



Northeastern

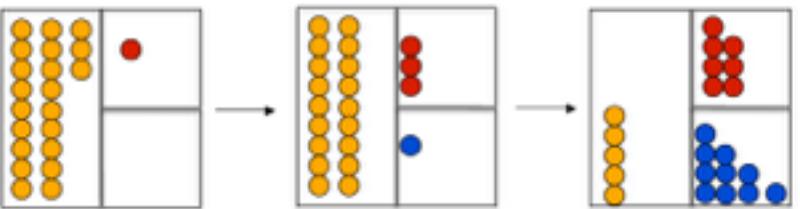
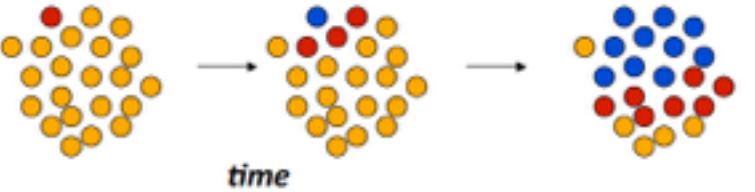
# MODELS AND DATA

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@alexvespi



MOBS LAB  
LABORATORY FOR THE MODELING OF BIOLOGICAL  
AND SOCIO-TECHNICAL SYSTEMS



Discrete individuals  
tracked one by one  
(Reed-Frost model)

State vector  
 $X = (x_1, x_2, \dots, x_N)$

Random process  
for each specific  
variable

# INDIVIDUAL BASED MODELS

time

ID	State
1	S
2	S
3	S
4	S
5	I
6	S
7	S
8	S
9	S
10	S
11	S
12	S
13	S
14	S
15	S
16	S
17	S
18	S
19	S
20	S
21	S
22	S

ID	State
1	S
2	I
3	S
4	S
5	R
6	S
7	S
8	S
9	S
10	S
11	S
12	I
13	I
14	S
15	S
16	S
17	S
18	S
19	S
20	S
21	S
22	S

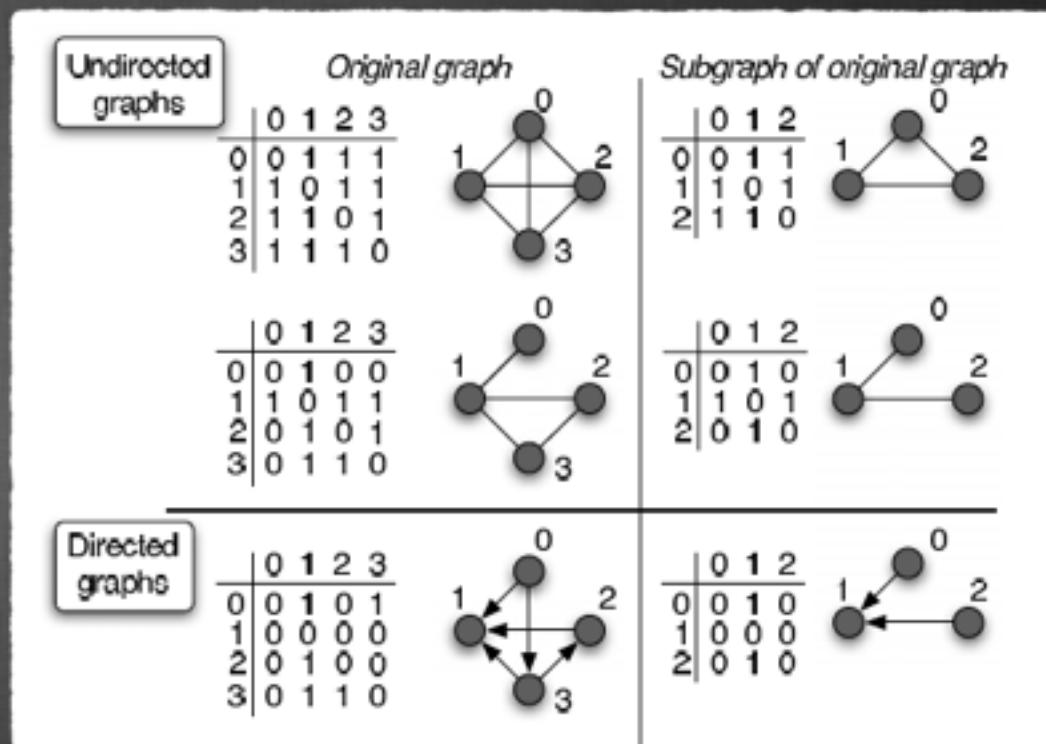
ID	State
1	I
2	R
3	I
4	R
5	R
6	R
7	S
8	R
9	R
10	S
11	S
12	R
13	R
14	I
15	S
16	R
17	I
18	S
19	I
20	I
21	I
22	R

# CONTACT NETWORKS

- Do  $i=1, n$ 
  - If  $a_i(t)=1$  with probability  $\mu$      $a_i(t+1)=2$
  - If  $a_i(t)=0$  then
  - Do  $j=1, n$ 
    - If  $A_{ij}=1$  and  $a_j(t)=1$
    - With probability  $\beta$          $a_i(t+1)=1$
  - End do
- End do

Repeat over time step until  $a_i$  is only  
0 or 2

At each time step measure the  
quantities of infected etc.



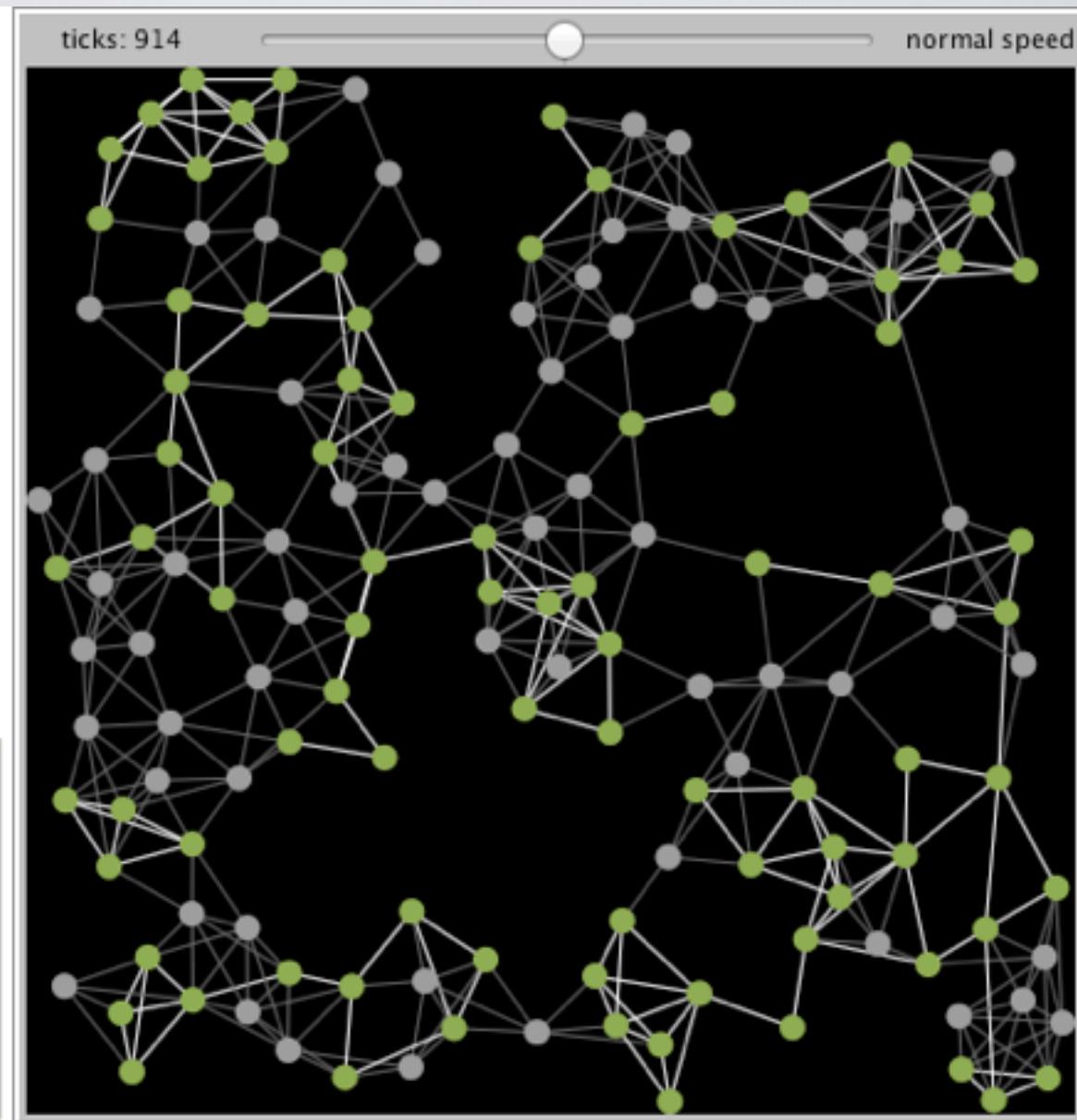
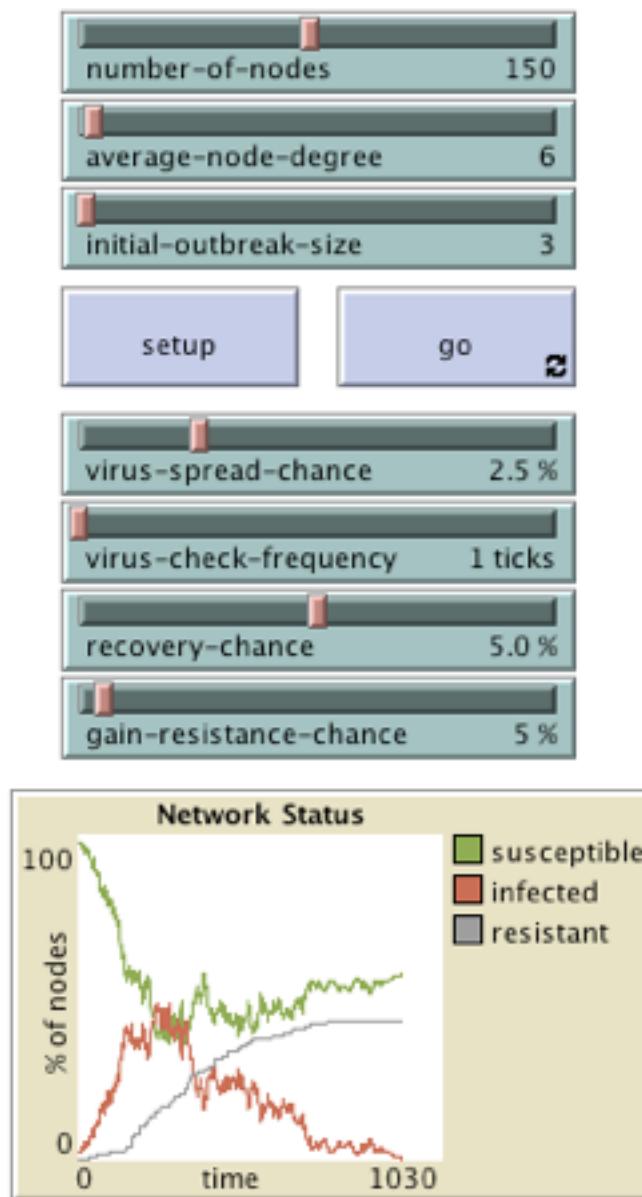
# SYNTHETIC POPULATIONS & "EFFECTIVE CONTACT" NETWORKS



# EPIDEMIC NETWORK MODELING

- Multiscale problem:
- Multiplexity
- Length scales: from local proximity to global travel patterns
- Time scale: From minutes to years.
- Interplay Network-Epidemic process:
  - Models generally considered in a time-scale separation regime
  - Process dynamics decoupled by network evolution
  - Frozen network (process time scale  $\ll$  network evolution time scale)
  - Random homogenous mixing (network evolution time scale  $\ll$  process time scale)

Review of Modern Physics 87, 925 (2015)



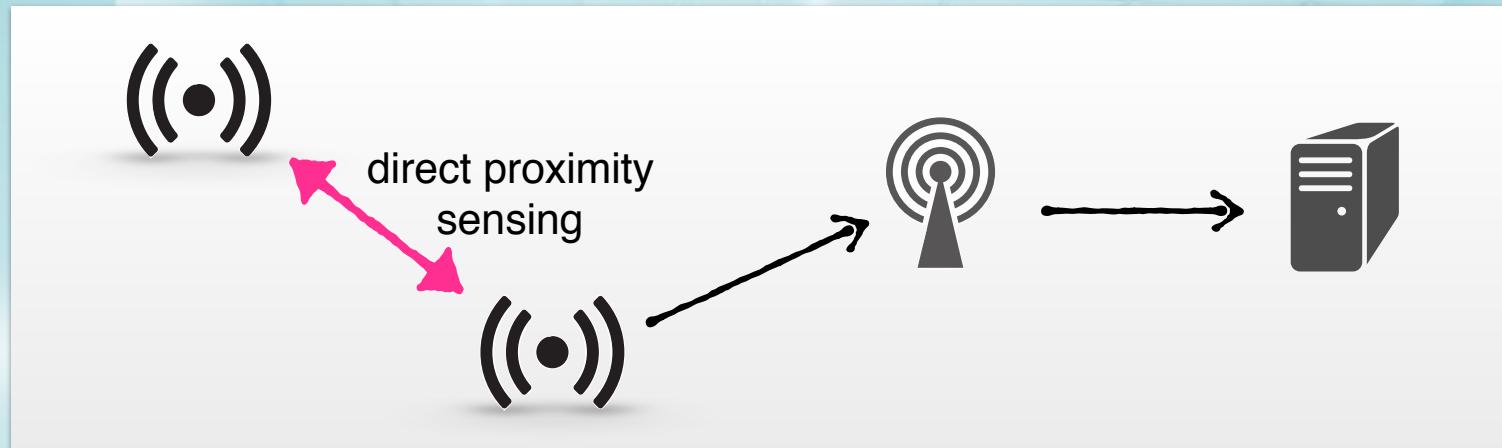
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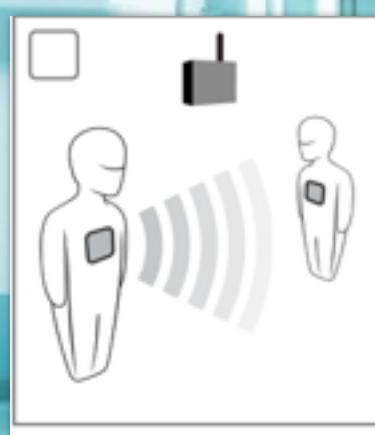
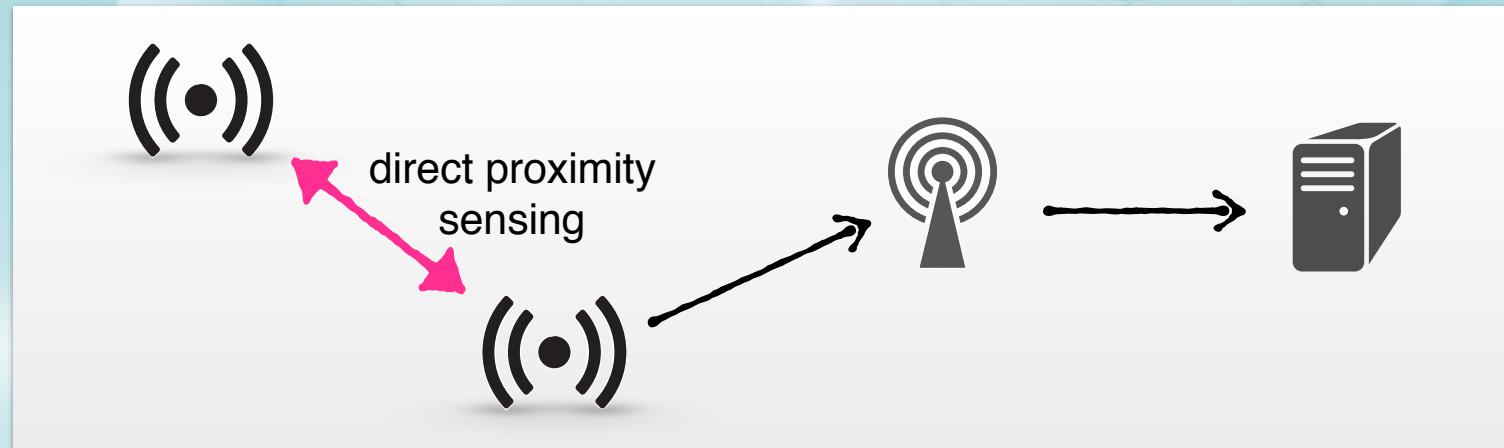
# manually

