_0\oplus, m_0⧂dm, j_0⧂dj, dm=dj, dm⏚, dj⏚, m_0⏚, j_0⏚,$

$\Leftrightarrow\ , m⦲j, Ins(t;j), m☮m_0, m_0\oplus, j☮j_0, j_0\oplus, m_0⦲j_0, m_0⏚, j_0⏚,$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}j,Ins(t;j),m\oslash m_0,j\oslash j_0,m_0\oplus,j_0\oplus,m_0\mathring{\bigcirc}j_0,m_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}j,Ins(t;j),m\oslash m_0,j\oslash j_0,m_0\mathring{\bigcirc}j_0,m_0\oplus,j_0\oplus,m_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}j,Ins(t;j),m\oslash m_0,j\oslash j_0,m_0\mathring{\bigcirc}j_0,m_0\oplus,m_0\textcircled{$\downarrow$},j_0\oplus,j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}j,Ins(t;j),m\oslash m_0,j\oslash j_0,m_0\mathring{\bigcirc}j_0,m_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}j,Ins(t;j),m\mathring{\bigcirc}j,m\oslash m_0,j\oslash j_0,m_0\textcircled{$\downarrow$},j_0\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}j,Ins(t;j),m\oslash m_0,m_0\textcircled{$\downarrow$},j\oslash j_0,j_0\textcircled{$\downarrow$},m\mathring{\bigcirc}j,$$

$$\Leftrightarrow\ ,m\mathring{\bigcirc}j,Ins(t;j),m\mathring{\bigcirc}j,$$

$$,Ins(t;j),m\mathring{\bigcirc}j,\ \Leftrightarrow\ ,m\mathring{\bigcirc}j,\sim,$$

proof:
$,Ins(t;j),m\mathring{\bigcirc}j,$

$$\Leftrightarrow\ ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},\\ ,\end{array}\right],Ins(t;j),m\mathring{\bigcirc}j,$$

$$\Leftrightarrow\ ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},m\mathring{\bigcirc}j,\\ ,m!\mathring{\bigcirc}j,\end{array}\right],Ins(t;j),m\mathring{\bigcirc}j,$$

$$\Leftrightarrow\ ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},m\mathring{\bigcirc}j,Ins(t;j),m\mathring{\bigcirc}j,\\ ,m!\mathring{\bigcirc}j,Ins(t;j),m\mathring{\bigcirc}j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},m\mathring{\bigcirc}j,Ins(t;j),m\mathring{\bigcirc}j,\\ ,m!\mathring{\bigcirc}j,Ins(t;j),m!\mathring{\bigcirc}j,m\mathring{\bigcirc}j,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},m\mathring{\bigcirc}j,Ins(t;j),m\mathring{\bigcirc}j,\\ ,m!\mathring{\bigcirc}j,Ins(t;j),\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l},m\mathring{\bigcirc}j,Ins(t;j),m\mathring{\bigcirc}j,\\ ,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ , if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, Ins(t;j), m\mathring{\bigcirc}j, \\ , \otimes, \end{array}\right],$$

$$\Leftrightarrow\ , m\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j,$$

$$, m\mathring{\bigcirc}j, Ins(t;j), \ \Leftrightarrow\ , Ins(t;j), m\mathring{\bigcirc}j,$$

proof:
$, m\mathring{\bigcirc}j, Ins(t;j),$

$$\Leftrightarrow\ , m\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j,$$

$$\Leftrightarrow\ , Ins(t;j), m\mathring{\bigcirc}j,$$

$$, m!\mathring{\bigcirc}j, Ins(t;j), \ \Leftrightarrow\ , Ins(t;j), m!\mathring{\bigcirc}j,$$

proof:
$, m!\mathring{\bigcirc}j, Ins(t;j),$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, Ins(t;j), if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ , \end{array}\right],$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, Ins(t;j), \left[\begin{array}{l}, m\mathring{\bigcirc}j, \\ , m!\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ , \left[\begin{array}{l}, m!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}j, \\ , m!\mathring{\bigcirc}j, Ins(t;j), m!\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ , \left[\begin{array}{l}, m!\mathring{\bigcirc}j, Ins(t;j), m!\mathring{\bigcirc}j, m\mathring{\bigcirc}j, \\ , m!\mathring{\bigcirc}j, Ins(t;j), m!\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ , \left[\begin{array}{l}, m!\mathring{\bigcirc}j, Ins(t;j), \otimes, \\ , m!\mathring{\bigcirc}j, Ins(t;j), m!\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ , \left[\begin{array}{l}, \otimes, \\ , m!\mathring{\bigcirc}j, Ins(t;j), m!\mathring{\bigcirc}j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,Ins(t;j),\otimes, \\ \\ ,m!♂j,Ins(t;j),m!♂j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,Ins(t;j),m♂j,m!♂j, \\ \\ ,m!♂j,Ins(t;j),m!♂j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,m♂j,Ins(t;j),m!♂j, \\ \\ ,m!♂j,Ins(t;j),m!♂j, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,m♂j, \\ \\ ,m!♂j, \end{array}\right],Ins(t;j),m!♂j,$$

$$\Leftrightarrow\ ,if(m♂j)\left[\begin{array}{l} , \\ \\ , \end{array}\right],Ins(t;j),m!♂j,$$

$$\Leftrightarrow\ ,Ins(t;j),m!♂j,$$

$$,m♂j,m♂n,Ins(t;j),\ \Leftrightarrow\ ,m♂j,Ins(t;j),m♂n,$$

proof:
$,m♂j,m♂n,Ins(t;j),$

$$\Leftrightarrow\ ,m♂j,j♂n,Ins(t;j),$$

$$\Leftrightarrow\ ,m♂j,n♂j,Ins(t;j),$$

$$\Leftrightarrow\ ,Ins(t;j),m♂j,n♂j,$$

$$\Leftrightarrow\ ,Ins(t;j),m♂j,j♂n,$$

$$\Leftrightarrow\ ,Ins(t;j),m♂j,m♂n,$$

$$\Leftrightarrow\ ,m♂j,Ins(t;j),m♂n,$$

$$,m!♂j,m♂n,Ins(t;j),\ \Leftrightarrow\ ,m!♂j,Ins(t;j),m♂n,$$

proof:
$, m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Ins(t;j),$

$\Leftrightarrow \;, m!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Ins(t;j),$

$\Leftrightarrow \;, m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Ins(t;j),$

$\Leftrightarrow \;, m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}n,$

$\Leftrightarrow \;, m!\mathring{\bigcirc}j, Ins(t;j), n!\mathring{\bigcirc}j, m\mathring{\bigcirc}n,$

$\Leftrightarrow \;, m!\mathring{\bigcirc}j, Ins(t;j), m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n,$

$\Leftrightarrow \;, m!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}n,$

$\Leftrightarrow \;, m!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}n,$

$$, m\mathring{\bigcirc}n, Ins(t;j), \;\Leftrightarrow\; , Ins(t;j), m\mathring{\bigcirc}n,$$

proof:
$, m\mathring{\bigcirc}n, Ins(t;j),$

$\Leftrightarrow \;, if(m\mathring{\bigcirc}j)\left[\begin{array}{l} , \\ , \end{array}\right], m\mathring{\bigcirc}n, Ins(t;j),$

$\Leftrightarrow \;, if(m\mathring{\bigcirc}j)\left[\begin{array}{l} , m\mathring{\bigcirc}j, \\ , m!\mathring{\bigcirc}j, \end{array}\right], m\mathring{\bigcirc}n, Ins(t;j),$

$\Leftrightarrow \;, if(m\mathring{\bigcirc}j)\left[\begin{array}{l} , m\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Ins(t;j), \\ , m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Ins(t;j), \end{array}\right],$

$\Leftrightarrow \;, if(m\mathring{\bigcirc}j)\left[\begin{array}{l} , m\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}n, \\ , m!\mathring{\bigcirc}j, Ins(t;j), m\mathring{\bigcirc}n, \end{array}\right],$

$\Leftrightarrow \;, if(m\mathring{\bigcirc}j)\left[\begin{array}{l} , m\mathring{\bigcirc}j, \\ , m!\mathring{\bigcirc}j, \end{array}\right], Ins(t;j), m\mathring{\bigcirc}n,$

$\Leftrightarrow \;, if(m\mathring{\bigcirc}j)\left[\begin{array}{l} , \\ , \end{array}\right], Ins(t;j), m\mathring{\bigcirc}n,$

$\Leftrightarrow\ , Ins(t;j), m\text{♂}n,$

$$, m!\text{♂}n, Ins(t;j),\ \Leftrightarrow\ , Ins(t;j), m!\text{♂}n,$$

25.4 Other

$$, j{\to}k, Ins(t;j),\ \Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\oplus, j_0{\to}k, j_0\ovee,$$

proof:
$, j{\to}k, Ins(t;j),$

$\Leftrightarrow\ , j\owedge m, m\oplus, m\text{♂}k, m\ovee, Ins(t;j),$

$\Leftrightarrow\ , j\owedge m, m\oplus, m\text{♂}k, Ins(t;j), m\ovee,$

$\Leftrightarrow\ , j\owedge m, m\oplus, Ins(t;j), m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , j\owedge m, m\text{♂}j, m\oplus, Ins(t;j), m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , j\owedge m, m\text{♂}j, Ins(t;j), m\oplus, m\oplus, m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , j\owedge m, Ins(t;j), m\oplus, m\oplus, m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , Ins(t;j), j\owedge m, m\oplus, m\oplus, m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\ovee, j\owedge m, m\oplus, m\oplus, m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\oplus, j_0\ovee, j\owedge m, m\oplus, m\oplus, m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\oplus, j_0\owedge j_1, j_1\ovee, j_0\ovee, j\owedge m, m\oplus, m\oplus, m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\oplus, j_0\owedge j_1, j_1\oplus, j_1\ovee, j_0\ovee, j\owedge m, m\oplus, m\oplus, m\text{♂}k, m\ovee,$

$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, j_0\owedge j_1, j_1\oplus, m\oplus, m\oplus, m\text{♂}k, m\ovee, j_1\ovee, j_0\ovee,$

$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, j_0\owedge j_1, j_1\oplus, m\oplus, m\text{♂}k, m\ovee, j_1\ovee, j_0\ovee,$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, m\varocircle j_0, j_0\oplus, m\oplus, j_0\owedge j_1, j_1\oplus, m\oplus, m\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, m\varocircle j_0, j_0\owedge j_1, j_1\oplus, m\oplus, m\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, m\varocircle j_0, j_0\owedge j_1, j_0\varocircle j_1, j_1\oplus, m\oplus, m\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, j_0\owedge j_1, j_0\varocircle j_1, m\varocircle j_0, j_1\oplus, m\oplus, m\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, j_0\owedge j_1, j_0\varocircle j_1, m\varocircle j_1, j_1\oplus, m\oplus, m\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, j_0\owedge j_1, j_0\varocircle j_1, j_1\oplus, m\oplus, m\varocircle j_1, m\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, j_0\owedge j_1, j_0\varocircle j_1, j_1\oplus, m\oplus, m\varocircle j_1, j_1\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, j_0\oplus, m\oplus, j_0\owedge j_1, j_1\oplus, m\oplus, j_1\varocircle k, m\ovee, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j\owedge m, m\oplus, m\oplus, m\ovee, j_0\oplus, j_0\owedge j_1, j_1\oplus, j_1\varocircle k, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\oplus, j_0\owedge j_1, j_1\oplus, j_1\varocircle k, j_1\ovee, j_0\ovee,$$

$$\Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\oplus, j_0{\rightarrow}k, j_0\ovee,$$

$$, j{\rightarrow}k, Ins(t;j), j\owedge j_0, j_0\oplus, \ \Leftrightarrow\ , Ins(t;j), j\owedge j_0, j_0\oplus, j_0{\rightarrow}k,$$

$$, m\,!{=}\varnothing, Ins(t;j), \ \Leftrightarrow\ , \sim, m\,!{=}\varnothing,$$

proof:
$, m\,!{=}\varnothing, Ins(t;j),$

$$\Leftrightarrow\ , if(m\varocircle j)\text{-}\left[\begin{array}{l} , \\ , \end{array}\right]\text{-}, m\,!{=}\varnothing, Ins(t;j),$$

$$\Leftrightarrow\ , if(m\varocircle j)\text{-}\left[\begin{array}{l} , m\varocircle j, \\ , m!\varocircle j, \end{array}\right]\text{-}, m\,!{=}\varnothing, Ins(t;j),$$

$$\Leftrightarrow\ , if(m\varocircle j)\text{-}\left[\begin{array}{l} , m\varocircle j, m\,!{=}\varnothing, Ins(t;j), \\ , m!\varocircle j, m\,!{=}\varnothing, Ins(t;j), \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}j)\left[\begin{array}{l},m\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),\\ \\ ,m!\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}j)\left[\begin{array}{l},m\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),j\,!{=}\varnothing,\\ \\ ,m!\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}j)\left[\begin{array}{l},m\,!{=}\varnothing,Ins(t;j),m\overset{\circ}{\bigcirc}j,j\,!{=}\varnothing,\\ \\ ,m!\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}j)\left[\begin{array}{l},m\,!{=}\varnothing,Ins(t;j),m\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,\\ \\ ,m!\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}j)\left[\begin{array}{l},m\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,\\ \\ ,m!\overset{\circ}{\bigcirc}j,m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,\end{array}\right],$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}j)\left[\begin{array}{l},m\overset{\circ}{\bigcirc}j,\\ \\ ,m!\overset{\circ}{\bigcirc}j,\end{array}\right],m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,$$

$$\Leftrightarrow\ ,if(m\overset{\circ}{\bigcirc}j)\left[\begin{array}{l},\\ \\ ,\end{array}\right],m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,$$

$$\Leftrightarrow\ ,m\,!{=}\varnothing,Ins(t;j),m\,!{=}\varnothing,$$

$$,m{=}t,Ins(t;j),\ \Leftrightarrow\ ,\sim,m{=}t,$$

proof:
$,m{=}t,Ins(t;j),$

$\Leftrightarrow\ ,m{=}t,Ins(t;j),t{=}j,$

$\Leftrightarrow\ ,m{=}t,Ins(m;j),t{=}j,$

$\Leftrightarrow\ ,m{=}t,Ins(m;j),m{=}j,t{=}j,$

$\Leftrightarrow\ ,m{=}t,Ins(m;j),m{=}j,t{=}m,$

$\Leftrightarrow \ , m\!=\!t, Ins(m;j), t\!=\!m,$

$\Leftrightarrow \ , m\!=\!t, Ins(t;j), m\!=\!t,$

$$, Ins(t;j), j\!\to\! i, \ \Leftrightarrow \ , \otimes,$$

proof:
$, Ins(t;j), j\!\to\! i,$

$\Leftrightarrow \ , Ins(t;j), j \, != \varnothing, j\!\to\! i,$

$\Leftrightarrow \ , Ins(t;j), j \, != \varnothing, j\!\to\! i, j\!\rightarrowtail\! i,$

$\Leftrightarrow \ , Ins(t;j), j \, != \varnothing, j\!\to\! i, j\!\rightarrowtail\! i, j!\mathring{\bigcirc} i,$

$\Leftrightarrow \ , Ins(t;j), j\!\to\! i, j!\mathring{\bigcirc} i,$

$\Leftrightarrow \ , Ins(t;j), j!\mathring{\bigcirc} i, j\!\to\! i,$

$\Leftrightarrow \ , j!\mathring{\bigcirc} i, Ins(t;j), j\!\to\! i,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, Ins(t;j), j\!\to\! i,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, Ins(t;j), j \owedge j_0, j_0 \oplus, j_0 \mathring{\bigcirc} i, j_0 \obot,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, Ins(t;j), j \owedge j_0, j_0 \oplus, j_0 \varowedge dj, i \varowedge di, dj\!=\!di, dj \obot, di \obot, j_0 \obot,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, Ins(t;j), j \owedge j_0, j_0 \oplus, j_0 \varowedge dj, j_0 \obot, i \varowedge di, dj\!=\!di, dj \obot, di \obot,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, j \varowedge dj, Ins(t;j), i \varowedge di, dj\!=\!di, dj \obot, di \obot,$

$\Leftrightarrow \ , j \varowedge dj, i!\mathring{\bigcirc} j, Ins(t;j), i \varowedge di, dj\!=\!di, dj \obot, di \obot,$

$\Leftrightarrow \ , j \varowedge dj, i!\mathring{\bigcirc} j, i \varowedge di, Ins(t;j), dj\!=\!di, dj \obot, di \obot,$

$\Leftrightarrow \ , j \varowedge dj, dj!\mathring{\bigcirc} j, i!\mathring{\bigcirc} j, i \varowedge di, di!\mathring{\bigcirc} j, Ins(t;j), dj\!=\!di, dj \obot, di \obot,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, j \varowedge dj, i \varowedge di, dj!\mathring{\bigcirc} j, di!\mathring{\bigcirc} j, Ins(t;j), dj\!=\!di, dj \obot, di \obot,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, j \varowedge dj, i \varowedge di, dj!\mathring{\bigcirc} j, di!\mathring{\bigcirc} j, dj\!=\!di, Ins(t;j), dj \obot, di \obot,$

$\Leftrightarrow \ , i!\mathring{\bigcirc} j, j \varowedge dj, i \varowedge di, dj\!=\!di, Ins(t;j), dj \obot, di \obot,$

$$\Leftrightarrow\ ,i!\overset{\circ}{\bigcirc}j, j\circledast dj, i\circledast di, dj=di, dj\odot, di\odot, Ins(t;j),$$

$$\Leftrightarrow\ ,i!\overset{\circ}{\bigcirc}j, j\overset{\circ}{\bigcirc}i, Ins(t;j),$$

$$\Leftrightarrow\ ,i!\overset{\circ}{\bigcirc}j, i\overset{\circ}{\bigcirc}j, Ins(t;j),$$

$$\Leftrightarrow\ ,\otimes, Ins(t;j),$$

$$\Leftrightarrow\ ,\otimes,$$

$$,Ins(t;j),\ \Leftrightarrow\ ,\sim, j!\rightarrow i,$$
$$,Ins(t;j), j\oplus,\ \Leftrightarrow\ \sim, j!\overset{\circ}{\bigcirc}i,$$

$$,m!\overset{\circ}{\bigcirc}j, j\,!{=}m, Ins(t;j),\ \Leftrightarrow\ ,m!\overset{\circ}{\bigcirc}j, Ins(t;j), j\circledast j_0, j_0\oplus, j_0\,!{=}m, j_0\odot,$$

proof:

$,m!\overset{\circ}{\bigcirc}j, j\,!{=}m, Ins(t;j),$

$$\Leftrightarrow\ ,m!\overset{\circ}{\bigcirc}j, j\,!{=}m, Ins(t;j), j\circledast j_0, j_0\odot,$$

$$\Leftrightarrow\ ,m!\overset{\circ}{\bigcirc}j, j\,!{=}m, Ins(t;j), j\circledast j_0, j_0\oplus, j_0\odot,$$

$$\Leftrightarrow\ ,m!\overset{\circ}{\bigcirc}j, j\,!{=}m, Ins(t;j), j\circledast j_0, j_0\oplus, if(j_0=m)\left[\begin{array}{l} , \\ , \end{array}\right], j_0\odot,$$

$$\Leftrightarrow\ ,m!\overset{\circ}{\bigcirc}j, j\,!{=}m, Ins(t;j), j\circledast j_0, j_0\oplus, \left[\begin{array}{l} ,j_0=m, \\ ,j_0\,!{=}m, \end{array}\right], j_0\odot,$$

$$\Leftrightarrow\ ,j\,!{=}m, m!\overset{\circ}{\bigcirc}j, Ins(t;j), j\circledast j_0, j_0\oplus, \left[\begin{array}{l} ,j_0=m, \\ ,j_0\,!{=}m, \end{array}\right], j_0\odot,$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,j\,!{=}m, m!\overset{\circ}{\bigcirc}j, Ins(t;j), j\circledast j_0, j_0\oplus, j_0=m, j_0\odot, \\ ,j\,!{=}m, m!\overset{\circ}{\bigcirc}j, Ins(t;j), j\circledast j_0, j_0\oplus, j_0\,!{=}m, j_0\odot, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,j\,!{=}m, m!\overset{\circ}{\bigcirc}j, j=m, Ins(t;j), \\ ,j\,!{=}m, m!\overset{\circ}{\bigcirc}j, Ins(t;j), j\circledast j_0, j_0\oplus, j_0\,!{=}m, j_0\odot, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,m!\mathring{\bigcirc}j, j\,!{=}\,m, j=m, Ins(t;j), \\ \\ ,j\,!{=}\,m, m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,m, j_0\obot, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,m!\mathring{\bigcirc}j, \otimes, Ins(t;j), \\ \\ ,j\,!{=}\,m, m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,m, j_0\obot, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,\otimes, \\ \\ ,j\,!{=}\,m, m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,m, j_0\obot, \end{array}\right],$$

$$\Leftrightarrow\ ,\left[\begin{array}{l} ,j\,!{=}\,m, m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, j_0=m, \otimes, \\ \\ ,j\,!{=}\,m, m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,m, j_0\obot, \end{array}\right],$$

$$\Leftrightarrow\ ,j\,!{=}\,m, m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, \left[\begin{array}{l} ,j_0=m, \otimes, \\ \\ ,j_0\,!{=}\,m, j_0\obot, \end{array}\right],$$

$$\Leftrightarrow\ ,j\,!{=}\,m, m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, if(j_0=m)\left[\begin{array}{l} ,\otimes, \\ \\ ,j_0\obot, \end{array}\right],$$

$$\Leftrightarrow\ ,m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,m, j_0\obot,$$

$$,m!\mathring{\bigcirc}j, j\,!{=}\,m, Ins(t;j), j\owedge j_0, j_0\oplus, \ \Leftrightarrow\ ,m!\mathring{\bigcirc}j, Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,m,$$

$$,m!\mathring{\bigcirc}j, j\,!{=}\,m, Ins(t;j), j\oplus, \ \Leftrightarrow\ ,m!\mathring{\bigcirc}j, Ins(t;j), j\oplus, j\,!{=}\,m,$$

$$,j\,!{=}\,\varnothing, Ins(t;j), \ \Leftrightarrow\ ,Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,\varnothing, j_0\obot,$$
$$,j=\varnothing, Ins(t;j), \ \Leftrightarrow\ ,Ins(t;j), j\owedge j_0, j_0\oplus, j_0=\varnothing, j_0\obot,$$

$$,j\,!{=}\,\varnothing, Ins(t;j), j\owedge j_0, j_0\oplus, \ \Leftrightarrow\ ,Ins(t;j), j\owedge j_0, j_0\oplus, j_0\,!{=}\,\varnothing,$$
$$,j=\varnothing, Ins(t;j), j\owedge j_0, j_0\oplus, \ \Leftrightarrow\ ,Ins(t;j), j\owedge j_0, j_0\oplus, j_0=\varnothing,$$

$$,j=\varnothing, Ins(t;j), j\oplus, \ \Leftrightarrow\ ,Ins(t;j), j\oplus, j=\varnothing,$$
$$,j\,!{=}\,\varnothing, Ins(t;j), j\oplus, \ \Leftrightarrow\ ,Ins(t;j), j\oplus, j\,!{=}\,\varnothing,$$

$$, i\overset{\circ}{\bigcirc}j, j=\varnothing, Ins(t;j), i\oplus, \iff , i\overset{\circ}{\bigcirc}j, Ins(t;j), i\oplus, i=\varnothing,$$

$$, i\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, Ins(t;j), i\oplus, \iff , i\overset{\circ}{\bigcirc}j, Ins(t;j), i\oplus, i\,!\!=\varnothing,$$

25.5 Swap with node connectivity propositions

25.5.1 Recursive Function R(i)

$$, i!\overset{\circ}{\bigcirc}{\circ}j, R(i), Ins(t;j), \iff , i!\overset{\circ}{\bigcirc}{\circ}j, Ins(t;j), R(i),$$

induction proof:

$premise\ 1:$

$, i=\varnothing, i!\overset{\circ}{\bigcirc}{\circ}j, R(i), Ins(t;j),$

$\iff , i!\overset{\circ}{\bigcirc}{\circ}j, i=\varnothing, R(i), Ins(t;j),$

$\iff , i!\overset{\circ}{\bigcirc}{\circ}j, i=\varnothing, Ins(t;j),$

$\iff , i!\overset{\circ}{\bigcirc}{\circ}j, i!\overset{\circ}{\bigcirc}j, i=\varnothing, Ins(t;j),$

$\iff , i!\overset{\circ}{\bigcirc}{\circ}j, i!\overset{\circ}{\bigcirc}j, Ins(t;j), i=\varnothing,$

$\iff , i!\overset{\circ}{\bigcirc}{\circ}j, i!\overset{\circ}{\bigcirc}j, Ins(t;j), i=\varnothing, R(i),$

$\iff , i!\overset{\circ}{\bigcirc}{\circ}j, i!\overset{\circ}{\bigcirc}j, i=\varnothing, Ins(t;j), R(i),$

$\iff , i!\overset{\circ}{\bigcirc}{\circ}j, i=\varnothing, Ins(t;j), R(i),$

$\iff , i=\varnothing, i!\overset{\circ}{\bigcirc}{\circ}j, Ins(t;j), R(i),$

$premise\ 2:$

$, \&SHi \to i, i!\overset{\circ}{\bigcirc}{\circ}j, R(i), Ins(t;j), \iff , \&SHi \to i, i!\overset{\circ}{\bigcirc}{\circ}j, Ins(t;j), R(i), \implies$

$, i\,!\!=\varnothing, \&SHi\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}j, R(i), Ins(t;j),$

$\iff , \&SHi\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}j, i\,!\!=\varnothing, R(i), Ins(t;j),$

$\iff , \&SHi\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}j, i\,!\!=\varnothing, i\oplus, R(i), Ins(t;j),$

$\iff , \&SHi\overset{\circ}{\bigcirc}i, i\,!\!=\varnothing, i\oplus, i!\overset{\circ}{\bigcirc}{\circ}j, R(i), Ins(t;j),$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oplus,\ \&SHi \to i, i!\mathring{\bigcirc}\!^{\circ} j, R(i), Ins(t;j),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oplus,\ \&SHi \to i, i!\mathring{\bigcirc}\!^{\circ} j, Ins(t;j), R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i\,!{=}\,\varnothing, i\oplus, i!\mathring{\bigcirc}\!^{\circ} j, Ins(t;j), R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i!\mathring{\bigcirc} j, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i\,!{=}\,\varnothing, i!\mathring{\bigcirc} j, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i\,!{=}\,\varnothing, i!\mathring{\bigcirc} j, Ins(t;j), i\oplus, R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i!\mathring{\bigcirc} j, i\,!{=}\,\varnothing, Ins(t;j), i\oplus, R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i!\mathring{\bigcirc} j, Ins(t;j), i\,!{=}\,\varnothing, i\oplus, R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i!\mathring{\bigcirc} j, Ins(t;j), i\,!{=}\,\varnothing, R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i!\mathring{\bigcirc} j, i\,!{=}\,\varnothing, Ins(t;j), R(i),$$

$$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, i\,!{=}\,\varnothing, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!^{\circ} j, Ins(t;j), R(i),$$

conclusion :
$, i!\mathring{\bigcirc}\!^{\circ} j, R(i), Ins(t;j),\ \Leftrightarrow\ , i!\mathring{\bigcirc}\!^{\circ} j, Ins(t;j), R(i),$

$$, i\mathring{\bigcirc}\!^{\circ} j, j=\varnothing, R(i), Ins(t;j),\ \Leftrightarrow\ , i\mathring{\bigcirc}\!^{\circ} j, j=\varnothing, Ins(t;j), R(i), i\ominus,$$

induction proof:
premise 1 :
$, i=\varnothing, i\mathring{\bigcirc}\!^{\circ} j, j=\varnothing, R(i), Ins(t;j),$

$$\Leftrightarrow\ , i\mathring{\bigcirc}\!^{\circ} j, j=\varnothing, i=\varnothing, R(i), Ins(t;j),$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}\!^{\circ} j, j=\varnothing, i=\varnothing, Ins(t;j),$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}\!^{\circ} j, i\mathring{\bigcirc}\!^{\circ} j, i=\varnothing, j=\varnothing, Ins(t;j),$$

$$\Leftrightarrow\ , i\mathring{\bigcirc}\!^{\circ} j, i\mathring{\bigcirc} j, i=\varnothing, j=\varnothing, Ins(t;j),$$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, i\overset{\circ}{\bigcirc}j, j=\varnothing, Ins(t;j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, i\overset{\circ}{\bigcirc}j, j=\varnothing, Ins(t;j), i\oplus, i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, i\overset{\circ}{\bigcirc}j, Ins(t;j), i\oplus, i=\varnothing, i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, i\overset{\circ}{\bigcirc}j, Ins(t;j), i\oplus, i=\varnothing, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, i\overset{\circ}{\bigcirc}j, j=\varnothing, Ins(t;j), i\oplus, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, i\overset{\circ}{\bigcirc}j, j=\varnothing, Ins(t;j), j\,!\!=\varnothing, i\oplus, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j=\varnothing, Ins(t;j), i\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, i\oplus, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j=\varnothing, Ins(t;j), i\overset{\circ}{\bigcirc}j, i\,!\!=\varnothing, i\oplus, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j=\varnothing, Ins(t;j), i\overset{\circ}{\bigcirc}j, i\,!\!=\varnothing, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j=\varnothing, Ins(t;j), i\overset{\circ}{\bigcirc}j, j\,!\!=\varnothing, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\overset{\circ}{\bigcirc}j, i=\varnothing, j=\varnothing, Ins(t;j), j\,!\!=\varnothing, R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\overset{\circ}{\bigcirc}j, i=\varnothing, j=\varnothing, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j=\varnothing, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j=\varnothing, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , i=\varnothing, i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), R(i), i\ominus,$

$premise\ 2:$

$, \ \&SHi\rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, R(i), Ins(t;j), \ \Leftrightarrow\ , \ \&SHi\rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), R(i), i\ominus, \ \Rightarrow$

$, i\,!\!=\varnothing, \ \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, R(i), Ins(t;j),$

$\Leftrightarrow\ , \ \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, i\,!\!=\varnothing, R(i), Ins(t;j),$

$\Leftrightarrow\ , \ \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, i\,!\!=\varnothing, i\oplus, R(i), Ins(t;j),$

$\Leftrightarrow\ , \ \&SHi\overset{\circ}{\bigcirc}i, j=\varnothing, i\,!\!=\varnothing, i\oplus, i\overset{\circ}{\bigcirc}{}^{\circ}j, R(i), Ins(t;j),$

$\Leftrightarrow\ , i\,!\!=\varnothing, i\oplus, \ \&SHi\rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, R(i), Ins(t;j),$

$\Leftrightarrow\ , i \,!\!= \varnothing, i\oplus, \ \&SHi \rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i \,!\!= \varnothing, i\oplus, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i \,!\!= \varnothing, j = \varnothing, i\oplus, i\overset{\circ}{\bigcirc}{}^{\circ}j, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, i \,!\!= \varnothing, j = \varnothing, i\oplus, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, i \,!\!= \varnothing, j = \varnothing, i!\mathring{\bigcirc}j, i\oplus, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, i \,!\!= \varnothing, j = \varnothing, i!\mathring{\bigcirc}j, Ins(t;j), i\oplus, R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, i!\mathring{\bigcirc}j, i \,!\!= \varnothing, Ins(t;j), i\oplus, R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, i!\mathring{\bigcirc}j, Ins(t;j), i \,!\!= \varnothing, i\oplus, R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, i!\mathring{\bigcirc}j, Ins(t;j), i \,!\!= \varnothing, R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, i!\mathring{\bigcirc}j, i \,!\!= \varnothing, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, i \,!\!= \varnothing, j = \varnothing, i!\mathring{\bigcirc}j, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, i \,!\!= \varnothing, j = \varnothing, Ins(t;j), R(i), i\ominus,$

$\Leftrightarrow\ , i \,!\!= \varnothing, \ \&SHi\,\mathring{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, Ins(t;j), R(i), i\ominus,$

conclusion :
$, i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, R(i), Ins(t;j), \ \Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j = \varnothing, Ins(t;j), R(i), i\ominus,$

$$, i\overset{\circ}{\bigcirc}{}^{\circ}j, j \,!\!= \varnothing, R(i), Ins(t;j), \ \Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j \,!\!= \varnothing, Ins(t;j), R(i),$$

induction proof:
premise 1 :
$, i = \varnothing, i\overset{\circ}{\bigcirc}{}^{\circ}j, j \,!\!= \varnothing, R(i), Ins(t;j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j \,!\!= \varnothing, i = \varnothing, R(i), Ins(t;j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i = \varnothing, j \,!\!= \varnothing, Ins(t;j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i = \varnothing, j \,!\!= \varnothing, i!\mathring{\bigcirc}j, Ins(t;j),$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j \,!\!= \varnothing, i!\mathring{\bigcirc}j, i = \varnothing, Ins(t;j),$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, i!\overset{\circ}{\bigcirc}j, Ins(t;j), i=\varnothing,$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, i!\overset{\circ}{\bigcirc}j, Ins(t;j), i=\varnothing, R(i),$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, i!\overset{\circ}{\bigcirc}j, i=\varnothing, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j\,!{=}\,\varnothing, i!\overset{\circ}{\bigcirc}j, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i=\varnothing, j\,!{=}\,\varnothing, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , i=\varnothing, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, Ins(t;j), R(i),$$

$premise\ 2:$

$$, \ \&SHi \rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, R(i), Ins(t;j), \ \Leftrightarrow\ , \ \&SHi \rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, Ins(t;j), R(i), \ \Longrightarrow$$

$$, i\,!{=}\,\varnothing, \ \&SHi\, \overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, R(i), Ins(t;j),$$

$$\Leftrightarrow\ , \ \&SHi\, \overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, R(i), Ins(t;j),$$

$$\Leftrightarrow\ , \ \&SHi\, \overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, R(i), Ins(t;j),$$

$$\Leftrightarrow\ , \ \&SHi\, \overset{\circ}{\bigcirc}i, i\,!{=}\,\varnothing, i\oplus, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, R(i), Ins(t;j),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oplus, \ \&SHi \rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, R(i), Ins(t;j),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\oplus, \ \&SHi \rightarrow i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , \ \&SHi\, \overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , \ \&SHi\, \overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right], j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , \ \&SHi\, \overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} , i\overset{\circ}{\bigcirc}j, \\ \\ , i!\overset{\circ}{\bigcirc}j, \end{array}\right], j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , \ \&SHi\, \overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}{}^{\circ}j, if(i\overset{\circ}{\bigcirc}j)\left[\begin{array}{l} , i\overset{\circ}{\bigcirc}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i), \\ \\ , i!\overset{\circ}{\bigcirc}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i), \end{array}\right],$$

$$\Leftrightarrow\ <1>$$

$$, i\text{♂}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\text{♂}j, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\text{♂}j, Ins(t;j), i\oplus, i\oplus, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\text{♂}j, j\,!{=}\,\varnothing, Ins(t;j), i\oplus, i\oplus, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\text{♂}j, Ins(t;j), i\oplus, i\,!{=}\,\varnothing, i\oplus, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\text{♂}j, Ins(t;j), i\oplus, i\,!{=}\,\varnothing, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\text{♂}j, j\,!{=}\,\varnothing, Ins(t;j), i\oplus, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, i\text{♂}j, j\,!{=}\,\varnothing, Ins(t;j), j\,!{=}\,\varnothing, i\oplus, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Ins(t;j), i\text{♂}j, j\,!{=}\,\varnothing, i\oplus, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Ins(t;j), i\text{♂}j, i\,!{=}\,\varnothing, i\oplus, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Ins(t;j), i\text{♂}j, i\,!{=}\,\varnothing, R(i),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Ins(t;j), i\text{♂}j, j\,!{=}\,\varnothing, R(i),$$

$$\Leftrightarrow\ , i\text{♂}j, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Ins(t;j), j\,!{=}\,\varnothing, R(i),$$

$$\Leftrightarrow\ , i\text{♂}j, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , i\text{♂}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, Ins(t;j), R(i),$$

$$, i!\text{♂}j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i!\text{♂}j, i\oplus, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i!\text{♂}j, Ins(t;j), i\oplus, R(i),$$

$$\Leftrightarrow\ , j\,!{=}\,\varnothing, i!\text{♂}j, i\,!{=}\,\varnothing, Ins(t;j), i\oplus, R(i),$$

$$\Leftrightarrow\ , j\,!{=}\,\varnothing, i!\text{♂}j, Ins(t;j), i\,!{=}\,\varnothing, i\oplus, R(i),$$

$\Leftrightarrow\ , j\,!\!=\varnothing, i!\mathring{\bigcirc}j, Ins(t;j), i\,!\!=\varnothing, R(i),$

$\Leftrightarrow\ , j\,!\!=\varnothing, i!\mathring{\bigcirc}j, i\,!\!=\varnothing, Ins(t;j), R(i),$

$\Leftrightarrow\ , i!\mathring{\bigcirc}j, j\,!\!=\varnothing, i\,!\!=\varnothing, Ins(t;j), R(i),$

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$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\mathring{\bigcirc}^{\circ}j, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\mathring{\bigcirc}j, j\,!\!=\varnothing, i\,!\!=\varnothing, Ins(t;j), R(i),\\ , i!\mathring{\bigcirc}j, j\,!\!=\varnothing, i\,!\!=\varnothing, Ins(t;j), R(i),\end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\mathring{\bigcirc}^{\circ}j, if(i\mathring{\bigcirc}j)\left[\begin{array}{l}, i\mathring{\bigcirc}j,\\ , i!\mathring{\bigcirc}j,\end{array}\right], j\,!\!=\varnothing, i\,!\!=\varnothing, Ins(t;j), R(i),$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\mathring{\bigcirc}^{\circ}j, if(i\mathring{\bigcirc}j)\left[\begin{array}{l},\\ ,\end{array}\right], j\,!\!=\varnothing, i\,!\!=\varnothing, Ins(t;j), R(i),$$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\mathring{\bigcirc}^{\circ}j, j\,!\!=\varnothing, i\,!\!=\varnothing, Ins(t;j), R(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\,\mathring{\bigcirc}i, i\mathring{\bigcirc}^{\circ}j, j\,!\!=\varnothing, Ins(t;j), R(i),$

$conclusion:$
$, i\mathring{\bigcirc}^{\circ}j, j\,!\!=\varnothing, R(i), Ins(t;j),\ \Leftrightarrow\ , i\mathring{\bigcirc}^{\circ}j, j\,!\!=\varnothing, Ins(t;j), R(i),$

25.5.2 $j=\varnothing$

$$, j=\varnothing, i\mathring{\bigcirc}^{\circ}j, Ins(t;j),\ \Leftrightarrow\ \sim, i\mathring{\bigcirc}^{\circ}j,$$

proof:
$, j=\varnothing, i\mathring{\bigcirc}^{\circ}j, Ins(t;j),$

$\Leftrightarrow\ , j=\varnothing, i\mathring{\bigcirc}^{\circ}j, i\mathring{\bigcirc}^{\circ}j, Ins(t;j),$

$\Leftrightarrow\ , j=\varnothing, i\mathring{\bigcirc}^{\circ}j, i\varowedge i_0, j\varowedge j_0, R(i_0), R(j_0), i_0\mathring{\bigcirc}j_0, i_0\obot, j_0\obot, Ins(t;j),$

$\Leftrightarrow\ , j=\varnothing, i\mathring{\bigcirc}^{\circ}j, i\varowedge i_0, j\varowedge j_0, R(i_0), R(j_0), i_0\mathring{\bigcirc}j_0, Ins(t;j), i_0\obot, j_0\obot,$

$\Leftrightarrow\ , j=\varnothing, i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, j\circledast j_0, R(i_0), R(j_0), Ins(t;j), i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , j=\varnothing, i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, j\circledast j_0, j\overset{\circ}{\bigcirc}j_0, R(i_0), R(j_0), Ins(t;j), i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , j=\varnothing, i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, j\circledast j_0, j\overset{\circ}{\bigcirc}j_0, j\overset{\circ}{\bigcirc}{}^{\circ}j_0, R(i_0), R(j_0), Ins(t;j), i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, j\circledast j_0, j\overset{\circ}{\bigcirc}j_0, R(i_0), j\overset{\circ}{\bigcirc}{}^{\circ}j_0, j=\varnothing, R(j_0), Ins(t;j), i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, j\circledast j_0, j\overset{\circ}{\bigcirc}j_0, R(i_0), j\overset{\circ}{\bigcirc}{}^{\circ}j_0, j=\varnothing, Ins(t;j), R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, j\circledast j_0, R(i_0), j=\varnothing, Ins(t;j), R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, j\circledast j_0, j=\varnothing, R(i_0), Ins(t;j), R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, i\circledast i_0, i\overset{\circ}{\bigcirc}i_0, j\circledast j_0, j=\varnothing, R(i_0), Ins(t;j), R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\circledast i_0, i\overset{\circ}{\bigcirc}i_0, i\overset{\circ}{\bigcirc}{}^{\circ}j, j\circledast j_0, j=\varnothing, R(i_0), Ins(t;j), R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\circledast i_0, i\overset{\circ}{\bigcirc}i_0, i_0\overset{\circ}{\bigcirc}{}^{\circ}j, j\circledast j_0, j=\varnothing, R(i_0), Ins(t;j), R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\circledast i_0, i\overset{\circ}{\bigcirc}i_0, j\circledast j_0, i_0\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, R(i_0), Ins(t;j), R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\circledast i_0, i\overset{\circ}{\bigcirc}i_0, j\circledast j_0, i_0\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), R(i_0), i_0\ominus, R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, i\circledast i_0, j\circledast j_0, Ins(t;j), R(i_0), i_0\ominus, R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), i\circledast i_0, j\circledast j_0, R(i_0), i_0\ominus, R(j_0), j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), i\circledast i_0, j\circledast j_0, R(i_0), R(j_0), i_0\ominus, j_0\ominus, i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), i\circledast i_0, j\circledast j_0, R(i_0), R(j_0), i_0\overset{\circ}{\bigcirc}j_0, i_0\ominus, j_0\ominus, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), i\circledast i_0, j\circledast j_0, R(i_0), R(j_0), i_0\overset{\circ}{\bigcirc}j_0, i_0\ominus, i_0\circledS, j_0\ominus, j_0\circledS,$

$\Leftrightarrow\ , i\overset{\circ}{\bigcirc}{}^{\circ}j, j=\varnothing, Ins(t;j), i\circledast i_0, j\circledast j_0, R(i_0), R(j_0), i_0\overset{\circ}{\bigcirc}j_0, i_0\circledS, j_0\circledS,$

$\Leftrightarrow\ , j=\varnothing, i\overset{\circ}{\bigcirc}{}^{\circ}j, Ins(t;j), i\overset{\circ}{\bigcirc}{}^{\circ}j,$

$$, j=\varnothing, i!\overset{\circ}{\bigcirc}{}^{\circ}j, Ins(t;j), \ \Leftrightarrow\ \sim, i!\overset{\circ}{\bigcirc}{}^{\circ}j,$$

proof:
$, j=\varnothing, i!\overset{\circ}{\bigcirc}{}^{\circ}j, Ins(t;j),$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, i\text{☮}i_0, i_0\text{⏚}, Ins(t;j),$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, i\text{☮}i_0, R(i_0), i_0\text{⏚}, Ins(t;j),$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, i\text{☮}i_0, R(i_0), Ins(t;j), i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, i\text{☮}i_0, R(i_0), Ins(t;j), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, i\text{☮}i_0, i\mathring{\bigcirc}i_0, R(i_0), Ins(t;j), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i\text{☮}i_0, i\mathring{\bigcirc}i_0, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, R(i_0), Ins(t;j), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i\text{☮}i_0, i\mathring{\bigcirc}i_0, i_0!\mathring{\bigcirc}\!{\scriptstyle\circ} j, R(i_0), Ins(t;j), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i\text{☮}i_0, i\mathring{\bigcirc}i_0, i_0!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), R(i_0), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, i\text{☮}i_0, Ins(t;j), R(i_0), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), i\text{☮}i_0, R(i_0), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\text{⏚}, i\text{☮}i_0, R(i_0), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, j_0\text{⏚}, i\text{☮}i_0, R(i_0), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, j\rightarrow j_0, j_0\text{⏚}, i\text{☮}i_0, R(i_0), j!\rightarrow i_0, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, i\text{☮}i_0, R(i_0), j\rightarrow j_0, j!\rightarrow i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, i\text{☮}i_0, R(i_0), j\rightarrow j_0, j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, j\rightarrow j_0, i\text{☮}i_0, R(i_0), j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, i\text{☮}i_0, R(i_0), j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, j_0=\varnothing, i\text{☮}i_0, R(i_0), j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, j_0=\varnothing, R(j_0), i\text{☮}i_0, R(i_0), j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\oplus, R(j_0), i\text{☮}i_0, R(i_0), j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\,!{=}\,\varnothing, j\text{☮}j_0, j_0\oplus, R(j_0), i\text{☮}i_0, R(i_0), j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j=\varnothing, i!\mathring{\bigcirc}\!{\scriptstyle\circ} j, Ins(t;j), j\text{☮}j_0, j_0\,!{=}\,\varnothing, j_0\oplus, R(j_0), i\text{☮}i_0, R(i_0), j_0!\mathring{\bigcirc}i_0, j_0\text{⏚}, i_0\text{⏚},$

$$\Leftrightarrow\ , j=\varnothing, i!\mathbin{\bigcirc\!\!\!\circ}j, Ins(t;j), j\circledast j_0, j_0 \mathrel{!=}\varnothing, R(j_0), i\circledast i_0, R(i_0), j_0!\overset{\circ}{\bigcirc}i_0, j_0\circleddownarrow, i_0\circleddownarrow,$$

$$\Leftrightarrow\ , j=\varnothing, i!\mathbin{\bigcirc\!\!\!\circ}j, Ins(t;j), j\circledast j_0, R(j_0), i\circledast i_0, R(i_0), j_0!\overset{\circ}{\bigcirc}i_0, j_0\circleddownarrow, i_0\circleddownarrow,$$

$$\Leftrightarrow\ , j=\varnothing, i!\mathbin{\bigcirc\!\!\!\circ}j, Ins(t;j), i\circledast i_0, j\circledast j_0, R(i_0), R(j_0), i_0!\overset{\circ}{\bigcirc}j_0, i_0\circleddownarrow, j_0\circleddownarrow,$$

$$\Leftrightarrow\ , j=\varnothing, i!\mathbin{\bigcirc\!\!\!\circ}j, Ins(t;j), i!\mathbin{\bigcirc\!\!\!\circ}j,$$

$$, j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j,\ \Leftrightarrow\ , i\mathbin{\bigcirc\!\!\!\circ}j, \sim,$$

proof:
$, j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j,$

$$\Leftrightarrow\ , if(i\mathbin{\bigcirc\!\!\!\circ}j)\left[\begin{array}{l} , \\ , \end{array}\right], j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j,$$

$$\Leftrightarrow\ , if(i\mathbin{\bigcirc\!\!\!\circ}j)\left[\begin{array}{l} , \\ , i!\mathbin{\bigcirc\!\!\!\circ}j, \end{array}\right], j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j,$$

$$\Leftrightarrow\ , if(i\mathbin{\bigcirc\!\!\!\circ}j)\left[\begin{array}{l} , j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j, \\ , i!\mathbin{\bigcirc\!\!\!\circ}j, j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathbin{\bigcirc\!\!\!\circ}j)\left[\begin{array}{l} , j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j, \\ , j=\varnothing, i!\mathbin{\bigcirc\!\!\!\circ}j, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathbin{\bigcirc\!\!\!\circ}j)\left[\begin{array}{l} , j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j, \\ , j=\varnothing, i!\mathbin{\bigcirc\!\!\!\circ}j, Ins(t;j), i!\mathbin{\bigcirc\!\!\!\circ}j, i\mathbin{\bigcirc\!\!\!\circ}j, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathbin{\bigcirc\!\!\!\circ}j)\left[\begin{array}{l} , j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j, \\ , j=\varnothing, i!\mathbin{\bigcirc\!\!\!\circ}j, Ins(t;j), \otimes, \end{array}\right],$$

$$\Leftrightarrow\ , if(i\mathbin{\bigcirc\!\!\!\circ}j)\left[\begin{array}{l} , j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j, \\ , \otimes, \end{array}\right],$$

$$\Leftrightarrow\ , i\mathbin{\bigcirc\!\!\!\circ}j, j=\varnothing, Ins(t;j), i\mathbin{\bigcirc\!\!\!\circ}j,$$

$$, j = \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, Ins(t;j), \;\Leftrightarrow\; , j = \varnothing, Ins(t;j), i \overset{\circ}{\bigcirc}\!{}^{\circ} j,$$

25.5.3 $j \mathrel{!=} \varnothing$

$$, j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, Ins(t;j), \;\Leftrightarrow\; \sim, i \overset{\circ}{\bigcirc}\!{}^{\circ} j,$$

proof:

$, j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, Ins(t;j),$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, Ins(t;j),$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, R(i_0), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot, Ins(t;j),$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, R(i_0), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, Ins(t;j), i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, R(i_0), R(j_0), Ins(t;j), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, j \overset{\circ}{\bigcirc} j_0, R(i_0), R(j_0), Ins(t;j), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, j \overset{\circ}{\bigcirc} j_0, j \overset{\circ}{\bigcirc}\!{}^{\circ} j_0, R(i_0), R(j_0), Ins(t;j), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, j \overset{\circ}{\bigcirc} j_0, R(i_0), j \mathrel{!=} \varnothing, j \overset{\circ}{\bigcirc}\!{}^{\circ} j_0, R(j_0), Ins(t;j), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, j \overset{\circ}{\bigcirc} j_0, R(i_0), j \mathrel{!=} \varnothing, j \overset{\circ}{\bigcirc}\!{}^{\circ} j_0, Ins(t;j), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, R(i_0), Ins(t;j), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, i \overset{\circ}{\bigcirc} i_0, j \owedge j_0, R(i_0), Ins(t;j), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \owedge i_0, i \overset{\circ}{\bigcirc} i_0, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, j \owedge j_0, R(i_0), Ins(t;j), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \owedge i_0, i \overset{\circ}{\bigcirc} i_0, i_0 \overset{\circ}{\bigcirc}\!{}^{\circ} j, j \owedge j_0, R(i_0), Ins(t;j), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , i \owedge i_0, i \overset{\circ}{\bigcirc} i_0, j \owedge j_0, j \mathrel{!=} \varnothing, i_0 \overset{\circ}{\bigcirc}\!{}^{\circ} j, R(i_0), Ins(t;j), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , i \owedge i_0, i \overset{\circ}{\bigcirc} i_0, j \owedge j_0, j \mathrel{!=} \varnothing, i_0 \overset{\circ}{\bigcirc}\!{}^{\circ} j, Ins(t;j), R(i_0), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, i \owedge i_0, j \owedge j_0, Ins(t;j), R(i_0), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, Ins(t;j), i \owedge i_0, j \owedge j_0, R(i_0), R(j_0), i_0 \overset{\circ}{\bigcirc} j_0, i_0 \obot, j_0 \obot,$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i \overset{\circ}{\bigcirc}\!{}^{\circ} j, Ins(t;j), i \overset{\circ}{\bigcirc}\!{}^{\circ} j,$

$$, j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, Ins(t;j), \;\Leftrightarrow\; \sim, i!\overset{\circ}{\bigcirc}{\circ} j,$$

proof:
$, j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, Ins(t;j),$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i!\overset{\circ}{\bigcirc}{\circ} j, Ins(t;j),$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, R(i_0), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚}, Ins(t;j),$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, R(i_0), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, Ins(t;j), i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, R(i_0), R(j_0), Ins(t;j), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, j\overset{\circ}{\bigcirc} j_0, R(i_0), R(j_0), Ins(t;j), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, j\overset{\circ}{\bigcirc} j_0, j\overset{\circ}{\bigcirc}{\circ} j_0, R(i_0), R(j_0), Ins(t;j), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, j\overset{\circ}{\bigcirc} j_0, R(i_0), j \mathrel{!=} \varnothing, j\overset{\circ}{\bigcirc}{\circ} j_0, R(j_0), Ins(t;j), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, j\overset{\circ}{\bigcirc} j_0, R(i_0), j \mathrel{!=} \varnothing, j\overset{\circ}{\bigcirc}{\circ} j_0, Ins(t;j), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, R(i_0), Ins(t;j), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, i\overset{\circ}{\bigcirc} i_0, j\text{☮}j_0, R(i_0), Ins(t;j), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i\text{☮}i_0, i\overset{\circ}{\bigcirc} i_0, i!\overset{\circ}{\bigcirc}{\circ} j, j\text{☮}j_0, R(i_0), Ins(t;j), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i\text{☮}i_0, i\overset{\circ}{\bigcirc} i_0, i_0!\overset{\circ}{\bigcirc}{\circ} j, j\text{☮}j_0, R(i_0), Ins(t;j), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i\text{☮}i_0, i\overset{\circ}{\bigcirc} i_0, j\text{☮}j_0, i_0!\overset{\circ}{\bigcirc}{\circ} j, R(i_0), Ins(t;j), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i\text{☮}i_0, i\overset{\circ}{\bigcirc} i_0, j\text{☮}j_0, i_0!\overset{\circ}{\bigcirc}{\circ} j, Ins(t;j), R(i_0), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, i\text{☮}i_0, j\text{☮}j_0, Ins(t;j), R(i_0), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, Ins(t;j), i\text{☮}i_0, j\text{☮}j_0, R(i_0), R(j_0), i_0!\overset{\circ}{\bigcirc} j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\; , j \mathrel{!=} \varnothing, i!\overset{\circ}{\bigcirc}{\circ} j, Ins(t;j), i!\overset{\circ}{\bigcirc}{\circ} j,$

$$, j \mathrel{!=} \varnothing, Ins(t;j), i\overset{\circ}{\bigcirc}{\circ} j, \;\Leftrightarrow\; i\overset{\circ}{\bigcirc}{\circ} j, \sim,$$

$$, j \mathrel{!=} \varnothing, i\overset{\circ}{\bigcirc}{\circ} j, Ins(t;j), \;\Leftrightarrow\; , j \mathrel{!=} \varnothing, Ins(t;j), i\overset{\circ}{\bigcirc}{\circ} j,$$

25.5.4 Total

$$,i\mathring{\bigcirc}\!\circ j, Ins(t;j), \iff ,Ins(t;j), i\mathring{\bigcirc}\!\circ j,$$
$$,i!\mathring{\bigcirc}\!\circ j, Ins(t;j), \iff ,Ins(t;j), i!\mathring{\bigcirc}\!\circ j,$$

$$,m\mathring{\bigcirc}\!\circ j, m\mathring{\bigcirc}\!\circ n, Ins(t;j), \iff ,m\mathring{\bigcirc}\!\circ j, Ins(t;j), m\mathring{\bigcirc}\!\circ n,$$

proof:
$,m\mathring{\bigcirc}\!\circ j, m\mathring{\bigcirc}\!\circ n, Ins(t;j),$

$\iff ,m\mathring{\bigcirc}\!\circ j, n\mathring{\bigcirc}\!\circ j, Ins(t;j),$

$\iff ,Ins(t;j), m\mathring{\bigcirc}\!\circ j, n\mathring{\bigcirc}\!\circ j,$

$\iff ,Ins(t;j), m\mathring{\bigcirc}\!\circ j, m\mathring{\bigcirc}\!\circ n,$

$\iff ,m\mathring{\bigcirc}\!\circ j, Ins(t;j), m\mathring{\bigcirc}\!\circ n,$

$$,m!\mathring{\bigcirc}\!\circ j, m\mathring{\bigcirc}\!\circ n, Ins(t;j), \iff ,m!\mathring{\bigcirc}\!\circ j, Ins(t;j), m\mathring{\bigcirc}\!\circ n,$$

proof:
$,m!\mathring{\bigcirc}\!\circ j, m\mathring{\bigcirc}\!\circ n, Ins(t;j),$

$\iff ,m!\mathring{\bigcirc}\!\circ j, m!\mathring{\bigcirc}\!\circ j, m\mathring{\bigcirc}\!\circ n, Ins(t;j),$

$\iff ,m!\mathring{\bigcirc}\!\circ j, n!\mathring{\bigcirc}\!\circ j, m\mathring{\bigcirc}\!\circ n, Ins(t;j),$

$\iff ,m!\mathring{\bigcirc}\!\circ j, n!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc} n_0, m_0\obot, n_0\obot, Ins(t;j),$

$\iff ,m!\mathring{\bigcirc}\!\circ j, n!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, R(m_0), R(n_0), m_0\mathring{\bigcirc} n_0, Ins(t;j), m_0\obot, n_0\obot,$

$\iff ,m!\mathring{\bigcirc}\!\circ j, n!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, R(m_0), R(n_0), Ins(t;j), m_0\mathring{\bigcirc} n_0, m_0\obot, n_0\obot,$

$\iff ,m!\mathring{\bigcirc}\!\circ j, n!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, n\mathring{\bigcirc} n_0, R(m_0), R(n_0), Ins(t;j), m_0\mathring{\bigcirc} n_0, m_0\obot, n_0\obot,$

$\iff ,m!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, n\mathring{\bigcirc} n_0, n!\mathring{\bigcirc}\!\circ j, R(m_0), R(n_0), Ins(t;j), m_0\mathring{\bigcirc} n_0, m_0\obot, n_0\obot,$

$\iff ,m!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, n\mathring{\bigcirc} n_0, n_0!\mathring{\bigcirc}\!\circ j, R(m_0), R(n_0), Ins(t;j), m_0\mathring{\bigcirc} n_0, m_0\obot, n_0\obot,$

$\iff ,m!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, n\mathring{\bigcirc} n_0, R(m_0), n_0!\mathring{\bigcirc}\!\circ j, R(n_0), Ins(t;j), m_0\mathring{\bigcirc} n_0, m_0\obot, n_0\obot,$

$\iff ,m!\mathring{\bigcirc}\!\circ j, m\circledast m_0, n\circledast n_0, n\mathring{\bigcirc} n_0, R(m_0), n_0!\mathring{\bigcirc}\!\circ j, Ins(t;j), R(n_0), m_0\mathring{\bigcirc} n_0, m_0\obot, n_0\obot,$

$$\Leftrightarrow \ , m!\overset{\circ}{\bigcirc}\!{\circ}j, n!\overset{\circ}{\bigcirc}\!{\circ}j, m\text{☮}m_0, n\text{☮}n_0, R(m_0), Ins(t;j), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , m!\overset{\circ}{\bigcirc}\!{\circ}j, n!\overset{\circ}{\bigcirc}\!{\circ}j, m\text{☮}m_0, m\overset{\circ}{\bigcirc}m_0, n\text{☮}n_0, R(m_0), Ins(t;j), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , n!\overset{\circ}{\bigcirc}\!{\circ}j, m\text{☮}m_0, m\overset{\circ}{\bigcirc}m_0, m!\overset{\circ}{\bigcirc}\!{\circ}j, n\text{☮}n_0, R(m_0), Ins(t;j), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , n!\overset{\circ}{\bigcirc}\!{\circ}j, m\text{☮}m_0, m\overset{\circ}{\bigcirc}m_0, m_0!\overset{\circ}{\bigcirc}\!{\circ}j, n\text{☮}n_0, R(m_0), Ins(t;j), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , n!\overset{\circ}{\bigcirc}\!{\circ}j, m\text{☮}m_0, m\overset{\circ}{\bigcirc}m_0, n\text{☮}n_0, m_0!\overset{\circ}{\bigcirc}\!{\circ}j, R(m_0), Ins(t;j), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , n!\overset{\circ}{\bigcirc}\!{\circ}j, m\text{☮}m_0, m\overset{\circ}{\bigcirc}m_0, n\text{☮}n_0, m_0!\overset{\circ}{\bigcirc}\!{\circ}j, Ins(t;j), R(m_0), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , m!\overset{\circ}{\bigcirc}\!{\circ}j, n!\overset{\circ}{\bigcirc}\!{\circ}j, m\text{☮}m_0, n\text{☮}n_0, Ins(t;j), R(m_0), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , m!\overset{\circ}{\bigcirc}\!{\circ}j, n!\overset{\circ}{\bigcirc}\!{\circ}j, Ins(t;j), m\text{☮}m_0, n\text{☮}n_0, R(m_0), R(n_0), m_0\overset{\circ}{\bigcirc}n_0, m_0\text{⏚}, n_0\text{⏚},$$

$$\Leftrightarrow \ , m!\overset{\circ}{\bigcirc}\!{\circ}j, n!\overset{\circ}{\bigcirc}\!{\circ}j, Ins(t;j), m\overset{\circ}{\bigcirc}\!{\circ}n,$$

$$\Leftrightarrow \ , Ins(t;j), m!\overset{\circ}{\bigcirc}\!{\circ}j, n!\overset{\circ}{\bigcirc}\!{\circ}j, m\overset{\circ}{\bigcirc}\!{\circ}n,$$

$$\Leftrightarrow \ , Ins(t;j), m!\overset{\circ}{\bigcirc}\!{\circ}j, m!\overset{\circ}{\bigcirc}\!{\circ}j, m\overset{\circ}{\bigcirc}\!{\circ}n,$$

$$\Leftrightarrow \ , Ins(t;j), m!\overset{\circ}{\bigcirc}\!{\circ}j, m\overset{\circ}{\bigcirc}\!{\circ}n,$$

$$\Leftrightarrow \ , m!\overset{\circ}{\bigcirc}\!{\circ}j, Ins(t;j), m\overset{\circ}{\bigcirc}\!{\circ}n,$$

$$\begin{aligned} , m\overset{\circ}{\bigcirc}\!{\circ}n, Ins(t;j), \ &\Leftrightarrow \ , Ins(t;j), m\overset{\circ}{\bigcirc}\!{\circ}n, \\ , m!\overset{\circ}{\bigcirc}\!{\circ}n, Ins(t;j), \ &\Leftrightarrow \ , Ins(t;j), m!\overset{\circ}{\bigcirc}\!{\circ}n, \end{aligned}$$

26 Theorems of Delete Node Function Del(j)

26.1 General theorems

26.1.1 Property

$$, Del(j) \Leftrightarrow , j \,!\!= \varnothing, Del(j)$$

26.1.2 Substitution

$$, j_1 \text{♂} j_2, Del(j_1), \ \Leftrightarrow \ , j_1 \text{♂} j_2, Del(j_2),$$

26.1.3 Swap with operator

$$, g\text{⏚}, Del(j), \ \Leftrightarrow \ , Del(j), g\text{⏚},$$

$$, \circledcirc g, Del(j), \ \Leftrightarrow \ , Del(j), \circledcirc g,$$

$$, m\text{☮}n, Del(j), \ \Leftrightarrow \ , Del(j), m\text{☮}n,$$

$$, j\text{☮}n, Del(j), \ \Leftrightarrow \ , Del(j), j\text{☮}n,$$

proof:
$, j\text{☮}n, Del(j),$

$\Leftrightarrow \ , j\text{☮}j_0, j_0\text{⏚}, j\text{☮}n, Del(j),$

$\Leftrightarrow \ , j\text{☮}j_0, j\text{☮}n, Del(j), j_0\text{⏚},$

$\Leftrightarrow \ , j\text{☮}j_0, j\text{♂}j_0, j\text{☮}n, Del(j), j_0\text{⏚},$

$\Leftrightarrow \ , j\text{☮}j_0, j\text{☮}n, j\text{♂}j_0, Del(j), j_0\text{⏚},$

$\Leftrightarrow \ , j\text{☮}j_0, j\text{☮}n, j\text{♂}j_0, Del(j_0), j_0\text{⏚},$

$\Leftrightarrow\ , j\text{☮}j_0, j\text{♂}j_0, Del(j_0), j\text{☮}n, j_0\text{⏚},$

$\Leftrightarrow\ , j\text{☮}j_0, j\text{♂}j_0, Del(j), j\text{☮}n, j_0\text{⏚},$

$\Leftrightarrow\ , j\text{☮}j_0, Del(j), j\text{☮}n, j_0\text{⏚},$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\text{⏚}, Del(j), j\text{☮}n,$

$\Leftrightarrow\ , Del(j), j\text{☮}n,$

$$, j{\to}m, m\ominus, Del(j), \ \Leftrightarrow\ , j{\to}m, Del(j),$$

proof:
$, j{\to}m, m\ominus, Del(j),$

$\Leftrightarrow\ , m\ominus, j\text{♂}m, Del(j),$

$\Leftrightarrow\ , m\ominus, j\text{♂}m, m\oplus, Del(j),$

$\Leftrightarrow\ , j{\to}m, m\ominus, m\oplus, Del(j),$

$\Leftrightarrow\ , j{\to}m, m\oplus, m\ominus, Del(j),$

$\Leftrightarrow\ , j{\to}m, Del(j),$

$$, j!{\to}m, m\ominus, Del(j), \ \Leftrightarrow\ , j!{\to}m, Del(j), m\ominus,$$

26.1.4 Swap with propositions

$$, m!\text{♂}j, n!\text{♂}j, m{=}n, Del(j), \ \Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, Del(j), m{=}n,$$

$$, m!\text{♂}j, n!\text{♂}j, m\,!{=}\,n, Del(j), \ \Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, Del(j), m\,!{=}\,n,$$

$$, m!\text{♂}j, m{=}\varnothing, Del(j), \ \Leftrightarrow\ , m!\text{♂}j, Del(j), m{=}\varnothing,$$

proof:
$, m!\text{♂}j, m{=}\varnothing, Del(j),$

$\Leftrightarrow\ , m!\text{♂}j, \circledcirc n, m{=}n, n\text{⏚}, Del(j),$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, \circledcirc n, n!\bigcirc^{\!\circ} j, m = n, n\textcircled{\scriptsize$\downarrow$}, Del(j),$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, \circledcirc n, n!\bigcirc^{\!\circ} j, m = n, Del(j), n\textcircled{\scriptsize$\downarrow$},$

$\Leftrightarrow\ , \circledcirc n, m!\bigcirc^{\!\circ} j, n!\bigcirc^{\!\circ} j, m = n, Del(j), n\textcircled{\scriptsize$\downarrow$},$

$\Leftrightarrow\ , \circledcirc n, m!\bigcirc^{\!\circ} j, n!\bigcirc^{\!\circ} j, Del(j), m = n, n\textcircled{\scriptsize$\downarrow$},$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, \circledcirc n, n!\bigcirc^{\!\circ} j, Del(j), m = n, n\textcircled{\scriptsize$\downarrow$},$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, \circledcirc n, Del(j), m = n, n\textcircled{\scriptsize$\downarrow$},$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, Del(j), \circledcirc n, m = n, n\textcircled{\scriptsize$\downarrow$},$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, Del(j), m = \varnothing,$

$$, m = \varnothing, Del(j), \quad \Leftrightarrow \quad , m!\bigcirc^{\!\circ} j, Del(j), m = \varnothing,$$

proof:
$, m = \varnothing, Del(j),$

$\Leftrightarrow\ , m = \varnothing, j\ != \varnothing, Del(j),$

$\Leftrightarrow\ , m = \varnothing, j\ != \varnothing, m!\bigcirc^{\!\circ} j, Del(j),$

$\Leftrightarrow\ , j\ != \varnothing, m!\bigcirc^{\!\circ} j, m = \varnothing, Del(j),$

$\Leftrightarrow\ , j\ != \varnothing, m!\bigcirc^{\!\circ} j, Del(j), m = \varnothing,$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, j\ != \varnothing, Del(j), m = \varnothing,$

$\Leftrightarrow\ , m!\bigcirc^{\!\circ} j, Del(j), m = \varnothing,$

$$, m = \varnothing, Del(j), \quad \Leftrightarrow \quad \sim, m = \varnothing,$$

proof:
$, m = \varnothing, Del(j),$

$\Leftrightarrow\ , m = \varnothing, j\ != \varnothing, Del(j),$

$\Leftrightarrow\ , m = \varnothing, j \mathrel{!=} \varnothing, m!\text{♂}j, Del(j),$

$\Leftrightarrow\ , m = \varnothing, m = \varnothing, j \mathrel{!=} \varnothing, m!\text{♂}j, Del(j),$

$\Leftrightarrow\ , m = \varnothing, j \mathrel{!=} \varnothing, m!\text{♂}j, m = \varnothing, Del(j),$

$\Leftrightarrow\ , m = \varnothing, j \mathrel{!=} \varnothing, m!\text{♂}j, Del(j), m = \varnothing,$

$\Leftrightarrow\ , m = \varnothing, j \mathrel{!=} \varnothing, Del(j), m = \varnothing,$

$\Leftrightarrow\ , m = \varnothing, Del(j), m = \varnothing,$

$$, m!\text{♂}j, m \mathrel{!=} \varnothing, Del(j), \ \Leftrightarrow\ , m!\text{♂}j, Del(j), m \mathrel{!=} \varnothing,$$

26.2 Swap with identical node propositions

$$, m!\text{♂}j, n!\text{♂}j, m\text{♂}n, Del(j), \ \Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, Del(j), m\text{♂}n,$$

proof:
$, m!\text{♂}j, n!\text{♂}j, m\text{♂}n, Del(j),$

$\Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, m\text{☮}dm, n\text{☮}dn, dm = dn, dm\text{⏚}, dn\text{⏚}, Del(j),$

$\Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, m\text{☮}dm, n\text{☮}dn, dm = dn, Del(j), dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, m\text{☮}dm, dm!\text{♂}j, n\text{☮}dn, dn!\text{♂}j, dm = dn, Del(j), dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, m\text{☮}dm, n\text{☮}dn, dm!\text{♂}j, dn!\text{♂}j, dm = dn, Del(j), dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, m\text{☮}dm, n\text{☮}dn, dm!\text{♂}j, dn!\text{♂}j, Del(j), dm = dn, dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , m!\text{♂}j, n!\text{♂}j, m\text{☮}dm, n\text{☮}dn, Del(j), dm = dn, dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , m!\text{♂}j, m\text{☮}dm, n!\text{♂}j, n\text{☮}dn, Del(j), dm = dn, dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , m!\text{♂}j, m\text{☮}dm, n!\text{♂}j, Del(j), n\text{☮}dn, dm = dn, dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , n!\text{♂}j, m!\text{♂}j, m\text{☮}dm, Del(j), n\text{☮}dn, dm = dn, dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , n!\text{♂}j, m!\text{♂}j, Del(j), m\text{☮}dm, n\text{☮}dn, dm = dn, dm\text{⏚}, dn\text{⏚},$

$\Leftrightarrow\ , m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}n,$

$$, m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, m!\mathring{\bigcirc}n, Del(j), \ \Leftrightarrow\ , m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, Del(j), m!\mathring{\bigcirc}n,$$

$$, j{\rightarrow}k, Del(j), \ \Leftrightarrow\ , k!\mathring{\bigcirc}j, Del(j), k\mathring{\bigcirc}j,$$

proof:
$, j{\rightarrow}k, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, k\mathring{\bigcirc}j_0, j_0\text{⏚}, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, k\mathring{\bigcirc}j_0, j_0\text{⏚}, j \mathrel{!=} \varnothing, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j{\rightarrow}j_0, k\mathring{\bigcirc}j_0, j_0\text{⏚}, j \mathrel{!=} \varnothing, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j \mathrel{!=} \varnothing, j{\rightarrow}j_0, k\mathring{\bigcirc}j_0, j_0\text{⏚}, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j \mathrel{!=} \varnothing, j{\rightarrow}j_0, j{>}j_0, k\mathring{\bigcirc}j_0, j_0\text{⏚}, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j \mathrel{!=} \varnothing, j{\rightarrow}j_0, j{>}j_0, j_0!\mathring{\bigcirc}j, k\mathring{\bigcirc}j_0, j_0\text{⏚}, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j \mathrel{!=} \varnothing, j{\rightarrow}j_0, j{>}j_0, j_0!\mathring{\bigcirc}j, j_0!\mathring{\bigcirc}j, k\mathring{\bigcirc}j_0, j_0\text{⏚}, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j \mathrel{!=} \varnothing, j{\rightarrow}j_0, j{>}j_0, j_0!\mathring{\bigcirc}j, k!\mathring{\bigcirc}j, k\mathring{\bigcirc}j_0, j_0\text{⏚}, Del(j),$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j \mathrel{!=} \varnothing, j{\rightarrow}j_0, j{>}j_0, j_0!\mathring{\bigcirc}j, k!\mathring{\bigcirc}j, k\mathring{\bigcirc}j_0, Del(j), j_0\text{⏚},$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, j \mathrel{!=} \varnothing, j{\rightarrow}j_0, j{>}j_0, j_0!\mathring{\bigcirc}j, k!\mathring{\bigcirc}j, Del(j), k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , j\text{☮}j_0, j_0\oplus, k!\mathring{\bigcirc}j, Del(j), k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, j\text{☮}j_0, j_0\oplus, Del(j), k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, j\text{☮}j_0, j\mathring{\bigcirc}j_0, j_0\oplus, Del(j), k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, j\text{☮}j_0, j\mathring{\bigcirc}j_0, Del(j), k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, j\text{☮}j_0, Del(j), k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, Del(j), j\text{☮}j_0, k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, Del(j), j\text{☮}j_0, j\mathring{\bigcirc}j_0, k\mathring{\bigcirc}j_0, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, Del(j), j\text{☮}j_0, j\mathring{\bigcirc}j_0, k\mathring{\bigcirc}j, j_0\text{⏚},$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, Del(j), j\text{☮}j_0, j\mathring{\bigcirc}j_0, j_0\text{⏚}, k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, Del(j), j\text{☮}j_0, j_0\text{⏚}, k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , k!\mathring{\bigcirc}j, Del(j), k\mathring{\bigcirc}j,$

$$, j{\rightarrow}k, Del(j), \ \Leftrightarrow\ \sim, k\mathring{\bigcirc}j,$$

proof:
$, j{\rightarrow}k, Del(j),$

$\Leftrightarrow\ , j{\rightarrow}k, j{\rightarrow}k, Del(j),$

$\Leftrightarrow\ , j{\rightarrow}k, k!\mathring{\bigcirc}j, Del(j), k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , j{\rightarrow}k, k!\mathring{\bigcirc}j, j \mathrel{!=} \varnothing, Del(j), k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing, j{\rightarrow}k, k!\mathring{\bigcirc}j, Del(j), k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing, j{\rightarrow}k, j{\succ}k, k!\mathring{\bigcirc}j, Del(j), k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing, j{\rightarrow}k, j{\succ}k, Del(j), k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing, j{\rightarrow}k, Del(j), k\mathring{\bigcirc}j,$

$\Leftrightarrow\ , j{\rightarrow}k, Del(j), k\mathring{\bigcirc}j,$

$$, m\mathring{\bigcirc}j, Del(j), \ \Leftrightarrow\ \sim, m\mathring{\bigcirc}j,$$

proof:
$, m\mathring{\bigcirc}j, Del(j),$

$\Leftrightarrow\ , m\mathring{\bigcirc}j, m\oplus, Del(j),$

$\Leftrightarrow\ , m\oplus, j{\rightarrow}m, Del(j),$

$\Leftrightarrow\ , m\oplus, j{\rightarrow}m, Del(j), m\mathring{\bigcirc}j,$

$\Leftrightarrow\ , m\mathring{\bigcirc}j, m\oplus, Del(j), m\mathring{\bigcirc}j,$

$\Leftrightarrow\ , m\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}j,$

$$, m\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j), \ \Leftrightarrow\ \sim, m\mathring{\bigcirc}n,$$

proof:
$, m\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j),$

$\Leftrightarrow\ , m\mathring{\bigcirc}j, n\mathring{\bigcirc}j, Del(j),$

$\Leftrightarrow\ , m\mathring{\bigcirc}j, n\mathring{\bigcirc}j, Del(j), n\mathring{\bigcirc}j,$

$\Leftrightarrow\ , n\mathring{\bigcirc}j, m\mathring{\bigcirc}j, Del(j), n\mathring{\bigcirc}j,$

$\Leftrightarrow\ , n\mathring{\bigcirc}j, m\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}j, n\mathring{\bigcirc}j,$

$\Leftrightarrow\ , n\mathring{\bigcirc}j, m\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}j, m\mathring{\bigcirc}n,$

$\Leftrightarrow\ , n\mathring{\bigcirc}j, m\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}n,$

$\Leftrightarrow\ , m\mathring{\bigcirc}j, n\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}n,$

$\Leftrightarrow\ , m\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j), m\mathring{\bigcirc}n,$

$$, m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j), \ \Leftrightarrow\ \sim, m\mathring{\bigcirc}n,$$

proof:
$, m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j),$

$\Leftrightarrow\ , m!\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, m\mathring{\bigcirc}n, Del(j),$

$\Leftrightarrow\ , m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j),$

$\Leftrightarrow\ , m!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, n!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j),$

$\Leftrightarrow\ , m\mathring{\bigcirc}n, m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, m\mathring{\bigcirc}n, Del(j),$

