

— ARIMA Modeling

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Learning Objectives

- Describe the purpose of the autoregressive and moving average components.
- Define hyperparameters p , d , and q .
- Describe AIC.
- Find the right value of p and q using AIC.
- Find the right value of d using the augmented Dickey-Fuller test.
- Complete a manual GridSearch.
- Fit an ARIMA model.

We have multiple approaches to work with time series data.

**Linear
Models**

**ARIMA
Models**

**Exponential
Smoothing
Methods**

**Recurrent
Neural
Networks
(RNNs)**

Note: This is not an exhaustive list of models, but lists the most common ones!



Why ARIMA?

- Among the most common approaches to time series modeling.
- Highly flexible; it can model time series with varying characteristics.
 - It takes information from **both long-term trends and sudden shocks!**
- Can easily be extended into more advanced models.
- Tends to perform well with moderate amounts of data.
 - It can be hard to get lots of time series data!

Downsides of ARIMA Models

- ARIMA models are best suited for **short-term forecasts**, but very quickly will start predicting the mean.
 - Some extensions to ARIMA models can handle this better.

What is an **ARIMA** model?

ARIMA

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ARIMA

Autoregressive Integrated Moving Average



What do you think the word “autoregressive” means?

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- Autocorrelation is the correlation of one variable with itself.
- An autobiography is a book written by a person, about that same person.
- An autotransplant is a surgical procedure in which an organ is transplanted from a person to that same person.
- **Autoregressive means we regress a variable on itself.**
 - We'll regress newer values on older values.



AR(p): An autoregressive model of order p

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdots + \beta_p Y_{t-p}$$

$$= \beta_0 + \sum_{k=1}^p \beta_k Y_{t-k}$$

AR(p): An autoregressive model of order p

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdots + \beta_p Y_{t-p} \\ &= \beta_0 + \sum_{k=1}^p \beta_k Y_{t-k} \end{aligned}$$

Purpose: An autoregressive model explains long-term trends in our data.

Hyperparameter: p , the number of previous values of Y to put into our model.



We'll **GridSearch** to find this value!

What is an ARIMA model?

ARIMA

Autoregressive Integrated Moving Average

Moving Average Models

- A moving average model takes **previous error terms as inputs**.
- The goal is to predict future values based on recent forecasting errors.
 - This isn't identical to boosting, but is similar in that fitting is driven by errors.
- *Annoying: this isn't the same thing as moving average smoothing.*



MA(q): A moving average model of order q

$$\begin{aligned} Y_t &= \mu + w_1 \varepsilon_{t-1} + w_2 \varepsilon_{t-2} + \cdots + w_p \varepsilon_{t-q} \\ &= \mu + \sum_{k=1}^q w_k \varepsilon_{t-k} \end{aligned}$$

$$\text{MA}(1) \Rightarrow Y_t = \mu + w_1 \varepsilon_{t-1}$$

MA(q): A moving average model of order q

$$\begin{aligned} Y_t &= \mu + w_1 \varepsilon_{t-1} + w_2 \varepsilon_{t-2} + \cdots + w_p \varepsilon_{t-q} \\ &= \mu + \sum_{k=1}^q w_k \varepsilon_{t-k} \end{aligned}$$

Purpose: A moving average model explains **sudden shocks** in our data.

Hyperparameter: q , the number of previous errors ε to put into our model.



We'll **GridSearch** to find this value!

How do we GridSearch to find the best values of p and q ?

- Because we're working in statsmodels, we will **manually GridSearch** values of p and q to see which gives us the **lowest AIC**.
- AIC, or Akaike Information Criterion, is a common way to evaluate time series models. (AIC is an attribute in **statsmodels**.)
- Remember that a model is a simplification of reality?
 - AIC attempts to measure how much information we lose when we simplify reality with a model.

$$AIC = 2 \times [\text{\# of model parameters}] - 2 \times \log(\text{likelihood})$$

What is an ARIMA model?

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At this point, it's helpful to see what an **ARIMA** model is.

$$Y_t^{(d)} = \beta_0 + \underbrace{\sum_{k=1}^p \beta_k Y_{t-k}^{(d)}}_{\text{AR part}} + \underbrace{\sum_{i=1}^q w_i \varepsilon_{t-i} + \varepsilon_t}_{\text{MA part}}$$

At this point, it's helpful to see what an **ARIMA** model is.

ARIMA

We literally just added together the AR(p) and MA(q) models.

$$Y_t^{(d)} = \beta_0 + \sum_{k=1}^p \beta_k Y_{t-k}^{(d)} + \sum_{i=1}^q w_i \varepsilon_{t-i} + \varepsilon_t$$

Autoregressive Integrated Moving Average

What is that $Y_t^{(d)}$?

- Onto the notebook!

...but first!

ARIMA Cheat Sheet

	AR	I	MA
Stands for:	Autoregressive	Integrated	Moving Average
Summary:	Regress future values on past values .	Differences our Y variable.	Regress future values on past errors .
Looks Like:	$\beta_0 + \sum_{k=1}^p \beta_k Y_{t-k}^{(d)}$	$Y_t^{(d)}$	$\beta_0 + \sum_{i=1}^q w_i \varepsilon_{t-i} + \varepsilon_t$
Purpose:	Long-term trends.	Ensure stationarity.	Sudden shocks.
Hyperparameter:	p	d	q
Find good value of hyperparameter by:	GridSearch	Augmented Dickey-Fuller Test	GridSearch