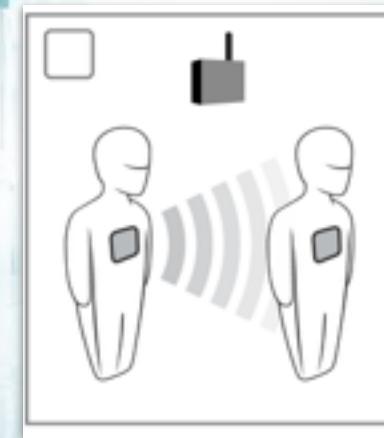
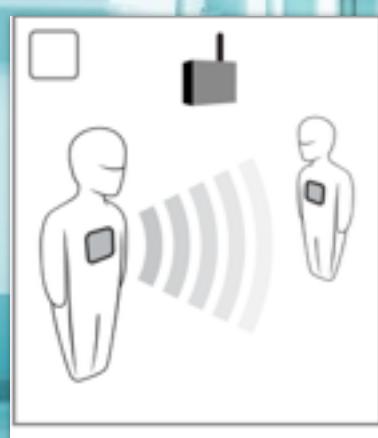
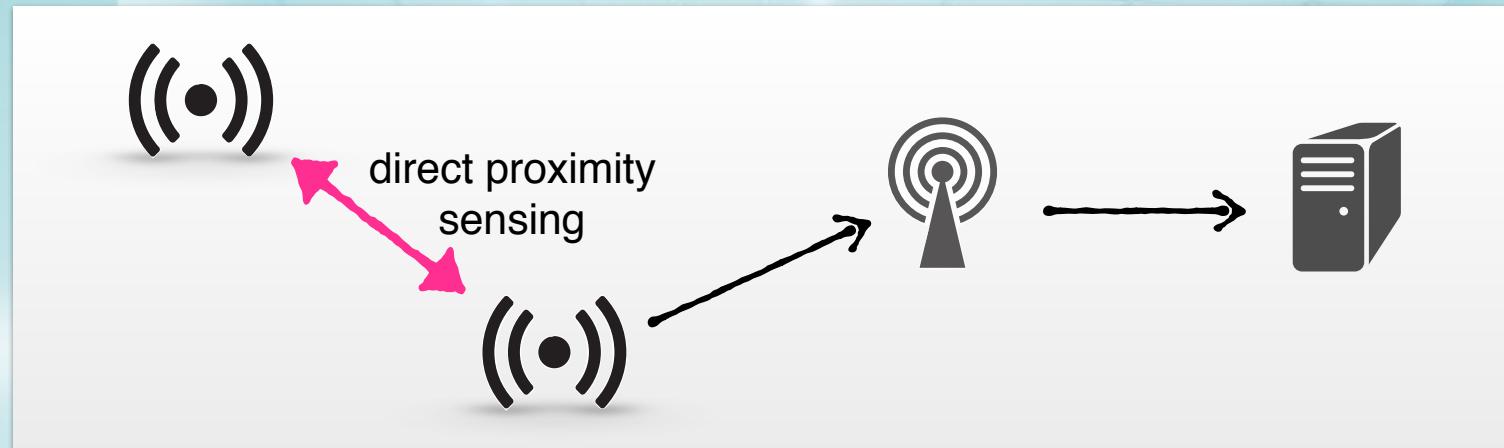
<p><b>Part A</b> Tear off and keep this section.</p> <p><b>MRC</b> Medical Research Council    <b>THE UNIVERSITY OF WARWICK</b>            To complete the survey online or for further information visit: <a href="http://www.contactsurvey.org">www.contactsurvey.org</a>          Helpline: 0844 2579900 (Local rate)</p> <p><b>1 Who did you meet?</b>          (name or description)</p> <p><b>Individual contacts</b></p> <p>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</p> <p><b>Groups of similar contacts</b></p> <p>Use this section to record groups of people who you met in a similar way. Again, only include people you had a conversation with or touched.</p> <p>See the instruction sheet for examples.</p>	<p><b>Part B</b> Please return this section in the envelope supplied.</p> <p>You should complete this survey about people you meet on the next:</p> <p style="text-align: center;"><b>Friday</b></p> <p>Your age: <input type="text"/> years          Your gender:  <input type="checkbox"/> female  <input type="checkbox"/> male</p> <p>Your main occupation: <input type="text"/></p> <p>How many people live in your home? <input type="text"/> people</p> <p>What is the first part of your postcode? <input type="text"/></p> <p>Remember to include yourself</p> <p>This is optional; you do not have to tell us.</p> <p><b>2 Did you touch?</b>          Mark with a 'X' for yes and leave blank for no</p> <p><b>3 Where did you meet?</b></p> <p>At my home Work or School Travelling Leisure / other</p> <p><b>4 How far from your home?</b></p> <p>Less than 2 miles or at my home 3 to 10 miles 11 to 50 miles 51 miles or more</p> <p><b>5 How long for?</b>          Over the entire day</p> <p>10 min or less 11 min to 30 min 31 min to 1 hour More than 1 hour</p> <p><b>6 How often?</b>          How often would you expect to meet each of your contacts?</p> <p>4 or more days a week 2 or 3 days a week Once a week Less often than once a week More than once a day</p> <p><b>7 Who met who?</b>          Which of your contacts met each other during this day or in the previous week?          Make sure you have listed all your contacts before you start this section.</p> <p>Each of your contacts has a code number in a circle (see the far left of the sheet). Below, on each row, are the code numbers for all the other contacts they could have met. For each of your contacts, put a cross through the corresponding code number of the contacts you think that this person has met during this day, or in the previous 7 days.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> <td>12</td> <td>13</td> <td>14</td> <td>15</td> <td>16</td> <td>17</td> <td>18</td> <td>19</td> <td>20</td> </tr> <tr> <td>2</td> <td>1</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> <td>12</td> <td>13</td> <td>14</td> <td>15</td> <td>16</td> <td>17</td> <td>18</td> <td>19</td> <td>20</td> </tr> <tr> <td>3</td> <td>1</td> <td>2</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> <td>12</td> <td>13</td> <td>14</td> <td>15</td> <td>16</td> <td>17</td> <td>18</td> <td>19</td> <td>20</td> </tr> <tr> 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Please write how many more contacts you made during this day?</p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	2	1	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	3	1	2	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	4	1	2	3	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	5	1	2	3	4	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	6	1	2	3	4	5	7	8	9	10	11	12	13	14	15	16	17	18	19	20	7	1	2	3	4	5	6	8	9	10	11	12	13	14	15	16	17	18	19	20	8	1	2	3	4	5	6	7	9	10	11	12	13	14	15	16	17	18	19	20	9	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18	19	20	10	1	2	3	4	5	6	7	8	9	11	12	13	14	15	16	17	18	19	20	11	1	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	19	20	12	1	2	3	4	5	6	7	8	9	10	11	13	14	15	16	17	18	19	20	13	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18	19	20	14	1	2	3	4	5	6	7	8	9	10	11	12	13	15	16	17	18	19	20	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	16	17	18	19	20	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	17	18	19	20	17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	19	20	18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	20	19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	20	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
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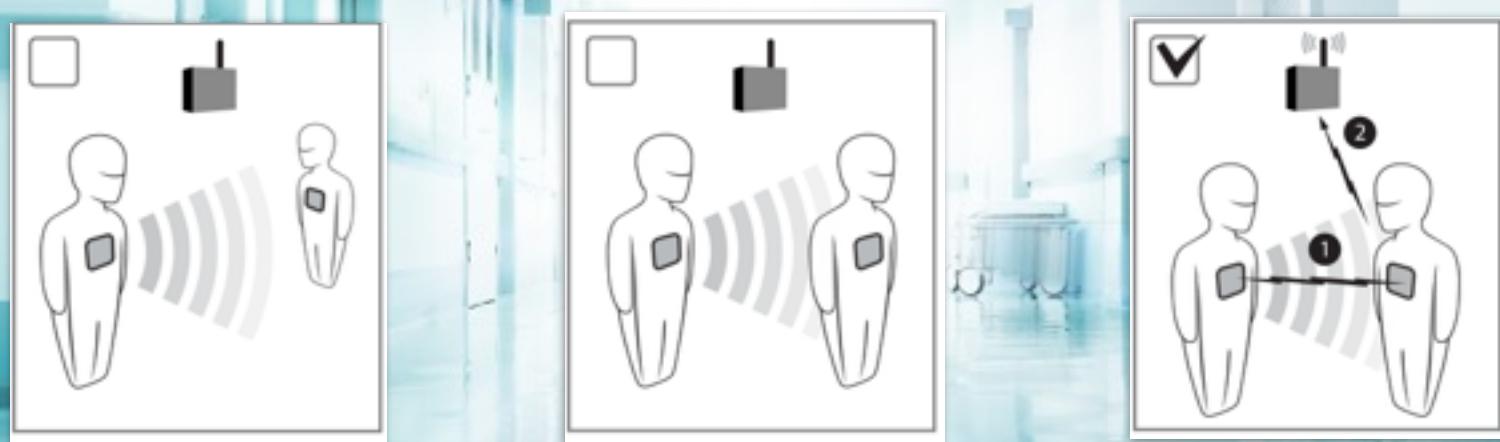
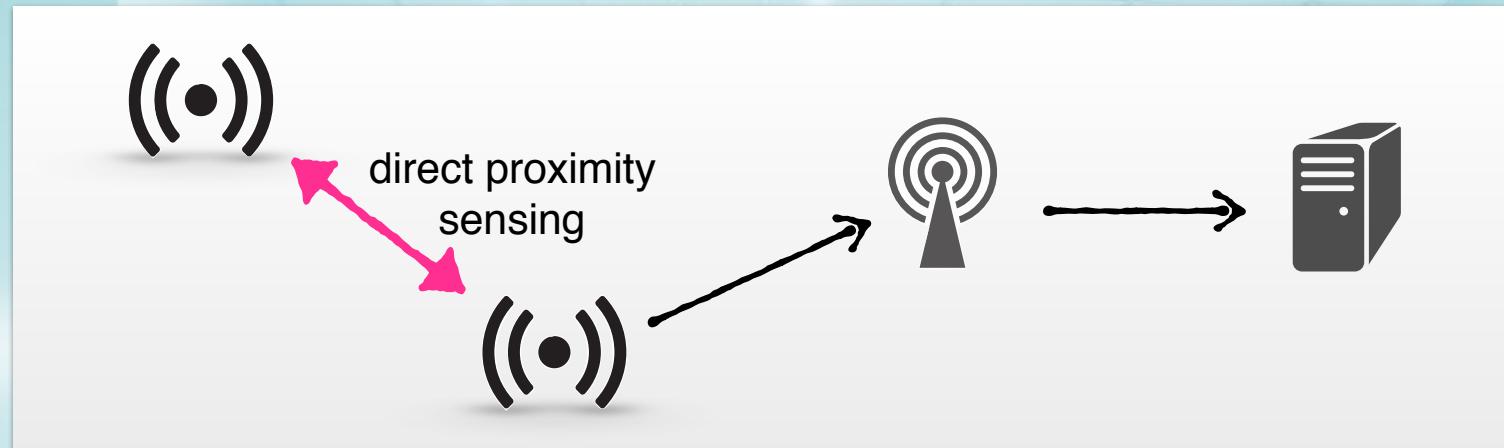
A black and white photograph of a proximity sensor module. The module is rectangular with a metal housing. On the top surface, there is a circular icon with three concentric arcs, representing a signal or sensor field. Below this icon, the text "proximity sensor" is printed. A flexible, flat cable is attached to the right side of the module, ending in a metal connector. The module is resting on a dark, textured surface.

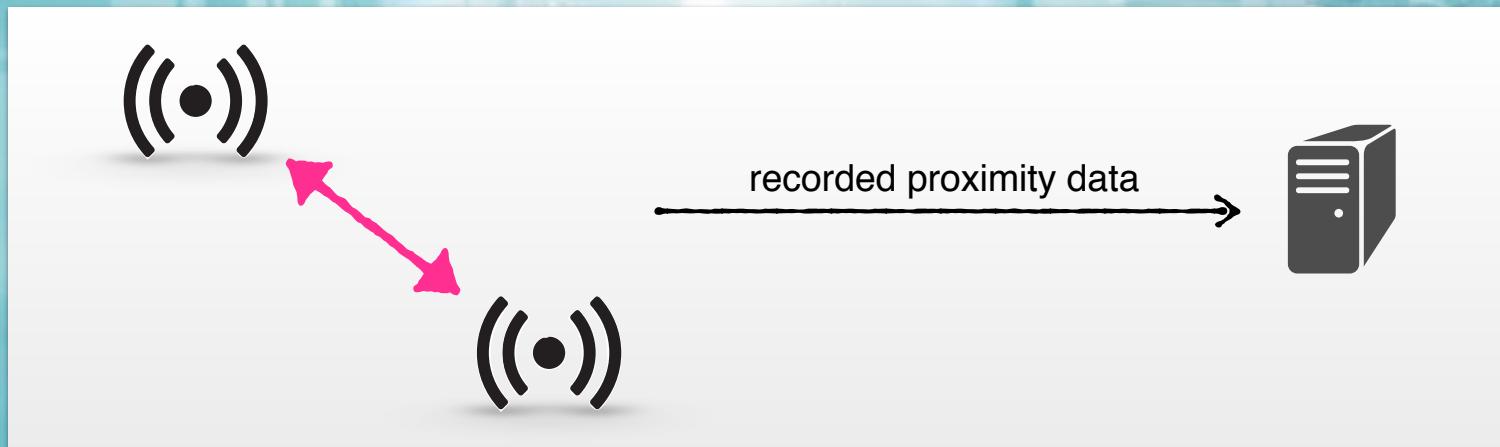
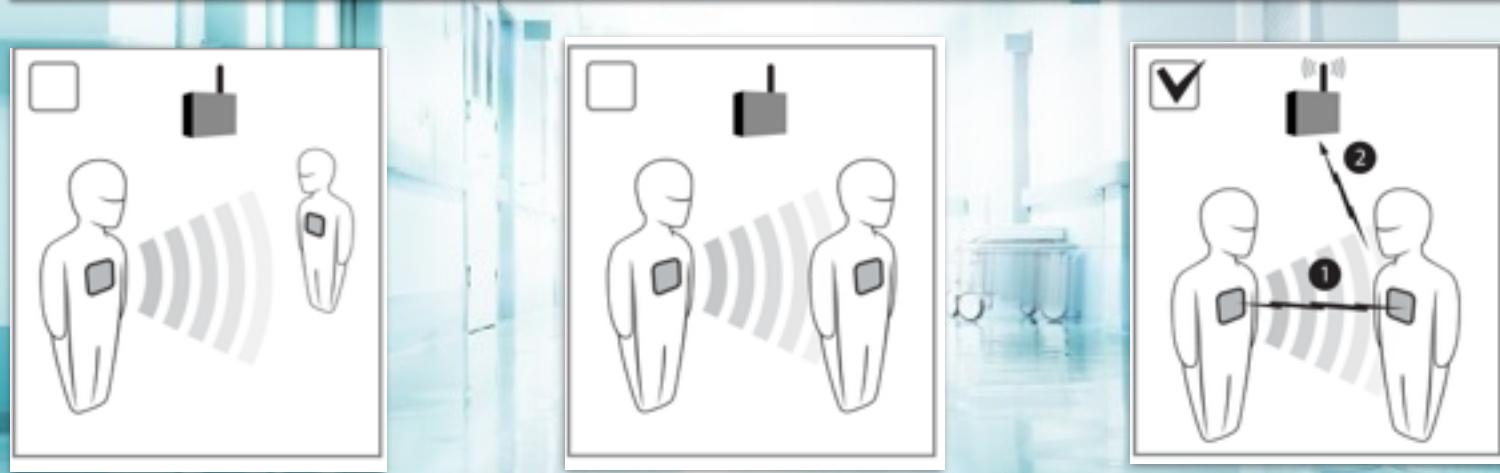
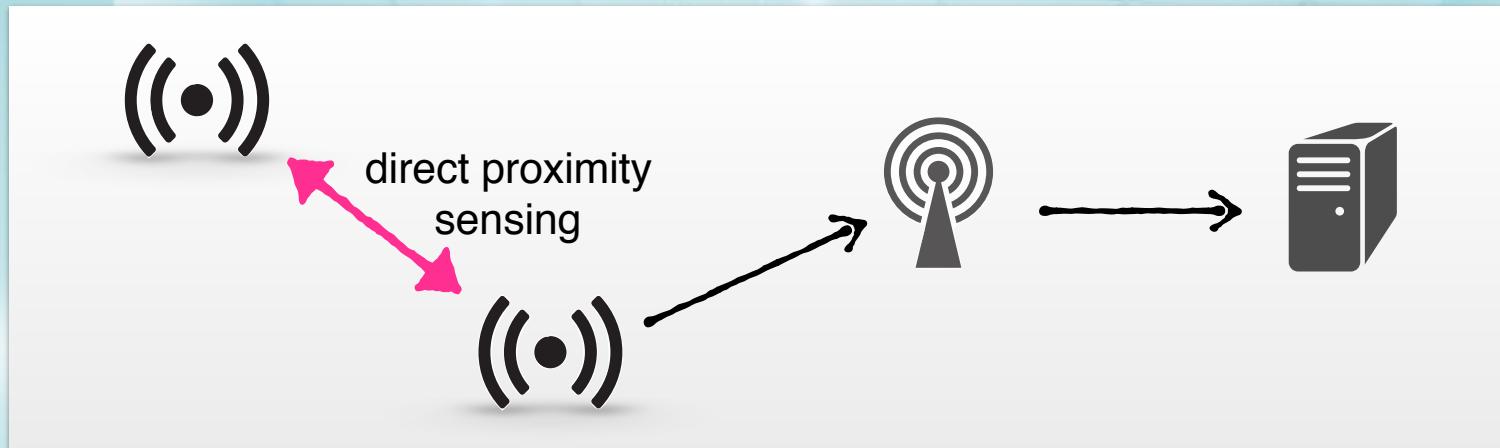
proximity sensors