$\Leftrightarrow\ , m\mathring{\bigcirc}n, m!\mathring{\bigcirc}j, n!\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}n,$

$$\Leftrightarrow\ , m!\mathring{\bigcirc} j, n!\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n,$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc} j, m!\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n,$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n,$$

$$, m\mathring{\bigcirc} n, Del(j),\ \Leftrightarrow\ \sim, m\mathring{\bigcirc} n,$$

proof:
$, m\mathring{\bigcirc} n, Del(j),$

$$\Leftrightarrow\ , if(m\mathring{\bigcirc} j) \left[\begin{array}{l} , \\ , \end{array}\right], m\mathring{\bigcirc} n, Del(j),$$

$$\Leftrightarrow\ , if(m\mathring{\bigcirc} j) \left[\begin{array}{l} , m\mathring{\bigcirc} n, Del(j), \\ , m\mathring{\bigcirc} n, Del(j), \end{array}\right],$$

$$\Leftrightarrow\ , if(m\mathring{\bigcirc} j) \left[\begin{array}{l} , m\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Del(j), \\ , m!\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Del(j), \end{array}\right],$$

$$\Leftrightarrow\ , if(m\mathring{\bigcirc} j) \left[\begin{array}{l} , m\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n, \\ , m!\mathring{\bigcirc} j, m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n, \end{array}\right],$$

$$\Leftrightarrow\ , if(m\mathring{\bigcirc} j) \left[\begin{array}{l} , m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n, \\ , m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n, \end{array}\right],$$

$$\Leftrightarrow\ , if(m\mathring{\bigcirc} j) \left[\begin{array}{l} , \\ , \end{array}\right], m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n,$$

$$\Leftrightarrow\ , m\mathring{\bigcirc} n, Del(j), m\mathring{\bigcirc} n,$$

$$, m!\mathring{\bigcirc} j, j!{\rightarrow} m, Del(j),\ \Leftrightarrow\ \sim, m!\mathring{\bigcirc} j,$$

proof:
$, m!\mathring{\bigcirc} j, j!{\rightarrow} m, Del(j),$

$$\Leftrightarrow\ , m!\mathring{\bigcirc} j, j!{\rightarrow} m, j \text{☮} k, k \text{⏚}, Del(j),$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, k\textcircled{$\downarrow$}, Del(j),$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, j{\rightarrow}k, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j\text{☮}k, k\oplus, j{\rightarrow}k, j!{\rightarrow}m, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j\text{☮}k, k\oplus, j{\rightarrow}k, j!{\rightarrow}m, m!\mathring{\bigcirc}k, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j\text{☮}k, k\oplus, j{\rightarrow}k, j!{\rightarrow}m, m!\mathring{\bigcirc}k, j\,!{=}\,\varnothing, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j\text{☮}k, k\oplus, j\,!{=}\,\varnothing, j{\rightarrow}k, j!{\rightarrow}m, m!\mathring{\bigcirc}k, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j\text{☮}k, k\oplus, j\,!{=}\,\varnothing, j{\rightarrow}k, j\succ k, j!{\rightarrow}m, m!\mathring{\bigcirc}k, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j\text{☮}k, k\oplus, j\,!{=}\,\varnothing, j{\rightarrow}k, j\succ k, k!\mathring{\bigcirc}j, j!{\rightarrow}m, m!\mathring{\bigcirc}k, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , j\text{☮}k, k\oplus, j\,!{=}\,\varnothing, j{\rightarrow}k, j\succ k, j!{\rightarrow}m, m!\mathring{\bigcirc}j, k!\mathring{\bigcirc}j, m!\mathring{\bigcirc}k, Del(j), k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , j\text{☮}k, k\oplus, j\,!{=}\,\varnothing, j{\rightarrow}k, j\succ k, j!{\rightarrow}m, m!\mathring{\bigcirc}j, k!\mathring{\bigcirc}j, Del(j), m!\mathring{\bigcirc}k, k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, j\,!{=}\,\varnothing, j{\rightarrow}k, j\succ k, k!\mathring{\bigcirc}j, Del(j), m!\mathring{\bigcirc}k, k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, j{\rightarrow}k, Del(j), m!\mathring{\bigcirc}k, k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, j{\rightarrow}k, Del(j), k\mathring{\bigcirc}j, m!\mathring{\bigcirc}k, k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, j{\rightarrow}k, Del(j), k\mathring{\bigcirc}j, m!\mathring{\bigcirc}j, k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, j{\rightarrow}k, Del(j), m!\mathring{\bigcirc}j, k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, Del(j), m!\mathring{\bigcirc}j, k\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\oplus, k\textcircled{$\downarrow$}, Del(j), m!\mathring{\bigcirc}j,$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, j\text{☮}k, k\textcircled{$\downarrow$}, Del(j), m!\mathring{\bigcirc}j,$$

$$\Leftrightarrow\ , m!\mathring{\bigcirc}j, j!{\rightarrow}m, Del(j), m!\mathring{\bigcirc}j,$$

$$,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ \\ ,j\to m,\end{array}\right], Del(j), \iff \sim, m\mathring{\bigcirc}j,$$

proof:

$$,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ \\ ,j\to m,\end{array}\right], Del(j),$$

$$\iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, Del(j), \\ \\ ,j\to m, Del(j),\end{array}\right],$$

$$\iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, m\mathring{\bigcirc}j, Del(j), \\ \\ ,j\to m, Del(j),\end{array}\right],$$

$$\iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, m\mathring{\bigcirc}j, Del(j), m\mathring{\bigcirc}j, \\ \\ ,j\to m, Del(j), m\mathring{\bigcirc}j,\end{array}\right],$$

$$\iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, m\mathring{\bigcirc}j, \\ \\ ,j\to m,\end{array}\right], Del(j), m\mathring{\bigcirc}j,$$

$$\iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ \\ ,j\to m,\end{array}\right], Del(j), m\mathring{\bigcirc}j,$$

$$,Del(j), m\mathring{\bigcirc}j, \iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ \\ ,j\to m,\end{array}\right],\sim$$

proof:

$$,Del(j), m\mathring{\bigcirc}j,$$

$$\iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ ,\end{array}\right], Del(j), m\mathring{\bigcirc}j,$$

$$\iff ,if(m\mathring{\bigcirc}j)\left[\begin{array}{l}, \\ \\ ,if(j\to m)\left[\begin{array}{l}, \\ ,\end{array}\right],\end{array}\right], Del(j), m\mathring{\bigcirc}j,$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} , \\ ,m!\text{♂}j,if(j{\rightarrow}m)\begin{bmatrix} , \\ , \end{bmatrix}, \end{bmatrix},Del(j),m\text{♂}j,$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} , \\ ,m!\text{♂}j,if(j{\rightarrow}m)\begin{bmatrix} , \\ ,j!{\rightarrow}m, \end{bmatrix}, \end{bmatrix},Del(j),m\text{♂}j,$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} ,Del(j),m\text{♂}j, \\ ,m!\text{♂}j,if(j{\rightarrow}m)\begin{bmatrix} ,Del(j),m\text{♂}j, \\ ,j!{\rightarrow}m,Del(j),m\text{♂}j, \end{bmatrix}, \end{bmatrix},$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} ,Del(j),m\text{♂}j, \\ ,if(j{\rightarrow}m)\begin{bmatrix} ,m!\text{♂}j,Del(j),m\text{♂}j, \\ ,m!\text{♂}j,j!{\rightarrow}m,Del(j),m\text{♂}j, \end{bmatrix}, \end{bmatrix},$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} ,Del(j),m\text{♂}j, \\ ,if(j{\rightarrow}m)\begin{bmatrix} ,m!\text{♂}j,Del(j),m\text{♂}j, \\ ,m!\text{♂}j,j!{\rightarrow}m,Del(j),m!\text{♂}j,m\text{♂}j, \end{bmatrix}, \end{bmatrix},$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} ,Del(j),m\text{♂}j, \\ ,if(j{\rightarrow}m)\begin{bmatrix} ,m!\text{♂}j,Del(j),m\text{♂}j, \\ ,m!\text{♂}j,j!{\rightarrow}m,Del(j),\otimes, \end{bmatrix}, \end{bmatrix},$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} ,Del(j),m\text{♂}j, \\ ,if(j{\rightarrow}m)\begin{bmatrix} ,m!\text{♂}j,Del(j),m\text{♂}j, \\ ,\otimes, \end{bmatrix}, \end{bmatrix},$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} ,Del(j),m\text{♂}j, \\ ,j{\rightarrow}m,m!\text{♂}j,Del(j),m\text{♂}j, \end{bmatrix},$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} , \\ ,j{\rightarrow}m,m!\text{♂}j, \end{bmatrix},Del(j),m\text{♂}j,$$

$$\Leftrightarrow\ ,if(m\text{♂}j)\begin{bmatrix} , \\ ,m!\text{♂}j,j{\rightarrow}m, \end{bmatrix},Del(j),m\text{♂}j,$$

$$\Leftrightarrow\ ,if(m\male j)\!-\!\left[\begin{array}{l} , \\ , j\to m, \end{array}\right]\!-, Del(j), m\male j,$$

$$,if(m\male j)\!-\!\left[\begin{array}{l} , \\ , j\to m, \end{array}\right]\!-, Del(j),\ \Leftrightarrow\ , Del(j), m\male j,$$

proof:

$$,if(m\male j)\!-\!\left[\begin{array}{l} , \\ , j\to m, \end{array}\right]\!-, Del(j),$$

$$\Leftrightarrow\ ,if(m\male j)\!-\!\left[\begin{array}{l} , \\ , j\to m, \end{array}\right]\!-, Del(j), m\male j,$$

$$\Leftrightarrow\ , Del(j), m\male j,$$

$$, m!\male j, j!\to m, Del(j),\ \Leftrightarrow\ , Del(j), m!\male j,$$

proof:

$$, Del(j), m!\male j,$$

$$\Leftrightarrow\ ,if(m\male j)\!-\!\left[\begin{array}{l} , \\ , \end{array}\right]\!-, Del(j), m!\male j,$$

$$\Leftrightarrow\ ,if(m\male j)\!-\!\left[\begin{array}{l} , \\ , if(j\to m)\!-\!\left[\begin{array}{l} , \\ , \end{array}\right]\!-, \end{array}\right]\!-, Del(j), m!\male j,$$

$$\Leftrightarrow\ ,if(m\male j)\!-\!\left[\begin{array}{l} , Del(j), m!\male j, \\ , if(j\to m)\!-\!\left[\begin{array}{l} , Del(j), m!\male j, \\ , Del(j), m!\male j, \end{array}\right]\!-, \end{array}\right]\!-,$$

$$\Leftrightarrow\ ,if(m\male j)\!-\!\left[\begin{array}{l} , m\male j, Del(j), m!\male j, \\ , if(j\to m)\!-\!\left[\begin{array}{l} , j\to m, Del(j), m!\male j, \\ , Del(j), m!\male j, \end{array}\right]\!-, \end{array}\right]\!-,$$

$$\Leftrightarrow\ ,if(m\male j)\!-\!\left[\begin{array}{l} , m\male j, Del(j), m\male j, m!\male j, \\ , if(j\to m)\!-\!\left[\begin{array}{l} , j\to m, Del(j), m\male j, m!\male j, \\ , Del(j), m!\male j, \end{array}\right]\!-, \end{array}\right]\!-,$$

$$\Leftrightarrow\ , if(m\male j)\left[\begin{array}{l} , m\male j, Del(j), \otimes, \\ , if(j\to m)\left[\begin{array}{l} , j\to m, Del(j), \otimes, \\ , Del(j), m!\male j, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , if(m\male j)\left[\begin{array}{l} , \otimes, \\ , if(j\to m)\left[\begin{array}{l} , \otimes, \\ , Del(j), m!\male j, \end{array}\right], \end{array}\right],$$

$$\Leftrightarrow\ , m!\male j, if(j\to m)\left[\begin{array}{l} , \otimes, \\ , Del(j), m!\male j, \end{array}\right],$$

$$\Leftrightarrow\ , m!\male j, j!\to m, Del(j), m!\male j,$$

$$\Leftrightarrow\ , m!\male j, j!\to m, Del(j),$$

26.3 Other

$$, i{=}j, i!\male j, Del(j), i\male j, \ \Leftrightarrow\ , \otimes,$$

proof:
$, i{=}j, i!\male j, Del(j), i\male j,$

$$\Leftrightarrow\ , if(j\to i)\left[\begin{array}{l} , \\ , \end{array}\right], i{=}j, i!\male j, Del(j), i\male j,$$

$$\Leftrightarrow\ , if(j\to i)\left[\begin{array}{l} , j\to i, \\ , \end{array}\right], i{=}j, i!\male j, Del(j), i\male j,$$

$$\Leftrightarrow\ , if(j\to i)\left[\begin{array}{l} , j\to i, i{=}j, i!\male j, Del(j), i\male j, \\ , i{=}j, i!\male j, Del(j), i\male j, \end{array}\right],$$

$$\Leftrightarrow\ , if(j\to i)\left[\begin{array}{l} , j\to i, i{=}j, i!\male j, j\ !{=}\ \varnothing, Del(j), i\male j, \\ , i{=}j, i!\male j, Del(j), i\male j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j{\to}i)\left[\begin{array}{l} ,j\,!{=}\varnothing, j{\to}i, i{=}j, i!\text{♂}j, Del(j), i\text{♂}j, \\ \\ ,i{=}j, i!\text{♂}j, Del(j), i\text{♂}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j{\to}i)\left[\begin{array}{l} ,j\,!{=}\varnothing, j{\to}i, j{>}i, i{=}j, i!\text{♂}j, Del(j), i\text{♂}j, \\ \\ ,i{=}j, i!\text{♂}j, Del(j), i\text{♂}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j{\to}i)\left[\begin{array}{l} ,j\,!{=}\varnothing, j{\to}i, \otimes, i!\text{♂}j, Del(j), i\text{♂}j, \\ \\ ,i{=}j, i!\text{♂}j, Del(j), i\text{♂}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(j{\to}i)\left[\begin{array}{l} ,\otimes, \\ \\ ,i{=}j, i!\text{♂}j, Del(j), i\text{♂}j, \end{array}\right],$$

$$\Leftrightarrow\ ,j!{\to}i, i{=}j, i!\text{♂}j, Del(j), i\text{♂}j,$$

$$\Leftrightarrow\ ,i{=}j, i!\text{♂}j, j!{\to}i, Del(j), i\text{♂}j,$$

$$\Leftrightarrow\ ,i{=}j, Del(j), i!\text{♂}j, i\text{♂}j,$$

$$\Leftrightarrow\ ,i{=}j, Del(j), \otimes,$$

$$\Leftrightarrow\ ,\otimes,$$

$$,i{=}j, Del(j), i\text{♂}j,\ \Leftrightarrow\ ,i{=}j, i\text{♂}j, Del(j),$$

proof:
$,i{=}j, Del(j), i\text{♂}j,$

$$\Leftrightarrow\ ,if(i\text{♂}j)\left[\begin{array}{l} , \\ \\ , \end{array}\right], i{=}j, Del(j), i\text{♂}j,$$

$$\Leftrightarrow\ ,if(i\text{♂}j)\left[\begin{array}{l} , \\ \\ ,i!\text{♂}j, \end{array}\right], i{=}j, Del(j), i\text{♂}j,$$

$$\Leftrightarrow\ ,if(i\mathrel{\bigcirc\!\!\!^{\circ}}j)\left[\begin{array}{l} ,i\mathrel{\text{⌶}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j, \\ \\ ,i!\mathrel{\bigcirc\!\!\!^{\circ}}j,i\mathrel{\text{⌶}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathrel{\bigcirc\!\!\!^{\circ}}j)\left[\begin{array}{l} ,i\mathrel{\text{⌶}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j, \\ \\ ,i\mathrel{\text{⌶}}j,i!\mathrel{\bigcirc\!\!\!^{\circ}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j, \end{array}\right],$$

$$\Leftrightarrow\ ,if(i\mathrel{\bigcirc\!\!\!^{\circ}}j)\left[\begin{array}{l} ,i\mathrel{\text{⌶}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j, \\ \\ ,\otimes, \end{array}\right],$$

$$\Leftrightarrow\ ,i\mathrel{\bigcirc\!\!\!^{\circ}}j,i\mathrel{\text{⌶}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j,$$

$$\Leftrightarrow\ ,i\mathrel{\text{⌶}}j,i\mathrel{\bigcirc\!\!\!^{\circ}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j,$$

$$\Leftrightarrow\ ,i\mathrel{\text{⌶}}j,i\mathrel{\bigcirc\!\!\!^{\circ}}j,Del(j),$$

$$,i\mathrel{\bigcirc\!\!\!^{\circ}}j,Del(j),\ \Leftrightarrow\ ,i\mathrel{\text{⌶}}j,Del(j),i\mathrel{\bigcirc\!\!\!^{\circ}}j,$$

$$,j\rightarrow k,j\mathrel{\text{☮}}n,Del(j),\ \Leftrightarrow\ ,j\rightarrow k,k\mathrel{\text{☮}}n,Del(j),$$

proof:
$,j\rightarrow k,j\mathrel{\text{☮}}n,Del(j),$

$$\Leftrightarrow\ ,j\rightarrow k,Del(j),j\mathrel{\text{☮}}n,$$

$$\Leftrightarrow\ ,j\rightarrow k,Del(j),j\mathrel{\bigcirc\!\!\!^{\circ}}k,j\mathrel{\text{☮}}n,$$

$$\Leftrightarrow\ ,j\rightarrow k,Del(j),j\mathrel{\bigcirc\!\!\!^{\circ}}k,k\mathrel{\text{☮}}n,$$

$$\Leftrightarrow\ ,j\rightarrow k,Del(j),k\mathrel{\text{☮}}n,$$

$$\Leftrightarrow\ ,j\rightarrow k,k\mathrel{\text{☮}}n,Del(j),$$

26.4 Swap with node connectivity propositions

$$,R(i),Del(j), \;\Leftrightarrow\; ,Del(j),R(i),$$

induction proof:

$premise\ 1:$

$,i=\varnothing,R(i),Del(j),$

$\Leftrightarrow\ ,i=\varnothing,Del(j),$

$\Leftrightarrow\ ,i=\varnothing,Del(j),i=\varnothing,$

$\Leftrightarrow\ ,i=\varnothing,Del(j),i=\varnothing,R(i),$

$\Leftrightarrow\ ,i=\varnothing,Del(j),R(i),$

$premise\ 2:$

$,\ \&SHi\rightarrow i,R(i),Del(j),\ \Leftrightarrow\ ,\ \&SHi\rightarrow i,Del(j),R(i),\ \Rightarrow$

$,i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i,R(i),Del(j),$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,R(i),Del(j),$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,i\oplus,R(i),Del(j),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,i\oplus,\ \&SHi\rightarrow i,R(i),Del(j),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,i\oplus,\ \&SHi\rightarrow i,Del(j),R(i),$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,i\,!\!=\varnothing,i\oplus,Del(j),R(i),$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},\\ ,\end{array}\right],i\,!\!=\varnothing,i\oplus,Del(j),R(i),$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},i\,!\!=\varnothing,i\oplus,Del(j),R(i),\\ ,i\,!\!=\varnothing,i\oplus,Del(j),R(i),\end{array}\right],$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},i\mathring{\bigcirc}j,i\,!\!=\varnothing,i\oplus,Del(j),R(i),\\ ,i\,!\!=\varnothing,i\oplus,Del(j),R(i),\end{array}\right],$

$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i,if(i\mathring{\bigcirc}j)\left[\begin{array}{l},i\,!\!=\varnothing,i\mathring{\bigcirc}j,i\oplus,Del(j),R(i),\\ ,i\,!\!=\varnothing,i\oplus,Del(j),R(i),\end{array}\right],$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, i\bigcirc^{\!\circ} j, Del(j), R(i),\\ \\ , i\,!{=}\,\varnothing, i\oplus, Del(j), R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\bigcirc^{\!\circ} j, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i\,!{=}\,\varnothing, i\oplus, Del(j), R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i\,!{=}\,\varnothing, i\oplus, Del(j), R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i!\bigcirc^{\!\circ} j, i\,!{=}\,\varnothing, i\oplus, Del(j), R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i\,!{=}\,\varnothing, i!\bigcirc^{\!\circ} j, i\oplus, Del(j), R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i\,!{=}\,\varnothing, i!\bigcirc^{\!\circ} j, Del(j), i\oplus, R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i!\bigcirc^{\!\circ} j, i\,!{=}\,\varnothing, Del(j), i\oplus, R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i!\bigcirc^{\!\circ} j, Del(j), i\,!{=}\,\varnothing, i\oplus, R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i!\bigcirc^{\!\circ} j, Del(j), i\,!{=}\,\varnothing, R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i!\bigcirc^{\!\circ} j, i\,!{=}\,\varnothing, Del(j), R(i),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\bigcirc^{\!\circ} i, if(i\bigcirc^{\!\circ} j)\left[\begin{array}{l}, i\,!{=}\,\varnothing, Del(j), R(i),\\ \\ , i\,!{=}\,\varnothing, Del(j), R(i),\end{array}\right],$$

$\Leftrightarrow\ ,\ \&SHi♂i, if(i♂j)\left[\begin{array}{c},\\ \\ ,\end{array}\right], i\,!\!=\varnothing, Del(j), R(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi♂i, Del(j), R(i),$

$conclusion:$
$, R(i), Del(j),\ \Leftrightarrow\ , Del(j), R(i),$

$$, m=\varnothing, n=\varnothing, m♂n, Del(j),\ \Leftrightarrow\ , m=\varnothing, n=\varnothing, Del(j), m♂n,$$

proof:
$, m=\varnothing, n=\varnothing, m♂n, Del(j),$

$\Leftrightarrow\ , m=\varnothing, n=\varnothing, m♂n, j\,!\!=\varnothing, Del(j),$

$\Leftrightarrow\ , m=\varnothing, j\,!\!=\varnothing, n=\varnothing, m♂n, Del(j),$

$\Leftrightarrow\ , m=\varnothing, j\,!\!=\varnothing, m!♂j, n=\varnothing, m♂n, Del(j),$

$\Leftrightarrow\ , m=\varnothing, m!♂j, n=\varnothing, j\,!\!=\varnothing, m♂n, Del(j),$

$\Leftrightarrow\ , m=\varnothing, m!♂j, n=\varnothing, j\,!\!=\varnothing, n!♂j, m♂n, Del(j),$

$\Leftrightarrow\ , m=\varnothing, n=\varnothing, j\,!\!=\varnothing, m!♂j, n!♂j, m♂n, Del(j),$

$\Leftrightarrow\ , m=\varnothing, n=\varnothing, j\,!\!=\varnothing, m!♂j, n!♂j, Del(j), m♂n,$

$\Leftrightarrow\ , m=\varnothing, n=\varnothing, j\,!\!=\varnothing, n!♂j, m!♂j, Del(j), m♂n,$

$\Leftrightarrow\ , m=\varnothing, n=\varnothing, j\,!\!=\varnothing, m!♂j, Del(j), m♂n,$

$\Leftrightarrow\ , n=\varnothing, m=\varnothing, j\,!\!=\varnothing, m!♂j, Del(j), m♂n,$

$\Leftrightarrow\ , n=\varnothing, m=\varnothing, j\,!\!=\varnothing, Del(j), m♂n,$

$\Leftrightarrow\ , n=\varnothing, m=\varnothing, Del(j), m♂n,$

$\Leftrightarrow\ , m=\varnothing, n=\varnothing, Del(j), m♂n,$

$$, m=\varnothing, n=\varnothing, m!♂n, Del(j),\ \Leftrightarrow\ , m=\varnothing, n=\varnothing, Del(j), m!♂n,$$

$$, m \overset{\circ}{\bigcirc}\!\circ n, Del(j), \;\Leftrightarrow\; , Del(j), m \overset{\circ}{\bigcirc}\!\circ n,$$

proof:
$, m \overset{\circ}{\bigcirc}\!\circ n, Del(j),$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), R(n_0), m_0 \overset{\circ}{\bigcirc} n_0, m_0 \text{⏚}, n_0 \text{⏚}, Del(j),$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), R(n_0), m_0 \overset{\circ}{\bigcirc} n_0, Del(j), m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), m_0 = \varnothing, R(n_0), m_0 \overset{\circ}{\bigcirc} n_0, Del(j), m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), m_0 = \varnothing, R(n_0), n_0 = \varnothing, m_0 \overset{\circ}{\bigcirc} n_0, Del(j), m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), R(n_0), m_0 = \varnothing, n_0 = \varnothing, m_0 \overset{\circ}{\bigcirc} n_0, Del(j), m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), R(n_0), m_0 = \varnothing, n_0 = \varnothing, Del(j), m_0 \overset{\circ}{\bigcirc} n_0, m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), m_0 = \varnothing, R(n_0), n_0 = \varnothing, Del(j), m_0 \overset{\circ}{\bigcirc} n_0, m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, R(m_0), R(n_0), Del(j), m_0 \overset{\circ}{\bigcirc} n_0, m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , m \text{☮} m_0, n \text{☮} n_0, Del(j), R(m_0), R(n_0), m_0 \overset{\circ}{\bigcirc} n_0, m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , Del(j), m \text{☮} m_0, n \text{☮} n_0, R(m_0), R(n_0), m_0 \overset{\circ}{\bigcirc} n_0, m_0 \text{⏚}, n_0 \text{⏚},$

$\Leftrightarrow\; , Del(j), m \overset{\circ}{\bigcirc}\!\circ n,$

$$, m! \overset{\circ}{\bigcirc}\!\circ n, Del(j), \;\Leftrightarrow\; , Del(j), m! \overset{\circ}{\bigcirc}\!\circ n,$$

$$, i \overset{\circ}{\bigcirc}\!\circ j, Del(j), \;\Leftrightarrow\; , Del(j), i \overset{\circ}{\bigcirc}\!\circ j,$$

proof:
$, i \overset{\circ}{\bigcirc}\!\circ j, Del(j),$

$\Leftrightarrow\; , j \text{☮} j_0, j_0 \text{⏚}, i \overset{\circ}{\bigcirc}\!\circ j, Del(j),$

$\Leftrightarrow\; , j \text{☮} j_0, j \overset{\circ}{\bigcirc} j_0, j_0 \text{⏚}, i \overset{\circ}{\bigcirc}\!\circ j, Del(j),$

$\Leftrightarrow\; , j \text{☮} j_0, i \overset{\circ}{\bigcirc}\!\circ j, j \overset{\circ}{\bigcirc} j_0, Del(j), j_0 \text{⏚},$

$\Leftrightarrow\; , j \text{☮} j_0, i \overset{\circ}{\bigcirc}\!\circ j, j \overset{\circ}{\bigcirc} j_0, Del(j_0), j_0 \text{⏚},$

$\Leftrightarrow\ , j\,\text{☮}\,j_0, j\mathring{\bigcirc}j_0, i\overset{\circ}{\bigcirc}\!\circ j, Del(j_0), j_0\text{⏚},$

$\Leftrightarrow\ , j\,\text{☮}\,j_0, j\mathring{\bigcirc}j_0, Del(j_0), i\overset{\circ}{\bigcirc}\!\circ j, j_0\text{⏚},$

$\Leftrightarrow\ , j\,\text{☮}\,j_0, j\mathring{\bigcirc}j_0, Del(j), i\overset{\circ}{\bigcirc}\!\circ j, j_0\text{⏚},$

$\Leftrightarrow\ , j\,\text{☮}\,j_0, Del(j), i\overset{\circ}{\bigcirc}\!\circ j, j_0\text{⏚},$

$\Leftrightarrow\ , j\,\text{☮}\,j_0, j_0\text{⏚}, Del(j), i\overset{\circ}{\bigcirc}\!\circ j,$

$\Leftrightarrow\ , Del(j), i\overset{\circ}{\bigcirc}\!\circ j,$

$$, i!\overset{\circ}{\bigcirc}\!\circ j, Del(j), \ \Leftrightarrow\ , Del(j), i!\overset{\circ}{\bigcirc}\!\circ j,$$

27 Theorems of Assign Operator

27.1 Unity

$$,t\text{\textcircled{$\Rightarrow$}}j, \;\Leftrightarrow\; ,if(t=\varnothing)\left[\begin{array}{l} ,if(j=\varnothing)\left[\begin{array}{l} , \\ ,Del(j), \end{array}\right], \\ ,Ins(t;j), \end{array}\right],$$

27.2 Swap with identical node propositions

$$,m!\bigcirc^{\!\circ}j,n!\bigcirc^{\!\circ}j,m\bigcirc^{\!\circ}n,t\text{\textcircled{$\Rightarrow$}}j, \;\Leftrightarrow\; ,m!\bigcirc^{\!\circ}j,n!\bigcirc^{\!\circ}j,t\text{\textcircled{$\Rightarrow$}}j,m\bigcirc^{\!\circ}n,$$

$$,m!\bigcirc^{\!\circ}j,n!\bigcirc^{\!\circ}j,m!\bigcirc^{\!\circ}n,t\text{\textcircled{$\Rightarrow$}}j, \;\Leftrightarrow\; ,m!\bigcirc^{\!\circ}j,n!\bigcirc^{\!\circ}j,t\text{\textcircled{$\Rightarrow$}}j,m!\bigcirc^{\!\circ}n,$$

$$,m\bigcirc^{\!\circ}j,t\text{\textcircled{$\Rightarrow$}}j, \;\Leftrightarrow\; \sim,m\bigcirc^{\!\circ}j,$$

$$,m\bigcirc^{\!\circ}n,t\text{\textcircled{$\Rightarrow$}}j, \;\Leftrightarrow\; \sim,m\bigcirc^{\!\circ}n,$$

27.3 Swap with R(i)

$$,i!\bigcirc^{\!\circ\circ}j,R(i),t\text{\textcircled{$\Rightarrow$}}j, \;\Leftrightarrow\; ,i!\bigcirc^{\!\circ\circ}j,t\text{\textcircled{$\Rightarrow$}}j,R(i),$$

$$,i\bigcirc^{\!\circ\circ}j,j\,!\!=\varnothing,R(i),t\text{\textcircled{$\Rightarrow$}}j, \;\Leftrightarrow\; ,i\bigcirc^{\!\circ\circ}j,j\,!\!=\varnothing,t\text{\textcircled{$\Rightarrow$}}j,R(i),$$

27.4 Swap with node connectivity propositions

$$, m\bigcirc^{\circ} j, t\textcircled{$\Rightarrow$} j, \;\Leftrightarrow\; , t\textcircled{$\Rightarrow$} j, m\bigcirc^{\circ} j,$$

$$, m!\bigcirc^{\circ} j, t\textcircled{$\Rightarrow$} j, \;\Leftrightarrow\; , t\textcircled{$\Rightarrow$} j, m!\bigcirc^{\circ} j,$$

$$, m\overset{\circ}{\bigcirc} n, t\textcircled{$\Rightarrow$} j, \;\Leftrightarrow\; , t\textcircled{$\Rightarrow$} j, m\overset{\circ}{\bigcirc} n,$$

$$, m!\overset{\circ}{\bigcirc} n, t\textcircled{$\Rightarrow$} j, \;\Leftrightarrow\; , t\textcircled{$\Rightarrow$} j, m!\overset{\circ}{\bigcirc} n,$$

27.5 Swap with self

27.5.1 Ins and Ins

$$, Ins(t; j_1), In(t; j_2), \;\Leftrightarrow\; , Ins(t; j_2), In(t; j_1),$$

proof:
$, Ins(t; j_1), In(t; j_2),$

$\Leftrightarrow \;, Ins(t; j_1), t\,!\!= \varnothing, t\textcircled{$\Rightarrow$} j_2,$

$\Leftrightarrow \;, Ins(t; j_1), t\textcircled{$\Rightarrow$} j_2,$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t_0\textcircled{$\downarrow$}, Ins(t; j_1), t\textcircled{$\Rightarrow$} j_2,$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, Ins(t; j_1), t\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t\overset{\circ}{\bigcirc} t_0, Ins(t; j_1), t\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t\overset{\circ}{\bigcirc} t_0, t = t_0, Ins(t; j_1), t\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t\overset{\circ}{\bigcirc} t_0, t = t_0, Ins(t; j_1), t = t_0, t\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t\overset{\circ}{\bigcirc} t_0, t = t_0, Ins(t; j_1), t = t_0, t_0\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t\overset{\circ}{\bigcirc} t_0, t = t_0, Ins(t; j_1), t_0\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t\overset{\circ}{\bigcirc} t_0, t = t_0, t\,!\!= \varnothing, t\textcircled{$\Rightarrow$} j_1, t_0\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \;, t\textcircled{$\curlywedge$} t_0, t\overset{\circ}{\bigcirc} t_0, t\,!\!= \varnothing, t = t_0, t\textcircled{$\Rightarrow$} j_1, t_0\textcircled{$\Rightarrow$} j_2, t_0\textcircled{$\downarrow$},$

$\Leftrightarrow \ , t \circledast t_0, t \mathring{\bigcirc} t_0, t \mathrel{!=} \varnothing, t \mathrel{!=} \varnothing, t = t_0, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\Rightarrow$}} j_2, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t \mathring{\bigcirc} t_0, t \mathrel{!=} \varnothing, t_0 \mathrel{!=} \varnothing, t = t_0, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\Rightarrow$}} j_2, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t \mathring{\bigcirc} t_0, t \mathrel{!=} \varnothing, t_0 \mathrel{!=} \varnothing, t = t_0, t_0 \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t \mathring{\bigcirc} t_0, t \mathrel{!=} \varnothing, t_0 \mathrel{!=} \varnothing, t = t_0, t \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t \mathring{\bigcirc} t_0, t \mathrel{!=} \varnothing, t \mathrel{!=} \varnothing, t = t_0, t \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t \mathring{\bigcirc} t_0, t \mathrel{!=} \varnothing, t = t_0, t \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t \mathring{\bigcirc} t_0, t = t_0, t \mathrel{!=} \varnothing, t \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t \mathrel{!=} \varnothing, t \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1, t_0 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , t \circledast t_0, t_0 \text{\textcircled{$\downarrow$}}, t \mathrel{!=} \varnothing, t \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1,$

$\Leftrightarrow \ , t \mathrel{!=} \varnothing, t \text{\textcircled{$\Rightarrow$}} j_2, t \text{\textcircled{$\Rightarrow$}} j_1,$

$\Leftrightarrow \ , In(t; j_2), t \text{\textcircled{$\Rightarrow$}} j_1,$

$\Leftrightarrow \ , In(t; j_2), t \mathrel{!=} \varnothing, t \text{\textcircled{$\Rightarrow$}} j_1,$

$\Leftrightarrow \ , Ins(t; j_2), In(t; j_1),$

$$\begin{aligned} &, i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, j_1 ! \mathring{\bigcirc} j_2, Ins(i_1; j_1), Ins(i_2; j_2), \ \Leftrightarrow \\ &\quad , i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, j_1 ! \mathring{\bigcirc} j_2, Ins(i_2; j_2), Ins(i_1; j_1), \end{aligned}$$

proof:
$, i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, j_1 ! \mathring{\bigcirc} j_2, Ins(i_1; j_1), Ins(i_2; j_2),$

$\Leftrightarrow \ , i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, j_1 ! \mathring{\bigcirc} j_2, Ins(i_1; j_1), i_2 \mathrel{!=} \varnothing, i_2 \text{\textcircled{$\Rightarrow$}} j_2,$

$\Leftrightarrow \ , i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, j_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, Ins(i_1; j_1), i_2 \mathrel{!=} \varnothing, i_2 \text{\textcircled{$\Rightarrow$}} j_2,$

$\Leftrightarrow \ , i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, j_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, i_2 \mathrel{!=} \varnothing, Ins(i_1; j_1), i_2 \text{\textcircled{$\Rightarrow$}} j_2,$

$\Leftrightarrow \ , i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, j_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, i_2 \mathrel{!=} \varnothing, i_1 \mathrel{!=} \varnothing, i_1 \text{\textcircled{$\Rightarrow$}} j_1, i_2 \text{\textcircled{$\Rightarrow$}} j_2,$

$\Leftrightarrow \ , i_1 \mathrel{!=} i_2, i_1 ! \mathring{\bigcirc} j_2, j_1 ! \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j_1, i_2 \mathrel{!=} \varnothing, i_1 \mathrel{!=} \varnothing, i_2 \text{\textcircled{$\Rightarrow$}} j_2, i_1 \text{\textcircled{$\Rightarrow$}} j_1,$

$\Leftrightarrow\ , i_1 \mathrel{!=} i_2, i_1 !\overset{\circ}{\bigcirc} j_2, j_1 !\overset{\circ}{\bigcirc} j_2, i_2 !\overset{\circ}{\bigcirc} j_1, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_2 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_2, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1,$

$\Leftrightarrow\ , i_1 \mathrel{!=} i_2, i_1 !\overset{\circ}{\bigcirc} j_2, j_1 !\overset{\circ}{\bigcirc} j_2, i_2 !\overset{\circ}{\bigcirc} j_1, i_1 \mathrel{!=} \varnothing, Ins(i_2; j_2), i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1,$

$\Leftrightarrow\ , i_1 \mathrel{!=} i_2, j_1 !\overset{\circ}{\bigcirc} j_2, i_2 !\overset{\circ}{\bigcirc} j_1, i_1 !\overset{\circ}{\bigcirc} j_2, i_1 \mathrel{!=} \varnothing, Ins(i_2; j_2), i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1,$

$\Leftrightarrow\ , i_1 \mathrel{!=} i_2, j_1 !\overset{\circ}{\bigcirc} j_2, i_2 !\overset{\circ}{\bigcirc} j_1, i_1 !\overset{\circ}{\bigcirc} j_2, Ins(i_2; j_2), i_1 \mathrel{!=} \varnothing, i_1 \text{\textcircled{$\scriptstyle\Rightarrow$}} j_1,$

$\Leftrightarrow\ , i_1 \mathrel{!=} i_2, i_1 !\overset{\circ}{\bigcirc} j_2, i_2 !\overset{\circ}{\bigcirc} j_1, j_1 !\overset{\circ}{\bigcirc} j_2, Ins(i_2; j_2), Ins(i_1; j_1),$

27.5.2 Del and Del

$$, Del(j_1), Del(j_2), \ \Leftrightarrow\ , Del(j_2), Del(j_1),$$

proof:
$, Del(j_1), Del(j_2),$

$\Leftrightarrow\ , if(j_1 \overset{\circ}{\bigcirc} j_2) \left[\begin{array}{l} , \\ , \end{array}\right], Del(j_1), Del(j_2),$

$\Leftrightarrow\ , if(j_1 \overset{\circ}{\bigcirc} j_2) \left[\begin{array}{l} , Del(j_1), Del(j_2), \\ , Del(j_1), Del(j_2), \end{array}\right],$

$\Leftrightarrow\ , if(j_1 \overset{\circ}{\bigcirc} j_2) \left[\begin{array}{l} , j_1 \overset{\circ}{\bigcirc} j_2, Del(j_1), Del(j_2), \\ , Del(j_1), Del(j_2), \end{array}\right],$

$\Leftrightarrow\ , if(j_1 \overset{\circ}{\bigcirc} j_2) \left[\begin{array}{l} , j_1 \overset{\circ}{\bigcirc} j_2, Del(j_1), j_1 \overset{\circ}{\bigcirc} j_2, Del(j_2), \\ , Del(j_1), Del(j_2), \end{array}\right],$

$\Leftrightarrow\ , if(j_1 \overset{\circ}{\bigcirc} j_2) \left[\begin{array}{l} , j_1 \overset{\circ}{\bigcirc} j_2, Del(j_1), j_1 \overset{\circ}{\bigcirc} j_2, Del(j_1), \\ , Del(j_1), Del(j_2), \end{array}\right],$

$\Leftrightarrow\ , if(j_1 \overset{\circ}{\bigcirc} j_2) \left[\begin{array}{l} , j_1 \overset{\circ}{\bigcirc} j_2, Del(j_1), Del(j_1), \\ , Del(j_1), Del(j_2), \end{array}\right],$

$\Leftrightarrow\ , if(j_1 \overset{\circ}{\bigcirc} j_2) \left[\begin{array}{l} , j_1 \overset{\circ}{\bigcirc} j_2, Del(j_2), Del(j_1), \\ , Del(j_1), Del(j_2), \end{array}\right],$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,Del(j_1),Del(j_2),\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,j_1!\overset{\circ}{\bigcirc}j_2,Del(j_1),Del(j_2),\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,j_1!\overset{\circ}{\bigcirc}j_2,Del(j_1),j_2\,!\!=\varnothing,\circledcirc t_2,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_2,j_1!\overset{\circ}{\bigcirc}j_2,Del(j_1),j_2\,!\!=\varnothing,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_2,j_1!\overset{\circ}{\bigcirc}j_2,j_2\,!\!=\varnothing,Del(j_1),t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_2,j_1!\overset{\circ}{\bigcirc}j_2,j_2\,!\!=\varnothing,j_1\,!\!=\varnothing,\circledcirc t_1,t_1\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_1,t_1\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,\circledcirc t_2,j_1!\overset{\circ}{\bigcirc}j_2,j_2\,!\!=\varnothing,j_1\,!\!=\varnothing,t_1\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_1,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_1\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,t_1=\varnothing,\circledcirc t_2,t_2=\varnothing,j_1!\overset{\circ}{\bigcirc}j_2,j_2\,!\!=\varnothing,j_1\,!\!=\varnothing,t_1\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_1,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_1\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,\circledcirc t_2,t_1=\varnothing,t_2=\varnothing,j_1!\overset{\circ}{\bigcirc}j_2,j_2\,!\!=\varnothing,j_1\,!\!=\varnothing,t_1\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_1,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_1\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,\circledcirc t_2,t_1=\varnothing,t_2=\varnothing,j_1!\overset{\circ}{\bigcirc}j_2,j_2\,!\!=\varnothing,j_1\,!\!=\varnothing,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_1\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_1,t_1\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\overset{\circ}{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,t_1=\varnothing,\circledcirc t_2,t_2=\varnothing,j_1!\overset{\circ}{\bigcirc}j_2,j_2\,!\!=\varnothing,j_1\,!\!=\varnothing,t_2\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_2,t_1\bigcirc\mkern-14mu{\scriptstyle\Rightarrow}\,j_1,t_1\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,t_2\bigcirc\mkern-14mu{\scriptstyle\downarrow}\,,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,\circledcirc t_2,j_1!\mathring{\bigcirc}j_2,j_2\,!\!=\varnothing,j_1\,!\!=\varnothing,t_2\circleddash j_2,t_1\circleddash j_1,t_1\circledast,t_2\circledast,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,j_1!\mathring{\bigcirc}j_2,j_1\,!\!=\varnothing,j_2\,!\!=\varnothing,\circledcirc t_2,t_2\circleddash j_2,t_2\circledast,t_1\circleddash j_1,t_1\circledast,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,j_1!\mathring{\bigcirc}j_2,j_1\,!\!=\varnothing,Del(j_2),t_1\circleddash j_1,t_1\circledast,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,\circledcirc t_1,j_1!\mathring{\bigcirc}j_2,Del(j_2),j_1\,!\!=\varnothing,t_1\circleddash j_1,t_1\circledast,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,j_1!\mathring{\bigcirc}j_2,Del(j_2),j_1\,!\!=\varnothing,\circledcirc t_1,t_1\circleddash j_1,t_1\circledast,\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,j_1!\mathring{\bigcirc}j_2,Del(j_2),Del(j_1),\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},Del(j_2),Del(j_1),\\ \\ ,Del(j_2),Del(j_1),\end{array}\right],$$

$$\Leftrightarrow\ ,if(j_1\mathring{\bigcirc}j_2)\left[\begin{array}{l},\\ \\ ,\end{array}\right],Del(j_2),Del(j_1),$$

$$\Leftrightarrow\ ,Del(j_2),Del(j_1),$$

27.5.3 Ins, Del

$$,i\,!\!=\varnothing,\ \Leftrightarrow\ ,Ins(i;j),Del(j),$$

proof:
$,i\,!\!=\varnothing,$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,j\oslash j_2,j_2\circledast,\circledcirc i_2,i_2\circledast,$$

$$\Leftrightarrow\ ,i\,!\!=\varnothing,j\oslash j_2,\circledcirc i_2,i_2=\varnothing,j_2\circledast,i_2\circledast,$$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \circledast j_2, \circledcirc i_2, i_2 ! \mathring{\bigcirc} j, i_2 = \varnothing, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \circledast j_2, j \mathring{\bigcirc} j_2, \circledcirc i_2, i_2 ! \mathring{\bigcirc} j, i_2 = \varnothing, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , j \circledast j_2, \circledcirc i_2, i \mathrel{!=} \varnothing, i_2 = \varnothing, j \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , j \circledast j_2, \circledcirc i_2, i \mathrel{!=} \varnothing, i_2 = \varnothing, j \mathring{\bigcirc} j_2, i_2 ! \mathring{\bigcirc} j, i \circleddash j, i_2 \circleddash j_2, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \circledast j_2, j \mathring{\bigcirc} j_2, \circledcirc i_2, i_2 = \varnothing, i_2 ! \mathring{\bigcirc} j, i \circleddash j, i_2 \circleddash j_2, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \circledast j_2, \circledcirc i_2, i \circleddash j, i_2 \circleddash j_2, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \circledcirc i_2, i \circleddash j, j \circledast j_2, i_2 \circleddash j_2, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \circledcirc i_2, i \circleddash j, j \circledast j_2, j \mathring{\bigcirc} j_2, i_2 \circleddash j_2, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \circledcirc i_2, i \circleddash j, j \circledast j_2, j \mathring{\bigcirc} j_2, i_2 \circleddash j, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \circledcirc i_2, i \circleddash j, j \circledast j_2, i_2 \circleddash j, j_2 \odot, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \circledcirc i_2, i \circleddash j, j \circledast j_2, j_2 \odot, i_2 \circleddash j, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \circledcirc i_2, i \circleddash j, i_2 \circleddash j, i_2 \odot,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i \circleddash j, \circledcirc i_2, i_2 \circleddash j, i_2 \odot,$

$\Leftrightarrow\ , Ins(i; j), \circledcirc i_2, i_2 \circleddash j, i_2 \odot,$

$\Leftrightarrow\ , Ins(i; j), j \mathrel{!=} \varnothing, \circledcirc i_2, i_2 \circleddash j, i_2 \odot,$

$\Leftrightarrow\ , Ins(i; j), Del(j),$

$$, j_1 ! \mathring{\bigcirc} j_2, i_1 ! \mathring{\bigcirc} j_2, Del(j_2), Ins(i_1; j_1), \ \Leftrightarrow\ , j_1 ! \mathring{\bigcirc} j_2, i_1 ! \mathring{\bigcirc} j_2, Ins(i_1; j_1), Del(j_2),$$

proof:
$, j_1 ! \mathring{\bigcirc} j_2, i_1 ! \mathring{\bigcirc} j_2, Del(j_2), Ins(i_1; j_1),$

$\Leftrightarrow\ , j_1 ! \mathring{\bigcirc} j_2, i_1 ! \mathring{\bigcirc} j_2, Del(j_2), i_1 \mathrel{!=} \varnothing, i_1 \circleddash j_1,$

$\Leftrightarrow\ , j_1 ! \mathring{\bigcirc} j_2, i_1 ! \mathring{\bigcirc} j_2, i_1 \mathrel{!=} \varnothing, Del(j_2), i_1 \circleddash j_1,$

$\Leftrightarrow\ , j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, \circledcirc i_2, i_2 \ominus j_2, i_2 \oplus, i_1 \ominus j_1,$

$\Leftrightarrow\ , \circledcirc i_2, j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_2 \ominus j_2, i_1 \ominus j_1, i_2 \oplus,$

$\Leftrightarrow\ , \circledcirc i_2, i_2 = \varnothing, j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_2 \ominus j_2, i_1 \ominus j_1, i_2 \oplus,$

$\Leftrightarrow\ , \circledcirc i_2, i_2!\mathring{\bigcirc}j_1, i_2 = \varnothing, j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, i_2 \ominus j_2, i_1 \ominus j_1, i_2 \oplus,$

$\Leftrightarrow\ , \circledcirc i_2, i_1 \mathrel{!=} \varnothing, i_2 = \varnothing, j_2 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_2!\mathring{\bigcirc}j_1, i_2 \ominus j_2, i_1 \ominus j_1, i_2 \oplus,$

$\Leftrightarrow\ , \circledcirc i_2, i_1 \mathrel{!=} \varnothing, i_2 = \varnothing, j_2 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_2!\mathring{\bigcirc}j_1, i_1 \ominus j_1, i_2 \ominus j_2, i_2 \oplus,$

$\Leftrightarrow\ , i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, j_2 \mathrel{!=} \varnothing, \circledcirc i_2, i_2 = \varnothing, i_2!\mathring{\bigcirc}j_1, i_1 \ominus j_1, i_2 \ominus j_2, i_2 \oplus,$

$\Leftrightarrow\ , i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, j_2 \mathrel{!=} \varnothing, \circledcirc i_2, i_2!\mathring{\bigcirc}j_1, i_1 \ominus j_1, i_2 \ominus j_2, i_2 \oplus,$

$\Leftrightarrow\ , i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, j_2 \mathrel{!=} \varnothing, \circledcirc i_2, i_1 \ominus j_1, i_2 \ominus j_2, i_2 \oplus,$

$\Leftrightarrow\ , i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, j_2 \mathrel{!=} \varnothing, i_1 \ominus j_1, \circledcirc i_2, i_2 \ominus j_2, i_2 \oplus,$

$\Leftrightarrow\ , i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, i_1 \ominus j_1, j_2 \mathrel{!=} \varnothing, \circledcirc i_2, i_2 \ominus j_2, i_2 \oplus,$

$\Leftrightarrow\ , i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, j_1!\mathring{\bigcirc}j_2, i_1 \ominus j_1, Del(j_2),$

$\Leftrightarrow\ , j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, i_1 \mathrel{!=} \varnothing, i_1 \ominus j_1, Del(j_2),$

$\Leftrightarrow\ , j_1!\mathring{\bigcirc}j_2, i_1!\mathring{\bigcirc}j_2, Ins(i_1; j_1), Del(j_2),$

27.5.4 Other

$$, i_1 = \varnothing, i_2 = \varnothing, , j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2, i_1 \ominus j_1, i_2 \ominus j_2, \ \Leftrightarrow$$
$$, i_1 = \varnothing, i_2 = \varnothing, , j_1 \mathrel{!=} \varnothing, j_2 \mathrel{!=} \varnothing, j_1\mathring{\bigcirc}j_2, i_2 \ominus j_2, i_1 \ominus j_1,$$

28 Function Cpo(r)

28.1 Definition of Cpo(r)

$$,Cpo(r), \iff ,r\circledast m, m\circledcirc r, m\circleddash,$$

28.2 Property

$$,Cpo(r), \iff ,r\circledast m, Ins(m;r), m\circleddash,$$

$$,Cpo(r), \iff \sim, r\,!{=}\,\varnothing,$$

$$,Cpo(r), r\oplus, \iff \sim, m!\mathring{\bigcirc}r,$$

$$,r=\varnothing, Cpo(r), r\oplus, \iff ,Cpo(r), r\oplus, r=\varnothing,$$

$$,r_1\mathring{\bigcirc}r_2, Cpo(r_1), \iff ,r_1\mathring{\bigcirc}r_2, Cpo(r_2),$$

proof:
$,r_1\mathring{\bigcirc}r_2, Cpo(r_1),$

$\iff ,r_1\mathring{\bigcirc}r_2, r_1\circledast m, m\circledcirc r_1, m\circleddash,$

$\iff ,r_1\circledast m, r_1\mathring{\bigcirc}r_2, m\circledcirc r_1, m\circleddash,$

$\iff ,r_1\circledast m, r_1\mathring{\bigcirc}r_2, m\circledcirc r_2, m\circleddash,$

$\iff ,r_1\mathring{\bigcirc}r_2, r_1\circledast m, m\circledcirc r_2, m\circleddash,$

$\iff ,r_1\mathring{\bigcirc}r_2, r_2\circledast m, m\circledcirc r_2, m\circleddash,$

$\iff ,r_1\mathring{\bigcirc}r_2, Cpo(r_2),$

28.3 Swap

28.3.1 Operator

$$, i\text{☮}m, Cpo(r), \iff , Cpo(r), i\text{☮}m,$$

$$, i\text{⏚}, Cpo(r), \iff , Cpo(r), i\text{⏚},$$

$$, \circledcirc m, Cpo(r), \iff , Cpo(r), \circledcirc m,$$

$$, i!\text{♂}r, i\text{☮}m, Cpo(r), \iff , i!\text{♂}r, Cpo(r), i\text{☮}m,$$

$$, m!\text{♂}r, m\oplus, Cpo(r), \iff , m!\text{♂}r, Cpo(r), m\oplus,$$

$$, m!\text{⚨}r, m\oplus, Cpo(r), \iff , m!\text{⚨}r, Cpo(r), m\oplus,$$

$$, r!\!\to\! m, m\ominus, Cpo(r), \iff , r!\!\to\! mCpo(r), m\ominus,$$

28.3.2 Propositions node null

$$, m!\text{♂}r, m \mathrel{!=} \varnothing, Cpo(r), m!\text{♂}r, Cpo(r), m \mathrel{!=} \varnothing,$$

$$, m!\text{♂}r, m = \varnothing, Cpo(r), m!\text{♂}r, Cpo(r), m = \varnothing,$$

$$, m \mathrel{!=} \varnothing, Cpo(r), \iff \sim, m \mathrel{!=} \varnothing,$$

$$, m!\text{⚨}r, m \mathrel{!=} \varnothing, Cpo(r), m!\text{⚨}r, Cpo(r), m \mathrel{!=} \varnothing,$$

$$, m!\text{⚨}r, m = \varnothing, Cpo(r), m!\text{⚨}r, Cpo(r), m = \varnothing,$$

28.3.3 Propositions identical node

$$, m\overset{\circ}{\bigcirc}r, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m\overset{\circ}{\bigcirc}r,$$

$$, m!\overset{\circ}{\bigcirc}r, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m!\overset{\circ}{\bigcirc}r,$$

$$, m\overset{\circ}{\bigcirc}n, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m\overset{\circ}{\bigcirc}n,$$

$$, m!\overset{\circ}{\bigcirc}n, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m!\overset{\circ}{\bigcirc}n,$$

28.3.4 Propositions node connectivity

$$, m\overset{\circ}{\bigcirc}\!\circ r, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m\overset{\circ}{\bigcirc}\!\circ r,$$

$$, m!\overset{\circ}{\bigcirc}\!\circ r, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m!\overset{\circ}{\bigcirc}\!\circ r,$$

$$, m\overset{\circ}{\bigcirc}\!\circ n, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m\overset{\circ}{\bigcirc}\!\circ n,$$

$$, m!\overset{\circ}{\bigcirc}\!\circ n, Cpo(r), \;\Leftrightarrow\; , Cpo(r), m!\overset{\circ}{\bigcirc}\!\circ n,$$

28.3.5 &SHi

$$, i!\overset{\circ}{\bigcirc}\!\circ r,\ \&SHi\,\overset{\circ}{\bigcirc}i, Cpo(r), \;\Leftrightarrow\; , i!\overset{\circ}{\bigcirc}\!\circ r, Cpo(r),\ \&SHi\,\overset{\circ}{\bigcirc}i,$$

$$, i!\overset{\circ}{\bigcirc}\!\circ r,\ \&SHi \rightarrow i, Cpo(r), \;\Leftrightarrow\; , i!\overset{\circ}{\bigcirc}\!\circ r, Cpo(r),\ \&SHi \rightarrow i,$$

28.3.6 Cpo

$$,r_1\mathring{\bigcirc}r_2, Cpo(r_1), Cpo(r_2), \;\Leftrightarrow\; ,r_1\mathring{\bigcirc}r_2, Cpo(r_2), Cpo(r_1),$$

proof:
$,r_1\mathring{\bigcirc}r_2, Cpo(r_1), Cpo(r_2),$

$\Leftrightarrow\;\; ,Cpo(r_1), r_1\mathring{\bigcirc}r_2, Cpo(r_2),$

$\Leftrightarrow\;\; ,Cpo(r_1), r_1\mathring{\bigcirc}r_2, Cpo(r_1),$

$\Leftrightarrow\;\; ,r_1\mathring{\bigcirc}r_2, Cpo(r_1), Cpo(r_1),$

$\Leftrightarrow\;\; ,r_1\mathring{\bigcirc}r_2, Cpo(r_2), Cpo(r_1),$

$$,r_1!\mathring{\bigcirc}r_2, r_1\circledast m_1, r_2\circledast m_2, \;\Leftrightarrow\; \sim, m_1 \mathrel{!=} m_2,$$

proof:
$,r_1!\mathring{\bigcirc}r_2, r_1\circledast m_1, r_2\circledast m_2,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_2\circledast n_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot, r_1\circledast m_1, r_2\circledast m_2,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, r_2\circledast n_2, r_2\circledast m_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, n_1 = m_1, r_2\circledast n_2, r_2\circledast m_2, n_2 = m_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, n_1 = m_1, r_2\circledast n_2, r_2\circledast m_2, n_2 = m_2, n_1 \mathrel{!=} n_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, n_1 = m_1, r_2\circledast n_2, r_2\circledast m_2, n_2 = m_2, n_1 \mathrel{!=} m_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, r_2\circledast n_2, r_2\circledast m_2, n_2 = m_2, n_1 = m_1, n_1 \mathrel{!=} m_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, r_2\circledast n_2, r_2\circledast m_2, n_2 = m_2, n_1 = m_1, m_1 \mathrel{!=} m_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, n_1 = m_1, r_2\circledast n_2, r_2\circledast m_2, n_2 = m_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot, m_1 \mathrel{!=} m_2,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_1\circledast m_1, r_2\circledast n_2, r_2\circledast m_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot, m_1 \mathrel{!=} m_2,$

$\Leftrightarrow\;\; ,r_1\circledast n_1, r_2\circledast n_2, n_1 \mathrel{!=} n_2, n_1\obot, n_2\obot, r_1\circledast m_1, r_2\circledast m_2, m_1 \mathrel{!=} m_2,$

$\Leftrightarrow\;\; ,r_1!\mathring{\bigcirc}r_2, r_1\circledast m_1, r_2\circledast m_2, m_1 \mathrel{!=} m_2,$

$$,r_1!\mathring{\bigcirc}r_2, Cpo(r_1), Cpo(r_2), \;\Leftrightarrow\; ,r_1!\mathring{\bigcirc}r_2, Cpo(r_2), Cpo(r_1),$$

proof:
$, r_1 !\mathring{\bigcirc} r_2, Cpo(r_1), Cpo(r_2),$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, Ins(m_1; r_1), m_1 \circledcirc, r_2 \circledast m_2, Ins(m_2; r_2), m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, Ins(m_1; r_1), r_2 \circledast m_2, Ins(m_2; r_2), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \circledast m_1, r_1 !\mathring{\bigcirc} r_2, Ins(m_1; r_1), r_2 \circledast m_2, Ins(m_2; r_2), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \circledast m_1, r_1 !\mathring{\bigcirc} r_2, r_2 \circledast m_2, Ins(m_1; r_1), Ins(m_2; r_2), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, r_2 \circledast m_2, Ins(m_1; r_1), Ins(m_2; r_2), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, r_2 \circledast m_2, m_1 \mathrel{!{=}} m_2, Ins(m_1; r_1), Ins(m_2; r_2), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, m_1 !\mathring{\bigcirc} r_2, r_2 \circledast m_2, m_2 !\mathring{\bigcirc} r_2, m_1 \mathrel{!{=}} m_2, Ins(m_1; r_1), Ins(m_2; r_2), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \circledast m_1, r_2 \circledast m_2, m_1 \mathrel{!{=}} m_2, r_1 !\mathring{\bigcirc} r_2, m_1 !\mathring{\bigcirc} r_2, m_2 !\mathring{\bigcirc} r_2, Ins(m_1; r_1), Ins(m_2; r_2), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \circledast m_1, r_2 \circledast m_2, m_1 \mathrel{!{=}} m_2, r_1 !\mathring{\bigcirc} r_2, m_1 !\mathring{\bigcirc} r_2, m_2 !\mathring{\bigcirc} r_2, Ins(m_2; r_2), Ins(m_1; r_1), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, m_1 !\mathring{\bigcirc} r_2, r_2 \circledast m_2, m_2 !\mathring{\bigcirc} r_2, m_1 \mathrel{!{=}} m_2, Ins(m_2; r_2), Ins(m_1; r_1), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, r_2 \circledast m_2, m_1 \mathrel{!{=}} m_2, Ins(m_2; r_2), Ins(m_1; r_1), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, r_2 \circledast m_2, Ins(m_2; r_2), Ins(m_1; r_1), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_2 \circledast m_2, r_1 !\mathring{\bigcirc} r_2, r_1 \circledast m_1, Ins(m_2; r_2), Ins(m_1; r_1), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_2 \circledast m_2, r_1 !\mathring{\bigcirc} r_2, Ins(m_2; r_2), r_1 \circledast m_1, Ins(m_1; r_1), m_1 \circledcirc, m_2 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, r_2 \circledast m_2, Ins(m_2; r_2), m_2 \circledcirc, r_1 \circledast m_1, Ins(m_1; r_1), m_1 \circledcirc,$

$\Leftrightarrow\ , r_1 !\mathring{\bigcirc} r_2, Cpo(r_2), Cpo(r_1),$

$$, Cpo(r_1), Cpo(r_2), \quad \Leftrightarrow \quad , Cpo(r_2), Cpo(r_1),$$

$$, r_1 !\mathring{\bigcirc}^{\circ} r_2, Cpo(r_1), r_1 \oplus, Cpo(r_2), r_2 \oplus, \quad \Leftrightarrow \quad , r_1 !\mathring{\bigcirc}^{\circ} r_2, Cpo(r_2), r_2 \oplus, Cpo(r_1), r_1 \oplus,$$

proof:
$, r_1 !\mathring{\bigcirc}^{\circ} r_2, Cpo(r_1), r_1 \oplus, Cpo(r_2), r_2 \oplus,$

$\Leftrightarrow\ , Cpo(r_1), r_1!\mathring{\bigcirc}\!\circ r_2, r_1\oplus, Cpo(r_2), r_2\oplus,$

$\Leftrightarrow\ , Cpo(r_1), r_1!\mathring{\bigcirc}\!\circ r_2, r_1!\mathring{\bigcirc} r_2, r_1\oplus, Cpo(r_2), r_2\oplus,$

$\Leftrightarrow\ , Cpo(r_1), r_1!\mathring{\bigcirc}\!\circ r_2, r_1!\mathring{\bigcirc} r_2, Cpo(r_2), r_1\oplus, r_2\oplus,$

$\Leftrightarrow\ , Cpo(r_1), r_1!\mathring{\bigcirc}\!\circ r_2, Cpo(r_2), r_1\oplus, r_2\oplus,$

$\Leftrightarrow\ , r_1!\mathring{\bigcirc}\!\circ r_2, Cpo(r_1), Cpo(r_2), r_1\oplus, r_2\oplus,$

$\Leftrightarrow\ , r_1!\mathring{\bigcirc}\!\circ r_2, Cpo(r_2), Cpo(r_1), r_1\oplus, r_2\oplus,$

$\Leftrightarrow\ , Cpo(r_2), r_1!\mathring{\bigcirc}\!\circ r_2, Cpo(r_1), r_1\oplus, r_2\oplus,$

$\Leftrightarrow\ , Cpo(r_2), r_1!\mathring{\bigcirc}\!\circ r_2, r_1!\mathring{\bigcirc} r_2, Cpo(r_1), r_1\oplus, r_2\oplus,$

$\Leftrightarrow\ , Cpo(r_2), r_1!\mathring{\bigcirc}\!\circ r_2, r_1!\mathring{\bigcirc} r_2, Cpo(r_1), r_2\oplus, r_1\oplus,$

$\Leftrightarrow\ , Cpo(r_2), r_1!\mathring{\bigcirc}\!\circ r_2, r_1!\mathring{\bigcirc} r_2, r_2\oplus, Cpo(r_1), r_1\oplus,$

$\Leftrightarrow\ , Cpo(r_2), r_1!\mathring{\bigcirc}\!\circ r_2, r_2\oplus, Cpo(r_1), r_1\oplus,$

$\Leftrightarrow\ , r_1!\mathring{\bigcirc}\!\circ r_2, Cpo(r_2), r_2\oplus, Cpo(r_1), r_1\oplus,$

28.3.7 R(i)

$$, i!\mathring{\bigcirc}\!\circ r, R(i), Cpo(r), \ \Leftrightarrow\ , i!\mathring{\bigcirc}\!\circ r, Cpo(r), R(i),$$

28.3.8 Rc(i;j)

$$, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, Rc(i;j), Cpo(r), \ \Leftrightarrow\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, Cpo(r), Rc(i;j),$$

induction proof:
premise 1 :
$, i=\varnothing, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, Rc(i;j), Cpo(r),$
$\Leftrightarrow\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i=\varnothing, Rc(i;j), Cpo(r),$
$\Leftrightarrow\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i=\varnothing, Cpo(r),$
$\Leftrightarrow\ , j!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}\!\circ r, i=\varnothing, Cpo(r),$

$\Leftrightarrow\ , j!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc} r, i=\varnothing, Cpo(r),$
$\Leftrightarrow\ , j!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc} r, Cpo(r), i=\varnothing,$
$\Leftrightarrow\ , j!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc} r, Cpo(r), i=\varnothing, Rc(i;j),$
$\Leftrightarrow\ , j!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc} r, i=\varnothing, Cpo(r), Rc(i;j),$
$\Leftrightarrow\ , j!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}\!\circ r, i=\varnothing, Cpo(r), Rc(i;j),$
$\Leftrightarrow\ , i=\varnothing, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, Cpo(r), Rc(i;j),$

$premise\ 2:$
$, \&SHi \rightarrow i, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, Rc(i;j), Cpo(r),\ \Leftrightarrow\ , \&SHi \rightarrow i, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, Cpo(r), Rc(i;j),\ \Longrightarrow$

$, i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, Rc(i;j), Cpo(r),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Rc(i;j), Cpo(r),$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , \\ , i\oplus, j\oplus, Rc(i;j), \end{array}\right], Cpo(r),$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , Cpo(r), \\ , i\oplus, j\oplus, Rc(i;j), Cpo(r), \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , j=\varnothing, Cpo(r), \\ , i\oplus, j\oplus, Rc(i;j), Cpo(r), \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , j!\mathring{\bigcirc}\!\circ r, j=\varnothing, Cpo(r), \\ , j!\mathring{\bigcirc}\!\circ r, i\oplus, j\oplus, Rc(i;j), Cpo(r), \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc} r, j=\varnothing, Cpo(r), \\ , j!\mathring{\bigcirc}\!\circ r, i\oplus, j\oplus, Rc(i;j), Cpo(r), \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc} r, Cpo(r), j=\varnothing, \\ , j!\mathring{\bigcirc}\!\circ r, i\oplus, j\oplus, Rc(i;j), Cpo(r), \end{array}\right],$$

$$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, i!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, if(j=\varnothing)\left[\begin{array}{l} , j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc} r, Cpo(r), j=\varnothing, Rc(i;j), \\ , j!\mathring{\bigcirc}\!\circ r, i\oplus, j\oplus, Rc(i;j), Cpo(r), \end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, if(j=\varnothing)-\left[\begin{array}{l}, j!\overset{\circ}{\bigcirc}{\circ}r, j!\bigcirc^{\circ}r, j=\varnothing, Cpo(r), Rc(i;j), \\ \\ , j!\overset{\circ}{\bigcirc}{\circ}r, i\oplus, j\oplus, Rc(i;j), Cpo(r),\end{array}\right]-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, if(j=\varnothing)-\left[\begin{array}{l}, j!\overset{\circ}{\bigcirc}{\circ}r, j=\varnothing, Cpo(r), Rc(i;j), \\ \\ , j!\overset{\circ}{\bigcirc}{\circ}r, i\oplus, j\oplus, Rc(i;j), Cpo(r),\end{array}\right]-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, if(j=\varnothing)-\left[\begin{array}{l}, j=\varnothing, j!\bigcirc^{\circ}r, Cpo(r), Rc(i;j), \\ \\ , j!\overset{\circ}{\bigcirc}{\circ}r, i\oplus, j\oplus, Rc(i;j), Cpo(r),\end{array}\right]-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, if(j=\varnothing)-\left[\begin{array}{l}, j!\overset{\circ}{\bigcirc}{\circ}r, Cpo(r), Rc(i;j), \\ \\ , j!\overset{\circ}{\bigcirc}{\circ}r, i\oplus, j\oplus, Rc(i;j), Cpo(r),\end{array}\right]-,$$

$$\Leftrightarrow\ , if(j=\varnothing)-\left[\begin{array}{l}, \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, Cpo(r), Rc(i;j), \\ \\ , \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), Cpo(r),\end{array}\right]-,$$

$$\Leftrightarrow\ , if(j=\varnothing)-\left[\begin{array}{l}, \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, Cpo(r), Rc(i;j), \\ \\ , j\,!\!=\varnothing, \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, i\oplus, j\oplus, Rc(i;j), Cpo(r),\end{array}\right]-,$$

$$\Leftrightarrow\ , if(j=\varnothing)-\left[\begin{array}{l}, \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, Cpo(r), Rc(i;j), \\ \\ , j\,!\!=\varnothing, i\,!\!=\varnothing, i\oplus, j\oplus, \ \&SHi \rightarrow i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, Rc(i;j), Cpo(r),\end{array}\right]-,$$

$$\Leftrightarrow\ , if(j=\varnothing)-\left[\begin{array}{l}, \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, Cpo(r), Rc(i;j), \\ \\ , j\,!\!=\varnothing, i\,!\!=\varnothing, i\oplus, j\oplus, \ \&SHi \rightarrow i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, Cpo(r), Rc(i;j),\end{array}\right]-,$$

$$\Leftrightarrow\ , if(j=\varnothing)-\left[\begin{array}{l}, \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, Cpo(r), Rc(i;j), \\ \\ , \ \&SHi\,\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, j\,!\!=\varnothing, i\,!\!=\varnothing, i\oplus, j\oplus, Cpo(r), Rc(i;j),\end{array}\right]-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, if(j=\varnothing)-\left[\begin{array}{l}, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, Cpo(r), Rc(i;j), \\ \\ , i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, j\,!\!=\varnothing, i\,!\!=\varnothing, i\oplus, j\oplus, Cpo(r), Rc(i;j),\end{array}\right]-,$$

$$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}i, if(j=\varnothing)-\left[\begin{array}{l}, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!\!=\varnothing, Cpo(r), Rc(i;j), \\ \\ , i!\overset{\circ}{\bigcirc}{\circ}r, i!\bigcirc^{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, j!\bigcirc^{\circ}r, j\,!\!=\varnothing, i\,!\!=\varnothing, i\oplus, j\oplus, Cpo(r), Rc(i;j),\end{array}\right]-,$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}r, j!\mathring{\bigcirc}\!\circ r, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, j!\mathring{\bigcirc}r, j\oplus, Cpo(r), Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}r, j!\mathring{\bigcirc}\!\circ r, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, i\oplus, j!\mathring{\bigcirc}r, Cpo(r), j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, i\oplus, j!\mathring{\bigcirc}r, j\,!{=}\,\varnothing, Cpo(r), j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, i!\mathring{\bigcirc}r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, i\oplus, j!\mathring{\bigcirc}r, Cpo(r), j\,!{=}\,\varnothing, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, j!\mathring{\bigcirc}r, i!\mathring{\bigcirc}r, i\oplus, Cpo(r), j\,!{=}\,\varnothing, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, j!\mathring{\bigcirc}r, i!\mathring{\bigcirc}r, Cpo(r), i\oplus, j\,!{=}\,\varnothing, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}r, i!\mathring{\bigcirc}r, i\,!{=}\,\varnothing, Cpo(r), i\oplus, j\,!{=}\,\varnothing, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}r, i!\mathring{\bigcirc}r, Cpo(r), i\,!{=}\,\varnothing, i\oplus, j\,!{=}\,\varnothing, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}r, i!\mathring{\bigcirc}r, Cpo(r), i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, i\oplus, j\oplus, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}r, i!\mathring{\bigcirc}r, Cpo(r), i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\mathring{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\\ , i!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}\!\circ r, j!\mathring{\bigcirc}r, i!\mathring{\bigcirc}r, i\,!{=}\,\varnothing, Cpo(r), j\,!{=}\,\varnothing, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j), \\ , i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i!\overset{\circ}{\bigcirc}r, i\,!{=}\,\varnothing, j!\overset{\circ}{\bigcirc}r, Cpo(r), j\,!{=}\,\varnothing, Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j), \\ , i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i!\overset{\circ}{\bigcirc}r, i\,!{=}\,\varnothing, j!\overset{\circ}{\bigcirc}r, j\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j), \\ , j\,!{=}\,\varnothing, i!\overset{\circ}{\bigcirc}{\circ}r, i!\overset{\circ}{\bigcirc}r, j!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, if(j=\varnothing)\left[\begin{array}{l}, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j), \\ , i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i\,!{=}\,\varnothing, Cpo(r), Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, if(j=\varnothing)\left[\begin{array}{l}, Cpo(r), Rc(i;j), \\ , Cpo(r), Rc(i;j),\end{array}\right],$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, if(j=\varnothing)\left[\begin{array}{l}, \\ ,\end{array}\right], Cpo(r), Rc(i;j),$$

$$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, Cpo(r), Rc(i;j),$$

$conclusion:$
$, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, Rc(i;j), Cpo(r),\ \Leftrightarrow\ , i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, Cpo(r), Rc(i;j),$

28.3.9 Propositions number comparison

$$, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i{=}j, Cpo(r),\ \Leftrightarrow\ , i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, Cpo(r), i{=}j,$$

$$, i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, i{>}j, Cpo(r),\ \Leftrightarrow\ , i!\overset{\circ}{\bigcirc}{\circ}r, j!\overset{\circ}{\bigcirc}{\circ}r, Cpo(r), i{>}j,$$

28.3.10 Other

$$,r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc}{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}{}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), \quad \Leftrightarrow \quad \sim, r_1 \mathrel{\text{⌶}} r_2,$$

induction proof:

$premise$ 1 :

$,r_1 = \varnothing, r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc}{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}{}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_1 = \varnothing, r_1 \mathring{\bigcirc}{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}{}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_1 = \varnothing, r_1 = \varnothing, r_1 \mathring{\bigcirc}{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}{}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_2 = \varnothing, r_1 = \varnothing, r_1 \mathring{\bigcirc}{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}{}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc}{}^{\circ} r_{10}, r_2 = \varnothing, r_{20} = \varnothing, r_2 \mathring{\bigcirc}{}^{\circ} r_{20},$

$Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, r_2 = \varnothing, r_{20} = \varnothing, r_2 \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_1 = \varnothing, r_{10} = \varnothing, r_2 = \varnothing, r_{20} = \varnothing,$

$r_1 \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_2 \mathring{\bigcirc} r_{20}, Cpo(r_{20}),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_1 ! \mathring{\bigcirc}{}^{\circ} r_2, r_1 = \varnothing, r_{10} = \varnothing, r_2 = \varnothing, r_{20} = \varnothing,$

$r_1 \mathring{\bigcirc} r_{10}, Cpo(r_1), r_2 \mathring{\bigcirc} r_{20}, Cpo(r_2),$

$\Leftrightarrow \; ,r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 = \varnothing, r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, r_2 = \varnothing, Cpo(r_1), Cpo(r_2),$

$\Leftrightarrow \; , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 = \varnothing, r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, r_1 ! \overset{\circ}{\bigcirc} r_2, r_2 = \varnothing, Cpo(r_1), Cpo(r_2),$

$\Leftrightarrow \; , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 = \varnothing, r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, r_1 ! \overset{\circ}{\bigcirc} r_2, Cpo(r_1), r_2 = \varnothing, Cpo(r_2),$

$\Leftrightarrow \; , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, r_1 = \varnothing, Cpo(r_1),$

$r_2 = \varnothing, Cpo(r_2),$

$\Leftrightarrow \; , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, Cpo(r_1), r_1 \text{☮} n_1, n_1 \oplus, n_1 = \varnothing, n_1 \text{⏚},$

$Cpo(r_2), r_2 \text{☮} n_2, n_2 \oplus, n_2 = \varnothing, n_2 \text{⏚},$

$\Leftrightarrow \; , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, r_1 \text{☮} n_1, n_1 \oplus, n_1 = \varnothing,$

$Cpo(r_2), r_2 \text{☮} n_2, n_2 \oplus, n_2 = \varnothing, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow \; , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \text{☮} n_1, n_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, n_1 \oplus, n_1 = \varnothing,$

$Cpo(r_2), r_2 \text{☮} n_2, n_2 \oplus, n_2 = \varnothing, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow \; , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \text{☮} n_1, n_1 \oplus, n_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r_2, n_1 = \varnothing,$

$Cpo(r_2), r_2 \circledwedge n_2, n_2 \oplus, n_2 = \varnothing, n_1 \text{\textcircled{$\downarrow$}}, n_2 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \circledwedge n_1, n_1 \oplus, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2, n_1 = \varnothing,$

$Cpo(r_2), r_2 \circledwedge n_2, n_2 \oplus, n_2 = \varnothing, n_1 \text{\textcircled{$\downarrow$}}, n_2 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \circledwedge n_1, n_1 \oplus, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2,$

$Cpo(r_2), n_1 = \varnothing, r_2 \circledwedge n_2, n_2 \oplus, n_2 = \varnothing, n_1 \text{\textcircled{$\downarrow$}}, n_2 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \circledwedge n_1, n_1 \oplus, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2,$

$Cpo(r_2), r_2 \circledwedge n_2, n_2 \oplus, n_1 = \varnothing, n_2 = \varnothing, n_1 \text{\textcircled{$\downarrow$}}, n_2 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \circledwedge n_1, n_1 \oplus, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2,$

$Cpo(r_2), r_2 \circledwedge n_2, n_2 \oplus, n_1 = \varnothing, n_2 = \varnothing, n_1 = n_2, n_1 \text{\textcircled{$\downarrow$}}, n_2 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \circledwedge n_1, n_1 \oplus, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2,$

$Cpo(r_2), r_2 \circledwedge n_2, n_2 \oplus, n_1 = \varnothing, n_2 = \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \text{\textcircled{$\downarrow$}}, n_2 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \circledwedge n_1, n_1 \oplus, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2,$

$Cpo(r_2), n_1 = \varnothing, r_2 \circledwedge n_2, n_2 \oplus, n_2 = \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \text{\textcircled{$\downarrow$}}, n_2 \text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 \oplus, n_1 ! \bigcirc\!\!^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2, n_1 = \varnothing,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \oplus, n_2 = \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 ! \bigcirc\!\!^{\circ} r_2, Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 \oplus, n_1 = \varnothing,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \oplus, n_2 = \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 ! \bigcirc\!\!^{\circ} r_2, r_1 = \varnothing, Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 \oplus,$

$r_2 = \varnothing, Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 = \varnothing, r_1 ! \bigcirc\!\!^{\circ} r_2, r_1 ! \mathring{\bigcirc} r_2, Cpo(r_1), r_2 = \varnothing, r_1 \mathrel{\text{☮}} n_1, n_1 \oplus,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$r_1 = \varnothing, r_1 ! \bigcirc\!\!^{\circ} r_2, r_1 ! \mathring{\bigcirc} r_2, r_2 = \varnothing, Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 \oplus,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \bigcirc\!\!^{\circ} r_2, Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 \oplus,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \overset{\circ}{\bigcirc}\!\circ\, r_2, Cpo(r_1), r_1 \,!\!= \varnothing, r_1 \mathrel{\text{☮}} n_1, n_1 \oplus,$

$Cpo(r_2), r_2 \,!\!= \varnothing, r_2 \mathrel{\text{☮}} n_2, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \overset{\circ}{\bigcirc}\!\circ\, r_2, Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 \,!\!= \varnothing, n_1 \oplus,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \,!\!= \varnothing, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 ! \overset{\circ}{\bigcirc}\!\circ\, r_2, n_1 \,!\!= \varnothing, n_1 \oplus,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \,!\!= \varnothing, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 ! \overset{\circ}{\bigcirc}\!\circ\, r_2, n_1 ! \overset{\circ}{\bigcirc} r_2, n_1 \,!\!= \varnothing, n_1 \oplus,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_2 \,!\!= \varnothing, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 ! \overset{\circ}{\bigcirc}\!\circ\, r_2, n_1 ! \overset{\circ}{\bigcirc} r_2,$

$Cpo(r_2), n_1 \,!\!= \varnothing, n_1 \oplus, r_2 \mathrel{\text{☮}} n_2, n_2 \,!\!= \varnothing, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$Cpo(r_1), r_1 \mathrel{\text{☮}} n_1, n_1 ! \overset{\circ}{\bigcirc}\!\circ\, r_2, n_1 ! \overset{\circ}{\bigcirc} r_2,$

$Cpo(r_2), r_2 \mathrel{\text{☮}} n_2, n_1 \,!\!= \varnothing, n_1 \oplus, n_2 \,!\!= \varnothing, n_2 \oplus, Rc(n_1; n_2), n_1 = n_2, n_1 \text{⏚}, n_2 \text{⏚},$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$Cpo(r_1), r_1 \otimes n_1, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2,$

$Cpo(r_2), r_2 \otimes n_2, n_1 \mathrel{!=} \varnothing, n_2 \mathrel{!=} \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \circledcirc, n_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \mp r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$Cpo(r_1), r_1 \otimes n_1, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2,$

$Cpo(r_2), n_1 \mathrel{!=} \varnothing, r_2 \otimes n_2, n_2 \mathrel{!=} \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \circledcirc, n_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \mp r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$Cpo(r_1), r_1 \otimes n_1, n_1 ! \mathring{\bigcirc}^{\circ} r_2, n_1 ! \mathring{\bigcirc} r_2, n_1 \mathrel{!=} \varnothing,$

$Cpo(r_2), r_2 \otimes n_2, n_2 \mathrel{!=} \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \circledcirc, n_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \mp r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \mathring{\bigcirc}^{\circ} r_2, Cpo(r_1), r_1 \otimes n_1, n_1 \mathrel{!=} \varnothing,$

$Cpo(r_2), r_2 \otimes n_2, n_2 \mathrel{!=} \varnothing, Rc(n_1; n_2), n_1 = n_2, n_1 \circledcirc, n_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \mp r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \mathring{\bigcirc}^{\circ} r_2, Cpo(r_1), r_1 \mathrel{!=} \varnothing, r_1 \otimes n_1,$

$Cpo(r_2), r_2 \mathrel{!=} \varnothing, r_2 \otimes n_2, Rc(n_1; n_2), n_1 = n_2, n_1 \circledcirc, n_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \mp r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \mathring{\bigcirc}^{\circ} r_2, Cpo(r_1), r_1 \otimes n_1,$

$Cpo(r_2), r_2 \otimes n_2, Rc(n_1; n_2), n_1 = n_2, n_1 \circledcirc, n_2 \circledcirc,$

$\Leftrightarrow\ , r_1 \mp r_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \mathring{\bigcirc}^{\circ} r_2, Cpo(r_1),$

$Cpo(r_2), r_1 \text{☮} n_1, r_2 \text{☮} n_2, Rc(n_1; n_2), n_1 = n_2, n_1 \textcircled{\downarrow}, n_2 \textcircled{\downarrow},$

$\Leftrightarrow\ , r_1 \text{⌶} r_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10} = \varnothing, r_{20} = \varnothing, r_1 = \varnothing, r_2 = \varnothing,$

$r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, Cpo(r_1), Cpo(r_2), r_1 \text{⌶} r_2,$

$\Leftrightarrow\ , r_1 \text{⌶} r_2, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 = \varnothing, r_{20} = \varnothing, r_2 \overset{\circ}{\bigcirc} r_{20},$

$r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, Cpo(r_1), Cpo(r_2), r_1 \text{⌶} r_2,$

$\Leftrightarrow\ , r_1 \text{⌶} r_2, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_2 = \varnothing, r_{20} = \varnothing, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20},$

$r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, Cpo(r_1), Cpo(r_2), r_1 \text{⌶} r_2,$

$\Leftrightarrow\ , r_1 \text{⌶} r_2, r_2 = \varnothing, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_{20} = \varnothing, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20},$

$r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, Cpo(r_1), Cpo(r_2), r_1 \text{⌶} r_2,$

$\Leftrightarrow\ , r_1 \text{⌶} r_2, r_1 = \varnothing, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_{20} = \varnothing, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20},$

$r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, Cpo(r_1), Cpo(r_2), r_1 \text{⌶} r_2,$

$\Leftrightarrow\ , r_1 \text{⌶} r_2, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_{20} = \varnothing, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20},$

$r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, Cpo(r_1), Cpo(r_2), r_1 \text{⌶} r_2,$

$\Leftrightarrow\ , r_1 = \varnothing, r_1 \text{⌶} r_2, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20}, r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \text{⌶} r_2,$

$premise\ 2:$
$, \ \&SHi \rightarrow r_1, r_1 \text{⌶} r_2, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20}, r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), \ \Leftrightarrow\ ,$

$\&SHi \rightarrow r_1, r_1 \text{⌶} r_2, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20}, r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \text{⌶} r_2, \ \Rightarrow$

$, r_1 \mathrel{!=} \varnothing, \ \&SHi \overset{\circ}{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20}, r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \ \&SHi \overset{\circ}{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathrel{!=} \varnothing, r_1 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{10}, r_2 \overset{\circ}{\bigcirc}\!{}^{\circ} r_{20}, r_1 ! \overset{\circ}{\bigcirc}\!{}^{\circ} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_1 \,!{=} \varnothing, r_1 \,!{=} \varnothing, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2,$

$r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_2 \,!{=} \varnothing, r_1 \,!{=} \varnothing, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2,$

$r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 \,!{=} \varnothing, r_{10} = \varnothing,$

$r_2 \,!{=} \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 \,!{=} \varnothing, r_{10} = \varnothing,$

$r_1 ! \mathring{\bigcirc} r_{10}, r_2 \,!{=} \varnothing, r_{20} = \varnothing, r_2 ! \mathring{\bigcirc} r_{20}, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 \,!{=} \varnothing,$

$r_{10} = \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \,!{=} \varnothing, r_{20} = \varnothing, r_2 ! \mathring{\bigcirc} r_{20}, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 \,!{=} \varnothing,$

$r_{10} = \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \,!{=} \varnothing, r_{20} = \varnothing, r_2 ! \mathring{\bigcirc} r_{20}, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} ! \mathring{\bigcirc}^{\circ} r_2, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2,$

$r_1 \,!{=} \varnothing, r_{10} = \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \,!{=} \varnothing, r_{20} = \varnothing, r_2 ! \mathring{\bigcirc} r_{20}, Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \overset{\top}{=} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 \,!{=} \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \,!{=} \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), Cpo(r_{20}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} r_1, r_1 \mathrel{\overline{\bot}} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathrel{!}= \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \mathrel{!}= \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), Cpo(r_{20}), r_1 \oplus, r_1 \ominus, r_2 \oplus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} r_1, r_1 \mathrel{\overline{\bot}} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 \mathrel{!}= \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \mathrel{!}= \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), Cpo(r_{20}), r_1 \oplus, r_1 \ominus, r_2 \oplus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} r_1, r_1 \mathrel{\overline{\bot}} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathrel{!}= \varnothing, r_2 \mathrel{!}= \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 ! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_1 ! \mathring{\bigcirc} r_{20}, Cpo(r_{20}), r_1 \oplus, r_1 \ominus, r_2 \oplus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} r_1, r_1 \mathrel{\overline{\bot}} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathrel{!}= \varnothing, r_2 \mathrel{!}= \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 ! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_1 ! \mathring{\bigcirc} r_{20}, r_1 \oplus, Cpo(r_{20}), r_1 \ominus, r_2 \oplus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} r_1, r_1 \mathrel{\overline{\bot}} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 \mathrel{!}= \varnothing, r_2 \mathrel{!}= \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 ! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_1 \oplus, Cpo(r_{20}), r_1 \ominus, r_2 \oplus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} r_1, r_1 \mathrel{\overline{\bot}} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 \mathrel{!}= \varnothing, r_2 \mathrel{!}= \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 ! \mathring{\bigcirc} r_{10}, r_1 \oplus, Cpo(r_{10}), Cpo(r_{20}), r_1 \ominus, r_2 \oplus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} r_1, r_1 \mathrel{\overline{\bot}} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 ! \mathring{\bigcirc} r_{10},$

$r_1 \oplus, r_2 ! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_2 ! \mathring{\bigcirc} r_{20}, Cpo(r_{20}), r_2 \oplus, r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow \; , \&SHi\, \mathring{\bigcirc} r_1, r_1 \underline{\top} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 ! \mathring{\bigcirc} r_{10},$

$r_1 \oplus, r_2 ! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_2 ! \mathring{\bigcirc} r_{20}, r_2 \oplus, Cpo(r_{20}), r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow \; , \&SHi\, \mathring{\bigcirc} r_1, r_1 \underline{\top} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 \oplus, r_2 ! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_2 \oplus, Cpo(r_{20}), r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow \; , \&SHi\, \mathring{\bigcirc} r_1, r_1 \underline{\top} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 \oplus, r_2 ! \mathring{\bigcirc} r_{10}, r_2 \oplus, Cpo(r_{10}), Cpo(r_{20}), r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow \; , \&SHi\, \mathring{\bigcirc} r_1, r_1 \underline{\top} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 \oplus, r_2 \oplus, Cpo(r_{10}), Cpo(r_{20}), r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow \; , r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$\&SHi\, \mathring{\bigcirc} r_1, r_1 \underline{\top} r_2, r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 \oplus, r_2 \oplus,$

$r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow \; , r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$\&SHi\, \mathring{\bigcirc} r_1, r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 \oplus, r_2 \oplus, r_1 \underline{\top} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20},$

$r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_2, r_{20}=\varnothing, r_{10}=\varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , r_2!\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{20},$

$r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, r_1\oplus, r_2\oplus,\ \&SHi \rightarrow r_1, r_1 \mathrel{\text{⌶}} r_2, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2\overset{\circ}{\bigcirc}{}^{\circ}r_{20},$

$r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_2, r_{20}=\varnothing, r_{10}=\varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , r_2!\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{20},$

$r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, r_1\oplus, r_2\oplus,\ \&SHi \rightarrow r_1, r_1 \mathrel{\text{⌶}} r_2, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2\overset{\circ}{\bigcirc}{}^{\circ}r_{20},$

$r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_2, r_{20}=\varnothing, r_{10}=\varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \mathrel{\text{⌶}} r_2, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, r_1 \mathrel{\text{⌶}} r_2, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_2, r_{20}=\varnothing, r_{10}=\varnothing,$

$r_2!\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{20},$

$r_1 \mathrel{!=} \varnothing, r_1\oplus, r_2!\overset{\circ}{\bigcirc}r_{10}, r_2 \mathrel{!=} \varnothing, r_2\oplus, Cpo(r_{10}), Cpo(r_{20}), r_1 \mathrel{\text{⌶}} r_2, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, r_1 \mathrel{\text{⌶}} r_2, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_2, r_{20}=\varnothing, r_{10}=\varnothing,$

$r_2!\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{20},$

$r_1 \mathrel{!=} \varnothing, r_1\oplus, r_2!\overset{\circ}{\bigcirc}r_{10}, Cpo(r_{10}), r_2 \mathrel{!=} \varnothing, r_2\oplus, Cpo(r_{20}), r_1 \mathrel{\text{⌶}} r_2, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, r_1 \mathrel{\text{⌶}} r_2, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_2, r_{20}=\varnothing, r_{10}=\varnothing,$

$r_2!\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{10},$

$r_1 \mathrel{!=} \varnothing, r_1\oplus, Cpo(r_{10}), r_2!\overset{\circ}{\bigcirc}r_{20}, r_2 \mathrel{!=} \varnothing, r_2\oplus, Cpo(r_{20}), r_1 \mathrel{\text{⌶}} r_2, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, r_1 \mathrel{\text{⌶}} r_2, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_2, r_{20}=\varnothing, r_{10}=\varnothing,$

$r_2!\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_2!\overset{\circ}{\bigcirc}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{20}, r_1!\overset{\circ}{\bigcirc}r_{10},$

$r_1 \mathrel{!=} \varnothing, r_1\oplus, Cpo(r_{10}), r_2!\overset{\circ}{\bigcirc}r_{20}, Cpo(r_{20}), r_2 \mathrel{!=} \varnothing, r_2\oplus, r_1 \mathrel{\text{⌶}} r_2, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 ! \mathring{\bigcirc} r_{10}, r_1 \,!\!= \varnothing, r_1 \oplus, Cpo(r_{10}), Cpo(r_{20}), r_2 \,!\!= \varnothing, r_2 \oplus, r_1 \text{⌶} r_2, r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 ! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_1 \,!\!= \varnothing, r_1 \oplus, Cpo(r_{20}), r_2 \,!\!= \varnothing, r_2 \oplus, r_1 \text{⌶} r_2, r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), r_1 ! \mathring{\bigcirc} r_{20}, r_1 \,!\!= \varnothing, r_1 \oplus, Cpo(r_{20}), r_2 \,!\!= \varnothing, r_2 \oplus, r_1 \text{⌶} r_2, r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), r_1 ! \mathring{\bigcirc} r_{20}, Cpo(r_{20}), r_1 \,!\!= \varnothing, r_1 \oplus, r_2 \,!\!= \varnothing, r_2 \oplus, r_1 \text{⌶} r_2, r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), r_1 ! \mathring{\bigcirc} r_{20}, Cpo(r_{20}), r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 \text{⌶} r_2, r_1 \oplus, r_2 \oplus, r_1 \ominus, r_2 \ominus,$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), r_1 ! \mathring{\bigcirc} r_{20}, Cpo(r_{20}), r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, r_1 \text{⌶} r_2,$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} r_1, r_1 \text{⌶} r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2!\male r_{10}, r_2!\male r_{10}, r_1!\male r_{20}, r_1!\male r_{10}, r_2!\male r_{20},$

$Cpo(r_{10}), r_1!\male r_{20}, r_1 != \varnothing, Cpo(r_{20}), r_2 != \varnothing, r_1 = r_2,$

$\Leftrightarrow$, $\&SHi\,\male r_1, r_1 = r_2, r_1\male r_{10}, r_2\male r_{20}, r_1!\male r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2!\male r_{10}, r_2!\male r_{10}, r_1!\male r_{20}, r_1!\male r_{20}, r_2!\male r_{20},$

$r_1!\male r_{10}, Cpo(r_{10}), r_1 != \varnothing, Cpo(r_{20}), r_2 != \varnothing, r_1 = r_2,$

$\Leftrightarrow$, $\&SHi\,\male r_1, r_1 = r_2, r_1\male r_{10}, r_2\male r_{20}, r_1!\male r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2!\male r_{10}, r_2!\male r_{10}, r_1!\male r_{20}, r_1!\male r_{20}, r_2!\male r_{20},$

$r_1!\male r_{10}, r_1 != \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_2 != \varnothing, r_1 = r_2,$

$\Leftrightarrow$, $\&SHi\,\male r_1, r_1 = r_2, r_1\male r_{10}, r_2\male r_{20}, r_1!\male r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2!\male r_{10}, r_1!\male r_{20}, r_1!\male r_{20}, r_1!\male r_{10},$

$r_1 != \varnothing, r_2!\male r_{10}, Cpo(r_{10}), r_2!\male r_{20}, Cpo(r_{20}), r_2 != \varnothing, r_1 = r_2,$

$\Leftrightarrow$, $\&SHi\,\male r_1, r_1 = r_2, r_1\male r_{10}, r_2\male r_{20}, r_1!\male r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2!\male r_{10}, r_1!\male r_{20}, r_1!\male r_{20}, r_1!\male r_{10},$

$r_1 != \varnothing, r_2!\male r_{10}, Cpo(r_{10}), r_2!\male r_{20}, r_2 != \varnothing, Cpo(r_{20}), r_1 = r_2,$

$\Leftrightarrow$, $\&SHi\,\male r_1, r_1 = r_2, r_1\male r_{10}, r_2\male r_{20}, r_1!\male r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2!\male r_{10}, r_1!\male r_{20}, r_1!\male r_{20}, r_1!\male r_{10}, r_2!\male r_{20},$

$r_1 != \varnothing, r_2!\male r_{10}, Cpo(r_{10}), r_2 != \varnothing, Cpo(r_{20}), r_1 = r_2,$

$\Leftrightarrow$, $\&SHi\,\male r_1, r_1 = r_2, r_1\male r_{10}, r_2\male r_{20}, r_1!\male r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2!\male r_{10}, r_1!\male r_{20}, r_1!\male r_{20}, r_1!\male r_{10}, r_2!\male r_{20},$

$r_1 \,!\!= \varnothing, r_2 ! \mathring{\bigcirc} r_{10}, r_2 \,!\!= \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \!=\! r_2,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} r_1, r_1 \!=\! r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \!=\! r_2,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} r_1, r_1 \!=\! r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_{20} = \varnothing, r_{10} = \varnothing,$

$r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc} r_{10}, r_2 ! \mathring{\bigcirc} r_{20},$

$r_1 \,!\!= \varnothing, r_2 \,!\!= \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \!=\! r_2,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} r_1, r_1 \!=\! r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2,$

$r_1 \,!\!= \varnothing, r_{10} = \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \,!\!= \varnothing, r_{20} = \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), Cpo(r_{20}), r_1 \!=\! r_2,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} r_1, r_1 \!=\! r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc}^{\circ} r_1, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 ! \mathring{\bigcirc}^{\circ} r_2,$

$r_1 \,!\!= \varnothing, r_{10} = \varnothing, r_1 ! \mathring{\bigcirc} r_{10}, r_2 \,!\!= \varnothing, r_{20} = \varnothing, r_2 ! \mathring{\bigcirc} r_{20},$

$Cpo(r_{10}), Cpo(r_{20}), r_1 \!=\! r_2,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} r_1, r_1 \!=\! r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 ! \mathring{\bigcirc}^{\circ} r_1, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2, r_1 ! \mathring{\bigcirc}^{\circ} r_2,$

$r_1 \,!\!= \varnothing, r_{10} = \varnothing, r_2 \,!\!= \varnothing, r_{20} = \varnothing,$

$Cpo(r_{10}), Cpo(r_{20}), r_1 \!=\! r_2,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} r_1, r_1 \!=\! r_2, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, r_1 ! \mathring{\bigcirc}^{\circ} r_2,$

$r_1 \,!\!= \varnothing, r_{10} = \varnothing, r_2 \,!\!= \varnothing, r_{20} = \varnothing,$

$Cpo(r_{10}), Cpo(r_{20}), r_1 \!=\! r_2,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, r_1 \mp r_2, r_2 \,!{=}\, \varnothing, r_1 \,!{=}\, \varnothing, r_1 \mathring{\mathring{\bigcirc}} r_{10}, r_2 \mathring{\mathring{\bigcirc}} r_{20}, r_1 ! \mathring{\mathring{\bigcirc}} r_2,$

$r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \mp r_2,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, r_1 \mp r_2, r_1 \,!{=}\, \varnothing, r_1 \,!{=}\, \varnothing, r_1 \mathring{\mathring{\bigcirc}} r_{10}, r_2 \mathring{\mathring{\bigcirc}} r_{20}, r_1 ! \mathring{\mathring{\bigcirc}} r_2,$

$r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \mp r_2,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, r_1 \mp r_2, r_1 \,!{=}\, \varnothing, r_1 \mathring{\mathring{\bigcirc}} r_{10}, r_2 \mathring{\mathring{\bigcirc}} r_{20}, r_1 ! \mathring{\mathring{\bigcirc}} r_2,$

$r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \mp r_2,$

$\Leftrightarrow\ , r_1 \,!{=}\, \varnothing,\ \&SHi\,\mathring{\bigcirc} r_1, r_1 \mp r_2, r_1 \mathring{\mathring{\bigcirc}} r_{10}, r_2 \mathring{\mathring{\bigcirc}} r_{20}, r_1 ! \mathring{\mathring{\bigcirc}} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \mp r_2,$

$conclusion:$
$, r_1 \mp r_2, r_1 \mathring{\mathring{\bigcirc}} r_{10}, r_2 \mathring{\mathring{\bigcirc}} r_{20}, r_1 ! \mathring{\mathring{\bigcirc}} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}),\ \Leftrightarrow$

$, r_1 \mp r_2, r_1 \mathring{\mathring{\bigcirc}} r_{10}, r_2 \mathring{\mathring{\bigcirc}} r_{20}, r_1 ! \mathring{\mathring{\bigcirc}} r_2, r_{10} = \varnothing, r_{20} = \varnothing, Cpo(r_{10}), Cpo(r_{20}), r_1 \mp r_2,$

29 Recursive Function Rcpo(i;r)

29.1 Definition of IsCpo(i;r)

$$, IsCpo(i;r), \iff , i!\bigcirc\!\!\!\!^{\circ}\!\circ r, r = \varnothing,$$

29.2 Property of IsCpo(i;r)

$$, IsCpo(i;r), \iff \sim, i!\bigcirc\!\!\!\!^{\circ}\!\circ r,$$

$$, IsCpo(i;r), \iff \sim, i!\bigcirc\!\!\!\!^{\circ} r,$$

$$, IsCpo(i;r), \iff \sim, r = \varnothing,$$

$$, IsCpo(i;r), \iff \sim, IsCpo(i;r),$$

$$, IsCpo(i;r), j!\bigcirc\!\!\!\!^{\circ}\!\circ r, \iff , IsCpo(i;r), IsCpo(j;r),$$

$$, IsCpo(i;r), i \circledwedge i_0, \iff , i \circledwedge i_0, IsCpo(i_0;r),$$

$$, i_1 \bigcirc\!\!\!\!^{\circ} i_2, IsCpo(i_1;r), \iff , i_1 \bigcirc\!\!\!\!^{\circ} i_2, IsCpo(i_2;r),$$

$$, IsCpo(i;r), Cpo(r), r\oplus, \iff , Cpo(r), r\oplus, IsCpo(i;r),$$

$$, IsCpo(i;r), i = \varnothing, Cpo(r), \iff , IsCpo(i;r), Cpo(r), i = \varnothing,$$

$$, IsCpo(i;r), i \mathrel{!=} \varnothing, Cpo(r), \iff , IsCpo(i;r), Cpo(r), i \mathrel{!=} \varnothing,$$

$$, IsCpo(i;r), i\oplus, Cpo(r), \iff , IsCpo(i;r), Cpo(r), i\oplus,$$

$$, IsCpo(i;r), i\oplus, \iff , i\oplus, IsCpo(i;r),$$

$$,IsCpo(i;r),\ \&SHi\mathring{\bigcirc}i, \;\Leftrightarrow\; ,\ \&SHi\mathring{\bigcirc}i, IsCpo(i;r),$$

$$,IsCpo(j;r),\ \&SHi\mathring{\bigcirc}i, \;\Leftrightarrow\; ,\ \&SHi\mathring{\bigcirc}i, IsCpo(j;r),$$

$$,IsCpo(i;r),\ \&SHi\mathring{\bigcirc}i, Cpo(r), \;\Leftrightarrow\; ,IsCpo(i;r), Cpo(r),\ \&SHi\mathring{\bigcirc}i,$$

$$,\circledcirc r, \;\Leftrightarrow\; \sim, IsCpo(i;r),$$

$$,r_1!\mathring{\bigcirc}r_2, IsCpo(i;r_1), Cpo(r_2), \;\Leftrightarrow\; r_1!\mathring{\bigcirc}r_2, Cpo(r_2), IsCpo(i;r_1),$$

$$,r_1!\mathring{\bigcirc}^{\circ}r_2, IsCpo(i;r_1), Cpo(r_2), \;\Leftrightarrow\; r_1!\mathring{\bigcirc}^{\circ}r_2, Cpo(r_2), IsCpo(i;r_1),$$

$$,i!\mathring{\bigcirc}^{\circ}r, j!\mathring{\bigcirc}^{\circ}r, i\mathrel{\shortmid\!\!=}j, Cpo(r), \;\Leftrightarrow\; ,i!\mathring{\bigcirc}^{\circ}r, j!\mathring{\bigcirc}^{\circ}r, Cpo(r), i\mathrel{\shortmid\!\!=}j,$$

$$,IsCpo(i;r), IsCpo(j;r), i\mathrel{\shortmid\!\!=}j, Cpo(r), \;\Leftrightarrow\; ,IsCpo(i;r), IsCpo(j;r), Cpo(r), i\mathrel{\shortmid\!\!=}j,$$

$$,i!\mathring{\bigcirc}^{\circ}r, j!\mathring{\bigcirc}^{\circ}r, i\succ j, Cpo(r), \;\Leftrightarrow\; ,i!\mathring{\bigcirc}^{\circ}r, j!\mathring{\bigcirc}^{\circ}r, Cpo(r), i\succ j,$$

$$,IsCpo(i;r), IsCpo(j;r), i\succ j, Cpo(r), \;\Leftrightarrow\; ,IsCpo(i;r), IsCpo(j;r), Cpo(r), i\succ j,$$

29.3 Definition of Rcpo(i;r)

$$,Rcpo(i;r), \;\Leftrightarrow\; ,if(i=\varnothing)\text{-}\begin{bmatrix} , \\ , Cpo(r), r\oplus, i\oplus, Rcpo(i;r), \end{bmatrix}\text{-},$$

29.4 Property of Rcpo(i;r)

$$,i=\varnothing,Rcpo(i;r),\ \Leftrightarrow\ ,i=\varnothing,$$

$$,i\,!\!=\varnothing,Rcpo(i;r),\ \Leftrightarrow\ ,i\,!\!=\varnothing,Cpo(r),r\oplus,i\oplus,Rcpo(i;r),$$

$$,IsCpo(i;r),Rcpo(i;r),\ \Leftrightarrow\ \sim,i=\varnothing,r=\varnothing,$$

induction proof:

$premise\ 1:$

$,i=\varnothing,IsCpo(i;r),Rcpo(i;r),$

$\Leftrightarrow\ ,IsCpo(i;r),i=\varnothing,Rcpo(i;r),$

$\Leftrightarrow\ ,IsCpo(i;r),i=\varnothing,$

$\Leftrightarrow\ ,IsCpo(i;r),r=\varnothing,i=\varnothing,i=\varnothing,$

$\Leftrightarrow\ ,IsCpo(i;r),i=\varnothing,i=\varnothing,r=\varnothing,$

$\Leftrightarrow\ ,IsCpo(i;r),i=\varnothing,Rcpo(i;r),i=\varnothing,r=\varnothing,$

$\Leftrightarrow\ ,i=\varnothing,IsCpo(i;r),Rcpo(i;r),i=\varnothing,r=\varnothing,$

$premise\ 2:$

$,\ \&SHi\to i,IsCpo(i;r),Rcpo(i;r),\ \Leftrightarrow\ ,\ \&SHi\to i,IsCpo(i;r),Rcpo(i;r),i=\varnothing,r=\varnothing,\ \Longrightarrow$

$,i\,!\!=\varnothing,\ \&SHi\,{\bigcirc}\!^{\circ}i,IsCpo(i;r),Rcpo(i;r),$

$\Leftrightarrow\ ,\ \&SHi\,{\bigcirc}\!^{\circ}i,IsCpo(i;r),i\,!\!=\varnothing,Rcpo(i;r),$

$\Leftrightarrow\ ,\ \&SHi\,{\bigcirc}\!^{\circ}i,IsCpo(i;r),i\,!\!=\varnothing,Cpo(r),r\oplus,i\oplus,Rcpo(i;r),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,IsCpo(i;r),\ \&SHi\,{\bigcirc}\!^{\circ}i,Cpo(r),r\oplus,i\oplus,Rcpo(i;r),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,IsCpo(i;r),Cpo(r),\ \&SHi\,{\bigcirc}\!^{\circ}i,r\oplus,i\oplus,Rcpo(i;r),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,IsCpo(i;r),Cpo(r),r\oplus,i\oplus,\ \&SHi\to i,Rcpo(i;r),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,Cpo(r),r\oplus,IsCpo(i;r),i\oplus,\ \&SHi\to i,Rcpo(i;r),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,Cpo(r),r\oplus,i\oplus,\ \&SHi\to i,IsCpo(i;r),Rcpo(i;r),$

$\Leftrightarrow\ ,i\,!\!=\varnothing,Cpo(r),r\oplus,i\oplus,\ \&SHi\to i,IsCpo(i;r),Rcpo(i;r),i=\varnothing,r=\varnothing,$

$\Leftrightarrow\ ,\&SHi\,\text{♂}i, IsCpo(i;r), i\,!\!=\varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), i=\varnothing, r=\varnothing,$

$\Leftrightarrow\ ,\&SHi\,\text{♂}i, IsCpo(i;r), i\,!\!=\varnothing, Rcpo(i;r), i=\varnothing, r=\varnothing,$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\text{♂}i, IsCpo(i;r), Rcpo(i;r), i=\varnothing, r=\varnothing,$

$conclusion:$

$, IsCpo(i;r), Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), i=\varnothing, r=\varnothing,$

$$
\begin{aligned}
, IsCpo(i;r), i\,!\!=\varnothing, Rcpo(i;r), &\ \Leftrightarrow\ \sim, m!\text{♂}r,\\
, IsCpo(i;r), m!\text{♂}r, Rcpo(i;r), &\ \Leftrightarrow\ \sim, m!\text{♂}r,
\end{aligned}
$$

$$, IsCpo(i;r), Rcpo(i;r), \otimes,\ \Leftrightarrow\ , \otimes,$$

29.5 Swap

29.5.1 Operator

$$, IsCpo(i;r), \circledcirc j, Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), \circledcirc j,$$

$$, IsCpo(i;r), j\text{☮}j_0, Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), j\text{☮}j_0,$$

$$, IsCpo(i;r), j\text{⏚}, Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), j\text{⏚},$$

$$, IsCpo(i;r), j!\text{♂}r, j\oplus, Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), j!\text{♂}r, Rcpo(i;r), j\oplus,$$

induction proof:
$premise\ 1:$
$, i=\varnothing, IsCpo(i;r), j!\text{♂}r, j\oplus, Rcpo(i;r),$
$\Leftrightarrow\ , IsCpo(i;r), j!\text{♂}r, j\oplus, i=\varnothing, Rcpo(i;r),$

$\Leftrightarrow\ , IsCpo(i;r), j!\text{♂}r, j\oplus, i=\varnothing,$

$\Leftrightarrow\ , IsCpo(i;r), j!\text{♂}r, i=\varnothing, j\oplus,$

$\Leftrightarrow\ , IsCpo(i;r), j!\text{♂}r, i=\varnothing, Rcpo(i;r), j\oplus,$

$\Leftrightarrow\ , i = \varnothing, IsCpo(i;r), j!\male r, Rcpo(i;r), j\oplus,$

$premise\ 2:$

$, \&SHi \rightarrow i, IsCpo(i;r), j!\male r, j\oplus, Rcpo(i;r),\ \Leftrightarrow\ , \&SHi \rightarrow i, IsCpo(i;r), j!\male r, Rcpo(i;r), j\oplus,\ \Rightarrow$

$, i != \varnothing, \&SHi \male i, IsCpo(i;r), j!\male r, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi \male i, IsCpo(i;r), j!\male r, j\oplus, i != \varnothing, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi \male i, IsCpo(i;r), j!\male r, j\oplus, i != \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, \&SHi \male i, IsCpo(i;r), j!\male r, j\oplus, Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, \&SHi \male i, IsCpo(i;r), j!\male r, Cpo(r), j\oplus, r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, \&SHi \male i, IsCpo(i;r), j!\male r, Cpo(r), r\oplus, i\oplus, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, \&SHi \male i, IsCpo(i;r), j!\male r, Cpo(r), r\oplus, j!\male r, i\oplus, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, j!\male r, IsCpo(i;r), \&SHi \male i, Cpo(r), r\oplus, i\oplus, j!\male r, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, j!\male r, IsCpo(i;r), Cpo(r), \&SHi \male i, r\oplus, i\oplus, j!\male r, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, j!\male r, IsCpo(i;r), Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, j!\male r, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, j!\male r, Cpo(r), r\oplus, IsCpo(i;r), i\oplus, \&SHi \rightarrow i, j!\male r, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, j!\male r, Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), j!\male r, j\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , i != \varnothing, j!\male r, Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), j!\male r, Rcpo(i;r), j\oplus,$

$\Leftrightarrow\ , \&SHi \male i, IsCpo(i;r), j!\male r, i != \varnothing, Cpo(r), r\oplus, j!\male r, i\oplus, Rcpo(i;r), j\oplus,$

$\Leftrightarrow\ , \&SHi \male i, IsCpo(i;r), j!\male r, i != \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), j\oplus,$

$\Leftrightarrow\ , \&SHi \male i, IsCpo(i;r), j!\male r, i != \varnothing, Rcpo(i;r), j\oplus,$

$\Leftrightarrow\ , i != \varnothing, \&SHi \male i, IsCpo(i;r), j!\male r, Rcpo(i;r), j\oplus,$

$conclusion:$

$, IsCpo(i;r), j!\male r, j\oplus, Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), j!\male r, Rcpo(i;r), j\oplus,$

$$,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, j\oplus, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, Rcpo(i;r), j\oplus,$$

$$,IsCpo(i;r), r!{\rightarrow}j, j\ominus, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), r!{\rightarrow}j, Rcpo(i;r), j\ominus,$$

$$,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, j\ominus, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, Rcpo(i;r), j\ominus,$$

29.5.2 Propositions node null

$$,IsCpo(i;r), j!\overset{\circ}{\bigcirc}r, j=\varnothing, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ}{\bigcirc}r, Rcpo(i;r), j=\varnothing,$$

$$,IsCpo(i;r), j!\overset{\circ}{\bigcirc}r, j\,!{=}\varnothing, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ}{\bigcirc}r, Rcpo(i;r), j\,!{=}\varnothing,$$

$$,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, j=\varnothing, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, Rcpo(i;r), j=\varnothing,$$

$$,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, j\,!{=}\varnothing, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, Rcpo(i;r), j\,!{=}\varnothing,$$

$$,IsCpo(i;r), j\,!{=}\varnothing, Rcpo(i;r), \;\Leftrightarrow\; \sim, j\,!{=}\varnothing,$$

$$,IsCpo(i;r), j\,!{=}\varnothing, j\oplus, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j\,!{=}\varnothing, Rcpo(i;r), j\,!{=}\varnothing, j\oplus,$$

$$,IsCpo(i;r), j!\overset{\circ}{\bigcirc}r, j\,!{=}\varnothing, j\oplus, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ}{\bigcirc}r, Rcpo(i;r), j\,!{=}\varnothing, j\oplus,$$

$$,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, j\,!{=}\varnothing, j\oplus, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), j!\overset{\circ\circ}{\bigcirc}r, Rcpo(i;r), j\,!{=}\varnothing, j\oplus,$$

29.5.3 Propositions identical node

$$,IsCpo(i;r), m\overset{\circ}{\bigcirc}n, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), m\overset{\circ}{\bigcirc}n,$$

$$,IsCpo(i;r), m!\overset{\circ}{\bigcirc}n, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), m!\overset{\circ}{\bigcirc}n,$$

29.5.4 Propositions node connectivity

$$,IsCpo(i;r), m\mathbin{\bigcirc^{\circ\circ}}n, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), m\mathbin{\bigcirc^{\circ\circ}}n,$$

$$,IsCpo(i;r), m!\mathbin{\bigcirc^{\circ\circ}}n, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), m!\mathbin{\bigcirc^{\circ\circ}}n,$$

$$,IsCpo(i;r), i!\mathbin{\bigcirc^{\circ\circ}}r, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), i!\mathbin{\bigcirc^{\circ\circ}}r,$$

$$,IsCpo(i;r), m\mathbin{\bigcirc^{\circ\circ}}r, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), m\mathbin{\bigcirc^{\circ\circ}}r,$$

$$,IsCpo(i;r), m!\mathbin{\bigcirc^{\circ\circ}}r, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), m!\mathbin{\bigcirc^{\circ\circ}}r,$$

29.5.5 IsCpo

$$,IsCpo(i;r), Rcpo(i;r), \;\Leftrightarrow\; \sim, IsCpo(i;r),$$

$$,IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), IsCpo(j;r),$$

$$,IsCpo(i;r_1), r_1!\mathbin{\bigcirc^{\circ\circ}}r_2, IsCpo(j;r_2), Rcpo(i;r_1), \;\Leftrightarrow$$
$$,IsCpo(i;r_1), r_1!\mathbin{\bigcirc^{\circ\circ}}r_2, Rcpo(i;r_1), IsCpo(j;r_2),$$

29.5.6 Cpo

$$,IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r), \;\Leftrightarrow\; ,IsCpo(i;r), Rcpo(i;r), Cpo(r), r\oplus,$$

induction proof:
$premise\ 1:$
$,i=\varnothing, IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r),$
$\Leftrightarrow\; ,IsCpo(i;r), i=\varnothing, Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow\; ,IsCpo(i;r), Cpo(r), i=\varnothing, r\oplus, Rcpo(i;r),$

$\Leftrightarrow\; ,IsCpo(i;r), Cpo(r), r\oplus, i=\varnothing, Rcpo(i;r),$

$\Leftrightarrow\; ,IsCpo(i;r), Cpo(r), r\oplus, i=\varnothing,$

$\Leftrightarrow\; ,IsCpo(i;r), i=\varnothing, Cpo(r), r\oplus,$

$\Leftrightarrow$ $, IsCpo(i;r), i = \varnothing, Rcpo(i;r), Cpo(r), r\oplus,$

$\Leftrightarrow$ $, i = \varnothing, IsCpo(i;r), Rcpo(i;r), Cpo(r), r\oplus,$

premise 2 :
$, \&SHi \rightarrow i, IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi \rightarrow i, IsCpo(i;r), Rcpo(i;r), Cpo(r), r\oplus,$ $\Rightarrow$

$, i \,!\!= \varnothing, \&SHi ♂ i, IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), i \,!\!= \varnothing, Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), Cpo(r), i \,!\!= \varnothing, r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), Cpo(r), r\oplus, i \,!\!= \varnothing, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), Cpo(r), r\oplus, i \,!\!= \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), Cpo(r), r\oplus, IsCpo(i;r), i \,!\!= \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), Cpo(r), i \,!\!= \varnothing, r\oplus, IsCpo(i;r), Cpo(r), i\oplus, r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), Cpo(r), i \,!\!= \varnothing, r\oplus, IsCpo(i;r), i\oplus, Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, IsCpo(i;r), \&SHi ♂ i, Cpo(r), i \,!\!= \varnothing, r\oplus, i\oplus, IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, IsCpo(i;r), Cpo(r), \&SHi ♂ i, i \,!\!= \varnothing, r\oplus, i\oplus, IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, IsCpo(i;r), Cpo(r), i \,!\!= \varnothing, r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r),$

$\Leftrightarrow$ $, IsCpo(i;r), Cpo(r), i \,!\!= \varnothing, r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), Rcpo(i;r), Cpo(r), r\oplus,$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), i \,!\!= \varnothing, Cpo(r), r\oplus, i\oplus, IsCpo(i;r), Rcpo(i;r), Cpo(r), r\oplus,$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), i \,!\!= \varnothing, Cpo(r), r\oplus, IsCpo(i;r), i\oplus, Rcpo(i;r), Cpo(r), r\oplus,$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), i \,!\!= \varnothing, IsCpo(i;r), Cpo(r), r\oplus, i\oplus, Rcpo(i;r), Cpo(r), r\oplus,$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), i \,!\!= \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), Cpo(r), r\oplus,$

$\Leftrightarrow$ $, \&SHi ♂ i, IsCpo(i;r), i \,!\!= \varnothing, Rcpo(i;r), Cpo(r), r\oplus,$

$\Leftrightarrow\ , i \mathop{!}= \varnothing,\ \&SHi \bigcirc i, IsCpo(i;r), Rcpo(i;r), Cpo(r), r\oplus,$

$conclusion:$

$, IsCpo(i;r), Cpo(r), r\oplus, Rcpo(i;r),\ \ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), Cpo(r), r\oplus,$

$$, IsCpo(i;r_1), i!\bigcirc r_2, r_1!\bigcirc r_2, Cpo(r_2), Rcpo(i;r_1),\ \ \Leftrightarrow$$
$$, IsCpo(i;r_1), i!\bigcirc r_2, r_1!\bigcirc r_2, Rcpo(i;r_1), Cpo(r_2),$$

induction proof:
$premise\ 1:$
$, i = \varnothing, IsCpo(i;r_1), i!\bigcirc r_2, r_1!\bigcirc r_2, Cpo(r_2), Rcpo(i;r_1),$
$\Leftrightarrow\ , IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, i = \varnothing, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, Cpo(r_2), i = \varnothing, Rcpo(i;r_1),$

$\Leftrightarrow\ , IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, Cpo(r_2), i = \varnothing,$

$\Leftrightarrow\ , IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, i = \varnothing, Cpo(r_2),$

$\Leftrightarrow\ , IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, i = \varnothing, Rcpo(i;r_1), Cpo(r_2),$

$\Leftrightarrow\ , i = \varnothing, IsCpo(i;r_1), i!\bigcirc r_2, r_1!\bigcirc r_2, Rcpo(i;r_1), Cpo(r_2),$

$premise\ 2:$
$,\ \&SHi \rightarrow i, IsCpo(i;r_1), i!\bigcirc r_2, r_1!\bigcirc r_2, Cpo(r_2), Rcpo(i;r_1),\ \ \Leftrightarrow$

$,\ \&SHi \rightarrow i, IsCpo(i;r_1), i!\bigcirc r_2, r_1!\bigcirc r_2, Rcpo(i;r_1), Cpo(r_2),\ \ \Rightarrow$

$, i \mathop{!}= \varnothing,\ \&SHi \bigcirc i, IsCpo(i;r_1), i!\bigcirc r_2, r_1!\bigcirc r_2, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi \bigcirc i, IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, i \mathop{!}= \varnothing, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi \bigcirc i, IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, Cpo(r_2), i \mathop{!}= \varnothing, Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi \bigcirc i, IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, Cpo(r_2), i \mathop{!}= \varnothing, Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi \bigcirc i, IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, i \mathop{!}= \varnothing, Cpo(r_2), Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi \bigcirc i, IsCpo(i;r_1), r_1!\bigcirc r_2, i!\bigcirc r_2, i \mathop{!}= \varnothing, Cpo(r_1), Cpo(r_2), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi \bigcirc i, IsCpo(i;r_1), i!\bigcirc r_2, i \mathop{!}= \varnothing, Cpo(r_1), r_1!\bigcirc r_2, Cpo(r_2), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpo(i;r_1), i!\text{♂}r_2, i\,!{=}\,\varnothing, Cpo(r_1), r_1!\text{♂}r_2, r_1\oplus, Cpo(r_2), i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpo(i;r_1), i\,!{=}\,\varnothing, Cpo(r_1), r_1\oplus, r_1!\text{♂}r_2, i!\text{♂}r_2, Cpo(r_2), i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpo(i;r_1), i\,!{=}\,\varnothing, Cpo(r_1), r_1\oplus, r_1!\text{♂}r_2, i!\text{♂}r_2, i\oplus, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, r_1!\text{♂}r_2, i!\text{♂}r_2, i\,!{=}\,\varnothing, IsCpo(i;r_1), Cpo(r_1), r_1\oplus, i\oplus, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , r_1!\text{♂}r_2, i!\text{♂}r_2, i\,!{=}\,\varnothing, IsCpo(i;r_1),\ \&SHi\,\text{♂}i, Cpo(r_1), r_1\oplus, i\oplus, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , r_1!\text{♂}r_2, i!\text{♂}r_2, i\,!{=}\,\varnothing, IsCpo(i;r_1), Cpo(r_1), r_1\oplus, i\oplus,\ \&SHi\rightarrow i, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , r_1!\text{♂}r_2, i!\text{♂}r_2, i\,!{=}\,\varnothing, Cpo(r_1), r_1\oplus, IsCpo(i;r_1), i\oplus,\ \&SHi\rightarrow i, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, Cpo(r_1), r_1\oplus, i\oplus,\ \&SHi\rightarrow i, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, Cpo(r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, Cpo(r_1), r_1\oplus, i\oplus,\ \&SHi\rightarrow i, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, Rcpo(i;r_1), Cpo(r_2),$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, i\,!{=}\,\varnothing, Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i;r_1), Cpo(r_2),$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, i\,!{=}\,\varnothing, Rcpo(i;r_1), Cpo(r_2),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\text{♂}i, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, Rcpo(i;r_1), Cpo(r_2),$

$conclusion:$

$, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, Cpo(r_2), Rcpo(i;r_1), \ \Leftrightarrow$

$, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, Rcpo(i;r_1), Cpo(r_2),$

$$, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, Cpo(r_2), r_2\oplus, Rcpo(i;r_1), \ \Leftrightarrow$$
$$, IsCpo(i;r_1), i!\text{♂}r_2, r_1!\text{♂}r_2, Rcpo(i;r_1), Cpo(r_2), r_2\oplus,$$

29.5.7 Rcpo

$$, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r), \ \Leftrightarrow$$
$$, IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),$$

induction proof:

$premise\ 1:$

$, i = \varnothing, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), i = \varnothing, Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , IsCpo(i;r), i!\overset{\circ}{\bigcirc}\!\circ r, IsCpo(j;r), i = \varnothing, Rcpo(j;r),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), i!\overset{\circ}{\bigcirc}\!\circ r, i = \varnothing, Rcpo(j;r),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), i!\overset{\circ}{\bigcirc}\!\circ r, Rcpo(j;r), i = \varnothing,$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), i!\overset{\circ}{\bigcirc}\!\circ r, Rcpo(j;r), i = \varnothing, Rcpo(i;r),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), i!\overset{\circ}{\bigcirc}\!\circ r, i = \varnothing, Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , i = \varnothing, IsCpo(i;r), i!\overset{\circ}{\bigcirc}\!\circ r, IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , i = \varnothing, IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),$

$premise\ 2:$

$, \&SHi \rightarrow i, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , \&SHi \rightarrow i, IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),\quad \Longrightarrow$

$, i\ != \varnothing, \&SHi\, \overset{\circ}{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , \&SHi\, \overset{\circ}{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), i\ != \varnothing, Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , \&SHi\, \overset{\circ}{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), i\ != \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , i\ != \varnothing, IsCpo(i;r), \&SHi\, \overset{\circ}{\bigcirc} i, Cpo(r), r\oplus, i\oplus, IsCpo(j;r), Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , i\ != \varnothing, IsCpo(i;r), Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(j;r), Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , i\ != \varnothing, Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , i\ != \varnothing, Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\, \overset{\circ}{\bigcirc} i, i\ != \varnothing, IsCpo(i;r), Cpo(r), r\oplus, IsCpo(j;r), i\oplus, Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\, \overset{\circ}{\bigcirc} i, i\ != \varnothing, IsCpo(i;r), i!\overset{\circ}{\bigcirc}\!\circ r, Cpo(r), r\oplus, IsCpo(j;r), i\oplus, Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\, \overset{\circ}{\bigcirc} i, i\ != \varnothing, IsCpo(i;r), Cpo(r), r\oplus, IsCpo(j;r), i!\overset{\circ}{\bigcirc}\!\circ r, i\oplus, Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!{=}\varnothing, IsCpo(i;r), Cpo(r), r\oplus, IsCpo(j;r), i!\ddot{\bigcirc}r, Rcpo(j;r), i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!{=}\varnothing, IsCpo(i;r), i!\ddot{\bigcirc}r, IsCpo(j;r), Cpo(r), r\oplus, Rcpo(j;r), i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!{=}\varnothing, IsCpo(i;r), i!\ddot{\bigcirc}r, IsCpo(j;r), Rcpo(j;r), Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i!\ddot{\bigcirc}r, i\,!{=}\varnothing, Rcpo(j;r), Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i!\ddot{\bigcirc}r, Rcpo(j;r), i\,!{=}\varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i!\ddot{\bigcirc}r, Rcpo(j;r), i\,!{=}\varnothing, Rcpo(i;r),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i!\ddot{\bigcirc}r, i\,!{=}\varnothing, Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHi\mathring{\bigcirc}i, IsCpo(i;r), i!\ddot{\bigcirc}r, IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),$

$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHi\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),$

$conclusion:$

$, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r),$

$$, IsCpo(i;r_1), IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, Rcpo(i;r_1), Rcpo(j;r_2), \ \Leftrightarrow$$
$$, IsCpo(i;r_1), IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, Rcpo(j;r_2), Rcpo(i;r_1),$$

induction proof:
$premise\ 1:$
$, i{=}\varnothing, IsCpo(i;r_1), IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, Rcpo(i;r_1), Rcpo(j;r_2),$
$\Leftrightarrow\ , IsCpo(i;r_1), IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, i{=}\varnothing, Rcpo(i;r_1), Rcpo(j;r_2),$

$\Leftrightarrow\ , IsCpo(i;r_1), IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, i{=}\varnothing, Rcpo(j;r_2),$

$\Leftrightarrow\ , IsCpo(i;r_1), j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, i{=}\varnothing, Rcpo(j;r_2),$

$\Leftrightarrow\ , IsCpo(i;r_1), j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, Rcpo(j;r_2), i{=}\varnothing,$

$\Leftrightarrow\ , IsCpo(i;r_1), j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, Rcpo(j;r_2), i{=}\varnothing, Rcpo(i;r_1),$

$\Leftrightarrow\ , IsCpo(i;r_1), j!\ddot{\bigcirc}r_1, r_1!\ddot{\bigcirc}r_2, IsCpo(j;r_2), i!\ddot{\bigcirc}r_2, i{=}\varnothing, Rcpo(j;r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , i = \varnothing, IsCpo(i; r_1), IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(j; r_2), Rcpo(i; r_1),$

$premise\ 2:$
$, \ \&SHi \rightarrow i, IsCpo(i; r_1), IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(i; r_1), Rcpo(j; r_2), \ \Leftrightarrow$

$, \ \&SHi \rightarrow i, IsCpo(i; r_1), IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(j; r_2), Rcpo(i; r_1), \ \Rightarrow$

$, i != \varnothing, \ \&SHi \bigcirc i, IsCpo(i; r_1), IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , \ \&SHi \bigcirc i, IsCpo(i; r_1), IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2,$

$i != \varnothing, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , \ \&SHi \bigcirc i, IsCpo(i; r_1), IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2,$

$i != \varnothing, Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , i != \varnothing, \ \&SHi \bigcirc i, IsCpo(i; r_1), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, IsCpo(j; r_2),$

$Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , i != \varnothing, \ \&SHi \bigcirc i, IsCpo(i; r_1), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2,$

$Cpo(r_1), IsCpo(j; r_2), r_1\oplus, i\oplus, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , i != \varnothing, IsCpo(i; r_1), \ \&SHi \bigcirc i, Cpo(r_1), r_1\oplus, i\oplus,$

$IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , i != \varnothing, IsCpo(i; r_1), Cpo(r_1), r_1\oplus, i\oplus,$

$\&SHi \rightarrow i, IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , i != \varnothing, Cpo(r_1), r_1\oplus, i\oplus,$

$\&SHi \rightarrow i, IsCpo(i; r_1), IsCpo(j; r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(i; r_1), Rcpo(j; r_2),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, Cpo(r_1), r_1\oplus, i\oplus,$

$\&SHi \to i, IsCpo(i;r_1), IsCpo(j;r_2), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2, Rcpo(j;r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), j!\circledcirc r_1, r_1!\circledcirc r_2,$

$Cpo(r_1), r_1\oplus, i\oplus, IsCpo(j;r_2), i!\circledcirc r_2, Rcpo(j;r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), j!\circledcirc r_1, r_1!\circledcirc r_2,$

$Cpo(r_1), r_1\oplus, IsCpo(j;r_2), i!\circledcirc r_2, i\oplus, Rcpo(j;r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), j!\circledcirc r_1, r_1!\circledcirc r_2,$

$Cpo(r_1), r_1\oplus, IsCpo(j;r_2), i!\circledcirc r_2, Rcpo(j;r_2), i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), i!\circledcirc r_2, j!\circledcirc r_1,$

$Cpo(r_1), IsCpo(j;r_2), r_1!\circledcirc r_2, r_1\oplus, Rcpo(j;r_2), i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), i!\circledcirc r_2, j!\circledcirc r_1,$

$Cpo(r_1), IsCpo(j;r_2), r_1!\circledcirc r_2, Rcpo(j;r_2), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2,$

$Cpo(r_1), IsCpo(j;r_2), Rcpo(j;r_2), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), i!\circledcirc r_2, j!\circledcirc r_1, r_1!\circledcirc r_2,$

$IsCpo(j;r_2), Cpo(r_1), Rcpo(j;r_2), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi \bigcirc i, i \mathrel{!=} \varnothing, IsCpo(i;r_1), i!\circledcirc r_2,$

$IsCpo(j;r_2), j!\circledcirc r_1, r_1!\circledcirc r_2, Cpo(r_1), Rcpo(j;r_2), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, i\,!{=}\,\varnothing, IsCpo(i;r_1), i!\text{♂}r_2,$

$IsCpo(j;r_2), j!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(j;r_2), Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpo(i;r_1), j!\text{♂}r_1, r_1!\text{♂}r_2,$

$IsCpo(j;r_2), i!\text{♂}r_2, i\,!{=}\,\varnothing, Rcpo(j;r_2), Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpo(i;r_1), j!\text{♂}r_1, r_1!\text{♂}r_2,$

$IsCpo(j;r_2), i!\text{♂}r_2, Rcpo(j;r_2), i\,!{=}\,\varnothing, Cpo(r_1), r_1\oplus, i\oplus, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpo(i;r_1), j!\text{♂}r_1, r_1!\text{♂}r_2,$

$IsCpo(j;r_2), i!\text{♂}r_2, Rcpo(j;r_2), i\,!{=}\,\varnothing, Rcpo(i;r_1),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpo(i;r_1), j!\text{♂}r_1, r_1!\text{♂}r_2,$

$IsCpo(j;r_2), i!\text{♂}r_2, i\,!{=}\,\varnothing, Rcpo(j;r_2), Rcpo(i;r_1),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\text{♂}i, IsCpo(i;r_1), IsCpo(j;r_2), i!\text{♂}r_2, j!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(j;r_2), Rcpo(i;r_1),$

$conclusion:$

$, IsCpo(i;r_1), IsCpo(j;r_2), i!\text{♂}r_2, j!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(i;r_1), Rcpo(j;r_2),$

$\Leftrightarrow\ , IsCpo(i;r_1), IsCpo(j;r_2), i!\text{♂}r_2, j!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(j;r_2), Rcpo(i;r_1),$

29.5.8 R(m)

$, IsCpo(i;r), m!\text{♂}r, R(m), Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), m!\text{♂}r, Rcpo(i;r), R(m),$

29.5.9 Rc(m;n)

$, IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, Rc(m;n), Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, Rcpo(i;r), Rc(m;n),$

29.5.10 Propositions number comparison

$, IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, m{=}n, Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, Rcpo(i;r), m{=}n,$

$, IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, m!{=}n, Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, Rcpo(i;r), m!{=}n,$

$, IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, m{>}n, Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, Rcpo(i;r), m{>}n,$

$, IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, m!{>}n, Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, n!\overset{\circ}{\bigcirc}{}^{\circ}r, Rcpo(i;r), m!{>}n,$

29.5.11 &SHi

$, IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, \ \&SHi\overset{\circ}{\bigcirc}m, Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, Rcpo(i;r), \ \&SHi\overset{\circ}{\bigcirc}m,$

29.6 Propositions number equal

$$, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), Rcpo(i_0;r_0), \;\Leftrightarrow\; \sim, r{=}i,$$

induction proof:
premise 1 :
$, i=\varnothing, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), Rcpo(i_0;r_0),$
$\Leftrightarrow\; , i\overset{\circ}{\bigcirc}i_0, i=\varnothing, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), Rcpo(i_0;r_0),$

$\Leftrightarrow\; , i\overset{\circ}{\bigcirc}i_0, i_0=\varnothing, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), Rcpo(i_0;r_0),$

$\Leftrightarrow\; , i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), i_0=\varnothing, Rcpo(i_0;r_0),$

$\Leftrightarrow\; , i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), r_0=\varnothing, i_0=\varnothing,$

$\Leftrightarrow\; , IsCpo(i_0;r_0), i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, r_0=\varnothing, i_0=\varnothing,$

$\Leftrightarrow\ , IsCpo(i_0; r_0), i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, r = \varnothing, i = \varnothing,$

$\Leftrightarrow\ , IsCpo(i_0; r_0), i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, r = \varnothing, i = \varnothing, r \mathrel{\perp\!\!\!\!\top} i,$

$\Leftrightarrow\ , IsCpo(i_0; r_0), i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, r_0 = \varnothing, i_0 = \varnothing, r \mathrel{\perp\!\!\!\!\top} i,$

$\Leftrightarrow\ , IsCpo(i_0; r_0), i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, r_0 = \varnothing, i_0 = \varnothing, Rcpo(i_0; r_0), r \mathrel{\perp\!\!\!\!\top} i,$

$\Leftrightarrow\ , i \mathring{\bigcirc} i_0, i_0 = \varnothing, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), r_0 = \varnothing, Rcpo(i_0; r_0), r \mathrel{\perp\!\!\!\!\top} i,$

$\Leftrightarrow\ , i \mathring{\bigcirc} i_0, i = \varnothing, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0), r \mathrel{\perp\!\!\!\!\top} i,$

$\Leftrightarrow\ , i = \varnothing, i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0), r \mathrel{\perp\!\!\!\!\top} i,$

$premise\ 2:$
$, \ \&SHi \rightarrow i, i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0), \ \Leftrightarrow$

$, \ \&SHi \rightarrow i, i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0), r \mathrel{\perp\!\!\!\!\top} i, \ \Rightarrow$

$, i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i, i \mathring{\bigcirc} i_0, i \mathrel{!=} \varnothing, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i, i \mathring{\bigcirc} i_0, i_0 \mathrel{!=} \varnothing, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i, i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), i_0 \mathrel{!=} \varnothing, Rcpo(i_0; r_0),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i, i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0 \oplus, i_0 \oplus,$

$Rcpo(i_0; r_0),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i, i \mathring{\bigcirc} i_0, r \mathring{\bigcirc} r_0, IsCpo(i_0; r_0), i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0 \oplus, i_0 \oplus,$

$Rcpo(i_0; r_0), i \oplus, i \ominus, r \oplus, r \ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, IsCpo(i_0; r_0), i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i!♂r_0, i_0\oplus,$

$Rcpo(i_0; r_0), i\oplus, r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, IsCpo(i_0; r_0), i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i_0\oplus,$

$i!♂r_0, Rcpo(i_0; r_0), i\oplus, r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), i!♂r_0, Rcpo(i_0; r_0), i\oplus, r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), i!♂r_0, i\oplus, Rcpo(i_0; r_0), r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i!♂r_0, i\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, r!♂r_0, i\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), r!♂r_0, Rcpo(i_0; r_0), r\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi ♂ i, i♂i_0, r♂r_0, i_0 \mathrel{!=} \varnothing, Cpo(r_0), r_0\oplus, i\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), r!♂r_0, r\oplus, Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi♂i, i♂i_0, r♂r_0, i_0 != \varnothing, Cpo(r_0), r_0\oplus, r!♂r_0, i\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), r\oplus, Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi♂i, i♂i_0, r♂r_0, i_0 != \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi♂i, IsCpo(i_0; r_0), i♂i_0, r♂r_0, i_0 != \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi♂i, IsCpo(i_0; r_0), i_0!♂r_0, i♂i_0, r♂r_0, i_0 != \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi♂i, IsCpo(i_0; r_0), i!♂r_0, i♂i_0, r♂r_0, i_0 != \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi♂i, i_0 != \varnothing, IsCpo(i_0; r_0), i!♂r_0, Cpo(r_0), i♂i_0, r♂r_0, r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi♂i, i_0 != \varnothing, IsCpo(i_0; r_0), i!♂r_0, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$i♂i_0, r♂r_0, Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $i_0 != \varnothing, IsCpo(i_0; r_0), i!♂r_0,$ $\&SHi♂i, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$i♂i_0, r♂r_0, Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow$, $i_0 != \varnothing, i!♂r_0, IsCpo(i_0; r_0), Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$\&SHi \rightarrow i, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow\ , i_0 \,!\!= \varnothing, i!\overset{\circ}{\bigcirc}{}^{\circ}r_0, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$\&SHi \rightarrow i, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0), i\ominus, r\ominus,$

$\Leftrightarrow\ , i_0 \,!\!= \varnothing, i!\overset{\circ}{\bigcirc}{}^{\circ}r_0, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$\&SHi \rightarrow i, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0; r_0), Rcpo(i_0; r_0), i\mathrel{\overline{\bot}}r, i\ominus, r\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpo(i_0; r_0), i!\overset{\circ}{\bigcirc}{}^{\circ}r_0, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\mathrel{\overline{\bot}}r, i\ominus, r\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpo(i_0; r_0), i_0!\overset{\circ}{\bigcirc}{}^{\circ}r_0, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\mathrel{\overline{\bot}}r, i\ominus, r\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpo(i_0; r_0), i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\mathrel{\overline{\bot}}r, i\ominus, r\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpo(i_0; r_0), i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, i!\overset{\circ}{\bigcirc}r_0, r\oplus, i\oplus, i_0\oplus,$

$Rcpo(i_0; r_0), i\mathrel{\overline{\bot}}r, i\ominus, r\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), i!\overset{\circ}{\bigcirc}r_0, i\oplus, Rcpo(i_0; r_0), i\mathrel{\overline{\bot}}r, i\ominus, r\ominus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), i!\overset{\circ}{\bigcirc}r_0, Rcpo(i_0; r_0), i\oplus, i\mathrel{\overline{\bot}}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, i!\eth r_0, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i_0 \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i \,!\!= \varnothing, Cpo(r_0), i \,!\!= \varnothing, r_0\oplus, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), i \,!\!= \varnothing, Rcpo(i_0; r_0), i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r\oplus, i_0\oplus,$

$IsCpo(i_0; r_0), i \,!\!= \varnothing, Rcpo(i_0; r_0), i \,!\!= \varnothing, i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i \,!\!= \varnothing, Cpo(r_0), r_0\oplus, r!\eth r_0, r\oplus, i_0\oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i \,!\!= \varnothing, i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i \,!\!= \varnothing, Cpo(r_0), r_0\oplus, i_0\oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), r!\eth r_0, r\oplus, Rcpo(i_0; r_0), i \,!\!= \varnothing, i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi\,\eth i, i\eth i_0, r\eth r_0, i \,!\!= \varnothing, Cpo(r_0), r_0\oplus, i_0\oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), r!\eth r_0, Rcpo(i_0; r_0), r\oplus, i \,!\!= \varnothing, i\oplus, i{=}r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, r \overset{\circ}{\bigcirc} r_0, i \,!\!= \varnothing, Cpo(r_0), r_0 \oplus, r! \overset{\circ}{\bigcirc} r_0, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i \,!\!= \varnothing, i\oplus, i \mathrel{\text{⌶}} r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, r \overset{\circ}{\bigcirc} r_0, i \,!\!= \varnothing, Cpo(r_0), r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i \,!\!= \varnothing, i\oplus, i \mathrel{\text{⌶}} r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, r \overset{\circ}{\bigcirc} r_0, i \,!\!= \varnothing, Cpo(r_0), r_0 \,!\!= \varnothing, r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i \,!\!= \varnothing, i\oplus, i \mathrel{\text{⌶}} r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, i \,!\!= \varnothing, Cpo(r_0), r \overset{\circ}{\bigcirc} r_0, r_0 \,!\!= \varnothing, r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i \,!\!= \varnothing, i\oplus, i \mathrel{\text{⌶}} r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, i \,!\!= \varnothing, Cpo(r_0), r \overset{\circ}{\bigcirc} r_0, r \,!\!= \varnothing, r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r\oplus, i \,!\!= \varnothing, i\oplus, i \mathrel{\text{⌶}} r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, i \,!\!= \varnothing, Cpo(r_0), r \overset{\circ}{\bigcirc} r_0, r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), r \,!\!= \varnothing, Rcpo(i_0; r_0), r\oplus, i \,!\!= \varnothing, i\oplus, i \mathrel{\text{⌶}} r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, i \,!\!= \varnothing, Cpo(r_0), r \overset{\circ}{\bigcirc} r_0, r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), r \,!\!= \varnothing, Rcpo(i_0; r_0), r \,!\!= \varnothing, r\oplus, i \,!\!= \varnothing, i\oplus, i \mathrel{\text{⌶}} r, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, i \,!\!= \varnothing, Cpo(r_0), r \overset{\circ}{\bigcirc} r_0, r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), r \,!\!= \varnothing, Rcpo(i_0; r_0), i \mathrel{\text{⌶}} r, r \,!\!= \varnothing, i \,!\!= \varnothing, r\oplus, i\oplus, i\ominus, r\ominus,$

$\Leftrightarrow$, $\&SHi \overset{\circ}{\bigcirc} i, i \overset{\circ}{\bigcirc} i_0, i \,!\!= \varnothing, Cpo(r_0), r \overset{\circ}{\bigcirc} r_0, r_0 \oplus, i_0 \oplus, i \,!\!= \varnothing,$

$IsCpo(i_0; r_0), r \,!\!= \varnothing, Rcpo(i_0; r_0), i \mathrel{\text{⌶}} r, r \,!\!= \varnothing, i \,!\!= \varnothing,$

$\Leftrightarrow \; , \&SHi \male i, i \male i_0, i \,!{=} \varnothing, Cpo(r_0), r \male r_0, r_0 \oplus, i_0 \oplus, i \,!{=} \varnothing,$

$IsCpo(i_0; r_0), r \,!{=} \varnothing, Rcpo(i_0; r_0), r \,!{=} \varnothing, i \,!{=} \varnothing, i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, i \male i_0, i \,!{=} \varnothing, Cpo(r_0), r \male r_0, r_0 \oplus, i_0 \oplus, i \,!{=} \varnothing,$

$IsCpo(i_0; r_0), r \,!{=} \varnothing, Rcpo(i_0; r_0), i \,!{=} \varnothing, i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, i \male i_0, i \,!{=} \varnothing, Cpo(r_0), r \male r_0, r \,!{=} \varnothing, r_0 \oplus, i_0 \oplus, i \,!{=} \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i \,!{=} \varnothing, i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, i \male i_0, i \,!{=} \varnothing, Cpo(r_0), r \male r_0, r_0 \,!{=} \varnothing, r_0 \oplus, i_0 \oplus, i \,!{=} \varnothing,$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0), i \,!{=} \varnothing, i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, i \male i_0, r \male r_0, i \,!{=} \varnothing, Cpo(r_0), r_0 \oplus, i_0 \oplus,$

$IsCpo(i_0; r_0), i \,!{=} \varnothing, Rcpo(i_0; r_0), i \,!{=} \varnothing, i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, i \male i_0, r \male r_0, i \,!{=} \varnothing, Cpo(r_0), r_0 \oplus, i_0 \oplus,$

$IsCpo(i_0; r_0), i \,!{=} \varnothing, Rcpo(i_0; r_0), i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, IsCpo(i_0; r_0), r \male r_0, i \male i_0, i \,!{=} \varnothing, Cpo(r_0), r_0 \oplus, i_0 \oplus,$

$Rcpo(i_0; r_0), i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, IsCpo(i_0; r_0), r \male r_0, i \male i_0, i_0 \,!{=} \varnothing, Cpo(r_0), r_0 \oplus, i_0 \oplus,$

$Rcpo(i_0; r_0), i \mathrel{\underline{\top}} r,$

$\Leftrightarrow \; , \&SHi \male i, IsCpo(i_0; r_0), r \male r_0, i \male i_0, i_0 \,!{=} \varnothing, Rcpo(i_0; r_0), i \mathrel{\underline{\top}} r,$

$\Leftrightarrow\ ,\ \&SHi\overset{\circ}{\bigcirc}i, IsCpo(i_0;r_0), r\overset{\circ}{\bigcirc}r_0, i\overset{\circ}{\bigcirc}i_0, i\,!{=}\,\varnothing, Rcpo(i_0;r_0), i\mathrel{=\!\!\!\mid}r,$

$\Leftrightarrow\ ,i\,!{=}\,\varnothing,\ \&SHi\overset{\circ}{\bigcirc}i, i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), Rcpo(i_0;r_0), r\mathrel{=\!\!\!\mid}i,$

$conclusion:$

$,i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), Rcpo(i_0;r_0),$

$\Leftrightarrow\ ,i\overset{\circ}{\bigcirc}i_0, r\overset{\circ}{\bigcirc}r_0, IsCpo(i_0;r_0), Rcpo(i_0;r_0), r\mathrel{=\!\!\!\mid}i,$

$,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, Rcpo(i;r), R(j),\ \Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, Rcpo(j;r), R(i),$

induction proof:

$premise\ 1:$

$,i=\varnothing, IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, Rcpo(i;r), R(j),$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, i=\varnothing, Rcpo(i;r), R(j),$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, i=\varnothing, R(j),$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, j=\varnothing, R(j),$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, j=\varnothing,$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, i=\varnothing,$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, i=\varnothing, R(i),$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, j=\varnothing, R(i),$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, j=\varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow\ ,IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, i=\varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow\ ,i=\varnothing, IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, Rcpo(j;r), R(i),$

$premise\ 2:$

$,\ \&SHi\to i, IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, Rcpo(i;r), R(j),$

$\Leftrightarrow\ ,\ \&SHi\to i, IsCpo(i;r), IsCpo(j;r), i\mathrel{=\!\!\!\mid}j, Rcpo(j;r), R(i), Rcpo(i;r),\ \Rightarrow$

$, i \,!{=}\, \varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), i \mathrel{⌶} j, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), i \mathrel{⌶} j, i \,!{=}\, \varnothing, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), i \mathrel{⌶} j, i \,!{=}\, \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), r\oplus, i\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), j!\mathring{\bigcirc}\!\circ r, i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), r\oplus, i\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), j!\mathring{\bigcirc}\!\circ r, i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), j \,!{=}\, \varnothing, r\oplus, i\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(j;r), i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), r\oplus, i\oplus, IsCpo(i;r), j!\mathring{\bigcirc}\!\circ r, j \,!{=}\, \varnothing, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(j;r), i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), r\oplus, i\oplus, IsCpo(i;r), j!\mathring{\bigcirc}\!\circ r, Rcpo(i;r), j \,!{=}\, \varnothing, R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(j;r), i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), r\oplus, i\oplus, IsCpo(i;r), j!\mathring{\bigcirc}\!\circ r, Rcpo(i;r), j \,!{=}\, \varnothing, j\oplus, R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(j;r), i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), r\oplus, i\oplus, IsCpo(i;r), j!\mathring{\bigcirc}\!\circ r, j \,!{=}\, \varnothing, j\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(j;r), j!\mathring{\bigcirc}\!\circ r, i \mathrel{⌶} j, j \,!{=}\, \varnothing,$

$Cpo(r), j \mathrel{!=} \varnothing, r\oplus, i\oplus, j\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi$ ♂ $i, IsCpo(i;r), IsCpo(j;r), i \mathrel{\text{I}} j, j \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,$

$Cpo(r), r\oplus, i\oplus, j\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , \&SHi$ ♂ $i, IsCpo(i;r), IsCpo(j;r), i \mathrel{\text{I}} j, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,$

$Cpo(r), i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, r\oplus, i\oplus, j\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,\ \&SHi$ ♂ $i, IsCpo(i;r), IsCpo(j;r), i \mathrel{\text{I}} j,$

$Cpo(r), r\oplus, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing,\ \&SHi$ ♂ $i, IsCpo(i;r), IsCpo(j;r),$

$Cpo(r), i \mathrel{\text{I}} j, r\oplus, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, IsCpo(i;r),\ \&SHi$ ♂ $i,$

$Cpo(r), r\oplus, IsCpo(j;r), i \mathrel{\text{I}} j, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, IsCpo(i;r),\ \&SHi$ ♂ $i, Cpo(r), r\oplus,$

$IsCpo(j;r), i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus, i \mathrel{\text{I}} j, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, Cpo(r), r\oplus,$

$i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus,\ \&SHi \to i, IsCpo(i;r), IsCpo(j;r), i \mathrel{\text{I}} j, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, Cpo(r), r\oplus,$

$i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, i\oplus, j\oplus,\ \&SHi \to i, IsCpo(i;r), IsCpo(j;r), i \mathrel{\text{I}} j, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , \&SHi$ ♂ $i, i \mathrel{!=} \varnothing, j \mathrel{!=} \varnothing, IsCpo(i;r), IsCpo(j;r), Cpo(r), r\oplus,$

$i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, i\oplus, j\oplus, i{\mp}j, Rcpo(j;r), R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, IsCpo(i;r), IsCpo(j;r), Cpo(r), i{\mp}j, r\oplus,$

$i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, i\oplus, j\oplus, Rcpo(j;r), R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, IsCpo(i;r), IsCpo(j;r), i{\mp}j, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, Cpo(r), r\oplus,$

$i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, i\oplus, j\oplus, Rcpo(j;r), R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, IsCpo(i;r), i!\text{♂}r, IsCpo(j;r), i{\mp}j, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, Cpo(r), r\oplus,$

$i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, i\oplus, j\oplus, Rcpo(j;r), R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, IsCpo(i;r), i{\mp}j, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, Cpo(r), r\oplus,$

$j \,!{=}\, \varnothing, j\oplus, IsCpo(j;r), i!\text{♂}r, i \,!{=}\, \varnothing, i\oplus, Rcpo(j;r), R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, IsCpo(i;r), i{\mp}j, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, Cpo(r), r\oplus,$

$j \,!{=}\, \varnothing, j\oplus, IsCpo(j;r), i!\text{♂}r, Rcpo(j;r), i \,!{=}\, \varnothing, i\oplus, R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, IsCpo(i;r), i{\mp}j, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, Cpo(r), r\oplus,$

$j \,!{=}\, \varnothing, j\oplus, IsCpo(j;r), i!\text{♂}r, Rcpo(j;r), i \,!{=}\, \varnothing, R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, IsCpo(i;r), i{\mp}j, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, Cpo(r), r\oplus,$

$j \,!{=}\, \varnothing, j\oplus, IsCpo(j;r), i!\text{♂}r, i \,!{=}\, \varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow \;, \&SHi\,\text{♂}i, IsCpo(i;r), i!\text{♂}r, IsCpo(j;r), i{\mp}j, i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, Cpo(r),$

$i \,!{=}\, \varnothing, j \,!{=}\, \varnothing, r\oplus, j\oplus, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), i!\mathring{\bigcirc}r, IsCpo(j;r), i⌶j, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Cpo(r), r\oplus, j\oplus, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), i!\mathring{\bigcirc}r, IsCpo(j;r), i⌶j, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i⌶j, i\,!{=}\,\varnothing, j\,!{=}\,\varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i⌶j, j\,!{=}\,\varnothing, i\,!{=}\,\varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i⌶j, i\,!{=}\,\varnothing, i\,!{=}\,\varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i⌶j, i\,!{=}\,\varnothing, Rcpo(j;r), R(i),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(j;r), i⌶j, Rcpo(j;r), R(i),$

$conclusion:$

$, IsCpo(i;r), IsCpo(j;r), i⌶j, Rcpo(i;r), R(j),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), i⌶j, Rcpo(j;r), R(i),$

$$, IsCpo(i;r), IsCpo(j;r), i⌶j, Rcpo(i;r), i⏚, j⏚,$$
$$\Leftrightarrow\ IsCpo(i;r), IsCpo(j;r), i⌶j, Rcpo(j;r), i⏚, j⏚,$$

29.7 &Tm(r)

$$, IsCpo(i;r), \&Fam(r), Rcpo(i;r),\ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), \&Fam(r),$$

induction proof:
$premise\ 1:$
$, i=\varnothing, IsCpo(i;r), \&Fam(r), Rcpo(i;r),$
$\Leftrightarrow\ , IsCpo(i;r), \&Fam(r), i=\varnothing, Rcpo(i;r),$

$\Leftrightarrow\ , IsCpo(i;r), \&Fam(r), i=\varnothing,$

$\Leftrightarrow\ , IsCpo(i;r), i=\varnothing, \&Fam(r),$

$\Leftrightarrow\ , IsCpo(i;r), i=\varnothing, Rcpo(i;r), \&Fam(r),$

$\Leftrightarrow\ , i=\varnothing, IsCpo(i;r), Rcpo(i;r), \&Fam(r),$

$premise\ 2:$
$, \&SHi \rightarrow i, IsCpo(i;r), \&Fam(r), Rcpo(i;r), \;\Leftrightarrow\; , \&SHi \rightarrow i, IsCpo(i;r), Rcpo(i;r), \&Fam(r), \;\Rightarrow$

$, i \mathrel{!=} \varnothing, \&SHi \mathring{\bigcirc} i, IsCpo(i;r), \&Fam(r), Rcpo(i;r),$

$\Leftrightarrow\; , \&SHi \mathring{\bigcirc} i, IsCpo(i;r), \&Fam(r), i \mathrel{!=} \varnothing, Rcpo(i;r),$

$\Leftrightarrow\; , \&SHi \mathring{\bigcirc} i, IsCpo(i;r), \&Fam(r), i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r),$

$\Leftrightarrow\; , \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, \&Fam(r), Rcpo(i;r),$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, IsCpo(i;r), \&SHi \mathring{\bigcirc} i, Cpo(r), r\oplus, i\oplus, \&Fam(r), Rcpo(i;r),$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), \&Fam(r), Rcpo(i;r),$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, \&SHi \rightarrow i, IsCpo(i;r), Rcpo(i;r), \&Fam(r),$

$\Leftrightarrow\; , \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), \&Fam(r),$

$\Leftrightarrow\; , \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i \mathrel{!=} \varnothing, Rcpo(i;r), \&Fam(r),$

$\Leftrightarrow\; , i \mathrel{!=} \varnothing, \&SHi \mathring{\bigcirc} i, IsCpo(i;r), Rcpo(i;r), \&Fam(r),$

$conclusion:$

$, IsCpo(i;r), \&Fam(r), Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), Rcpo(i;r), \&Fam(r),$

$$, IsCpo(i;r), \&Fam(m), Rcpo(i;r), \;\Leftrightarrow\; , IsCpo(i;r), Rcpo(i;r), \&Fam(m),$$

$$, \&Tm(r), \;\Leftrightarrow\; , Cpo(r), \&Tm(r),$$

proof:

$, \&Tm(r),$

$\Leftrightarrow\; , r \textcircled{\curlyvee} m, m\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\; , r \textcircled{\curlyvee} m, m \textcircled{\Rightarrow} r, m\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , Cpo(r), \&Tm(r),$

$$, \&Tm(r), \ \Leftrightarrow\ , Cpo(r), r\oplus, \&Tm(r),$$

$$, IsCpo(i;r), i\textcircled{\downarrow}, \&Tm(r), \ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$$

induction proof:

premise 1 :

$, i = \varnothing, IsCpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , IsCpo(i;r), i = \varnothing, i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , IsCpo(i;r), i = \varnothing, Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , i = \varnothing, IsCpo(i;r), Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

premise 2 :

$, \ \&SHi \rightarrow i, IsCpo(i;r), i\textcircled{\downarrow}, \&Tm(r), \ \Leftrightarrow\ , \ \&SHi \rightarrow i, IsCpo(i;r), Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r), \ \Longrightarrow$

$, i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i\textcircled{\downarrow}, Cpo(r), r\oplus, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i\oplus, i\textcircled{\downarrow}, Cpo(r), r\oplus, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i\oplus, Cpo(r), r\oplus, i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), Cpo(r), i\oplus, r\oplus, i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, IsCpo(i;r), \ \&SHi \mathring{\bigcirc} i, Cpo(r), r\oplus, i\oplus, i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, \ \&SHi \rightarrow i, IsCpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, \ \&SHi \rightarrow i, IsCpo(i;r), Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i \mathrel{!=} \varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), i \mathrel{!=} \varnothing, Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, \ \&SHi \mathring{\bigcirc} i, IsCpo(i;r), Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

conclusion :

$, IsCpo(i;r), i\textcircled{\downarrow}, \&Tm(r), \ \Leftrightarrow\ , IsCpo(i;r), Rcpo(i;r), i\textcircled{\downarrow}, \&Tm(r),$

$$, IsCpo(i;r), R(i), \&Tm(r), \iff , IsCpo(i;r), Rcpo(i;r), \&Tm(r),$$

$$, IsCpo(i;r), IsCpo(j;r), i⭳, j⭳, \&Tm(r), \iff$$
$$, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r), i⭳, j⭳, \&Tm(r),$$

proof:

$, IsCpo(i;r), IsCpo(j;r), i⭳, j⭳, \&Tm(r),$

$\iff , IsCpo(i;r), i⭳, IsCpo(j;r), j⭳, \&Tm(r),$

$\iff , IsCpo(i;r), i⭳, IsCpo(j;r), Rcpo(j;r), j⭳, \&Tm(r),$

$\iff , IsCpo(i;r), IsCpo(j;r), i⭳, Rcpo(j;r), j⭳, \&Tm(r),$

$\iff , IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), i⭳, j⭳, \&Tm(r),$

$\iff , IsCpo(j;r), IsCpo(i;r), Rcpo(j;r), i⭳, j⭳, \&Tm(r),$

$\iff , IsCpo(j;r), Rcpo(j;r), IsCpo(i;r), i⭳, j⭳, \&Tm(r),$

$\iff , IsCpo(j;r), Rcpo(j;r), j⭳, IsCpo(i;r), i⭳, \&Tm(r),$

$\iff , IsCpo(j;r), Rcpo(j;r), j⭳, IsCpo(i;r), Rcpo(i;r), i⭳, \&Tm(r),$

$\iff , IsCpo(j;r), Rcpo(j;r), IsCpo(i;r), j⭳, Rcpo(i;r), i⭳, \&Tm(r),$

$\iff , IsCpo(j;r), Rcpo(j;r), IsCpo(i;r), Rcpo(i;r), j⭳, i⭳, \&Tm(r),$

$\iff , IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r), j⭳, i⭳, \&Tm(r),$

$\iff , IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r), i⭳, j⭳, \&Tm(r),$

$$, IsCpo(i;r), IsCpo(j;r), R(i), R(j), \&Tm(r), \iff$$
$$, IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r), \&Tm(r),$$

$$, \circledcirc r, i⭳, r⭳, \iff , \circledcirc r, i⭳, \&Tm(r),$$

$$, \circledcirc r, Rcpo(i;r), i⭳, r⭳, \iff , \circledcirc r, Rcpo(i;r), i⭳, \&Tm(r),$$

$$,\circledcirc r, Rcpo(i;r), Rcpo(j;r), i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow}, \;\Leftrightarrow\; ,\circledcirc r, Rcpo(i;r), Rcpo(j;r), i\textcircled{\downarrow}, j\textcircled{\downarrow}, \&Tm(r),$$

$$,\circledcirc r, r\textcircled{\downarrow}, \;\Leftrightarrow\; ,\circledcirc r, \&Tm(r),$$

$$,\circledcirc r, Rcpo(i;r), r\textcircled{\downarrow}, \;\Leftrightarrow\; ,\circledcirc r, Rcpo(i;r), \&Tm(r),$$

$$,\circledcirc r, Rcpo(i;r), Rcpo(j;r), r\textcircled{\downarrow}, \;\Leftrightarrow\; ,\circledcirc r, Rcpo(i;r), Rcpo(j;r), \&Tm(r),$$

$$, i\textcircled{\downarrow}, \;\Leftrightarrow\; ,\circledcirc r, Rcpo(i;r), i\textcircled{\downarrow}, r\textcircled{\downarrow},$$

$$, R(i), \;\Leftrightarrow\; ,\circledcirc r, Rcpo(i;r), r\textcircled{\downarrow},$$

$$, i\textcircled{\downarrow}, j\textcircled{\downarrow}, \;\Leftrightarrow\; ,\circledcirc r, Rcpo(i;r), Rcpo(j;r), i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow},$$

$$, R(i), R(j), \;\Leftrightarrow\; ,\circledcirc r, Rcpo(i;r), Rcpo(j;r), r\textcircled{\downarrow},$$

$$,\circledcirc r, r\textcircled{\curlywedge}r_0, i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow}, r_0\textcircled{\downarrow},$$
$$\Leftrightarrow\; ,\circledcirc r, r\textcircled{\curlywedge}r_0, i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow}, \&Tm(r_0),$$

$$,\circledcirc r, r\textcircled{\curlywedge}r_0, Rcpo(i;r_0), Rcpo(j;r_0), i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow}, r_0\textcircled{\downarrow},$$
$$\Leftrightarrow\; ,\circledcirc r, r\textcircled{\curlywedge}r_0, Rcpo(i;r_0), Rcpo(j;r_0), i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow}, \&Tm(r_0),$$

$$,\circledcirc r, r\textcircled{\curlywedge}r_0, i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow}, r_0\textcircled{\downarrow},$$
$$\Leftrightarrow\; ,\circledcirc r, r\textcircled{\curlywedge}r_0, Rcpo(i;r_0), Rcpo(j;r_0), i\textcircled{\downarrow}, j\textcircled{\downarrow}, r\textcircled{\downarrow}, r_0\textcircled{\downarrow},$$