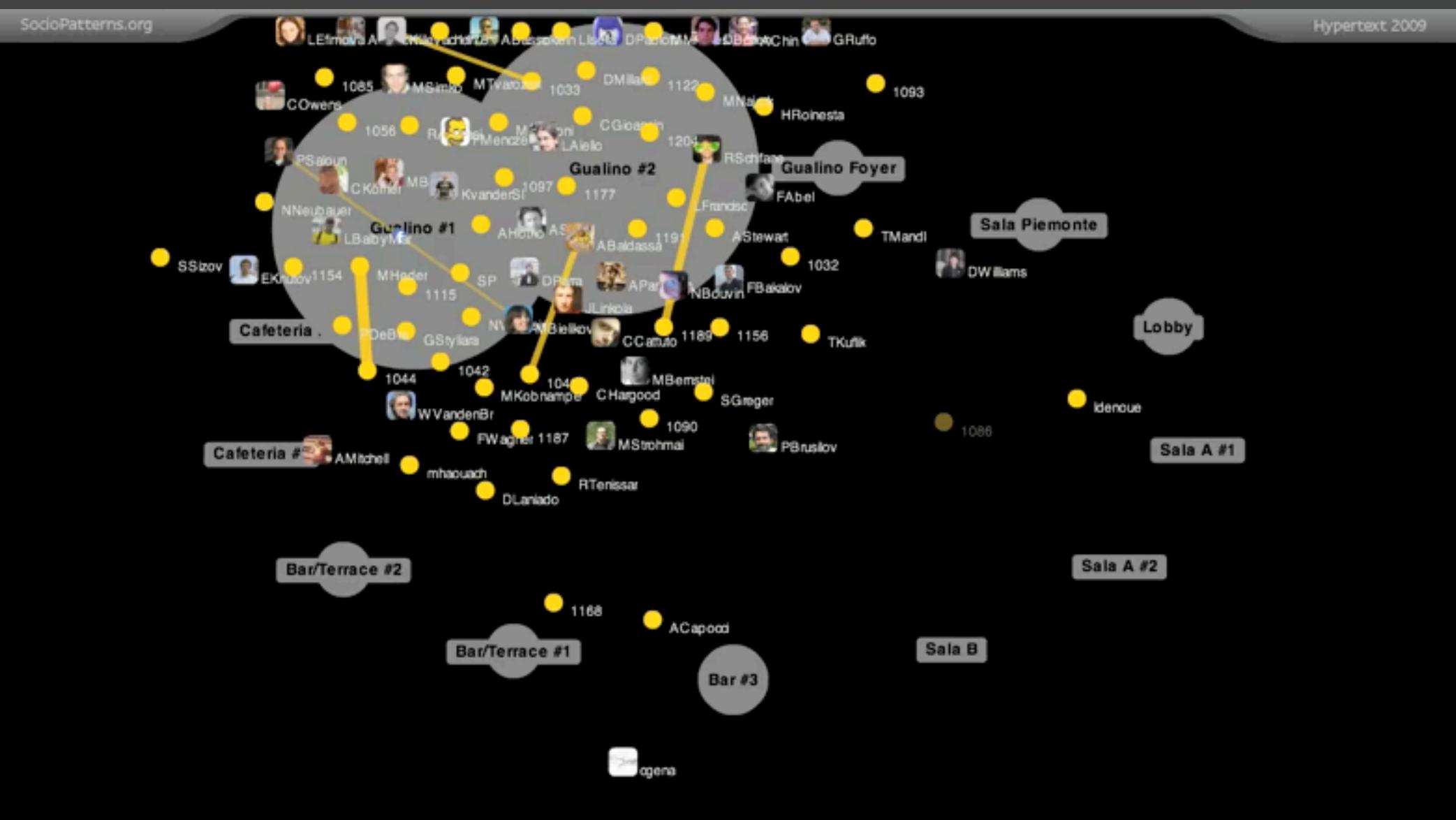




# proximity sensors

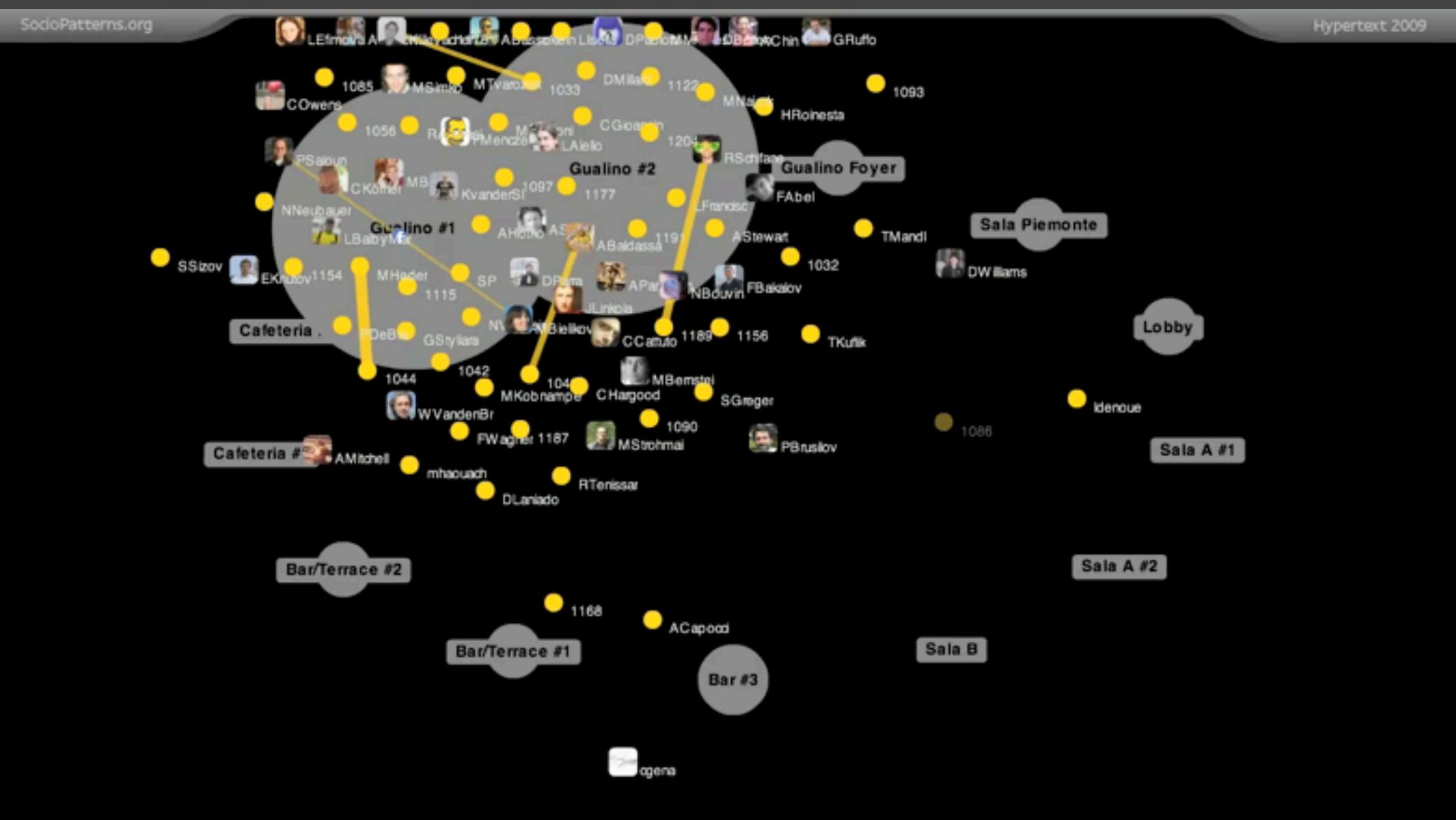
- ▶ **unobtrusive**
- ▶ highly **scalable**: 1000s devices simultaneously
- ▶ very **low cost**
- ▶ low-power 2.4 GHz ISM band operation
- ▶ **high proximity resolution** (1-1.5 meters)
- ▶ **face-to-face** presence detection
- ▶ high temporal resolution (~ 10 seconds)
- ▶ battery duration ~ 1-4 months
- ▶ **flexible backend** architecture for data analytics
  
- ▶ 3 operational modalities:
  - **centralized monitoring** via LAN readers
  - **distributed logging** via readers with SD storage
  - **fully distributed operation** with on-board data storage and no installed infrastructure