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$$, IsCpo(j;r), j\ominus, j \mathrel{!\!=} \varnothing, j\circledwedge j_0, Rcpo(j_0;r), j_0\circledvee, \quad \Leftrightarrow$$

$$, IsCpo(j;r), Cpo(r), r\oplus, j\circledwedge j_1, Rcpo(j_1;r), j_1\circledvee, j\ominus, j \mathrel{!\!=} \varnothing,$$

proof:

$, IsCpo(j;r), j\ominus, j \mathrel{!\!=} \varnothing, j\circledwedge j_0, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , IsCpo(j;r), j\ominus, j\circledwedge j_0, j_0 \mathrel{!\!=} \varnothing, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , IsCpo(j;r), j\ominus, j\circledwedge j_0, j_0 \mathrel{!\!=} \varnothing, Cpo(r), r\oplus, j_0\oplus, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , IsCpo(j;r), j\ominus, j \mathrel{!\!=} \varnothing, j\circledwedge j_0, Cpo(r), r\oplus, j_0\oplus, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , IsCpo(j;r), j\ominus, j \mathrel{!\!=} \varnothing, Cpo(r), j\circledwedge j_0, r\oplus, j_0\oplus, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , IsCpo(j;r), Cpo(r), j\ominus, j \mathrel{!\!=} \varnothing, j\circledwedge j_0, r\oplus, j_0\oplus, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , IsCpo(j;r), Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j_0\oplus, j \mathrel{!\!=} \varnothing, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , IsCpo(j;r), Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j_0\oplus, j\oplus, j\ominus, j \mathrel{!\!=} \varnothing, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, IsCpo(j;r), j\circledwedge j_0, j_0\oplus, j\oplus, j\ominus, j \mathrel{!\!=} \varnothing, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, IsCpo(j;r), j!\ddot{\bigcirc}r, j\circledwedge j_0, j_0\oplus, j\oplus, j\ominus, j \mathrel{!\!=} \varnothing, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, IsCpo(j_0;r), j_0\oplus, j\oplus, j!\ddot{\bigcirc}r, j\ominus, j \mathrel{!\!=} \varnothing, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j_0\oplus, j\oplus, IsCpo(j_0;r), j!\ddot{\bigcirc}r, j\ominus, j \mathrel{!\!=} \varnothing, Rcpo(j_0;r), j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j_0\oplus, j\oplus, IsCpo(j_0;r), j!\ddot{\bigcirc}r, Rcpo(j_0;r), j\ominus, j \mathrel{!\!=} \varnothing, j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j_0\oplus, j\oplus, j!\ddot{\bigcirc}r, j\circledwedge j_1, IsCpo(j_0;r), j_1\circledvee, Rcpo(j_0;r), j\ominus, j \mathrel{!\!=} \varnothing, j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j_0\oplus, j\oplus, j!\ddot{\bigcirc}r, j\circledwedge j_1, IsCpo(j_0;r), Rcpo(j_0;r), j_1\circledvee, j\ominus, j \mathrel{!\!=} \varnothing, j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j\mathring{\bigcirc}j_0, j_0\oplus, j\oplus, j!\ddot{\bigcirc}r, j\circledwedge j_1, IsCpo(j_0;r), Rcpo(j_0;r),$

$j_1\circledvee, j\ominus, j \mathrel{!\!=} \varnothing, j_0\circledvee,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledwedge j_0, j_0\oplus, j\oplus, j\mathring{\bigcirc}j_0, j\circledwedge j_1, j_1!\ddot{\bigcirc}r, IsCpo(j_0;r), Rcpo(j_0;r),$

$j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledcirc j_0, j\circledast j_1, j_1!\circledcirc r, IsCpo(j_0;r), r=\varnothing,$

$Rcpo(j_0;r), j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledcirc j_0, j\circledast j_1, IsCpo(j_0;r), j_1!\circledcirc r, r=\varnothing,$

$Rcpo(j_0;r), j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, IsCpo(j_0;r), IsCpo(j_1;r),$

$Rcpo(j_0;r), j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, j_1 = j_0, IsCpo(j_0;r), IsCpo(j_1;r),$

$Rcpo(j_0;r), j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, IsCpo(j_0;r), IsCpo(j_1;r), j_1 = j_0,$

$Rcpo(j_0;r), j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, IsCpo(j_0;r), IsCpo(j_1;r), j_1 = j_0,$

$Rcpo(j_0;r), R(j_1), j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, IsCpo(j_0;r), IsCpo(j_1;r), j_1 = j_0,$

$Rcpo(j_1;r), R(j_0), j_1\odot, j\ominus, j \mathrel{!=} \varnothing, j_0\odot,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, j_1 = j_0, IsCpo(j_0;r), IsCpo(j_1;r),$

$Rcpo(j_1;r), R(j_0), j_0\odot, j_1\odot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, IsCpo(j_0;r), IsCpo(j_1;r),$

$Rcpo(j_1;r), j_0\odot, j_1\odot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, IsCpo(j_0;r), IsCpo(j_1;r), j_0\odot,$

$Rcpo(j_1;r), j_1\odot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\circledast j_0, j_0\oplus, j\oplus, j\circledast j_1, j_1\circledcirc j_0, IsCpo(j_1;r), IsCpo(j_1;r), j_0\odot,$

$Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\varowedge j_0, j_0\oplus, j\oplus, j\varowedge j_1, j_1\mathring{\bigcirc} j_0, IsCpo(j_1;r), j_0\obot,$

$Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\varowedge j_0, j_0\oplus, j\oplus, j\mathring{\bigcirc} j_0, j\varowedge j_1, IsCpo(j_1;r), j_0\obot,$

$Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\varowedge j_0, j_0\oplus, j\oplus, j\varowedge j_1, IsCpo(j_1;r), j_0\obot,$

$Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , Cpo(r), r\oplus, j\ominus, j\varowedge j_0, j_0\oplus, j\oplus, IsCpo(j;r), j\varowedge j_1, j_0\obot,$

$Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , IsCpo(j;r), Cpo(r), r\oplus, j\ominus, j\varowedge j_0, j_0\oplus, j_0\obot, j\oplus, j\varowedge j_1,$

$Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , IsCpo(j;r), Cpo(r), r\oplus, j\ominus, j\oplus, j\varowedge j_1, Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , IsCpo(j;r), Cpo(r), r\oplus, j\varowedge j_1, Rcpo(j_1;r), j_1\obot, j\ominus, j \mathrel{!=} \varnothing,$

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$$, IsCpo(r_1;r), r_1\mathring{\mathring{\bigcirc}} r_{10}, r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus, \quad \Leftrightarrow$$
$$, IsCpo(r_1;r), r_1\mathring{\mathring{\bigcirc}} r_{10}, r_{10}=\varnothing, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$$

induction proof:
premise 1 :
$, r_1=\varnothing, IsCpo(r_1;r), r_1\mathring{\mathring{\bigcirc}} r_{10}, r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$
$\Leftrightarrow\ , r_1\mathring{\mathring{\bigcirc}} r_{10}, r_1=\varnothing, r_{10}=\varnothing, IsCpo(r_1;r), Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1\mathring{\mathring{\bigcirc}} r_{10}, r_1=\varnothing, r_{10}=\varnothing, r_1\mathring{\bigcirc} r_{10}, IsCpo(r_1;r), Cpo(r),$

$r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1\mathring{\mathring{\bigcirc}} r_{10}, r_{10}=\varnothing, r_1\mathring{\bigcirc} r_{10}, IsCpo(r_1;r), r_1=\varnothing, Cpo(r),$

$r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, IsCpo(r_1;r), Cpo(r), r_1 = \varnothing,$

$r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, IsCpo(r_1;r), Cpo(r),$

$r\oplus, r_1 = \varnothing, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, IsCpo(r_1;r), Cpo(r),$

$r\oplus, r_1 = \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, IsCpo(r_1;r), Cpo(r),$

$r\oplus, r_1 \overset{\circ}{\bigcirc} r_{10}, r_1 = \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, IsCpo(r_1;r), Cpo(r),$

$r\oplus, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, IsCpo(r_1;r), Cpo(r),$

$r\oplus, r_1 \overset{\circ}{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_{10} = \varnothing, r_1\oplus,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, IsCpo(r_1;r), Cpo(r),$

$r\oplus, r_1 \overset{\circ}{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_{10} = \varnothing,$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} = \varnothing, IsCpo(r_1;r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10}\oplus, r_1\oplus, r_{10} = \varnothing,$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_1 \mathring{\bigcirc} r_{10}, r_{10} = \varnothing,$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_1 \mathring{\bigcirc} r_{10}, r_1 = \varnothing,$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_1 \mathring{\bigcirc} r_{10}, r_1 = \varnothing, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_1 \mathring{\bigcirc} r_{10}, r_{10} = \varnothing, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\, != \varnothing, r_{10}\oplus, r_1\oplus, r_1 \mathring{\bigcirc} r_{10}, r_{10} = \varnothing, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_1 \mathring{\bigcirc} r_{10}, r_{10}\, != \varnothing, r_{10}\oplus, r_1\oplus, r_{10} = \varnothing, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_1 \mathring{\bigcirc} r_{10}, r_1\, != \varnothing, r_{10}\oplus, r_1\oplus, r_{10} = \varnothing, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, r_1 \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_{10} = \varnothing, r_1\, != \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, r_1 \mathring{\bigcirc} r_{10}, r_{10} = \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1\, != \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r),$

$r\oplus, r_1 \mathring{\bigcirc} r_{10}, r_1 = \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1; r), Cpo(r), r_1 = \varnothing,$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1; r), r_1 = \varnothing, Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1; r), Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1; r), r_1 ! \mathring{\bigcirc}\!\circ r, Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1; r), r_{10} ! \mathring{\bigcirc}\!\circ r, Cpo(r),$

$r\oplus, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1; r), r_{10} ! \mathring{\bigcirc}\!\circ r, Cpo(r_{10}), r_{10}\oplus,$

$Cpo(r), r\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, r_{10} ! \mathring{\bigcirc}\!\circ r, IsCpo(r_1; r), Cpo(r_{10}), r_{10}\oplus,$

$Cpo(r), r\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}\!\circ r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, r_{10} ! \mathring{\bigcirc}\!\circ r, r_1 ! \mathring{\bigcirc}\!\circ r, r = \varnothing, Cpo(r_{10}), r_{10}\oplus,$

$Cpo(r), r\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, r = \varnothing, Cpo(r_{10}), r_{10}\oplus,$

$r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r), r\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r_{10}), r = \varnothing, r_{10}\oplus,$

$r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r), r\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r_{10}), r_{10}\oplus,$

$r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r, r = \varnothing, Cpo(r), r\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r_{10}), r_{10}\oplus,$

$IsCpo(r_1; r), Cpo(r), r\oplus, r_1 \mathrel{!=} \varnothing, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r_{10}), r_{10}\oplus,$

$IsCpo(r_1; r), Cpo(r), r_1 \mathrel{!=} \varnothing, r\oplus, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r_{10}), r_{10}\oplus,$

$IsCpo(r_1; r), r_1 \mathrel{!=} \varnothing, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, Cpo(r_{10}), r_{10}\oplus,$

$IsCpo(r_1; r), r_1 \mathrel{!=} \varnothing, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \overset{\circ}{\bigcirc} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, IsCpo(r_1; r),$

$Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r, IsCpo(r_1; r),$

$Cpo(r_{10}), r_1 \mathring{\bigcirc} r_{10}, r_1 \,!{=}\, \varnothing, r_{10}\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_{10}! \mathring{\bigcirc}^{\circ} r, IsCpo(r_1; r),$

$Cpo(r_{10}), r_1 \mathring{\bigcirc} r_{10}, r_{10} \,!{=}\, \varnothing, r_{10}\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, r_{10}! \mathring{\bigcirc}^{\circ} r, IsCpo(r_1; r),$

$Cpo(r_{10}), r_{10} \,!{=}\, \varnothing, r_{10}\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, r_{10}! \mathring{\bigcirc}^{\circ} r, IsCpo(r_1; r),$

$Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, r_1! \mathring{\bigcirc}^{\circ} r, IsCpo(r_1; r),$

$Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1; r), Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1 = \varnothing, r_{10} = \varnothing, IsCpo(r_1; r), Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1; r),$

$\Leftrightarrow\ , r_1 = \varnothing, IsCpo(r_1; r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1; r),$

$premise\ 2:$
$, \ \&SHi \rightarrow r_1, IsCpo(r_1; r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r), r\oplus, Rcpo(r_1; r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus, \ \Leftrightarrow$

$, \ \&SHi \rightarrow r_1, IsCpo(r_1; r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1; r), \ \Rightarrow$

$, r_1 \,!{=}\, \varnothing, \ \&SHi \mathring{\bigcirc} r_1, IsCpo(r_1; r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r), r\oplus, Rcpo(r_1; r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \,!{=}\, \varnothing, \ \&SHi \mathring{\bigcirc} r_1, IsCpo(r_1; r), r_1! \mathring{\bigcirc}^{\circ} r, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r), r\oplus,$
$Rcpo(r_1; r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} r_1, IsCpo(r_1; r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, r_1! \mathring{\bigcirc}^{\circ} r, r_1 \,!{=}\, \varnothing, Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r_1 \mathrel{!}= \varnothing, r\oplus,$
$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus,$
$r_1 \mathrel{!}= \varnothing, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus,$
$r_1 \mathrel{!}= \varnothing, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, Cpo(r), r\oplus,$
$r_1 \mathrel{!}= \varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r_1\oplus, r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, Cpo(r), r\oplus,$
$r_1 \mathrel{!}= \varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, r_1\oplus, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r_1 \mathrel{!}= \varnothing, r\oplus, r_1\oplus,$
$Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, r_1 \mathrel{!}= \varnothing, Cpo(r), r\oplus, r_1\oplus,$
$Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \mathrel{!}= \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, r_{10}=\varnothing, Cpo(r), r\oplus, r_1\oplus,$
$Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \mathrel{!}= \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r, r_{10}=\varnothing, Cpo(r), r\oplus, r_1\oplus,$
$Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \mathrel{!}= \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r_{10}=\varnothing, r\oplus, r_1\oplus,$
$Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \mathrel{!}= \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r, IsCpo(r_1;r), Cpo(r), r\oplus, r_1\oplus,$
$r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \mathrel{!}= \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1\oplus,$
$IsCpo(r_1;r), r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \mathrel{!}= \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1\oplus,$
$IsCpo(r_1;r), r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \mathrel{!}= \varnothing, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, Cpo(r), r\oplus, r_1\oplus,$
$IsCpo(r_1;r), r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \,!{=}\, \varnothing, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r),\ \&SHi\,\overset{\circ}{\bigcirc}r_1, r\oplus, r_1\oplus,$
$IsCpo(r_1;r), r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \,!{=}\, \varnothing, r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1\oplus,\ \&SHi \rightarrow r_1,$
$IsCpo(r_1;r), r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \,!{=}\, \varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1\oplus,\ \&SHi \rightarrow r_1, IsCpo(r_1;r),$
$r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , r_1 \,!{=}\, \varnothing, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1\oplus,\ \&SHi \rightarrow r_1, IsCpo(r_1;r),$
$r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}=\varnothing, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, r_1 \,!{=}\, \varnothing, Cpo(r), r_{10}=\varnothing, r\oplus, r_1\oplus,$
$Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r_1 \,!{=}\, \varnothing, r_{10}=\varnothing, r\oplus, r_1\oplus,$
$Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$
$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1 \,!{=}\, \varnothing, r_{10}=\varnothing,$
$r_1\oplus, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1 \,!{=}\, \varnothing, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}r_{10},$
$r_1\oplus, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1 \,!{=}\, \varnothing, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}r_{10},$
$Cpo(r_{10}), r_1\oplus, r_{10}\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1 \,!{=}\, \varnothing, r_{10}=\varnothing, r_1!\overset{\circ}{\bigcirc}r_{10},$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r\oplus, r_1 \,!{=}\, \varnothing, r_{10}=\varnothing,$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r_1 \,!{=}\, \varnothing, r_{10}=\varnothing, r\oplus,$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_1!\overset{\circ}{\bigcirc}{}^{\circ}r, r_1 \,!{=}\, \varnothing, Cpo(r), r_{10}=\varnothing, r\oplus,$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r, r_1 \,!{=}\, \varnothing, Cpo(r), r_{10}=\varnothing, r\oplus,$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ , r_1 \,!{=}\, \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}r_1, IsCpo(r_1;r), r_1\overset{\circ}{\bigcirc}{}^{\circ}r_{10}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r, Cpo(r), r_{10}=\varnothing, r\oplus,$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ , r_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10}!\mathring{\bigcirc}^{\circ} r, r_{10} = \varnothing, Cpo(r), r\oplus,$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ , r_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_{10} = \varnothing, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10}!\mathring{\bigcirc}^{\circ} r, Cpo(r), r\oplus,$
$Cpo(r_{10}), r_{10}\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ , r_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_{10} = \varnothing, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10}!\mathring{\bigcirc}^{\circ} r,$
$Cpo(r_{10}), r_{10}\oplus, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ , r_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_{10} = \varnothing, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_1!\mathring{\bigcirc}^{\circ} r,$
$Cpo(r_{10}), r_{10}\oplus, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1!\mathring{\bigcirc}^{\circ} r, r_1 \mathring{\bigcirc}^{\circ} r_{10},$
$r_{10} = \varnothing, r_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing,$
$r_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing,$
$r_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_1 \mathrel{!=} \varnothing, r_{10}\oplus, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing,$
$r_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, Cpo(r), r\oplus, r_1\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, r_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1 \mathrel{!=} \varnothing, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, r_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_1 \mathrel{!=} \varnothing, r_{10}\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, r_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$

$\Leftrightarrow\ , r_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} r_1, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$

$conclusion:$

$, IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r), r\oplus, Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$\Leftrightarrow\ , IsCpo(r_1;r), r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_{10} = \varnothing, Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r),$

$$, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10},$$
$$Rcpo(i;r), r_1 \bigcirc\!\!\!\!\!\downarrow, \&Tm(r_{10}),\ \Leftrightarrow$$

$$, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10},$$
$$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$$

proof:

$, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10},$

$Rcpo(i;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10},$

$i\text{☮}i_0, i_0\text{⏚}, Rcpo(i;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10},$

$i\text{☮}i_0, IsCpo(i;r), i_0\text{⏚}, Rcpo(i;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1!\text{⚲}r, r_1\text{⚲}r_{10},$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_1!\text{⚲}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_{10}!\text{⚲}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), r_{10}=\varnothing, IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_{10}!\text{⚲}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10},$

$i\text{☮}i_0, IsCpo(i;r), r_{10}!\text{⚲}r, r_{10}=\varnothing, Rcpo(i;r), i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10},$

$i\text{☮}i_0, IsCpo(i;r), r_{10}!\text{⚯}r, Rcpo(i;r), r_{10}=\varnothing, i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), i!\text{⚯}r_{10}, IsCpo(r_1;r), r_1\text{⧂}r_{10},$

$i\text{☮}i_0, IsCpo(i;r), r_{10}!\text{⚯}r, Rcpo(i;r), r_{10}=\varnothing, i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10}, r_{10}!\text{⚯}r,$

$i!\text{⚯}r_{10}, i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), r_{10}=\varnothing, i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10}, r_{10}!\text{⚯}r,$

$i\text{☮}i_0, i_0!\text{⚯}r_{10}, IsCpo(i;r), Rcpo(i;r), r_{10}=\varnothing, i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10}, r_{10}!\text{⚯}r,$

$i\text{☮}i_0, IsCpo(i;r), i_0!\text{⚯}r_{10}, Rcpo(i;r), r_{10}=\varnothing, i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10}, r_{10}!\text{⚯}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), i_0!\text{⚯}r_{10}, r_{10}=\varnothing, i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10}, r_{10}!\text{⚯}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), r_1\text{⏚}, IsCpo(i_0;r_{10}), i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10}, r_{10}!\text{⚯}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), r_1\text{⏚}, IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⧂}r_{10}, r_{10}!\text{⚯}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), IsCpo(i_0;r_{10}), r_1\text{⏚}, Rcpo(i_0;r_{10}), i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_{10}!\text{⚨}r,$

$i\text{☮}i_0, IsCpo(i;r), Rcpo(i;r), IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), i!\text{⚨}r_{10}, IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_{10}!\text{⚨}r,$

$i\text{☮}i_0, IsCpo(i;r), i!\text{⚨}r, IsCpo(i_0;r_{10}), Rcpo(i;r), Rcpo(i_0;r_{10}), r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), i!\text{⚨}r_{10}, IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_{10}!\text{⚨}r,$

$i\text{☮}i_0, IsCpo(i;r), i_0!\text{⚨}r, IsCpo(i_0;r_{10}), Rcpo(i;r), Rcpo(i_0;r_{10}), r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10}, i\text{☮}i_0,$

$IsCpo(i;r), IsCpo(i_0;r_{10}), i!\text{⚨}r_{10}, i_0!\text{⚨}r, r_{10}!\text{⚨}r, Rcpo(i;r), Rcpo(i_0;r_{10}),$

$r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10}, i\text{☮}i_0,$

$IsCpo(i;r), IsCpo(i_0;r_{10}), i!\text{⚨}r_{10}, i_0!\text{⚨}r, r_{10}!\text{⚨}r, Rcpo(i_0;r_{10}), Rcpo(i;r),$

$r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_{10}!\text{⚨}r, i\text{☮}i_0,$

$IsCpo(i;r), IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), Rcpo(i;r),$

$r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\text{⚲}r_{10}, r_{10}!\text{⚨}r, i\text{☮}i_0, i\text{⚲}i_0,$

$IsCpo(i;r), IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), Rcpo(i;r),$

$r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_{10}!\mathring{\bigcirc}^{\circ}r, i\owedge i_0,$

$IsCpo(i;r), r_1\mathring{\bigcirc}r_{10}, i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), Rcpo(i;r),$

$r_1\obot, i_0\obot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_{10}!\mathring{\bigcirc}^{\circ}r, i\owedge i_0,$

$IsCpo(i;r), r_1\mathring{\bigcirc}r_{10}, i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), i\underline{\top}r_1, Rcpo(i;r),$

$r_1\obot, i_0\obot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r_{10}!\mathring{\bigcirc}^{\circ}r, i\owedge i_0,$

$i!\mathring{\bigcirc}^{\circ}r, r=\varnothing, r_1\mathring{\bigcirc}r_{10}, i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), i\underline{\top}r_1, Rcpo(i;r),$

$r_1\obot, i_0\obot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\owedge i_0, i!\mathring{\bigcirc}^{\circ}r, r_1\mathring{\bigcirc}r_{10}, i\mathring{\bigcirc}i_0,$

$IsCpo(i_0;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, r=\varnothing, Rcpo(i_0;r_{10}), i\underline{\top}r_1, Rcpo(i;r),$

$r_1\obot, i_0\obot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\owedge i_0, i!\mathring{\bigcirc}^{\circ}r, r_1\mathring{\bigcirc}r_{10}, i\mathring{\bigcirc}i_0,$

$IsCpo(i_0;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i_0;r_{10}), r=\varnothing, i\underline{\top}r_1, Rcpo(i;r),$

$r_1\obot, i_0\obot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\owedge i_0, r_1\mathring{\bigcirc}r_{10}, i\mathring{\bigcirc}i_0, r_{10}!\mathring{\bigcirc}^{\circ}r,$

$IsCpo(i_0;r_{10}), i!\mathring{\bigcirc}^{\circ}r, Rcpo(i_0;r_{10}), r=\varnothing, i\underline{\top}r_1, Rcpo(i;r),$

$r_1\obot, i_0\obot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\circledwedge i_0, r_1\mathring{\bigcirc} r_{10}, i\mathring{\bigcirc} i_0, r_{10}!\mathring{\bigcirc}^{\circ} r,$

$IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), i!\mathring{\bigcirc}^{\circ} r, r=\varnothing, i\models r_1, Rcpo(i;r),$

$r_1\circledbot, i_0\circledbot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\circledwedge i_0, r_1\mathring{\bigcirc} r_{10}, i\mathring{\bigcirc} i_0, r_{10}!\mathring{\bigcirc}^{\circ} r,$

$IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), IsCpo(i;r), i\models r_1, Rcpo(i;r),$

$r_1\circledbot, i_0\circledbot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), r!\mathring{\bigcirc}^{\circ} r_1, i\circledwedge i_0, r_1\mathring{\bigcirc} r_{10}, i\mathring{\bigcirc} i_0, r_{10}!\mathring{\bigcirc}^{\circ} r,$

$IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), IsCpo(i;r), i\models r_1, Rcpo(i;r),$

$r_1\circledbot, i_0\circledbot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\circledwedge i_0, r_1\mathring{\bigcirc} r_{10}, i\mathring{\bigcirc} i_0, r_{10}!\mathring{\bigcirc}^{\circ} r,$

$IsCpo(i_0;r_{10}), r!\mathring{\bigcirc}^{\circ} r_1, Rcpo(i_0;r_{10}), IsCpo(i;r), i\models r_1, Rcpo(i;r),$

$r_1\circledbot, i_0\circledbot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\circledwedge i_0, r_1\mathring{\bigcirc} r_{10}, i\mathring{\bigcirc} i_0, r_{10}!\mathring{\bigcirc}^{\circ} r,$

$IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), r!\mathring{\bigcirc}^{\circ} r_1, IsCpo(i;r), i\models r_1, Rcpo(i;r),$

$r_1\circledbot, i_0\circledbot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\circledwedge i_0, r_1\mathring{\bigcirc} r_{10}, i\mathring{\bigcirc} i_0, r_{10}!\mathring{\bigcirc}^{\circ} r,$

$IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}), IsCpo(r_1;r), IsCpo(i;r), i\models r_1, Rcpo(i;r),$

$r_1\circledbot, i_0\circledbot, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(r_1;r), i\circledwedge i_0, r_1\mathring{\bigcirc} r_{10}, i\mathring{\bigcirc} i_0, r_{10}!\mathring{\bigcirc}^{\circ} r,$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), IsCpo(r_1; r), IsCpo(i; r), i \text{⫧} r_1, Rcpo(i; r), R(r_1),$

$r_1 \text{⏚}, i_0 \text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(i; r_{10}), IsCpo(r_1; r), i \text{☮} i_0, r_1 \text{⚲} r_{10}, i \text{⚲} i_0, r_{10}! \text{⚨} r,$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), IsCpo(r_1; r), IsCpo(i; r), i \text{⫧} r_1, Rcpo(r_1; r), R(i),$

$r_1 \text{⏚}, i_0 \text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(i; r_{10}), IsCpo(r_1; r), i \text{☮} i_0, r_1 \text{⚲} r_{10}, i \text{⚲} i_0, r_{10}! \text{⚨} r,$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), r! \text{⚨} r_1, IsCpo(i; r), i \text{⫧} r_1, Rcpo(r_1; r), R(i),$

$r_1 \text{⏚}, i_0 \text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(i; r_{10}), IsCpo(r_1; r), i \text{☮} i_0, r_1 \text{⚲} r_{10}, i \text{⚲} i_0, r_{10}! \text{⚨} r,$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), r! \text{⚨} r_1, IsCpo(i; r), i \text{⫧} r_1, R(i), Rcpo(r_1; r),$

$r_1 \text{⏚}, i_0 \text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(i; r_{10}), IsCpo(r_1; r), r! \text{⚨} r_1, i \text{☮} i_0, r_{10}! \text{⚨} r,$

$IsCpo(i; r), r_1 \text{⚲} r_{10}, i \text{⚲} i_0, IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), i \text{⫧} r_1, R(i),$

$Rcpo(r_1; r), r_1 \text{⏚}, i_0 \text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(r_1; r), r! \text{⚨} r_1, i \text{☮} i_0, r_{10}! \text{⚨} r, IsCpo(i; r), r_1 \text{⚲} r_{10},$

$i \text{⚲} i_0, IsCpo(i; r_{10}), IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), R(i),$

$Rcpo(r_1; r), r_1 \text{⏚}, i_0 \text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(r_1; r), r! \text{⚨} r_1, i \text{☮} i_0, r_{10}! \text{⚨} r, IsCpo(i; r), r_1 \text{⚲} r_{10},$

$i \text{⚲} i_0, i \text{⫧} i_0, IsCpo(i; r_{10}), IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), R(i),$

$Rcpo(r_1;r), r_1\circledast, i_0\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\circledcirc r_1, i\otimes i_0, r_{10}!\circledcirc r, IsCpo(i;r), r_1\bigcirc r_{10},$

$i\bigcirc i_0, IsCpo(i;r_{10}), IsCpo(i_0;r_{10}), i\asymp i_0, Rcpo(i_0;r_{10}), R(i),$

$Rcpo(r_1;r), r_1\circledast, i_0\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\circledcirc r_1, i\otimes i_0, r_{10}!\circledcirc r, IsCpo(i;r), r_1\bigcirc r_{10},$

$i\bigcirc i_0, IsCpo(i;r_{10}), IsCpo(i_0;r_{10}), i\asymp i_0, Rcpo(i;r_{10}), R(i_0),$

$Rcpo(r_1;r), r_1\circledast, i_0\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\circledcirc r_1, i\otimes i_0, r_{10}!\circledcirc r, IsCpo(i;r), r_1\bigcirc r_{10},$

$i\bigcirc i_0, i\asymp i_0, IsCpo(i;r_{10}), IsCpo(i_0;r_{10}), Rcpo(i;r_{10}), R(i_0),$

$Rcpo(r_1;r), r_1\circledast, i_0\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\circledcirc r_1, i\otimes i_0, r_{10}!\circledcirc r, IsCpo(i;r), r_1\bigcirc r_{10},$

$i\bigcirc i_0, IsCpo(i;r_{10}), IsCpo(i_0;r_{10}), Rcpo(i;r_{10}), R(i_0),$

$Rcpo(r_1;r), r_1\circledast, i_0\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r=\varnothing, r!\circledcirc r_1, i\otimes i_0, r_{10}!\circledcirc r, IsCpo(i;r), r_1\bigcirc r_{10},$

$i\bigcirc i_0, IsCpo(i;r_{10}), IsCpo(i_0;r_{10}), Rcpo(i;r_{10}), R(i_0),$

$Rcpo(r_1;r), r_1\circledast, i_0\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\circledcirc r_1, i\otimes i_0, r_{10}!\circledcirc r, IsCpo(i;r), r_1\bigcirc r_{10},$

$i\bigcirc i_0, IsCpo(i;r_{10}), IsCpo(i_0;r_{10}), r=\varnothing, Rcpo(i;r_{10}), R(i_0),$

$Rcpo(r_1;r), r_1\circledast, i_0\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}^{\circ}r_1, i\text{☮}i_0, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, r=\varnothing, Rcpo(i;r_{10}), R(i_0),$

$Rcpo(r_1;r), r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}^{\circ}r_1, i\text{☮}i_0, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), r=\varnothing, R(i_0),$

$Rcpo(r_1;r), r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), i\text{☮}i_0, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), r!\mathring{\bigcirc}^{\circ}r_1, r=\varnothing, R(i_0),$

$Rcpo(r_1;r), r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), i\text{☮}i_0, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), R(i_0), r!\mathring{\bigcirc}^{\circ}r_1, r=\varnothing,$

$Rcpo(r_1;r), r_1\text{⏚}, i_0\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), i\text{☮}i_0, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), R(i_0), IsCpo(r_1;r),$

$Rcpo(r_1;r), i_0\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), i\text{☮}i_0, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), R(i_0), IsCpo(r_1;r),$

$i_0\text{⏚}, Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}^{\circ}r_1, i\text{☮}i_0, r_{10}!\mathring{\bigcirc}^{\circ}r, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), i\mathring{\bigcirc}i_0, IsCpo(i_0;r_{10}), Rcpo(i;r_{10}), R(i_0), i_0\text{⏚},$

$Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}\!\circ r_1, i\text{☮}i_0, r_{10}!\mathring{\bigcirc}\!\circ r, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), IsCpo(i;r_{10}), i\mathring{\bigcirc}i_0, Rcpo(i;r_{10}), R(i_0), i_0\text{⏚},$

$Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}\!\circ r_1, i\text{☮}i_0, r_{10}!\mathring{\bigcirc}\!\circ r, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), i\mathring{\bigcirc}i_0, Rcpo(i;r_{10}), R(i_0), i_0\text{⏚},$

$Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}\!\circ r_1, r_{10}!\mathring{\bigcirc}\!\circ r, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), i\text{☮}i_0, i\mathring{\bigcirc}i_0, Rcpo(i;r_{10}), R(i_0), i_0\text{⏚},$

$Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}\!\circ r_1, r_{10}!\mathring{\bigcirc}\!\circ r, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), i\text{☮}i_0, Rcpo(i;r_{10}), i_0\text{⏚},$

$Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}\!\circ r_1, r_{10}!\mathring{\bigcirc}\!\circ r, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\text{☮}i_0, IsCpo(i;r_{10}), Rcpo(i;r_{10}), i_0\text{⏚},$

$Rcpo(r_1;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1;r), r!\mathring{\bigcirc}\!\circ r_1, r_{10}!\mathring{\bigcirc}\!\circ r, IsCpo(i;r), r_1\mathring{\bigcirc}r_{10},$

$i\text{☮}i_0, IsCpo(i;r_{10}), i_0\text{⏚}, Rcpo(i;r_{10}),$

$Rcpo(r_1; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1; r), r!\text{⚨}r_1, r_{10}!\text{⚨}r, IsCpo(i; r), r_1\text{♂}r_{10},$

$IsCpo(i; r_{10}), i\text{☮}i_0, i_0\text{⏚}, Rcpo(i; r_{10}),$

$Rcpo(r_1; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1; r), r!\text{⚨}r_1, r_{10}!\text{⚨}r, IsCpo(i; r), r_1\text{♂}r_{10},$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}), Rcpo(r_1; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1; r), r!\text{⚨}r_1, r_{10}!\text{⚨}r, r_1\text{♂}r_{10}, IsCpo(i; r),$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}), Rcpo(r_1; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1; r), r!\text{⚨}r_1, r_1!\text{⚨}r, r_1\text{♂}r_{10}, IsCpo(i; r),$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}), Rcpo(r_1; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(r_1; r), r_1\text{♂}r_{10}, IsCpo(i; r),$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}), Rcpo(r_1; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i; r), IsCpo(i; r_{10}), IsCpo(r_1; r), r_1\text{♂}r_{10},$

$Rcpo(i; r_{10}), Rcpo(r_1; r), r_1\text{⏚}, \&Tm(r_{10}),$

$$, IsCpo(i; r), IsCpo(i; r_{10}), IsCpo(r_1; r), r_1\text{♂}r_{10},$$
$$Rcpo(i; r), i\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}), \quad \Leftrightarrow$$
$$, IsCpo(i; r), IsCpo(i; r_{10}), IsCpo(r_1; r), r_1\text{♂}r_{10},$$
$$Rcpo(i; r_{10}), Rcpo(r_1; r), i\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$$

$$, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10},$$

$$Rcpo(i;r), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}), \quad \Leftrightarrow$$

$$, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10},$$

$$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$$

induction proof:

premise 1 :

$, i = \varnothing, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, Rcpo(i;r), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, i = \varnothing, Rcpo(i;r), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, i = \varnothing,$

$Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow \; , IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, i = \varnothing,$

$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow \; , i = \varnothing, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10},$

$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

premise 2 :

$, \&SHi \rightarrow i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, Rcpo(i;r), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow \; , \&SHi \rightarrow i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10},$

$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}), \quad \Rightarrow$

$, i \mathrel{!=} \varnothing, \&SHi \mathring{\bigcirc} i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10},$

$Rcpo(i;r), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow \; , \&SHi \mathring{\bigcirc} i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10},$

$i \mathrel{!=} \varnothing, Rcpo(i;r), Rcpo(r_1;r), r_1 \textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10},$

$i\,!{=}\varnothing, Cpo(r), r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1!\bigcirc^{\circ}r, r_1\bigcirc^{\circ}r_{10},$

$Cpo(r), r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_1!\bigcirc^{\circ}r,$

$Cpo(r), r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_{10}!\bigcirc^{\circ}r,$

$Cpo(r), r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_{10}!\bigcirc^{\circ}r, IsCpo(i;r_{10}), Cpo(r),$

$r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_{10}!\bigcirc^{\circ}r, Cpo(r),$

$IsCpo(i;r_{10}), r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing, IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_{10}!\bigcirc^{\circ}r, IsCpo(i;r),\ \&SHi\,\mathring{\bigcirc}i, Cpo(r),$

$IsCpo(i;r_{10}), r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing, IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_{10}!\bigcirc^{\circ}r, IsCpo(i;r), Cpo(r),$

$\&SHi\,\mathring{\bigcirc}i, IsCpo(i;r_{10}), r\oplus, i\oplus, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\varnothing, IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_{10}!\bigcirc^{\circ}r, IsCpo(i;r), Cpo(r),$

$r\oplus, i\oplus,\ \&SHi\rightarrow i, IsCpo(i;r_{10}), Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , i \mathrel{!}= \varnothing, r_{10} \mathrel{!}\text{♂} r, Cpo(r), r\oplus, i\oplus,$

$\&SHi \to i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \text{♂} r_{10}, Rcpo(i;r), Rcpo(r_1;r), r_1\text{⦽}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i \mathrel{!}= \varnothing, r_{10} \mathrel{!}\text{♂} r, Cpo(r), r\oplus, i\oplus,$

$\&SHi \to i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \text{♂} r_{10}, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\text{⦽}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} i, i \mathrel{!}= \varnothing, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \text{♂} r_{10}, r_{10} \mathrel{!}\text{♂} r,$

$Cpo(r), r\oplus, i\oplus, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\text{⦽}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} i, i \mathrel{!}= \varnothing, IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \text{♂} r_{10}, r_{10} \mathrel{!}\text{♂} r, IsCpo(i;r),$

$Cpo(r), i\oplus, r\oplus, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\text{⦽}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} i, i \mathrel{!}= \varnothing, IsCpo(i;r_{10}), IsCpo(r_1;r), r_1 \text{♂} r_{10}, r_{10} \mathrel{!}\text{♂} r, IsCpo(i;r),$

$i\oplus, Cpo(r), r\oplus, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\text{⦽}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} i, i \mathrel{!}= \varnothing, IsCpo(r_1;r), r_1 \text{♂} r_{10}, IsCpo(i;r), i\oplus,$

$IsCpo(i;r_{10}), r_{10} \mathrel{!}\text{♂} r, Cpo(r), r\oplus, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\text{⦽}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} i, i \mathrel{!}= \varnothing, IsCpo(r_1;r), r_1 \text{♂} r_{10}, IsCpo(i;r), i \mathrel{!}\text{♂} r, i\oplus,$

$IsCpo(i;r_{10}), r_{10} \mathrel{!}\text{♂} r, Cpo(r), r\oplus, Rcpo(i;r_{10}),$

$Rcpo(r_1;r), r_1\text{⦽}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\, \mathring{\bigcirc} i, i \mathrel{!}= \varnothing, IsCpo(r_1;r), r_1 \text{♂} r_{10}, IsCpo(i;r), i\oplus,$

$IsCpo(i;r_{10}), i \mathrel{!}\text{♂} r, r_{10} \mathrel{!}\text{♂} r, Cpo(r), r\oplus, Rcpo(i;r_{10}),$

$Rcpo(r_1;r), r_1\bigcirc\!\!\!\!\!\downarrow, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(r_1;r), r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(i;r), i\oplus,$

$IsCpo(i;r_{10}), i!\mathring{\bigcirc}^{\circ}r, r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), Cpo(r), r\oplus,$

$Rcpo(r_1;r), r_1\bigcirc\!\!\!\!\!\downarrow, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(r_1;r), r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(i;r), i\oplus,$

$IsCpo(i;r_{10}), i!\mathring{\bigcirc}^{\circ}r, r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), Cpo(r), r\oplus,$

$Rcpo(r_1;r), r_1\bigcirc\!\!\!\!\!\downarrow, Cpo(r_{10}), \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(r_1;r), r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(i;r), i\oplus,$

$IsCpo(i;r_{10}), i!\mathring{\bigcirc}^{\circ}r, r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), Cpo(r), r\oplus,$

$Rcpo(r_1;r), r_1\bigcirc\!\!\!\!\!\downarrow, Cpo(r_{10}), r_{10}\oplus, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(r_1;r), r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(i;r), i\oplus,$

$IsCpo(i;r_{10}), i!\mathring{\bigcirc}^{\circ}r, r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\bigcirc\!\!\!\!\!\downarrow, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(i;r), i!\mathring{\bigcirc}^{\circ}r, i\oplus,$

$IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, IsCpo(r_1;r), Rcpo(i;r_{10}), Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\bigcirc\!\!\!\!\!\downarrow, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, i\,!\!=\varnothing, r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(i;r), i!\mathring{\bigcirc}^{\circ}r, i\oplus,$

$IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}r, Rcpo(i;r_{10}), IsCpo(r_1;r), Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\bigcirc\!\!\!\!\!\downarrow, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, r_1\mathring{\bigcirc}{}^{\circ}r_{10}, IsCpo(i;r), i\oplus,$

$IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc}{}^{\circ}r, Rcpo(i;r_{10}), IsCpo(r_1;r), Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(i;r), i\oplus, r_{10}!\mathring{\bigcirc}{}^{\circ}r,$

$IsCpo(i;r_{10}), r_1\mathring{\bigcirc}{}^{\circ}r_{10}, Rcpo(i;r_{10}), IsCpo(r_1;r), Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_1\text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(i;r), i\oplus, r_{10}!\mathring{\bigcirc}{}^{\circ}r,$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}), r_1\mathring{\bigcirc}{}^{\circ}r_{10}, IsCpo(r_1;r), Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_1\text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(i;r), i\oplus, r_{10}!\mathring{\bigcirc}{}^{\circ}r,$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}), r_{10}=\varnothing, r_1\mathring{\bigcirc}{}^{\circ}r_{10}, IsCpo(r_1;r), Cpo(r), r\oplus,$

$Rcpo(r_1;r), Cpo(r_{10}), r_{10}\oplus, r_1\oplus, r_1\text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(i;r), i\oplus, r_{10}!\mathring{\bigcirc}{}^{\circ}r,$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}), r_{10}=\varnothing, r_1\mathring{\bigcirc}{}^{\circ}r_{10}, IsCpo(r_1;r),$

$Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r), r_1\text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(i;r), i\oplus, r_{10}!\mathring{\bigcirc}{}^{\circ}r, r_1\mathring{\bigcirc}{}^{\circ}r_{10}, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}),$

$Cpo(r_{10}), r_{10}\oplus, Rcpo(r_1;r), r_1\text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!\!=\varnothing, IsCpo(i;r), i\oplus, r_{10}!\mathring{\bigcirc}{}^{\circ}r, r_1\mathring{\bigcirc}{}^{\circ}r_{10}, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), Cpo(r_{10}), r_{10}\oplus, Rcpo(i;r_{10}),$

$Rcpo(r_1;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!{=}\,\varnothing, IsCpo(i;r), r_{10}!\mathring{\bigcirc}^{\circ}r, r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), i\oplus, Cpo(r_{10}), r_{10}\oplus, Rcpo(i;r_{10}),$

$Rcpo(r_1;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, i\,!{=}\,\varnothing, IsCpo(i;r), r_{10}!\mathring{\bigcirc}^{\circ}r, r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), Cpo(r_{10}), r_{10}\oplus, i\oplus, Rcpo(i;r_{10}),$

$Rcpo(r_1;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), r_{10}!\mathring{\bigcirc}^{\circ}r, r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), i\,!{=}\,\varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, Rcpo(i;r_{10}),$

$Rcpo(r_1;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), r_{10}!\mathring{\bigcirc}^{\circ}r, r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), i\,!{=}\,\varnothing, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), r_1\mathring{\bigcirc}^{\circ}r_{10}, r_{10}!\mathring{\bigcirc}^{\circ}r, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), i\,!{=}\,\varnothing, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), r_1\mathring{\bigcirc}^{\circ}r_{10}, r_1!\mathring{\bigcirc}^{\circ}r, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), i\,!{=}\,\varnothing, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpo(i;r), r_1\mathring{\bigcirc}^{\circ}r_{10}, IsCpo(r_1;r),$

$IsCpo(i;r_{10}), i\,!\!=\varnothing, Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\mathring{\bigcirc} r_{10},$

$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$conclusion:$

$, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\mathring{\bigcirc} r_{10}, Rcpo(i;r), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\mathring{\bigcirc} r_{10},$

$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$$\begin{gathered}, IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\mathring{\bigcirc} r_{10},\\ Rcpo(i;r), Rcpo(r_1;r), i\textcircled{$\downarrow$}, r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),\quad \Leftrightarrow\\ , IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(r_1;r), r_1\mathring{\bigcirc} r_{10},\\ Rcpo(i;r_{10}), Rcpo(r_1;r), i\textcircled{$\downarrow$}, r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),\end{gathered}$$

$$\begin{gathered}, IsCpo(i;r), IsCpo(j;r), IsCpo(k;r), IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k;r_{10}), IsCpo(r_1;r),\\ , r_1\mathring{\bigcirc} r_{10}, Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpo(r_1;r), Rcpo(k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),\quad \Leftrightarrow\\ , IsCpo(i;r), IsCpo(j;r), IsCpo(k;r), IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k;r_{10}), IsCpo(r_1;r),\\ , r_1\mathring{\bigcirc} r_{10}, Rcpo(i;r), Rcpo(j;r), Rcpo(k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),\end{gathered}$$

proof:

$, IsCpo(i;r), IsCpo(j;r), IsCpo(k;r), IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k;r_{10}), IsCpo(r_1;r),$

$r_1\mathring{\bigcirc} r_{10}, Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpo(r_1;r), Rcpo(k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r), IsCpo(j;r), IsCpo(k;r),$

$IsCpo(r_1;r), r_1\mathring{\bigcirc} r_{10},$

$IsCpo(i;r_{10}), IsCpo(j;r_{10}), Rcpo(i;r_{10}), Rcpo(j;r_{10}),$

$Rcpo(r_1; r), Rcpo(k; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}), IsCpo(i; r), IsCpo(j; r), IsCpo(k; r),$

$IsCpo(r_1; r), r_1 \overset{\circ}{\bigcirc} r_{10},$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$Rcpo(r_1; r), Rcpo(k; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}), IsCpo(i; r), IsCpo(j; r), IsCpo(k; r),$

$IsCpo(r_1; r), r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r, r_1 \overset{\circ}{\bigcirc} r_{10}, r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10},$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$Rcpo(r_1; r), Rcpo(k; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}), IsCpo(i; r), IsCpo(j; r), IsCpo(k; r),$

$IsCpo(r_1; r), r_1 \overset{\circ}{\bigcirc} r_{10}, r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_1 ! \overset{\circ}{\bigcirc}{}^{\circ} r,$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$Rcpo(r_1; r), Rcpo(k; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}), IsCpo(i; r), IsCpo(j; r), IsCpo(k; r),$

$IsCpo(r_1; r), r_1 \overset{\circ}{\bigcirc} r_{10}, r_1 \overset{\circ}{\bigcirc}{}^{\circ} r_{10}, r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r,$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j;r_{10}), Rcpo(j;r_{10}),$

$Rcpo(r_1;r), Rcpo(k;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r), IsCpo(j;r), IsCpo(k;r),$

$IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_1\bigcirc^{\circ\circ}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\bigcirc^{\circ\circ}r, Rcpo(i;r_{10}),$

$IsCpo(j;r_{10}), r_{10}!\bigcirc^{\circ\circ}r, Rcpo(j;r_{10}),$

$Rcpo(r_1;r), Rcpo(k;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r), IsCpo(j;r),$

$r_1\bigcirc^{\circ}r_{10}, r_1\bigcirc^{\circ\circ}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\bigcirc^{\circ\circ}r, Rcpo(i;r_{10}),$

$IsCpo(j;r_{10}), r_{10}!\bigcirc^{\circ\circ}r, Rcpo(j;r_{10}),$

$IsCpo(r_1;r), IsCpo(k;r), Rcpo(r_1;r), Rcpo(k;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r), IsCpo(j;r),$

$r_1\bigcirc^{\circ}r_{10}, r_1\bigcirc^{\circ\circ}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\bigcirc^{\circ\circ}r, Rcpo(i;r_{10}),$

$IsCpo(j;r_{10}), r_{10}!\bigcirc^{\circ\circ}r, Rcpo(j;r_{10}),$

$IsCpo(r_1;r), IsCpo(k;r), Rcpo(k;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r), IsCpo(j;r),$

$IsCpo(r_1;r), r_1\bigcirc^{\circ}r_{10}, r_1\bigcirc^{\circ\circ}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\bigcirc^{\circ\circ}r, Rcpo(i;r_{10}),$

$IsCpo(j;r_{10}), r_{10}!\ddot{\bigcirc}r, IsCpo(k;r), Rcpo(j;r_{10}), Rcpo(k;r),$

$Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r),$

$IsCpo(r_1;r), r_1\mathring{\bigcirc}r_{10}, r_1\ddot{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\ddot{\bigcirc}r, Rcpo(i;r_{10}),$

$IsCpo(j;r_{10}), IsCpo(k;r_{10}), r_{10}!\ddot{\bigcirc}r, IsCpo(k;r), IsCpo(j;r),$

$Rcpo(j;r_{10}), Rcpo(k;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r),$

$IsCpo(r_1;r), r_1\mathring{\bigcirc}r_{10}, r_1\ddot{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\ddot{\bigcirc}r, Rcpo(i;r_{10}),$

$IsCpo(j;r_{10}), IsCpo(k;r_{10}), r_{10}!\ddot{\bigcirc}r, IsCpo(k;r), IsCpo(j;r),$

$Rcpo(k;r), Rcpo(j;r_{10}), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r),$

$r_1\mathring{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\ddot{\bigcirc}r, Rcpo(i;r_{10}),$

$IsCpo(k;r_{10}), r_{10}!\ddot{\bigcirc}r, IsCpo(k;r), Rcpo(k;r),$

$IsCpo(j;r_{10}), IsCpo(j;r), IsCpo(r_1;r), r_1\ddot{\bigcirc}r_{10},$

$Rcpo(j;r_{10}), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r),$

$r_1\mathring{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\overset{\circ\circ}{\bigcirc}r, Rcpo(i;r_{10}),$

$IsCpo(k;r_{10}), r_{10}!\overset{\circ\circ}{\bigcirc}r, IsCpo(k;r), Rcpo(k;r),$

$IsCpo(j;r_{10}), IsCpo(j;r), IsCpo(r_1;r), r_1\overset{\circ\circ}{\bigcirc}r_{10},$

$Rcpo(j;r), Rcpo(r_1;r), r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r_{10}),$

$IsCpo(j;r), IsCpo(r_1;r), r_1\mathring{\bigcirc}r_{10}, r_1\overset{\circ\circ}{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), r_{10}!\overset{\circ\circ}{\bigcirc}r, Rcpo(i;r_{10}),$

$IsCpo(k;r_{10}), r_{10}!\overset{\circ\circ}{\bigcirc}r, IsCpo(k;r), Rcpo(k;r),$

$Rcpo(j;r), Rcpo(r_1;r), r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(j;r_{10}),$

$IsCpo(j;r), IsCpo(r_1;r), r_1\mathring{\bigcirc}r_{10}, r_1\overset{\circ\circ}{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), IsCpo(i;r), r_{10}!\overset{\circ\circ}{\bigcirc}r, IsCpo(k;r_{10}), IsCpo(k;r),$

$Rcpo(i;r_{10}), Rcpo(k;r),$

$Rcpo(j;r), Rcpo(r_1;r), r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(j;r_{10}),$

$IsCpo(j;r), IsCpo(r_1;r), r_1\mathring{\bigcirc}r_{10}, r_1\overset{\circ\circ}{\bigcirc}r_{10},$

$IsCpo(i;r_{10}), IsCpo(i;r), r_{10}!\overset{\circ\circ}{\bigcirc}r, IsCpo(k;r_{10}), IsCpo(k;r),$

$Rcpo(k;r), Rcpo(i;r_{10}),$

$Rcpo(j;r), Rcpo(r_1;r), r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(j;r_{10}),$

$IsCpo(j;r), IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, r_1 \mathring{\bigcirc}{\circ} r_{10},$

$IsCpo(i;r_{10}), IsCpo(i;r),$

$IsCpo(k;r), r_{10}! \mathring{\bigcirc}{\circ} r, Rcpo(k;r),$

$Rcpo(i;r_{10}),$

$Rcpo(j;r), Rcpo(r_1;r), r_1 \bigcirc\kern-0.8em\downarrow, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}),$

$IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, r_1 \mathring{\bigcirc}{\circ} r_{10},$

$IsCpo(k;r), r_{10}! \mathring{\bigcirc}{\circ} r, Rcpo(k;r),$

$IsCpo(i;r_{10}), IsCpo(i;r), r_{10}! \mathring{\bigcirc}{\circ} r, IsCpo(j;r), IsCpo(j;r_{10}),$

$Rcpo(i;r_{10}), Rcpo(j;r), Rcpo(r_1;r), r_1 \bigcirc\kern-0.8em\downarrow, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}),$

$IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, r_1 \mathring{\bigcirc}{\circ} r_{10},$

$IsCpo(k;r), r_{10}! \mathring{\bigcirc}{\circ} r, Rcpo(k;r),$

$IsCpo(i;r_{10}), IsCpo(i;r), r_{10}! \mathring{\bigcirc}{\circ} r, IsCpo(j;r), IsCpo(j;r_{10}),$

$Rcpo(j;r), Rcpo(i;r_{10}), Rcpo(r_1;r), r_1 \bigcirc\kern-0.8em\downarrow, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}),$

$IsCpo(r_1;r), r_1 \mathring{\bigcirc} r_{10}, r_1 \mathring{\bigcirc}{\circ} r_{10},$

$IsCpo(k;r), r_{10}! \mathring{\bigcirc}{\circ} r, Rcpo(k;r),$

$IsCpo(i;r), IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(j;r), r_{10}! \mathring{\bigcirc}{\circ} r, Rcpo(j;r),$

$Rcpo(i;r_{10}), Rcpo(r_1;r), r_1 \bigcirc\kern-0.8em\downarrow, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}),$

$r_1 \mathring{\bigcirc} r_{10}, r_1 \mathring{\bigcirc}\!\circ r_{10},$

$IsCpo(k; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(k; r),$

$IsCpo(j; r_{10}), IsCpo(j; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(j; r),$

$IsCpo(i; r), IsCpo(i; r_{10}), r_1 \mathring{\bigcirc}\!\circ r_{10}, IsCpo(r_1; r), Rcpo(i; r_{10}), Rcpo(r_1; r), r_1 \text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}),$

$r_1 \mathring{\bigcirc} r_{10},$

$IsCpo(k; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(k; r),$

$IsCpo(j; r_{10}), IsCpo(j; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(j; r),$

$IsCpo(i; r), IsCpo(i; r_{10}), r_1 \mathring{\bigcirc}\!\circ r_{10}, IsCpo(r_1; r), Rcpo(i; r), Rcpo(r_1; r), r_1 \text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}),$

$r_1 \mathring{\bigcirc} r_{10}, r_1 \mathring{\bigcirc}\!\circ r_{10},$

$IsCpo(k; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(k; r),$

$IsCpo(j; r_{10}), IsCpo(j; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(j; r),$

$IsCpo(i; r), IsCpo(i; r_{10}), IsCpo(r_1; r), Rcpo(i; r), Rcpo(r_1; r), r_1 \text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}), r_1 \mathring{\bigcirc} r_{10},$

$IsCpo(k; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(k; r),$

$IsCpo(j; r_{10}), IsCpo(j; r), r_{10}! \mathring{\bigcirc}\!\circ r, Rcpo(j; r),$

$IsCpo(i; r), IsCpo(i; r_{10}), IsCpo(r_1; r), Rcpo(i; r), Rcpo(r_1; r), r_1 \text{\textcircled{$\downarrow$}}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k; r_{10}), IsCpo(i; r_{10}), IsCpo(j; r_{10}), r_1 \mathring{\bigcirc} r_{10},$

$IsCpo(k;r), r_{10}!\mathring{\bigcirc}r, Rcpo(k;r),$

$IsCpo(j;r), r_{10}!\mathring{\bigcirc}r, Rcpo(j;r),$

$IsCpo(i;r), IsCpo(r_1;r), Rcpo(i;r), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\mathring{\bigcirc}r_{10}, r_{10}!\mathring{\bigcirc}r,$

$IsCpo(k;r), Rcpo(k;r),$

$IsCpo(j;r), Rcpo(j;r),$

$IsCpo(i;r), IsCpo(r_1;r), Rcpo(i;r), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\mathring{\bigcirc}r_{10}, r_1!\mathring{\bigcirc}r,$

$IsCpo(k;r), Rcpo(k;r),$

$IsCpo(j;r), Rcpo(j;r),$

$IsCpo(i;r), IsCpo(r_1;r), Rcpo(i;r), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\mathring{\bigcirc}r_{10}, r_1!\mathring{\bigcirc}r,$

$IsCpo(r_1;r), IsCpo(k;r), Rcpo(k;r),$

$IsCpo(j;r), Rcpo(j;r),$

$IsCpo(i;r), Rcpo(i;r), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\mathring{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1!\mathring{\bigcirc}r, Rcpo(k;r),$

$IsCpo(j;r), r_1!\mathring{\bigcirc}r, Rcpo(j;r),$

$IsCpo(i;r), r_1!\mathring{\bigcirc}r, Rcpo(i;r), Rcpo(r_1;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_{10}=\varnothing, r_1\mathring{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1!\mathring{\bigcirc}r, Rcpo(k;r),$

$IsCpo(j;r), r_1!\mathring{\bigcirc}r, Rcpo(j;r),$

$IsCpo(i;r), r_1!\mathring{\bigcirc}r, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1 = \varnothing, r_1\mathring{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1!\mathring{\bigcirc}r, Rcpo(k;r),$

$IsCpo(j;r), r_1!\mathring{\bigcirc}r, Rcpo(j;r),$

$IsCpo(i;r), r_1!\mathring{\bigcirc}r, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\mathring{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1!\mathring{\bigcirc}r, r_1 = \varnothing, Rcpo(k;r),$

$IsCpo(j;r), r_1!\mathring{\bigcirc}r, Rcpo(j;r),$

$IsCpo(i;r), r_1!\mathring{\bigcirc}r, Rcpo(i;r), Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\mathring{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1!\mathring{\bigcirc}r, Rcpo(k;r),$

$IsCpo(j;r), r_1!\mathring{\bigcirc}r, Rcpo(j;r),$

$IsCpo(i;r), r_1!\mathring{\bigcirc}r, Rcpo(i;r), r_1 = \varnothing, Rcpo(r_1;r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\mathring{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1!\mathring{\bigcirc}r, Rcpo(k;r),$

$IsCpo(j;r), r_1!\mathring{\bigcirc}r, Rcpo(j;r),$

$IsCpo(i;r), r_1!\mathring{\bigcirc}r, Rcpo(i;r), r_1 = \varnothing, r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1 = \varnothing, r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1 ! \mathring{\bigcirc} r, Rcpo(k;r),$

$IsCpo(j;r), r_1 ! \mathring{\bigcirc} r, Rcpo(j;r),$

$IsCpo(i;r), r_1 ! \mathring{\bigcirc} r, Rcpo(i;r), r_1 \circledast, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), r_1 ! \mathring{\bigcirc} r, Rcpo(k;r),$

$IsCpo(j;r), r_1 ! \mathring{\bigcirc} r, Rcpo(j;r),$

$IsCpo(i;r), r_1 ! \mathring{\bigcirc} r, Rcpo(i;r), r_1 \circledast, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1;r), r_1 ! \mathring{\bigcirc} r,$

$IsCpo(k;r), Rcpo(k;r),$

$IsCpo(j;r), Rcpo(j;r),$

$IsCpo(i;r), Rcpo(i;r), r_1 \circledast, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), Rcpo(k;r),$

$IsCpo(j;r), Rcpo(j;r),$

$IsCpo(i;r), Rcpo(i;r), r_1 \circledast, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), IsCpo(j;r), Rcpo(k;r), Rcpo(j;r),$

$IsCpo(i;r), Rcpo(i;r), r_1 \circledast, \& Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1 \mathring{\bigcirc} r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), IsCpo(j;r), Rcpo(j;r), Rcpo(k;r),$

$IsCpo(i;r), Rcpo(i;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\overset{\circ}{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(j;r), Rcpo(j;r), IsCpo(k;r), Rcpo(k;r),$

$IsCpo(i;r), Rcpo(i;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\overset{\circ}{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(j;r), Rcpo(j;r), IsCpo(k;r), IsCpo(i;r), Rcpo(k;r), Rcpo(i;r),$

$r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\overset{\circ}{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), IsCpo(i;r), IsCpo(j;r), Rcpo(j;r), Rcpo(i;r), Rcpo(k;r),$

$r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(k;r_{10}), IsCpo(i;r_{10}), IsCpo(j;r_{10}), r_1\overset{\circ}{\bigcirc}r_{10}, IsCpo(r_1;r),$

$IsCpo(k;r), IsCpo(i;r), IsCpo(j;r), Rcpo(i;r), Rcpo(j;r), Rcpo(k;r),$

$r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpo(i;r), IsCpo(j;r), IsCpo(k;r), IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k;r_{10}), IsCpo(r_1;r),$

$r_1\overset{\circ}{\bigcirc}r_{10}, Rcpo(i;r), Rcpo(j;r), Rcpo(k;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

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$$, IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\overset{\circ}{\bigcirc}\!{\circ}\, r_{20}, i_2!\overset{\circ}{\bigcirc}\!{\circ}\, r_{10}, r_1\overset{\circ}{\bigcirc}\!{\circ}\, r_{10}, r_2\overset{\circ}{\bigcirc}\!{\circ}\, r_{20}, r_{10}!\overset{\circ}{\bigcirc}\!{\circ}\, r_{20},$$

$$, i_1 = i_2, r_1 = r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), \quad \Leftrightarrow \quad \sim, r_1 = r_2,$$

induction proof:
premise 1 :
$, i_1 = \varnothing, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, r_1 = r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, r_1 = r_2, i_1 = \varnothing, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, r_1 = r_2, i_1 = \varnothing, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, i_1 = \varnothing, r_1 = r_2, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, i_2 = \varnothing, r_1 = r_2, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, r_1 = r_2, i_2 = \varnothing, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, r_1 = r_2, i_2 = \varnothing,$

$\Leftrightarrow \; , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \bigcirc r_{20}, i_2 ! \bigcirc r_{10}, r_{10} ! \bigcirc r_{20},$

$r_1 \bigcirc r_{10}, r_2 \bigcirc r_{20}, i_1 = i_2, r_1 = r_2, r_1 = r_2, i_2 = \varnothing,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, r_1\mathrel{\text{⌶}}r_2, i_2=\varnothing, r_1\mathrel{\text{⌶}}r_2,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, r_1\mathrel{\text{⌶}}r_2, i_2=\varnothing, Rcpo(i_2;r_{20}), r_1\mathrel{\text{⌶}}r_2,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, i_2=\varnothing, r_1\mathrel{\text{⌶}}r_2, Rcpo(i_2;r_{20}), r_1\mathrel{\text{⌶}}r_2,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, i_1=\varnothing, r_1\mathrel{\text{⌶}}r_2, Rcpo(i_2;r_{20}), r_1\mathrel{\text{⌶}}r_2,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, r_1\mathrel{\text{⌶}}r_2, i_1=\varnothing, Rcpo(i_2;r_{20}), r_1\mathrel{\text{⌶}}r_2,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, r_1\mathrel{\text{⌶}}r_2, i_1=\varnothing, Rcpo(i_1;r_{10}), Rcpo(i_2;r_{20}), r_1\mathrel{\text{⌶}}r_2,$

$\Leftrightarrow\ , i_1=\varnothing, IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, r_1\mathrel{\text{⌶}}r_2, Rcpo(i_1;r_{10}), Rcpo(i_2;r_{20}), r_1\mathrel{\text{⌶}}r_2,$

premise 2 :
$, \&SHi\rightarrow i_1, IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, r_1\mathrel{\text{⌶}}r_2, Rcpo(i_1;r_{10}), Rcpo(i_2;r_{20}),$

$\Leftrightarrow\ , \&SHi\rightarrow i_1, IsCpo(i_1;r_{10}), IsCpo(i_2;r_{20}), i_1!\bigcirc r_{20}, i_2!\bigcirc r_{10}, r_{10}!\bigcirc r_{20},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1\mathrel{\text{⌶}}i_2, r_1\mathrel{\text{⌶}}r_2, Rcpo(i_1;r_{10}), Rcpo(i_2;r_{20}),\ \Rightarrow$

$, i_1 \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} i, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_1 \mathrel{\overline{\perp}} i_2, r_1 \mathrel{\overline{\perp}} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}),$

$i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_1 \mathrel{\overline{\perp}} i_2, r_1 \mathrel{\overline{\perp}} r_2,$

$i_1 \mathrel{!=} \varnothing, Rcpo(i_1; r_{10}),$

$Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}),$

$i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_1 \mathrel{\overline{\perp}} i_2, r_1 \mathrel{\overline{\perp}} r_2,$

$i_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10} \oplus, i_1 \oplus, Rcpo(i_1; r_{10}),$

$Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_1 \mathrel{\overline{\perp}} i_2, i_1 \mathrel{!=} \varnothing, r_1 \mathrel{\overline{\perp}} r_2,$

$i_2 ! \mathring{\bigcirc}^{\circ} r_{10}, Cpo(r_{10}), r_{10} \oplus, i_1 \oplus,$

$IsCpo(i_1; r_{10}), Rcpo(i_1; r_{10}),$

$Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1{=}i_2, i_2 \,!{=}\, \varnothing, r_1{=}r_2,$

$i_2! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), Rcpo(i_1; r_{10}),$

$Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$i_1! \mathring{\bigcirc} r_{20}, r_{10}! \mathring{\bigcirc} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1{=}i_2, r_1{=}r_2,$

$i_2! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), i_2 \,!{=}\, \varnothing, Rcpo(i_1; r_{10}),$

$Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$i_1! \mathring{\bigcirc} r_{20}, r_{10}! \mathring{\bigcirc} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1{=}i_2, r_1{=}r_2,$

$i_2! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), i_2 \,!{=}\, \varnothing, Rcpo(i_1; r_{10}),$

$i_2 \,!{=}\, \varnothing, Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$i_1! \mathring{\bigcirc} r_{20}, r_{10}! \mathring{\bigcirc} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1{=}i_2, r_1{=}r_2,$

$i_2! \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), i_2 \, != \varnothing, Rcpo(i_1; r_{10}),$

$i_2 \, != \varnothing, Cpo(r_{20}), r_{20}\oplus, i_2\oplus, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \ , \&SHi \, \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$i_1 ! \mathring{\bigcirc}^{\!\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\!\circ} r_{20},$

$r_1 \mathring{\bigcirc}^{\!\circ} r_{10}, r_2 \mathring{\bigcirc}^{\!\circ} r_{20}, i_1 \text{⌶} i_2, i_2 \, != \varnothing, r_1 \text{⌶} r_2,$

$i_2 ! \mathring{\bigcirc}^{\!\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), Rcpo(i_1; r_{10}),$

$, Cpo(r_{20}), r_{20}\oplus, i_2\oplus, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \ , \&SHi \, \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$r_1 \mathring{\bigcirc}^{\!\circ} r_{10}, r_2 \mathring{\bigcirc}^{\!\circ} r_{20}, i_1 \text{⌶} i_2, i_2 \, != \varnothing, r_1 \text{⌶} r_2,$

$i_2 ! \mathring{\bigcirc}^{\!\circ} r_{10}, r_{10} ! \mathring{\bigcirc}^{\!\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), i_1 ! \mathring{\bigcirc}^{\!\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\!\circ} r_{20}, Rcpo(i_1; r_{10}),$

$, Cpo(r_{20}), r_{20}\oplus, i_2\oplus, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \ , \&SHi \, \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$r_1 \mathring{\bigcirc}^{\!\circ} r_{10}, r_2 \mathring{\bigcirc}^{\!\circ} r_{20}, i_1 \text{⌶} i_2, i_2 \, != \varnothing, r_1 \text{⌶} r_2,$

$i_2 ! \mathring{\bigcirc}^{\!\circ} r_{10}, r_{10} ! \mathring{\bigcirc}^{\!\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), i_1 ! \mathring{\bigcirc}^{\!\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\!\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus,$

$Rcpo(i_1; r_{10}), i_2\oplus, Rcpo(i_2; r_{20}),$

$\Leftrightarrow \ , \&SHi \, \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$r_1 \mathring{\bigcirc}^{\!\circ} r_{10}, r_2 \mathring{\bigcirc}^{\!\circ} r_{20}, i_1 \text{⌶} i_2, i_2 \, != \varnothing, r_1 \text{⌶} r_2,$

$r_{10}!\circlearrowright r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$i_1!\circlearrowright r_{20}, r_{10}!\circlearrowright r_{20}, IsCpo(i_1; r_{10}), Cpo(r_{20}), r_{20}\oplus,$

$i_2!\circlearrowright r_{10}, Rcpo(i_1; r_{10}), i_2\oplus, Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \circlearrowright i_1, IsCpo(i_2; r_{20}),$

$r_1\circlearrowright r_{10}, r_2\circlearrowright r_{20}, i_1{=}i_2, i_2 \mathrel{!=} \varnothing, r_1{=}r_2,$

$r_{10}!\circlearrowright r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$i_1!\circlearrowright r_{20}, r_{10}!\circlearrowright r_{20}, Cpo(r_{20}), r_{20}\oplus,$

$IsCpo(i_1; r_{10}), i_2!\circlearrowright r_{10}, Rcpo(i_1; r_{10}), i_2\oplus, Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \circlearrowright i_1, IsCpo(i_2; r_{20}),$

$r_1\circlearrowright r_{10}, r_2\circlearrowright r_{20}, i_1{=}i_2, i_2 \mathrel{!=} \varnothing, r_1{=}r_2,$

$r_{10}!\circlearrowright r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$i_1!\circlearrowright r_{20}, r_{10}!\circlearrowright r_{20}, Cpo(r_{20}), r_{20}\oplus,$

$IsCpo(i_1; r_{10}), i_2!\circlearrowright r_{10}, i_2\oplus, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \circlearrowright i_1, IsCpo(i_2; r_{20}),$

$r_1\circlearrowright r_{10}, r_2\circlearrowright r_{20}, i_1{=}i_2, i_2 \mathrel{!=} \varnothing, r_1{=}r_2,$

$r_{10}!\circlearrowright r_{20}, Cpo(r_{10}), r_{10}\oplus,$

$i_1!\circlearrowright r_{20}, r_{10}!\circlearrowright r_{20}, i_1\oplus, Cpo(r_{20}), r_{20}\oplus, i_2\oplus,$

$IsCpo(i_1; r_{10}), i_2!\circlearrowright r_{10}, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \circlearrowright i_1, IsCpo(i_2; r_{20}),$

$r_1\circlearrowright r_{10}, r_2\circlearrowright r_{20}, i_1{=}i_2, i_2 \mathrel{!=} \varnothing, r_1{=}r_2,$

$r_{10}!\bigcirc r_{20}, Cpo(r_{10}), r_{10}\oplus,$

$i_1!\bigcirc r_{20}, r_{10}!\bigcirc r_{20}, Cpo(r_{20}), r_{20}\oplus, i_1\oplus, i_2\oplus,$

$IsCpo(i_1; r_{10}), i_2!\bigcirc r_{10}, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi\,\bigcirc i_1, IsCpo(i_2; r_{20}), i_2!\bigcirc r_{10},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1{=}i_2, i_2 \mathrel{!{=}} \varnothing, r_1{=}r_2,$

$IsCpo(i_1; r_{10}), r_{10}!\bigcirc r_{20}, Cpo(r_{10}),$

$i_1!\bigcirc r_{20}, r_{10}!\bigcirc r_{20}, r_{10}\oplus, Cpo(r_{20}), r_{20}\oplus,$

$i_1\oplus, i_2\oplus, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi\,\bigcirc i_1, IsCpo(i_2; r_{20}), i_2!\bigcirc r_{10},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1{=}i_2, i_2 \mathrel{!{=}} \varnothing, r_1{=}r_2,$

$IsCpo(i_1; r_{10}), r_{10}!\bigcirc r_{20}, Cpo(r_{10}),$

$i_1!\bigcirc r_{20}, r_{10}!\bigcirc r_{20}, Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi\,\bigcirc i_1, IsCpo(i_2; r_{20}), i_2!\bigcirc r_{10},$

$r_1\bigcirc r_{10}, r_2\bigcirc r_{20}, i_1{=}i_2, i_2 \mathrel{!{=}} \varnothing, r_1{=}r_2,$

$IsCpo(i_1; r_{10}), r_{10}!\bigcirc r_{20}, i_1!\bigcirc r_{20}, r_{10}!\bigcirc r_{20},$

$Cpo(r_{10}), Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi\,\bigcirc i_1, IsCpo(i_2; r_{20}), i_2!\bigcirc r_{10},$

$i_1{=}i_2, i_2 \mathrel{!{=}} \varnothing, IsCpo(i_1; r_{10}), r_{10}!\bigcirc r_{20}, i_1!\bigcirc r_{20},$

$r_1 \vdash\!\dashv r_2, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_{10}), Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}), i_2 ! \circledcirc r_{10},$

$i_1 \vdash\!\dashv i_2, i_2 \mathrel{!=} \varnothing, IsCpo(i_1; r_{10}), r_{10} ! \circledcirc r_{20}, i_1 ! \circledcirc r_{20},$

$r_1 \vdash\!\dashv r_2, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, r_{10} = \varnothing, r_{20} = \varnothing,$

$Cpo(r_{10}), Cpo(r_{20}), r_1 \vdash\!\dashv r_2,$

$r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

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$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}), i_2 ! \circledcirc r_{10},$

$i_1 \vdash\!\dashv i_2, i_2 \mathrel{!=} \varnothing, IsCpo(i_1; r_{10}), r_{10} ! \circledcirc r_{20}, i_1 ! \circledcirc r_{20},$

$r_1 \vdash\!\dashv r_2, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, r_{10} ! \circledcirc r_{20},$

$Cpo(r_{10}), Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus,$

$r_1 \vdash\!\dashv r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} i_1, IsCpo(i_2; r_{20}),$

$i_1 \vdash\!\dashv i_2, i_2 \mathrel{!=} \varnothing, i_1 ! \circledcirc r_{20},$

$r_1 \vdash\!\dashv r_2, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20}, Cpo(r_{10}),$

$r_{10} ! \circledcirc r_{20}, Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus,$

$r_1 \mathrel{⌶} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} i_1,$

$i_1 \mathrel{⌶} i_2, i_2 \,!\!= \varnothing,$

$r_1 \mathrel{⌶} r_2, r_1 \bigcirc\!\!\circ\, r_{10}, r_2 \bigcirc\!\!\circ\, r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \bigcirc\!\!\circ\, r_{10}, r_{10} ! \bigcirc\!\!\circ\, r_{20}, Cpo(r_{10}),$

$IsCpo(i_2; r_{20}), i_1 ! \bigcirc\!\!\circ\, r_{20}, r_{10} ! \bigcirc\!\!\circ\, r_{20}, Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus,$

$r_1 \mathrel{⌶} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} i_1,$

$r_1 \mathrel{⌶} r_2, r_1 \bigcirc\!\!\circ\, r_{10}, r_2 \bigcirc\!\!\circ\, r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \bigcirc\!\!\circ\, r_{10}, r_{10} ! \bigcirc\!\!\circ\, r_{20}, Cpo(r_{10}),$

$IsCpo(i_2; r_{20}), i_1 ! \bigcirc\!\!\circ\, r_{20}, r_{10} ! \bigcirc\!\!\circ\, r_{20}, Cpo(r_{20}),$

$i_1 \mathrel{⌶} i_2, i_2 \,!\!= \varnothing, r_{10}\oplus, r_{20}\oplus, i_1\oplus, i_2\oplus,$

$r_1 \mathrel{⌶} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} i_1,$

$r_1 \mathrel{⌶} r_2, r_1 \bigcirc\!\!\circ\, r_{10}, r_2 \bigcirc\!\!\circ\, r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \bigcirc\!\!\circ\, r_{10}, r_{10} ! \bigcirc\!\!\circ\, r_{20}, Cpo(r_{10}),$

$IsCpo(i_2; r_{20}), i_1 ! \bigcirc\!\!\circ\, r_{20}, r_{10} ! \bigcirc\!\!\circ\, r_{20}, Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1 \,!\!= \varnothing, i_2 \,!\!= \varnothing, i_1\oplus, i_2\oplus, i_1 \mathrel{⌶} i_2,$

$r_1 \mathrel{⌶} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathrel{\bigcirc^{\circ\circ}} r_{10}, r_2 \mathrel{\bigcirc^{\circ\circ}} r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \mathrel{\bigcirc^{\circ\circ}} r_{10}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{10}),$

$IsCpo(i_2; r_{20}), i_1 ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{20}),\ \&SHi \mathrel{\bigcirc^{\circ}} i_1,$

$r_{10}\oplus, r_{20}\oplus, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus,$

$i_1 \mathrel{\text{⌶}} i_2, r_1 \mathrel{\text{⌶}} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathrel{\bigcirc^{\circ\circ}} r_{10}, r_2 \mathrel{\bigcirc^{\circ\circ}} r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \mathrel{\bigcirc^{\circ\circ}} r_{10}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{10}),$

$IsCpo(i_2; r_{20}), i_1 ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus,$

$\&SHi \rightarrow i_1,$

$i_1 \mathrel{\text{⌶}} i_2, r_1 \mathrel{\text{⌶}} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2, r_1 \mathrel{\bigcirc^{\circ\circ}} r_{10}, r_2 \mathrel{\bigcirc^{\circ\circ}} r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \mathrel{\bigcirc^{\circ\circ}} r_{10}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{10}), r_{10}\oplus,$

$IsCpo(i_2; r_{20}), i_1 ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{20}),$

$r_{20}\oplus, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus,$

$\&SHi \rightarrow i_1,$

$i_1 \mathrel{\text{⌶}} i_2, r_1 \mathrel{\text{⌶}} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , r_1 \mathrel{\text{⌶}} r_2,$

$IsCpo(i_1; r_{10}), i_2 ! \mathrel{\bigcirc^{\circ\circ}} r_{10}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{10}), r_{10}\oplus,$

$IsCpo(i_2; r_{20}), i_1 ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, r_{10} ! \mathrel{\bigcirc^{\circ\circ}} r_{20}, Cpo(r_{20}),$

$r_{20}\oplus, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus,$

$\&SHi \rightarrow i_1, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}),$

$i_1 ! \circledcirc r_{20}, i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20},$

$r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, i_1 \mathrel{\overline{\bot}} i_2, r_1 \mathrel{\overline{\bot}} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , r_1 \mathrel{\overline{\bot}} r_2,$

$IsCpo(i_1; r_{10}), i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20}, Cpo(r_{10}), r_{10} \oplus,$

$IsCpo(i_2; r_{20}), i_1 ! \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, Cpo(r_{20}),$

$r_{20} \oplus, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1 \oplus, i_2 \oplus,$

$\&SHi \rightarrow i_1, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}),$

$i_1 ! \circledcirc r_{20}, i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20},$

$r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, i_1 \mathrel{\overline{\bot}} i_2, r_1 \mathrel{\overline{\bot}} r_2,$

$Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 \mathrel{\overline{\bot}} r_2,$

$\Leftrightarrow\ , r_1 \mathrel{\overline{\bot}} r_2, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20}, Cpo(r_{10}), r_{10} \oplus,$

$IsCpo(i_2; r_{20}), i_1 ! \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, Cpo(r_{20}),$

$r_{20} \oplus, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1 \oplus, i_2 \oplus,$

$\&SHi \rightarrow i_1, i_1 \mathrel{\overline{\bot}} i_2, r_1 \mathrel{\overline{\bot}} r_2,$

$Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 \mathrel{\overline{\bot}} r_2,$

$\Leftrightarrow\ , r_1 \mathrel{\overline{\bot}} r_2, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20},$

$IsCpo(i_1; r_{10}), i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20}, Cpo(r_{10}),$

$IsCpo(i_2; r_{20}), i_1 ! \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, Cpo(r_{20}),$

$r_{10}\oplus, r_{20}\oplus, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus,$

$\&SHi \rightarrow i_1, i_1 \mathrel{\underline{\top}} i_2, r_1 \mathrel{\underline{\top}} r_2,$

$Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 \mathrel{\underline{\top}} r_2,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}\!{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}\!{}^{\circ} r_{20},$

$r_1 \mathrel{\underline{\top}} r_2, i_1 \mathrel{\underline{\top}} i_2, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing,$

$r_{10}! \mathring{\bigcirc}\!{}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_2; r_{20}), i_1! \mathring{\bigcirc}\!{}^{\circ} r_{20}, r_{10}! \mathring{\bigcirc}\!{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus,$

$IsCpo(i_1; r_{10}), i_2! \mathring{\bigcirc}\!{}^{\circ} r_{10}, i_2\oplus,$

$Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 \mathrel{\underline{\top}} r_2,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}\!{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}\!{}^{\circ} r_{20},$

$r_1 \mathrel{\underline{\top}} r_2, i_1 \mathrel{\underline{\top}} i_2, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing,$

$r_{10}! \mathring{\bigcirc}\!{}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_2; r_{20}), i_1! \mathring{\bigcirc}\!{}^{\circ} r_{20}, r_{10}! \mathring{\bigcirc}\!{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus,$

$IsCpo(i_1; r_{10}), i_2! \mathring{\bigcirc}\!{}^{\circ} r_{10},$

$Rcpo(i_1; r_{10}), i_2\oplus, Rcpo(i_2; r_{20}), r_1 \mathrel{\underline{\top}} r_2,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}\!{}^{\circ} r_{10}, r_2 \mathring{\bigcirc}\!{}^{\circ} r_{20}, i_2! \mathring{\bigcirc}\!{}^{\circ} r_{10},$

$IsCpo(i_2; r_{20}), r_1 \mathrel{\underline{\top}} r_2, i_1 \mathrel{\underline{\top}} i_2, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing,$

$r_{10}! \mathring{\bigcirc}\!{}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, i_1\oplus,$

$IsCpo(i_1; r_{10}), i_1! \mathring{\bigcirc}\!{}^{\circ} r_{20}, r_{10}! \mathring{\bigcirc}\!{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus,$

$Rcpo(i_1; r_{10}), i_2\oplus, Rcpo(i_2; r_{20}), r_1 \mathrel{\underline{\top}} r_2,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10},$

$IsCpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2, i_1 \mathrel{\top\!\!\!\perp} i_2, i_1 \mathrel{!=} \varnothing, i_2 \mathrel{!=} \varnothing,$

$r_{10} ! \mathring{\bigcirc}^{\circ} r_{20}, Cpo(r_{10}), r_{10} \oplus, i_1 \oplus,$

$IsCpo(i_1; r_{10}), i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20},$

$Rcpo(i_1; r_{10}), Cpo(r_{20}), r_{20} \oplus, i_2 \oplus, Rcpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10},$

$IsCpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2, i_1 \mathrel{\top\!\!\!\perp} i_2, i_2 \mathrel{!=} \varnothing,$

$IsCpo(i_1; r_{10}), i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20},$

$i_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10} \oplus, i_1 \oplus,$

$Rcpo(i_1; r_{10}), Cpo(r_{20}), r_{20} \oplus, i_2 \oplus, Rcpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10},$

$IsCpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2, i_1 \mathrel{\top\!\!\!\perp} i_2, i_2 \mathrel{!=} \varnothing,$

$IsCpo(i_1; r_{10}), i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20}, i_1 \mathrel{!=} \varnothing,$

$Rcpo(i_1; r_{10}), Cpo(r_{20}), r_{20} \oplus, i_2 \oplus, Rcpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10},$

$IsCpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2, i_1 \mathrel{\top\!\!\!\perp} i_2, i_1 \mathrel{!=} \varnothing,$

$i_1 ! \mathring{\bigcirc}^{\circ} r_{20}, r_{10} ! \mathring{\bigcirc}^{\circ} r_{20}, IsCpo(i_1; r_{10}), i_2 \mathrel{!=} \varnothing,$

$Rcpo(i_1; r_{10}), Cpo(r_{20}), r_{20} \oplus, i_2 \oplus, Rcpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i_1, r_1 \mathring{\bigcirc}^{\circ} r_{10}, r_2 \mathring{\bigcirc}^{\circ} r_{20}, i_2 ! \mathring{\bigcirc}^{\circ} r_{10},$

$IsCpo(i_2; r_{20}), r_1 \mathrel{\top\!\!\!\perp} r_2, i_1 \mathrel{\top\!\!\!\perp} i_2, i_1 \mathrel{!=} \varnothing,$

$i_1 ! \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, IsCpo(i_1; r_{10}), i_2 != \varnothing,$

$Rcpo(i_1; r_{10}), i_2 != \varnothing, Cpo(r_{20}), r_{20}\oplus, i_2\oplus, Rcpo(i_2; r_{20}), r_1 = r_2,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, i_2 ! \circledcirc r_{10},$

$IsCpo(i_2; r_{20}), r_1 = r_2, i_1 = i_2, i_1 != \varnothing,$

$i_1 ! \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, IsCpo(i_1; r_{10}), i_2 != \varnothing,$

$Rcpo(i_1; r_{10}), i_2 != \varnothing, Rcpo(i_2; r_{20}), r_1 = r_2,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, i_2 ! \circledcirc r_{10},$

$IsCpo(i_2; r_{20}), r_1 = r_2, i_1 = i_2, i_2 != \varnothing, i_1 != \varnothing,$

$i_1 ! \circledcirc r_{20}, r_{10} ! \circledcirc r_{20}, IsCpo(i_1; r_{10}),$

$Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 = r_2,$

$\Leftrightarrow\ , i_1 != \varnothing,\ \&SHi \mathring{\bigcirc} i_1, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}),$

$i_1 ! \circledcirc r_{20}, i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20}, r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20},$

$r_1 = r_2, i_1 = i_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 = r_2,$

$conclusion:$

$, IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \circledcirc r_{20}, i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20},$

$r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, i_1 = i_2, r_1 = r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1; r_{10}), IsCpo(i_2; r_{20}), i_1 ! \circledcirc r_{20}, i_2 ! \circledcirc r_{10}, r_{10} ! \circledcirc r_{20},$

$r_1 \circledcirc r_{10}, r_2 \circledcirc r_{20}, i_1 = i_2, r_1 = r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 = r_2,$

$$, IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(j_1; r_{20}),$$
$$r_{10} ! \circledcirc r_{20}, i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20},$$

$$Rcpo(i_1;r_{10}), Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}), \quad \Leftrightarrow \quad \sim, r_1 \mathrel{\overline{\bot}} r_2,$$

proof:

$, IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$r_{10} ! \mathring{\bigcirc} r_{20}, i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20},$

$Rcpo(i_1;r_{10}), Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20},$

$IsCpo(i_2;r_{10}), IsCpo(j_2;r_{10}), IsCpo(i_2;r_{20}), IsCpo(j_2;r_{20}),$

$r_{10} ! \mathring{\bigcirc} r_{20}, Rcpo(i_1;r_{10}),$

$Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$IsCpo(i_2;r_{10}), IsCpo(j_2;r_{10}), IsCpo(i_2;r_{20}), IsCpo(j_2;r_{20}),$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20},$

$IsCpo(i_1;r_{10}), r_{10} ! \mathring{\bigcirc} r_{20}, Rcpo(i_1;r_{10}),$

$Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}),$

$\Leftrightarrow \; , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$IsCpo(i_2;r_{10}), IsCpo(j_2;r_{10}), IsCpo(i_2;r_{20}), IsCpo(j_2;r_{20}),$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20},$

$IsCpo(i_1;r_{10}), r_{10} ! \mathring{\bigcirc} r_{20}, Rcpo(i_1;r_{10}),$

$IsCpo(j_1;r_{10}), IsCpo(i_2;r_{20}), r_{10} ! \mathring{\bigcirc} r_{20}, IsCpo(j_1;r_{20}), IsCpo(i_2;r_{10}),$

$Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$IsCpo(i_2;r_{10}), IsCpo(j_2;r_{10}), IsCpo(i_2;r_{20}), IsCpo(j_2;r_{20}),$

$i_1 \overset{\circ}{\bigcirc} i_2, j_1 \overset{\circ}{\bigcirc} j_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20},$

$IsCpo(i_1;r_{10}), r_{10}! \overset{\circ\circ}{\bigcirc} r_{20}, Rcpo(i_1;r_{10}),$

$IsCpo(j_1;r_{10}), IsCpo(i_2;r_{20}), r_{10}! \overset{\circ\circ}{\bigcirc} r_{20}, IsCpo(j_1;r_{20}), IsCpo(i_2;r_{10}),$

$Rcpo(i_2;r_{20}), Rcpo(j_1;r_{10}), Rcpo(j_2;r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$IsCpo(i_2;r_{10}), IsCpo(j_2;r_{10}), IsCpo(i_2;r_{20}), IsCpo(j_2;r_{20}),$

$i_1 \overset{\circ}{\bigcirc} i_2, j_1 \overset{\circ}{\bigcirc} j_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10}! \overset{\circ\circ}{\bigcirc} r_{20},$

$i_1 = i_2, j_1 = j_2, r_1 = \varnothing, r_2 = \varnothing, r_1 = r_2, r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_2 \overset{\circ\circ}{\bigcirc} r_{20},$

$IsCpo(i_1;r_{10}), r_{10}! \overset{\circ\circ}{\bigcirc} r_{20},$

$Rcpo(i_1;r_{10}), Rcpo(i_2;r_{20}),$

$Rcpo(j_1;r_{10}), Rcpo(j_2;r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$IsCpo(i_2;r_{10}), IsCpo(j_2;r_{10}), IsCpo(i_2;r_{20}), IsCpo(j_2;r_{20}),$

$i_1 \overset{\circ}{\bigcirc} i_2, j_1 \overset{\circ}{\bigcirc} j_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, r_{10}! \overset{\circ\circ}{\bigcirc} r_{20},$

$i_1 = i_2, j_1 = j_2, r_1 = r_2, r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_2 \overset{\circ\circ}{\bigcirc} r_{20},$

$IsCpo(i_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(i_2;r_{10}), IsCpo(i_2;r_{20}), r_{10}! \overset{\circ\circ}{\bigcirc} r_{20}, r_1 \overset{\circ\circ}{\bigcirc} r_{10}, r_2 \overset{\circ\circ}{\bigcirc} r_{20},$

$i_1 = i_2, r_1 = r_2, Rcpo(i_1;r_{10}), Rcpo(i_2;r_{20}),$

$Rcpo(j_1; r_{10}), Rcpo(j_2; r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(j_1; r_{20}),$

$IsCpo(i_2; r_{10}), IsCpo(j_2; r_{10}), IsCpo(i_2; r_{20}), IsCpo(j_2; r_{20}),$

$i_1 \text{♂} i_2, j_1 \text{♂} j_2, r_1 \text{♂} r_{10}, r_2 \text{♂} r_{20}, r_{10} ! \text{⚦} r_{20},$

$i_1 \text{⌶} i_2, j_1 \text{⌶} j_2, r_1 \text{⌶} r_2, r_1 \text{⚦} r_{10}, r_2 \text{⚦} r_{20},$

$IsCpo(i_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(i_2; r_{10}), IsCpo(i_2; r_{20}), r_{10} ! \text{⚦} r_{20}, r_1 \text{⚦} r_{10}, r_2 \text{⚦} r_{20},$

$i_1 \text{⌶} i_2, r_1 \text{⌶} r_2, Rcpo(i_1; r_{10}), Rcpo(i_2; r_{20}), r_1 \text{⌶} r_2,$

$Rcpo(j_1; r_{10}), Rcpo(j_2; r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(j_1; r_{20}),$

$IsCpo(i_2; r_{10}), IsCpo(j_2; r_{10}), IsCpo(i_2; r_{20}), IsCpo(j_2; r_{20}),$

$i_1 \text{♂} i_2, j_1 \text{♂} j_2, r_1 \text{♂} r_{10}, r_2 \text{♂} r_{20}, r_{10} ! \text{⚦} r_{20},$

$i_1 \text{⌶} i_2, j_1 \text{⌶} j_2, r_1 \text{⌶} r_2, r_1 \text{⚦} r_{10}, r_2 \text{⚦} r_{20},$

$IsCpo(i_1; r_{10}), r_{10} ! \text{⚦} r_{20}, Rcpo(i_1; r_{10}),$

$Rcpo(i_2; r_{20}), r_1 \text{⌶} r_2,$

$Rcpo(j_1; r_{10}), Rcpo(j_2; r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(j_1; r_{20}),$

$IsCpo(i_2; r_{10}), IsCpo(j_2; r_{10}), IsCpo(i_2; r_{20}), IsCpo(j_2; r_{20}),$

$i_1 \text{♂} i_2, j_1 \text{♂} j_2, r_1 \text{♂} r_{10}, r_2 \text{♂} r_{20}, r_{10} ! \text{⚦} r_{20},$

$i_1 \text{⌶} i_2, j_1 \text{⌶} j_2, r_1 \text{⌶} r_2, r_1 \text{⚦} r_{10}, r_2 \text{⚦} r_{20},$

$IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(j_2; r_{10}), r_{10} ! \text{⚦} r_{20}, Rcpo(i_1; r_{10}),$

$IsCpo(i_2; r_{20}), IsCpo(j_1; r_{20}), IsCpo(j_2; r_{20}), r_{10} ! \text{⚦} r_{20}, Rcpo(i_2; r_{20}),$

$r_1 = r_2, Rcpo(j_1; r_{10}), Rcpo(j_2; r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(j_1; r_{20}),$

$IsCpo(i_2; r_{10}), IsCpo(j_2; r_{10}), IsCpo(i_2; r_{20}), IsCpo(j_2; r_{20}),$

$i_1 \mathrel{\bigcirc} i_2, j_1 \mathrel{\bigcirc} j_2, r_1 \mathrel{\bigcirc} r_{10}, r_2 \mathrel{\bigcirc} r_{20}, r_{10} ! \mathrel{\bigcirc\!\circ} r_{20},$

$i_1 = i_2, j_1 = j_2, r_1 = r_2, r_1 \mathrel{\bigcirc\!\circ} r_{10}, r_2 \mathrel{\bigcirc\!\circ} r_{20},$

$IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(j_2; r_{10}), r_{10} ! \mathrel{\bigcirc\!\circ} r_{20}, Rcpo(i_1; r_{10}),$

$IsCpo(i_2; r_{20}), IsCpo(j_1; r_{20}), IsCpo(j_2; r_{20}), r_{10} ! \mathrel{\bigcirc\!\circ} r_{20}, Rcpo(i_2; r_{20}),$

$j_1 = j_2, r_1 = r_2, Rcpo(j_1; r_{10}), Rcpo(j_2; r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(j_1; r_{20}),$

$IsCpo(i_2; r_{10}), IsCpo(j_2; r_{10}), IsCpo(i_2; r_{20}), IsCpo(j_2; r_{20}),$

$i_1 \mathrel{\bigcirc} i_2, j_1 \mathrel{\bigcirc} j_2, r_1 \mathrel{\bigcirc} r_{10}, r_2 \mathrel{\bigcirc} r_{20}, r_{10} ! \mathrel{\bigcirc\!\circ} r_{20},$

$i_1 = i_2, j_1 = j_2, r_1 = r_2, r_1 \mathrel{\bigcirc\!\circ} r_{10}, r_2 \mathrel{\bigcirc\!\circ} r_{20},$

$IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(j_2; r_{10}), r_{10} ! \mathrel{\bigcirc\!\circ} r_{20}, Rcpo(i_1; r_{10}),$

$IsCpo(i_2; r_{20}), IsCpo(j_1; r_{20}), IsCpo(j_2; r_{20}), r_{10} ! \mathrel{\bigcirc\!\circ} r_{20}, Rcpo(i_2; r_{20}),$

$IsCpo(j_1; r_{10}), IsCpo(j_1; r_{20}), IsCpo(j_2; r_{10}), IsCpo(j_2; r_{20}), r_{10} ! \mathrel{\bigcirc\!\circ} r_{20},$

$r_1 \mathrel{\bigcirc\!\circ} r_{10}, r_2 \mathrel{\bigcirc\!\circ} r_{20}, j_1 = j_2, r_1 = r_2, Rcpo(j_1; r_{10}), Rcpo(j_2; r_{20}),$

$\Leftrightarrow\ , IsCpo(i_1; r_{10}), IsCpo(j_1; r_{10}), IsCpo(i_1; r_{20}), IsCpo(j_1; r_{20}),$

$IsCpo(i_2; r_{10}), IsCpo(j_2; r_{10}), IsCpo(i_2; r_{20}), IsCpo(j_2; r_{20}),$

$i_1 \mathrel{\bigcirc} i_2, j_1 \mathrel{\bigcirc} j_2, r_1 \mathrel{\bigcirc} r_{10}, r_2 \mathrel{\bigcirc} r_{20}, r_{10} ! \mathrel{\bigcirc\!\circ} r_{20},$

$i_1 = i_2, j_1 = j_2, r_1 = r_2, r_1 \mathrel{\bigcirc\!\circ} r_{10}, r_2 \mathrel{\bigcirc\!\circ} r_{20},$

$IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(j_2;r_{10}), r_{10}!\mathring{\bigcirc}\!\!\circ r_{20}, Rcpo(i_1;r_{10}),$

$IsCpo(i_2;r_{20}), IsCpo(j_1;r_{20}), IsCpo(j_2;r_{20}), r_{10}!\mathring{\bigcirc}\!\!\circ r_{20}, Rcpo(i_2;r_{20}),$

$IsCpo(j_1;r_{10}), IsCpo(j_1;r_{20}), IsCpo(j_2;r_{10}), IsCpo(j_2;r_{20}), r_{10}!\mathring{\bigcirc}\!\!\circ r_{20},$

$r_1\mathring{\bigcirc}\!\!\circ r_{10}, r_2\mathring{\bigcirc}\!\!\circ r_{20}, j_1\text{工}j_2, r_1\text{工}r_2,$

$Rcpo(j_1;r_{10}), Rcpo(j_2;r_{20}), r_1\text{工}r_2,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$IsCpo(i_2;r_{10}), IsCpo(j_2;r_{10}), IsCpo(i_2;r_{20}), IsCpo(j_2;r_{20}),$

$i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, r_{10}!\mathring{\bigcirc}\!\!\circ r_{20},$

$i_1\text{工}i_2, j_1\text{工}j_2, r_1\text{工}r_2, r_1\mathring{\bigcirc}\!\!\circ r_{10}, r_2\mathring{\bigcirc}\!\!\circ r_{20},$

$IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(j_2;r_{10}), r_{10}!\mathring{\bigcirc}\!\!\circ r_{20}, Rcpo(i_1;r_{10}),$

$IsCpo(i_2;r_{20}), IsCpo(j_1;r_{20}), IsCpo(j_2;r_{20}), r_{10}!\mathring{\bigcirc}\!\!\circ r_{20}, Rcpo(i_2;r_{20}),$

$Rcpo(j_1;r_{10}), Rcpo(j_2;r_{20}), r_1\text{工}r_2,$

$\Leftrightarrow\ , IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}\!\!\circ r_{20}, i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20},$

$Rcpo(i_1;r_{10}), Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}), r_1\text{工}r_2,$

$$, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}), IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$$

$$, r_{10}!\mathring{\bigcirc}\!\!\circ r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\gtrdot j,$$

$$, Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),\quad \Leftrightarrow \quad \sim, r_1\gtrdot r_2,$$

induction proof:
premise 1 :
$, k_1=\varnothing, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}), IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i\gtrdot j,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i\gtrdot j,$

$k_1=\varnothing, Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i\gtrdot j,$

$k_1=\varnothing, Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$IsCpo(i;r_{10}), k_2!\mathring{\bigcirc} r_{10}, k_2=\varnothing, Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$IsCpo(i;r_{10}), k_2!\mathring{\bigcirc} r_{10}, Rcpo(i;r_{10}), k_2=\varnothing, Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\bigcirc\!\!^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, k_1 = \varnothing, i \succ j,$

$IsCpo(i;r_{10}), k_2!\bigcirc\!\!^{\circ} r_{10}, Rcpo(i;r_{10}), k_2 = \varnothing, Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\bigcirc\!\!^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, k_1 = \varnothing, i \succ j,$

$i \text{☮} i_0, j \text{☮} j_0, i_0 \succ j_0, i_0 \text{⏚}, j_0 \text{⏚},$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}),$

$Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\bigcirc\!\!^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, k_1 = \varnothing, i \succ j,$

$i \text{☮} i_0, j \text{☮} j_0, i_0 \succ j_0,$

$IsCpo(i;r_{10}), i \mathring{\bigcirc} i_0, r_1 \mathring{\bigcirc} r_{10}, Rcpo(i;r_{10}),$

$i_0 \text{⏚}, j_0 \text{⏚}, Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\bigcirc\!\!^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, k_1 = \varnothing, i \succ j,$

$i \text{☮} i_0, j \text{☮} j_0, i_0 \succ j_0,$

$IsCpo(i;r_{10}), i \mathring{\bigcirc} i_0, r_1 \mathring{\bigcirc} r_{10}, Rcpo(i;r_{10}), i_0 \text{⌶} r_1,$

$i_0 \text{⏚}, j_0 \text{⏚}, Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\text{⚨}r_{20}, r_1\text{⚲}r_{10}, r_2\text{⚲}r_{20}, k_1\text{⚲}k_2, k_1 = \varnothing, i\succ j,$

$i\text{☮}i_0, j\text{☮}j_0, i_0\succ j_0,$

$IsCpo(i;r_{10}), r_{10}!\text{⚨}r_{20}, Rcpo(i;r_{10}),$

$i_0\text{⟛}r_1, IsCpo(j;r_{20}), i_0\text{⏚}, j_0\text{⏚}, Rcpo(j;r_{20}),$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\text{⚨}r_{20}, r_1\text{⚲}r_{10}, r_2\text{⚲}r_{20}, k_1\text{⚲}k_2, k_1 = \varnothing, i\succ j,$

$i\text{☮}i_0, j\text{☮}j_0, i_0\succ j_0,$

$IsCpo(i;r_{10}), r_{10}!\text{⚨}r_{20}, Rcpo(i;r_{10}),$

$IsCpo(j;r_{20}), i_0!\text{⚨}r_{20}, r_1!\text{⚨}r_{20}, i_0\text{⟛}r_1, Rcpo(j;r_{20}), i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\text{⚨}r_{20}, r_1\text{⚲}r_{10}, r_2\text{⚲}r_{20}, k_1\text{⚲}k_2, k_1 = \varnothing, i\succ j,$

$i\text{☮}i_0, j\text{☮}j_0, i_0\succ j_0,$

$IsCpo(i;r_{10}), r_{10}!\text{⚨}r_{20}, Rcpo(i;r_{10}),$

$IsCpo(j;r_{20}), i_0!\text{⚨}r_{20}, r_1!\text{⚨}r_{20}, Rcpo(j;r_{20}),$

$i_0\text{⟛}r_1, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$i\circledast i_0, j\circledast j_0, i_0\gtrdot j_0,$

$IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc} r_{20}, Rcpo(i;r_{10}),$

$IsCpo(j;r_{20}), j\mathring{\bigcirc} j_0, r_2\mathring{\bigcirc} r_{20}, Rcpo(j;r_{20}),$

$i_0 \mathrel{\text{⌶}} r_1, i_0\textcircled{\downarrow}, j_0\textcircled{\downarrow},$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$i\circledast i_0, j\circledast j_0, i_0\gtrdot j_0,$

$IsCpo(i;r_{10}), r_{10}!\mathring{\bigcirc} r_{20}, Rcpo(i;r_{10}),$

$IsCpo(j;r_{20}), j\mathring{\bigcirc} j_0, r_2\mathring{\bigcirc} r_{20}, Rcpo(j;r_{20}),$

$j_0 \mathrel{\text{⌶}} r_2, i_0 \mathrel{\text{⌶}} r_1, i_0\textcircled{\downarrow}, j_0\textcircled{\downarrow},$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$i\circledast i_0, j\circledast j_0, i_0\gtrdot j_0,$

$IsCpo(i;r_{10}), i_0!\mathring{\bigcirc} r_{10}, j_0!\mathring{\bigcirc} r_{10}, i_0\gtrdot j_0, Rcpo(i;r_{10}),$

$IsCpo(j;r_{20}), i_0!\mathring{\bigcirc} r_{20}, j_0!\mathring{\bigcirc} r_{20}, Rcpo(j;r_{20}),$

$j_0 \mathrel{\text{⌶}} r_2, i_0 \mathrel{\text{⌶}} r_1, i_0\textcircled{\downarrow}, j_0\textcircled{\downarrow},$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}\!\circ\, r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$i\text{☮}i_0, j\text{☮}j_0, i_0\gtrdot j_0,$

$IsCpo(i;r_{10}), i_0!\mathring{\bigcirc}\!\circ\, r_{10}, j_0!\mathring{\bigcirc}\!\circ\, r_{10}, Rcpo(i;r_{10}),$

$IsCpo(j;r_{20}), i_0!\mathring{\bigcirc}\!\circ\, r_{20}, j_0!\mathring{\bigcirc}\!\circ\, r_{20}, Rcpo(j;r_{20}),$

$j_0\text{⌶}r_2, i_0\text{⌶}r_1, i_0\gtrdot j_0, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}\!\circ\, r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$i\text{☮}i_0, j\text{☮}j_0, i_0\gtrdot j_0,$

$IsCpo(i;r_{10}), i_0!\mathring{\bigcirc}\!\circ\, r_{10}, j_0!\mathring{\bigcirc}\!\circ\, r_{10}, Rcpo(i;r_{10}),$

$IsCpo(j;r_{20}), i_0!\mathring{\bigcirc}\!\circ\, r_{20}, j_0!\mathring{\bigcirc}\!\circ\, r_{20}, Rcpo(j;r_{20}),$

$j_0\text{⌶}r_2, i_0\text{⌶}r_1, r_1\gtrdot r_2, i_0\text{⏚}, j_0\text{⏚},$

$\Leftrightarrow\ , IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}\!\circ\, r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, k_1=\varnothing, i\gtrdot j,$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}), Rcpo(j;r_{20}),$

$r_1\gtrdot r_2,$

$\Leftrightarrow\ , k_1=\varnothing, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$r_1\rhd r_2,$

premise 2 :
$, \&SHi \rightarrow k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$\Leftrightarrow\ , \&SHi \rightarrow k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}), i\rhd j,\ \Longrightarrow$

$, k_1 \mathrel{!=} \varnothing, \&SHi\, \mathring{\bigcirc} k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing, \&SHi\, \mathring{\bigcirc} k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$k_1 \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, k_1\oplus, Rcpo(k_1; r_{10}),$

$Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$\Leftrightarrow\ , k_1 \,!\!= \varnothing,\ \&SHi\,\mathring{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, k_1\oplus,$

$Rcpo(k_1;r_{10}),$

$Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , k_1 \,!\!= \varnothing,\ \&SHi\,\mathring{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, k_1\oplus,$

$IsCpo(k_1;r_{10}), k_2 \,!\!= \varnothing, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}),$

$Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , k_1 \,!\!= \varnothing,\ \&SHi\,\mathring{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, k_1\oplus,$

$IsCpo(k_1;r_{10}), k_2 \,!\!= \varnothing, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), k_2 \,!\!= \varnothing, Rcpo(i;r_{10}),$

$k_2 \,!\!= \varnothing, Cpo(r_{20}), r_{20}\oplus, k_2\oplus, Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , k_1 \,!{=}\, \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1\overset{\circ}{\bigcirc} r_{10}, r_2\overset{\circ}{\bigcirc} r_{20}, k_1\overset{\circ}{\bigcirc} k_2, i \rhd j,$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, k_1\oplus,$

$IsCpo(k_1;r_{10}), k_1!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), i!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Rcpo(i;r_{10}),$

$Cpo(r_{20}), r_{20}\oplus, k_2\oplus, Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , k_1 \,!{=}\, \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1\overset{\circ}{\bigcirc} r_{10}, r_2\overset{\circ}{\bigcirc} r_{20}, k_1\overset{\circ}{\bigcirc} k_2, i \rhd j,$

$IsCpo(k_1;r_{10}), r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, k_1\oplus,$

$IsCpo(k_1;r_{10}), k_1!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), i!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Rcpo(i;r_{10}),$

$k_2\oplus, Rcpo(k_2;r_{20}), Rcpo(j;r_{20}),$

$\Leftrightarrow\ , k_1 \,!{=}\, \varnothing,\ \&SHi\,\overset{\circ}{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1\overset{\circ}{\bigcirc} r_{10}, r_2\overset{\circ}{\bigcirc} r_{20}, k_1\overset{\circ}{\bigcirc} k_2, i \rhd j,$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus, k_1\oplus,$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus,$

$IsCpo(k_1;r_{10}), k_2!\overset{\circ}{\bigcirc}{}^{\circ} r_{10}, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), k_2!\overset{\circ}{\bigcirc}{}^{\circ} r_{10}, Rcpo(i;r_{10}),$

$k_2 \oplus, Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\, \mathring{\bigcirc} k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{10}), r_{10} \oplus, k_1 \oplus,$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{20}), r_{20} \oplus, k_2 \oplus,$

$IsCpo(k_1; r_{10}), k_2!\mathring{\bigcirc}{}^{\circ} r_{10}, Rcpo(k_1; r_{10}),$

$IsCpo(i; r_{10}), k_2!\mathring{\bigcirc}{}^{\circ} r_{10}, Rcpo(i; r_{10}),$

$Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\, \mathring{\bigcirc} k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, k_1!\mathring{\bigcirc}{}^{\circ} r_{10}, Cpo(r_{10}), r_{10} \oplus,$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, k_1!\mathring{\bigcirc}{}^{\circ} r_{20}, k_1 \oplus, Cpo(r_{20}), r_{20} \oplus, k_2 \oplus,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\, \mathring{\bigcirc} k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, k_1!\mathring{\bigcirc}{}^{\circ} r_{10}, Cpo(r_{10}), r_{10} \oplus, r_1!\mathring{\bigcirc} r_{10},$

$r_{10}!\mathring{\bigcirc}{}^{\circ} r_{20}, k_1!\mathring{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{20}), r_{20} \oplus, r_2!\mathring{\bigcirc} r_{20},$

$k_1 \oplus, k_2 \oplus,$

$IsCpo(k_1;r_{10}), r_1!\mathring{\bigcirc} r_{10}, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), r_1!\mathring{\bigcirc} r_{10}, Rcpo(i;r_{10}),$

$IsCpo(k_2;r_{20}), r_1!\mathring{\bigcirc}^{\circ} r_{20}, Rcpo(k_2;r_{20}),$

$IsCpo(j;r_{20}), r_1!\mathring{\bigcirc}^{\circ} r_{20}, Rcpo(j;r_{20}), r_1\oplus, r_2\oplus, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i > j,$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, k_1!\mathring{\bigcirc}^{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1!\mathring{\bigcirc} r_{10},$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, k_1!\mathring{\bigcirc}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2!\mathring{\bigcirc} r_{20},$

$k_1\oplus, k_2\oplus, r_1\oplus,$

$IsCpo(k_1;r_{10}), r_1!\mathring{\bigcirc} r_{10}, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), r_1!\mathring{\bigcirc} r_{10}, Rcpo(i;r_{10}),$

$IsCpo(k_2;r_{20}), r_1!\mathring{\bigcirc}^{\circ} r_{20}, Rcpo(k_2;r_{20}),$

$IsCpo(j;r_{20}), r_1!\mathring{\bigcirc}^{\circ} r_{20}, Rcpo(j;r_{20}), r_2\oplus, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i > j,$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, k_1!\mathring{\bigcirc}^{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1!\mathring{\bigcirc} r_{10},$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, k_1!\mathring{\bigcirc}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2!\mathring{\bigcirc} r_{20},$

$k_1\oplus, k_2\oplus, r_1\oplus,$

$IsCpo(k_1;r_{10}), r_2!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), r_2!\mathring{\bigcirc}^{\!\circ} r_{10}, Rcpo(i;r_{10}),$

$IsCpo(k_2;r_{20}), r_2!\mathring{\bigcirc} r_{20}, Rcpo(k_2;r_{20}),$

$IsCpo(j;r_{20}), r_2!\mathring{\bigcirc} r_{20}, Rcpo(j;r_{20}), r_2\oplus, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}^{\!\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}^{\!\circ} r_{20}, k_1!\mathring{\bigcirc}^{\!\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1!\mathring{\bigcirc} r_{10},$

$r_{10}!\mathring{\bigcirc}^{\!\circ} r_{20}, k_1!\mathring{\bigcirc}^{\!\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2!\mathring{\bigcirc} r_{20},$

$k_1\oplus, k_2\oplus, r_1\oplus, r_2\oplus,$

$IsCpo(k_1;r_{10}), r_2!\mathring{\bigcirc}^{\!\circ} r_{10}, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), r_2!\mathring{\bigcirc}^{\!\circ} r_{10}, Rcpo(i;r_{10}),$

$IsCpo(k_2;r_{20}), r_2!\mathring{\bigcirc} r_{20}, Rcpo(k_2;r_{20}),$

$IsCpo(j;r_{20}), r_2!\mathring{\bigcirc} r_{20}, Rcpo(j;r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}^{\!\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, k_1\mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc}^{\!\circ} r_{20}, Cpo(r_{10}), r_{10}\oplus,$

$r_{10}!\mathring{\bigcirc}^{\!\circ} r_{20}, r_1!\mathring{\bigcirc}^{\!\circ} r_{20}, Cpo(r_{20}), r_1\oplus, r_{20}\oplus, r_2\oplus,$

$k_1\oplus, k_2\oplus,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \,!{=}\, \varnothing,\ \&SHi\,\mathring{\bigcirc}k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1!\mathring{\bigcirc}\!\circ r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2\oplus,$

$k_1\oplus, k_2\oplus,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \,!{=}\, \varnothing,\ \&SHi\,\mathring{\bigcirc}k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2\oplus, k_1\oplus, k_2\oplus,$

$r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \,!{=}\, \varnothing,\ \&SHi\,\mathring{\bigcirc}k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i\rhd j,$

$i!\mathring{\bigcirc}\!\circ r_{10}, j!\mathring{\bigcirc}\!\circ r_{10}, i\rhd j, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$i!\mathring{\bigcirc}\!\circ r_{20}, j!\mathring{\bigcirc}\!\circ r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2\oplus, k_1\oplus, k_2\oplus,$

$r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \,!\!= \varnothing,\ \&SHi \overset{\circ}{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, k_1 \overset{\circ}{\bigcirc} k_2, i \gtrdot j,$

$i!\overset{\circ}{\bigcirc}{}^{\circ} r_{10}, j!\overset{\circ}{\bigcirc}{}^{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$i!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, j!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2\oplus, k_1\oplus, k_2\oplus,$

$r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, k_1 \overset{\circ}{\bigcirc} k_2, i \gtrdot j,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \,!\!= \varnothing,\ \&SHi \overset{\circ}{\bigcirc} k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, k_1 \overset{\circ}{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, k_1!\overset{\circ}{\bigcirc}{}^{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, k_1!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2\oplus, k_1\oplus, k_2\oplus,$

$r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, k_1 \overset{\circ}{\bigcirc} k_2, i \gtrdot j,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \,!\!= \varnothing, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, k_1 \overset{\circ}{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, k_1!\overset{\circ}{\bigcirc}{}^{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, k_1!\overset{\circ}{\bigcirc}{}^{\circ} r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2\oplus, k_1\oplus, k_2\oplus,$

$\&SHi \rightarrow k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, i \gtrdot j,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}), r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 != \varnothing, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, i \gtrdot j,$

$r_{10}!\mathring{\bigcirc} r_{20}, k_1!\mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$r_{10}!\mathring{\bigcirc} r_{20}, k_1!\mathring{\bigcirc} r_{20}, Cpo(r_{20}), r_{20}\oplus, r_2\oplus, k_1\oplus, k_2\oplus,$

$\&SHi \rightarrow k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, i \gtrdot j,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}), i \gtrdot j, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 != \varnothing,\ \&SHi \mathring{\bigcirc} k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, k_1 \mathring{\bigcirc} k_2, i \gtrdot j,$

$r_1 \mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_1 != \varnothing, r_{10}\oplus,$

$r_1!\mathring{\bigcirc} r_{20}, r_2 \mathring{\bigcirc} r_{20}, Cpo(r_{20}), r_2 != \varnothing, r_{20}\oplus, k_1\oplus, k_2\oplus,$

$Rcpo(k_1; r_{10}), Rcpo(i; r_{10}), Rcpo(k_2; r_{20}), Rcpo(j; r_{20}),$

$r_1\oplus, r_2\oplus, i \gtrdot j, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 != \varnothing,\ \&SHi \mathring{\bigcirc} k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1\overset{\circ}{\bigcirc}r_{10}, r_2\overset{\circ}{\bigcirc}r_{20}, k_1\overset{\circ}{\bigcirc}k_2, i\gtrdot j,$

$r_1\overset{\circ}{\bigcirc}r_{10}, Cpo(r_{10}), r_{10}\oplus,$

$r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_2\overset{\circ}{\bigcirc}r_{20}, Cpo(r_{20}), r_{20}\oplus, k_1\oplus, k_2\oplus,$

$IsCpo(k_1;r_{10}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}),$

$IsCpo(k_2;r_{20}), Rcpo(k_2;r_{20}),$

$IsCpo(j;r_{20}), Rcpo(j;r_{20}),$

$r_1\oplus, r_2\oplus, i\gtrdot j, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\overset{\circ}{\bigcirc}k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_1\overset{\circ}{\bigcirc}r_{10}, r_2\overset{\circ}{\bigcirc}r_{20}, k_1\overset{\circ}{\bigcirc}k_2, i\gtrdot j,$

$r_1\overset{\circ}{\bigcirc}r_{10}, Cpo(r_{10}), r_{10}\oplus,$

$r_1!\overset{\circ}{\bigcirc}{}^{\circ}r_{20}, r_2\overset{\circ}{\bigcirc}r_{20}, Cpo(r_{20}), r_{20}\oplus, k_1\oplus, k_2\oplus,$

$IsCpo(k_1;r_{10}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(k_1;r_{10}),$

$IsCpo(i;r_{10}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(i;r_{10}),$

$IsCpo(k_2;r_{20}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(k_2;r_{20}),$

$IsCpo(j;r_{20}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(j;r_{20}),$

$r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, r_1\oplus, r_2\oplus, i\gtrdot j, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\overset{\circ}{\bigcirc}k_1, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i{>}j,$

$r_1\mathring{\bigcirc}r_{10}, Cpo(r_{10}), r_{10}\oplus,$

$r_1!\mathring{\bigcirc}r_{20}, r_2\mathring{\bigcirc}r_{20}, Cpo(r_{20}), r_{20}\oplus, k_1\oplus, k_2\oplus,$

$IsCpo(k_1; r_{10}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(k_1; r_{10}),$

$IsCpo(i; r_{10}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(i; r_{10}),$

$IsCpo(k_2; r_{20}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(k_2; r_{20}),$

$IsCpo(j; r_{20}), r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, Rcpo(j; r_{20}),$

$i{>}j, r_1 \mathrel{!=} \varnothing, r_2 \mathrel{!=} \varnothing, r_1\oplus, r_2\oplus, r_1\ominus, r_2\ominus,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc}k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i{>}j,$

$r_1\mathring{\bigcirc}r_{10}, Cpo(r_{10}), r_{10}\oplus,$

$r_1!\mathring{\bigcirc}r_{20}, r_2\mathring{\bigcirc}r_{20}, Cpo(r_{20}), r_{20}\oplus, k_1\oplus, k_2\oplus,$

$IsCpo(k_1; r_{10}), Rcpo(k_1; r_{10}),$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(k_2; r_{20}), Rcpo(k_2; r_{20}),$

$IsCpo(j; r_{20}), Rcpo(j; r_{20}),$

$i{>}j,$

$\Leftrightarrow\ , k_1 \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc}k_1, IsCpo(i; r_{10}), IsCpo(j; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i; r_{20}), IsCpo(j; r_{20}), IsCpo(k_1; r_{20}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, k_1\mathring{\bigcirc}k_2, i{>}j,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), i \gtrdot j,$

$conclusion:$
$, IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, k_1 \overset{\circ}{\bigcirc} k_2, i \gtrdot j,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), \quad \Leftrightarrow \quad ,$

$IsCpo(i;r_{10}), IsCpo(j;r_{10}), IsCpo(k_1;r_{10}),$

$IsCpo(i;r_{20}), IsCpo(j;r_{20}), IsCpo(k_1;r_{20}),$

$r_{10} ! \overset{\circ}{\bigcirc}{}^{\circ} r_{20}, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, k_1 \overset{\circ}{\bigcirc} k_2, i \gtrdot j,$

$Rcpo(k_1;r_{10}), Rcpo(i;r_{10}), Rcpo(k_2;r_{20}), Rcpo(j;r_{20}), i \gtrdot j,$

30 Addition

30.1 Definition

$$,i+j:r, \iff ,\circledcirc r, i\text{☮}i_0, j\text{☮}j_0, r\text{☮}r_0, Rcpo(i_0;r_0), Rcpo(j_0;r_0), i_0\text{⏚}, j_0\text{⏚}, r_0\text{⏚},$$

30.2 Swap

30.2.1 Operator

$$,i+j:r, m\text{☮}n, \iff ,m\text{☮}n, i+j:r,$$

$$,i+j:r, m\text{☮}'n, \iff ,m\text{☮}'n, i+j:r,$$

$$,i+j:r, m\text{☮}^{\circ}n, \iff ,m\text{☮}^{\circ}n, i+j:r,$$

$$,i+j:r, \circledcirc m, \iff ,\circledcirc m, i+j:r,$$

$$,i+j:r, \odot m, \iff ,\odot m, i+j:r,$$

$$,i+j:r, m\text{⏚}, \iff ,m\text{⏚}, i+j:r,$$

$$,i+j:r, m\oplus, \iff ,m\oplus, i+j:r,$$

$$,i+j:r, m\ominus, \iff ,m\ominus, i+j:r,$$

$$,i+j:r, m\text{⊜}n\begin{cases} , \\ , \end{cases} \iff ,m\text{⊜}n\begin{cases} ,i+j:r, \\ ,i+j:r, \end{cases}$$

30.2.2 Recursive Function

$$,i+j:r,R(m), \iff ,R(m),i+j:r,$$

$$,i+j:r,Rc(m;n), \iff ,Rc(m;n),i+j:r,$$

30.2.3 Propositions

$$,i+j:r,m\!=\!n, \iff ,m\!=\!n,i+j:r,$$

$$,i+j:r,m\!=\!\varnothing, \iff ,m\!=\!\varnothing,i+j:r,$$

$$,i+j:r,m\bigcirc^{\!\!\circ}n, \iff ,m\bigcirc^{\!\!\circ}n,i+j:r,$$

$$,i+j:r,m\bigcirc^{\!\!\circ\circ}n, \iff ,m\bigcirc^{\!\!\circ\circ}n,i+j:r,$$

$$,i+j:r,m\!\rightarrow\!n, \iff ,m\!\rightarrow\!n,i+j:r,$$

$$,i+j:r,m\bot\!\!\!\!\!\bigcirc n, \iff ,m\bot\!\!\!\!\!\bigcirc n,i+j:r,$$

$$,i+j:r,m\top\!\!\!\!\bot n, \iff ,m\top\!\!\!\!\bot n,i+j:r,$$

$$,i+j:r,m\rhd n, \iff ,m\rhd n,i+j:r,$$

$$,i+j:r,m\,!\!=\!n, \iff ,m\,!\!=\!n,i+j:r,$$

$$,i+j:r,m\,!\!=\!\varnothing, \iff ,m\,!\!=\!\varnothing,i+j:r,$$

$$,i+j:r,m!\bigcirc^{\!\!\circ}n, \iff ,m!\bigcirc^{\!\!\circ}n,i+j:r,$$

$$,i+j:r,m!\bigcirc^{\!\!\circ\circ}n, \iff ,m!\bigcirc^{\!\!\circ\circ}n,i+j:r,$$

$$,i+j:r,m!\!\rightarrow\!n, \iff ,m!\!\rightarrow\!n,i+j:r,$$

$$,i+j:r,m!\bot\!\!\!\!\!\bigcirc n, \iff ,m!\bot\!\!\!\!\!\bigcirc n,i+j:r,$$

$$, i + j : r, m!{=}n, \;\Leftrightarrow\; , m!{=}n, i + j : r,$$

$$, i + j : r, m!{>}n, \;\Leftrightarrow\; , m!{>}n, i + j : r,$$

30.2.4 Itself

$$, i_1 + j : r_1, i_2 + j : r_2, \;\Leftrightarrow\; , i_2 + j : r_2, i_1 + j : r_1,$$

$$, i_1 + j_1 : r_1, i_2 + j_2 : r_2, \;\Leftrightarrow\; , i_2 + j_2 : r_2, i_1 + j_1 : r_1,$$

$$, i + j : r_1, i + j : r_2, \;\Leftrightarrow\; , i + j : r_2, i + j : r_1,$$

30.2.5 Rcpo

$$, IsCpo(m; n), i!\text{⚲}n, j!\text{⚲}n, i + j : r, Rcpo(m; n),$$
$$\Leftrightarrow\; , IsCpo(m; n), i!\text{⚲}n, j!\text{⚲}n, Rcpo(m; n), i + j : r,$$

30.2.6 The same operand

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30.3 General property

$$, \;\Leftrightarrow\; , i + j : r, r\text{⏚},$$

proof:

,

$\Leftrightarrow\;, \circledcirc r, i\text{☮}i_0, j\text{☮}j_0, r\text{☮}r_0, i_0\text{⏚}, j_0\text{⏚}, r\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\;, \circledcirc r, i\text{☮}i_0, j\text{☮}j_0, r\text{☮}r_0, i_0\text{⏚}, j_0\text{⏚}, r\text{⏚}, \&Tm(r_0),$

$\Leftrightarrow\;, \circledcirc r, IsCpo(i; r), IsCpo(j; r), i\text{☮}i_0, j\text{☮}j_0, r\text{☮}r_0, r\text{⏚},$

$i_0⏚, j_0⏚, \&Tm(r_0),$

$\Leftrightarrow\ , \circledcirc r, i☮i_0, j☮j_0, r☮r_0, r⏚,$

$IsCpo(i_0; r_0), IsCpo(j_0; r_0), i_0⏚, j_0⏚, \&Tm(r_0),$

$\Leftrightarrow\ , \circledcirc r, i☮i_0, j☮j_0, r☮r_0, r⏚,$

$IsCpo(i_0; r_0), IsCpo(j_0; r_0), Rcpo(i_0; r_0), Rcpo(j_0; r_0), i_0⏚, j_0⏚, \&Tm(r_0),$

$\Leftrightarrow\ , \circledcirc r, i☮i_0, j☮j_0, r☮r_0, r⏚,$

$IsCpo(i_0; r_0), IsCpo(j_0; r_0), Rcpo(i_0; r_0), Rcpo(j_0; r_0), i_0⏚, j_0⏚, r_0⏚,$

$\Leftrightarrow\ , \circledcirc r, i☮i_0, j☮j_0, r☮r_0, r⏚,$

$Rcpo(i_0; r_0), Rcpo(j_0; r_0), i_0⏚, j_0⏚, r_0⏚,$

$\Leftrightarrow\ , i + j : r, r⏚,$

$$, i + j : r, \otimes, \ \Leftrightarrow\ , \otimes,$$

$$, i + j : r_1, i + j : r_2, \ \Leftrightarrow\ \sim, r_1{=}r_2,$$

proof:
$, i + j : r_1, i + j : r_2,$

$\Leftrightarrow\ , \circledcirc r_1, i☮i_1, j☮j_1, r_1☮r_{10}, Rcpo(i_1; r_{10}), Rcpo(j_1; r_{10}), i_1⏚, j_1⏚, r_{10}⏚,$

$\circledcirc r_2, i☮i_2, j☮j_2, r_2☮r_{20}, Rcpo(i_2; r_{20}), Rcpo(j_2; r_{20}), i_2⏚, j_2⏚, r_{20}⏚,$

$\Leftrightarrow\ , \circledcirc r_1, i☮i_1, j☮j_1, r_1☮r_{10},$

$IsCpo(i_1; r_{10}), Rcpo(i_1; r_{10}),$

$IsCpo(j_1; r_{10}), Rcpo(j_1; r_{10}),$

$i_1⏚, j_1⏚, r_{10}⏚, \circledcirc r_2, i☮i_2, j☮j_2, r_2☮r_{20},$

$IsCpo(i_2; r_{20}), Rcpo(i_2; r_{20}),$

$IsCpo(j_2;r_{20}), Rcpo(j_2;r_{20}), i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpo(i_1;r_{10}), Rcpo(i_1;r_{10}),$

$IsCpo(j_1;r_{10}), Rcpo(j_1;r_{10}),$

$IsCpo(i_2;r_{20}), Rcpo(i_2;r_{20}),$

$IsCpo(j_2;r_{20}), Rcpo(j_2;r_{20}), i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpo(i_1;r_{10}), r_{10}!\bigcirc^{\circ\circ} r_{20}, Rcpo(i_1;r_{10}),$

$IsCpo(j_1;r_{10}), r_{10}!\bigcirc^{\circ\circ} r_{20}, Rcpo(j_1;r_{10}),$

$IsCpo(i_2;r_{20}), Rcpo(i_2;r_{20}),$

$IsCpo(j_2;r_{20}), Rcpo(j_2;r_{20}), i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}), r_{10}!\bigcirc^{\circ\circ} r_{20},$

$i_1\bigcirc^{\circ} i_2, j_1\bigcirc^{\circ} j_2, r_1\bigcirc^{\circ} r_{10}, r_2\bigcirc^{\circ} r_{20},$

$Rcpo(i_1;r_{10}), Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}),$

$i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpo(i_1;r_{10}), IsCpo(j_1;r_{10}), IsCpo(i_1;r_{20}), IsCpo(j_1;r_{20}), r_{10}!\bigcirc^{\circ\circ} r_{20},$

$i_1\bigcirc^{\circ} i_2, j_1\bigcirc^{\circ} j_2, r_1\bigcirc^{\circ} r_{10}, r_2\bigcirc^{\circ} r_{20},$

$Rcpo(i_1;r_{10}), Rcpo(j_1;r_{10}), Rcpo(i_2;r_{20}), Rcpo(j_2;r_{20}), r_1{=}r_2,$

$i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, Rcpo(i_1; r_{10}), Rcpo(j_1; r_{10}), i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚},$

$\circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20}, Rcpo(i_2; r_{20}), Rcpo(j_2; r_{20}), i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚}, r_1\text{⌶}r_2,$

$\Leftrightarrow\ , i + j : r_1, i + j : r_2, r_1\text{⌶}r_2,$

$$, i_1\text{⌶}i_2, i_1 + j : r, \ \Leftrightarrow\ , i_1\text{⌶}i_2, i_2 + j : r,$$

proof:
$, i_1\text{⌶}i_2, i_1 + j : r,$

$\Leftrightarrow\ , i_1\text{⌶}i_2, \circledcirc r, i_1\text{☮}i_{10}, j\text{☮}j_0, r\text{☮}r_0,$

$Rcpo(i_{10}; r_0), Rcpo(j_0; r_0), i_{10}\text{⏚}, j_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , i_1\text{⌶}i_2, \circledcirc r, i_1\text{☮}i_{10}, j\text{☮}j_0, r\text{☮}r_0,$

$IsCpo(i_{10}; r_0), Rcpo(i_{10}; r_0),$

$IsCpo(j_0; r_0), Rcpo(j_0; r_0), i_{10}\text{⏚}, j_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , i_1\text{⌶}i_2, \circledcirc r, i_1\text{☮}i_{10}, j\text{☮}j_0, r\text{☮}r_0,$

$IsCpo(i_{10}; r_0), Rcpo(i_{10}; r_0), i_{10}\text{⏚},$

$IsCpo(j_0; r_0), Rcpo(j_0; r_0), j_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , i_1\text{⌶}i_2, \circledcirc r, i_1\text{☮}i_{10}, j\text{☮}j_0, r\text{☮}r_0, i_2\text{☮}i_{20}, IsCpo(i_{20}; r_0), i_{10}\text{⌶}i_{20}, i_{20}\text{⏚},$

$IsCpo(i_{10}; r_0), Rcpo(i_{10}; r_0), i_{10}\text{⏚},$

$IsCpo(j_0; r_0), Rcpo(j_0; r_0), j_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow \ , i_1 \mathrel{\bar\bot} i_2, \circledcirc r, i_1 \varowedge i_{10}, j \varowedge j_0, r \varowedge r_0, i_2 \varowedge i_{20},$

$IsCpo(i_{10}; r_0), IsCpo(i_{20}; r_0), i_{10} \mathrel{\bar\bot} i_{20}, Rcpo(i_{10}; r_0), i_{10} \circledbot, i_{20} \circledbot,$

$IsCpo(j_0; r_0), Rcpo(j_0; r_0), j_0 \circledbot, r_0 \circledbot,$

$\Leftrightarrow \ , i_1 \mathrel{\bar\bot} i_2, \circledcirc r, i_1 \varowedge i_{10}, j \varowedge j_0, r \varowedge r_0, i_2 \varowedge i_{20},$

$IsCpo(i_{10}; r_0), IsCpo(i_{20}; r_0), i_{10} \mathrel{\bar\bot} i_{20}, Rcpo(i_{20}; r_0), i_{10} \circledbot, i_{20} \circledbot,$

$IsCpo(j_0; r_0), Rcpo(j_0; r_0), j_0 \circledbot, r_0 \circledbot,$

$\Leftrightarrow \ , i_1 \mathrel{\bar\bot} i_2, \circledcirc r, j \varowedge j_0, r \varowedge r_0, i_2 \varowedge i_{20}, i_1 \varowedge i_{10}, IsCpo(i_{10}; r_0), i_{10} \mathrel{\bar\bot} i_{20}, i_{10} \circledbot,$

$IsCpo(i_{20}; r_0), Rcpo(i_{20}; r_0), i_{20} \circledbot,$

$IsCpo(j_0; r_0), Rcpo(j_0; r_0), j_0 \circledbot, r_0 \circledbot,$

$\Leftrightarrow \ , i_1 \mathrel{\bar\bot} i_2, \circledcirc r, j \varowedge j_0, r \varowedge r_0, i_2 \varowedge i_{20},$

$IsCpo(i_{20}; r_0), Rcpo(i_{20}; r_0),$

$IsCpo(j_0; r_0), Rcpo(j_0; r_0), i_{20} \circledbot, j_0 \circledbot, r_0 \circledbot,$

$\Leftrightarrow \ , i_1 \mathrel{\bar\bot} i_2, \circledcirc r, i_2 \varowedge i_{20}, j \varowedge j_0, r \varowedge r_0,$

$Rcpo(i_{20}; r_0), Rcpo(j_0; r_0), i_{20} \circledbot, j_0 \circledbot, r_0 \circledbot,$

$\Leftrightarrow \ , i_1 \mathrel{\bar\bot} i_2, i_2 + j : r,$

$$, k = \varnothing, i + k : r, \ \Leftrightarrow \ \sim, i \mathrel{\bar\bot} r,$$

proof:
$, k = \varnothing, i + k : r,$

$\Leftrightarrow \ , k = \varnothing, \circledcirc r, r \varowedge r_0, i \varowedge i_0, k \varowedge k_0, Rcpo(i_0; r_0), Rcpo(k_0; r_0), r_0 \circledbot, i_0 \circledbot, k_0 \circledbot,$

$$\Leftrightarrow\ \ , k = \varnothing, \circledcirc r, r \text{☮} r_0, i \text{☮} i_0, k \text{☮} k_0,$$

$$IsCpo(i_0; r_0), k_0 ! \text{♂} r_0, k_0 = \varnothing, Rcpo(i_0; r_0), Rcpo(k_0; r_0), r_0 \text{⏚}, i_0 \text{⏚}, k_0 \text{⏚},$$

$$\Leftrightarrow\ \ , k = \varnothing, \circledcirc r, r \text{☮} r_0, i \text{☮} i_0, k \text{☮} k_0,$$

$$IsCpo(i_0; r_0), k_0 ! \text{♂} r_0, Rcpo(i_0; r_0), k_0 = \varnothing, Rcpo(k_0; r_0), r_0 \text{⏚}, i_0 \text{⏚}, k_0 \text{⏚},$$

$$\Leftrightarrow\ \ , k = \varnothing, \circledcirc r, r \text{☮} r_0, i \text{☮} i_0, k \text{☮} k_0,$$

$$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r_0 \text{⏚}, i_0 \text{⏚}, k_0 \text{⏚},$$

$$\Leftrightarrow\ \ , k = \varnothing, \circledcirc r, r \text{☮} r_0, i \text{☮} i_0, k \text{☮} k_0,$$

$$IsCpo(i_0; r_0), i \text{♂} i_0, r \text{♂} r_0, Rcpo(i_0; r_0), r_0 \text{⏚}, i_0 \text{⏚}, k_0 \text{⏚},$$

$$\Leftrightarrow\ \ , k = \varnothing, \circledcirc r, r \text{☮} r_0, i \text{☮} i_0, k \text{☮} k_0,$$

$$IsCpo(i_0; r_0), i \text{♂} i_0, r \text{♂} r_0, Rcpo(i_0; r_0), i \text{⌶} r, r_0 \text{⏚}, i_0 \text{⏚}, k_0 \text{⏚},$$

$$\Leftrightarrow\ \ , k = \varnothing, \circledcirc r, r \text{☮} r_0, i \text{☮} i_0, k \text{☮} k_0,$$

$$IsCpo(i_0; r_0), Rcpo(i_0; r_0), r_0 \text{⏚}, i_0 \text{⏚}, k_0 \text{⏚}, i \text{⌶} r,$$

$$\Leftrightarrow\ \ , k = \varnothing, i + k : r, i \text{⌶} r,$$

$$, i + j : r, i \text{⏚}, j \text{⏚},\ \ \Leftrightarrow\ \ , \circledcirc r, r \text{☮} r_0, Rcpo(i; r_0), Rcpo(j; r_0), i \text{⏚}, j \text{⏚}, r_0 \text{⏚},$$

$$, i + j : r, i \text{⏚},\ \ \Leftrightarrow\ \ , \circledcirc r, r \text{☮} r_0, j \text{☮} j_0, Rcpo(i; r_0), Rcpo(j_0; r_0), i \text{⏚}, j_0 \text{⏚}, r_0 \text{⏚},$$

$$, i + j : r, j \text{⏚},\ \ \Leftrightarrow\ \ , \circledcirc r, r \text{☮} r_0, i \text{☮} i_0, Rcpo(i_0; r_0), Rcpo(j; r_0), i_0 \text{⏚}, j \text{⏚}, r_0 \text{⏚},$$

30.4 Additive commutativity

$$,i+j:r, \;\Leftrightarrow\; ,j+i:r,$$

proof:
$,i+j:r,$

$\Leftrightarrow\;,\circledcirc r, i\owedge i_0, j\owedge j_0, r\owedge r_0, Rcpo(i_0;r_0), Rcpo(j_0;r_0), i_0\obot, j_0\obot, r_0\obot,$

$\Leftrightarrow\;,\circledcirc r, i\owedge i_0, j\owedge j_0, r\owedge r_0,$

$IsCpo(i_0;r_0), IsCpo(j_0;r_0), Rcpo(i_0;r_0), Rcpo(j_0;r_0), i_0\obot, j_0\obot, r_0\obot,$

$\Leftrightarrow\;,\circledcirc r, i\owedge i_0, j\owedge j_0, r\owedge r_0,$

$IsCpo(i_0;r_0), IsCpo(j_0;r_0), Rcpo(j_0;r_0), Rcpo(i_0;r_0), i_0\obot, j_0\obot, r_0\obot,$

$\Leftrightarrow\;,\circledcirc r, i\owedge i_0, j\owedge j_0, r\owedge r_0, Rcpo(j_0;r_0), Rcpo(i_0;r_0), i_0\obot, j_0\obot, r_0\obot,$

$\Leftrightarrow\;,\circledcirc r, j\owedge j_0, i\owedge i_0, r\owedge r_0, Rcpo(j_0;r_0), Rcpo(i_0;r_0), j_0\obot, i_0\obot, r_0\obot,$

$\Leftrightarrow\;,j+i:r,$

$$,i+j:r_1, j+i:r_2, \;\Leftrightarrow\; \sim, r_1 \mathrel{\text{⌶}} r_2,$$

proof:
$,i+j:r_1, j+i:r_2,$

$\Leftrightarrow\;,i+j:r_1, i+j:r_2,$

$\Leftrightarrow\;,i+j:r_1, i+j:r_2, r_1 \mathrel{\text{⌶}} r_2,$

$\Leftrightarrow\;,i+j:r_1, j+i:r_2, r_1 \mathrel{\text{⌶}} r_2,$

30.5 Additive associativity

$$, i + j : r_1, r_1 + k : r, r_1\text{⍗}, \ \Leftrightarrow \ , j + k : r_1, r_1 + i : r, r_1\text{⍗},$$

proof:
$, i + j : r_1, r_1 + k : r, r_1\text{⍗},$

$\Leftrightarrow \ , \circledcirc r_1, i\text{☮}i_0, j\text{☮}j_0, r_1\text{☮}r_{10}, Rcpo(i_0; r_{10}), Rcpo(j_0; r_{10}), i_0\text{⍗}, j_0\text{⍗}, r_{10}\text{⍗},$

$\circledcirc r, k\text{☮}k_0, r\text{☮}r_0, Rcpo(r_1; r_0), Rcpo(k_0; r_0), r_1\text{⍗}, r_0\text{⍗}, k_0\text{⍗},$

$\Leftrightarrow \ , \circledcirc r_1, i\text{☮}i_0, j\text{☮}j_0, r_1\text{☮}r_{10},$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}),$

$IsCpo(j_0; r_{10}), Rcpo(j_0; r_{10}), i_0\text{⍗}, j_0\text{⍗}, r_{10}\text{⍗}, \circledcirc r, k\text{☮}k_0, r\text{☮}r_0,$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(k_0; r_0), Rcpo(k_0; r_0), r_1\text{⍗}, r_0\text{⍗}, k_0\text{⍗},$

$\Leftrightarrow \ , \circledcirc r_1, i\text{☮}i_0, j\text{☮}j_0, r_1\text{☮}r_{10}, \circledcirc r, k\text{☮}k_0, r\text{☮}r_0,$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}),$

$IsCpo(j_0; r_{10}), Rcpo(j_0; r_{10}),$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(k_0; r_0), Rcpo(k_0; r_0), i_0\text{⍗}, j_0\text{⍗}, r_{10}\text{⍗}, r_1\text{⍗}, r_0\text{⍗}, k_0\text{⍗},$

$\Leftrightarrow \ , \circledcirc r_1, i\text{☮}i_0, j\text{☮}j_0, r_1\text{☮}r_{10}, \circledcirc r, k\text{☮}k_0, r\text{☮}r_0,$

$IsCpo(i_0; r_{10}), r_0!\text{⚲}r_{10}, Rcpo(i_0; r_{10}),$

$IsCpo(j_0; r_{10}), r_0!\text{⚲}r_{10}, Rcpo(j_0; r_{10}),$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(k_0; r_0), Rcpo(k_0; r_0), r_1\text{⍗}, r_{10}\text{⍗}, i_0\text{⍗}, j_0\text{⍗}, r_0\text{⍗}, k_0\text{⍗},$

$\Leftrightarrow \quad < 1 >$

$\circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(i_0; r_{10}), r_0 ! \mathring{\bigcirc}^{\circ} r_{10}, Rcpo(i_0; r_{10}),$

$IsCpo(j_0; r_{10}), r_0 ! \mathring{\bigcirc}^{\circ} r_{10}, Rcpo(j_0; r_{10}),$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(k_0; r_0), Rcpo(k_0; r_0), r_1 \text{⏚}, r_{10} \text{⏚},$

$\Leftrightarrow \quad , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(i_0; r_{10}), r_0 ! \mathring{\bigcirc}^{\circ} r_{10}, Rcpo(i_0; r_{10}),$

$IsCpo(j_0; r_{10}), r_0 ! \mathring{\bigcirc}^{\circ} r_{10}, Rcpo(j_0; r_{10}),$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(k_0; r_0), Rcpo(k_0; r_0), r_1 \text{⏚}, \& Tm(r_{10}),$

$\Leftrightarrow \quad , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(i_0; r_{10}), IsCpo(j_0; r_{10}), IsCpo(k_0; r_{10}),$

$IsCpo(i_0; r_0), IsCpo(j_0; r_0), IsCpo(k_0; r_0), IsCpo(r_1; r_0), r_1 \mathring{\bigcirc} r_{10},$

$Rcpo(i_0; r_{10}), Rcpo(j_0; r_{10}), Rcpo(r_1; r_0), Rcpo(k_0; r_0),$

$r_1 \text{⏚}, \& Tm(r_{10}),$

$\Leftrightarrow \quad , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(i_0; r_{10}), IsCpo(j_0; r_{10}), IsCpo(k_0; r_{10}),$

$IsCpo(i_0; r_0), IsCpo(j_0; r_0), IsCpo(k_0; r_0), IsCpo(r_1; r_0), r_1 \mathring{\bigcirc} r_{10},$

$Rcpo(i_0; r_0), Rcpo(j_0; r_0), Rcpo(k_0; r_0),$

$r_1 \text{⏚}, \& Tm(r_{10}),$

$\Leftrightarrow \ , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(i_0; r_{10}), IsCpo(j_0; r_{10}), IsCpo(k_0; r_{10}),$

$IsCpo(i_0; r_0), IsCpo(j_0; r_0), IsCpo(k_0; r_0), IsCpo(r_1; r_0), r_1 \overset{\circ}{\bigcirc} r_{10},$

$Rcpo(j_0; r_0), Rcpo(k_0; r_0), Rcpo(i_0; r_0),$

$r_1 \text{⏚}, \& Tm(r_{10}),$

$\Leftrightarrow \ , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(i_0; r_{10}), IsCpo(j_0; r_{10}), IsCpo(k_0; r_{10}),$

$IsCpo(i_0; r_0), IsCpo(j_0; r_0), IsCpo(k_0; r_0), IsCpo(r_1; r_0), r_1 \overset{\circ}{\bigcirc} r_{10},$

$Rcpo(j_0; r_{10}), Rcpo(k_0; r_{10}), Rcpo(r_1; r_0), Rcpo(i_0; r_0),$

$r_1 \text{⏚}, \& Tm(r_{10}),$

$\Leftrightarrow \ , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(j_0; r_{10}), r_0 ! \overset{\circ\circ}{\bigcirc} r_{10}, Rcpo(j_0; r_{10}),$

$IsCpo(k_0; r_{10}), r_0 ! \overset{\circ\circ}{\bigcirc} r_{10}, Rcpo(k_0; r_{10}),$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0),$

$r_1 \text{⏚}, \& Tm(r_{10}),$

$\Leftrightarrow \ , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, k \text{☮} k_0, r \text{☮} r_0,$

$IsCpo(j_0; r_{10}), r_0 ! \overset{\circ\circ}{\bigcirc} r_{10}, Rcpo(j_0; r_{10}),$

$IsCpo(k_0; r_{10}), r_0 ! \overset{\circ\circ}{\bigcirc} r_{10}, Rcpo(k_0; r_{10}),$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0),$

$r_1\text{⏚}, r_{10}\text{⏚},$

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$\Leftrightarrow\ , \circledcirc r_1, i\text{☮}i_0, j\text{☮}j_0, r_1\text{☮}r_{10}, \circledcirc r, k\text{☮}k_0, r\text{☮}r_0,$

$IsCpo(j_0; r_{10}), r_0!\bigcirc^{\circ} r_{10}, Rcpo(j_0; r_{10}),$

$IsCpo(k_0; r_{10}), r_0!\bigcirc^{\circ} r_{10}, Rcpo(k_0; r_{10}),$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0),$

$r_1\text{⏚}, r_{10}\text{⏚}, i_0\text{⏚}, j_0\text{⏚}, r_0\text{⏚}, k_0\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, j\text{☮}j_0, r_1\text{☮}r_{10}, k\text{☮}k_0,$

$IsCpo(j_0; r_{10}), Rcpo(j_0; r_{10}),$

$IsCpo(k_0; r_{10}), Rcpo(k_0; r_{10}), j_0\text{⏚}, k_0\text{⏚}, r_{10}\text{⏚}, \circledcirc r, r\text{☮}r_0, i\text{☮}i_0,$

$IsCpo(r_1; r_0), Rcpo(r_1; r_0),$

$IsCpo(i_0; r_0), Rcpo(i_0; r_0),$

$r_1\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, j\text{☮}j_0, r_1\text{☮}r_{10}, k\text{☮}k_0, Rcpo(j_0; r_{10}), Rcpo(k_0; r_{10}), j_0\text{⏚}, k_0\text{⏚}, r_{10}\text{⏚},$

$\circledcirc r, r\text{☮}r_0, i\text{☮}i_0, , Rcpo(r_1; r_0), Rcpo(i_0; r_0), r_1\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , j + k : r_1, r_1 + i : r, r_1\text{⏚},$

$$,i+j:r,r+k:r_1,r\text{\textcircled{$\downarrow$}},j+k:r,r+i:r_2,r\text{\textcircled{$\downarrow$}}, \quad\Leftrightarrow\quad \sim,r_1{=}r_2,$$

proof:

$,i+j:r,r+k:r_1,r\text{\textcircled{$\downarrow$}},j+k:r,r+i:r_2,r\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,i+j:r,r+k:r_1,r\text{\textcircled{$\downarrow$}},i+j:r,r+k:r_2,r\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,i+j:r_3,r_3+k:r_1,r_3\text{\textcircled{$\downarrow$}},i+j:r_4,r_4+k:r_2,r_4\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,i+j:r_3,i+j:r_4,r_3{=}r_4,r_3+k:r_1,r_4+k:r_2,r_3\text{\textcircled{$\downarrow$}},r_4\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,i+j:r_3,i+j:r_4,r_3{=}r_4,r_4+k:r_1,r_4+k:r_2,r_3\text{\textcircled{$\downarrow$}},r_4\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,i+j:r_3,i+j:r_4,r_3{=}r_4,r_4+k:r_1,r_4+k:r_2,r_1{=}r_2,r_3\text{\textcircled{$\downarrow$}},r_4\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,i+j:r_3,i+j:r_4,r_3{=}r_4,r_4+k:r_1,r_4+k:r_2,r_3\text{\textcircled{$\downarrow$}},r_4\text{\textcircled{$\downarrow$}},r_1{=}r_2,$

$\Leftrightarrow\ ,i+j:r,r+k:r_1,r\text{\textcircled{$\downarrow$}},j+k:r,r+i:r_2,r\text{\textcircled{$\downarrow$}},r_1{=}r_2,$

30.6 Additive monotonicity

$$,i{\succ}j,i+k:r_1,j+k:r_2, \quad\Leftrightarrow\quad \sim,r_1{\succ}r_2,$$

proof:

$,i{\succ}j,i+k:r_1,j+k:r_2,$

$\Leftrightarrow\ ,i{\succ}j,\circledcirc r_1,r_1\owedge r_{10},i\owedge i_0,k\owedge k_1,Rcpo(i_0;r_{10}),Rcpo(k_1;r_{10}),i_0\text{\textcircled{$\downarrow$}},r_{10}\text{\textcircled{$\downarrow$}},k_1\text{\textcircled{$\downarrow$}},$

$\circledcirc r_2,r_2\owedge r_{20},j\owedge j_0,k\owedge k_2,Rcpo(j_0;r_{20}),Rcpo(k_2;r_{20}),j_0\text{\textcircled{$\downarrow$}},r_{20}\text{\textcircled{$\downarrow$}},k_2\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,i{\succ}j,\circledcirc r_1,r_1\owedge r_{10},i\owedge i_0,k\owedge k_1,$

$IsCpo(i_0;r_{10}),Rcpo(i_0;r_{10}),IsCpo(k_1;r_{10}),Rcpo(k_1;r_{10}),i_0\text{\textcircled{$\downarrow$}},r_{10}\text{\textcircled{$\downarrow$}},k_1\text{\textcircled{$\downarrow$}},$

$\circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2,$

$IsCpo(j_0; r_{20}), Rcpo(j_0; r_{20}), IsCpo(k_2; r_{20}), Rcpo(k_2; r_{20}), j_0\text{⦽}, r_{20}\text{⦽}, k_2\text{⦽},$

$\Leftrightarrow \;, i \succ j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1, \circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2,$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), IsCpo(k_1; r_{10}), Rcpo(k_1; r_{10}),$

$IsCpo(j_0; r_{20}), Rcpo(j_0; r_{20}), IsCpo(k_2; r_{20}), Rcpo(k_2; r_{20}),$

$i_0\text{⦽}, r_{10}\text{⦽}, k_1\text{⦽}, j_0\text{⦽}, r_{20}\text{⦽}, k_2\text{⦽},$

$\Leftrightarrow \;, i \succ j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1, \circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2,$

$IsCpo(i_0; r_{10}), r_{10}!\text{♂} r_{20}, Rcpo(i_0; r_{10}), IsCpo(k_1; r_{10}), r_{10}!\text{♂} r_{20}, Rcpo(k_1; r_{10}),$

$IsCpo(j_0; r_{20}), Rcpo(j_0; r_{20}), IsCpo(k_2; r_{20}), Rcpo(k_2; r_{20}),$

$i_0\text{⦽}, r_{10}\text{⦽}, k_1\text{⦽}, j_0\text{⦽}, r_{20}\text{⦽}, k_2\text{⦽},$

$\Leftrightarrow \;, i \succ j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1, \circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2,$

$IsCpo(i_0; r_{10}), IsCpo(k_1; r_{10}), r_{10}!\text{♂} r_{20},$

$IsCpo(j_0; r_{20}), IsCpo(k_2; r_{20}),$

$Rcpo(i_0; r_{10}), Rcpo(k_1; r_{10}), Rcpo(j_0; r_{20}), Rcpo(k_2; r_{20}),$

$i_0\text{⦽}, r_{10}\text{⦽}, k_1\text{⦽}, j_0\text{⦽}, r_{20}\text{⦽}, k_2\text{⦽},$

$\Leftrightarrow \;, i \succ j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1, \circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2,$

$IsCpo(i_0; r_{10}), IsCpo(j_0; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i_0; r_{20}), IsCpo(j_0; r_{20}), IsCpo(k_2; r_{20}),$

$r_{10}!\text{♂} r_{20}, k_1 \text{♂} k_2, r_1 \text{♂} r_{10}, r_2 \text{♂} r_{20}, i_0 \succ j_0,$

$Rcpo(i_0; r_{10}), Rcpo(k_1; r_{10}), Rcpo(j_0; r_{20}), Rcpo(k_2; r_{20}),$

$i_0\text{↓⃝}, r_{10}\text{↓⃝}, k_1\text{↓⃝}, j_0\text{↓⃝}, r_{20}\text{↓⃝}, k_2\text{↓⃝},$

$\Leftrightarrow \;\; , i \rhd j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1, \circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2,$

$IsCpo(i_0; r_{10}), IsCpo(j_0; r_{10}), IsCpo(k_1; r_{10}),$

$IsCpo(i_0; r_{20}), IsCpo(j_0; r_{20}), IsCpo(k_2; r_{20}),$

$r_{10}!\text{♂}r_{20}, k_1\text{♂}k_2, r_1\text{♂}r_{10}, r_2\text{♂}r_{20}, i_0 \rhd j_0,$

$Rcpo(i_0; r_{10}), Rcpo(k_1; r_{10}), Rcpo(j_0; r_{20}), Rcpo(k_2; r_{20}), r_1 \rhd r_2,$

$i_0\text{↓⃝}, r_{10}\text{↓⃝}, k_1\text{↓⃝}, j_0\text{↓⃝}, r_{20}\text{↓⃝}, k_2\text{↓⃝},$

$\Leftrightarrow \;\; , i \rhd j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1, \circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2,$

$IsCpo(i_0; r_{10}), r_{10}!\text{♂}r_{20}, Rcpo(i_0; r_{10}), IsCpo(k_1; r_{10}), r_{10}!\text{♂}r_{20}, Rcpo(k_1; r_{10}),$

$IsCpo(j_0; r_{20}), Rcpo(j_0; r_{20}), IsCpo(k_2; r_{20}), Rcpo(k_2; r_{20}),$

$i_0\text{↓⃝}, r_{10}\text{↓⃝}, k_1\text{↓⃝}, j_0\text{↓⃝}, r_{20}\text{↓⃝}, k_2\text{↓⃝}, r_1 \rhd r_2,$

$\Leftrightarrow \;\; , i \rhd j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1,$

$IsCpo(i_0; r_{10}), Rcpo(i_0; r_{10}), IsCpo(k_1; r_{10}), Rcpo(k_1; r_{10}), i_0\text{↓⃝}, r_{10}\text{↓⃝}, k_1\text{↓⃝},$

$\circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2, IsCpo(j_0; r_{20}), Rcpo(j_0; r_{20}), IsCpo(k_2; r_{20}), Rcpo(k_2; r_{20}),$

$j_0\text{↓⃝}, r_{20}\text{↓⃝}, k_2\text{↓⃝}, r_1 \rhd r_2,$

$\Leftrightarrow \;\; , i \rhd j, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, k \text{☮} k_1, Rcpo(i_0; r_{10}), Rcpo(k_1; r_{10}), i_0\text{↓⃝}, r_{10}\text{↓⃝}, k_1\text{↓⃝},$

$\circledcirc r_2, r_2 \text{☮} r_{20}, j \text{☮} j_0, k \text{☮} k_2, Rcpo(j_0; r_{20}), Rcpo(k_2; r_{20}), j_0\text{↓⃝}, r_{20}\text{↓⃝}, k_2\text{↓⃝}, r_1 \rhd r_2,$

$\Leftrightarrow \;\; , i \rhd j, i + k : r_1, j + k : r_2, r_1 \rhd r_2,$

$$, k \,!\!= \varnothing, i + k : r, \;\; \Leftrightarrow \;\; \sim, r \succ i,$$

proof:

$, k \,!\!= \varnothing, i + k : r,$

$\Leftrightarrow \;, \circledcirc m, m = \varnothing, k \,!\!= \varnothing, i + k : r, m\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc m, m = \varnothing, k \,!\!= \varnothing, k \succ m, i + k : r, m\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc m, m = \varnothing, k \,!\!= \varnothing, k \succ m, i + k : r, i + m : r_1, r_1\text{\textcircled{$\downarrow$}}, m\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc m, m = \varnothing, k \,!\!= \varnothing, k \succ m, i + k : r, i + m : r_1, r \succ r_1, r_1\text{\textcircled{$\downarrow$}}, m\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc m, k \,!\!= \varnothing, k \succ m, i + k : r, m = \varnothing, i + m : r_1, r \succ r_1, r_1\text{\textcircled{$\downarrow$}}, m\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc m, k \,!\!= \varnothing, k \succ m, i + k : r, m = \varnothing, i + m : r_1, i \mathrel{\overline{\underline{\vert}}} r_1, r \succ r_1, r_1\text{\textcircled{$\downarrow$}}, m\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc m, k \,!\!= \varnothing, k \succ m, i + k : r, m = \varnothing, i + m : r_1, i \mathrel{\overline{\underline{\vert}}} r_1, r \succ i, r_1\text{\textcircled{$\downarrow$}}, m\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, \circledcirc m, m = \varnothing, k \,!\!= \varnothing, i + k : r, i + m : r_1, r_1\text{\textcircled{$\downarrow$}}, m\text{\textcircled{$\downarrow$}}, r \succ i,$