# time-varying proximity network



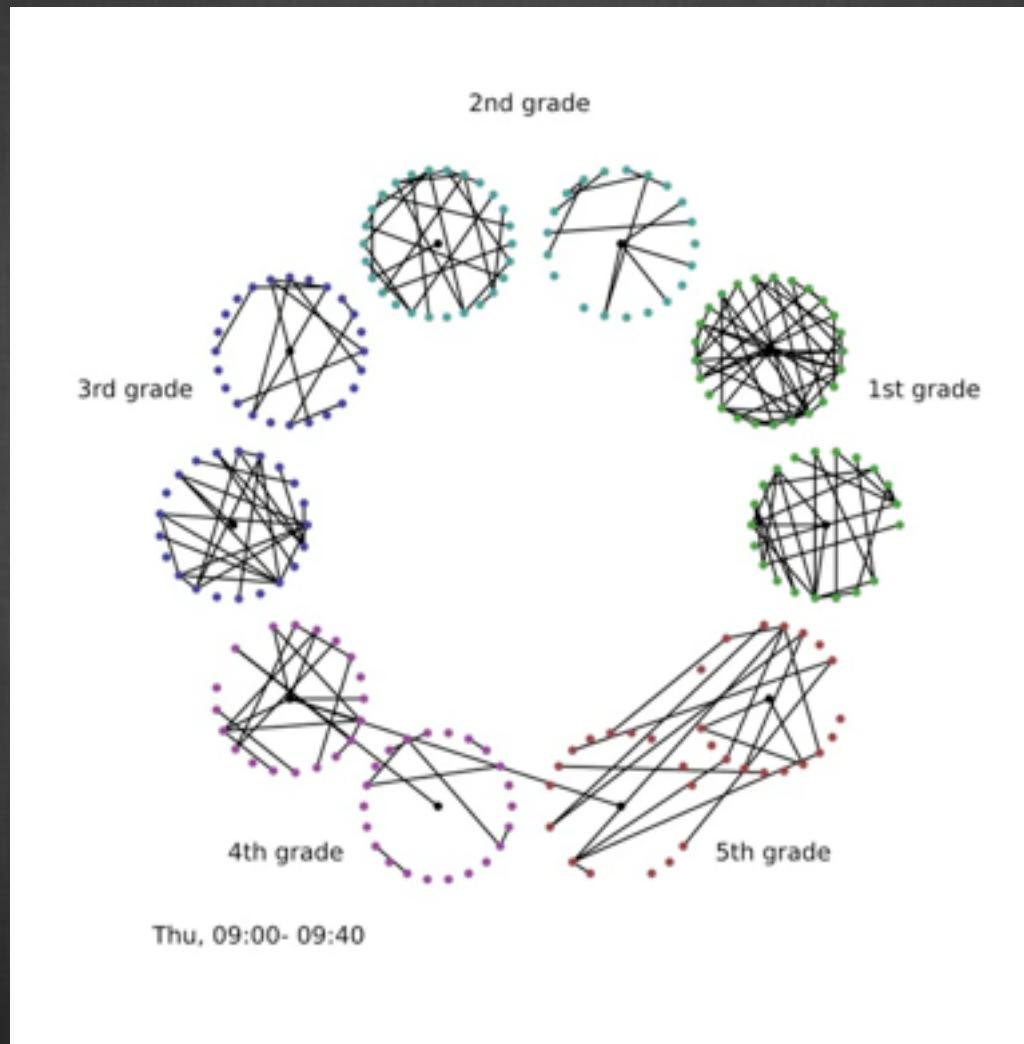
<http://www.vimeo.com/6590604>

# time-varying proximity network



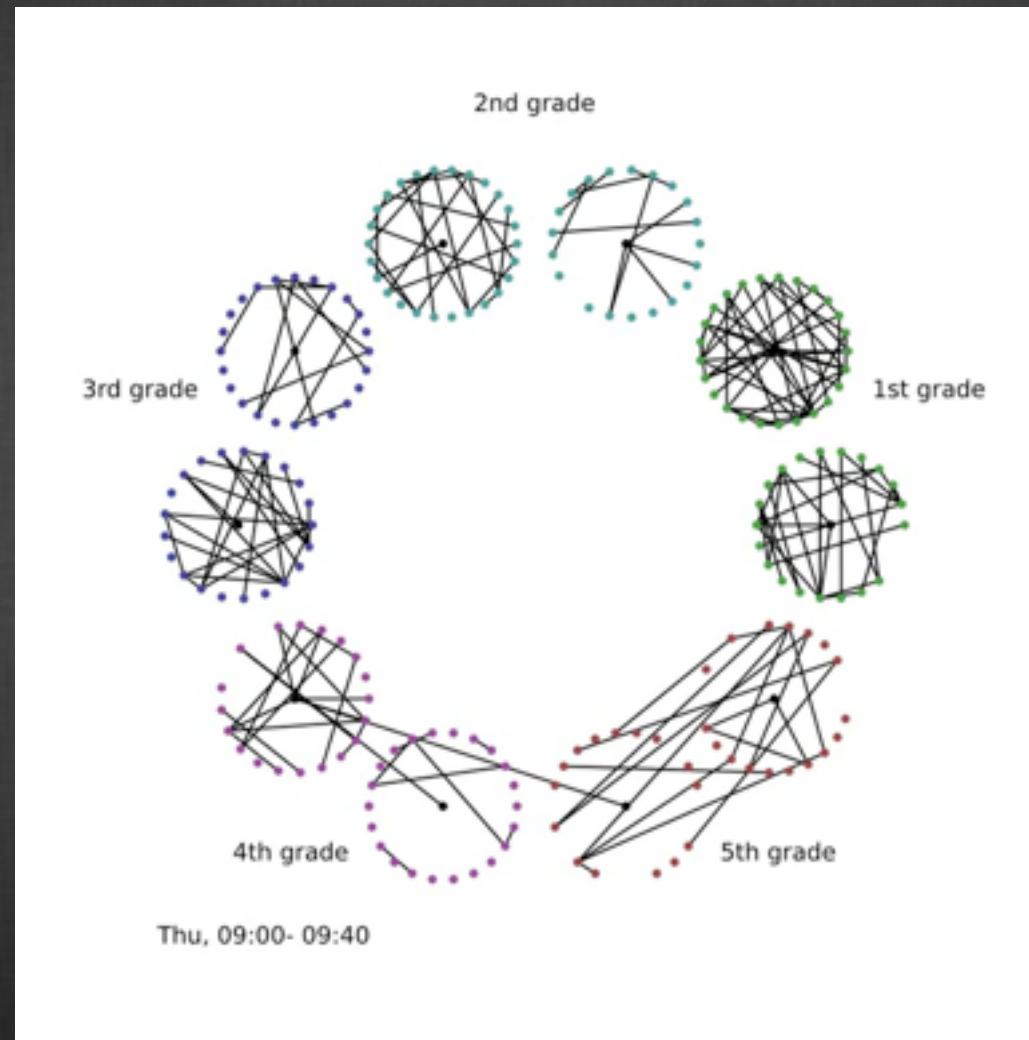
<http://www.vimeo.com/6590604>

# MICRO-SCALE: PROXIMITY INTERACTIONS IN CONFINED ENVIRONMENTS



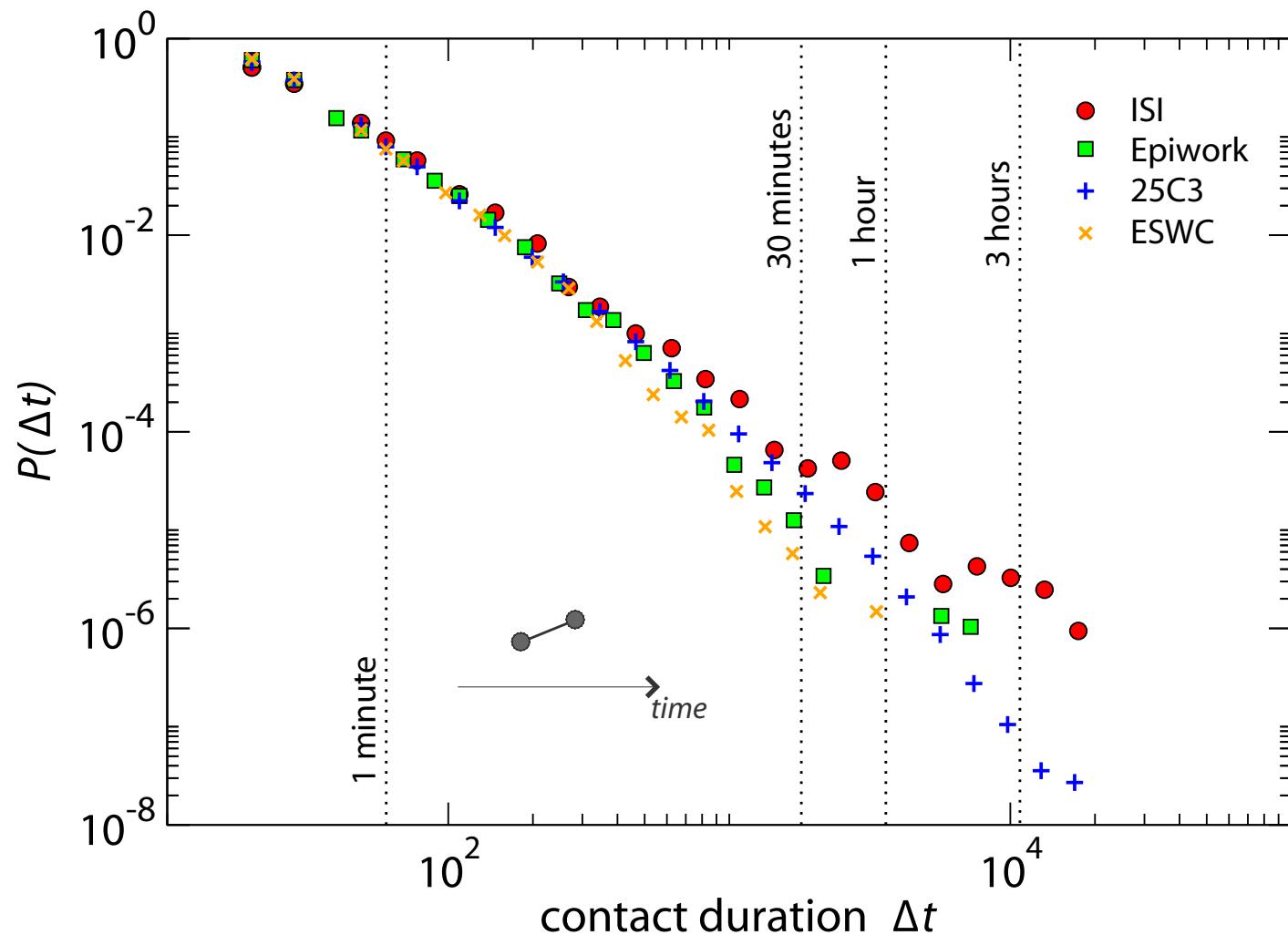
Sociopatterns experiment in school

# MICRO-SCALE: PROXIMITY INTERACTIONS IN CONFINED ENVIRONMENTS

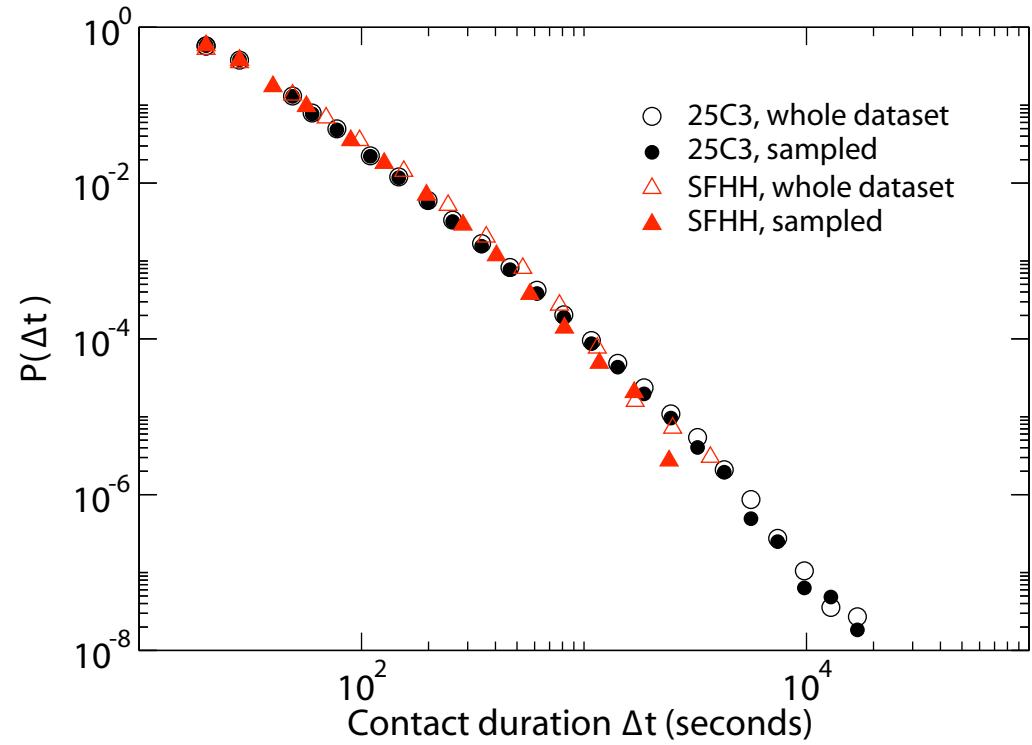
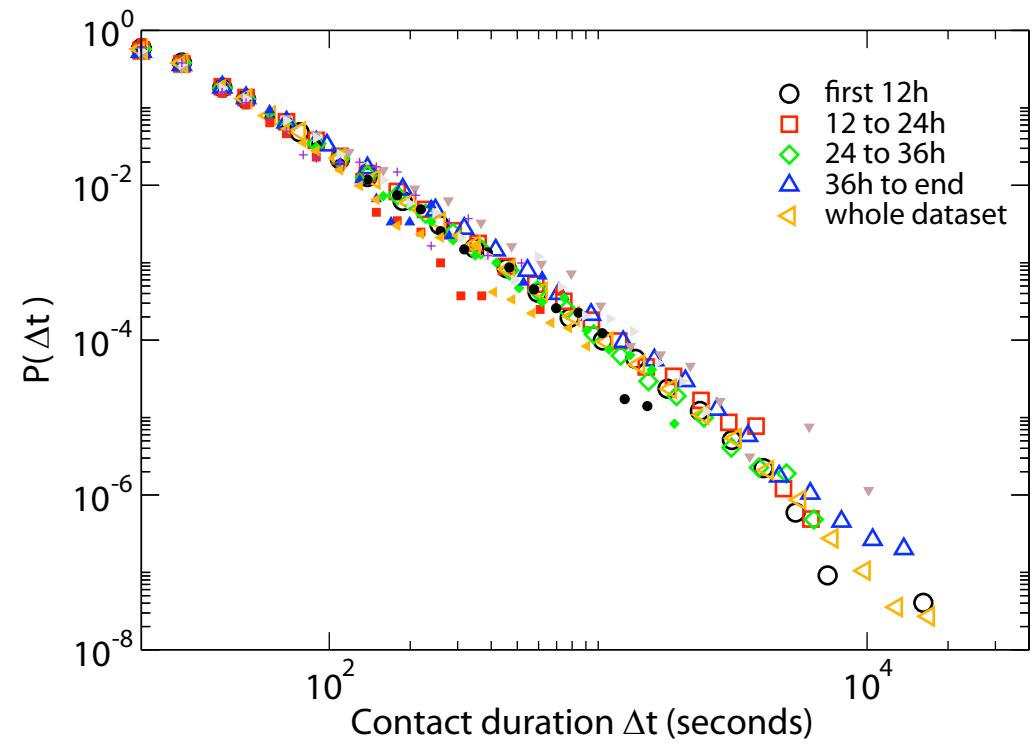


Sociopatterns experiment in school

# contact durations



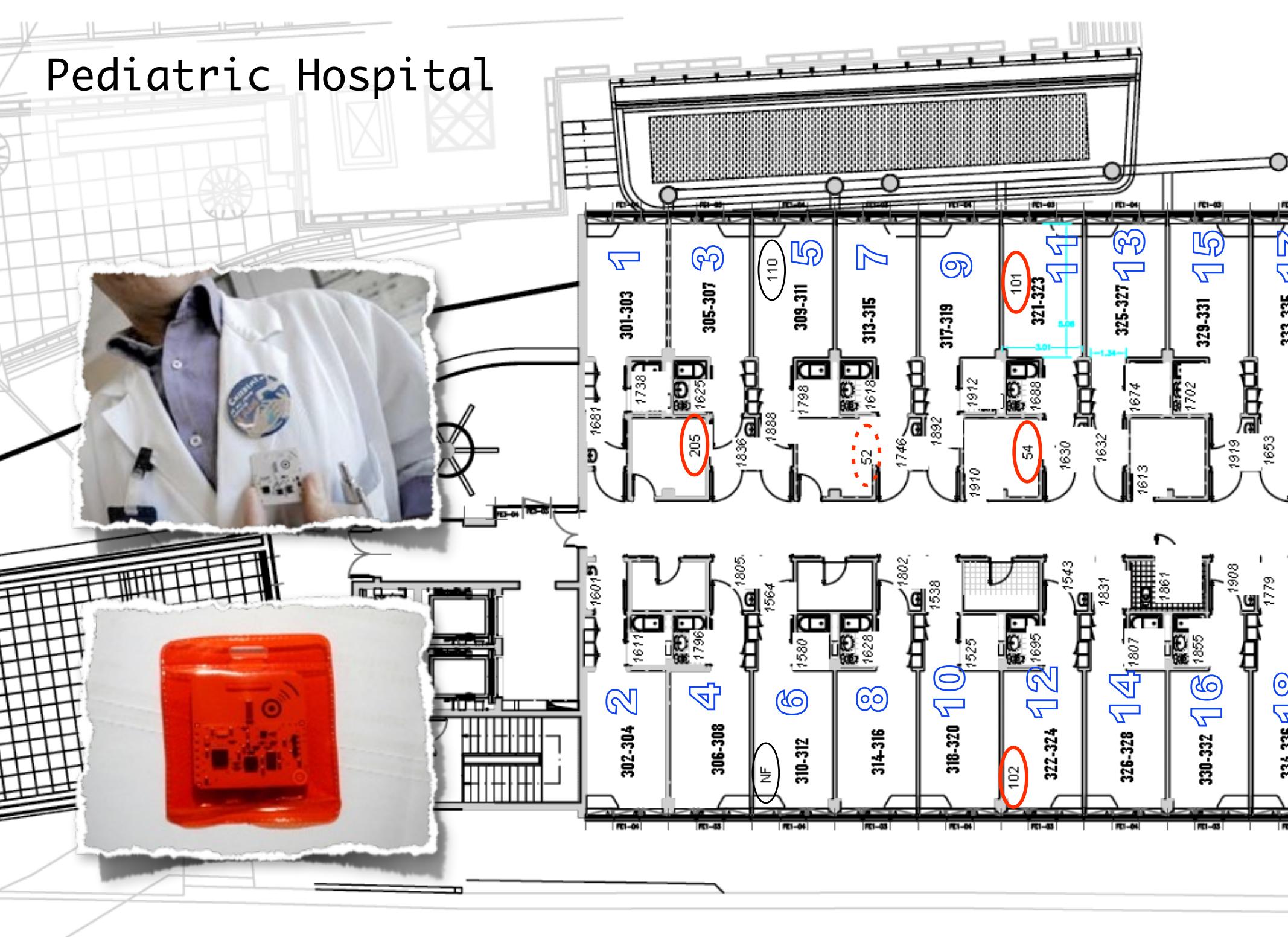
# contact durations: stationarity & sampling



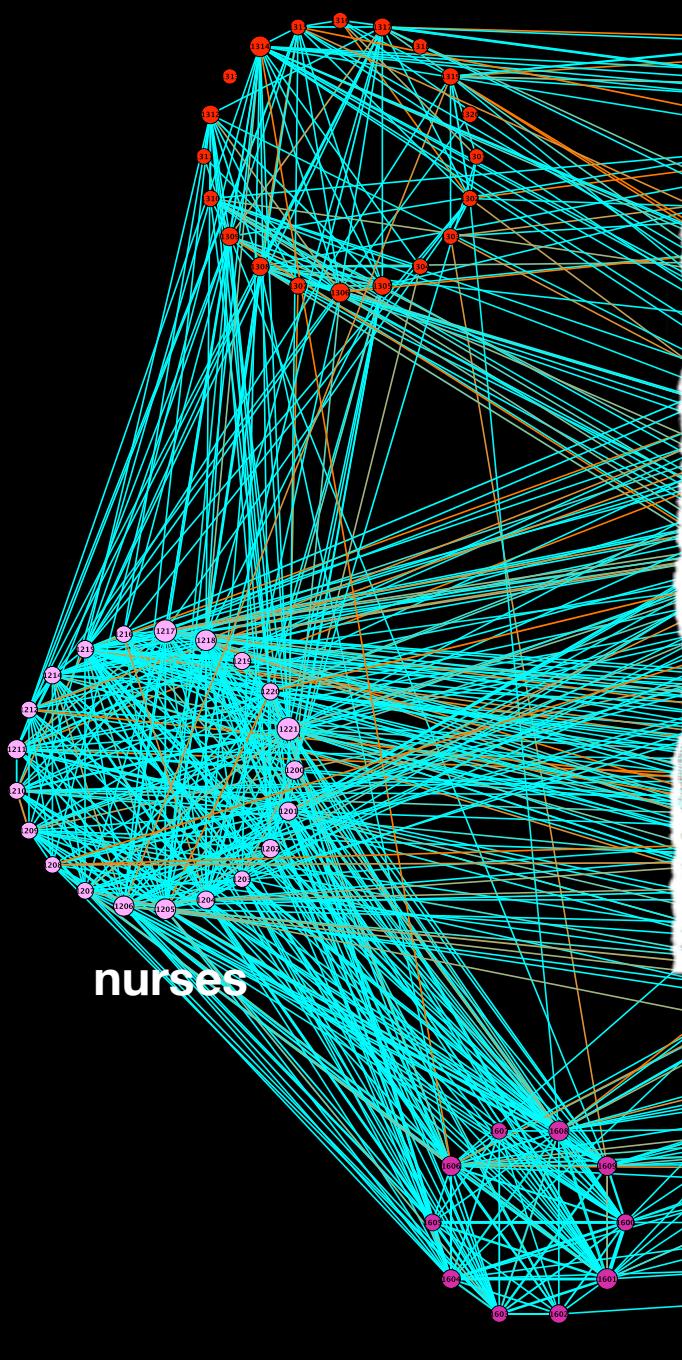
# heterogeneities and correlations

- ▶ network **topology** (connectivity, communities, ...)
- ▶ heterogeneity of **weights**
  - e.g., broad weight distributions
- ▶ bursty nature of **human dynamics**
  - non-Poissonian inter-event distributions
  - power-law temporal correlations
- ▶ **daily**, weekly, and **organizational** rhythms
- ▶ weight-topology correlations
- ▶ topology-activity correlations

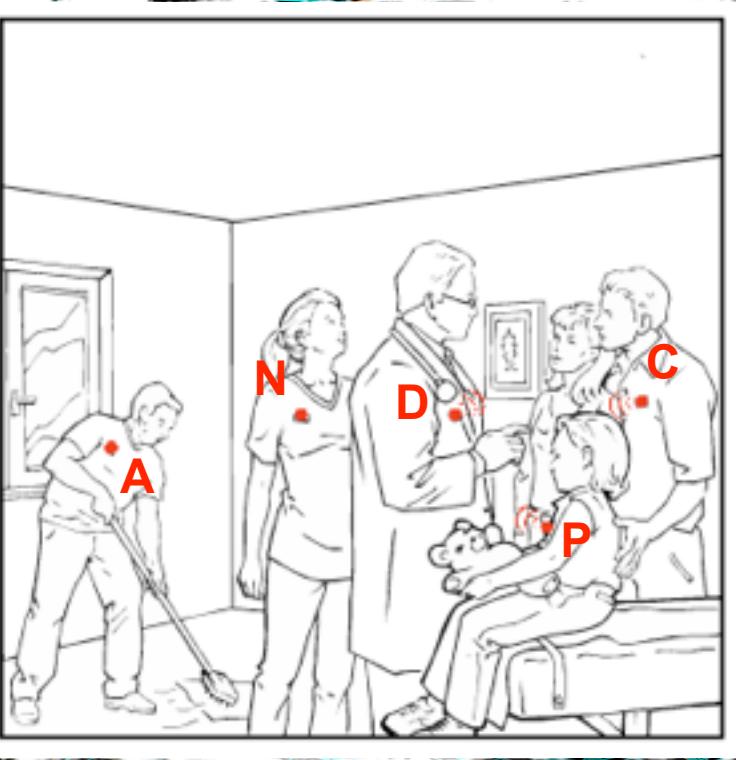
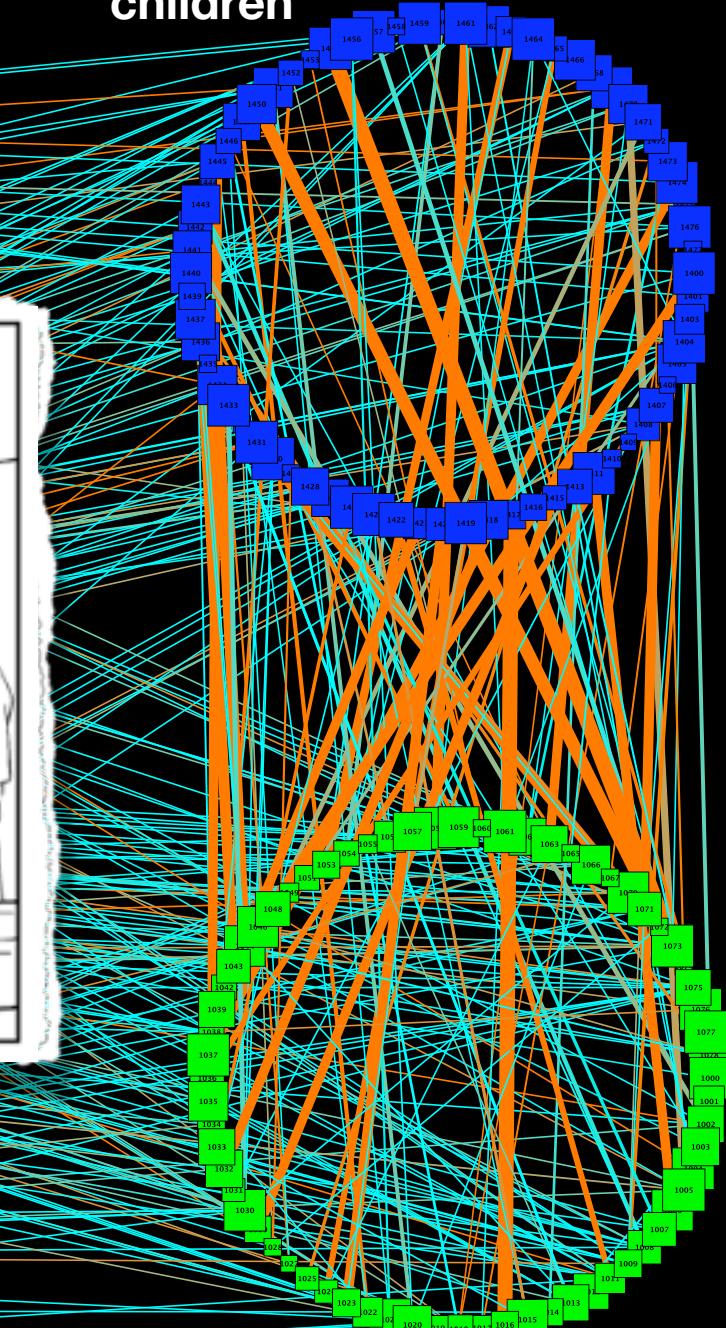
# Pediatric Hospital



doctors



children



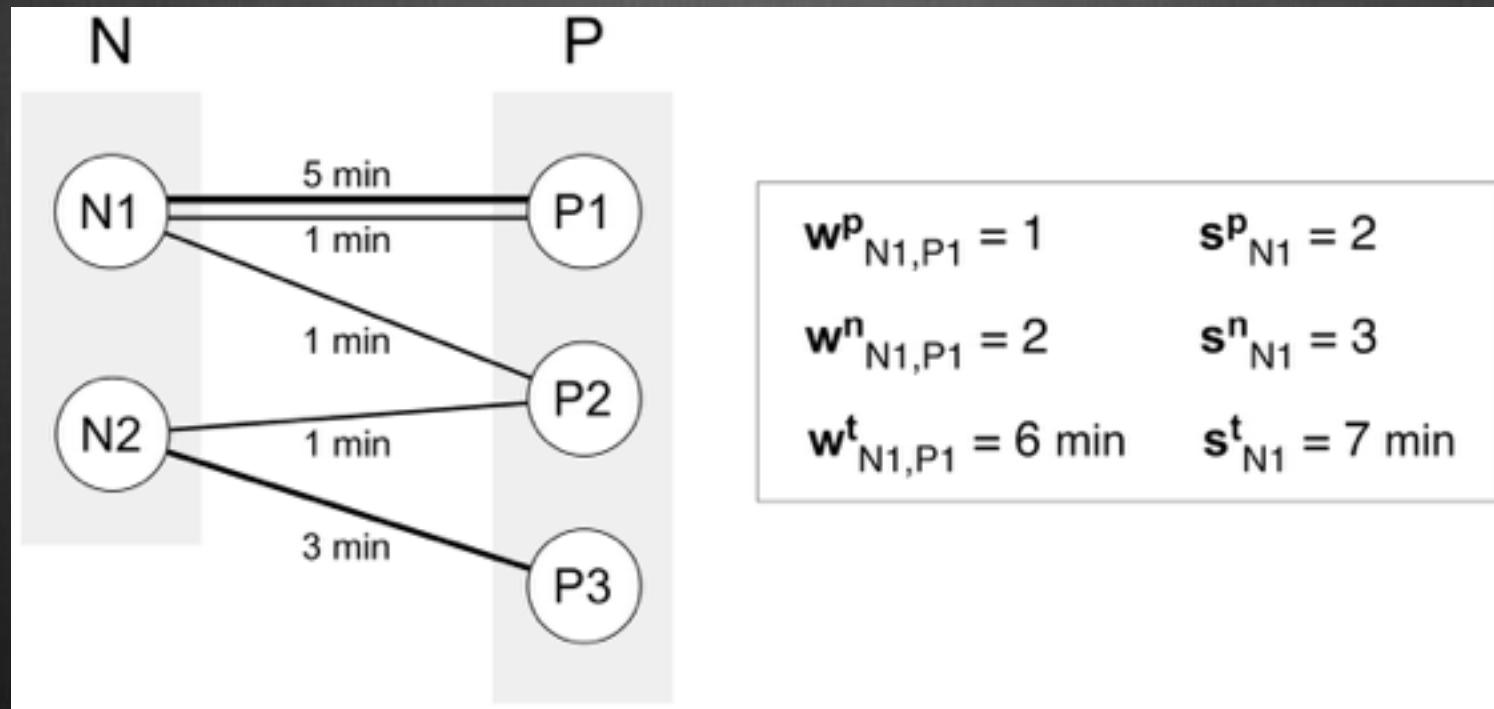
nurses

auxiliaries

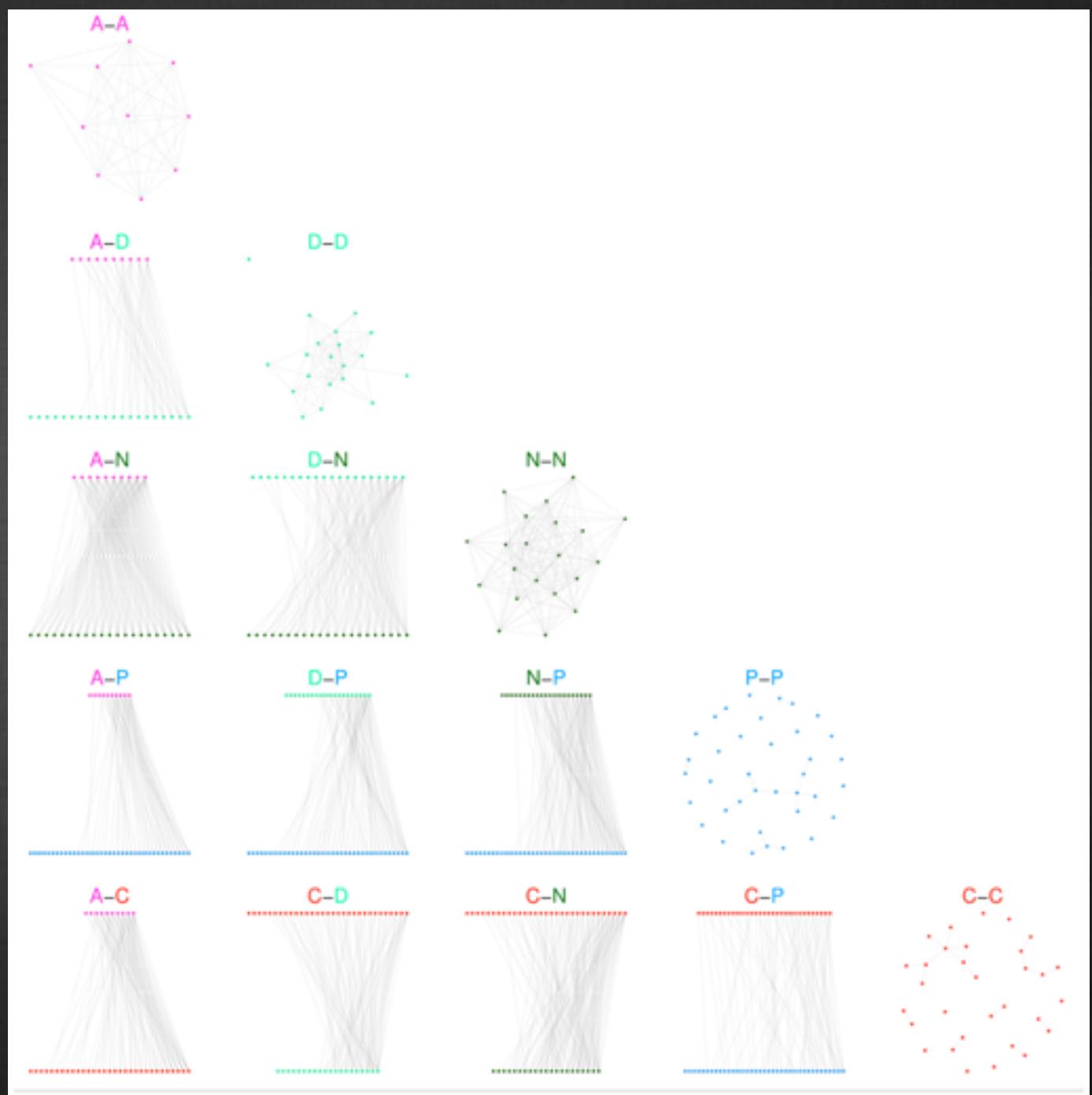
parents

# NETWORKS IN ACTION

- Close Encounters in a Pediatric Ward: Measuring Face-to-Face Proximity and Mixing Patterns with Wearable Sensors

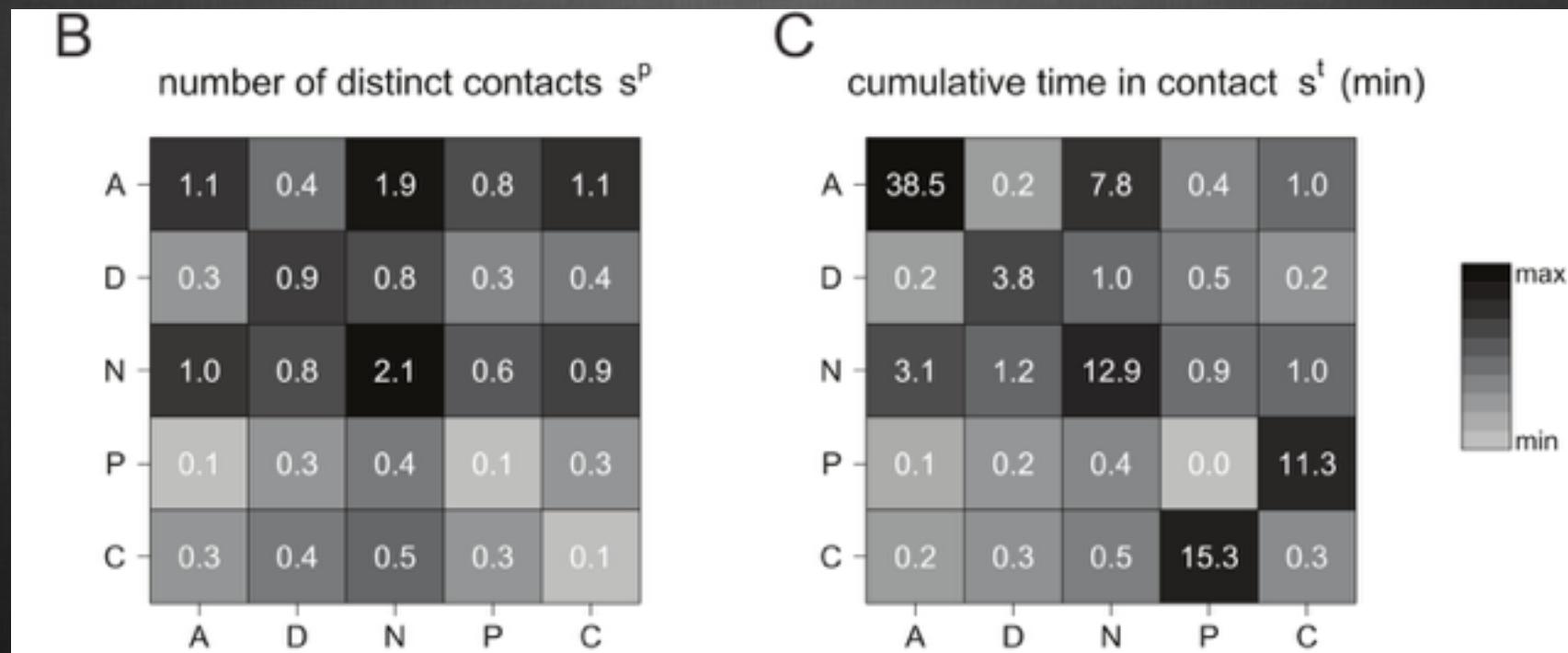


- A= ward Assistant
- D=doctors
- N=nurses
- P=patients
- C= Caregivers (family)