$\Leftrightarrow \;, k \,!\!= \varnothing, i + k : r, r \succ i,$

$$, i_1 \succ j_1, i_2 \succ j_2, i_1 + i_2 : r_1, j_1 + j_2 : r_2, \;\; \Leftrightarrow \;\; \sim, r_1 \succ r_2,$$

proof:

$, i_1 \succ j_1, i_2 \succ j_2, i_1 + i_2 : r_1, j_1 + j_2 : r_2,$

$\Leftrightarrow \;, i_1 \succ j_1, i_2 \succ j_2, i_1 + i_2 : r_1, i_1 + j_2 : r_3, r_3\text{\textcircled{$\downarrow$}}, j_1 + j_2 : r_2,$

$\Leftrightarrow \;, i_1 \succ j_1, i_2 \succ j_2, i_2 + i_1 : r_1, j_2 + i_1 : r_3, j_1 + j_2 : r_2, r_3\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, i_1 \succ j_1, i_2 \succ j_2, i_2 + i_1 : r_1, j_2 + i_1 : r_3, r_1 \succ r_3, j_1 + j_2 : r_2, r_3\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, i_2 \succ j_2, i_1 + i_2 : r_1, i_1 \succ j_1, i_1 + j_2 : r_3, j_1 + j_2 : r_2, r_1 \succ r_3, r_3\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, i_2 \succ j_2, i_1 + i_2 : r_1, i_1 \succ j_1, i_1 + j_2 : r_3, j_1 + j_2 : r_2, r_3 \succ r_2, r_1 \succ r_3, r_3\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow \;, i_2 \succ j_2, i_1 + i_2 : r_1, i_1 \succ j_1, i_1 + j_2 : r_3, j_1 + j_2 : r_2, r_1 \succ r_3, r_3 \succ r_2, r_1 \succ r_2, r_3\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , i_1 \gtrdot j_1, i_2 \gtrdot j_2, i_1 + i_2 : r_1, j_1 + j_2 : r_2, r_1 \gtrdot r_2,$

31 Recursive Function Rcpm(i;j;r)

31.1 Definition of IsCpm(i;j;r)

$$,IsCpm(i;j;r), \;\Leftrightarrow\; ,i!\bigcirc\!\!^{\circ}\!\!\circ r, j!\bigcirc\!\!^{\circ}\!\!\circ r, r=\varnothing,$$

31.2 Property of IsCpm(i;j;r)

$$,IsCpm(i;j;r), \;\Leftrightarrow\; ,IsCpo(i;r), IsCpo(j;r),$$

$$,IsCpm(i;j;r), \;\Leftrightarrow\; ,IsCpm(j;i;r),$$

$$,\circledcirc r, \;\Leftrightarrow\; \sim, IsCpm(i;j;r),$$

31.3 Swap of IsCpm(i;j;r)

$$,IsCpm(i;j;r), Cpo(r), r\oplus, \;\Leftrightarrow\; ,Cpo(r), r\oplus, IsCpm(i;j;r),$$

$$,IsCpm(i;j;r),\ \&SHi\,\mathring{\bigcirc}i, \;\Leftrightarrow\; ,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r),$$

$$,IsCpm(i;j;r),\ \&SHi\,\mathring{\bigcirc}j, \;\Leftrightarrow\; ,\ \&SHi\,\mathring{\bigcirc}j, IsCpm(i;j;r),$$

$$,IsCpm(i;j;r),\ \&SHi\,\mathring{\bigcirc}m, \;\Leftrightarrow\; ,\ \&SHi\,\mathring{\bigcirc}m, IsCpm(i;j;r),$$

$$,IsCpm(i;j;r), Rcpo(j;r), \;\Leftrightarrow\; \sim, IsCpm(i;j;r),$$

$$,IsCpo(m;r), IsCpm(i;j;r), Rcpo(m;r), \;\Leftrightarrow\; ,IsCpo(m;r), Rcpo(m;r), IsCpm(i;j;r),$$

$$,IsCpo(m;r_1), r_1!\bigcirc\!\!^{\circ}\!\!\circ r_2, IsCpm(i;j;r_2), Rcpo(m;r_1), \;\Leftrightarrow\; ,IsCpo(m;r_1), r_1!\bigcirc\!\!^{\circ}\!\!\circ r_2, Rcpo(m;r_1), IsCpm(i;j;r_2),$$

31.4 Definition of Rcpm(i;j;r)

$$,Rcpm(i;j;r), \iff ,if(i=\varnothing)\begin{bmatrix} , \\ \\ ,j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r), \end{bmatrix},$$

31.5 Property of Rcpm(i;j;r)

$$,i=\varnothing, Rcpm(i;j;r), \iff ,i=\varnothing,$$

$$,i\,!=\varnothing, Rcpm(i;j;r), \iff ,i\,!=\varnothing, j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$$

$$,j=\varnothing, Rcpm(i;j;r), \iff ,j=\varnothing, R(i),$$

induction proof:
premise 1 :
$,i=\varnothing, j=\varnothing, Rcpm(i;j;r),$
$\iff ,j=\varnothing, i=\varnothing,$

$\iff ,j=\varnothing, i=\varnothing, R(i),$

$\iff ,i=\varnothing, j=\varnothing, R(i),$

premise 2 :
$, \&SHi \to i, j=\varnothing, Rcpm(i;j;r), \iff$

$, \&SHi \to i, j=\varnothing, R(i), \implies$

$,i\,!=\varnothing, \&SHi\text{♂}i, j=\varnothing, Rcpm(i;j;r),$

$\iff , \&SHi\text{♂}i, j=\varnothing, i\,!=\varnothing, j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$

$\iff , \&SHi\text{♂}i, j=\varnothing, i\,!=\varnothing, j\text{☮}j_0, j_0=\varnothing, Rcpo(j_0;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$

$\iff , \&SHi\text{♂}i, j=\varnothing, i\,!=\varnothing, i\oplus, Rcpm(i;j;r),$

$\iff ,i\,!=\varnothing, i\oplus, \&SHi \to i, j=\varnothing, Rcpm(i;j;r),$

$\iff ,i\,!=\varnothing, i\oplus, \&SHi \to i, j=\varnothing, R(i),$

$\iff , \&SHi\text{♂}i, j=\varnothing, i\,!=\varnothing, i\oplus, R(i),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, j=\varnothing, R(i),$

$conclusion:$

$, j=\varnothing, Rcpm(i;j;r),\ \Leftrightarrow\ , j=\varnothing, R(i),$

$$IsCpm(i;j;r), Rcpm(i;j;r),\ \Leftrightarrow\ \sim, i=\varnothing, r=\varnothing,$$

induction proof:

$premise\ 1:$

$, i=\varnothing, IsCpm(i;j;r), Rcpm(i;j;r),$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, i=\varnothing, r=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), Rcpm(i;j;r), i=\varnothing, r=\varnothing,$

$premise\ 2:$

$,\ \&SHi\rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r),\ \Leftrightarrow$

$,\ \&SHi\rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), i=\varnothing, r=\varnothing,\ \dashrightarrow$

$, i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r), Rcpm(i;j;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r), i\,!\!=\varnothing, j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⊻}, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\text{☮}j_0,$

$IsCpo(j_0;r), i!\mathring{\bigcirc}r,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r), Rcpo(j_0;r), j_0\text{⊻}, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\text{☮}j_0, IsCpo(j_0;r), i!\mathring{\bigcirc}r, Rcpo(j_0;r), j_0\text{⊻}, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\text{☮}j_0, IsCpo(j_0;r), i!\mathring{\bigcirc}r, Rcpo(j_0;r), j_0\text{⊻}, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), i=\varnothing, r=\varnothing,$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, IsCpm(i;j;r), i\,!\!=\varnothing, j\circledast j_0, Rcpo(j_0;r), j_0\circleddash, i\oplus,$

$Rcpm(i;j;r), i=\varnothing, r=\varnothing,$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i, IsCpm(i;j;r), Rcpm(i;j;r), i=\varnothing, r=\varnothing,$

$conclusion:$

$, IsCpm(i;j;r), Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), i=\varnothing, r=\varnothing,$

$$IsCpm(i;j;r), Rcpm(i;j;r), \otimes,\ \Leftrightarrow\ , \otimes,$$

$$IsCpm(i;j;r), m!\mathring{\bigcirc}r, Rcpm(i;j;r),\ \Leftrightarrow\ \sim, m!\mathring{\bigcirc}r,$$

31.6 Swap of Rcpm(i;j;r)

31.6.1 Operator

$$, IsCpm(i;j;r), \circledcirc m, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), \circledcirc m,$$

$$, IsCpm(i;j;r), m\circleddash, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), m\circleddash,$$

$$, IsCpm(i;j;r), m\circledast m_0, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), m\circledast m_0,$$

$$, IsCpm(i;j;r), j\circledast j_0, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), j\circledast j_0,$$

$$, IsCpm(i;j;r), m!\mathring{\bigcirc}r, m\oplus, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\mathring{\bigcirc}r, Rcpm(i;j;r), m\oplus,$$

induction proof:
$premise\ 1:$
$, i=\varnothing, IsCpm(i;j;r), m!\mathring{\bigcirc}r, m\oplus, Rcpm(i;j;r),$
$\Leftrightarrow\ , IsCpm(i;j;r), m!\mathring{\bigcirc}r, m\oplus, i=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\ , IsCpm(i;j;r), m!♂r, m\oplus, i=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j;r), m!♂r, i=\varnothing, m\oplus,$

$\Leftrightarrow\ , IsCpm(i;j;r), m!♂r, i=\varnothing, Rcpm(i;j;r), m\oplus,$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), m\oplus,$

$premise\ 2:$
$, \&SHi \rightarrow i, IsCpm(i;j;r), m!♂r, m\oplus, Rcpm(i;j;r), \ \Leftrightarrow$

$, \&SHi \rightarrow i, IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), m\oplus, \ \Rightarrow$

$, i\,!\!=\varnothing, \&SHi\, ♂i, IsCpm(i;j;r), m!♂r, m\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , \&SHi\, ♂i, IsCpm(i;j;r), m!♂r, m\oplus,$

$i\,!\!=\varnothing, j⦸j_0, Rcpo(j_0;r), j_0⊕, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , \&SHi\, ♂i, IsCpm(i;j;r), i\,!\!=\varnothing, j⦸j_0,$

$IsCpo(j_0;r), m!♂r, m\oplus, Rcpo(j_0;r), j_0⊕, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , \&SHi\, ♂i, IsCpm(i;j;r), i\,!\!=\varnothing, j⦸j_0,$

$IsCpo(j_0;r), m!♂r, Rcpo(j_0;r), j_0⊕, i\oplus, m!♂r, m\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j⦸j_0, m!♂r, IsCpo(j_0;r), i!⚦r, \&SHi\, ♂i, IsCpm(i;j;r), Rcpo(j_0;r),$

$j_0⊕, i\oplus, m!♂r, m\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j⦸j_0, m!♂r, IsCpo(j_0;r), i!⚦r, Rcpo(j_0;r), j_0⊕, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), m!♂r, m\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j⦸j_0, m!♂r, IsCpo(j_0;r), i!⚦r, Rcpo(j_0;r), j_0⊕, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), m\oplus,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), m!\mathring{\bigcirc} r, i \mathrel{!=} \varnothing, j\text{☮}j_0, IsCpo(j_0;r), i!\bigcirc^{\circ} r, Rcpo(j_0;r), j_0\text{⏚}, i\oplus,$

$Rcpm(i;j;r), m\oplus,$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), m!\mathring{\bigcirc} r, i \mathrel{!=} \varnothing, j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⏚}, i\oplus,$

$Rcpm(i;j;r), m\oplus,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), m!\mathring{\bigcirc} r, Rcpm(i;j;r), m\oplus,$

$conclusion:$

$, IsCpm(i;j;r), m!\mathring{\bigcirc} r, m\oplus, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\mathring{\bigcirc} r, Rcpm(i;j;r), m\oplus,$

$, IsCpm(i;j;r), m!\bigcirc^{\circ} r, m\oplus, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\bigcirc^{\circ} r, Rcpm(i;j;r), m\oplus,$

$, IsCpm(i;j;r), r!\rightarrow m, m\ominus, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), r!\rightarrow m, Rcpm(i;j;r), m\ominus,$

$, IsCpm(i;j;r), m!\bigcirc^{\circ} r, m\ominus, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\bigcirc^{\circ} r, Rcpm(i;j;r), m\ominus,$

31.6.2 Propositions node null

$, IsCpm(i;j;r), m!\mathring{\bigcirc} r, m=\varnothing, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\mathring{\bigcirc} r, Rcpm(i;j;r), m=\varnothing,$

$, IsCpm(i;j;r), m!\mathring{\bigcirc} r, m \mathrel{!=} \varnothing, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\mathring{\bigcirc} r, Rcpm(i;j;r), m \mathrel{!=} \varnothing,$

$, IsCpm(i;j;r), m!\bigcirc^{\circ} r, m=\varnothing, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\bigcirc^{\circ} r, Rcpm(i;j;r), m=\varnothing,$

$, IsCpm(i;j;r), m!\bigcirc^{\circ} r, m \mathrel{!=} \varnothing, Rcpm(i;j;r),\ \Leftrightarrow\ , IsCpm(i;j;r), m!\bigcirc^{\circ} r, Rcpm(i;j;r), m \mathrel{!=} \varnothing,$

$$, IsCpm(i;j;r), m \mathrel{!=} \varnothing, Rcpm(i;j;r), \;\Leftrightarrow\; \sim, m \mathrel{!=} \varnothing,$$

$$, IsCpm(i;j;r), m \mathrel{!=} \varnothing, m\oplus, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m \mathrel{!=} \varnothing, Rcpm(i;j;r), m \mathrel{!=} \varnothing, m\oplus,$$

$$, IsCpm(i;j;r), m!\overset{\circ}{\bigcirc}r, m \mathrel{!=} \varnothing, m\oplus, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m!\overset{\circ}{\bigcirc}r, Rcpm(i;j;r), m \mathrel{!=} \varnothing, m\oplus,$$

$$, IsCpm(i;j;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, m \mathrel{!=} \varnothing, m\oplus, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m!\overset{\circ}{\bigcirc}{}^{\circ}r, Rcpm(i;j;r), m \mathrel{!=} \varnothing, m\oplus,$$

$$, IsCpm(i;j;r), j \mathrel{!=} \varnothing, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), j \mathrel{!=} \varnothing,$$

31.6.3 Propositions identical node

$$, IsCpm(i;j;r), m\overset{\circ}{\bigcirc}j, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), m\overset{\circ}{\bigcirc}j,$$

$$, IsCpm(i;j;r), m!\overset{\circ}{\bigcirc}j, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), m!\overset{\circ}{\bigcirc}j,$$

$$, IsCpm(i;j;r), m\overset{\circ}{\bigcirc}n, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), m\overset{\circ}{\bigcirc}n,$$

$$, IsCpm(i;j;r), m!\overset{\circ}{\bigcirc}n, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), m!\overset{\circ}{\bigcirc}n,$$

31.6.4 Propositions node connectivity

$$,IsCpm(i;j;r), m♂n, Rcpm(i;j;r), \;\Leftrightarrow\; ,IsCpm(i;j;r), Rcpm(i;j;r), m♂n,$$

$$,IsCpm(i;j;r), m!♂n, Rcpm(i;j;r), \;\Leftrightarrow\; ,IsCpm(i;j;r), Rcpm(i;j;r), m!♂n,$$

$$,IsCpm(i;j;r), m♂r, Rcpm(i;j;r), \;\Leftrightarrow\; ,IsCpm(i;j;r), Rcpm(i;j;r), m♂r,$$

induction proof:
premise 1 :
$,i=\varnothing, IsCpm(i;j;r), m♂r, Rcpm(i;j;r),$
$\Leftrightarrow\; ,IsCpm(i;j;r), m♂r, i=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\; ,IsCpm(i;j;r), m♂r, i=\varnothing,$

$\Leftrightarrow\; ,IsCpm(i;j;r), i=\varnothing, m♂r,$

$\Leftrightarrow\; ,IsCpm(i;j;r), i=\varnothing, Rcpm(i;j;r), m♂r,$

$\Leftrightarrow\; ,i=\varnothing, IsCpm(i;j;r), Rcpm(i;j;r), m♂r,$

premise 2 :
$, \&SHi \rightarrow i, IsCpm(i;j;r), m♂r, Rcpm(i;j;r), \;\Leftrightarrow$

$, \&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), m♂r, \;\Rightarrow$

$,i\,!\!=\varnothing, \&SHi ♂i, IsCpm(i;j;r), m♂r, Rcpm(i;j;r),$

$\Leftrightarrow\; , \&SHi ♂i, IsCpm(i;j;r), m♂r,$

$i\,!\!=\varnothing, j\circledast j_0, Rcpo(j_0;r), j_0\ominus, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\; , \&SHi ♂i, IsCpm(i;j;r), m♂r, i\,!\!=\varnothing, j\circledast j_0,$

$IsCpo(j_0;r), i!♂r, Rcpo(j_0;r), j_0\ominus, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\; ,i\,!\!=\varnothing, j\circledast j_0,$

$IsCpo(j_0;r), i!♂r, \&SHi ♂i, IsCpm(i;j;r), m♂r, Rcpo(j_0;r), j_0\ominus, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j☮j_0, IsCpo(j_0;r), i!♂r, Rcpo(j_0;r), j_0\textcircled{\downarrow}, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), m♂r, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j☮j_0, IsCpo(j_0;r), i!♂r, Rcpo(j_0;r), j_0\textcircled{\downarrow}, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), m♂r,$

$\Leftrightarrow\ , \&SHi\,\mathring{O}i, IsCpm(i;j;r), i\,!\!=\varnothing, j☮j_0, IsCpo(j_0;r), i!♂r, Rcpo(j_0;r), j_0\textcircled{\downarrow}, i\oplus,$

$Rcpm(i;j;r), m♂r,$

$\Leftrightarrow\ , \&SHi\,\mathring{O}i, IsCpm(i;j;r), i\,!\!=\varnothing, j☮j_0, Rcpo(j_0;r), j_0\textcircled{\downarrow}, i\oplus,$

$Rcpm(i;j;r), m♂r,$

$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\,\mathring{O}i, IsCpm(i;j;r), Rcpm(i;j;r), m♂r,$

$conclusion:$

$, IsCpm(i;j;r), m♂r, Rcpm(i;j;r), \ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), m♂r,$

$$, IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), \ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), m!♂r,$$

$$, IsCpm(i;j;r), i!♂r, Rcpm(i;j;r), \ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), i!♂r,$$

$$, IsCpm(i;j;r), j!♂r, Rcpm(i;j;r), \ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), j!♂r,$$

31.6.5 IsCpo

$$, IsCpm(i;j;r), IsCpo(m;r), Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), IsCpo(m;r),$$

$$, IsCpm(i;j;r_1), r_1!\mathring{\bigcirc}r_2, IsCpo(m;r_2), Rcpm(i;j;r_1), \;\Leftrightarrow$$
$$, IsCpm(i;j;r_1), r_1!\mathring{\bigcirc}r_2, Rcpm(i;j;r_1), IsCpo(m;r_2),$$

31.6.6 IsCpm

$$, IsCpm(i;j;r), Rcpm(i;j;r), \;\Leftrightarrow\; \sim, IsCpm(i;j;r),$$

$$, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), IsCpm(m;n;r),$$

$$, IsCpm(i;j;r_1), r_1!\mathring{\bigcirc}r_2, IsCpm(m;n;r_2), Rcpm(i;j;r_1), \;\Leftrightarrow$$
$$, IsCpm(i;j;r_1), r_1!\mathring{\bigcirc}r_2, Rcpm(i;j;r_1), IsCpm(m;n;r_2),$$

31.6.7 Cpo

$$, IsCpm(i;j;r), Cpo(r), r\oplus, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), Rcpm(i;j;r), Cpo(r), r\oplus,$$

induction proof:
$premise$ 1 :
$, i=\varnothing, IsCpm(i;j;r), Cpo(r), r\oplus, Rcpm(i;j;r),$
$\Leftrightarrow\; , IsCpm(i;j;r), i!\mathring{\bigcirc}r, i=\varnothing, Cpo(r), r\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\; , IsCpm(i;j;r), i!\mathring{\bigcirc}r, Cpo(r), r\oplus, i=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\; , IsCpm(i;j;r), i!\mathring{\bigcirc}r, Cpo(r), r\oplus, i=\varnothing,$

$\Leftrightarrow\; , IsCpm(i;j;r), i!\mathring{\bigcirc}r, i=\varnothing, Cpo(r), r\oplus,$

$\Leftrightarrow\; , IsCpm(i;j;r), i!\mathring{\bigcirc}r, i=\varnothing, Rcpm(i;j;r), Cpo(r), r\oplus,$

$\Leftrightarrow\; , i=\varnothing, IsCpm(i;j;r), Rcpm(i;j;r), Cpo(r), r\oplus,$

$premise$ 2 :
$, \&SHi \rightarrow i, IsCpm(i;j;r), Cpo(r), r\oplus, Rcpm(i;j;r), \;\Leftrightarrow$

$, \&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), Cpo(r), r\oplus, \Rightarrow$

$, i \,!\!= \varnothing, \&SHi ♂ i, IsCpm(i;j;r), Cpo(r), r\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, \&SHi ♂ i, IsCpm(i;j;r), i!♂r, i \,!\!= \varnothing, Cpo(r), r\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, \&SHi ♂ i, IsCpm(i;j;r), i!♂r, Cpo(r), r\oplus,$

$i \,!\!= \varnothing, j\oslash j_0, Rcpo(j_0;r), j_0⊎, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, i \,!\!= \varnothing, \&SHi ♂ i, IsCpm(i;j;r), j\oslash j_0,$

$IsCpo(j_0;r), Cpo(r), r\oplus, Rcpo(j_0;r), j_0⊎, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, i \,!\!= \varnothing, \&SHi ♂ i, IsCpm(i;j;r), j\oslash j_0,$

$IsCpo(j_0;r), Rcpo(j_0;r), j_0⊎, Cpo(r), r\oplus, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, i \,!\!= \varnothing, \&SHi ♂ i, IsCpm(i;j;r), j\oslash j_0,$

$IsCpo(j_0;r), i!♂r, Rcpo(j_0;r), j_0⊎, Cpo(r), r\oplus, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, i \,!\!= \varnothing, \&SHi ♂ i, IsCpm(i;j;r), j\oslash j_0,$

$IsCpo(j_0;r), Rcpo(j_0;r), j_0⊎, i!♂r, Cpo(r), i\oplus, r\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, i \,!\!= \varnothing, \&SHi ♂ i, IsCpm(i;j;r), j\oslash j_0,$

$IsCpo(j_0;r), Rcpo(j_0;r), j_0⊎, i!♂r, i\oplus, Cpo(r), r\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, i \,!\!= \varnothing, j\oslash j_0, IsCpo(j_0;r), i!♂r, \&SHi ♂ i, IsCpm(i;j;r), Rcpo(j_0;r),$

$j_0⊎, i\oplus, Cpo(r), r\oplus, Rcpm(i;j;r),$

$\Leftrightarrow \;, i \,!\!= \varnothing, j\oslash j_0, IsCpo(j_0;r), i!♂r, Rcpo(j_0;r), j_0⊎, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), Cpo(r), r\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i \,!{=}\, \varnothing, j \oslash j_0, IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0 \circledast, i \oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), Cpo(r), r\oplus,$

$\Leftrightarrow\ , \&SHi\, \text{♂} i, IsCpm(i;j;r), i \,!{=}\, \varnothing, j \oslash j_0, IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0 \circledast, i \oplus,$

$Rcpm(i;j;r), Cpo(r), r\oplus,$

$\Leftrightarrow\ , \&SHi\, \text{♂} i, IsCpm(i;j;r), i \,!{=}\, \varnothing, j \oslash j_0, Rcpo(j_0;r), j_0 \circledast, i \oplus,$

$Rcpm(i;j;r), Cpo(r), r\oplus,$

$\Leftrightarrow\ , i \,!{=}\, \varnothing,\ \&SHi\, \text{♂} i, IsCpm(i;j;r), Rcpm(i;j;r), Cpo(r), r\oplus,$

$conclusion:$

$, IsCpm(i;j;r), Cpo(r), r\oplus, Rcpm(i;j;r), \ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), Cpo(r), r\oplus,$

$$, IsCpm(i;j;r_1), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, Cpo(r_2), Rcpm(i;j;r_1), \ \Leftrightarrow$$
$$, IsCpm(i;j;r_1), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, Rcpm(i;j;r_1), Cpo(r_2),$$

induction proof:

$premise\ 1:$

$, i=\varnothing, IsCpm(i;j;r_1), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, Cpo(r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1!\text{♂}r_2, j!\text{♂}r_2, i!\text{♂}r_2, i=\varnothing, Cpo(r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1!\text{♂}r_2, j!\text{♂}r_2, i!\text{♂}r_2, Cpo(r_2), i=\varnothing, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1!\text{♂}r_2, j!\text{♂}r_2, i!\text{♂}r_2, Cpo(r_2), i=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1!\text{♂}r_2, j!\text{♂}r_2, i!\text{♂}r_2, i=\varnothing, Cpo(r_2),$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1!\text{♂}r_2, j!\text{♂}r_2, i!\text{♂}r_2, i=\varnothing, Rcpm(i;j;r_1), Cpo(r_2),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r_1), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, Rcpm(i;j;r_1), Cpo(r_2),$

premise 2 :

$, \&SHi \rightarrow i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, Cpo(r_2), Rcpm(i;j;r_1), \;\; \Leftrightarrow$

$, \&SHi \rightarrow i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, Rcpm(i;j;r_1), Cpo(r_2), \;\; \Rightarrow$

$, i \mathrel{!=} \varnothing, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, Cpo(r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow \;, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, i \mathrel{!=} \varnothing, Cpo(r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow \;, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, i \mathrel{!=} \varnothing, Cpo(r_2),$

$i \mathrel{!=} \varnothing, j ⊘ j_0, Rcpo(j_0;r_1), j_0 ⦷, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \;, i \mathrel{!=} \varnothing, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, j ⊘ j_0,$

$IsCpo(j_0;r_1), r_1!♂r_2, j_0!♂r_2, Cpo(r_2), Rcpo(j_0;r_1), j_0 ⦷, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \;, i \mathrel{!=} \varnothing, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, j ⊘ j_0,$

$IsCpo(j_0;r_1), r_1!♂r_2, j_0!♂r_2, Rcpo(j_0;r_1), j_0 ⦷, Cpo(r_2), i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \;, i \mathrel{!=} \varnothing, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, j ⊘ j_0,$

$IsCpo(j_0;r_1), i!♂r_2, Rcpo(j_0;r_1), j_0 ⦷, Cpo(r_2), i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \;, i \mathrel{!=} \varnothing, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, j ⊘ j_0,$

$IsCpo(j_0;r_1), Rcpo(j_0;r_1), j_0 ⦷, i!♂r_2, Cpo(r_2), i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \;, i \mathrel{!=} \varnothing, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, j ⊘ j_0,$

$IsCpo(j_0;r_1), Rcpo(j_0;r_1), j_0 ⦷, i!♂r_2, i \oplus, Cpo(r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow \;, i \mathrel{!=} \varnothing, \&SHi\, ♂ i, IsCpm(i;j;r_1), r_1!♂r_2, i!♂r_2, j!♂r_2, j ⊘ j_0,$

$IsCpo(j_0;r_1), i!♂r_1, Rcpo(j_0;r_1), j_0 ⦷, i \oplus, Cpo(r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing,\ \&SHi\,\mathring{\bigcirc}i, j\text{☮}j_0,$

$IsCpo(j_0; r_1), i!\mathring{\bigcirc}\!\circ r_1, Rcpo(j_0; r_1), j_0\text{⏬}, i\oplus,$

$IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, Cpo(r_2), Rcpm(i; j; r_1),$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing, j\text{☮}j_0,$

$IsCpo(j_0; r_1), i!\mathring{\bigcirc}\!\circ r_1,\ \&SHi\,\mathring{\bigcirc}i, Rcpo(j_0; r_1), j_0\text{⏬}, i\oplus,$

$IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, Cpo(r_2), Rcpm(i; j; r_1),$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing, j\text{☮}j_0, IsCpo(j_0; r_1), i!\mathring{\bigcirc}\!\circ r_1, Rcpo(j_0; r_1), j_0\text{⏬}, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, Cpo(r_2), Rcpm(i; j; r_1),$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing, j\text{☮}j_0, IsCpo(j_0; r_1), i!\mathring{\bigcirc}\!\circ r_1, Rcpo(j_0; r_1), j_0\text{⏬}, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, Rcpm(i; j; r_1), Cpo(r_2),$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2,$
$j\text{☮}j_0, IsCpo(j_0; r_1), i!\mathring{\bigcirc}\!\circ r_1, Rcpo(j_0; r_1), j_0\text{⏬}, i\oplus,$

$Rcpm(i; j; r_1), Cpo(r_2),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, i \mathrel{!\!=} \varnothing,$
$j\text{☮}j_0, Rcpo(j_0; r_1), j_0\text{⏬}, i\oplus, Rcpm(i; j; r_1), Cpo(r_2),$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, Rcpm(i; j; r_1), Cpo(r_2),$

$conclusion:$

$, IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, Cpo(r_2), Rcpm(i; j; r_1),\ \Leftrightarrow$

$, IsCpm(i; j; r_1), r_1!\mathring{\bigcirc}\!\circ r_2, i!\mathring{\bigcirc}\!\circ r_2, j!\mathring{\bigcirc}\!\circ r_2, Rcpm(i; j; r_1), Cpo(r_2),$

$$, IsCpm(i;j;r_1), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, Cpo(r_2), r_2\oplus, Rcpm(i;j;r_1), \;\Leftrightarrow$$
$$, IsCpm(i;j;r_1), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, Rcpm(i;j;r_1), Cpo(r_2), r_2\oplus,$$

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$$, IsCpm(i;j;r), IsCpo(m;r), Rcpo(m;r), Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), IsCpo(m;r), Rcpm(i;j;r), Rcpo(m;r),$$

induction proof:
premise 1 :
$, i=\varnothing, IsCpm(i;j;r), IsCpo(m;r), Rcpo(m;r), Rcpm(i;j;r),$
$\Leftrightarrow\;, IsCpm(i;j;r), IsCpo(m;r), i!\text{♂}r, i=\varnothing, Rcpo(m;r), Rcpm(i;j;r),$

$\Leftrightarrow\;, IsCpm(i;j;r), IsCpo(m;r), i!\text{♂}r, Rcpo(m;r), i=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\;, IsCpm(i;j;r), IsCpo(m;r), i!\text{♂}r, Rcpo(m;r), i=\varnothing,$

$\Leftrightarrow\;, IsCpm(i;j;r), IsCpo(m;r), i!\text{♂}r, i=\varnothing, Rcpo(m;r),$

$\Leftrightarrow\;, IsCpm(i;j;r), IsCpo(m;r), i!\text{♂}r, i=\varnothing, Rcpm(i;j;r), Rcpo(m;r),$

$\Leftrightarrow\;, i=\varnothing, IsCpm(i;j;r), IsCpo(m;r), Rcpm(i;j;r), Rcpo(m;r),$

premise 2 :
$, \&SHi\rightarrow i, IsCpm(i;j;r), IsCpo(m;r), Rcpo(m;r), Rcpm(i;j;r), \;\Leftrightarrow$

$, \&SHi\rightarrow i, IsCpm(i;j;r), IsCpo(m;r), Rcpm(i;j;r), Rcpo(m;r), \;\Rightarrow$

$, i\,!\!=\varnothing, \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpo(m;r), Rcpo(m;r), Rcpm(i;j;r),$

$\Leftrightarrow\;, \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpo(m;r), i\,!\!=\varnothing, Rcpo(m;r), Rcpm(i;j;r),$

$\Leftrightarrow\;, \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpo(m;r), i\,!\!=\varnothing, Rcpo(m;r), i\,!\!=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\;, \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpo(m;r), i\,!\!=\varnothing, Rcpo(m;r),$

$i\,!\!=\varnothing, j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\;, i\,!\!=\varnothing, j\text{☮}j_0, \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r),$

$IsCpo(m;r), IsCpo(j_0;r), Rcpo(m;r), Rcpo(j_0;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\text{☮}j_0,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r),$

$IsCpo(m;r), IsCpo(j_0;r), Rcpo(j_0;r), Rcpo(m;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\text{☮}j_0,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r),$

$IsCpo(j_0;r), Rcpo(j_0;r),$

$IsCpo(m;r), Rcpo(m;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\text{☮}j_0,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r),$

$IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0\text{⏚},$

$IsCpo(m;r), i!\text{♂}r, Rcpo(m;r), i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\text{☮}j_0,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r),$

$IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0\text{⏚}, i\oplus,$

$IsCpo(m;r), i!\text{♂}r, Rcpo(m;r), Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\text{☮}j_0,$

$IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0\text{⏚}, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), IsCpo(m;r), Rcpo(m;r), Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\text{☮}j_0,$

$IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0\text{⏚}, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), IsCpo(m;r), Rcpm(i;j;r), Rcpo(m;r),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r), IsCpo(m;r), i\,!{=}\,\varnothing, j\text{☮}j_0,$

$IsCpo(j_0; r), i!\text{♂}r, Rcpo(j_0; r), j_0\text{⏚}, i\oplus,$

$Rcpm(i; j; r), Rcpo(m; r),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpm(i; j; r), IsCpo(m; r), i\,!\!=\varnothing, j\text{☮}j_0,$

$Rcpo(j_0; r), j_0\text{⏚}, i\oplus, Rcpm(i; j; r), Rcpo(m; r),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i; j; r), IsCpo(m; r), Rcpm(i; j; r), Rcpo(m; r),$

conclusion :

$, IsCpm(i; j; r), IsCpo(m; r), Rcpo(m; r), Rcpm(i; j; r),\ \ \Leftrightarrow$

$, IsCpm(i; j; r), IsCpo(m; r), Rcpm(i; j; r), Rcpo(m; r),$

$$
\begin{gathered}
, IsCpm(i; j; r_1), IsCpo(m; r_2), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, m!\text{♂}r_1, \\
Rcpo(m; r_2), Rcpm(i; j; r_1),\ \ \Leftrightarrow \\
, IsCpm(i; j; r_1), IsCpo(m; r_2), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, m!\text{♂}r_1, \\
Rcpm(i; j; r_1), Rcpo(m; r_2),
\end{gathered}
$$

induction proof:
premise 1 :
$, i=\varnothing, IsCpm(i; j; r_1), IsCpo(m; r_2), r_1!\text{♂}r_2, i!\text{♂}r_2, j!\text{♂}r_2, m!\text{♂}r_1,$

$Rcpo(m; r_2), Rcpm(i; j; r_1),$

$\Leftrightarrow\ , IsCpm(i; j; r_1), r_1!\text{♂}r_2, j!\text{♂}r_2, m!\text{♂}r_1,$

$IsCpo(m; r_2), i!\text{♂}r_2, i=\varnothing, Rcpo(m; r_2), Rcpm(i; j; r_1),$

$\Leftrightarrow\ , IsCpm(i; j; r_1), r_1!\text{♂}r_2, j!\text{♂}r_2, m!\text{♂}r_1,$

$IsCpo(m; r_2), i!\text{♂}r_2, Rcpo(m; r_2), i=\varnothing, Rcpm(i; j; r_1),$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$IsCpo(m;r_2), i\text{!♂}r_2, Rcpo(m;r_2), i=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$IsCpo(m;r_2), i\text{!♂}r_2, i=\varnothing, Rcpo(m;r_2),$

$\Leftrightarrow\ , IsCpm(i;j;r_1), r_1\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$IsCpo(m;r_2), i\text{!♂}r_2, i=\varnothing, Rcpm(i;j;r_1), Rcpo(m;r_2),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1\text{!♂}r_2, i\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$Rcpm(i;j;r_1), Rcpo(m;r_2),$

$premise\ 2:$
$,\ \&SHi \rightarrow i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1\text{!♂}r_2, i\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$Rcpo(m;r_2), Rcpm(i;j;r_1),\ \ \Leftrightarrow$

$,\ \&SHi \rightarrow i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1\text{!♂}r_2, i\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$Rcpm(i;j;r_1)Rcpo(m;r_2),,\ \ \Rightarrow$

$, i\,!\!=\varnothing,\ \&SHi\ \text{♂}i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1\text{!♂}r_2, i\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$Rcpo(m;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ ,\ \&SHi\ \text{♂}i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1\text{!♂}r_2, i\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$IsCpo(m;r_2), i\,!\!=\varnothing, Rcpo(m;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ ,\ \&SHi\ \text{♂}i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1\text{!♂}r_2, i\text{!♂}r_2, j\text{!♂}r_2, m\text{!♂}r_1,$

$IsCpo(m;r_2), i \,!{=}\, \varnothing, Rcpo(m;r_2),$

$i \,!{=}\, \varnothing, j \oslash j_0, Rcpo(j_0;r_1), j_0 \circleddash, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i \,!{=}\, \varnothing,\ \&SHi\, \mathring{\bigcirc} i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$j \oslash j_0, IsCpo(m;r_2), Rcpo(m;r_2),$

$Rcpo(j_0;r_1), j_0 \circleddash, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i \,!{=}\, \varnothing,\ \&SHi\, \mathring{\bigcirc} i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$j \oslash j_0, IsCpo(m;r_2), IsCpo(j_0;r_1), r_1 ! \mathring{\bigcirc} r_2, j_0 ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1, Rcpo(m;r_2),$

$Rcpo(j_0;r_1), j_0 \circleddash, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i \,!{=}\, \varnothing,\ \&SHi\, \mathring{\bigcirc} i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$j \oslash j_0, IsCpo(m;r_2), IsCpo(j_0;r_1), r_1 ! \mathring{\bigcirc} r_2, j_0 ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1, Rcpo(j_0;r_1),$

$Rcpo(m;r_2), j_0 \circleddash, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i \,!{=}\, \varnothing,\ \&SHi\, \mathring{\bigcirc} i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$j \oslash j_0, IsCpo(j_0;r_1), r_1 ! \mathring{\bigcirc} r_2, Rcpo(j_0;r_1),$

$IsCpo(m;r_2), Rcpo(m;r_2), j_0 \circleddash, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i \,!{=}\, \varnothing,\ \&SHi\, \mathring{\bigcirc} i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$j \oslash j_0, IsCpo(j_0;r_1), r_1 ! \mathring{\bigcirc} r_2, Rcpo(j_0;r_1), j_0 \circleddash,$

$IsCpo(m;r_2), i ! \mathring{\bigcirc} r_2, Rcpo(m;r_2), i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i \,!{=}\, \varnothing,\ \&SHi\, \mathring{\bigcirc} i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$j \oslash j_0, IsCpo(j_0;r_1), r_1 ! \mathring{\bigcirc} r_2, Rcpo(j_0;r_1), j_0 \circleddash, i \oplus,$

$IsCpo(m;r_2), i!$♂$r_2, Rcpo(m;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow$ $, i \mathrel{!=} \varnothing,$ $\&SHi$ ♂$i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1!$♂$r_2, i!$♂$r_2, j!$♂$r_2, m!$♂$r_1,$

j⊘$j_0, IsCpo(j_0;r_1), r_1!$♂$r_2, i!$♂$r_1, Rcpo(j_0;r_1), j_0$⊕$, i\oplus,$

$IsCpo(m;r_2), Rcpo(m;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow$ $, i \mathrel{!=} \varnothing, j$⊘$j_0, IsCpo(j_0;r_1), r_1!$♂$r_2, i!$♂$r_1, Rcpo(j_0;r_1), j_0$⊕$, i\oplus,$

$\&SHi \to i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1!$♂$r_2, i!$♂$r_2, j!$♂$r_2, m!$♂$r_1,$

$Rcpo(m;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow$ $, i \mathrel{!=} \varnothing, j$⊘$j_0, IsCpo(j_0;r_1), r_1!$♂$r_2, i!$♂$r_1, Rcpo(j_0;r_1), j_0$⊕$, i\oplus,$

$\&SHi \to i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1!$♂$r_2, i!$♂$r_2, j!$♂$r_2, m!$♂$r_1,$

$Rcpm(i;j;r_1), Rcpo(m;r_2),$

$\Leftrightarrow$ $, i \mathrel{!=} \varnothing,$ $\&SHi$ ♂$i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1!$♂$r_2, i!$♂$r_2, j!$♂$r_2, m!$♂$r_1,$

j⊘$j_0, IsCpo(j_0;r_1), r_1!$♂$r_2, i!$♂$r_1, Rcpo(j_0;r_1), j_0$⊕$, i\oplus,$

$Rcpm(i;j;r_1), Rcpo(m;r_2),$

$\Leftrightarrow$ $,$ $\&SHi$ ♂$i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1!$♂$r_2, i!$♂$r_2, j!$♂$r_2, m!$♂$r_1,$

$i \mathrel{!=} \varnothing, j$⊘$j_0, Rcpo(j_0;r_1), j_0$⊕$, i\oplus,$

$Rcpm(i;j;r_1), Rcpo(m;r_2),$

$\Leftrightarrow$ $, i \mathrel{!=} \varnothing,$ $\&SHi$ ♂$i, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1!$♂$r_2, i!$♂$r_2, j!$♂$r_2, m!$♂$r_1,$

$Rcpm(i;j;r_1), Rcpo(m;r_2),$

$conclusion:$

$, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$Rcpo(m;r_2), Rcpm(i;j;r_1), \ \Leftrightarrow$

$, IsCpm(i;j;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2, i ! \mathring{\bigcirc} r_2, j ! \mathring{\bigcirc} r_2, m ! \mathring{\bigcirc} r_1,$

$Rcpm(i;j;r_1), Rcpo(m;r_2),$

$$, IsCpm(i;j;r_1), IsCpm(i;j;r_2), IsCpo(m;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2,$$
$$Rcpo(m;r_2), Rcpm(i;j;r_1), \ \Leftrightarrow$$
$$, IsCpm(i;j;r_1), IsCpm(i;j;r_2), IsCpo(m;r_1), IsCpo(m;r_2), r_1 ! \mathring{\bigcirc} r_2,$$
$$Rcpm(i;j;r_1), Rcpo(m;r_2),$$

31.6.9 Rcpm

$$, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(m;n;r), Rcpm(i;j;r), \ \Leftrightarrow$$
$$, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(i;j;r), Rcpm(m;n;r),$$

induction proof:
premise 1 :
$, i = \varnothing, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(m;n;r), Rcpm(i;j;r),$
$\Leftrightarrow \ , IsCpm(i;j;r), IsCpm(m;n;r), i ! \mathring{\bigcirc} r, i = \varnothing, Rcpm(m;n;r), Rcpm(i;j;r),$

$\Leftrightarrow \ , IsCpm(i;j;r), IsCpm(m;n;r), i ! \mathring{\bigcirc} r, Rcpm(m;n;r), i = \varnothing, Rcpm(i;j;r),$

$\Leftrightarrow \ , IsCpm(i;j;r), IsCpm(m;n;r), i ! \mathring{\bigcirc} r, Rcpm(m;n;r), i = \varnothing,$

$\Leftrightarrow \ , IsCpm(i;j;r), IsCpm(m;n;r), i ! \mathring{\bigcirc} r, i = \varnothing, Rcpm(m;n;r),$

$\Leftrightarrow \ , IsCpm(i;j;r), IsCpm(m;n;r), i ! \mathring{\bigcirc} r, i = \varnothing, Rcpm(i;j;r), Rcpm(m;n;r),$

$\Leftrightarrow \ , i = \varnothing, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(i;j;r), Rcpm(m;n;r),$

premise 2 :
$, \ \&SHi \rightarrow i, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(m;n;r), Rcpm(i;j;r), \ \Leftrightarrow$

$, \ \&SHi \rightarrow i, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(i;j;r), Rcpm(m;n;r), \ \Rightarrow$

$, i \, != \varnothing, \ \&SHi \, \mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(m;n;r), Rcpm(i;j;r),$

$\Leftrightarrow$, &SHi ♂$i, IsCpm(i;j;r), IsCpm(m;n;r), i\,!\!=\varnothing, Rcpm(m;n;r), Rcpm(i;j;r),$

$\Leftrightarrow$, &SHi ♂$i, IsCpm(i;j;r), IsCpm(m;n;r), i\,!\!=\varnothing, Rcpm(m;n;r),$

$i\,!\!=\varnothing, j$☮$j_0, Rcpo(j_0;r), j_0$⊕$, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow$ $, i\,!\!=\varnothing,$ &SHi ♂$i, IsCpm(i;j;r), j$☮$j_0,$

$IsCpm(m;n;r), IsCpo(j_0;r), Rcpm(m;n;r),$

$Rcpo(j_0;r), j_0$⊕$, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow$ $, i\,!\!=\varnothing,$ &SHi ♂$i, IsCpm(i;j;r), j$☮$j_0,$

$IsCpm(m;n;r), IsCpo(j_0;r), Rcpo(j_0;r),$

$Rcpm(m;n;r), j_0$⊕$, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow$ $, i\,!\!=\varnothing,$ &SHi ♂$i, IsCpm(i;j;r), j$☮$j_0,$

$IsCpo(j_0;r), Rcpo(j_0;r),$

$IsCpm(m;n;r), i!$♂$r, Rcpm(m;n;r), j_0$⊕$, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow$ $, i\,!\!=\varnothing,$ &SHi ♂$i, IsCpm(i;j;r), j$☮$j_0,$

$IsCpo(j_0;r), Rcpo(j_0;r), j_0$⊕$, i\oplus,$

$IsCpm(m;n;r), i!$♂$r, Rcpm(m;n;r), Rcpm(i;j;r),$

$\Leftrightarrow$ $, i\,!\!=\varnothing,$ &SHi ♂$i, IsCpm(i;j;r), j$☮$j_0,$

$IsCpo(j_0;r), i!$♂$r, Rcpo(j_0;r), j_0$⊕$, i\oplus,$

$IsCpm(m;n;r), Rcpm(m;n;r), Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\text{☮}j_0,$

$IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0\circledast, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(m;n;r), Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\text{☮}j_0,$

$IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0\circledast, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(i;j;r), Rcpm(m;n;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r), IsCpm(m;n;r), j\text{☮}j_0,$

$IsCpo(j_0;r), i!\text{♂}r, Rcpo(j_0;r), j_0\circledast, i\oplus,$

$Rcpm(i;j;r), Rcpm(m;n;r),$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpm(i;j;r), IsCpm(m;n;r), i\,!\!=\varnothing, j\text{☮}j_0,$

$Rcpo(j_0;r), j_0\circledast, i\oplus, Rcpm(i;j;r), Rcpm(m;n;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\text{♂}iIsCpm(i;j;r), IsCpm(m;n;r), Rcpm(i;j;r), Rcpm(m;n;r),$

$conclusion:$

$, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(m;n;r), Rcpm(i;j;r),\ \Leftrightarrow$

$, IsCpm(i;j;r), IsCpm(m;n;r), Rcpm(i;j;r), Rcpm(m;n;r),$

$$, IsCpm(i;k;r), IsCpm(m;k;r), Rcpm(m;k;r), Rcpm(i;k;r),\ \Leftrightarrow$$
$$, IsCpm(i;k;r), IsCpm(m;k;r), Rcpm(i;k;r), Rcpm(m;k;r),$$

$$, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1!\text{♂}r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$$
$$Rcpm(m;n;r_2), Rcpm(i;j;r_1),\ \Leftrightarrow$$

$$, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$$
$$Rcpm(i;j;r_1), Rcpm(m;n;r_2),$$

induction proof:
premise 1 :
$, i = \varnothing, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(m;n;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow \; , IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$IsCpo(m;r_2), i ! \circlearrowleft r_2, i = \varnothing, Rcpm(m;n;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow \; , IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$IsCpo(m;r_2), i ! \circlearrowleft r_2, Rcpm(m;n;r_2), i = \varnothing, Rcpm(i;j;r_1),$

$\Leftrightarrow \; , IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$IsCpo(m;r_2), i ! \circlearrowleft r_2, Rcpm(m;n;r_2), i = \varnothing,$

$\Leftrightarrow \; , IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$IsCpo(m;r_2), i ! \circlearrowleft r_2, i = \varnothing, Rcpm(m;n;r_2),$

$\Leftrightarrow \; , IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$IsCpo(m;r_2), i ! \circlearrowleft r_2, i = \varnothing, Rcpm(i;j;r_1), Rcpm(m;n;r_2),$

$\Leftrightarrow \; , i = \varnothing, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \circlearrowleft r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(i;j;r_1), Rcpm(m;n;r_2),$

$premise\ 2:$

$, \ \&SHi \rightarrow i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(m;n;r_2), Rcpm(i;j;r_1), \quad \Leftrightarrow$

$, \ \&SHi \rightarrow i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(i;j;r_1), Rcpm(m;n;r_2), \quad \Longrightarrow$

$, i \,!{=}\, \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(m;n;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$IsCpm(m;n;r_2), i \,!{=}\, \varnothing, Rcpm(m;n;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow \ , \ \&SHi \overset{\circ}{\bigcirc} i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$IsCpm(m;n;r_2), i \,!{=}\, \varnothing, Rcpm(m;n;r_2),$

$i \,!{=}\, \varnothing, j \text{☮} j_0, Rcpo(j_0;r_1), j_0 \text{⏚}, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \ , i \,!{=}\, \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$j \text{☮} j_0, IsCpm(m;n;r_2), Rcpm(m;n;r_2),$

$Rcpo(j_0;r_1), j_0 \text{⏚}, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \ , i \,!{=}\, \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$j \text{☮} j_0, IsCpm(m;n;r_2), IsCpm(m;n;r_1), IsCpo(j_0;r_1), IsCpo(j_0;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, Rcpm(m;n;r_2),$

$Rcpo(j_0;r_1), j_0 \text{⏚}, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow \ , i \,!{=}\, \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$j \text{☮} j_0, IsCpm(m;n;r_2), IsCpm(m;n;r_1), IsCpo(j_0;r_1), IsCpo(j_0;r_2), r_1 ! \bigcirc\!\!^{\circ} r_2, Rcpo(j_0;r_1),$

$Rcpm(m;n;r_2), j_0 \text{⏚}, i \oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1!\text{♂}r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$j\oslash j_0, IsCpo(j_0;r_1), r_1!\text{♂}r_2, Rcpo(j_0;r_1),$

$IsCpm(m;n;r_2), i!\text{♂}r_2, Rcpm(m;n;r_2), j_0\oplus, i\oplus, Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1!\text{♂}r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$j\oslash j_0, IsCpo(j_0;r_1), r_1!\text{♂}r_2, Rcpo(j_0;r_1), j_0\oplus, i\oplus,$

$IsCpm(m;n;r_2), i!\text{♂}r_2, Rcpm(m;n;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1!\text{♂}r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$j\oslash j_0, IsCpo(j_0;r_1), i!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(j_0;r_1), j_0\oplus, i\oplus,$

$IsCpm(m;n;r_2), Rcpm(m;n;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\oslash j_0, IsCpo(j_0;r_1), i!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(j_0;r_1), j_0\oplus, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1!\text{♂}r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(m;n;r_2), Rcpm(i;j;r_1),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\oslash j_0, IsCpo(j_0;r_1), i!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(j_0;r_1), j_0\oplus, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1!\text{♂}r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(i;j;r_1), Rcpm(m;n;r_2),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1!\text{♂}r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$j\oslash j_0, IsCpo(j_0;r_1), i!\text{♂}r_1, r_1!\text{♂}r_2, Rcpo(j_0;r_1), j_0\oplus, i\oplus,$

$Rcpm(i;j;r_1), Rcpm(m;n;r_2),$

$\Leftrightarrow$ $, \&SHi \text{♁}i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 !\text{♁} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$i\,!\!= \varnothing, j\text{☮}j_0, Rcpo(j_0;r_1), j_0\text{⏚}, i\oplus,$

$Rcpm(i;j;r_1), Rcpm(m;n;r_2),$

$\Leftrightarrow$ $, i\,!\!= \varnothing, \&SHi \text{♁}i, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 !\text{♁} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(i;j;r_1), Rcpm(m;n;r_2),$

conclusion :

$, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 !\text{♁} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(m;n;r_2), Rcpm(i;j;r_1), \ \Leftrightarrow$

$, IsCpm(i;j;r_1), IsCpm(m;n;r_2), r_1 !\text{♁} r_2, IsCpm(i;j;r_2), IsCpm(m;n;r_1),$

$Rcpm(i;j;r_1), Rcpm(m;n;r_2),$

$$, IsCpm(i;k;r_1), IsCpm(m;k;r_2), r_1 !\text{♁} r_2, IsCpm(i;k;r_2), IsCpm(m;k;r_1),$$
$$Rcpm(m;k;r_2), Rcpm(i;k;r_1), \ \Leftrightarrow$$
$$, IsCpm(i;k;r_1), IsCpm(m;k;r_2), r_1 !\text{♁} r_2, IsCpm(i;k;r_2), IsCpm(m;k;r_1),$$
$$Rcpm(i;k;r_1), Rcpm(m;k;r_2),$$

31.6.10 R(m)

$, IsCpm(i;j;r), m!\text{♁}r, R(m), Rcpm(i;j;r), \ \Leftrightarrow \ , IsCpm(i;j;r), m!\text{♁}r, Rcpm(i;j;r), R(m),$

31.6.11 Rc(m;n)

$$, IsCpm(i;j;r), m!\text{♁}r, n!\text{♁}r, Rc(m;n), Rcpm(i;j;r), \ \Leftrightarrow$$
$$, IsCpm(i;j;r), m!\text{♁}r, n!\text{♁}r, Rcpm(i;j;r), Rc(m;n),$$

31.6.12 Propositions number comparison

$$, IsCpm(i;j;r), m!♂r, n!♂r, m{=}n, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m!♂r, n!♂r, Rcpm(i;j;r), m{=}n,$$

$$, IsCpm(i;j;r), m!♂r, n!♂r, m!{=}n, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m!♂r, n!♂r, Rcpm(i;j;r), m!{=}n,$$

$$, IsCpm(i;j;r), m!♂r, n!♂r, m{>}n, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m!♂r, n!♂r, Rcpm(i;j;r), m{>}n,$$

$$, IsCpm(i;j;r), m!♂r, n!♂r, m!{>}n, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m!♂r, n!♂r, Rcpm(i;j;r), m!{>}n,$$

$$, IsCpm(i;j;r), m!♂r, m{=}j, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), m{=}j,$$

$$, IsCpm(i;j;r), m!♂r, m!{=}j, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), m!{=}j,$$

$$, IsCpm(i;j;r), m!♂r, m{>}j, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), m{>}j,$$

$$, IsCpm(i;j;r), m!♂r, m!{>}j, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), m!{>}j,$$

$$, IsCpm(i;j;r), m!♂r, j{>}m, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), j{>}m,$$

$$, IsCpm(i;j;r), m!♂r, j!{>}m, Rcpm(i;j;r), \;\Leftrightarrow\; , IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), j!{>}m,$$

31.6.13 &SHi

$$, IsCpm(i;j;r), m!♂r, \&SHi ○m, Rcpm(i;j;r), \;\Leftrightarrow$$
$$, IsCpm(i;j;r), m!♂r, Rcpm(i;j;r), \&SHi ○m,$$

31.6.14 Swap in Rcpm

$$, IsCpm(i;j;r), j\ominus, j\,!\!=\varnothing, Rcpm(i;j;r), \quad \Leftrightarrow$$

$$, IsCpm(i;j;r), i\text{☮}i_0, Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j\,!\!=\varnothing,$$

induction proof:

$premise$ 1 :

$, i=\varnothing, IsCpm(i;j;r), j\ominus, j\,!\!=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\ , IsCpm(i;j;r), j\ominus, j\,!\!=\varnothing, i=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\ , IsCpm(i;j;r), j\ominus, j\,!\!=\varnothing, i=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, j\ominus, j\,!\!=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, Rcpm(i;j;r), j\ominus, j\,!\!=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, i\text{☮}i_0, Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j\,!\!=\varnothing,$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), i\text{☮}i_0, Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j\,!\!=\varnothing,$

$premise$ 2 :

$, \&SHi\rightarrow i, IsCpm(i;j;r), j\ominus, j\,!\!=\varnothing, Rcpm(i;j;r), \quad \Leftrightarrow$

$, \&SHi\rightarrow i, IsCpm(i;j;r), i\text{☮}i_0, Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j\,!\!=\varnothing, \quad \Rightarrow$

$, i\,!\!=\varnothing, \&SHi\,\text{♂}i, IsCpm(i;j;r), j\ominus, j\,!\!=\varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpm(i;j;r), j\ominus, j\,!\!=\varnothing,$

$i\,!\!=\varnothing, j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⏚}, i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\,\text{♂}i, IsCpm(i;j;r),$

$IsCpo(j;r), j\ominus, j\,!\!=\varnothing, j\text{☮}j_0, Rcpo(j_0;r), j_0\text{⏚},$

$i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i\,!\!=\varnothing, \&SHi\,\text{♂}i, IsCpm(i;j;r),$

$IsCpo(j;r), Cpo(r), r\oplus, j\text{☮}j_1, Rcpo(j_1;r), j_1\text{⏚}, j\ominus, j\,!\!=\varnothing,$

$i\oplus, Rcpm(i;j;r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} i, IsCpm(i;j;r),$

$i!\bigcirc\!\!^{\circ\circ} r, Cpo(r), r\oplus, j\circledast j_1,$

$IsCpo(j_1;r), i!\bigcirc\!\!^{\circ\circ} r, Rcpo(j_1;r), j_1\circledcirc, i\oplus,$

$j\ominus, j \mathrel{!=} \varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,$

$i!\bigcirc\!\!^{\circ\circ} r, Cpo(r), r\oplus, j\circledast j_1,$

$IsCpo(j_1;r), i!\bigcirc\!\!^{\circ\circ} r, Rcpo(j_1;r), j_1\circledcirc, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing, Rcpm(i;j;r),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,$

$i!\bigcirc\!\!^{\circ\circ} r, Cpo(r), r\oplus, j\circledast j_1,$

$IsCpo(j_1;r), i!\bigcirc\!\!^{\circ\circ} r, Rcpo(j_1;r), j_1\circledcirc, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), i\circledast i_0, Rcpo(i_0;r), i_0\circledcirc, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} i, IsCpm(i;j;r),$

$i!\bigcirc\!\!^{\circ\circ} r, Cpo(r), r\oplus, j\circledast j_1,$

$IsCpo(j_1;r), i!\bigcirc\!\!^{\circ\circ} r, Rcpo(j_1;r), j_1\circledcirc, i\oplus,$

$i\circledast i_0, IsCpo(i_0;r), Rcpo(i_0;r), i_0\circledcirc, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} i, IsCpm(i;j;r), j\circledast j_1,$

$IsCpo(j_1;r), Cpo(r), r\oplus, Rcpo(j_1;r), j_1\circledcirc, i\oplus,$

$i\circledast i_0, IsCpo(i_0;r), Rcpo(i_0;r), i_0\circledcirc, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \text{⚲} i, IsCpm(i;j;r), j \text{☮} j_1,$

$IsCpo(j_1;r), Rcpo(j_1;r), j_1\text{⏚}, Cpo(r), r\oplus, i\oplus,$

$i\text{☮}i_0, IsCpo(i_0;r), Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \text{⚲} i, IsCpm(i;j;r), j \text{☮} j_1,$

$Rcpo(j_1;r), j_1\text{⏚}, Cpo(r), r\oplus, i\text{☮}i_1, i_1\oplus, i_1\text{⏚}, i\oplus,$

$i\text{☮}i_0, IsCpo(i_0;r), Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \text{⚲} i, IsCpm(i;j;r), j \text{☮} j_1,$

$Rcpo(j_1;r), j_1\text{⏚}, Cpo(r), r\oplus, i\text{☮}i_1, i_1\oplus, i\oplus,$

$i\text{☮}i_0, i_0{=}i_1, IsCpo(i_0;r), IsCpo(i_1;r), Rcpo(i_0;r), i_1\text{⏚}, i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \text{⚲} i, IsCpm(i;j;r), j \text{☮} j_1,$

$Rcpo(j_1;r), j_1\text{⏚}, Cpo(r), r\oplus, i\text{☮}i_1, i_1\oplus, i\oplus,$

$i\text{☮}i_0, i_0{=}i_1, IsCpo(i_0;r), IsCpo(i_1;r), Rcpo(i_1;r), i_1\text{⏚}, i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \text{⚲} i, IsCpm(i;j;r), j \text{☮} j_1,$

$Rcpo(j_1;r), j_1\text{⏚}, Cpo(r), r\oplus, i\text{☮}i_1, i_1\oplus, i\oplus,$

$IsCpo(i_1;r), Rcpo(i_1;r), i_1\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \text{⚲} i, IsCpm(i;j;r), j \text{☮} j_1,$

$Rcpo(j_1;r), j_1\text{⏚}, Cpo(r), r\oplus, i\text{☮}i_1, i_1\oplus, i\oplus,$

$IsCpo(i_1;r), i!\text{⚲}r, Rcpo(i_1;r), i_1\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi \text{⚲} i, IsCpm(i;j;r), j \text{☮} j_1,$

$Rcpo(j_1;r), j_1\circledast, Cpo(r), r\oplus, i\otimes i_1, i_1\oplus,$

$IsCpo(i_1;r), i!\mathring{O}r, Rcpo(i_1;r), i_1\circledast, i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , \&SHi\,\mathring{O}i, IsCpm(i;j;r), j\otimes j_1, IsCpo(j_1;r), i \mathrel{!=} \varnothing, Rcpo(j_1;r), i \mathrel{!=} \varnothing, j_1\circledast, i\otimes i_1,$

$IsCpo(i_1;r), i_1 \mathrel{!=} \varnothing, Cpo(r), r\oplus, i_1\oplus, Rcpo(i_1;r),$

$i_1\circledast, i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), j\otimes j_1, IsCpo(j_1;r), Rcpo(j_1;r), j_1\circledast, i\otimes i_1,$

$IsCpo(i_1;r), Rcpo(i_1;r),$

$i_1\circledast, i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), j\otimes j_1, IsCpo(j_1;r), i\otimes i_1, Rcpo(j_1;r),$

$IsCpo(i_1;r), Rcpo(i_1;r), j_1\circledast,$

$i_1\circledast, i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), j\otimes j_1, i\otimes i_1, IsCpo(j_1;r), IsCpo(i_1;r),$

$Rcpo(j_1;r), Rcpo(i_1;r), j_1\circledast,$

$i_1\circledast, i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), j\otimes j_1, i\otimes i_1, IsCpo(j_1;r), IsCpo(i_1;r),$

$Rcpo(i_1;r), Rcpo(j_1;r), j_1\circledast,$

$i_1\circledast, i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), j\otimes j_1, i\otimes i_1, IsCpo(i_1;r),$

$Rcpo(i_1;r), IsCpo(j_1;r), Rcpo(j_1;r), j_1\circledast,$

$i_1\text{⏚}, i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r), i\text{☮}i_1, IsCpo(i_1;r),$

$Rcpo(i_1;r), i_1\text{⏚}, j\text{☮}j_1, IsCpo(j_1;r), Rcpo(j_1;r), j_1\text{⏚},$

$i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing,$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpm(i;j;r), i\text{☮}i_1, IsCpo(i_1;r), i \mathrel{!\!=} \varnothing,$

$Rcpo(i_1;r), i_1\text{⏚}, i \mathrel{!\!=} \varnothing, j\text{☮}j_1, Rcpo(j_1;r), j_1\text{⏚},$

$i\oplus, Rcpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing,$

$\Leftrightarrow\ ,\ \&SHi\,\text{♂}i, IsCpm(i;j;r), i\text{☮}i_1, IsCpo(i_1;r), i \mathrel{!\!=} \varnothing,$

$Rcpo(i_1;r), i_1\text{⏚}, i \mathrel{!\!=} \varnothing, Rcpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r), i\text{☮}i_1,$

$Rcpo(i_1;r), i_1\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing,$

$\Leftrightarrow\ , i \mathrel{!\!=} \varnothing,\ \&SHi\,\text{♂}i, IsCpm(i;j;r), i\text{☮}i_0, Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing,$

$conclusion:$

$, IsCpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing, Rcpm(i;j;r),\ \Leftrightarrow$

$, IsCpm(i;j;r), i\text{☮}i_0, Rcpo(i_0;r), i_0\text{⏚}, Rcpm(i;j;r), j\ominus, j \mathrel{!\!=} \varnothing,$

$$, IsCpm(i;j;r), Rcpm(i;j;r), R(j),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(j;i;r), R(i),$$

induction proof:
$premise\ 1:$
$, i = \varnothing, IsCpm(i;j;r), Rcpm(i;j;r), R(j),$
$\Leftrightarrow\ , IsCpm(i;j;r), i = \varnothing, Rcpm(i;j;r), R(j),$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, R(j),$

$\Leftrightarrow\ , IsCpm(i;j;r), R(j), i=\varnothing, R(i),$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, R(j), R(i),$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, Rcpm(j;i;r), R(i),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), Rcpm(j;i;r), R(i),$

$premise\ 2:$
$, \&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), R(j),\ \Leftrightarrow$

$, \&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(j;i;r), R(i),\ \Rightarrow$

$, i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), Rcpm(i;j;r), R(j),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r),$

$i\,!{=}\,\varnothing, j\circledast j_0, Rcpo(j_0;r), j_0\circledcirc, i\oplus, Rcpm(i;j;r), R(j),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), j\circledast j_0,$

$IsCpo(j_0;r), i!\mathring{\bigcirc}^{\circ} r, Rcpo(j_0;r), j_0\circledcirc, i\oplus, Rcpm(i;j;r), R(j),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\circledast j_0, IsCpo(j_0;r), i!\mathring{\bigcirc}^{\circ} r, Rcpo(j_0;r), j_0\circledcirc, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), R(j),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing, j\circledast j_0, IsCpo(j_0;r), i!\mathring{\bigcirc}^{\circ} r, Rcpo(j_0;r), j_0\circledcirc, i\oplus,$

$\&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(j;i;r), R(i),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, j\circledast j_0, IsCpo(j_0;r), i!\mathring{\bigcirc}^{\circ} r, Rcpo(j_0;r), j_0\circledcirc,$

$i\,!{=}\,\varnothing, i\oplus, IsCpm(i;j;r), Rcpm(j;i;r), R(i),$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} i, j\circledast j_0, IsCpo(j_0;r), i!\mathring{\bigcirc}^{\circ} r, Rcpo(j_0;r), j_0\circledcirc,$

$i\, != \varnothing, i \oslash i_1, i_1 \oplus, i_1 \ominus, i_1\, != \varnothing, i_1 \circledcirc, i \oplus, IsCpm(i;j;r), Rcpm(j;i;r), R(i),$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing, i \oslash i_1,$

$i_1 \oplus, i \oplus, IsCpm(i;j;r), i_1! \male r, i_1 \ominus, i_1\, != \varnothing, i_1 \circledcirc, Rcpm(j;i;r), R(i),$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing, i \oslash i_1,$

$i_1 \oplus, i \oplus, IsCpm(i;j;r), i_1! \male r, Rcpm(j;i;r), i_1 \ominus, i_1\, != \varnothing, R(i), i_1 \circledcirc,$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing, i \oslash i_1,$

$i_1 \oplus, i \oplus, IsCpm(i;j;r), i_1 \male i, Rcpm(j;i;r), i_1 \ominus, i_1\, != \varnothing, R(i), i_1 \circledcirc,$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing, i \oslash i_1,$

$i_1 \oplus, i \oplus, IsCpm(i;j;r), Rcpm(j;i;r), i_1 \male i, i_1 \ominus, i_1\, != \varnothing, R(i), i_1 \circledcirc,$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing, i \oslash i_1,$

$i_1 \oplus, i \oplus, IsCpm(i;j;r), Rcpm(j;i;r), i_1 \male i, i_1 \ominus, i \ominus, i_1\, != \varnothing, i \oplus, R(i), i_1 \circledcirc,$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing, i \oslash i_1,$

$i_1 \oplus, i \oplus, IsCpm(i;j;r), Rcpm(j;i;r), i_1 \male i, i_1 \ominus, i \ominus, i\, != \varnothing, i \oplus, R(i), i_1 \circledcirc,$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing,$

$i \oplus, IsCpm(i;j;r), Rcpm(j;i;r), i \ominus, i\, != \varnothing, i \oplus, R(i),$

$\Leftrightarrow\ , \&SHi \male i, j \oslash j_0, IsCpo(j_0;r), i! \male r, Rcpo(j_0;r), j_0 \circledcirc, i\, != \varnothing,$

$i \oplus, IsCpm(i;j;r), Rcpm(j;i;r), i \ominus, i\, != \varnothing, R(i),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), i\oplus, j\circledwedge j_0, IsCpo(j_0;r), i!\mathring{\bigcirc} r, Rcpo(j_0;r), j_0\textcircled{$\downarrow$},$

$Rcpm(j;i;r), i\ominus, i\,!{=}\,\varnothing, R(i),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), i\oplus,$

$j\circledwedge j_0, Rcpo(j_0;r), j_0\textcircled{$\downarrow$}, Rcpm(j;i;r), i\ominus, i\,!{=}\,\varnothing, R(i),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), i\oplus, i\ominus, i\,!{=}\,\varnothing, Rcpm(j;i;r), R(i),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), Rcpm(j;i;r), R(i),$

$conclusion:$

$, IsCpm(i;j;r), Rcpm(i;j;r), R(j),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(j;i;r), R(i),$

$$, IsCpm(i;j;r), Rcpm(i;j;r), i\textcircled{$\downarrow$}, j\textcircled{$\downarrow$},\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(j;i;r), i\textcircled{$\downarrow$}, j\textcircled{$\downarrow$},$$

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$$, IsCpm(i;j;r), i\textcircled{$\downarrow$}, \&Tm(r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), i\textcircled{$\downarrow$}, \&Tm(r),$$

$$, IsCpm(i;j;r), R(i), \&Tm(r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), \&Tm(r),$$

induction proof:
$premise\ 1:$
$, i=\varnothing, IsCpm(i;j;r), R(i), \&Tm(r),$
$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, R(i), \&Tm(r),$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, \&Tm(r),$

$\Leftrightarrow\ , IsCpm(i;j;r), i=\varnothing, Rcpm(i;j;r), \&Tm(r),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), Rcpm(i;j;r), \&Tm(r),$

$premise\ 2:$
$, \&SHi \rightarrow i, IsCpm(i;j;r), R(i), \&Tm(r), \ \Leftrightarrow$

$, \&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), \&Tm(r), \ \Rightarrow$

$, i\,!{=}\,\varnothing, \ \&SHi ♂ i, IsCpm(i;j;r), R(i), \&Tm(r),$

$\Leftrightarrow \ , \&SHi ♂ i, IsCpm(i;j;r), i\,!{=}\,\varnothing, R(i),$

$j \oslash j_0, IsCpo(j_0;r), j_0 \oplus, \&Tm(r),$

$\Leftrightarrow \ , \&SHi ♂ i, IsCpm(i;j;r), i\,!{=}\,\varnothing, R(i),$

$j \oslash j_0, IsCpo(j_0;r), Rcpo(j_0;r), j_0 \oplus, \&Tm(r),$

$\Leftrightarrow \ , \&SHi ♂ i, IsCpm(i;j;r), i\,!{=}\,\varnothing, R(i),$

$j \oslash j_0, IsCpo(j_0;r), i!♂ r, Rcpo(j_0;r), j_0 \oplus, \&Tm(r),$

$\Leftrightarrow \ , j \oslash j_0, IsCpo(j_0;r), i!♂ r, Rcpo(j_0;r), j_0 \oplus,$

$\&SHi ♂ i, IsCpm(i;j;r), i\,!{=}\,\varnothing, R(i), \&Tm(r),$

$\Leftrightarrow \ , j \oslash j_0, IsCpo(j_0;r), i!♂ r, Rcpo(j_0;r), j_0 \oplus,$

$\&SHi ♂ i, IsCpm(i;j;r), i\,!{=}\,\varnothing, i \oplus, R(i), \&Tm(r),$

$\Leftrightarrow \ , j \oslash j_0, IsCpo(j_0;r), i!♂ r, Rcpo(j_0;r), j_0 \oplus,$

$i\,!{=}\,\varnothing, i \oplus, \ \&SHi \rightarrow i, IsCpm(i;j;r), R(i), \&Tm(r),$

$\Leftrightarrow \ , j \oslash j_0, IsCpo(j_0;r), i!♂ r, Rcpo(j_0;r), j_0 \oplus,$

$i\,!{=}\,\varnothing, i \oplus, \ \&SHi \rightarrow i, IsCpm(i;j;r), Rcpm(i;j;r), \&Tm(r),$

$\Leftrightarrow \ , \&SHi ♂ i, IsCpm(i;j;r), i\,!{=}\,\varnothing,$

$j☮j_0, IsCpo(j_0;r), i!♂r, Rcpo(j_0;r), j_0⏚, i\oplus, Rcpm(i;j;r), \&Tm(r),$

$\Leftrightarrow\ , \&SHi\,♂i, IsCpm(i;j;r), i\,!\!=\varnothing,$

$j☮j_0, Rcpo(j_0;r), j_0⏚, i\oplus, Rcpm(i;j;r), \&Tm(r),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,♂i, IsCpm(i;j;r), Rcpm(i;j;r), \&Tm(r),$

conclusion :

$, IsCpm(i;j;r), R(i), \&Tm(r),\ \Leftrightarrow\ , IsCpm(i;j;r), Rcpm(i;j;r), \&Tm(r),$

$$, i⏚,\ \Leftrightarrow\ , \circledcirc r, Rcpm(i;j;r), i⏚, r⏚,$$

$$, R(i),\ \Leftrightarrow\ , \circledcirc r, Rcpm(i;j;r), r⏚,$$

31.8 Substitution

$$, IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, Rcpm(i;j_1;r),\ \Leftrightarrow$$
$$, IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, Rcpm(i;j_2;r),$$

induction proof:

premise 1 :

$, i=\varnothing, IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, Rcpm(i;j_1;r),$

$\Leftrightarrow\ , IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, i=\varnothing, Rcpm(i;j_1;r),$

$\Leftrightarrow\ , IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, i=\varnothing,$

$\Leftrightarrow\ , IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, i=\varnothing, Rcpm(i;j_2;r),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, Rcpm(i;j_2;r),$

premise 2 :

$, \&SHi\rightarrow i, IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, Rcpm(i;j_1;r),\ \Leftrightarrow$

$, \&SHi\rightarrow i, IsCpm(i;j_1;r), IsCpm(i;j_2;r), j_1{=}j_2, Rcpm(i;j_2;r),\ \Rightarrow$

$, i \,!\!= \varnothing,\ \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, Rcpm(i; j_1; r),$

$\Leftrightarrow\ , \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2,$

$i \,!\!= \varnothing, j_1 \otimes j_0, Rcpo(j_0; r), j_0 \circleddash, i \oplus, Rcpm(i; j_1; r),$

$\Leftrightarrow\ , i \,!\!= \varnothing, j_1 \otimes j_0,\ \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2,$

$IsCpo(j_0; r), i! \circlearrowright r, j_1! \circlearrowright r, j_2! \circlearrowright r, Rcpo(j_0; r), j_0 \circleddash, i \oplus, Rcpm(i; j_1; r),$

$\Leftrightarrow\ , i \,!\!= \varnothing, j_1 \otimes j_0, IsCpo(j_0; r), i! \circlearrowright r, j_1! \circlearrowright r, j_2! \circlearrowright r, Rcpo(j_0; r), j_0 \circleddash, i \oplus,$

$\&SHi \to i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, Rcpm(i; j_1; r),$

$\Leftrightarrow\ , i \,!\!= \varnothing, j_1 \otimes j_0, IsCpo(j_0; r), i! \circlearrowright r, j_1! \circlearrowright r, j_2! \circlearrowright r, Rcpo(j_0; r), j_0 \circleddash, i \oplus,$

$\&SHi \to i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, Rcpm(i; j_2; r),$

$\Leftrightarrow\ , \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, i \,!\!= \varnothing, j_1 \otimes j_0,$

$IsCpo(j_0; r), i! \circlearrowright r, j_1! \circlearrowright r, j_2! \circlearrowright r, Rcpo(j_0; r), j_0 \circleddash, i \oplus, Rcpm(i; j_2; r),$

$\Leftrightarrow\ , \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, i \,!\!= \varnothing, j_1 \otimes j_0, j_2 \otimes j_3, j_3 \circleddash,$

$IsCpo(j_0; r), Rcpo(j_0; r), j_0 \circleddash, i \oplus, Rcpm(i; j_2; r),$

$\Leftrightarrow\ , \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, i \,!\!= \varnothing, j_1 \otimes j_0, j_2 \otimes j_3,$

$IsCpo(j_0; r), IsCpo(j_3; r), j_0 \mp j_3, Rcpo(j_0; r), j_0 \circleddash, j_3 \circleddash, i \oplus, Rcpm(i; j_2; r),$

$\Leftrightarrow\ , \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, i \,!\!= \varnothing, j_1 \otimes j_0, j_2 \otimes j_3,$

$IsCpo(j_0; r), IsCpo(j_3; r), j_0 \mp j_3, Rcpo(j_3; r), j_0 \circleddash, j_3 \circleddash, i \oplus, Rcpm(i; j_2; r),$

$\Leftrightarrow\ , \&SHi \circlearrowleft i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1 \mp j_2, i \,!\!= \varnothing,$

j_2☮$j_3, Rcpo(j_3; r), j_3$⏚$, i\oplus, Rcpm(i; j_2; r),$

$\Leftrightarrow\ , i \,!\!= \varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1{=}j_2, Rcpm(i; j_2; r),$

$conclusion:$

$, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1{=}j_2, Rcpm(i; j_1; r),\ \ \Leftrightarrow$

$, IsCpm(i; j_1; r), IsCpm(i; j_2; r), j_1{=}j_2, Rcpm(i; j_2; r),$

$$, IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1{=}i_2, Rcpm(i_1; j; r), R(i_2),\ \ \Leftrightarrow$$
$$, IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1{=}i_2, Rcpm(i_2; j; r), R(i_1),$$

proof:

$, IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1{=}i_2, Rcpm(i_1; j; r), R(i_2),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1{=}i_2,$

j☮$j_0, j{=}j_0, j_0$⏚$, Rcpm(i_1; j; r), R(i_2),$

$\Leftrightarrow\ , IsCpm(i_2; j; r), i_1{=}i_2, j$☮$j_0,$

$IsCpm(i_1; j; r), j{=}j_0, j_0$⏚$, Rcpm(i_1; j; r), R(i_2),$

$\Leftrightarrow\ , IsCpm(i_2; j; r), i_1{=}i_2, j$☮$j_0,$

$IsCpm(i_1; j; r), IsCpm(i_1; j_0; r), j{=}j_0, Rcpm(i_1; j; r), j_0$⏚$, R(i_2),$

$\Leftrightarrow\ , IsCpm(i_2; j; r), i_1{=}i_2, j$☮$j_0,$

$IsCpm(i_1; j; r), IsCpm(i_1; j_0; r), j{=}j_0, Rcpm(i_1; j_0; r), j_0$⏚$, R(i_2),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1{=}i_2, j$☮$j_0,$

$IsCpm(i_1; j_0; r), Rcpm(i_1; j_0; r), R(j_0), j_0\text{⏚}, R(i_2),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1\text{⌶}i_2, j\text{☮}j_0,$

$IsCpm(i_1; j_0; r), Rcpm(j_0; i_1; r), R(i_1), j_0\text{⏚}, R(i_2),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), j\text{☮}j_0,$