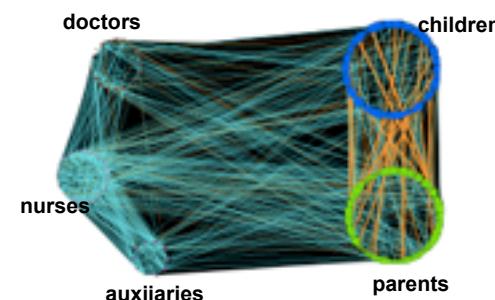


# NETWORKS IN ACTION

- Close Encounters in a Pediatric Ward: Measuring Face-to-Face Proximity and Mixing Patterns with Wearable Sensors



## intervention design by means of data-driven simulation



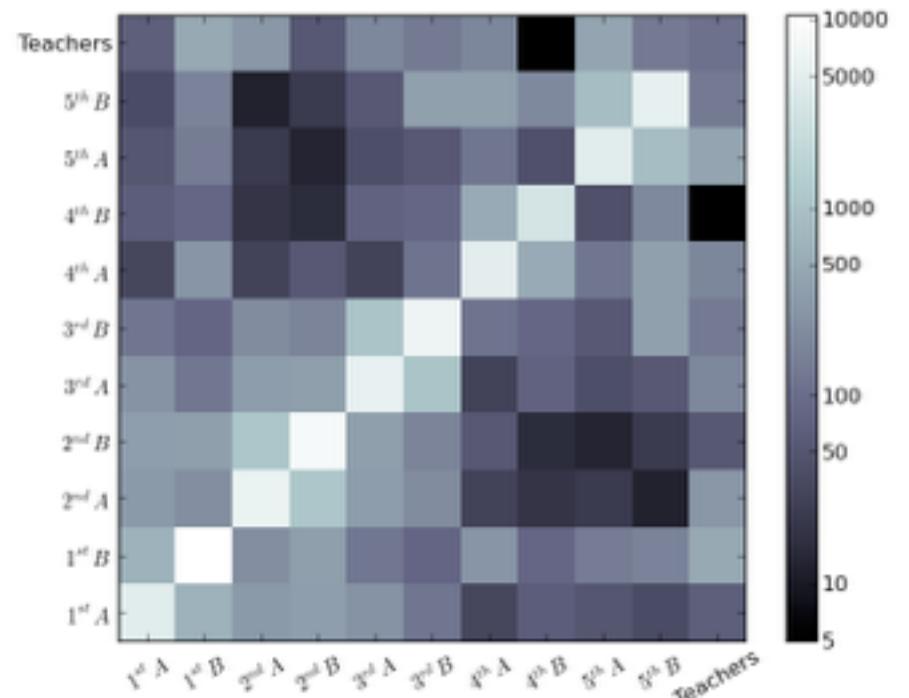
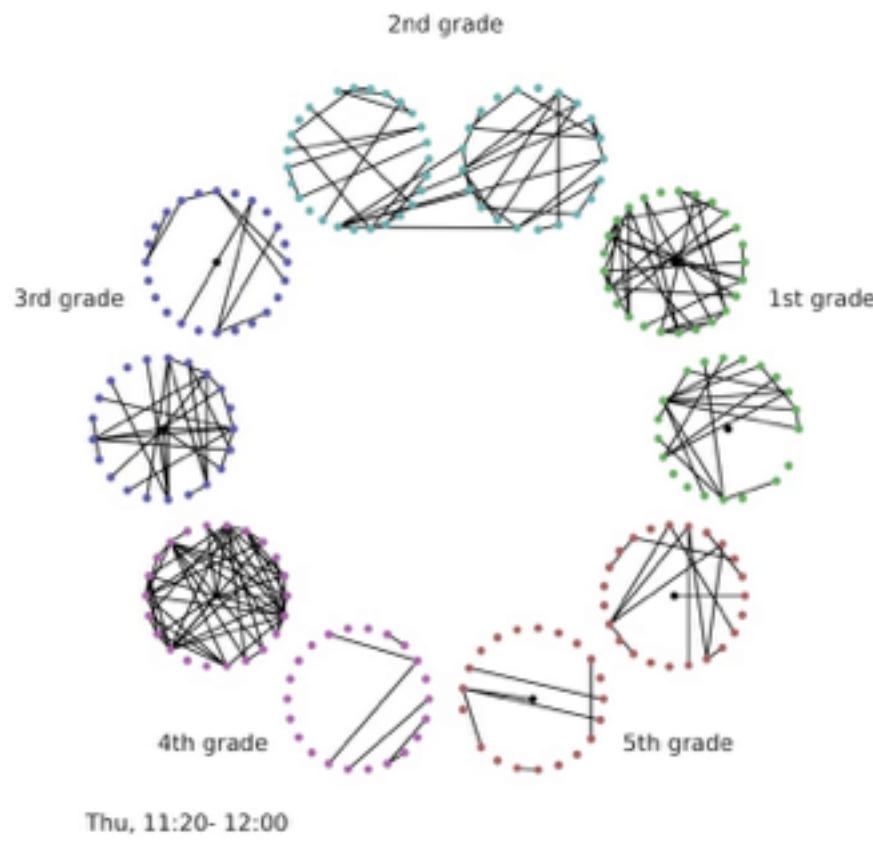
+



---

simulation

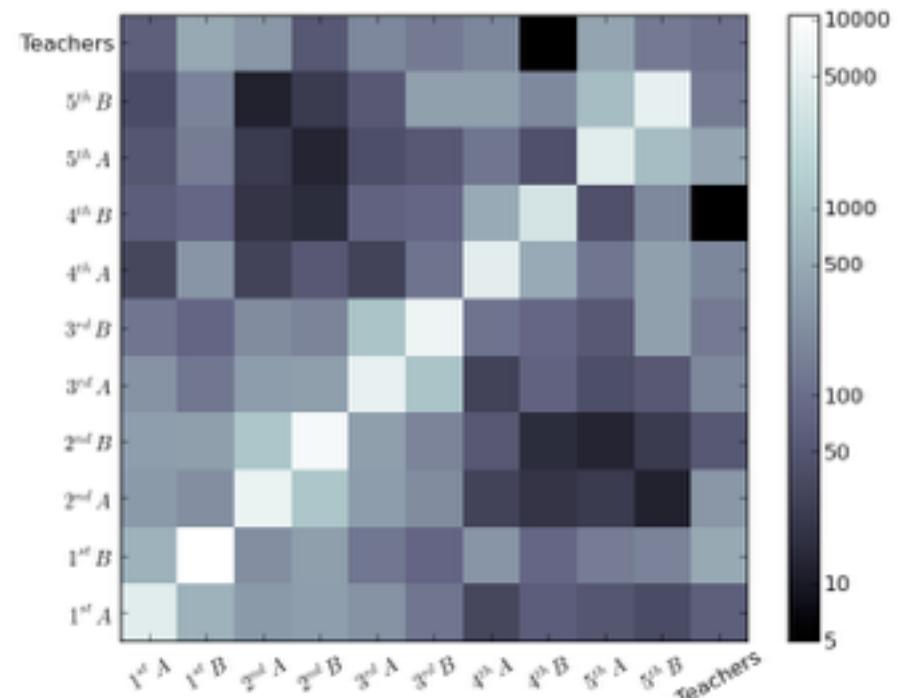
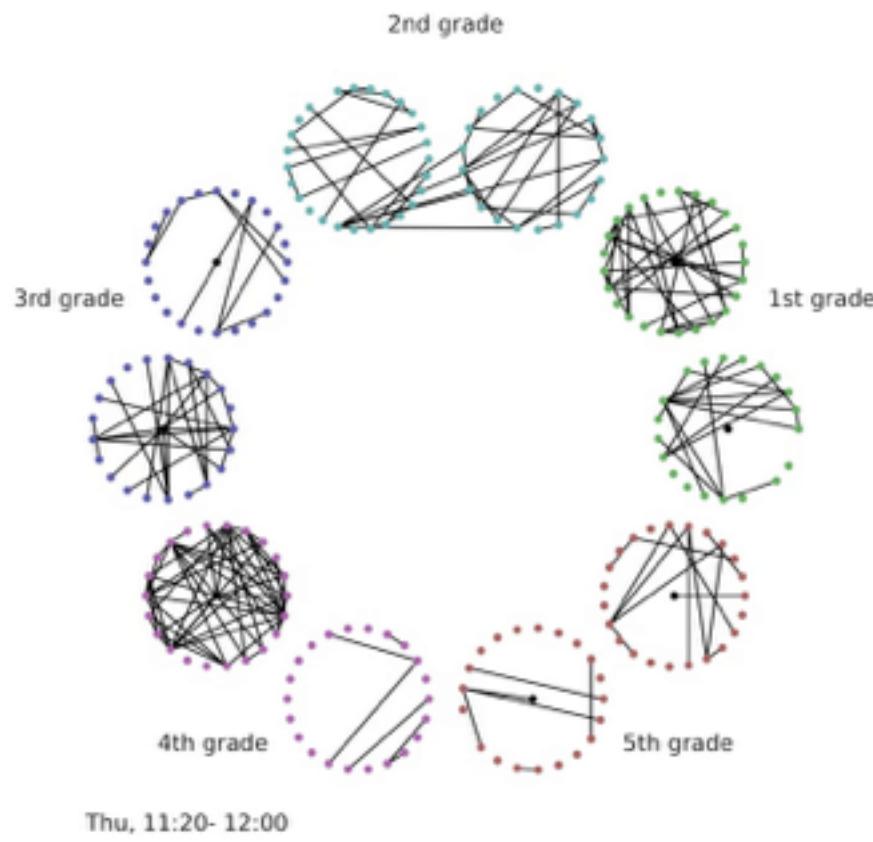
# epidemic models & micro-interventions



epidemic model  
simulated  
using high-resolution  
contact network

interventions based on  
observed cases +  
contact matrix

# epidemic models & micro-interventions

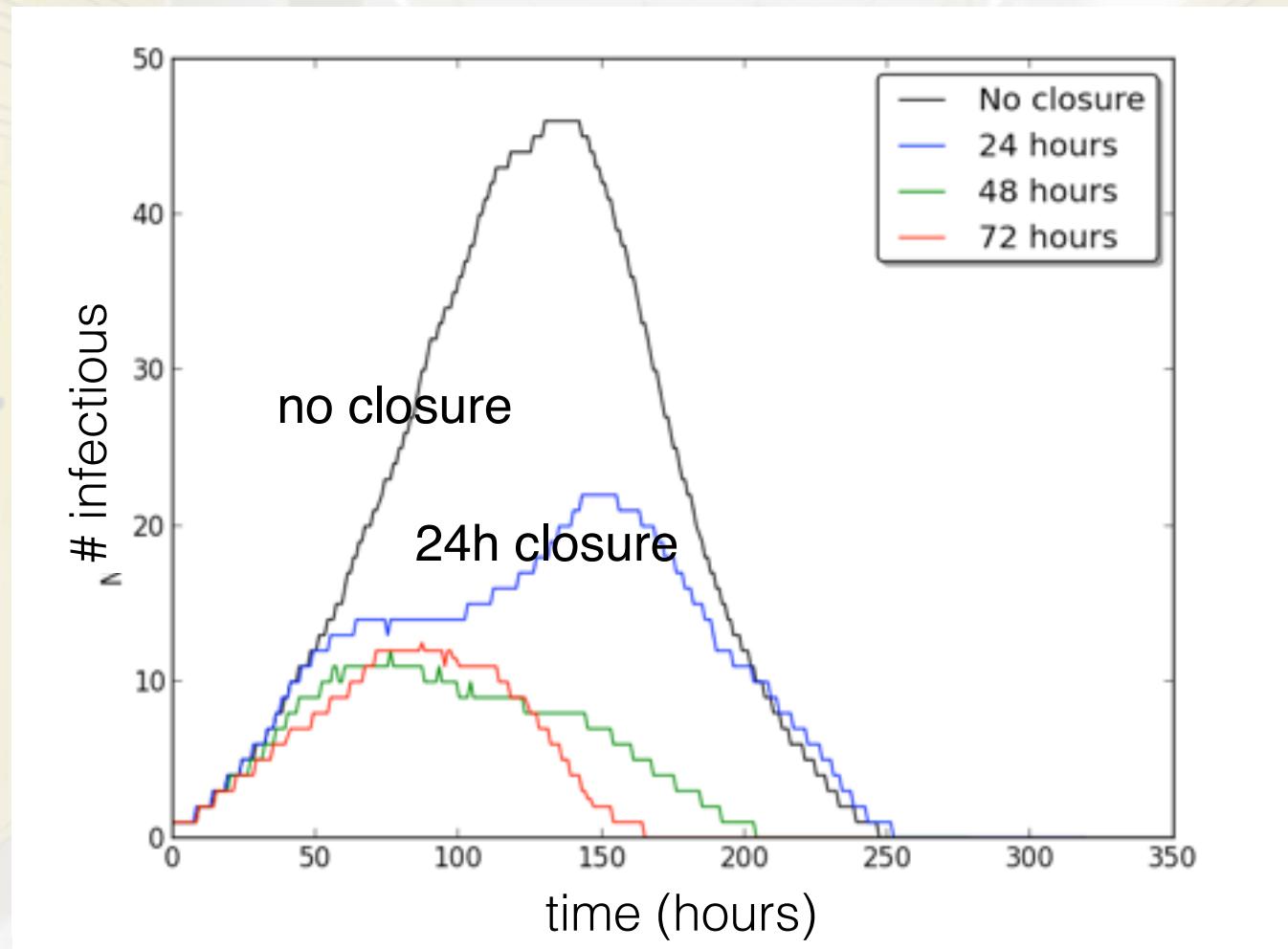


epidemic model  
simulated  
using high-resolution  
contact network

interventions based on  
observed cases +  
contact matrix

# policy 1: school closure

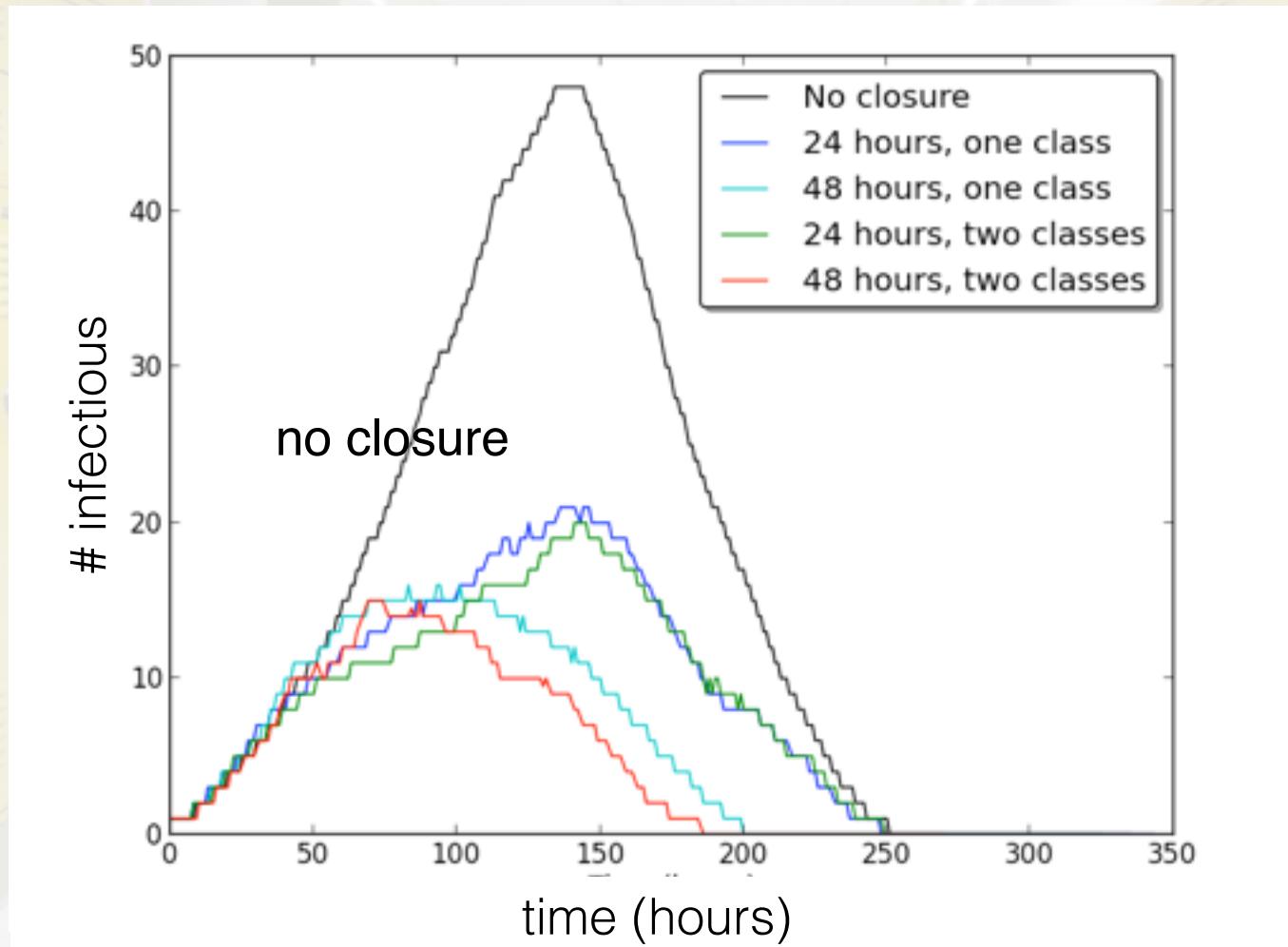
- close the school on observing a total number of symptomatic cases in excess of a fixed threshold
- close the school for a fixed time interval (24, 48, 72 hours)