$IsCpm(i_1; j_0; r), IsCpm(i_2; j_0; r), i_1\text{⌶}i_2, Rcpm(j_0; i_1; r), j_0\text{⏚}, R(i_2), R(i_1),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), j\text{☮}j_0,$

$IsCpm(i_1; j_0; r), IsCpm(i_2; j_0; r), i_1\text{⌶}i_2, Rcpm(j_0; i_2; r), j_0\text{⏚}, R(i_2), R(i_1),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1\text{⌶}i_2, j\text{☮}j_0,$

$IsCpm(i_2; j_0; r), Rcpm(j_0; i_2; r), R(i_2), j_0\text{⏚}, R(i_1),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1\text{⌶}i_2, j\text{☮}j_0,$

$IsCpm(i_2; j_0; r), Rcpm(i_2; j_0; r), R(j_0), j_0\text{⏚}, R(i_1),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1\text{⌶}i_2, j\text{☮}j_0,$

$IsCpm(i_2; j_0; r), IsCpm(i_2; j; r), j\text{⌶}j_0, Rcpm(i_2; j_0; r), j_0\text{⏚}, R(i_1),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1\text{⌶}i_2, j\text{☮}j_0,$

$IsCpm(i_2; j_0; r), IsCpm(i_2; j; r), j\text{⌶}j_0, Rcpm(i_2; j; r), j_0\text{⏚}, R(i_1),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1\text{⌶}i_2, j\text{☮}j_0,$

$IsCpm(i_2; j; r), Rcpm(i_2; j; r), j_0\text{⏚}, R(i_1),$

$\Leftrightarrow\ , IsCpm(i_1; j; r), IsCpm(i_2; j; r), i_1\text{⌶}i_2, Rcpm(i_2; j; r), R(i_1),$

$$,IsCpm(i_1;j;r),IsCpm(i_2;j;r),i_1{=}i_2,Rcpm(i_1;j;r),i_1\text{\textcircled{$\downarrow$}},i_2\text{\textcircled{$\downarrow$}},\ \Leftrightarrow$$
$$,IsCpm(i_1;j;r),IsCpm(i_2;j;r),i_1{=}i_2,Rcpm(i_2;j;r),i_1\text{\textcircled{$\downarrow$}},i_2\text{\textcircled{$\downarrow$}},$$

31.9 Expand

$$,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$$
$$Rcpm(i;j;r),r_1\text{\textcircled{$\downarrow$}},\&Tm(r_{10}),\ \Leftrightarrow$$
$$,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$$
$$Rcpm(i;j;r_{10}),Rcpo(r_1;r),r_1\text{\textcircled{$\downarrow$}},\&Tm(r_{10}),$$

induction proof:
premise 1 :
$,i=\varnothing,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$

$Rcpm(i;j;r),r_1\text{\textcircled{$\downarrow$}},\&Tm(r_{10}),$

$\Leftrightarrow\ ,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$

$i=\varnothing,Rcpm(i;j;r),r_1\text{\textcircled{$\downarrow$}},\&Tm(r_{10}),$

$\Leftrightarrow\ ,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$

$i=\varnothing,r_1\text{\textcircled{$\downarrow$}},\&Tm(r_{10}),$

$\Leftrightarrow\ ,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$

$i=\varnothing,r_1=\varnothing,r_1\text{\textcircled{$\downarrow$}},\&Tm(r_{10}),$

$\Leftrightarrow\ ,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$

$i=\varnothing,r_1=\varnothing,Rcpo(r_1;r),r_1\text{\textcircled{$\downarrow$}},\&Tm(r_{10}),$

$\Leftrightarrow\ ,IsCpm(i;j;r),IsCpm(i;j;r_{10}),r!\overset{\circ\circ}{\bigcirc}r_{10},r_1\overset{\circ}{\bigcirc}r_{10},$

$i=\varnothing, Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\circleddash, \&Tm(r_{10}),$

$premise\ 2:$
$, \&SHi\rightarrow i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r), r_1\circleddash, \&Tm(r_{10}),\ \ \Leftrightarrow$

$, \&SHi\rightarrow i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\circleddash, \&Tm(r_{10}),\ \ \Longrightarrow$

$, i\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$i\,!\!=\varnothing, j\circledast j_0, Rcpo(j_0;r), j_0\circleddash, i\oplus, Rcpm(i;j;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\circledast j_0,\ \&SHi\,\mathring{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$IsCpo(j_0;r), r!\mathring{\bigcirc}^{\circ}r_{10}, i!\mathring{\bigcirc}^{\circ}r, Rcpo(j_0;r), j_0\circleddash, i\oplus,$

$Rcpm(i;j;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\circledast j_0, IsCpo(j_0;r), r!\mathring{\bigcirc}^{\circ}r_{10}, i!\mathring{\bigcirc}^{\circ}r, Rcpo(j_0;r), j_0\circleddash, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!\!=\varnothing, j\circledast j_0, IsCpo(j_0;r), r!\mathring{\bigcirc}^{\circ}r_{10}, i!\mathring{\bigcirc}^{\circ}r, Rcpo(j_0;r), j_0\circleddash, i\oplus,$

$\&SHi\rightarrow i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$j\text{☮}j_0, IsCpo(j_0;r), r!\mathring{\bigcirc}{\circ} r_{10}, i!\mathring{\bigcirc}{\circ} r, Rcpo(j_0;r), j_0\text{⍗}, i\oplus,$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10}, i\oplus,$

$j\text{☮}j_0, IsCpo(j_0;r), r!\mathring{\bigcirc}{\circ} r_{10}, i!\mathring{\bigcirc}{\circ} r, Rcpo(j_0;r), j_0\text{⍗},$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10}, i\oplus,$

$j\text{☮}j_0, IsCpo(j_0;r), r!\mathring{\bigcirc}{\circ} r_{10}, i!\mathring{\bigcirc}{\circ} r, Rcpo(j_0;r), j_0\text{⍗},$

$IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, Rcpm(i;j;r_{10}),$

$IsCpm(r_1;r), Rcpo(r_1;r), r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10}, i\oplus,$

$j\text{☮}j_0, IsCpo(j_0;r), r!\mathring{\bigcirc}{\circ} r_{10}, i!\mathring{\bigcirc}{\circ} r, Rcpo(j_0;r),$

$IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, Rcpm(i;j;r_{10}),$

$IsCpm(r_1;r), Rcpo(r_1;r), j_0\text{⍗}, r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10}, i\oplus, j\text{☮}j_0,$

$IsCpo(j_0;r), IsCpo(j_0;r_{10}), IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10},$

$Rcpo(j_0;r), Rcpm(i;j;r_{10}),$

$Rcpo(r_1;r), j_0\text{⍗}, r_1\text{⍗}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10}, i\oplus, j\text{☮}j_0,$

$IsCpo(j_0; r), IsCpo(j_0; r_{10}), IsCpm(i; j; r), IsCpm(i; j; r_{10}), r!\bigcirc\!\!^{\circ}r_{10},$

$Rcpm(i; j; r_{10}), Rcpo(j_0; r),$

$Rcpo(r_1; r), j_0\textcircled{$\downarrow$}, r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), r!\bigcirc\!\!^{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10}, i\oplus, j\circledast j_0,$

$IsCpm(i; j; r), IsCpm(i; j; r_{10}), r!\bigcirc\!\!^{\circ}r_{10}, Rcpm(i; j; r_{10}),$

$IsCpo(j_0; r), IsCpo(j_0; r_{10}), IsCpo(r_1; r), r_1\bigcirc\!\!^{\circ}r_{10}, Rcpo(j_0; r),$

$Rcpo(r_1; r), j_0\textcircled{$\downarrow$}, r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), r!\bigcirc\!\!^{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10}, i\oplus, j\circledast j_0,$

$IsCpm(i; j; r), IsCpm(i; j; r_{10}), r!\bigcirc\!\!^{\circ}r_{10}, Rcpm(i; j; r_{10}),$

$IsCpo(j_0; r), IsCpo(j_0; r_{10}), IsCpo(r_1; r), r_1\bigcirc\!\!^{\circ}r_{10}, Rcpo(j_0; r_{10}),$

$Rcpo(r_1; r), j_0\textcircled{$\downarrow$}, r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), r!\bigcirc\!\!^{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10}, i\oplus, j\circledast j_0,$

$IsCpm(i; j; r_{10}), IsCpo(j_0; r_{10}),$

$Rcpm(i; j; r_{10}), Rcpo(j_0; r_{10}),$

$Rcpo(r_1; r), j_0\textcircled{$\downarrow$}, r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), r!\bigcirc\!\!^{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10}, i\oplus, j\circledast j_0,$

$IsCpm(i; j; r_{10}), IsCpo(j_0; r_{10}),$

$Rcpo(j_0; r_{10}), Rcpm(i; j; r_{10}),$

$Rcpo(r_1; r), j_0\textcircled{$\downarrow$}, r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\mathring{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10}, i\,!\!=\varnothing,$

$j\oslash j_0, Rcpo(j_0;r_{10}), j_0\ominus, i\oplus,$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\mathring{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\ominus, \&Tm(r_{10}),$

$conclusion:$

$, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r), r_1\ominus, \&Tm(r_{10}),\ \Leftrightarrow$

$, IsCpm(i;j;r), IsCpm(i;j;r_{10}), r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;j;r_{10}), Rcpo(r_1;r), r_1\ominus, \&Tm(r_{10}),$

$$, IsCpm(i;j;r), Rcpm(i;j;r),\ \Leftrightarrow$$
$$, IsCpm(i;j;r), \circledcirc r_1, r_1\oslash r_{10}, Rcpm(i;j;r_{10}), , Rcpo(r_1;r), r_1\ominus, r_{10}\ominus,$$

31.10 Distributivity

$$, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}^{\circ}r, k!\mathring{\bigcirc}^{\circ}r_{10}, r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$$
$$Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\ominus, \&Tm(r_{10}),\ \Leftrightarrow$$
$$, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}^{\circ}r, k!\mathring{\bigcirc}^{\circ}r_{10}, r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$$
$$Rcpm(i;k;r), Rcpm(j;k;r), r_1\ominus, \&Tm(r_{10}),$$

induction proof:
$premise\ 1:$
$, i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}^{\circ}r, k!\mathring{\bigcirc}^{\circ}r_{10}, r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}{\circ}r, k!\overset{\circ}{\bigcirc}{\circ}r_{10}, r!\overset{\circ}{\bigcirc}{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$i=\varnothing, Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}{\circ}r, k!\overset{\circ}{\bigcirc}{\circ}r_{10}, r!\overset{\circ}{\bigcirc}{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$IsCpo(j;r_{10}), r!\overset{\circ}{\bigcirc}{\circ}r_{10}, Rcpo(j;r_{10}),$

$IsCpm(r_1;k;r), Rcpm(r_1;k;r), r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}{\circ}r, k!\overset{\circ}{\bigcirc}{\circ}r_{10}, r!\overset{\circ}{\bigcirc}{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$j\otimes j_1, IsCpo(j;r_{10}), Rcpo(j;r_{10}),$

$IsCpm(r_1;k;r), Rcpm(r_1;k;r), j_1\ominus, r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}{\circ}r, k!\overset{\circ}{\bigcirc}{\circ}r_{10}, r!\overset{\circ}{\bigcirc}{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$j\otimes j_1, IsCpo(j;r_{10}), j\overset{\circ}{\bigcirc}j_1, r_1\overset{\circ}{\bigcirc}r_{10}, Rcpo(j;r_{10}),$

$IsCpm(r_1;k;r), Rcpm(r_1;k;r), j_1\ominus, r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}{\circ}r, k!\overset{\circ}{\bigcirc}{\circ}r_{10}, r!\overset{\circ}{\bigcirc}{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$j\otimes j_1, IsCpo(j;r_{10}), j\overset{\circ}{\bigcirc}j_1, r_1\overset{\circ}{\bigcirc}r_{10}, Rcpo(j;r_{10}), r_1\doteq j_1,$

$IsCpm(r_1;k;r), Rcpm(r_1;k;r), j_1\ominus, r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}{\circ}r, k!\overset{\circ}{\bigcirc}{\circ}r_{10}, r!\overset{\circ}{\bigcirc}{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$j\otimes j_1, IsCpo(j;r_{10}), Rcpo(j;r_{10}), IsCpm(r_1;k;r), IsCpm(j_1;k;r),$

$r_1\doteq j_1, Rcpm(r_1;k;r), R(j_1), j_1\ominus, r_1\ominus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}{\circ}r, k!\overset{\circ}{\bigcirc}{\circ}r_{10}, r!\overset{\circ}{\bigcirc}{\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$j\text{☮}j_1, IsCpo(j;r_{10}), Rcpo(j;r_{10}), IsCpm(r_1;k;r), IsCpm(j_1;k;r),$

$r_1 \mp j_1, Rcpm(j_1;k;r), R(r_1), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \ , i = \varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{♂}r, k!\text{♂}r_{10}, r!\text{♂}r_{10}, r_1\text{♁}r_{10},$

$j\text{☮}j_1, IsCpo(j;r_{10}), Rcpo(j;r_{10}),$

$IsCpm(j_1;k;r), Rcpm(j_1;k;r), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \ , i = \varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{♂}r, k!\text{♂}r_{10}, r!\text{♂}r_{10}, r_1\text{♁}r_{10},$

$j\text{☮}j_1, IsCpo(j;r_{10}), r!\text{♂}r_{10}, Rcpo(j;r_{10}),$

$IsCpm(j_1;k;r), Rcpm(j_1;k;r), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \ , i = \varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{♂}r, k!\text{♂}r_{10}, r!\text{♂}r_{10}, r_1\text{♁}r_{10},$

$j\text{☮}j_1, IsCpo(j;r_{10}), IsCpo(j;r), IsCpm(j_1;k;r), IsCpm(j_1;k;r_{10}), r!\text{♂}r_{10},$

$Rcpo(j;r_{10}), Rcpm(j_1;k;r), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \ , i = \varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{♂}r, k!\text{♂}r_{10}, r!\text{♂}r_{10}, r_1\text{♁}r_{10},$

$j\text{☮}j_1, IsCpo(j;r_{10}), IsCpo(j;r), IsCpm(j_1;k;r), IsCpm(j_1;k;r_{10}), r!\text{♂}r_{10},$

$Rcpm(j_1;k;r), Rcpo(j;r_{10}), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \ , i = \varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{♂}r, k!\text{♂}r_{10}, r!\text{♂}r_{10}, r_1\text{♁}r_{10},$

$j\text{☮}j_1, IsCpm(j_1;k;r), r!\text{♂}r_{10}, Rcpm(j_1;k;r),$

$IsCpo(j;r_{10}), Rcpo(j;r_{10}), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow \ , i = \varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{♂}r, k!\text{♂}r_{10}, r!\text{♂}r_{10}, r_1\text{♁}r_{10},$

$j\text{☮}j_1, IsCpm(j_1;k;r), r!\text{♂}r_{10}, Rcpm(j_1;k;r),$

$IsCpo(j;r_{10}), R(j), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$j\text{☮}j_1, IsCpm(j_1;k;r), IsCpm(j;k;r), j_1\text{⫧}j,$

$Rcpm(j_1;k;r), R(j), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$j\text{☮}j_1, IsCpm(j_1;k;r), IsCpm(j;k;r), j_1\text{⫧}j,$

$Rcpm(j;k;r), R(j_1), j_1\text{⏚}, r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$i=\varnothing, Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$i=\varnothing, Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i=\varnothing, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

premise 2 :
$, \&SHi \to i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\text{⏚}, \&Tm(r_{10}),\quad \Leftrightarrow$

$, \&SHi \to i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),\quad \Longrightarrow$

$, i\,!\!=\varnothing, \ \&SHi\,\text{♂}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}\!{\scriptstyle\circ} r, k!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i\,!\!= \varnothing, Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}\!{\scriptstyle\circ} r, k!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i\,!\!= \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, Rcpo(i;r_{10}),$
$Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}\!{\scriptstyle\circ} r, k!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i\,!\!= \varnothing, r_1\mathring{\bigcirc} r_{10}, Cpo(r_{10}), r_1\mathring{\bigcirc} r_{10}, r_{10}\,!\!= \varnothing, r_1\,!\!= \varnothing, r_{10}\oplus, i\oplus,$

$Rcpo(i;r_{10}), Rcpo(j;r_{10}),$

$Rcpm(r_1;k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}\!{\scriptstyle\circ} r, k!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i\,!\!= \varnothing, Cpo(r_{10}), r_1\,!\!= \varnothing, r_{10}\oplus, i\oplus,$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}), IsCpo(j;r_{10}), Rcpo(j;r_{10}),$

$Rcpm(r_1;k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}\!{\scriptstyle\circ} r, k!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i\,!\!= \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus,$

$IsCpo(i;r_{10}), r_1\,!\!= \varnothing, Rcpo(i;r_{10}), IsCpo(j;r_{10}), r_1\,!\!= \varnothing, Rcpo(j;r_{10}),$

$r_1\,!\!= \varnothing, k\textcircled{$\curlywedge$}k_0, Rcpo(k_0;r), k_0\textcircled{$\downarrow$}, r_1\oplus, Rcpm(r_1;k;r), r_1\textcircled{$\downarrow$}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}\!{\scriptstyle\circ} r, k!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r!\mathring{\bigcirc}\!{\scriptstyle\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i\,!\!= \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, r_1\,!\!= \varnothing, k\textcircled{$\curlywedge$}k_0,$

$IsCpo(i;r_{10}), Rcpo(i;r_{10}), IsCpo(j;r_{10}), Rcpo(j;r_{10}),$

$Rcpo(k_0; r), k_0\circledast, r_1\oplus, Rcpm(r_1; k; r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\bigcirc}^{\circ} r, k!\mathring{\bigcirc}^{\circ} r_{10}, r!\mathring{\bigcirc}^{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, k\text{☮}k_0,$

$IsCpo(i; r_{10}), IsCpo(i; r), IsCpo(k_0; r_{10}), IsCpo(k_0; r), r!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), IsCpo(j; r), IsCpo(k_0; r_{10}), IsCpo(k_0; r), r!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(j; r_{10}),$

$Rcpo(k_0; r), k_0\circledast, r_1\oplus, Rcpm(r_1; k; r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\bigcirc}^{\circ} r, k!\mathring{\bigcirc}^{\circ} r_{10}, r!\mathring{\bigcirc}^{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, k\text{☮}k_0,$

$IsCpo(i; r_{10}), IsCpo(i; r), IsCpo(k_0; r_{10}), IsCpo(k_0; r), r!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), IsCpo(j; r), IsCpo(k_0; r_{10}), IsCpo(k_0; r), r!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(k_0; r), Rcpo(j; r_{10}),$

$k_0\circledast, r_1\oplus, Rcpm(r_1; k; r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\bigcirc}^{\circ} r, k!\mathring{\bigcirc}^{\circ} r_{10}, r!\mathring{\bigcirc}^{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, k\text{☮}k_0,$

$IsCpo(i; r_{10}), IsCpo(i; r), IsCpo(k_0; r_{10}), IsCpo(k_0; r), r!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(i; r_{10}),$

$Rcpo(k_0; r), Rcpo(j; r_{10}),$

$k_0\circledast, r_1\oplus, Rcpm(r_1; k; r), r_1\circledast, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc} i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\bigcirc}^{\circ} r, k!\mathring{\bigcirc}^{\circ} r_{10}, r!\mathring{\bigcirc}^{\circ} r_{10}, r_1\mathring{\bigcirc} r_{10},$

$i \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, k\text{☮}k_0,$

$IsCpo(i; r_{10}), IsCpo(i; r), IsCpo(k_0; r_{10}), IsCpo(k_0; r), r!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(k_0; r),$

$Rcpo(i; r_{10}), Rcpo(j; r_{10}),$

$k_0⦽, r_1\oplus, Rcpm(r_1; k; r), r_1⦽, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi♂i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!⚣r, k!⚣r_{10}, r!⚣r_{10}, r_1♂r_{10},$

$i != \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, k☮k_0,$

$IsCpo(k_0; r), r!⚣r_{10}, Rcpo(k_0; r),$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$k_0⦽, r_1\oplus, Rcpm(r_1; k; r), r_1⦽, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi♂i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!⚣r, k!⚣r_{10}, r!⚣r_{10}, r_1♂r_{10},$

$i != \varnothing, Cpo(r_{10}), r_{10}\oplus, i\oplus, k☮k_0,$

$IsCpo(k_0; r), r!⚣r_{10}, Rcpo(k_0; r), k_0⦽,$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$r_1\oplus, Rcpm(r_1; k; r), r_1⦽, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi♂i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!⚣r, k!⚣r_{10}, r!⚣r_{10}, r_1♂r_{10},$

$i != \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1!♂r_{10}, i\oplus, k☮k_0,$

$IsCpo(k_0; r), r!⚣r_{10}, r_1!♂r_{10}, Rcpo(k_0; r), k_0⦽,$

$IsCpo(i; r_{10}), r_1!♂r_{10}, Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), r_1!♂r_{10}, Rcpo(j; r_{10}),$

$r_1\oplus, Rcpm(r_1; k; r), r_1⦽, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi♂i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!⚣r, k!⚣r_{10}, r!⚣r_{10}, r_1♂r_{10},$

$i \,!\!= \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1!\text{♂}r_{10}, i\oplus, k\text{☮}k_0,$

$IsCpo(k_0; r), r!\text{⚨}r_{10}, r_1!\text{♂}r_{10}, Rcpo(k_0; r), k_0\text{⏚},$

$IsCpo(i; r_{10}), r_1!\text{♂}r_{10}, Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), r_1!\text{♂}r_{10}, r_1\oplus, Rcpo(j; r_{10}),$

$Rcpm(r_1; k; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$i \,!\!= \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1!\text{♂}r_{10}, i\oplus, k\text{☮}k_0,$

$IsCpo(k_0; r), r!\text{⚨}r_{10}, r_1!\text{♂}r_{10}, Rcpo(k_0; r), k_0\text{⏚},$

$IsCpo(i; r_{10}), r_1!\text{♂}r_{10}, r_1\oplus, Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$Rcpm(r_1; k; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$i \,!\!= \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1!\text{♂}r_{10}, i\oplus, k\text{☮}k_0,$

$IsCpo(k_0; r), r!\text{⚨}r_1, Rcpo(k_0; r), r_1\oplus, k_0\text{⏚},$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$Rcpm(r_1; k; r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\text{♂}i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\text{⚨}r, k!\text{⚨}r_{10}, r!\text{⚨}r_{10}, r_1\text{♂}r_{10},$

$i \,!\!= \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1!\text{♂}r_{10}, i\oplus, k\text{☮}k_0,$

$IsCpo(k_0; r), r!\text{⚨}r_1, r_1\oplus, Rcpo(k_0; r), k_0\text{⏚},$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$Rcpm(r_1; k; r), r_1\circledcirc, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi \mathring{\circ} i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\circ} r, k!\mathring{\circ} r_{10}, r!\mathring{\circ} r_{10}, r_1\mathring{\circ} r_{10},$

$i \mathrel{!=} \varnothing, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, i\oplus, k\oslash k_0,$

$IsCpo(k_0; r), r!\mathring{\circ} r_{10}, Rcpo(k_0; r), k_0\circledcirc,$

$IsCpo(i; r_{10}), Rcpo(i; r_{10}),$

$IsCpo(j; r_{10}), Rcpo(j; r_{10}),$

$Rcpm(r_1; k; r), r_1\circledcirc, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi \mathring{\circ} i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\circ} r, k!\mathring{\circ} r_{10}, r!\mathring{\circ} r_{10}, r_1\mathring{\circ} r_{10},$

$i \mathrel{!=} \varnothing, i!\mathring{\circ} r_{10}, r!\mathring{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, i\oplus, k\oslash k_0,$

$IsCpo(k_0; r), r!\mathring{\circ} r_{10}, i!\mathring{\circ} r, Rcpo(k_0; r), k_0\circledcirc,$

$Rcpo(i; r_{10}), Rcpo(j; r_{10}), Rcpm(r_1; k; r), r_1\circledcirc, \&Tm(r_{10}),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i!\mathring{\circ} r_{10}, r!\mathring{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, i\oplus, k\oslash k_0,$

$IsCpo(k_0; r), r!\mathring{\circ} r_{10}, i!\mathring{\circ} r, Rcpo(k_0; r), k_0\circledcirc,$

$\&SHi \rightarrow i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\circ} r, k!\mathring{\circ} r_{10}, r!\mathring{\circ} r_{10}, r_1\mathring{\circ} r_{10},$

$Rcpo(i; r_{10}), Rcpo(j; r_{10}), Rcpm(r_1; k; r), r_1\circledcirc, \&Tm(r_{10}),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing, i!\mathring{\circ} r_{10}, r!\mathring{\circ} r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, i\oplus, k\oslash k_0,$

$IsCpo(k_0; r), r!\mathring{\circ} r_{10}, i!\mathring{\circ} r, Rcpo(k_0; r), k_0\circledcirc,$

$\&SHi \rightarrow i, IsCpm(i; j; r), IsCpm(i; j; r_{10}), k!\mathring{\circ} r, k!\mathring{\circ} r_{10}, r!\mathring{\circ} r_{10}, r_1\mathring{\circ} r_{10},$

$Rcpm(i; k; r), Rcpm(j; k; r), r_1\circledcirc, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$i\,!\!=\varnothing, i!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, i\oplus, k\text{☮}k_0,$

$IsCpo(k_0;r), r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, i!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, Rcpo(k_0;r), k_0\text{⏚},$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$i\,!\!=\varnothing, i!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, k\text{☮}k_0,$

$IsCpo(k_0;r), r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, i!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, Rcpo(k_0;r), k_0\text{⏚}, i\oplus,$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$i!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, i\,!\!=\varnothing,$

$k\text{☮}k_0, Rcpo(k_0;r), k_0\text{⏚}, i\oplus,$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$i!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus, i\,!\!=\varnothing,$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, Cpo(r_{10}), r_{10}\oplus, r_1\oplus,$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\text{⏚}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!\!=\varnothing,\ \&SHi\,\overset{\circ}{\bigcirc}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r, k!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r!\overset{\circ}{\bigcirc}\!{\scriptstyle\circ}r_{10}, r_1\overset{\circ}{\bigcirc}r_{10},$

$r!\circledcirc r_{10}, Cpo(r_{10}), r_{10}\oplus,$

$IsCpm(i;k;r), r!\circledcirc r_1, r_1\oplus, Rcpm(i;k;r), IsCpm(j;k;r), r!\circledcirc r_1, Rcpm(j;k;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\circledcirc r, k!\circledcirc r_{10}, r!\circledcirc r_{10}, r_1\mathring{O}r_{10},$

$r!\circledcirc r_{10}, Cpo(r_{10}), r_{10}\oplus,$

$IsCpm(i;k;r), r!\circledcirc r_1, Rcpm(i;k;r), IsCpm(j;k;r), r!\circledcirc r_1, Rcpm(j;k;r), r_1\oplus, r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\circledcirc r, k!\circledcirc r_{10}, r!\circledcirc r_{10}, r_1\mathring{O}r_{10},$

$r!\circledcirc r_{10}, Cpo(r_{10}), r_{10}\oplus,$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\circledcirc r, k!\circledcirc r_{10}, r!\circledcirc r_{10}, r_1\mathring{O}r_{10},$

$IsCpm(i;k;r), r!\circledcirc r_{10}, i!\circledcirc r_{10}, k!\circledcirc r_{10}, Cpo(r_{10}), r_{10}\oplus, Rcpm(i;k;r),$

$Rcpm(j;k;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\circledcirc r, k!\circledcirc r_{10}, r!\circledcirc r_{10}, r_1\mathring{O}r_{10},$

$IsCpm(i;k;r), r!\circledcirc r_{10}, i!\circledcirc r_{10}, k!\circledcirc r_{10}, Rcpm(i;k;r), Cpo(r_{10}), r_{10}\oplus,$

$Rcpm(j;k;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\circledcirc r, k!\circledcirc r_{10}, r!\circledcirc r_{10}, r_1\mathring{O}r_{10},$

$IsCpm(i;k;r), Rcpm(i;k;r),$

$IsCpm(j;k;r), r!\circledcirc r_{10}, j!\circledcirc r_{10}, k!\circledcirc r_{10},$

$Cpo(r_{10}), r_{10}\oplus, Rcpm(j;k;r), r_1\circleddash, \&Tm(r_{10}),$

$\Leftrightarrow\ , i\,!{=}\,\varnothing,\ \&SHi\,\mathring{O}i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\circledcirc r, k!\circledcirc r_{10}, r!\circledcirc r_{10}, r_1\mathring{O}r_{10},$

$IsCpm(i;k;r), Rcpm(i;k;r),$

$IsCpm(j;k;r), r!\mathring{\bigcirc}^{\circ}r_{10}, j!\mathring{\bigcirc}^{\circ}r_{10}, k!\mathring{\bigcirc}^{\circ}r_{10},$

$Rcpm(j;k;r), Cpo(r_{10}), r_{10}\oplus, r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}^{\circ}r, k!\mathring{\bigcirc}^{\circ}r_{10}, r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$IsCpm(i;k;r), Rcpm(i;k;r),$

$Rcpm(j;k;r), r_1\textcircled{\downarrow}, Cpo(r_{10}), r_{10}\oplus, \&Tm(r_{10}),$

$\Leftrightarrow\ , i \mathrel{!=} \varnothing,\ \&SHi\,\mathring{\bigcirc} i, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}^{\circ}r, k!\mathring{\bigcirc}^{\circ}r_{10}, r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$conclusion:$

$, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}^{\circ}r, k!\mathring{\bigcirc}^{\circ}r_{10}, r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),\ \Leftrightarrow$

$, IsCpm(i;j;r), IsCpm(i;j;r_{10}), k!\mathring{\bigcirc}^{\circ}r, k!\mathring{\bigcirc}^{\circ}r_{10}, r!\mathring{\bigcirc}^{\circ}r_{10}, r_1\mathring{\bigcirc}r_{10},$

$Rcpm(i;k;r), Rcpm(j;k;r), r_1\textcircled{\downarrow}, \&Tm(r_{10}),$

$$, IsCpm(i;j;r), k!\mathring{\bigcirc}^{\circ}r, Rcpm(i;k;r), Rcpm(j;k;r),\ \Leftrightarrow$$
$$, IsCpm(i;j;r), k!\mathring{\bigcirc}^{\circ}r, \circledcirc r_1, r_1\text{☮}r_{10},$$
$$Rcpo(i;r_{10}), Rcpo(j;r_{10}), Rcpm(r_1;k;r), r_1\textcircled{\downarrow}, r_{10}\textcircled{\downarrow},$$

31.11 Result

$$, IsCpm(i_1;j_1;r_{10}), IsCpm(i_2;j_2;r_{20}), i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_{10}!\mathring{\bigcirc}^{\circ}r_{20},$$
$$r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, Rcpm(i_1;j_1;r_{10}), Rcpm(i_2;j_2;r_{20}),\ \Leftrightarrow\ \sim, r_1{=}r_2,$$

induction proof:
premise 1 :
$, i_1 = \varnothing, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 ♂ i_2, j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 ♂ r_{10}, r_2 ♂ r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 ♂ i_2, i_1 = \varnothing, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 ♂ i_2, i_2 = \varnothing, Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 ♂ i_2, i_2 = \varnothing,$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} = \varnothing, r_{20} = \varnothing, r_{10} ⌶ r_{20}, j_1 ♂ j_2,$

$r_{10} !♂ r_{20}, r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 ♂ i_2, i_2 = \varnothing,$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} = \varnothing, r_{20} = \varnothing, j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 ♂ r_{10}, r_2 ♂ r_{20}, r_{10} ⌶ r_{20}, i_1 ♂ i_2, i_2 = \varnothing,$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} = \varnothing, r_{20} = \varnothing, j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 ♂ r_{10}, r_2 ♂ r_{20}, r_1 ⌶ r_2, i_1 ♂ i_2, i_2 = \varnothing,$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} = \varnothing, r_{20} = \varnothing, j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 ♂ i_2, i_2 = \varnothing, r_1 ⌶ r_2,$

$\Leftrightarrow \ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} = \varnothing, r_{20} = \varnothing, j_1 ♂ j_2, r_{10} !♂ r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1 \mathring{\bigcirc} i_2, i_2 = \varnothing, Rcpm(i_2; j_2; r_{20}), r_1 {=} r_2,$

$\Leftrightarrow\ , IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} = \varnothing, r_{20} = \varnothing, j_1 \mathring{\bigcirc} j_2, r_{10} !\text{♂} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1 \mathring{\bigcirc} i_2, i_1 = \varnothing, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), r_1 {=} r_2,$

$\Leftrightarrow\ , i_1 = \varnothing, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_{10} !\text{♂} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), r_1 {=} r_2,$

premise 2 :
$, \ \&SHi \to i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_{10} !\text{♂} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), \ \Leftrightarrow$

$, \ \&SHi \to i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_{10} !\text{♂} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), r_1 {=} r_2, \ \Rightarrow$

$, i_1 \mathrel{!}= \varnothing, \ \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_{10} !\text{♂} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} !\text{♂} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1 \mathrel{!}= \varnothing,$

$j_1 \text{☮} j_{10}, Rcpo(j_{10}; r_{10}), j_{10}\text{⏚}, i_1 \oplus,$

$Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} !\text{♂} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1 \mathrel{!}= \varnothing,$

$j_1 \text{☮} j_{10}, IsCpo(j_{10}; r_{10}), i_1 !\text{♂} r_{10}, Rcpo(j_{10}; r_{10}), j_{10}\text{⏚}, i_1 \oplus,$

$Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow\ , \ \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} !\text{♂} r_{20},$

$i_1 ♂ i_2, j_1 ♂ j_2, r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 != \varnothing,$

$j_1 ☮ j_{10}, IsCpo(j_{10}; r_{10}), i_1 !♂ r_{10}, i_1 \oplus, Rcpo(j_{10}; r_{10}), j_{10} ⏚,$

$Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow\ , \&SHi ♂ i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} !♂ r_{20},$

$i_1 ♂ i_2, j_1 ♂ j_2, r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 != \varnothing, i_1 \oplus,$

$j_1 ☮ j_{10}, IsCpo(j_{10}; r_{10}), r_{10} !♂ r_{20}, Rcpo(j_{10}; r_{10}), j_{10} ⏚,$

$IsCpm(i_1; j_1; r_{10}), r_{10} !♂ r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), Rcpm(i_2; j_2; r_{20}),$

$\Leftrightarrow\ , \&SHi ♂ i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} !♂ r_{20},$

$i_1 ♂ i_2, j_1 ♂ j_2, r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 != \varnothing, i_1 \oplus,$

$j_1 ☮ j_{10}, IsCpo(j_{10}; r_{10}), r_{10} !♂ r_{20}, Rcpo(j_{10}; r_{10}),$

$IsCpm(i_1; j_1; r_{10}), r_{10} !♂ r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), Rcpm(i_2; j_2; r_{20}), j_{10} ⏚,$

$\Leftrightarrow\ , \&SHi ♂ i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} !♂ r_{20},$

$i_1 ♂ i_2, j_1 ♂ j_2, r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 != \varnothing, i_1 \oplus,$

$j_1 ☮ j_{10}, IsCpm(i_1; j_1; r_{10}), IsCpo(j_{10}; r_{10}),$

$Rcpo(j_{10}; r_{10}), Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), j_{10} ⏚,$

$\Leftrightarrow\ , \&SHi ♂ i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} !♂ r_{20},$

$i_1 ♂ i_2, j_1 ♂ j_2, r_1 ♂ r_{10}, r_2 ♂ r_{20}, i_1 != \varnothing, i_1 \oplus,$

$j_1 ☮ j_{10}, IsCpm(i_1; j_1; r_{10}), IsCpo(j_{10}; r_{10}),$

$Rcpm(i_1; j_1; r_{10}), Rcpo(j_{10}; r_{10}), Rcpm(i_2; j_2; r_{20}), j_{10}\obot,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1 != \varnothing, i_1\oplus,$

$j_1 \varovee j_{10}, IsCpm(i_1; j_1; r_{10}), r_{10}!\bigcirc^{\circ} r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), IsCpm(i_2; j_2; r_{10}), IsCpo(j_{10}; r_{10}), IsCpo(j_{10}; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$Rcpo(j_{10}; r_{10}), Rcpm(i_2; j_2; r_{20}), j_{10}\obot,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1 != \varnothing, i_1\oplus,$

$j_1 \varovee j_{10}, IsCpm(i_1; j_1; r_{10}), r_{10}!\bigcirc^{\circ} r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), IsCpm(i_2; j_2; r_{10}), IsCpo(j_{10}; r_{10}), IsCpo(j_{10}; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$Rcpm(i_2; j_2; r_{20}), Rcpo(j_{10}; r_{10}), j_{10}\obot,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_2 != \varnothing, i_1\oplus,$

$IsCpm(i_1; j_1; r_{10}), Rcpm(i_1; j_1; r_{10}),$

$Rcpm(i_2; j_2; r_{20}),$

$j_1 \varovee j_{10}, Rcpo(j_{10}; r_{10}), j_{10}\obot,$

$\Leftrightarrow\ , \&SHi \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_1\oplus,$

$IsCpm(i_1; j_1; r_{10}), i_2 != \varnothing, Rcpm(i_1; j_1; r_{10}), i_2 != \varnothing,$

$j_2 \varovee j_{20}, Rcpo(j_{20}; r_{20}), j_{20}\obot, i_2\oplus, Rcpm(i_2; j_2; r_{20}),$

$j_1 \otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10} \circledcirc,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_2 \mathrel{!}= \varnothing, i_1 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2 \mathring{\bigcirc} r_{20}, i_2!\bigcirc^{\circ} r_{10}, Rcpm(i_1; j_1; r_{10}),$

$j_2 \otimes j_{20}, IsCpo(j_{20}; r_{20}), i_2!\bigcirc^{\circ} r_{20}, Rcpo(j_{20}; r_{20}), i_2 \oplus, j_{20} \circledcirc,$

$Rcpm(i_2; j_2; r_{20}), j_1 \otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10} \circledcirc,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_2 \mathrel{!}= \varnothing, i_1 \oplus, i_2 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2 \mathring{\bigcirc} r_{20}, i_2!\bigcirc^{\circ} r_{10}, Rcpm(i_1; j_1; r_{10}),$

$j_2 \otimes j_{20}, IsCpo(j_{20}; r_{20}), i_2!\bigcirc^{\circ} r_{20}, Rcpo(j_{20}; r_{20}), j_{20} \circledcirc,$

$Rcpm(i_2; j_2; r_{20}), j_1 \otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10} \circledcirc,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_2 \mathrel{!}= \varnothing, i_1 \oplus, i_2 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}),$

$j_2 \otimes j_{20}, IsCpo(j_{20}; r_{20}), Rcpo(j_{20}; r_{20}), IsCpm(i_2; j_2; r_{20}), j_{20} \circledcirc,$

$Rcpm(i_2; j_2; r_{20}), j_1 \otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10} \circledcirc,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\bigcirc^{\circ} r_{20},$

$i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i_2 \mathrel{!}= \varnothing, i_1 \oplus, i_2 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}),$

$j_2 \otimes j_{20}, IsCpo(j_{20}; r_{20}), Rcpo(j_{20}; r_{20}), IsCpm(i_2; j_2; r_{20}),$

$Rcpm(i_2; j_2; r_{20}), j_{20}\circledast, j_1\otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10}\circledast,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\mathring{\bigcirc}r_{20},$

$i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i_2 \,!\!= \varnothing, i_1\oplus, i_2\oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2\mathring{\bigcirc}r_{20}, Rcpm(i_1; j_1; r_{10}),$

$j_2\otimes j_{20}, IsCpm(i_2; j_2; r_{20}), IsCpo(j_{20}; r_{20}), Rcpo(j_{20}; r_{20}),$

$Rcpm(i_2; j_2; r_{20}), j_{20}\circledast, j_1\otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10}\circledast,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\mathring{\bigcirc}r_{20},$

$i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i_2 \,!\!= \varnothing, i_1\oplus, i_2\oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2\mathring{\bigcirc}r_{20}, Rcpm(i_1; j_1; r_{10}),$

$j_2\otimes j_{20}, IsCpm(i_2; j_2; r_{20}), IsCpo(j_{20}; r_{20}),$

$Rcpm(i_2; j_2; r_{20}), Rcpo(j_{20}; r_{20}), j_{20}\circledast, j_1\otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10}\circledast,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\mathring{\bigcirc}r_{20},$

$i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i_2 \,!\!= \varnothing, i_1\oplus, i_2\oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2\mathring{\bigcirc}r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), j_2\otimes j_{20},$

$Rcpm(i_2; j_2; r_{20}), Rcpo(j_{20}; r_{20}), j_{20}\circledast, j_1\otimes j_{10}, Rcpo(j_{10}; r_{10}), j_{10}\circledast,$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!\mathring{\bigcirc}r_{20},$

$i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i_2 \,!\!= \varnothing, i_1\oplus, i_2\oplus,$

$IsCpm(i_1; j_1; r_{10}), r_2\mathring{\bigcirc}r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), Rcpm(i_2; j_2; r_{20}),$

j_2☮$j_{20}, Rcpo(j_{20}; r_{20}), j_{20}$⊍$, j_1$☮$j_{10}, Rcpo(j_{10}; r_{10}), j_{10}$⊍,

$\Leftrightarrow$, &*SHi* ♁$i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}$!♂$r_{20}$,

i_1♁i_2, j_1♁j_2, r_1♁r_{10}, r_2♁$r_{20}, i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus$,

$Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20})$,

j_2☮$j_{20}, Rcpo(j_{20}; r_{20}), j_{20}$⊍$, j_1$☮$j_{10}, Rcpo(j_{10}; r_{10}), j_{10}$⊍,

$\Leftrightarrow$, $i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus$, &*SHi* $\rightarrow i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}$!♂$r_{20}$,

i_1♁i_2, j_1♁j_2, r_1♁r_{10}, r_2♁r_{20},

$Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20})$,

j_2☮$j_{20}, Rcpo(j_{20}; r_{20}), j_{20}$⊍$, j_1$☮$j_{10}, Rcpo(j_{10}; r_{10}), j_{10}$⊍,

$\Leftrightarrow$, $i_2 \mathrel{!=} \varnothing, i_1\oplus, i_2\oplus$, &*SHi* $\rightarrow i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}$!♂$r_{20}$,

i_1♁i_2, j_1♁j_2, r_1♁r_{10}, r_2♁r_{20},

$Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), r_1 \mathrel{=} r_2$,

j_2☮$j_{20}, Rcpo(j_{20}; r_{20}), j_{20}$⊍$, j_1$☮$j_{10}, Rcpo(j_{10}; r_{10}), j_{10}$⊍,

$\Leftrightarrow$, $i_2 \mathrel{!=} \varnothing$, &*SHi* ♁$i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}$!♂$r_{20}$,

i_1♁i_2, j_1♁j_2, r_1♁r_{10}, r_2♁$r_{20}, i_1\oplus, i_2\oplus$,

$IsCpm(i_1; j_1; r_{10}), r_{10}$!♂$r_{20}, Rcpm(i_1; j_1; r_{10})$,

$IsCpm(i_2; j_2; r_{20}), r_{10}$!♂$r_{20}, Rcpm(i_2; j_2; r_{20}), r_1 \mathrel{=} r_2$,

j_2☮$j_{20}, Rcpo(j_{20}; r_{20}), j_{20}$⊍$, j_1$☮$j_{10}, Rcpo(j_{10}; r_{10}), j_{10}$⊍,

$\Leftrightarrow$, $i_2 \mathrel{!=} \varnothing$, &*SHi* ♁$i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}$!♂$r_{20}$,

$i_1 \overset{\circ}{\bigcirc} i_2, j_1 \overset{\circ}{\bigcirc} j_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, i_1 \oplus, i_2 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpm(i_2; j_2; r_{20}), r_1 = r_2,$

$j_2 \otimes j_{20}, IsCpo(j_{20}; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpo(j_{20}; r_{20}), j_{20} \ominus,$

$j_1 \otimes j_{10}, IsCpo(j_{10}; r_{10}), Rcpo(j_{10}; r_{10}), j_{10} \ominus,$

$\Leftrightarrow \;, i_2 \,!\!= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20},$

$i_1 \overset{\circ}{\bigcirc} i_2, j_1 \overset{\circ}{\bigcirc} j_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, i_1 \oplus, i_2 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpm(i_2; j_2; r_{20}), r_1 = r_2,$

$j_1 \otimes j_{10}, j_2 \otimes j_{20}, IsCpo(j_{20}; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpo(j_{20}; r_{20}),$

$IsCpo(j_{10}; r_{10}), Rcpo(j_{10}; r_{10}), j_{20} \ominus, j_{10} \ominus,$

$\Leftrightarrow \;, i_2 \,!\!= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20},$

$i_1 \overset{\circ}{\bigcirc} i_2, j_1 \overset{\circ}{\bigcirc} j_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, i_1 \oplus, i_2 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_{10} ! \bigcirc^{\circ} r_{20}, j_1 \overset{\circ}{\bigcirc} j_2, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpm(i_2; j_2; r_{20}), r_1 = r_2,$

$j_1 \otimes j_{10}, j_2 \otimes j_{20}, IsCpo(j_{20}; r_{20}), IsCpo(j_{10}; r_{10}), r_{10} ! \bigcirc^{\circ} r_{20},$

$Rcpo(j_{20}; r_{20}), Rcpo(j_{10}; r_{10}), j_{20} \ominus, j_{10} \ominus,$

$\Leftrightarrow \;, i_2 \,!\!= \varnothing, \ \&SHi \overset{\circ}{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20},$

$i_1 \overset{\circ}{\bigcirc} i_2, j_1 \overset{\circ}{\bigcirc} j_2, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, i_1 \oplus, i_2 \oplus,$

$IsCpm(i_1; j_1; r_{10}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), r_{10} ! \bigcirc^{\circ} r_{20}, Rcpm(i_2; j_2; r_{20}), r_1 = r_2,$

$j_1☮j_{10}, j_2☮j_{20}, j_1♂j_2, j_{10}{=}j_{20},$

$IsCpo(j_{20}; r_{20}), IsCpo(j_{10}; r_{10}), IsCpo(j_{20}; r_{10}), IsCpo(j_{10}; r_{20}), r_{10}!⚨r_{20},$

$r_1⚨r_{10}, r_2⚨r_{20},$

$Rcpo(j_{20}; r_{20}), Rcpo(j_{10}; r_{10}), j_{20}⏚, j_{10}⏚,$

$\Leftrightarrow\ , i_2 \,!{=}\, \varnothing,\ \&SHi♂i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!⚨r_{20},$

$i_1♂i_2, j_1♂j_2, r_1♂r_{10}, r_2♂r_{20}, i_1⊕, i_2⊕,$

$IsCpm(i_1; j_1; r_{10}), r_{10}!⚨r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), r_{10}!⚨r_{20}, Rcpm(i_2; j_2; r_{20}),$

$j_1☮j_{10}, j_2☮j_{20}, j_1♂j_2,$

$IsCpo(j_{20}; r_{20}), IsCpo(j_{10}; r_{10}), IsCpo(j_{20}; r_{10}), IsCpo(j_{10}; r_{20}), r_{10}!⚨r_{20},$

$r_1⚨r_{10}, r_2⚨r_{20}, j_{10}{=}j_{20}, r_1{=}r_2,$

$Rcpo(j_{20}; r_{20}), Rcpo(j_{10}; r_{10}), j_{20}⏚, j_{10}⏚,$

$\Leftrightarrow\ , i_2 \,!{=}\, \varnothing,\ \&SHi♂i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), r_{10}!⚨r_{20},$

$i_1♂i_2, j_1♂j_2, r_1♂r_{10}, r_2♂r_{20}, i_1⊕, i_2⊕,$

$IsCpm(i_1; j_1; r_{10}), r_{10}!⚨r_{20}, Rcpm(i_1; j_1; r_{10}),$

$IsCpm(i_2; j_2; r_{20}), r_{10}!⚨r_{20}, Rcpm(i_2; j_2; r_{20}),$

$j_1☮j_{10}, j_2☮j_{20}, j_1♂j_2,$

$IsCpo(j_{20}; r_{20}), IsCpo(j_{10}; r_{10}), IsCpo(j_{20}; r_{10}), IsCpo(j_{10}; r_{20}), r_{10}!⚨r_{20},$

$r_1⚨r_{10}, r_2⚨r_{20}, j_{10}{=}j_{20}, r_1{=}r_2,$

$Rcpo(j_{20}; r_{20}), Rcpo(j_{10}; r_{10}), r_1{=}r_2, j_{20}⏚, j_{10}⏚,$

$\Leftrightarrow\ , i_1 \mathrel{!}= \varnothing,\ \&SHi\,\mathring{\bigcirc} i_1, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_{10} \mathrel{!}\mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), r_1 \mp r_2,$

$conclusion:$

$, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_{10} \mathrel{!}\mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}),\ \ \Leftrightarrow$

$, IsCpm(i_1; j_1; r_{10}), IsCpm(i_2; j_2; r_{20}), i_1 \mathring{\bigcirc} i_2, j_1 \mathring{\bigcirc} j_2, r_{10} \mathrel{!}\mathring{\bigcirc}^{\circ} r_{20},$

$r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, Rcpm(i_1; j_1; r_{10}), Rcpm(i_2; j_2; r_{20}), r_1 \mp r_2,$

31.12 Associativity

$$, IsCpm(i; j; r), k \mathrel{!}\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1 \circledast r_{10},$$
$$Rcpm(j; i; r_{10}), Rcpm(r_1; k; r), r_1 \circledbot, r_{10} \circledbot,\ \ \Leftrightarrow$$
$$, IsCpm(i; j; r), k \mathrel{!}\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1 \circledast r_{10},$$
$$Rcpm(j; k; r_{10}), Rcpm(r_1; i; r), r_1 \circledbot, r_{10} \circledbot,$$

induction proof:
$premise\ 1:$
$, j = \varnothing, IsCpm(i; j; r), k \mathrel{!}\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1 \circledast r_{10},$

$Rcpm(j; i; r_{10}), Rcpm(r_1; k; r), r_1 \circledbot, r_{10} \circledbot,$

$\Leftrightarrow\ , IsCpm(i; j; r), k \mathrel{!}\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1 \circledast r_{10},$

$j = \varnothing, Rcpm(j; i; r_{10}), Rcpm(r_1; k; r), r_1 \circledbot, r_{10} \circledbot,$

$\Leftrightarrow\ , IsCpm(i; j; r), k \mathrel{!}\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1 \circledast r_{10},$

$j = \varnothing, r_1 = \varnothing, Rcpm(r_1; k; r), r_1 \circledbot, r_{10} \circledbot,$

$\Leftrightarrow\ , IsCpm(i; j; r), k \mathrel{!}\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1 \circledast r_{10},$

$j = \varnothing, r_1 = \varnothing, r_1 \circledast, r_{10} \circledast,$

$\Leftrightarrow \; , IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$j = \varnothing, r_1 = \varnothing, Rcpm(r_1;i;r), r_1 \circledast, r_{10} \circledast,$

$\Leftrightarrow \; , IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$j = \varnothing, Rcpm(j;k;r_{10}), Rcpm(r_1;i;r), r_1 \circledast, r_{10} \circledast,$

$\Leftrightarrow \; , j = \varnothing, IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$Rcpm(j;k;r_{10}), Rcpm(r_1;i;r), r_1 \circledast, r_{10} \circledast,$

$premise\ 2:$
$, \&SHi \rightarrow j, IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$Rcpm(j;i;r_{10}), Rcpm(r_1;k;r), r_1 \circledast, r_{10} \circledast, \quad \Leftrightarrow$

$, \&SHi \rightarrow j, IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$Rcpm(j;k;r_{10}), Rcpm(r_1;i;r), r_1 \circledast, r_{10} \circledast, \quad \Rightarrow$

$, j \mathrel{!}= \varnothing, \&SHi \male j, IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$Rcpm(j;i;r_{10}), Rcpm(r_1;k;r), r_1 \circledast, r_{10} \circledast,$

$\Leftrightarrow \; , \&SHi \male j, IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$j \mathrel{!}= \varnothing, i \oslash i_0, Rcpo(i_0;r_{10}), i_0 \circledast, j \oplus,$

$Rcpm(j;i;r_{10}), Rcpm(r_1;k;r), r_1 \circledast, r_{10} \circledast,$

$\Leftrightarrow \; , \&SHi \male j, IsCpm(i;j;r), k! \male r, \circledcirc r_1, r_1 \oslash r_{10},$

$j \mathrel{!}= \varnothing, i \oslash i_0, IsCpo(i_0;r_{10}), j! \male r_{10}, Rcpo(i_0;r_{10}), i_0 \circledast, j \oplus,$

$Rcpm(j;i;r_{10}), Rcpm(r_1;k;r), r_1 \circledast, r_{10} \circledast,$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc} r, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, j\oplus,$

$IsCpo(i_0; r_{10}), j!\mathring{\bigcirc} r_{10}, Rcpo(i_0; r_{10}), i_0\text{⏚},$

$Rcpm(j; i; r_{10}), Rcpm(r_1; k; r), r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc} r, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, j\oplus,$

$IsCpo(i_0; r_{10}), r!\mathring{\bigcirc} r_{10}, Rcpo(i_0; r_{10}), i_0\text{⏚},$

$IsCpm(j; i; r_{10}), r!\mathring{\bigcirc} r_{10}, Rcpm(j; i; r_{10}), IsCpm(r_1; k; r), Rcpm(r_1; k; r),$

$r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc} r, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, j\oplus,$

$IsCpo(i_0; r_{10}), r!\mathring{\bigcirc} r_{10}, Rcpo(i_0; r_{10}),$

$IsCpm(j; i; r_{10}), r!\mathring{\bigcirc} r_{10}, Rcpm(j; i; r_{10}), IsCpm(r_1; k; r), Rcpm(r_1; k; r),$

$i_0\text{⏚}, r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc} r, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, j\oplus,$

$IsCpo(i_0; r_{10}), IsCpm(j; i; r_{10}), Rcpo(i_0; r_{10}),$

$Rcpm(j; i; r_{10}), Rcpm(r_1; k; r),$

$i_0\text{⏚}, r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc} r, \circledcirc r_1, r_1 \text{☮} r_{10}, i \text{☮} i_0, j\oplus,$

$IsCpo(i_0; r_{10}), IsCpm(j; i; r_{10}), Rcpm(j; i; r_{10}),$

$Rcpo(i_0; r_{10}), Rcpm(r_1; k; r),$

$i_0\text{⏚}, r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\mathring{\bigcirc}r, \circledcirc r_1, r_1 \circledast r_{10}, i \circledast i_0, j\oplus,$

$\circledcirc r_2, r_2 \circledast r_{20}, IsCpm(j;i;r_{20}), Rcpm(j;i;r_{20}), Rcpo(r_2;r_{10}), r_2\circleddash, r_{20}\circleddash,$

$Rcpo(i_0;r_{10}), Rcpm(r_1;k;r),$

$i_0\circleddash, r_1\circleddash, r_{10}\circleddash,$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\mathring{\bigcirc}r, \circledcirc r_1, r_1 \circledast r_{10}, i \circledast i_0, j\oplus,$

$\circledcirc r_2, r_2 \circledast r_{20}, IsCpm(j;i;r_{20}), r!\mathring{\bigcirc}r_{20}, r_{10}!\mathring{\bigcirc}r_{20}, Rcpm(j;i;r_{20}),$

$IsCpo(r_2;r_{10}), r_{10}!\mathring{\bigcirc}r, Rcpo(r_2;r_{10}), r_2\circleddash, r_{20}\circleddash,$

$IsCpo(i_0;r_{10}), r_{10}!\mathring{\bigcirc}r, Rcpo(i_0;r_{10}), IsCpm(r_1;k;r), Rcpm(r_1;k;r),$

$i_0\circleddash, r_1\circleddash, r_{10}\circleddash,$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\mathring{\bigcirc}r, \circledcirc r_1, r_1 \circledast r_{10}, i \circledast i_0, j\oplus,$

$\circledcirc r_2, r_2 \circledast r_{20}, IsCpm(j;i;r_{20}), r!\mathring{\bigcirc}r_{20}, r_{10}!\mathring{\bigcirc}r_{20}, Rcpm(j;i;r_{20}),$

$IsCpo(r_2;r_{10}), r_{10}!\mathring{\bigcirc}r, Rcpo(r_2;r_{10}),$

$IsCpo(i_0;r_{10}), r_{10}!\mathring{\bigcirc}r, Rcpo(i_0;r_{10}), IsCpm(r_1;k;r), Rcpm(r_1;k;r),$

$r_2\circleddash, r_{20}\circleddash, i_0\circleddash, r_1\circleddash, r_{10}\circleddash,$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\mathring{\bigcirc}r, i \circledast i_0, j\oplus,$

$\circledcirc r_2, r_2 \circledast r_{20}, IsCpm(j;i;r_{20}), r!\mathring{\bigcirc}r_{20}, Rcpm(j;i;r_{20}),$

$IsCpm(i_0;k;r), r_2!\mathring{\bigcirc}r, \circledcirc r_1, r_1 \circledast r_{10}, Rcpo(r_2;r_{10}),$

$Rcpo(i_0;r_{10}), Rcpm(r_1;k;r),$

$r_2\circleddash, r_{20}\circleddash, i_0\circleddash, r_1\circleddash, r_{10}\circleddash,$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\mathring{\bigcirc}r, i \circledast i_0, j\oplus,$

$\circledcirc r_2, r_2 \text{☮} r_{20}, IsCpm(j;i;r_{20}), r!\text{⚲} r_{20}, Rcpm(j;i;r_{20}),$

$IsCpm(i_0;k;r), r_2!\text{⚲} r, Rcpm(r_2;k;r), Rcpm(i_0;k;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi\,\text{⚲}\,j, IsCpm(i;j;r), k!\text{⚲} r, i\text{☮} i_0, j\oplus,$

$\circledcirc r_2, r_2 \text{☮} r_{20}, IsCpm(j;i;r_{20}), r!\text{⚲} r_{20}, Rcpm(j;i;r_{20}),$

$IsCpm(i_0;k;r), r_2!\text{⚲} r, Rcpm(i_0;k;r), Rcpm(r_2;k;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi\,\text{⚲}\,j, IsCpm(i;j;r), k!\text{⚲} r, i\text{☮} i_0, j\oplus, \circledcirc r_2, r_2 \text{☮} r_{20},$

$IsCpm(j;i;r_{20}), IsCpm(i_0;k;r), IsCpm(j;i;r), IsCpm(i_0;k;r_{20}), r!\text{⚲} r_{20},$

$Rcpm(j;i;r_{20}), Rcpm(i_0;k;r), Rcpm(r_2;k;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi\,\text{⚲}\,j, IsCpm(i;j;r), k!\text{⚲} r, i\text{☮} i_0, j\oplus, \circledcirc r_2, r_2 \text{☮} r_{20},$

$IsCpm(j;i;r_{20}), IsCpm(i_0;k;r), IsCpm(j;i;r), IsCpm(i_0;k;r_{20}), r!\text{⚲} r_{20},$

$Rcpm(i_0;k;r), Rcpm(j;i;r_{20}), Rcpm(r_2;k;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing,\ \&SHi\,\text{⚲}\,j, IsCpm(i;j;r), k!\text{⚲} r, \circledcirc r_2, r_2 \text{☮} r_{20}, i\text{☮} i_0, j\oplus,$

$IsCpm(i_0;k;r), j!\text{⚲} r, Rcpm(i_0;k;r),$

$Rcpm(j;i;r_{20}), Rcpm(r_2;k;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!}= \varnothing, i\text{☮} i_0, j\oplus,$

$IsCpm(i_0;k;r), j!\text{♂}r, Rcpm(i_0;k;r),$

$\&SHi \rightarrow j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_2, r_2\text{☮}r_{20},$

$Rcpm(j;i;r_{20}), Rcpm(r_2;k;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing, i\text{☮}i_0, j\oplus,$

$IsCpm(i_0;k;r), j!\text{♂}r, Rcpm(i_0;k;r),$

$\&SHi \rightarrow j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_2, r_2\text{☮}r_{20},$

$Rcpm(j;k;r_{20}), Rcpm(r_2;i;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi\,\text{⚲}j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_2, r_2\text{☮}r_{20}, i\text{☮}i_0, j\oplus,$

$IsCpm(i_0;k;r), j!\text{♂}r, Rcpm(i_0;k;r),$

$Rcpm(j;k;r_{20}), Rcpm(r_2;i;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi\,\text{⚲}j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_2, r_2\text{☮}r_{20}, i\text{☮}i_0, j\oplus,$

$IsCpm(i_0;k;r), r!\text{♂}r_{20}, Rcpm(i_0;k;r),$

$IsCpm(j;k;r_{20}), r!\text{♂}r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2\text{⏚}, r_{20}\text{⏚}, i_0\text{⏚},$

$\Leftrightarrow\ , j \mathrel{!=} \varnothing,\ \&SHi\,\text{⚲}j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_2, r_2\text{☮}r_{20}, i\text{☮}i_0, j\oplus,$

$IsCpm(i_0;k;r), k\text{☮}k_0, k_0\text{⏚}, Rcpm(i_0;k;r), i_0\text{⏚},$

$IsCpm(j;k;r_{20}), r!\text{♂}r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⏚, r_{20}⏚,$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi♂j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, i☮i_0, j⊕, k☮k_0,$

$IsCpm(i_0;k;r), IsCpm(i_0;k_0;r), k⫧k_0, Rcpm(i_0;k;r), i_0⏚, k_0⏚,$

$IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⏚, r_{20}⏚,$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi♂j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, i☮i_0, j⊕, k☮k_0,$

$IsCpm(i_0;k;r), IsCpm(i_0;k_0;r), k⫧k_0, Rcpm(i_0;k_0;r), i_0⏚, k_0⏚,$

$IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⏚, r_{20}⏚,$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi♂j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, i☮i_0, j⊕, k☮k_0,$

$IsCpm(i_0;k_0;r), Rcpm(i_0;k_0;r), i_0⏚, k_0⏚,$

$IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⏚, r_{20}⏚,$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi♂j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, i☮i_0, j⊕, k☮k_0,$

$IsCpm(i_0;k_0;r), Rcpm(k_0;i_0;r), i_0⏚, k_0⏚,$

$IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⏚, r_{20}⏚,$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi♂j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, i☮i_0, j⊕, k☮k_0,$

$IsCpm(i_0;k_0;r), IsCpm(i;k_0;r), i⫧i_0, Rcpm(k_0;i_0;r), i_0⏚, k_0⏚,$

$IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⦹, r_{20}⦹,$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi⚨j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, i☮i_0, j\oplus, k☮k_0,$

$IsCpm(i_0;k_0;r), IsCpm(i;k_0;r), i⫟i_0, Rcpm(k_0;i;r), i_0⦹, k_0⦹,$

$IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⦹, r_{20}⦹,$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi⚨j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, i☮i_0, j\oplus, k☮k_0,$

$IsCpm(k_0;i;r), i_0⦹, Rcpm(k_0;i;r), k_0⦹,$

$IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}), IsCpm(r_2;i;r), Rcpm(r_2;i;r),$

$r_2⦹, r_{20}⦹,$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi⚨j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, j\oplus, k☮k_0,$

$IsCpm(k_0;i;r), IsCpm(j;k;r_{20}), IsCpm(k_0;i;r_{20}), IsCpm(j;k;r), r!♂r_{20},$

$Rcpm(k_0;i;r), Rcpm(j;k;r_{20}), Rcpm(r_2;i;r),$

$k_0⦹, r_2⦹, r_{20}⦹,$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi⚨j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, j\oplus, k☮k_0,$

$IsCpm(k_0;i;r), IsCpm(j;k;r_{20}), IsCpm(k_0;i;r_{20}), IsCpm(j;k;r), r!♂r_{20},$

$Rcpm(j;k;r_{20}), Rcpm(k_0;i;r), Rcpm(r_2;i;r),$

$k_0⦹, r_2⦹, r_{20}⦹,$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi⚨j, IsCpm(i;j;r), k!♂r, ◎r_2, r_2☮r_{20}, j\oplus, k☮k_0,$

$IsCpm(k_0;i;r), IsCpm(k_0;i;r_{20}), IsCpm(j;k;r_{20}), r!♂r_{20}, Rcpm(j;k;r_{20}),$

$IsCpm(k_0; i; r), IsCpm(r_2; i; r), Rcpm(k_0; i; r), Rcpm(r_2; i; r),$

$k_0\textcircled{\downarrow}, r_2\textcircled{\downarrow}, r_{20}\textcircled{\downarrow},$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi\,\text{⚲}j, IsCpm(i; j; r), k!\text{♂}r, \circledcirc r_2, r_2\text{☮}r_{20}, j\oplus, k\text{☮}k_0,$

$IsCpm(k_0; i; r), IsCpm(k_0; i; r_{20}), IsCpm(j; k; r_{20}), r!\text{♂}r_{20}, Rcpm(j; k; r_{20}),$

$IsCpm(k_0; i; r), IsCpm(r_2; i; r), Rcpm(r_2; i; r), Rcpm(k_0; i; r),$

$k_0\textcircled{\downarrow}, r_2\textcircled{\downarrow}, r_{20}\textcircled{\downarrow},$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi\,\text{⚲}j, IsCpm(i; j; r), k!\text{♂}r, \circledcirc r_2, r_2\text{☮}r_{20}, j\oplus, k\text{☮}k_0,$

$IsCpm(k_0; i; r), IsCpm(k_0; i; r_{20}), IsCpm(j; k; r_{20}), r!\text{♂}r_{20}, Rcpm(j; k; r_{20}),$

$IsCpm(k_0; i; r), IsCpm(r_2; i; r), \circledcirc r_1, r_1\text{☮}r_{10}, Rcpo(r_2; r_{10}), Rcpo(k_0; r_{10}), Rcpm(r_1; i; r),$

$r_1\textcircled{\downarrow}, r_{10}, k_0\textcircled{\downarrow}, r_2\textcircled{\downarrow}, r_{20}\textcircled{\downarrow},$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi\,\text{⚲}j, IsCpm(i; j; r), k!\text{♂}r, \circledcirc r_1, r_1\text{☮}r_{10}, j\oplus, k\text{☮}k_0,$

$\circledcirc r_2, r_2\text{☮}r_{20}, IsCpm(j; k; r_{20}), r!\text{♂}r_{20}, Rcpm(j; k; r_{20}),$

$IsCpo(r_2; r_{10}), IsCpo(k_0; r_{10}), Rcpo(r_2; r_{10}), Rcpo(k_0; r_{10}),$

$Rcpm(r_1; i; r),$

$r_1\textcircled{\downarrow}, r_{10}, k_0\textcircled{\downarrow}, r_2\textcircled{\downarrow}, r_{20}\textcircled{\downarrow},$

$\Leftrightarrow\ , j\,!{=}\,\varnothing,\ \&SHi\,\text{⚲}j, IsCpm(i; j; r), k!\text{♂}r, \circledcirc r_1, r_1\text{☮}r_{10}, j\oplus, k\text{☮}k_0,$

$\circledcirc r_2, r_2\text{☮}r_{20}, IsCpm(j; k; r_{20}), r!\text{♂}r_{20}, Rcpm(j; k; r_{20}),$

$IsCpo(r_2; r_{10}), r!\text{♂}r_{10}, Rcpo(r_2; r_{10}), IsCpo(k_0; r_{10}), r!\text{♂}r_{10}, Rcpo(k_0; r_{10}),$

$IsCpm(r_1; i; r), Rcpm(r_1; i; r),$

$r_1\textcircled{\downarrow}, r_{10}, k_0\textcircled{\downarrow}, r_2\textcircled{\downarrow}, r_{20}\textcircled{\downarrow},$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_1, r_1 \text{☮} r_{10}, j\oplus, k\text{☮}k_0,$

$\circledcirc r_2, r_2\text{☮}r_{20}, IsCpm(j;k;r_{20}), r!\text{♂}r_{20}, Rcpm(j;k;r_{20}),$

$IsCpo(r_2;r_{10}), r!\text{♂}r_{10}, Rcpo(r_2;r_{10}), r_2\text{⏚}, r_{20}\text{⏚},$

$IsCpo(k_0;r_{10}), r!\text{♂}r_{10}, Rcpo(k_0;r_{10}), k_0\text{⏚}, IsCpm(r_1;i;r), Rcpm(r_1;i;r),$

$r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_1, r_1 \text{☮} r_{10}, j\oplus, k\text{☮}k_0,$

$IsCpm(j;k;r_{10}), \circledcirc r_2, r_2\text{☮}r_{20}, Rcpm(j;k;r_{20}),$

$Rcpo(r_2;r_{10}), r_2\text{⏚}, r_{20}\text{⏚},$

$Rcpo(k_0;r_{10}), k_0\text{⏚}, Rcpm(r_1;i;r), r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_1, r_1 \text{☮} r_{10}, j\oplus, k\text{☮}k_0,$

$IsCpm(j;k;r_{10}), Rcpm(j;k;r_{10}),$

$Rcpo(k_0;r_{10}), k_0\text{⏚}, Rcpm(r_1;i;r), r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_1, r_1 \text{☮} r_{10}, j\oplus, k\text{☮}k_0,$

$IsCpm(j;k;r_{10}), IsCpo(k_0;r_{10}), Rcpm(j;k;r_{10}),$

$Rcpo(k_0;r_{10}), k_0\text{⏚}, Rcpm(r_1;i;r), r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_1, r_1 \text{☮} r_{10}, j\oplus, k\text{☮}k_0,$

$IsCpm(j;k;r_{10}), IsCpo(k_0;r_{10}), Rcpo(k_0;r_{10}), k_0\text{⏚}, Rcpm(j;k;r_{10}),$

$Rcpm(r_1;i;r), r_1\text{⏚}, r_{10}\text{⏚},$

$\Leftrightarrow\ , j\ != \varnothing,\ \&SHi \mathring{\bigcirc} j, IsCpm(i;j;r), k!\text{♂}r, \circledcirc r_1, r_1 \text{☮} r_{10}, k\text{☮}k_0,$

$IsCpo(k_0; r_{10}), j!\mathring{\bigcirc}^{\circ} r_{10}, j\oplus, Rcpo(k_0; r_{10}), k_0\text{\textcircled{$\downarrow$}}, Rcpm(j; k; r_{10}),$

$Rcpm(r_1; i; r), r_1\text{\textcircled{$\downarrow$}}, r_{10}\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , j\, !{=}\, \varnothing,\ \&SHi\,\mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1\text{☮} r_{10}, k\text{☮} k_0,$

$IsCpo(k_0; r_{10}), j!\mathring{\bigcirc}^{\circ} r_{10}, Rcpo(k_0; r_{10}), k_0\text{\textcircled{$\downarrow$}}, j\oplus, Rcpm(j; k; r_{10}),$

$Rcpm(r_1; i; r), r_1\text{\textcircled{$\downarrow$}}, r_{10}\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ ,\ \&SHi\,\mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1\text{☮} r_{10}, j\, !{=}\, \varnothing,$

$k\text{☮} k_0, Rcpo(k_0; r_{10}), k_0\text{\textcircled{$\downarrow$}}, j\oplus, Rcpm(j; k; r_{10}),$

$Rcpm(r_1; i; r), r_1\text{\textcircled{$\downarrow$}}, r_{10}\text{\textcircled{$\downarrow$}},$

$\Leftrightarrow\ , j\, !{=}\, \varnothing,\ \&SHi\,\mathring{\bigcirc} j, IsCpm(i; j; r), k!\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1\text{☮} r_{10},$

$Rcpm(j; k; r_{10}), Rcpm(r_1; i; r), r_1\text{\textcircled{$\downarrow$}}, r_{10}\text{\textcircled{$\downarrow$}},$

$conclusion:$

$, IsCpm(i; j; r), k!\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1\text{☮} r_{10}\text{\textcircled{$\downarrow$}},$

$Rcpm(j; i; r_{10}), Rcpm(r_1; k; r), r_1\text{\textcircled{$\downarrow$}}, r_{10}\text{\textcircled{$\downarrow$}},\ \ \Leftrightarrow$

$, IsCpm(i; j; r), k!\mathring{\bigcirc}^{\circ} r, \circledcirc r_1, r_1\text{☮} r_{10}\text{\textcircled{$\downarrow$}},$

$Rcpm(j; k; r_{10}), Rcpm(r_1; i; r), r_1\text{\textcircled{$\downarrow$}}, r_{10}\text{\textcircled{$\downarrow$}},$

31.13 Monotonicity

$$, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$$
$$, r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1\mathring{\bigcirc} r_{10}, r_2\mathring{\bigcirc} r_{20}, i\gtrdot j, k_1\text{⌶} k_2,$$
$$, k_1\, !{=}\, \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),\ \ \Leftrightarrow\ \ \sim, r_1\gtrdot r_2,$$

induction proof:

$premise\ 1:$

$, k_1 = \varnothing, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \mp k_2,$

$k_1 \,!\!= \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow\ , IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \mp k_2,$

$k_1 = \varnothing, k_1 \,!\!= \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow\ , \otimes,$

$\Leftrightarrow\ , \otimes, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \mp k_2,$

$Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}), r_1 \gtrdot r_2,$

$\Leftrightarrow\ , k_1 = \varnothing, k_1 \,!\!= \varnothing, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \mp k_2,$

$Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}), r_1 \gtrdot r_2,$

$\Leftrightarrow\ , k_1 = \varnothing, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \mp k_2,$

$k_1 \,!\!= \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}), r_1 \gtrdot r_2,$

$premise\ 2:$

$, \&SHi \rightarrow k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \mp k_2,$

$k_1 \,!\!= \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}), \quad \Leftrightarrow$

$, \ \&SHi \rightarrow k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \text{⌶} k_2,$

$k_1 \,!\!= \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}), r_1 \gtrdot r_2, \quad \Longrightarrow$

$, k_1 \,!\!= \varnothing, \ \&SHi \, \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \text{⌶} k_2,$

$k_1 \,!\!= \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow \ , \ \&SHi \, \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \text{⌶} k_2, k_1 \,!\!= \varnothing,$

$i \text{☮} i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, Rcpo(i_0; r_{10}), i_0 \text{⏚}, k_1 \oplus,$

$IsCpm(i; k_1; r_{10}), Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow \ , \ \&SHi \, \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \text{⌶} k_2, k_2 \,!\!= \varnothing,$

$i \text{☮} i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, Rcpo(i_0; r_{10}), i_0 \text{⏚}, k_1 \oplus,$

$IsCpm(i; k_1; r_{10}), k_2 \,!\!= \varnothing, Rcpm(k_1; i; r_{10}),$

$k_2 \,!\!= \varnothing, j \text{☮} j_0, Rcpo(j_0; r_{20}), j_0 \text{⏚}, k_2 \oplus, Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow \ , \ \&SHi \, \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j, k_1 \text{⌶} k_2, k_2 \,!\!= \varnothing,$

$i \text{☮} i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, Rcpo(i_0; r_{10}), i_0 \text{⏚}, k_1 \oplus,$

$j \text{☮} j_0, IsCpm(i; k_1; r_{10}), IsCpm(i; k_1; r_{20}),$

$IsCpo(j_0; r_{20}), IsCpo(j_0; r_{10}), r_{10}!\mathring{\bigcirc}^{\circ} r_{20}, Rcpm(k_1; i; r_{10}),$

$Rcpo(j_0; r_{20}), j_0\circleddash, k_2\oplus, Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\gtrdot j, k_1\asymp k_2, k_2 \mathrel{!=} \varnothing,$

$i\otimes i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}r_{20}, Rcpo(i_0; r_{10}), i_0\circleddash, k_1\oplus,$

$j\otimes j_0, IsCpm(i; k_1; r_{10}), IsCpm(i; k_1; r_{20}),$

$IsCpo(j_0; r_{20}), IsCpo(j_0; r_{10}), r_{10}!\mathring{\bigcirc}r_{20}, Rcpo(j_0; r_{20}),$

$Rcpm(k_1; i; r_{10}), j_0\circleddash, k_2\oplus, Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\gtrdot j, k_1\asymp k_2, k_2 \mathrel{!=} \varnothing,$

$i\otimes i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}r_{20}, Rcpo(i_0; r_{10}), i_0\circleddash, k_1\oplus,$

$j\otimes j_0, IsCpo(j_0; r_{20}), r_{10}!\mathring{\bigcirc}r_{20}, Rcpo(j_0; r_{20}),$

$IsCpm(i; k_1; r_{10}), k_2!\mathring{\bigcirc}r_{10}, Rcpm(k_1; i; r_{10}), j_0\circleddash, k_2\oplus, Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\gtrdot j, k_1\asymp k_2, k_2 \mathrel{!=} \varnothing,$

$i\otimes i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}r_{20}, Rcpo(i_0; r_{10}), i_0\circleddash, k_1\oplus,$

$j\otimes j_0, IsCpo(j_0; r_{20}), r_{10}!\mathring{\bigcirc}r_{20}, Rcpo(j_0; r_{20}), j_0\circleddash, k_2\oplus,$

$IsCpm(i; k_1; r_{10}), k_2!\mathring{\bigcirc}r_{10}, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow\ , \&SHi\,\mathring{\bigcirc}k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\gtrdot j, k_1\asymp k_2, k_2 \mathrel{!=} \varnothing,$

$i\otimes i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}k_1, Rcpo(i_0; r_{10}), i_0\circleddash, k_1\oplus,$

j☮$j_0, IsCpo(j_0; r_{20}), r_{10}!$⚥$r_{20}, Rcpo(j_0; r_{20}), j_0$⏚$, k_2$⊕,

$IsCpm(i; k_1; r_{10}), k_2!$⚥$r_{10}, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20})$,

$\Leftrightarrow$, &SHi⚲$k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10})$,

$r_{10}!$⚥r_{20}, r_1⚲r_{10}, r_2⚲$r_{20}, i \gtrdot j$,

$k_1 \,!\!= \varnothing, k_1 \mp k_2, k_1$⊕,

i☮$i_0, IsCpo(i_0; r_{10}), r_{10}!$⚥$k_1, Rcpo(i_0; r_{10}), i_0$⏚,

j☮$j_0, IsCpo(j_0; r_{20}), r_{10}!$⚥$r_{20}, Rcpo(j_0; r_{20}), j_0$⏚$, k_2$⊕,

$IsCpm(i; k_1; r_{10}), k_2!$⚥$r_{10}, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20})$,

$\Leftrightarrow$, &SHi⚲$k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10})$,

$r_{10}!$⚥r_{20}, r_1⚲r_{10}, r_2⚲$r_{20}, i \gtrdot j$,

$k_1 \,!\!= \varnothing, k_1 \mp k_2, k_1$⊕,

i☮$i_0, IsCpo(i_0; r_{10}), r_{10}!$⚥$k_2, Rcpo(i_0; r_{10}), i_0$⏚,

j☮$j_0, IsCpo(j_0; r_{20}), k_2!$⚥$r_{20}, Rcpo(j_0; r_{20}), j_0$⏚$, k_2$⊕,

$IsCpm(i; k_1; r_{10}), k_2!$⚥$r_{10}, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20})$,

$\Leftrightarrow$, &SHi⚲$k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10})$,

$r_{10}!$⚥r_{20}, r_1⚲r_{10}, r_2⚲$r_{20}, i \gtrdot j$,

$k_1 \,!\!= \varnothing, k_1 \mp k_2, k_1$⊕$, k_2$⊕,

i☮$i_0, IsCpo(i_0; r_{10}), r_{10}!$⚥$k_2, r_{10}!$⚥$k_1, Rcpo(i_0; r_{10}), i_0$⏚,

j☮$j_0, IsCpo(j_0; r_{20}), k_2!$⚥$r_{20}, k_1!$⚥$r_{20}, Rcpo(j_0; r_{20}), j_0$⏚,

$Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20})$,

$\Leftrightarrow$, &*SHi* ♂$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10})$,

r_{10}!♂r_{20}, r_1♂r_{10}, r_2♂$r_{20}, i \gtrdot j$,

$k_1 \,!\!= \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus$,

i☮$i_0, IsCpo(i_0;r_{10}), r_{10}$!♂$k_2, r_{10}$!♂$k_1, Rcpo(i_0;r_{10}), i_0$⊕̣,

j☮$j_0, IsCpo(j_0;r_{20}), k_2$!♂$r_{20}, k_1$!♂$r_{20}, Rcpo(j_0;r_{20}), j_0$⊕̣,