( flu-like + 1/3 asymptomatic subjects + off-school infection probability )

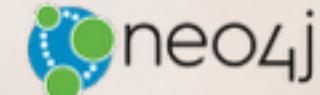
# policy 2: targeted class closure

- close a class when observing there a number of symptomatic cases in excess of a fixed threshold (1, 2, 3, ...)
- close the class for a fixed time interval (24, 48, 72 hours)



# Ticino2015 IoT setup

- SocioPatterns proximity-detecting tags with local storage on:
  - all individuals
  - ambulances
  - field hospital rooms
- fully distributed system operation, full interaction history
- real-time monitoring of hospital
- real-time Web-based views in operation centers
- integration of person/object/space metadata
- technology stack based on Nordic nRF51 SoC, OpenBeacon tags, SocioPatterns firmware, IBM Node-RED, Mosquitto MQTT broker, Neo4j, D3.js, Docker, ...

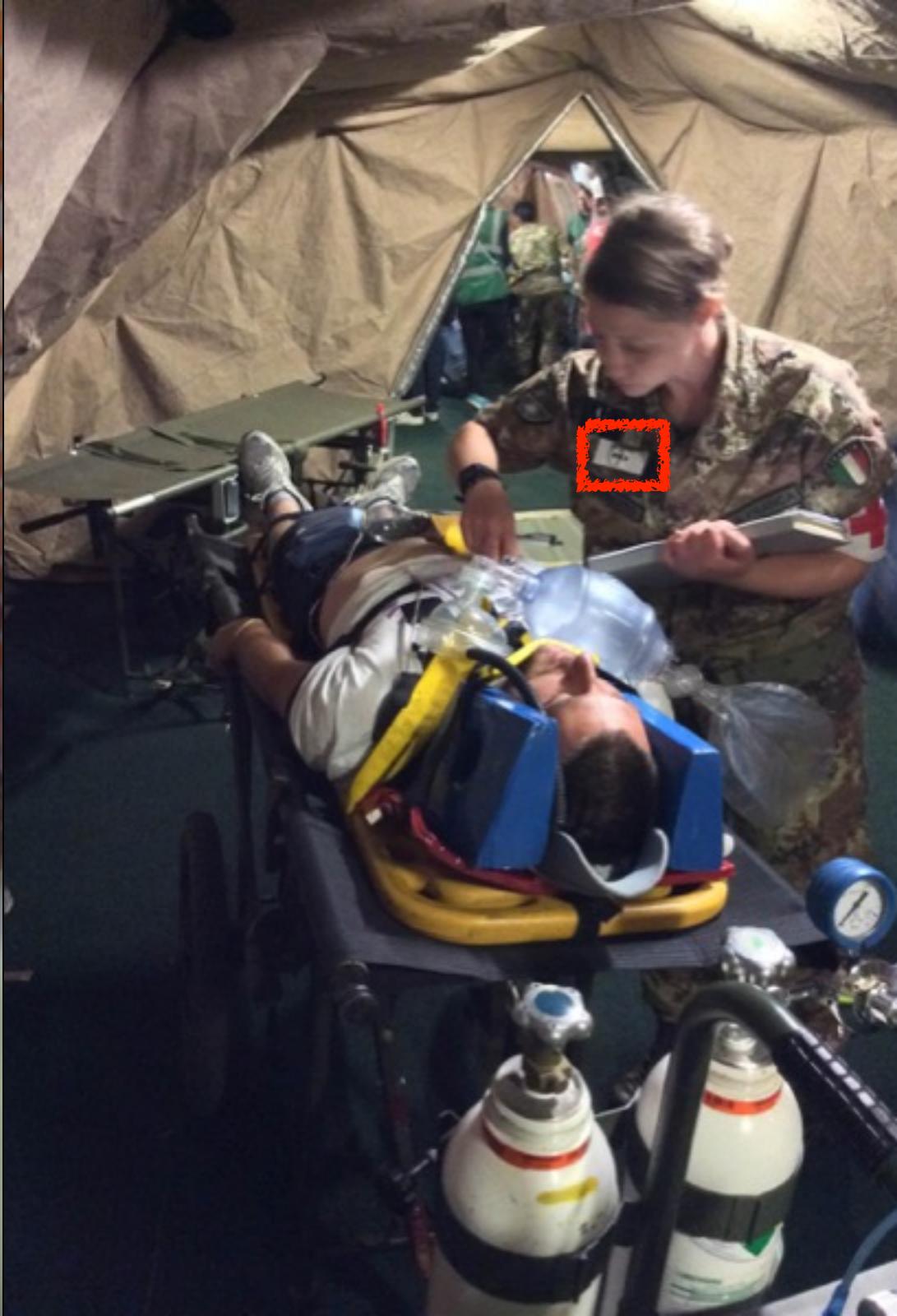






## PATIENT FLOW







26 Field Hospital  
Director + 26

27 Surgery + 27  
Preparation room

45 Service  
unit

28 Post Anesthesia  
care unit + 28

40

ICU Service unit

48

Oxygen suppl

19 Emergency  
First aid + 21

20 Triage + 20

21 Emergency  
Room + 21

22 Surgical  
clinic + 22

23 Doctor on duty  
Reception Ward + 23

24 Men ward  
6 PAX + 24

25

Men ward

6-PAX + 25

10 Admission + 10

11 Waiting room + 11

12 Waiting room + 12

13 Orthopedics + 13

14 Cardiology  
Ophthalmology  
ENT + 14

15 Women ward + 15  
6 PAX  
Pediatrics: 2 PAX

16

Women

6 PAX + 16

30 Pharmacy

31 Fridge

1 Blood  
sampling  
room + 1

2 CR019  
Ultrasound + 2

3 Radiology + 3

4 Ginecology + 4

46 Dental clinic

32 Fridge

33 Ultrasound

A + + O

CASUALTY

DOCTOR

NURSE

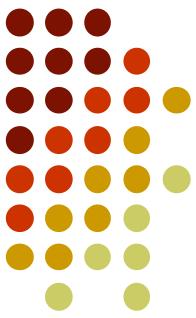
EMT

ARMY PERSONELL

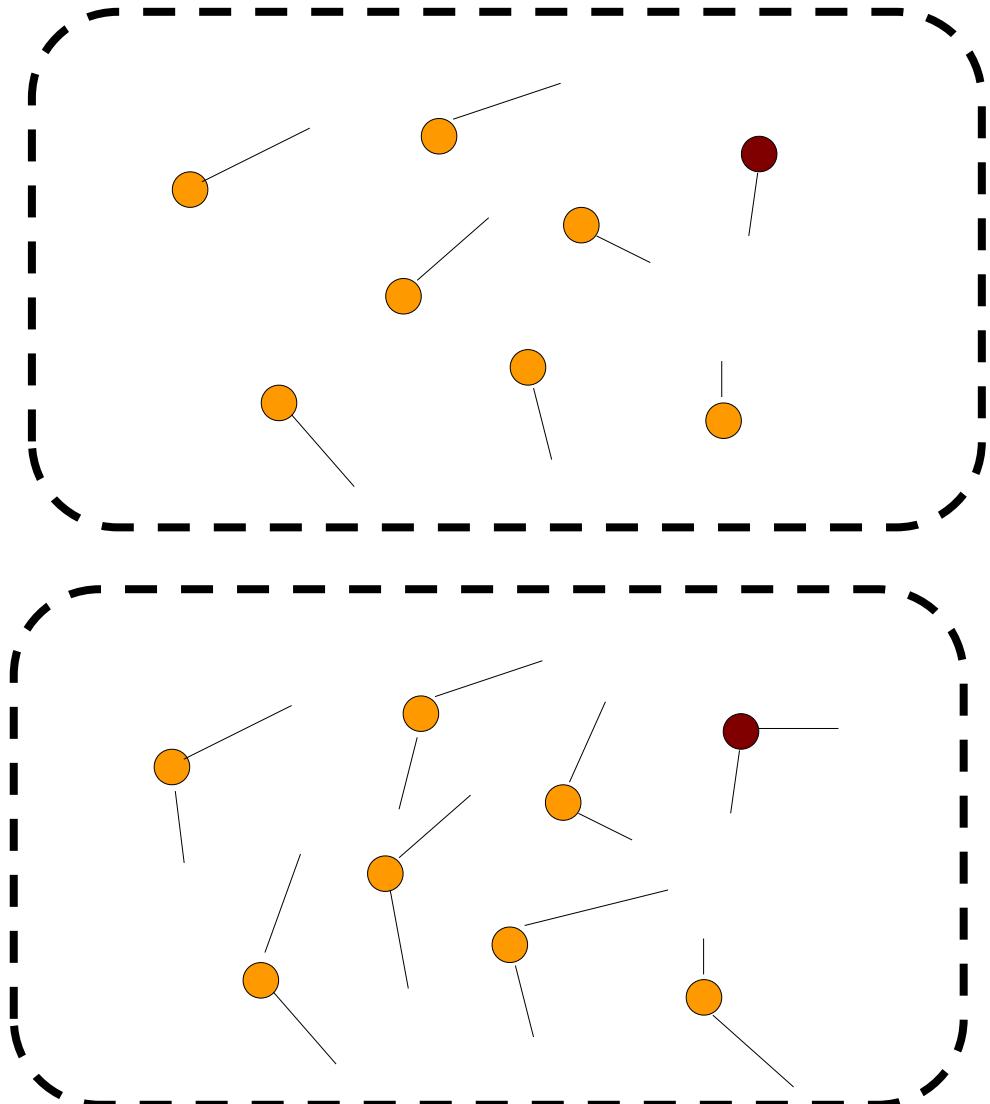
MEDICAL DIRECTOR [ARMY]

MEDICAL DIRECTOR [EMOM]

FIRST RESPONDER



# Degree based representation

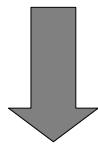


**Class of individuals with degree  
 $k=1$**

**Class of individuals with degree  
 $k=2$**



## Class of individuals with degree k



**S**  
 $k$

**I**  
 $k$

**R**  
 $k$

$S_k$

$I_k$

$R_k$

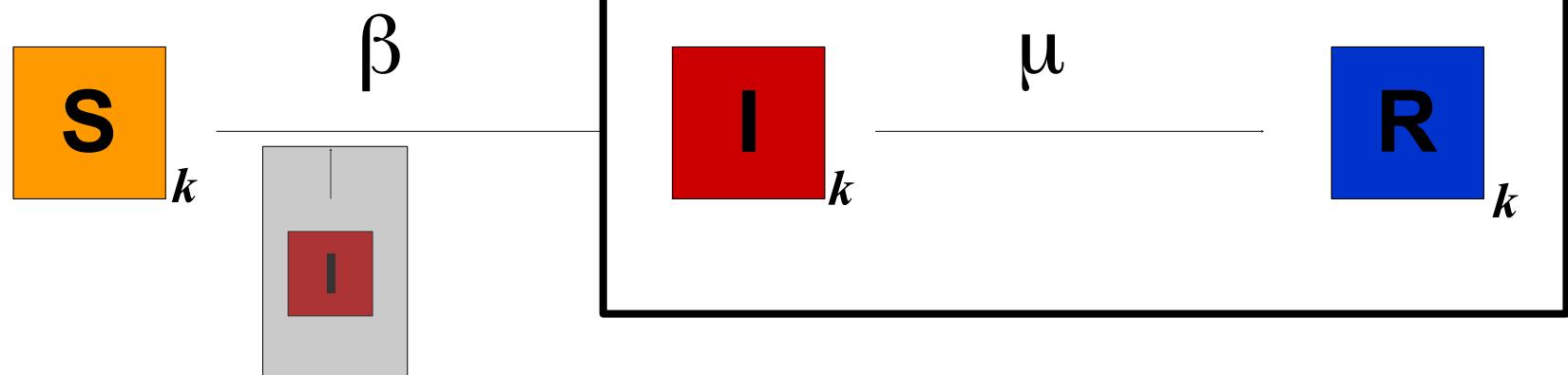
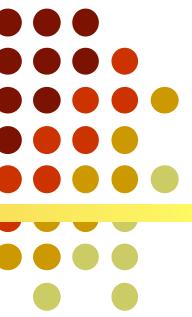
$$S_k(t+1) = S_k(t) - \beta S_k(t) \frac{I(t)}{N}$$

$$I_k(t+1) = I_k(t) + \beta S_k(t) \frac{I(t)}{N} - \mu I(t)$$

$$R_k(t+1) = R_k(t) + \mu I(t)$$

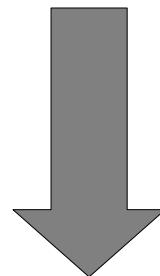
?