$k_1 \doteq k_2, Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20})$,

$\Leftrightarrow$, &*SHi* ♂$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10})$,

r_{10}!♂r_{20}, r_1♂r_{10}, r_2♂$r_{20}, i \gtrdot j$,

$k_1 \,!\!= \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus$,

i☮$i_0, IsCpo(i_0;r_{10}), r_{10}$!♂$k_2, r_{10}$!♂$k_1, Rcpo(i_0;r_{10}), i_0$⊕̣,

j☮$j_0, IsCpo(j_0;r_{20}), k_2$!♂$r_{20}, k_1$!♂$r_{20}, Rcpo(j_0;r_{20}), j_0$⊕̣,

$$k_1 \doteq k_2, if(k_1 = \varnothing) \left[\begin{array}{l} , Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}), \\ , Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}), \end{array} \right],$$

$\Leftrightarrow$ $, < 1 >,$

, &*SHi* ♂$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10})$,

r_{10}!♂r_{20}, r_1♂r_{10}, r_2♂$r_{20}, i \gtrdot j$,

$k_1 \,!\!= \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus$,

i☮$i_0, IsCpo(i_0;r_{10}), r_{10}$!♂$k_2, r_{10}$!♂$k_1, Rcpo(i_0;r_{10}), i_0$⊕̣,

j☮$j_0, IsCpo(j_0;r_{20}), k_2$!♂$r_{20}, k_1$!♂$r_{20}, Rcpo(j_0;r_{20}), j_0$⊕̣,

$k_1 \doteq k_2, k_1 = \varnothing, Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20})$,

$\Leftrightarrow$, &*SHi* ♂$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10})$,

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 \,!\!= \varnothing, k_1 \asymp k_2, k_1 \oplus, k_2 \oplus,$

$i \otimes i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc} k_2, r_{10}!\mathring{\bigcirc} k_1, Rcpo(i_0; r_{10}), i_0 \circledast,$

$j \otimes j_0, IsCpo(j_0; r_{20}), k_2!\mathring{\bigcirc} r_{20}, k_1!\mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j_0 \circledast,$

$k_1 \asymp k_2, k_1 = \varnothing, Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 \,!\!= \varnothing, k_1 \asymp k_2, k_1 \oplus, k_2 \oplus,$

$i \otimes i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc} k_2, r_{10}!\mathring{\bigcirc} k_1, Rcpo(i_0; r_{10}), i_0 \circledast,$

$j \otimes j_0, IsCpo(j_0; r_{20}), k_2!\mathring{\bigcirc} r_{20}, k_1!\mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j_0 \circledast,$

$k_1 \asymp k_2, k_2 = \varnothing, Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 \,!\!= \varnothing, k_1 \asymp k_2, k_1 \oplus, k_2 \oplus,$

$i \otimes i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc} k_2, r_{10}!\mathring{\bigcirc} k_1, Rcpo(i_0; r_{10}), i_0 \circledast,$

$j \otimes j_0, IsCpo(j_0; r_{20}), k_2!\mathring{\bigcirc} r_{20}, k_1!\mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j_0 \circledast,$

$k_1 \asymp k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 \,!\!= \varnothing, k_1 \asymp k_2, k_1 \oplus, k_2 \oplus,$

$i \circledast i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}\!\circ i, r_{10}!\mathring{\bigcirc}\!\circ j, Rcpo(i_0; r_{10}), i_0 \circleddash,$

$j \circledast j_0, IsCpo(j_0; r_{20}), i!\mathring{\bigcirc}\!\circ r_{20}, j!\mathring{\bigcirc}\!\circ r_{20}, Rcpo(j_0; r_{20}), j_0 \circleddash,$

$k_1 \vdash\!\dashv k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j,$

$k_1 \mathrel{!=} \varnothing, k_1 \vdash\!\dashv k_2, k_1 \oplus, k_2 \oplus,$

$i \circledast i_0, IsCpo(i_0; r_{10}), r_{10}!\mathring{\bigcirc}\!\circ i, r_{10}!\mathring{\bigcirc}\!\circ j, Rcpo(i_0; r_{10}), i_0 \circleddash,$

$j \circledast j_0, IsCpo(j_0; r_{20}), i!\mathring{\bigcirc}\!\circ r_{20}, j!\mathring{\bigcirc}\!\circ r_{20}, Rcpo(j_0; r_{20}), j_0 \circleddash,$

$i \gtrdot j, k_1 \vdash\!\dashv k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j,$

$k_1 \mathrel{!=} \varnothing, k_1 \vdash\!\dashv k_2, k_1 \oplus, k_2 \oplus,$

$i \circledast i_0, IsCpo(i_0; r_{10}), r_1 \mathring{\bigcirc} r_{10}, Rcpo(i_0; r_{10}), i_0 \circleddash,$

$j \circledast j_0, IsCpo(j_0; r_{20}), r_2 \mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j_0 \circleddash,$

$i \gtrdot j, k_1 \vdash\!\dashv k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi\, \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \gtrdot j,$

$k_1 \mathrel{!=} \varnothing, k_1 \vdash\!\dashv k_2, k_1 \oplus, k_2 \oplus,$

$i \circledast i_0, IsCpo(i_0; r_{10}), r_1 \mathring{\bigcirc} r_{10}, Rcpo(i_0; r_{10}), i \vdash\!\dashv r_1, i_0 \circleddash,$

$j \circledast j_0, IsCpo(j_0; r_{20}), r_2 \mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j \vdash\!\dashv r_2, j_0 \circleddash,$

$i \gtrdot j, k_1 \vdash\!\dashv k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi$ ♁$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

r_{10}!♂r_{20}, r_1♁r_{10}, r_2♁$r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus,$

i☮$i_0, IsCpo(i_0;r_{10}), r_1$♁$r_{10}, Rcpo(i_0;r_{10}), i \doteq r_1, i_0$⏀,

j☮$j_0, IsCpo(j_0;r_{20}), i$!♂$r_{20}, r_1$!♂$r_{20}, Rcpo(j_0;r_{20}), j \doteq r_2, j_0$⏀,

$i \succ j, k_1 \doteq k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi$ ♁$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

r_{10}!♂r_{20}, r_1♁r_{10}, r_2♁$r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus,$

i☮$i_0, IsCpo(i_0;r_{10}), r_1$♁$r_{10}, Rcpo(i_0;r_{10}), i_0$⏀,

j☮$j_0, IsCpo(j_0;r_{20}), i$!♂$r_{20}, r_1$!♂$r_{20}, Rcpo(j_0;r_{20}), j_0$⏀,

$i \doteq r_1, j \doteq r_2, i \succ j, k_1 \doteq k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi$ ♁$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

r_{10}!♂r_{20}, r_1♁r_{10}, r_2♁$r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus,$

i☮$i_0, IsCpo(i_0;r_{10}), r_1$♁$r_{10}, Rcpo(i_0;r_{10}), i_0$⏀,

j☮$j_0, IsCpo(j_0;r_{20}), i$!♂$r_{20}, r_1$!♂$r_{20}, Rcpo(j_0;r_{20}), j_0$⏀,

$i \doteq r_1, j \doteq r_2, r_1 \succ r_2, k_1 \doteq k_2, k_2 = \varnothing,$

$\Leftrightarrow$, $\&SHi$ ♁$k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

r_{10}!♂r_{20}, r_1♁r_{10}, r_2♁$r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteqdot k_2, k_1 \oplus, k_2 \oplus,$

$i \oslash i_0, IsCpo(i_0; r_{10}), r_{10} ! \text{♂} k_2, r_{10} ! \text{♂} k_1, Rcpo(i_0; r_{10}), i_0 \circledast,$

$j \oslash j_0, IsCpo(j_0; r_{20}), k_2 ! \text{♂} r_{20}, k_1 ! \text{♂} r_{20}, Rcpo(j_0; r_{20}), j_0 \circledast,$

$k_1 \doteqdot k_2, k_1 = \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}), r_1 \succ r_2,$

$, \&SHi \text{♂} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} ! \text{♂} r_{20}, r_1 \text{♂} r_{10}, r_2 \text{♂} r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteqdot k_2, k_1 \oplus, k_2 \oplus,$

$i \oslash i_0, IsCpo(i_0; r_{10}), r_{10} ! \text{♂} i, r_{10} ! \text{♂} j, Rcpo(i_0; r_{10}), i_0 \circledast,$

$j \oslash j_0, IsCpo(j_0; r_{20}), i ! \text{♂} r_{20}, j ! \text{♂} r_{20}, Rcpo(j_0; r_{20}), j_0 \circledast,$

$k_1 \doteqdot k_2, k_1 \mathrel{!=} \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow \; , \&SHi \text{♂} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} ! \text{♂} r_{20}, r_1 \text{♂} r_{10}, r_2 \text{♂} r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteqdot k_2, k_1 \oplus, k_2 \oplus,$

$i \oslash i_0, IsCpo(i_0; r_{10}), r_{10} ! \text{♂} i, r_{10} ! \text{♂} j, Rcpo(i_0; r_{10}), i_0 \circledast,$

$j \oslash j_0, IsCpo(j_0; r_{20}), i ! \text{♂} r_{20}, j ! \text{♂} r_{20}, Rcpo(j_0; r_{20}), j_0 \circledast,$

$i \succ j, k_1 \doteqdot k_2, k_1 \mathrel{!=} \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}),$

$\Leftrightarrow \; , \&SHi \text{♂} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} ! \text{♂} r_{20}, r_1 \text{♂} r_{10}, r_2 \text{♂} r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteqdot k_2, k_1 \oplus, k_2 \oplus,$

$i\text{☮}i_0, IsCpo(i_0;r_{10}), r_{10}!\text{⚩}k_1, r_{10}!\text{⚩}r_{20}, Rcpo(i_0;r_{10}), i_0\text{⏚},$

$j\text{☮}j_0, IsCpo(j_0;r_{20}), k_1!\text{⚩}r_{20}, r_{10}!\text{⚩}r_{20}, Rcpo(j_0;r_{20}), j_0\text{⏚},$

$i\gtrdot j, k_1\text{⊢}k_2, k_1 \mathrel{!=} \varnothing, Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}),$

$\Leftrightarrow\ , IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\text{⚩}r_{20}, r_1\text{♂}r_{10}, r_2\text{♂}r_{20}, i\gtrdot j,$

$k_1 \mathrel{!=} \varnothing, k_1\text{⊢}k_2, k_1\oplus, k_2\oplus,$

$i\text{☮}i_0, IsCpo(i_0;r_{10}), r_{10}!\text{⚩}k_1, r_{10}!\text{⚩}r_{20}, Rcpo(i_0;r_{10}), i_0\text{⏚},$

$j\text{☮}j_0, IsCpo(j_0;r_{20}), k_1!\text{⚩}r_{20}, r_{10}!\text{⚩}r_{20}, Rcpo(j_0;r_{20}), j_0\text{⏚},$

$\&SHi\rightarrow k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\text{⚩}r_{20}, i\gtrdot j, k_1\text{⊢}k_2, k_1 \mathrel{!=} \varnothing, Rcpm(k_1;i;r_{10}),$

$IsCpm(j;k_2;r_{20}), Rcpm(k_2;j;r_{20}),$

$\Leftrightarrow\ , IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\text{⚩}r_{20}, r_1\text{♂}r_{10}, r_2\text{♂}r_{20}, i\gtrdot j,$

$k_1 \mathrel{!=} \varnothing, k_1\text{⊢}k_2, k_1\oplus, k_2\oplus,$

$i\text{☮}i_0, IsCpo(i_0;r_{10}), r_{10}!\text{⚩}k_1, r_{10}!\text{⚩}r_{20}, Rcpo(i_0;r_{10}), i_0\text{⏚},$

$j\text{☮}j_0, IsCpo(j_0;r_{20}), k_1!\text{⚩}r_{20}, r_{10}!\text{⚩}r_{20}, Rcpo(j_0;r_{20}), j_0\text{⏚},$

$\&SHi\rightarrow k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\text{⚩}r_{20}, i\gtrdot j, k_1\text{⊢}k_2, k_1 \mathrel{!=} \varnothing,$

$r_{10}\text{☮}r_{11}, r_{11}\text{⏚}, r_{20}\text{☮}r_{21}, r_{21}\text{⏚}, Rcpm(k_1;i;r_{10}),$

$IsCpm(j;k_2;r_{20}), Rcpm(k_2;j;r_{20}),$

$\Leftrightarrow\ , IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i \rhd j,$

$k_1 \mathrel{!{=}} \varnothing, k_1 \mp k_2, k_1\oplus, k_2\oplus,$

$i\circledast i_0, IsCpo(i_0;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}k_1, r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, Rcpo(i_0;r_{10}), i_0\obot,$

$j\circledast j_0, IsCpo(j_0;r_{20}), k_1!\mathring{\bigcirc}^{\circ}r_{20}, r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, Rcpo(j_0;r_{20}), j_0\obot,$

$\&SHi \rightarrow k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, i \rhd j, k_1 \mp k_2, k_1 \mathrel{!{=}} \varnothing,$

$r_{10}\circledast r_{11}, r_{20}\circledast r_{21}, Rcpm(k_1;i;r_{10}),$

$IsCpm(j;k_2;r_{20}), Rcpm(k_2;j;r_{20}), r_{11}\obot, r_{21}\obot,$

$\Leftrightarrow\ , IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i \rhd j,$

$k_1 \mathrel{!{=}} \varnothing, k_1 \mp k_2, k_1\oplus, k_2\oplus,$

$i\circledast i_0, IsCpo(i_0;r_{10}), r_{10}!\mathring{\bigcirc}^{\circ}k_1, r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, Rcpo(i_0;r_{10}), i_0\obot,$

$j\circledast j_0, IsCpo(j_0;r_{20}), k_1!\mathring{\bigcirc}^{\circ}r_{20}, r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, Rcpo(j_0;r_{20}), j_0\obot,$

$r_{10}\circledast r_{11}, r_{20}\circledast r_{21},$

$\&SHi \rightarrow k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, i \rhd j, k_1 \mp k_2, k_1 \mathrel{!{=}} \varnothing,$

$r_{10}\mathring{\bigcirc}r_{11}, r_{20}\mathring{\bigcirc}r_{21}, Rcpm(k_1;i;r_{10}),$

$Rcpm(k_2;j;r_{20}), r_{11}\obot, r_{21}\obot,$

$\Leftrightarrow\ , IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}^{\circ}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i \rhd j,$

$k_1 \mathrel{!}= \varnothing, k_1 \text{⫧} k_2, k_1 \oplus, k_2 \oplus,$

$i \text{☮} i_0, IsCpo(i_0; r_{10}), r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ k_1, r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, Rcpo(i_0; r_{10}), i_0 \text{⏚},$

$j \text{☮} j_0, IsCpo(j_0; r_{20}), k_1 \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, Rcpo(j_0; r_{20}), j_0 \text{⏚},$

$r_{10} \text{☮} r_{11}, r_{20} \text{☮} r_{21},$

$\&SHi \rightarrow k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, i \gtrdot j, k_1 \text{⫧} k_2, k_1 \mathrel{!}= \varnothing,$

$r_{10} \overset{\circ}{\bigcirc} r_{11}, r_{20} \overset{\circ}{\bigcirc} r_{21}, Rcpm(k_1; i; r_{10}),$

$Rcpm(k_2; j; r_{20}), r_{11} \succ r_{21}, r_{11} \text{⏚}, r_{21} \text{⏚},$

$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, i \gtrdot j,$

$k_1 \mathrel{!}= \varnothing, k_1 \text{⫧} k_2, k_1 \oplus, k_2 \oplus,$

$i \text{☮} i_0, IsCpo(i_0; r_{10}), r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ k_1, r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, Rcpo(i_0; r_{10}), i_0 \text{⏚},$

$j \text{☮} j_0, IsCpo(j_0; r_{20}), k_1 \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, Rcpo(j_0; r_{20}), j_0 \text{⏚},$

$r_{10} \text{☮} r_{11}, r_{20} \text{☮} r_{21}, k_1 \mathrel{!}= \varnothing,$

$IsCpm(i; k_1; r_{10}), \circledcirc n_1, n_1 \text{☮} n_{10}, Rcpm(k_1; i; n_{10}), Rcpo(n_1; r_{10}), n_1 \text{⏚}, n_{10} \text{⏚},$

$IsCpm(j; k_2; r_{20}), \circledcirc n_2, n_2 \text{☮} n_{20}, Rcpm(k_2; j; n_{20}), Rcpo(n_2; r_{20}), n_2 \text{⏚}, n_{20} \text{⏚},$

$r_{11} \succ r_{21}, r_{11} \text{⏚}, r_{21} \text{⏚},$

$\Leftrightarrow\ , \&SHi \overset{\circ}{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, r_1 \overset{\circ}{\bigcirc} r_{10}, r_2 \overset{\circ}{\bigcirc} r_{20}, i \gtrdot j,$

$k_1 \mathrel{!}= \varnothing, k_1 \text{⫧} k_2, k_1 \oplus, k_2 \oplus,$

$i \text{☮} i_0, IsCpo(i_0; r_{10}), r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ k_1, r_{10} \mathrel{!}\overset{\circ}{\bigcirc}\!\circ r_{20}, Rcpo(i_0; r_{10}), i_0 \text{⏚},$

$j \circledast j_0, IsCpo(j_0; r_{20}), k_1 !\mathring{\bigcirc} r_{20}, r_{10} !\mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j_0 \circleddash,$

$r_{10} \circledast r_{11}, r_{20} \circledast r_{21}, k_1 \mathrel{!=} \varnothing,$

$\circledcirc n_1, n_1 \circledast n_{10}, \circledcirc n_2, n_2 \circledast n_{20},$

$IsCpm(i; k_1; n_{10}), Rcpm(k_1; i; n_{10}),$

$IsCpo(n_1; r_{10}), n_1 \circledast n_{11}, r_{10} \mathring{\bigcirc} r_{11}, Rcpo(n_1; r_{10}), n_{11} \doteq r_{11}, n_{11} \circleddash,$

$IsCpm(j; k_2; n_{20}), Rcpm(k_2; j; n_{20}),$

$IsCpo(n_2; r_{20}), n_2 \circledast n_{21}, r_{20} \mathring{\bigcirc} r_{21}, Rcpo(n_2; r_{20}), n_{21} \doteq r_{21}, n_{21} \circleddash,$

$r_{11} \succ r_{21}, n_1 \circleddash, n_{10} \circleddash, n_2 \circleddash, n_{20} \circleddash, r_{11} \circleddash, r_{21} \circleddash,$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} !\mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteq k_2, k_1 \oplus, k_2 \oplus,$

$i \circledast i_0, IsCpo(i_0; r_{10}), r_{10} !\mathring{\bigcirc} k_1, r_{10} !\mathring{\bigcirc} r_{20}, Rcpo(i_0; r_{10}), i_0 \circleddash,$

$j \circledast j_0, IsCpo(j_0; r_{20}), k_1 !\mathring{\bigcirc} r_{20}, r_{10} !\mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j_0 \circleddash,$

$r_{10} \circledast r_{11}, r_{20} \circledast r_{21}, k_1 \mathrel{!=} \varnothing,$

$\circledcirc n_1, n_1 \circledast n_{10}, n_1 \circledast n_{11}, \circledcirc n_2, n_2 \circledast n_{20}, n_2 \circledast n_{21},$

$IsCpm(i; k_1; n_{10}), Rcpm(k_1; i; n_{10}),$

$IsCpo(n_1; r_{10}), r_{10} \mathring{\bigcirc} r_{11}, Rcpo(n_1; r_{10}),$

$IsCpm(j; k_2; n_{20}), Rcpm(k_2; j; n_{20}),$

$IsCpo(n_2; r_{20}), r_{20} \mathring{\bigcirc} r_{21}, Rcpo(n_2; r_{20}),$

$n_{11} \doteq r_{11}, n_{21} \doteq r_{21}, r_{11} \succ r_{21},$

$n_{11} \circleddash, n_{21} \circleddash, n_1 \circleddash, n_{10} \circleddash, n_2 \circleddash, n_{20} \circleddash, r_{11} \circleddash, r_{21} \circleddash,$

$\Leftrightarrow$, $\&SHi\,\text{♂}k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\text{⚩}r_{20}, r_1\text{♂}r_{10}, r_2\text{♂}r_{20}, i\succ j,$

$k_1 \mathrel{!=} \varnothing, k_1{=}k_2, k_1\oplus, k_2\oplus,$

$i\text{☮}i_0, IsCpo(i_0;r_{10}), r_{10}!\text{⚩}k_1, r_{10}!\text{⚩}r_{20}, Rcpo(i_0;r_{10}), i_0\text{⦽},$

$j\text{☮}j_0, IsCpo(j_0;r_{20}), k_1!\text{⚩}r_{20}, r_{10}!\text{⚩}r_{20}, Rcpo(j_0;r_{20}), j_0\text{⦽},$

$r_{10}\text{☮}r_{11}, r_{20}\text{☮}r_{21}, k_1 \mathrel{!=} \varnothing,$

$\text{◎}n_1, n_1\text{☮}n_{10}, n_1\text{☮}n_{11}, \text{◎}n_2, n_2\text{☮}n_{20}, n_2\text{☮}n_{21},$

$IsCpm(i;k_1;n_{10}), Rcpm(k_1;i;n_{10}),$

$IsCpo(n_1;r_{10}), r_{10}\text{♂}r_{11}, Rcpo(n_1;r_{10}),$

$IsCpm(j;k_2;n_{20}), Rcpm(k_2;j;n_{20}),$

$IsCpo(n_2;r_{20}), r_{20}\text{♂}r_{21}, Rcpo(n_2;r_{20}),$

$n_{11}{=}r_{11}, n_{21}{=}r_{21}, n_{11}\succ n_{21},$

$n_{11}\text{⦽}, n_{21}\text{⦽}, n_1\text{⦽}, n_{10}\text{⦽}, n_2\text{⦽}, n_{20}\text{⦽}, r_{11}\text{⦽}, r_{21}\text{⦽},$

$\Leftrightarrow$, $\&SHi\,\text{♂}k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\text{⚩}r_{20}, r_1\text{♂}r_{10}, r_2\text{♂}r_{20}, i\succ j,$

$k_1 \mathrel{!=} \varnothing, k_1{=}k_2, k_1\oplus, k_2\oplus, k_1 \mathrel{!=} \varnothing,$

$\text{◎}n_1, n_1\text{☮}n_{10}, n_1\text{☮}n_{11}, \text{◎}n_2, n_2\text{☮}n_{20}, n_2\text{☮}n_{21},$

$i\text{☮}i_0, j\text{☮}j_0,$

$IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}),$

$IsCpo(j_0;r_{20}), Rcpo(j_0;r_{20}),$

$IsCpm(i;k_1;n_{10}), Rcpm(k_1;i;n_{10}),$

$IsCpo(n_1;r_{10}), Rcpo(n_1;r_{10}),$

$IsCpm(j;k_2;n_{20}), Rcpm(k_2;j;n_{20}),$

$IsCpo(n_2;r_{20}), Rcpo(n_2;r_{20}),$

$n_{11} \succ n_{21},$

$i_0 \circledast, j_0 \circledast, n_{11} \circledast, n_{21} \circledast, n_1 \circledast, n_{10} \circledast, n_2 \circledast, n_{20} \circledast,$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10} !\mathring{\bigcirc} r_{20}, r_1 \bigcirc r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 != \varnothing, k_1 \mp k_2, k_1 \oplus, k_2 \oplus, k_1 != \varnothing,$

$\circledcirc n_1, n_1 \oslash n_{10}, n_1 \oslash n_{11}, \circledcirc n_2, n_2 \oslash n_{20}, n_2 \oslash n_{21},$

$i \oslash i_0, j \oslash j_0,$

$IsCpm(i;k_1;n_{10}), Rcpm(k_1;i;n_{10}),$

$IsCpm(j;k_2;n_{20}), Rcpm(k_2;j;n_{20}),$

$IsCpo(i_0;r_{10}), Rcpo(i_0;r_{10}),$

$IsCpo(n_1;r_{10}), Rcpo(n_1;r_{10}),$

$IsCpo(j_0;r_{20}), Rcpo(j_0;r_{20}),$

$IsCpo(n_2;r_{20}), Rcpo(n_2;r_{20}),$

$n_{11} \succ n_{21},$

$i_0 \circledast, j_0 \circledast, n_{11} \circledast, n_{21} \circledast, n_1 \circledast, n_{10} \circledast, n_2 \circledast, n_{20} \circledast,$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10} !\mathring{\bigcirc} r_{20}, r_1 \bigcirc r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 != \varnothing, k_1 \mp k_2, k_1 \oplus, k_2 \oplus, k_1 != \varnothing,$

$\circledcirc n_1, n_1 \oslash n_{10}, n_1 \oslash n_{11}, \circledcirc n_2, n_2 \oslash n_{20}, n_2 \oslash n_{21},$

$i\text{☮}i_0, j\text{☮}j_0,$

$IsCpm(i; k_1; n_{10}), Rcpm(k_1; i; n_{10}),$

$IsCpm(j; k_2; n_{20}), Rcpm(k_2; j; n_{20}),$

$IsCpo(i_0; r_{10}), IsCpo(n_1; r_{10}), i\text{⧂}i_0, n_1\text{⧂}n_{11}, r_1\text{⧂}r_{10},$

$Rcpo(i_0; r_{10}), Rcpo(n_1; r_{10}), i + n_{11} : t_1, t_1\text{⊻},$

$IsCpo(j_0; r_{20}), IsCpo(n_2; r_{20}), j\text{⧂}j_0, n_2\text{⧂}n_{21}, r_2\text{⧂}r_{20},$

$Rcpo(j_0; r_{20}), Rcpo(n_2; r_{20}), j + n_{21} : t_2, t_2\text{⊻},$

$n_{11}\text{⪧}n_{21},$

$i_0\text{⊻}, j_0\text{⊻}, n_{11}\text{⊻}, n_{21}\text{⊻}, n_1\text{⊻}, n_{10}\text{⊻}, n_2\text{⊻}, n_{20}\text{⊻},$

$\Leftrightarrow$, &SHi $\text{⧂}k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\text{⧂}r_{20}, r_1\text{⧂}r_{10}, r_2\text{⧂}r_{20}, i\text{⪧}j,$

$k_1 != \varnothing, k_1\text{⫧}k_2, k_1\oplus, k_2\oplus, k_1 != \varnothing,$

$\circledcirc n_1, n_1\text{☮}n_{10}, n_1\text{☮}n_{11}, \circledcirc n_2, n_2\text{☮}n_{20}, n_2\text{☮}n_{21},$

$i\text{☮}i_0, j\text{☮}j_0,$

$IsCpm(i; k_1; n_{10}), Rcpm(k_1; i; n_{10}),$

$IsCpm(j; k_2; n_{20}), Rcpm(k_2; j; n_{20}),$

$IsCpo(i_0; r_{10}), IsCpo(n_1; r_{10}), i\text{⧂}i_0, n_1\text{⧂}n_{11}, r_1\text{⧂}r_{10},$

$Rcpo(i_0; r_{10}), Rcpo(n_1; r_{10}), i + n_{11} : t_1, r_1\text{⫧}t_1, t_1\text{⊻},$

$IsCpo(j_0; r_{20}), IsCpo(n_2; r_{20}), j\text{⧂}j_0, n_2\text{⧂}n_{21}, r_2\text{⧂}r_{20},$

$Rcpo(j_0; r_{20}), Rcpo(n_2; r_{20}), j + n_{21} : t_2, r_2\text{⫧}t_2, t_2\text{⊻},$

$n_{11}\text{⪧}n_{21},$

$i_0\circledast, j_0\circledast, n_{11}\circledast, n_{21}\circledast, n_1\circledast, n_{10}\circledast, n_2\circledast, n_{20}\circledast,$

$\Leftrightarrow$, &SHi $\mathring{\bigcirc} k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i \succ j,$

$k_1 \mathrel{!}= \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus, k_1 \mathrel{!}= \varnothing,$

$\circledcirc n_1, n_1\otimes n_{10}, n_1\otimes n_{11}, \circledcirc n_2, n_2\otimes n_{20}, n_2\otimes n_{21},$

$i\otimes i_0, j\otimes j_0,$

$IsCpm(i;k_1;n_{10}), Rcpm(k_1;i;n_{10}),$

$IsCpm(j;k_2;n_{20}), Rcpm(k_2;j;n_{20}),$

$IsCpo(i_0;r_{10}), IsCpo(n_1;r_{10}), i\mathring{\bigcirc}i_0, n_1\mathring{\bigcirc}n_{11}, r_1\mathring{\bigcirc}r_{10},$

$Rcpo(i_0;r_{10}), Rcpo(n_1;r_{10}),$

$IsCpo(j_0;r_{20}), IsCpo(n_2;r_{20}), j\mathring{\bigcirc}j_0, n_2\mathring{\bigcirc}n_{21}, r_2\mathring{\bigcirc}r_{20},$

$Rcpo(j_0;r_{20}), Rcpo(n_2;r_{20}),$

$i + n_{11} : t_1, r_1 \doteq t_1, j + n_{21} : t_2, r_2 \doteq t_2, n_{11} \succ n_{21},$

$t_1\circledast, t_2\circledast, i_0\circledast, j_0\circledast, n_{11}\circledast, n_{21}\circledast, n_1\circledast, n_{10}\circledast, n_2\circledast, n_{20}\circledast,$

$\Leftrightarrow$, &SHi $\mathring{\bigcirc} k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i \succ j,$

$k_1 \mathrel{!}= \varnothing, k_1 \doteq k_2, k_1\oplus, k_2\oplus, k_1 \mathrel{!}= \varnothing,$

$\circledcirc n_1, n_1\otimes n_{10}, n_1\otimes n_{11}, \circledcirc n_2, n_2\otimes n_{20}, n_2\otimes n_{21},$

$i\otimes i_0, j\otimes j_0,$

$IsCpm(i;k_1;n_{10}), Rcpm(k_1;i;n_{10}),$

$IsCpm(j;k_2;n_{20}), Rcpm(k_2;j;n_{20}),$

$IsCpo(i_0; r_{10}), IsCpo(n_1; r_{10}), i\mathring{\bigcirc}i_0, n_1\mathring{\bigcirc}n_{11}, r_1\mathring{\bigcirc}r_{10},$

$Rcpo(i_0; r_{10}), Rcpo(n_1; r_{10}),$

$IsCpo(j_0; r_{20}), IsCpo(n_2; r_{20}), j\mathring{\bigcirc}j_0, n_2\mathring{\bigcirc}n_{21}, r_2\mathring{\bigcirc}r_{20},$

$Rcpo(j_0; r_{20}), Rcpo(n_2; r_{20}),$

$i\succ j, n_{11}\succ n_{21}, i + n_{11} : t_1, j + n_{21} : t_2, r_1\mp t_1, r_2\mp t_2,$

$t_1\ovee, t_2\ovee, i_0\ovee, j_0\ovee, n_{11}\ovee, n_{21}\ovee, n_1\ovee, n_{10}\ovee, n_2\ovee, n_{20}\ovee,$

$\Leftrightarrow$, $\&SHi\,\mathring{\bigcirc}k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\succ j,$

$k_1 != \varnothing, k_1\mp k_2, k_1\oplus, k_2\oplus, k_1 != \varnothing,$

$\circledcirc n_1, n_1\owedge n_{10}, n_1\owedge n_{11}, \circledcirc n_2, n_2\owedge n_{20}, n_2\owedge n_{21},$

$i\owedge i_0, j\owedge j_0,$

$IsCpm(i; k_1; n_{10}), Rcpm(k_1; i; n_{10}),$

$IsCpm(j; k_2; n_{20}), Rcpm(k_2; j; n_{20}),$

$IsCpo(i_0; r_{10}), IsCpo(n_1; r_{10}), i\mathring{\bigcirc}i_0, n_1\mathring{\bigcirc}n_{11}, r_1\mathring{\bigcirc}r_{10},$

$Rcpo(i_0; r_{10}), Rcpo(n_1; r_{10}),$

$IsCpo(j_0; r_{20}), IsCpo(n_2; r_{20}), j\mathring{\bigcirc}j_0, n_2\mathring{\bigcirc}n_{21}, r_2\mathring{\bigcirc}r_{20},$

$Rcpo(j_0; r_{20}), Rcpo(n_2; r_{20}),$

$i\succ j, n_{11}\succ n_{21}, i + n_{11} : t_1, j + n_{21} : t_2, t_1\succ t_2, r_1\mp t_1, r_2\mp t_2,$

$t_1\ovee, t_2\ovee, i_0\ovee, j_0\ovee, n_{11}\ovee, n_{21}\ovee, n_1\ovee, n_{10}\ovee, n_2\ovee, n_{20}\ovee,$

$\Leftrightarrow$, $\&SHi\,\mathring{\bigcirc}k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10}!\mathring{\bigcirc}r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteq k_2, k_1 \oplus, k_2 \oplus, k_1 \mathrel{!=} \varnothing,$

$\circledcirc n_1, n_1 \text{☮} n_{10}, n_1 \text{☮} n_{11}, \circledcirc n_2, n_2 \text{☮} n_{20}, n_2 \text{☮} n_{21},$

$i \text{☮} i_0, j \text{☮} j_0,$

$IsCpm(i; k_1; n_{10}), Rcpm(k_1; i; n_{10}),$

$IsCpm(j; k_2; n_{20}), Rcpm(k_2; j; n_{20}),$

$IsCpo(i_0; r_{10}), IsCpo(n_1; r_{10}), i \mathring{\bigcirc} i_0, n_1 \mathring{\bigcirc} n_{11}, r_1 \mathring{\bigcirc} r_{10},$

$Rcpo(i_0; r_{10}), Rcpo(n_1; r_{10}),$

$IsCpo(j_0; r_{20}), IsCpo(n_2; r_{20}), j \mathring{\bigcirc} j_0, n_2 \mathring{\bigcirc} n_{21}, r_2 \mathring{\bigcirc} r_{20},$

$Rcpo(j_0; r_{20}), Rcpo(n_2; r_{20}),$

$i \succ j, n_{11} \succ n_{21}, i + n_{11} : t_1, j + n_{21} : t_2, r_1 \succ r_2, r_1 \doteq t_1, r_2 \doteq t_2,$

$t_1 \ominus, t_2 \ominus, i_0 \ominus, j_0 \ominus, n_{11} \ominus, n_{21} \ominus, n_1 \ominus, n_{10} \ominus, n_2 \ominus, n_{20} \ominus,$

$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} ! \mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteq k_2, k_1 \oplus, k_2 \oplus,$

$i \text{☮} i_0, IsCpo(i_0; r_{10}), r_{10} ! \mathring{\bigcirc} i, r_{10} ! \mathring{\bigcirc} j, Rcpo(i_0; r_{10}), i_0 \ominus,$

$j \text{☮} j_0, IsCpo(j_0; r_{20}), i ! \mathring{\bigcirc} r_{20}, j ! \mathring{\bigcirc} r_{20}, Rcpo(j_0; r_{20}), j_0 \ominus,$

$k_1 \doteq k_2, k_1 \mathrel{!=} \varnothing, Rcpm(k_1; i; r_{10}), Rcpm(k_2; j; r_{20}), r_1 \succ r_2,$

$, < 1 >,$
$\Leftrightarrow$, $\&SHi \mathring{\bigcirc} k_1, IsCpm(i; k_1; r_{10}), IsCpm(j; k_2; r_{20}), IsCpm(i; k_1; r_{20}), IsCpm(j; k_2; r_{10}),$

$r_{10} ! \mathring{\bigcirc} r_{20}, r_1 \mathring{\bigcirc} r_{10}, r_2 \mathring{\bigcirc} r_{20}, i \succ j,$

$k_1 \mathrel{!=} \varnothing, k_1 \doteq k_2, k_1 \oplus, k_2 \oplus,$

$i\text{☮}i_0, IsCpo(i_0;r_{10}), r_{10}!\mathring{\bigcirc}\!\circ k_2, r_{10}!\mathring{\bigcirc}\!\circ k_1, Rcpo(i_0;r_{10}), i_0\text{⏚},$

$j\text{☮}j_0, IsCpo(j_0;r_{20}), k_2!\mathring{\bigcirc}\!\circ r_{20}, k_1!\mathring{\bigcirc}\!\circ r_{20}, Rcpo(j_0;r_{20}), j_0\text{⏚},$

$$k_1\text{⌶}k_2, if(k_1=\varnothing)\left[\begin{array}{l} , Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}), r_1\succ r_2, \\ \\ , Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}), r_1\succ r_2, \end{array}\right],$$

$\Leftrightarrow\ , k_1\,!\!=\varnothing,\ \&SHi\,\mathring{\bigcirc}k_1, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\succ j, k_1\text{⌶}k_2,$

$k_1\,!\!=\varnothing, Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}), r_1\succ r_2,$

$conclusion:$

$, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\succ j, k_1\text{⌶}k_2,$

$k_1\,!\!=\varnothing, Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}),\ \Leftrightarrow$

$, IsCpm(i;k_1;r_{10}), IsCpm(j;k_2;r_{20}), IsCpm(i;k_1;r_{20}), IsCpm(j;k_2;r_{10}),$

$r_{10}!\mathring{\bigcirc}\!\circ r_{20}, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20}, i\succ j, k_1\text{⌶}k_2,$

$k_1\,!\!=\varnothing, Rcpm(k_1;i;r_{10}), Rcpm(k_2;j;r_{20}), r_1\succ r_2,$

32 Multiplication

32.1 Definition

$$,i \times j : r, \iff ,\circledcirc r, r\text{☮} r_0, i\text{☮} i_0, Rcpm(i_0; j; r_0), i_0\text{⏚}, r_0\text{⏚},$$

32.2 Swap

32.2.1 Operator

$$,i \times j : r, m\text{☮} n, \iff ,m\text{☮} n, i \times j : r,$$

$$,i \times j : r, m\text{☮}_{\lrcorner} n, \iff ,m\text{☮}_{\lrcorner} n, i \times j : r,$$

$$,i \times j : r, m\text{☮}_{\circ} n, \iff ,m\text{☮}_{\circ} n, i \times j : r,$$

$$,i \times j : r, \circledcirc m, \iff ,\circledcirc m, i \times j : r,$$

$$,i \times j : r, \odot m, \iff ,\odot m, i \times j : r,$$

$$,i \times j : r, m\text{⏚}, \iff ,m\text{⏚}, i \times j : r,$$

$$,i \times j : r, m\oplus, \iff ,m\oplus, i \times j : r,$$

$$,i \times j : r, m\ominus, \iff ,m\ominus, i \times j : r,$$

$$,i \times j : r, m\text{⊜} n\text{-}\left[\begin{array}{l} , \\ , \end{array}\right. \iff ,m\text{⊜} n\text{-}\left[\begin{array}{l} ,i\times j{:}r, \\ ,i\times j{:}r, \end{array}\right.$$

32.2.2 Recursive Function

$$, i \times j : r, R(m), \;\Leftrightarrow\; , R(m), i \times j : r,$$

$$, i \times j : r, Rc(m;n), \;\Leftrightarrow\; , Rc(m;n), i \times j : r,$$

32.2.3 Propositions

$$, i \times j : r, m{=}n, \;\Leftrightarrow\; , m{=}n, i \times j : r,$$

$$, i \times j : r, m{=}\varnothing, \;\Leftrightarrow\; , m{=}\varnothing, i \times j : r,$$

$$, i \times j : r, m\overset{\circ}{\bigcirc}n, \;\Leftrightarrow\; , m\overset{\circ}{\bigcirc}n, i \times j : r,$$

$$, i \times j : r, m\overset{\circ\circ}{\bigcirc}n, \;\Leftrightarrow\; , m\overset{\circ\circ}{\bigcirc}n, i \times j : r,$$

$$, i \times j : r, m{\rightarrow}n, \;\Leftrightarrow\; , m{\rightarrow}n, i \times j : r,$$

$$, i \times j : r, m\obot n, \;\Leftrightarrow\; , m\obot n, i \times j : r,$$

$$, i \times j : r, m{\perp\!\!\!\!\top}n, \;\Leftrightarrow\; , m{\perp\!\!\!\!\top}n, i \times j : r,$$

$$, i \times j : r, m{\succ}n, \;\Leftrightarrow\; , m{\succ}n, i \times j : r,$$

$$, i \times j : r, m\,!{=}\,n, \;\Leftrightarrow\; , m\,!{=}\,n, i \times j : r,$$

$$, i \times j : r, m\,!{=}\,\varnothing, \;\Leftrightarrow\; , m\,!{=}\,\varnothing, i \times j : r,$$

$$, i \times j : r, m!\overset{\circ}{\bigcirc}n, \;\Leftrightarrow\; , m!\overset{\circ}{\bigcirc}n, i \times j : r,$$

$$, i \times j : r, m!\overset{\circ\circ}{\bigcirc}n, \;\Leftrightarrow\; , m!\overset{\circ\circ}{\bigcirc}n, i \times j : r,$$

$$, i \times j : r, m!{\rightarrow}n, \;\Leftrightarrow\; , m!{\rightarrow}n, i \times j : r,$$

$$, i \times j : r, m!\obot n, \;\Leftrightarrow\; , m!\obot n, i \times j : r,$$

$$, i \times j : r, m!\!=\!n, \iff , m!\!=\!n, i \times j : r,$$

$$, i \times j : r, m!\!>\!n, \iff , m!\!>\!n, i \times j : r,$$

32.2.4 Itself

$$, i_1 \times j : r_1, i_2 \times j : r_2, \iff , i_2 \times j : r_2, i_1 \times j : r_1,$$

$$, i_1 \times j_1 : r_1, i_2 \times j_2 : r_2, \iff , i_2 \times j_2 : r_2, i_1 \times j_1 : r_1,$$

$$, i_1 \times j : r_1, i_2 + j : r_2, \iff , i_2 + j : r_2, i_1 \times j : r_1,$$

$$, i_1 \times j_1 : r_1, i_2 + j_2 : r_2, \iff , i_2 + j_2 : r_2, i_1 \times j_1 : r_1,$$

$$, i \times j : r_1, i + j : r_2, \iff , i + j : r_2, i \times j : r_1,$$

$$, i \times j : r_1, i \times j : r_2, \iff , i \times j : r_2, i \times j : r_1,$$

32.2.5 The same operand

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32.3 General property

$$, \iff , i \times j : r, r\textcircled{$\downarrow$},$$

proof:
$, i \times j : r, r\textcircled{$\downarrow$},$

$\iff , \circledcirc r, i \circledwedge i_0, r \circledwedge r_0, Rcpm(i_0; j; r_0), i_0\textcircled{$\downarrow$}, r_0\textcircled{$\downarrow$}, r\textcircled{$\downarrow$},$

$\iff , \circledcirc r, i \circledwedge i_0, r \circledwedge r_0, IsCpm(i_0; j; r_0), Rcpm(i_0; j; r_0), i_0\textcircled{$\downarrow$}, r_0\textcircled{$\downarrow$}, r\textcircled{$\downarrow$},$

$\iff , \circledcirc r, i \circledwedge i_0, r \circledwedge r_0, IsCpm(i_0; j; r_0), Rcpm(i_0; j; r_0), i_0\textcircled{$\downarrow$}, r\textcircled{$\downarrow$}, r_0\textcircled{$\downarrow$},$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, IsCpm(i_0;j;r_0), Rcpm(i_0;j;r_0), i_0\text{⏚}, r\text{⏚}, \&Tm(r_0),$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, r\text{⏚}, IsCpm(i_0;j;r_0), Rcpm(i_0;j;r_0), i_0\text{⏚}, \&Tm(r_0),$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, r\text{⏚}, IsCpm(i_0;j;r_0), i_0\text{⏚}, \&Tm(r_0),$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, r\text{⏚}, IsCpm(i_0;j;r_0), i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, i_0\text{⏚}, r\text{☮}r_0, r_0\text{⏚}, r\text{⏚},$

$\Leftrightarrow\ ,$

$$,i\times j:r,\ \Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, j\text{☮}j_0, r\text{☮}r_0, Rcpm(i_0;j_0;r_0), i_0\text{⏚}, j_0\text{⏚}, r_0\text{⏚},$$

$$,i\times j:r,\otimes,\ \Leftrightarrow\ ,\otimes,$$

$$,i\times j:r_1, i\times j:r_2,\ \Leftrightarrow\ \sim, r_1 = r_2,$$

proof:
$,i\times j:r_1, i\times j:r_2,$

$\Leftrightarrow\ ,\circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, Rcpm(i_1;j_1;r_{10}), i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚},$

$\circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20}, Rcpm(i_2;j_2;r_{20}), i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, IsCpm(i_1;j_1;r_{10}), Rcpm(i_1;j_1;r_{10}), i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚},$

$\circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20}, IsCpm(i_2;j_2;r_{20}), Rcpm(i_2;j_2;r_{20}), i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpm(i_1;j_1;r_{10}), Rcpm(i_1;j_1;r_{10}),$

$IsCpm(i_2;j_2;r_{20}), Rcpm(i_2;j_2;r_{20}),$

$i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpm(i_1;j_1;r_{10}), r_{10}!\text{⚲}r_{20}, Rcpm(i_1;j_1;r_{10}),$

$IsCpm(i_2;j_2;r_{20}), Rcpm(i_2;j_2;r_{20}),$

$i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpm(i_1;j_1;r_{10}), IsCpm(i_2;j_2;r_{20}), r_{10}!\text{⚲}r_{20},$

$Rcpm(i_1;j_1;r_{10}), Rcpm(i_2;j_2;r_{20}),$

$i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpm(i_1;j_1;r_{10}), IsCpm(i_2;j_2;r_{20}), r_{10}!\text{⚲}r_{20},$

$i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20},$

$Rcpm(i_1;j_1;r_{10}), Rcpm(i_2;j_2;r_{20}),$

$i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r_1, i\text{☮}i_1, j\text{☮}j_1, r_1\text{☮}r_{10}, \circledcirc r_2, i\text{☮}i_2, j\text{☮}j_2, r_2\text{☮}r_{20},$

$IsCpm(i_1;j_1;r_{10}), IsCpm(i_2;j_2;r_{20}), r_{10}!\text{⚲}r_{20},$

$i_1\mathring{\bigcirc}i_2, j_1\mathring{\bigcirc}j_2, r_1\mathring{\bigcirc}r_{10}, r_2\mathring{\bigcirc}r_{20},$

$Rcpm(i_1;j_1;r_{10}), Rcpm(i_2;j_2;r_{20}), r_1 \doteq r_2,$

$i_1\text{⏚}, j_1\text{⏚}, r_{10}\text{⏚}, i_2\text{⏚}, j_2\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , i\times j : r_1, i\times j : r_2, r_1 \doteq r_2,$

$$, i_1 ⌶ i_2, i_1 \times j : r, \iff , i_1 ⌶ i_2, i_2 \times j : r,$$

proof:
$, i_1 ⌶ i_2, i_1 \times j : r,$

$\iff , i_1 ⌶ i_2, \circledcirc r, i_1 ☮ i_{10}, r ☮ r_0,$

$Rcpm(i_{10}; j; r_0), i_{10} ⏚, r_0 ⏚,$

$\iff , i_1 ⌶ i_2, \circledcirc r, i_1 ☮ i_{10}, r ☮ r_0, i_2 ☮ i_{20}, i_{20} ⏚,$

$IsCpm(i_{10}; j; r_0),$

$Rcpm(i_{10}; j; r_0), i_{10} ⏚, r_0 ⏚,$

$\iff , i_1 ⌶ i_2, \circledcirc r, i_1 ☮ i_{10}, r ☮ r_0, i_2 ☮ i_{20},$

$IsCpm(i_{10}; j; r_0), IsCpm(i_{20}; j; r_0), i_{10} ⌶ i_{20},$

$Rcpm(i_{10}; j; r_0), i_{10} ⏚, i_{20} ⏚, r_0 ⏚,$

$\iff , i_1 ⌶ i_2, \circledcirc r, i_1 ☮ i_{10}, r ☮ r_0, i_2 ☮ i_{20},$

$IsCpm(i_{10}; j; r_0), IsCpm(i_{20}; j; r_0), i_{10} ⌶ i_{20},$

$Rcpm(i_{20}; j; r_0), i_{10} ⏚, i_{20} ⏚, r_0 ⏚,$

$\iff , i_1 ⌶ i_2, \circledcirc r, r ☮ r_0, i_2 ☮ i_{20},$

$IsCpm(i_{20}; j; r_0),$

$Rcpm(i_{20}; j; r_0), i_{20} ⏚, r_0 ⏚,$

$\iff , i_1 ⌶ i_2, i_2 \times j : r,$

$$,i \times j : r, i\text{⏚}, \;\Leftrightarrow\; ,\circledcirc r, r\text{☮}r_0, Rcpm(i;j;r_0), i\text{⏚}, r_0\text{⏚},$$

$$,i = \varnothing, i \times j : r, \;\Leftrightarrow\; \sim, r = \varnothing,$$

$$,j = \varnothing, i \times j : r, \;\Leftrightarrow\; \sim, r = \varnothing,$$

32.4 Commutativity

$$,i \times j : r, \;\Leftrightarrow\; ,j \times i : r,$$

proof:
$,i \times j : r,$

$\Leftrightarrow\; ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, Rcpo(i_0;j;r_0), i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\; ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, j\text{☮}j_0, j_0\text{⏚},$

$IsCpm(i_0;j;r_0), Rcpm(i_0;j;r_0), i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\; ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, j\text{☮}j_0,$

$IsCpm(i_0;j;r_0), IsCpm(i_0;j_0;r_0), j = j_0,$

$Rcpm(i_0;j;r_0), j_0\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\; ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, j\text{☮}j_0,$

$IsCpm(i_0;j;r_0), IsCpm(i_0;j_0;r_0), j = j_0,$

$Rcpm(i_0;j_0;r_0), j_0\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\; ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, j\text{☮}j_0,$

$IsCpm(i_0;j_0;r_0),$

$Rcpm(i_0;j_0;r_0), R(j_0), j_0\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, j\text{☮}j_0,$

$IsCpm(i_0; j_0; r_0),$

$Rcpm(j_0; i_0; r_0), R(i_0), j_0\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, j\text{☮}j_0,$

$IsCpm(i_0; j_0; r_0), IsCpm(i; j_0; r_0), i{=}i_0,$

$Rcpm(j_0; i_0; r_0), j_0\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r, i\text{☮}i_0, r\text{☮}r_0, j\text{☮}j_0,$

$IsCpm(i_0; j_0; r_0), IsCpm(i; j_0; r_0), i{=}i_0,$

$Rcpm(j_0; i; r_0), j_0\text{⏚}, i_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ ,\circledcirc r, r\text{☮}r_0, j\text{☮}j_0,$

$IsCpm(j_0; i; r_0), Rcpm(j_0; i; r_0), j_0\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , j \times i : r,$

$$, i \times j : r_1, j \times i : r_2, \ \Leftrightarrow\ \sim, r_1{=}r_2,$$

proof:
$, i \times j : r_1, j \times i : r_2,$

$\Leftrightarrow\ , i \times j : r_1, i \times j : r_2,$

$\Leftrightarrow\ , i \times j : r_1, i \times j : r_2, r_1{=}r_2,$

$\Leftrightarrow\ , i \times j : r_1, j \times i : r_2, r_1{=}r_2,$

32.5 Distributivity

$$,i+j:r_1,r_1\times k:r,r_1\text{⊕}, \;\Leftrightarrow\; ,i\times k:r_1,j\times k:r_2,r_1+r_2:r,r_1\text{⊕},r_2\text{⊕},$$

proof:

$,i+j:r_1,r_1\times k:r,r_1\text{⊕},$

$\Leftrightarrow\ ,\circledcirc r_1,i\text{☮}i_0,j\text{☮}j_0,r_1\text{☮}r_{10},$

$Rcpo(i_0;r_{10}),Rcpo(j_0;r_{10}),i_0\text{⊕},j_0\text{⊕},r_{10}\text{⊕},$

$\circledcirc r,r\text{☮}r_0,Rcpm(r_1;k;r_0),r_1\text{⊕},r_0\text{⊕},$

$\Leftrightarrow\ ,\circledcirc r_1,i\text{☮}i_0,j\text{☮}j_0,r_1\text{☮}r_{10},$

$IsCpo(i_0;r_{10}),Rcpo(i_0;r_{10}),IsCpo(j_0;r_{10}),Rcpo(j_0;r_{10}),$

$\circledcirc r,r\text{☮}r_0,IsCpm(r_1;k;r_0),Rcpm(r_1;k;r_0),$

$i_0\text{⊕},j_0\text{⊕},r_{10}\text{⊕},r_1\text{⊕},r_0\text{⊕},$

$\Leftrightarrow\ ,\circledcirc r_1,i\text{☮}i_0,j\text{☮}j_0,r_1\text{☮}r_{10},\circledcirc r,r\text{☮}r_0,$

$IsCpo(i_0;r_{10}),Rcpo(i_0;r_{10}),IsCpo(j_0;r_{10}),Rcpo(j_0;r_{10}),$

$IsCpm(r_1;k;r_0),Rcpm(r_1;k;r_0),$

$i_0\text{⊕},j_0\text{⊕},r_{10}\text{⊕},r_1\text{⊕},r_0\text{⊕},$

$\Leftrightarrow\ ,\circledcirc r_1,i\text{☮}i_0,j\text{☮}j_0,r_1\text{☮}r_{10},\circledcirc r,r\text{☮}r_0,$

$IsCpo(i_0;r_{10}),r_{10}!\text{⚲}r_0,Rcpo(i_0;r_{10}),IsCpo(j_0;r_{10}),r_{10}!\text{⚲}r_0,Rcpo(j_0;r_{10}),$

$IsCpm(r_1;k;r_0),Rcpm(r_1;k;r_0),$

$i_0 \text{⏚}, j_0 \text{⏚}, r_{10} \text{⏚}, r_1 \text{⏚}, r_0 \text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, r \text{☮} r_0,$

$IsCpm(i_0; j_0; r_{10}), IsCpm(i_0; j_0; r_0), k!\text{⚇} r_0, k!\text{⚇} r_{10}, r_0!\text{⚇} r_{10}, r_1 \text{⚲} r_{10},$

$Rcpo(i_0; r_{10}), Rcpo(j_0; r_{10}), Rcpm(r_1; k; r_0),$

$i_0 \text{⏚}, j_0 \text{⏚}, r_{10} \text{⏚}, r_1 \text{⏚}, r_0 \text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, i \text{☮} i_0, j \text{☮} j_0, r_1 \text{☮} r_{10}, \circledcirc r, r \text{☮} r_0,$

$IsCpm(i_0; j_0; r_{10}), IsCpm(i_0; j_0; r_0), k!\text{⚇} r_0, k!\text{⚇} r_{10}, r_0!\text{⚇} r_{10}, r_1 \text{⚲} r_{10},$

$Rcpm(i_0; k; r_0), Rcpm(j_0; k; r_0),$

$i_0 \text{⏚}, j_0 \text{⏚}, r_{10} \text{⏚}, r_1 \text{⏚}, r_0 \text{⏚},$

$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, \circledcirc r, r \text{☮} r_0,$

$IsCpm(i_0; k; r_0), Rcpm(i_0; k; r_0),$

$IsCpm(j_0; k; r_0), Rcpm(j_0; k; r_0),$

$i_0 \text{⏚}, j_0 \text{⏚}, r_0 \text{⏚},$

$\Leftrightarrow\ , i \text{☮} i_0, j \text{☮} j_0, \circledcirc r, r \text{☮} r_0,$

$IsCpm(i_0; k; r_0), \circledcirc r_1, r_1 \text{☮} r_{10}, Rcpm(i_0; k; r_{10}), Rcpo(r_1; r_0), r_1 \text{⏚}, r_{10} \text{⏚},$

$IsCpm(j_0; k; r_0), \circledcirc r_2, r_2 \text{☮} r_{20}, Rcpm(j_0; k; r_{20}), Rcpo(r_2; r_0), r_2 \text{⏚}, r_{20} \text{⏚},$

$i_0 \text{⏚}, j_0 \text{⏚}, r_0 \text{⏚},$

$\Leftrightarrow\ , i \text{☮} i_0, \circledcirc r_1, r_1 \text{☮} r_{10}, Rcpm(i_0; k; r_{10}), r_{10} \text{⏚}, i_0 \text{⏚},$

$j \text{☮} j_0, \circledcirc r_2, r_2 \text{☮} r_{20}, Rcpm(j_0; k; r_{20}), r_{20} \text{⏚}, j_0 \text{⏚},$

$\circledcirc r, r \text{☮} r_0, Rcpo(r_1; r_0), Rcpo(r_2; r_0), r_2 \text{⏚}, r_1 \text{⏚}, r_0 \text{⏚},$

$\Leftrightarrow\ , i \times k : r_1, j \times k : r_2,$

$\circledcirc r, r \text{⦸} r_0, Rcpo(r_1; r_0), Rcpo(r_2; r_0), r_2\text{⏚}, r_1\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , i \times k : r_1, j \times k : r_2, r_1 + r_2 : r, r_1\text{⏚}, r_2\text{⏚},$

$$\begin{array}{c} , i + j : r_3, r_3 \times k : r_1, r_3\text{⏚}, \\ i \times k : r_4, j \times k : r_5, r_4 + r_5 : r_2, r_4\text{⏚}, r_5\text{⏚}, \end{array} \ \Leftrightarrow\ \sim, r_1 \text{⌶} r_2,$$

proof:

$, i + j : r_3, r_3 \times k : r_1, r_3\text{⏚},$

$i \times k : r_4, j \times k : r_5, r_4 + r_5 : r_2, r_4\text{⏚}, r_5\text{⏚},$

$\Leftrightarrow\ , i \times k : r_6, j \times k : r_7, r_6 + r_7 : r_1, r_6\text{⏚}, r_7\text{⏚},$

$i \times k : r_4, j \times k : r_5, r_4 + r_5 : r_2, r_4\text{⏚}, r_5\text{⏚},$

$\Leftrightarrow\ , i \times k : r_6, i \times k : r_4, j \times k : r_7, j \times k : r_5,$

$r_6 + r_7 : r_1, r_4 + r_5 : r_2, r_6\text{⏚}, r_7\text{⏚}, r_4\text{⏚}, r_5\text{⏚},$

$\Leftrightarrow\ , i \times k : r_6, i \times k : r_4, r_6 \text{⌶} r_4, j \times k : r_7, j \times k : r_5, r_7 \text{⌶} r_5,$

$r_6 + r_7 : r_1, r_4 + r_5 : r_2, r_6\text{⏚}, r_7\text{⏚}, r_4\text{⏚}, r_5\text{⏚},$

$\Leftrightarrow\ , i \times k : r_6, i \times k : r_4, j \times k : r_7, j \times k : r_5, r_6 \text{⌶} r_4, r_7 \text{⌶} r_5,$

$r_6 + r_7 : r_1, r_4 + r_5 : r_2, r_6\text{⏚}, r_7\text{⏚}, r_4\text{⏚}, r_5\text{⏚},$

$\Leftrightarrow\ , i \times k : r_6, i \times k : r_4, j \times k : r_7, j \times k : r_5, r_6 \text{⌶} r_4, r_7 \text{⌶} r_5,$

$r_4 + r_5 : r_1, r_4 + r_5 : r_2, r_6\text{⏚}, r_7\text{⏚}, r_4\text{⏚}, r_5\text{⏚},$

$\Leftrightarrow\ , i \times k : r_6, i \times k : r_4, j \times k : r_7, j \times k : r_5, r_6 \text{⌶} r_4, r_7 \text{⌶} r_5,$

$r_4 + r_5 : r_1, r_4 + r_5 : r_2, r_1 \text{⌶} r_2, r_6\text{⏚}, r_7\text{⏚}, r_4\text{⏚}, r_5\text{⏚},$

$\Leftrightarrow\ , i + j : r_3, r_3 \times k : r_1, r_3\text{⏚},$

$i \times k : r_4, j \times k : r_5, r_4 + r_5 : r_2, r_4\text{⏚}, r_5\text{⏚}, r_1 \text{⌶} r_2,$

32.6 Associativity

$$, j \times i : r_1, r_1 \times k : r, r_1\text{⏚}, \ \Leftrightarrow\ , j \times k : r_1, r_1 \times i : r, r_1\text{⏚},$$

proof:
$, j \times i : r_1, r_1 \times k : r, r_1\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, r_1 \text{☮} r_{10}, j \text{☮} j_0, Rcpm(j_0; i; r_{10}), j_0\text{⏚}, r_{10}\text{⏚},$

$\circledcirc r, r \text{☮} r_0, Rcpm(r_1; k; r_0), r_1\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, r_1 \text{☮} r_{10}, j \text{☮} j_0,$

$IsCpm(j_0; i; r_{10}), Rcpm(j_0; i; r_{10}), j_0\text{⏚}, r_{10}\text{⏚},$

$\circledcirc r, r \text{☮} r_0, IsCpm(r_1; k; r_0), Rcpm(r_1; k; r_0), r_1\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, r_1 \text{☮} r_{10}, j \text{☮} j_0, \circledcirc r, r \text{☮} r_0,$

$IsCpm(j_0; i; r_{10}), Rcpm(j_0; i; r_{10}),$

$IsCpm(r_1; k; r_0), Rcpm(r_1; k; r_0), j_0\text{⏚}, r_{10}\text{⏚}, r_1\text{⏚}, r_0\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, r_1 \text{☮} r_{10}, j \text{☮} j_0, \circledcirc r, r \text{☮} r_0,$

$IsCpm(j_0; i; r_{10}), r_0 ! \text{⚲} r_{10}, Rcpm(j_0; i; r_{10}),$

$IsCpm(r_1;k;r_0), Rcpm(r_1;k;r_0), j_0⏚, r_{10}⏚, r_1⏚, r_0⏚,$

$\Leftrightarrow\ , \circledcirc r_1, r_1☮r_{10}, j☮j_0, \circledcirc r, r☮r_0,$

$IsCpm(j_0;i;r_{10}), IsCpm(j_0;i;r_0), k!♂r_0, k!♂r_{10}, r_0!♂r_{10},$

$Rcpm(j_0;i;r_{10}), Rcpm(r_1;k;r_0), j_0⏚, r_{10}⏚, r_1⏚, r_0⏚,$

$\Leftrightarrow\ , \circledcirc r_1, r_1☮r_{10}, j☮j_0, \circledcirc r, r☮r_0,$

$IsCpm(j_0;i;r_{10}), IsCpm(j_0;i;r_0), k!♂r_0, k!♂r_{10}, r_0!♂r_{10},$

$Rcpm(j_0;k;r_{10}), Rcpm(r_1;i;r_0), j_0⏚, r_{10}⏚, r_1⏚, r_0⏚,$

$\Leftrightarrow\ , j\times k : r_1, r_1\times i : r, r_1⏚,$

$$, j\times i : r_3, r_3\times k : r_1, r_3⏚,$$
$$, j\times k : r_4, r_4\times i : r_2, r_4⏚, \ \Leftrightarrow\ \sim, r_1⌶r_2,$$

proof:
$, j\times i : r_3, r_3\times k : r_1, r_3⏚,$

$j\times k : r_4, r_4\times i : r_2, r_4⏚,$

$\Leftrightarrow\ , j\times i : r_3, r_3\times k : r_1, r_3⏚,$

$j\times i : r_4, r_4\times k : r_2, r_4⏚,$

$\Leftrightarrow\ , j\times i : r_3, j\times i : r_4,$

$r_3\times k : r_1, r_4\times k : r_2, r_3⏚, r_4⏚,$

$\Leftrightarrow\ , j\times i : r_3, j\times i : r_4, r_3⌶r_4,$

$r_3\times k : r_1, r_4\times k : r_2, r_3⏚, r_4⏚,$

$\Leftrightarrow\ , j \times i : r_3, j \times i : r_4, r_3 \doteq r_4,$

$r_4 \times k : r_1, r_4 \times k : r_2, r_3 ⊕, r_4 ⊕,$

$\Leftrightarrow\ , j \times i : r_3, j \times i : r_4, r_3 \doteq r_4,$

$r_4 \times k : r_1, r_4 \times k : r_2, r_1 \doteq r_2, r_3 ⊕, r_4 ⊕,$

$\Leftrightarrow\ , j \times i : r_3, r_3 \times k : r_1, r_3 ⊕,$

$j \times k : r_4, r_4 \times i : r_2, r_4 ⊕, r_1 \doteq r_2,$

32.7 Monotonicity

$$, i \gtrdot j, k_1 \doteq k_2, k_1 \mathrel{!}= \varnothing, i \times k_1 : r_1, j \times k_2 : r_2, \quad \Leftrightarrow \quad \sim, r_1 \gtrdot r_2,$$

proof:
$, i \gtrdot j, k_1 \doteq k_2, k_1 \mathrel{!}= \varnothing, i \times k_1 : r_1, j \times k_2 : r_2,$

$\Leftrightarrow\ , i \gtrdot j, k_1 \doteq k_2, k_1 \mathrel{!}= \varnothing, k_1 \times i : r_1, k_2 \times j : r_2,$

$\Leftrightarrow\ , i \gtrdot j, k_1 \doteq k_2, k_1 \mathrel{!}= \varnothing,$

$⊚ r_1, r_1 ☮ r_{10}, k ☮ k_{10}, Rcpm(k_{10}; i; r_{10}), k_{10} ⊕, r_{10} ⊕,$

$⊚ r_2, r_2 ☮ r_{20}, k ☮ k_{20}, Rcpm(k_{20}; j; r_{20}), k_{20} ⊕, r_{20} ⊕,$

$\Leftrightarrow\ , i \gtrdot j, k_1 \doteq k_2, k_1 \mathrel{!}= \varnothing, ⊚ r_1, r_1 ☮ r_{10}, k ☮ k_{10},$

$IsCpm(k_{10}; i; r_{10}), Rcpm(k_{10}; i; r_{10}), k_{10} ⊕, r_{10} ⊕,$

$⊚ r_2, r_2 ☮ r_{20}, k ☮ k_{20}, IsCpm(k_{20}; j; r_{20}), Rcpm(k_{20}; j; r_{20}), k_{20} ⊕, r_{20} ⊕,$

$\Leftrightarrow\ , i \gtrdot j, k_1 \text{⌶} k_2, k_1 \mathrel{!=} \varnothing, \circledcirc r_1, r_1 \text{☮} r_{10}, k \text{☮} k_{10}, \circledcirc r_2, r_2 \text{☮} r_{20}, k \text{☮} k_{20},$

$IsCpm(k_{10}; i; r_{10}), Rcpm(k_{10}; i; r_{10}),$

$IsCpm(k_{20}; j; r_{20}), Rcpm(k_{20}; j; r_{20}), k_{10}\text{⏚}, r_{10}\text{⏚}, k_{20}\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , i \gtrdot j, k_1 \text{⌶} k_2, k_1 \mathrel{!=} \varnothing, \circledcirc r_1, r_1 \text{☮} r_{10}, k \text{☮} k_{10}, \circledcirc r_2, r_2 \text{☮} r_{20}, k \text{☮} k_{20},$

$IsCpm(k_{10}; i; r_{10}), r_{10} ! \text{⚨} r_{20}, Rcpm(k_{10}; i; r_{10}),$

$IsCpm(k_{20}; j; r_{20}), Rcpm(k_{20}; j; r_{20}), k_{10}\text{⏚}, r_{10}\text{⏚}, k_{20}\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, r_1 \text{☮} r_{10}, k \text{☮} k_{10}, \circledcirc r_2, r_2 \text{☮} r_{20}, k \text{☮} k_{20},$

$IsCpm(k_{10}; i; r_{10}), IsCpm(k_{10}; i; r_{20}), IsCpm(k_{20}; j; r_{20}), IsCpm(k_{20}; j; r_{10}), r_{10} ! \text{⚨} r_{20},$

$r_1 \text{⚨} r_{10}, r_2 \text{⚨} r_{20}, i \gtrdot j, k_{10} \text{⌶} k_{20}, k_{10} \mathrel{!=} \varnothing,$

$Rcpm(k_{10}; i; r_{10}), Rcpm(k_{20}; j; r_{20}), k_{10}\text{⏚}, r_{10}\text{⏚}, k_{20}\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , \circledcirc r_1, r_1 \text{☮} r_{10}, k \text{☮} k_{10}, \circledcirc r_2, r_2 \text{☮} r_{20}, k \text{☮} k_{20},$

$IsCpm(k_{10}; i; r_{10}), IsCpm(k_{10}; i; r_{20}), IsCpm(k_{20}; j; r_{20}), IsCpm(k_{20}; j; r_{10}), r_{10} ! \text{⚨} r_{20},$

$r_1 \text{⚨} r_{10}, r_2 \text{⚨} r_{20}, i \gtrdot j, k_{10} \text{⌶} k_{20}, k_{10} \mathrel{!=} \varnothing,$

$Rcpm(k_{10}; i; r_{10}), Rcpm(k_{20}; j; r_{20}), r_1 \gtrdot r_2, k_{10}\text{⏚}, r_{10}\text{⏚}, k_{20}\text{⏚}, r_{20}\text{⏚},$

$\Leftrightarrow\ , i \gtrdot j, k_1 \text{⌶} k_2, k_1 \mathrel{!=} \varnothing, i \times k_1 : r_1, j \times k_2 : r_2, r_1 \gtrdot r_2,$

$$,i\succ j,k\,!\!=\varnothing,i\times k:r_1,j\times k:r_2,\quad\Leftrightarrow\quad\sim,r_1\succ r_2,$$

proof:

$,i\succ j,k\,!\!=\varnothing,i\times k:r_1,j\times k:r_2,$

$\Leftrightarrow\ ,i\succ j,k\,!\!=\varnothing,k\text{☮}k_1,k_1\text{⏚},i\times k:r_1,j\times k:r_2,$

$\Leftrightarrow\ ,k\text{☮}k_1,i\succ j,k\,!\!=\varnothing,k\text{⌶}k_1,i\times k:r_1,j\times k:r_2,k_1\text{⏚},$

$\Leftrightarrow\ ,k\text{☮}k_1,i\succ j,k\,!\!=\varnothing,k\text{⌶}k_1,i\times k_1:r_1,j\times k:r_2,k_1\text{⏚},$

$\Leftrightarrow\ ,k\text{☮}k_1,i\succ j,k\,!\!=\varnothing,k\text{⌶}k_1,i\times k_1:r_1,j\times k:r_2,r_1\succ r_2,k_1\text{⏚},$

$\Leftrightarrow\ ,i\succ j,k\,!\!=\varnothing,i\times k:r_1,j\times k:r_2,r_1\succ r_2,$

$$,i\succ j,k_1\succ k_2,i\times k_1:r_1,j\times k_2:r_2,\quad\Leftrightarrow\quad\sim,r_1\succ r_2,$$

proof:

$,i\succ j,k_1\succ k_2,i\times k_1:r_1,j\times k_2:r_2,$

$\Leftrightarrow\ ,i\succ j,k_1\succ k_2,i\times k_1:r_1,j\times k_1:r_3,r_3\text{⏚},j\times k_2:r_2,$

$\Leftrightarrow\ ,i\succ j,k_1\succ k_2,k_1\,!\!=\varnothing,i\times k_1:r_1,j\times k_1:r_3,j\times k_2:r_2,r_3\text{⏚},$

$\Leftrightarrow\ ,i\succ j,k_1\succ k_2,k_1\,!\!=\varnothing,i\times k_1:r_1,j\times k_1:r_3,r_1\succ r_3,j\times k_2:r_2,r_3\text{⏚},$

$\Leftrightarrow\ ,i\times k_1:r_1,i\succ j,if(j=\varnothing)\left[\begin{array}{l},\\ ,\end{array}\right],k_1\succ k_2,j\times k_1:r_3,j\times k_2:r_2,r_1\succ r_3,r_3\text{⏚},$

$\Leftrightarrow\ ,i\times k_1:r_1,i\succ j,$

$$if(j=\varnothing)\left[\begin{array}{l},j=\varnothing,k_1\succ k_2,j\times k_1:r_3,j\times k_2:r_2,r_1\succ r_3,\\ ,k_1\succ k_2,j\times k_1:r_3,j\times k_2:r_2,r_1\succ r_3,\end{array}\right],r_3\text{⏚},$$

$\Leftrightarrow\ ,i\times k_1:r_1,i\succ j,$

$$if(j=\varnothing)\left[\begin{array}{l},k_1\succ k_2,j\times k_1:r_3,j=\varnothing,j\times k_2:r_2,r_2=\varnothing,r_1\succ r_3,r_1\,!\!=\varnothing,r_1\succ r_2,\\ ,k_1\succ k_2,j\times k_1:r_3,j\times k_2:r_2,r_1\succ r_3,\end{array}\right],r_3\text{⏚},$$

$$\Leftrightarrow\ , i \times k_1 : r_1, i \succ j,$$
$$if(j = \varnothing)\left[\begin{array}{l} , k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_1 \succ r_3, r_1 \succ r_2, \\ \\ , k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_1 \succ r_3, \end{array}\right], r_3 \textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , i \times k_1 : r_1, i \succ j,$$
$$if(j = \varnothing)\left[\begin{array}{l} , k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_1 \succ r_3, r_1 \succ r_2, \\ \\ , j\ != \varnothing, k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_1 \succ r_3, \end{array}\right], r_3 \textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , i \times k_1 : r_1, i \succ j,$$
$$if(j = \varnothing)\left[\begin{array}{l} , k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_1 \succ r_3, r_1 \succ r_2, \\ \\ , j\ != \varnothing, k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_3 \succ r_2, r_1 \succ r_3, r_1 \succ r_2, \end{array}\right], r_3 \textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , i \times k_1 : r_1, i \succ j,$$
$$if(j = \varnothing)\left[\begin{array}{l} , k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_1 \succ r_3, r_1 \succ r_2, \\ \\ , k_1 \succ k_2, j \times k_1 : r_3, j \times k_2 : r_2, r_1 \succ r_3, r_1 \succ r_2, \end{array}\right], r_3 \textcircled{$\downarrow$},$$

$$\Leftrightarrow\ , i \succ j, k_1 \succ k_2, i \times k_1 : r_1, j \times k_2 : r_2, r_1 \succ r_2,$$

33 Paradox

33.1 Theorems of contradiction

$$, \;\Leftrightarrow\; ,\otimes, \;\Rightarrow\; ,©c_1, \;\Leftrightarrow\; ,©c_2,$$

proof:
$, \;\Leftrightarrow\; ,\otimes, \;\Rightarrow$

$,©c_1,$

$\Leftrightarrow\; ,\otimes,©c_1,$

$\Leftrightarrow\; ,\otimes,$

$\Leftrightarrow\; ,\otimes,©c_2,$

$\Leftrightarrow\; ,©c_2,$

$$,i=j, \;\Leftrightarrow\; ,i\,!\!=j, \;\Rightarrow\; , \;\Leftrightarrow\; ,\otimes,$$

proof:
$,i=j, \;\Leftrightarrow\; ,i\,!\!=j, \;\Rightarrow$

$,$

$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} , \\ \\ , \end{array}\right],$

$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,i=j, \\ \\ ,i\,!\!=j, \end{array}\right],$

$\Leftrightarrow\; ,if(i=j)\left[\begin{array}{l} ,i=j,i=j, \\ \\ ,i\,!\!=j,i\,!\!=j, \end{array}\right],$

$$\Leftrightarrow \; , if(i=j) \left[\begin{array}{l} , i=j, i\,!\!=j, \\ \\ , i=j, i\,!\!=j, \end{array} \right],$$

$$\Leftrightarrow \; , if(i=j) \left[\begin{array}{l} , \otimes, \\ \\ , \otimes, \end{array} \right],$$

$$\Leftrightarrow \; , if(i=j) \left[\begin{array}{l} , \\ \\ , \end{array} \right], \otimes,$$

$$\Leftrightarrow \; , \otimes,$$

$$, i=\varnothing, \;\; \Leftrightarrow \;\; , i\,!\!=\varnothing, \;\; \Rightarrow \;\; , \;\; \Leftrightarrow \;\; , \otimes,$$

$$, i \bigcirc\!\!\!\!^{\circ}\; j, \;\; \Leftrightarrow \;\; , i! \bigcirc\!\!\!\!^{\circ}\; j, \;\; \Rightarrow \;\; , \;\; \Leftrightarrow \;\; , \otimes,$$

$$, i \bigcirc\!\!\!\!^{\circ}{}^{\circ} j, \;\; \Leftrightarrow \;\; , i! \bigcirc\!\!\!\!^{\circ}{}^{\circ} j, \;\; \Rightarrow \;\; , \;\; \Leftrightarrow \;\; , \otimes,$$

$$, i \rightarrow j, \;\; \Leftrightarrow \;\; , i! \rightarrow j, \;\; \Rightarrow \;\; , \;\; \Leftrightarrow \;\; , \otimes,$$

$$, i \bigcirc\!\!\!\!\!\perp j, \;\; \Leftrightarrow \;\; , i! \bigcirc\!\!\!\!\!\perp j, \;\; \Rightarrow \;\; , \;\; \Leftrightarrow \;\; , \otimes,$$

$$, i \dashv\!\vdash j, \;\; \Leftrightarrow \;\; , i! \dashv\!\vdash j, \;\; \Rightarrow \;\; , \;\; \Leftrightarrow \;\; , \otimes,$$

$$, i \succ j, \;\; \Leftrightarrow \;\; , i! \succ j, \;\; \Rightarrow \;\; , \;\; \Leftrightarrow \;\; , \otimes,$$

33.2 Definition of paradox

paradox: " This statement is false."

$$,if(Pdx)\left[\begin{array}{l},\\ \\,\end{array}\right. \Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\circledcirc m, m\text{☮}n, m\textcircled{$\downarrow$},\\ \\,\circledcirc n,\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\textcircled{$\downarrow$},\\ \\,n\textcircled{$\downarrow$},\end{array}\right.$$

$$,Pdx,\ \Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\\ \\,\otimes,\end{array}\right],$$

$$,!Pdx,\ \Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\otimes,\\ \\,\end{array}\right],$$

33.3 Theorems of paradox propositions

$$,Pdx,\ \Leftrightarrow\ ,!Pdx,$$

proof:
$,Pdx,$

$$\Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\\ \\,\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\circledcirc m, m\text{☮}n, m\textcircled{$\downarrow$},\\ \\,\circledcirc n,\end{array}\right],if(n=\varnothing)\left[\begin{array}{l},n\textcircled{$\downarrow$},\\ \\,n\textcircled{$\downarrow$},\otimes,\end{array}\right],$$

$$\Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\circledcirc m, m\text{☮}n, m\textcircled{$\downarrow$},\\ \\,\circledcirc n,\end{array}\right],n=\varnothing,n\textcircled{$\downarrow$},$$

$$\Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\circledcirc m, m\text{☮}n, n=\varnothing, n\textcircled{$\downarrow$}, m\textcircled{$\downarrow$},\\ \\,\circledcirc n, n=\varnothing, n\textcircled{$\downarrow$},\end{array}\right],$$

$$\Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\circledcirc m, m\text{☮}n, n\,!\!=\varnothing, n=\varnothing, n\textcircled{$\downarrow$}, m\textcircled{$\downarrow$},\\ \\,\circledcirc n, n\textcircled{$\downarrow$},\end{array}\right],$$

$$\Leftrightarrow\ ,if(Pdx)\left[\begin{array}{l},\circledcirc m, m\text{☮}n, \otimes, n\textcircled{$\downarrow$}, m\textcircled{$\downarrow$},\\ \\,\circledcirc n, n\textcircled{$\downarrow$},\end{array}\right],$$

$$\Leftrightarrow\ ,if(Pdx)\text{-}\left[\begin{array}{l} ,\circledcirc m, m\circledast n, \otimes, \\ \\ , \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,if(Pdx)\text{-}\left[\begin{array}{l} ,\otimes, \\ \\ , \end{array}\right]\text{-},$$

$$\Leftrightarrow\ ,!Pdx,$$

33.4 Proof of paradox

Because the recursive function if(pdx) is infinite,rule

$$,\ \Leftrightarrow\ ,if(Pdx)\text{-}\left[\begin{array}{l} , \\ \\ , \end{array}\right]\text{-},$$

does not exist. So we can't get rule of contradiction:

$$,\ \Leftrightarrow\ ,\otimes,$$

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