# homogeneous mixing



Interactions with  
all possible class  
k

No changes



$$\Delta I_k = -\mu I_k \Delta t$$

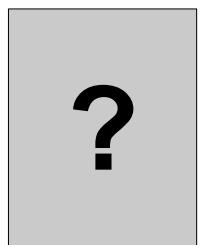


$$S_k(t+1) = S_k(t) - \beta S_k(t)$$

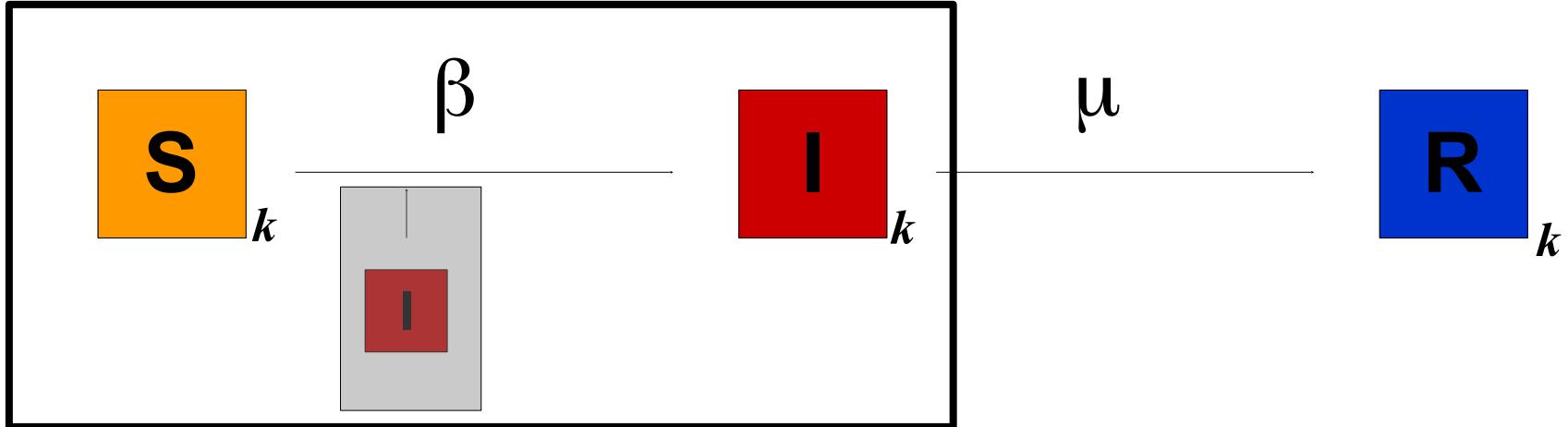
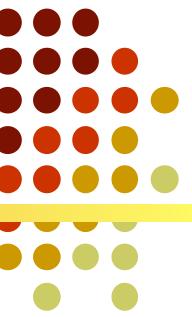
$$\frac{I(t)}{N}$$

$$I_k(t+1) = I_k(t) + \beta S_k(t) \frac{I(t)}{N} - \mu I_k(t)$$

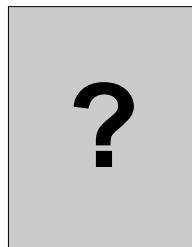
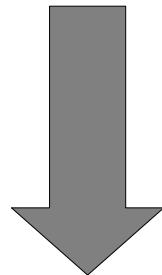
$$R_k(t+1) = R_k(t) + \mu I_k(t)$$



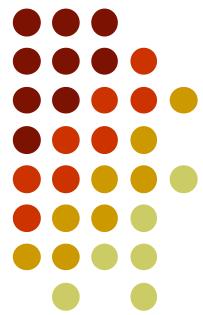
# homogeneous mixing



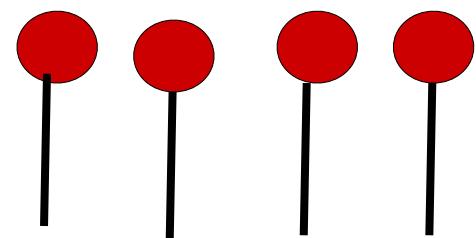
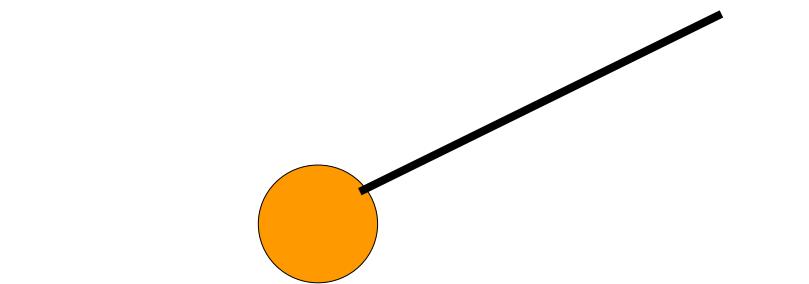
Interactions with  
all possible class  
k



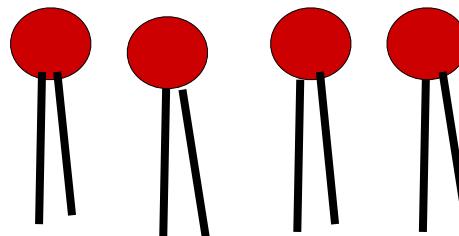
$$p_{S_k \rightarrow I_k} = \Delta t \beta k \sum_{k'} k' \frac{P(k')}{\langle k \rangle} \frac{I_{k'}}{N_{k'}}$$



What is the probability that  
this link points to an infected  
individuals?



.....  $N_{k=1}$



.....  $N_{k=2}$

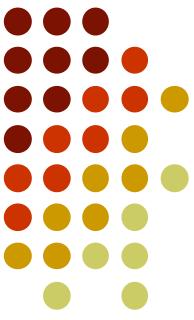


Probability to=  
a node of  
degree  $k$

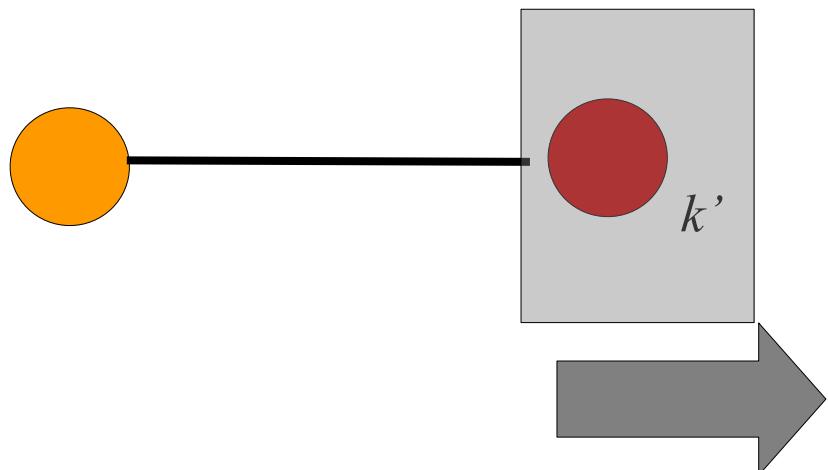
$$\frac{kN_k}{N < k >}$$

.....

$$\frac{k' N_{k'}}{N \langle k \rangle} = \frac{k' P(k')}{\langle k \rangle}$$



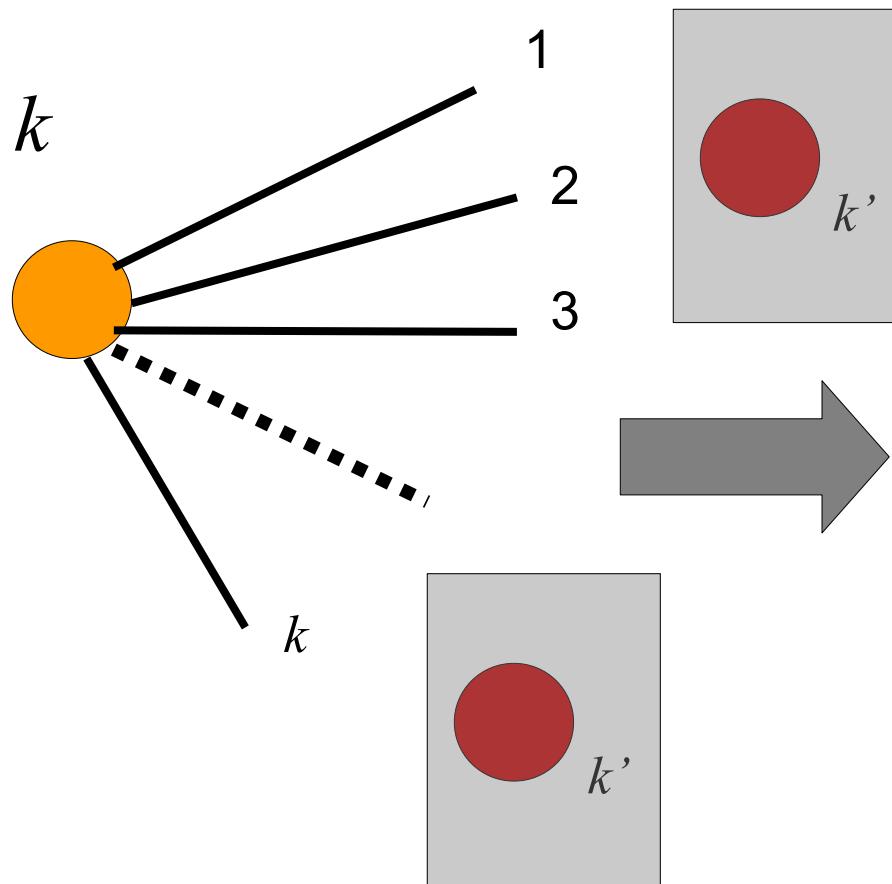
All possible  $k'$



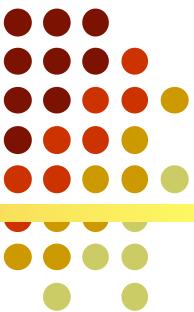
$$\sum_{k'} \frac{k' P(k')}{\langle k \rangle} \frac{I_{k'}}{N_{k'}}$$



Each susceptible of degree  $k$  has  $k$  possible connections



$$\frac{k' P(k')}{\langle k \rangle} \frac{I_{k'}}{N_{k'}}$$



# SIR model

$$S_{k,t+\Delta t} = S_{k,t} - \text{Binom}(S_{k,t}, p_{S_k \rightarrow I_k})$$

$$I_{k,t+\Delta t} = I_{k,t} + \text{Binom}(S_{k,t}, p_{S_k \rightarrow I_k}) - \text{Binom}(I_{k,t}, \mu \Delta t)$$

$$R_{k,t+\Delta t} = R_{k,t} + \text{Binom}(I_{k,t}, \mu \Delta t)$$

*summing....*

$$S_{k,t+\Delta t} + I_{k,t+\Delta t} + R_{k,t+\Delta t} = S_{k,t} + I_{k,t} + R_{k,t}$$

constant  
population

!



# Final set of equations

A set of  $3 \times k$  equations

$$S_k(t+1) = S_k(t) - \beta S_k(t) k \sum_{k'} \frac{k' P(k')}{{<} k {>}} \frac{I_{k'}(t)}{N_{k'}}$$

$$I_k(t+1) = I_k(t) + \beta S_k(t) k \sum_{k'} \frac{k' P(k')}{{<} k {>}} \frac{I_{k'}(t)}{N_{k'}} - \mu I_k(t)$$

$$R_k(t+1) = R_k(t) + \mu I_k(t)$$

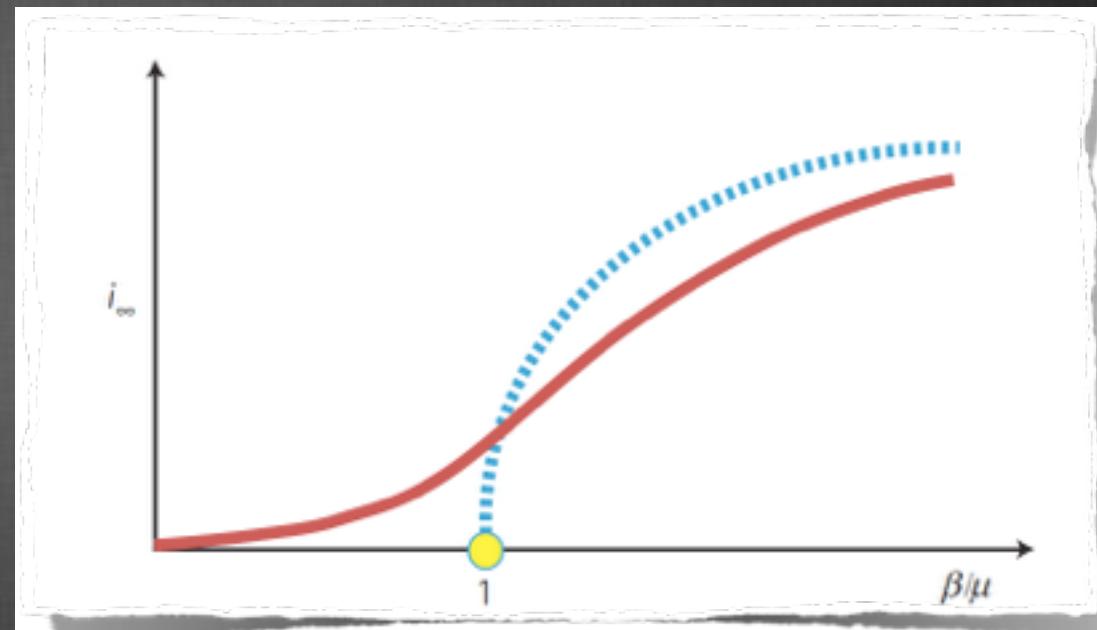
- Network heterogeneity tend to suppress the spreading threshold

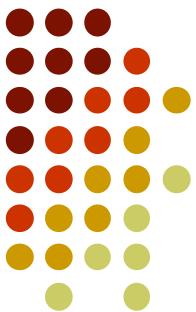
- **Random-annealed network patterns have a vanishing epidemic threshold**

$$\beta/\mu = \langle k \rangle^2 / \langle k^2 \rangle$$

- Quenched networks have an epidemic threshold vanishing as the inverse of the largest eigenvalue of the connectivity matrix.

- Structure may change the results, however, assortativity/disassortativity do not impact the qualitative behavior concerning threshold.

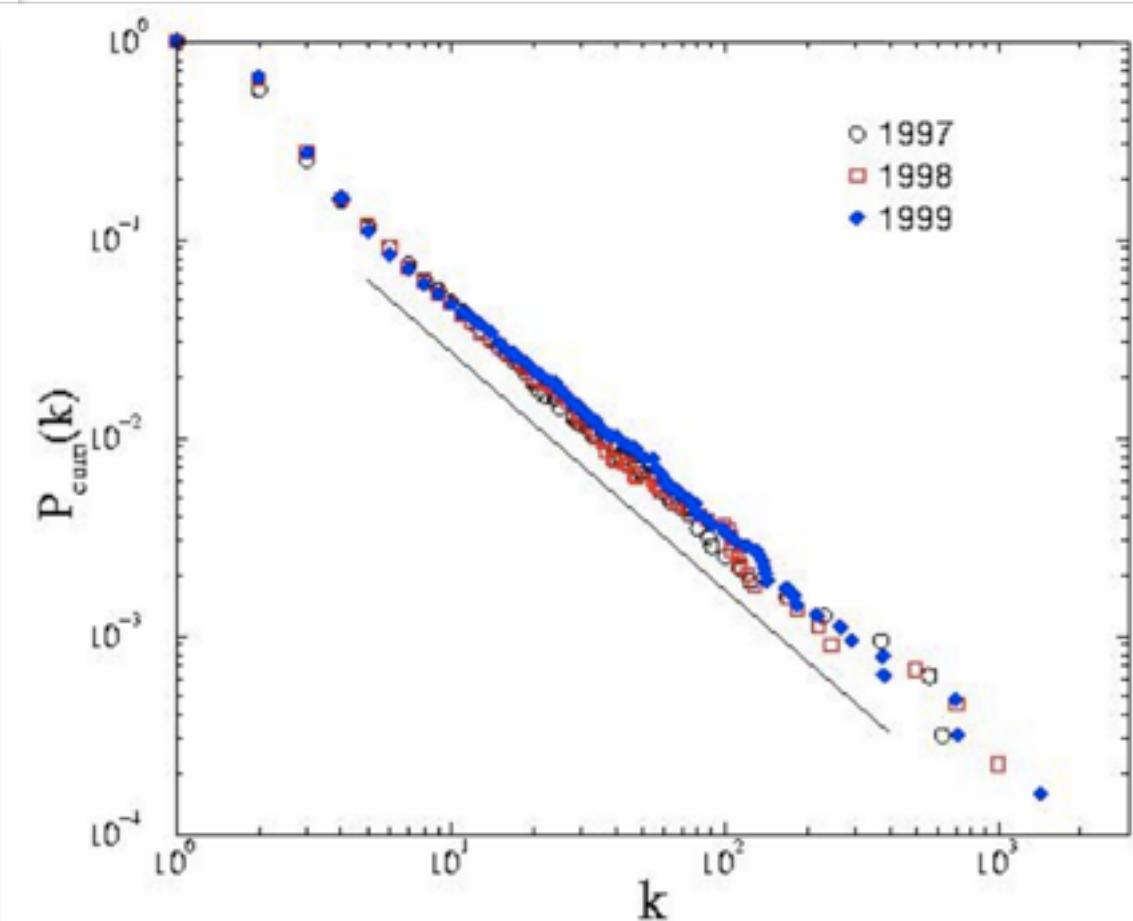




# Heterogeneous networks

## Statistical distributions

- ✖ Skewed
- ✖ Heterogeneity and high variability
- ✖ Very large fluctuations (variance>>average)



# MICRO/INDIVIDUAL BASED LARGE-SCALE COMPUTATIONAL MODELS FOR INFECTIOUS DISEASE SPREADING

## Individual based,structured metapopulation models

- Collection of sub-populations coupled by traveling individuals. [Ravchev, Longini. Mathematical Biosciences (1985) 50 urban areas worldwide; Viboud et al., Valleron et al., Brockmann et al., Colizza et al., Balcan et al.]

## Agent based models

- (highly detailed individual based simulation– household+work+mobility detailed representation)
  - Eubank&Barrett, Longini, Halloran, Ferguson, Burke, Merler and the FBK

## Challenges

- Data hungry models,
- Lack of information=assumption
- Set of parameters usually not available initially (vaccination rate, pre-exposure immunity etc.)

