TABLE 14-6 Recovery Rates by Original Debt Seniority

Debt Seniority	Recovery Rate Average	
	Investment Grade	Sub-Investment Grade
Senior secured	54.8%	56.4%
Senior unsecured	48.2%	48.7%
Senior subordinated	32.7%	39.9%
Subordinated	31.9%	31.7%
Discount and zero-coupon	24.1%	24.4%
Total	41.0%	

Source: Altman and Kishore (1996)

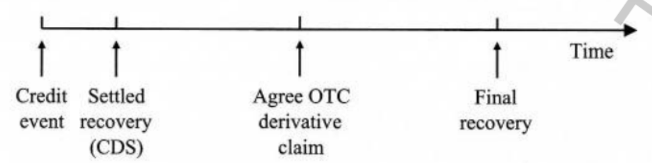


FIGURE 14-11 Schematic illustration of recovery settlement after a credit event.

The settled recovery rate is achieved very close to the credit event time (for example, by participating in the CDS auction). The final recovery occurs when the company has been completely wound up. The actual recovery for a derivative claim may be realised sometime between the settled and final recoveries.

- *Actual recovery.* This is the actual recovery paid on the debt following a bankruptcy or similar process.

In theory, settled and actual recoveries should be very similar, but in reality, since bankruptcy processes can take many years, they may differ materially. This is illustrated in Figure 14-11. It should be possible to agree on the claim with the bankruptcy administrators prior to the actual recovery, although this process may take many months. This would allow an institution to sell the claim and monetize the recovery value as early as possible. In the case of the Lehman Brothers bankruptcy, the settled recovery was around 9%, whereas some actual recoveries traded to date have been substantially higher (in the region of 30–40%).

CREDIT DEFAULT SWAPS

The credit derivatives market has grown quickly in recent years, fuelled by the need to transfer credit risk efficiently and develop ever-more sophisticated products for investors. A credit derivative is an agreement designed to shift credit risk between parties and its value is derived from the credit performance of a corporation, sovereign entity or security. Credit derivatives can be traded on a single-name basis (referencing a single component such as a corporate) or a portfolio basis (referencing many components such as 125 corporate names).

Credit derivatives instruments are important since they represent opportunities for trading, hedging and diversification of counterparty risk. However, credit derivatives as a product class give rise to a significant amount of counterparty risk. Indeed, the continued development of the credit derivative market is contingent on control of this counterparty risk. This is a key role of CCPs.

Basics of CDSs

Many credit derivatives take the form of a CDS, which transfers the default risk of one or more corporations or sovereign entities from one party to another. In a single-name CDS, the protection buyer pays an upfront and/or periodic fee (the premium) to the protection seller for a certain notional amount of debt for a specified reference entity. If the reference entity specified undergoes a credit event, then the protection seller must compensate the protection buyer for the associated loss by means of a pre-specified settlement procedure (the protection buyer must also typically pay an accrued premium at this point as compensation, due to the fact that premiums are paid in arrears). The premium is paid until either the maturity date or the credit event time, whichever comes first. The

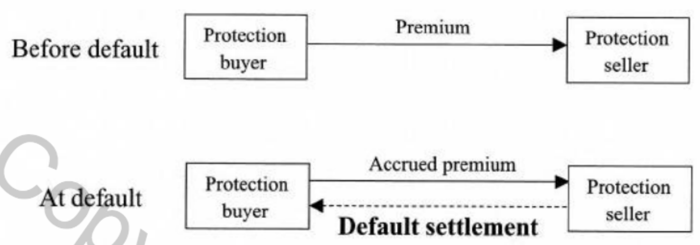


FIGURE 14-12 Illustration of a typical CDS contract on a single reference entity.

reference entity is not a party to the contract, and it is not necessary for the buyer or seller to obtain the reference entity's consent to enter into a CDS. The mechanics of a single-name CDS contract are shown in Figure 14-12 (index contracts are discussed later).

CDS contracts trade with both fixed premiums and upfront payments. This reduces annuity risk in the hedge and unwinding of CDS contracts. Although it is not compulsory, the standard is that a CDS, on investment-grade reference entities, typically has a fixed premium of 100 basis points whilst high-yield reference entities trade at 500 basis points.[11] The scheduled termination dates of CDSs are March 20th, June 20th, September 20th or December 20th.

CDS documentation refers to a reference obligation and reference entity. The reference entity may be a corporate, a sovereign or any other form of legal entity which has incurred debt.[12] The reference obligation defines the seniority of the debt that can be delivered. Commonly, all obligations of the same or better seniority can be delivered (in the case of no reference obligation being specified, the seniority is senior unsecured).

Credit Events

Generally, the term "default" is used (as in default probability, for example) instead of the more accurate generic term "credit event." There are various credit events that can all potentially lead to losses for creditors. Some credit events are well-defined, such as Chapter 11 bankruptcy in the US, whereas some other technical credit events, for example involving a breach of some contractual terms, are less so.

The three most important credit events are:

- *Bankruptcy.* This will be triggered by a variety of events associated with bankruptcy or insolvency proceedings, such as winding up, administration and receivership, under English and New York law or analogous events under other insolvency laws.
- *Failure to pay.* This event covers the failure to make a payment of principal or interest. A minimum threshold amount must be exceeded before this event is triggered (default value $1m). Failure to make a collateral posting even after the relevant grace period falls into this category, as discussed in Chapter 11.
- *Restructuring.* This covers the restructuring of debt causing a material adverse change in creditworthiness.

A significant risk when hedging with CDS contracts is that there is an economic loss but the credit event in the contract is not triggered.[13] Obvious examples of this may be restructuring-type credit events such as a debt-to-equity swap, a distressed exchange or another form of restructuring. The voluntary haircuts taken by most holders of Greek debt in 2012 were not enough to trigger a credit event. Whilst the exercise by Greece of the "Collective Action Clause" forcing all bondholders to participate did eventually trigger a restructuring credit event, this illustrates that default losses and the triggering of a credit event are in danger of being misaligned. CDSs may well appear to be good hedges for counterparty risk but may completely or partially fail when the credit event actually occurs.

CDS Settlement

The fundamental aim of a CDS is to compensate the protection buyer for the loss of par value on a defaulted security such as a bond. However, debt securities will typically not be worth zero when there has been a credit event, but will rather trade at some recovery value. Hence, the protection buyer needs to be paid par minus this

[11] Fixed premiums of 25, 100, 500 and 1000 (and 300 and 750) basis points may also trade. Historically, CDSs traded without any upfront payment, leading to many different running premiums traded at any one time.

[12] Occasionally, CDSs trade on names with little or no outstanding debt, often in the case of sovereign entities.

[13] In 2008, the conservatorship of Fannie Mae/Freddie Mac gave the reverse case by triggering the bankruptcy clause without a bankruptcy filing. However, the bonds traded very close to par due to a delivery squeeze and the explicit guarantee by the US government. In this case, sellers of CDS protection faced costs due to settlement even though there was no economic loss.

recovery value. There are fundamentally two ways in which this payoff has been achieved in CDSs:

- *Physical settlement.* In this case, the protection buyer will deliver to the protection seller defaulted securities of the reference entity with a par value equal to the notional amount of the CDS contract. In return, the protection seller must make a payment of par in cash. For example, an investor buying a bond and holding CDS protection for the same notional may deliver the defaulted bond against receiving par. This mechanism is clearly attractive since no other parties need to be involved and there can be limited dispute over payments.
- *Cash settlement.* Here, the protection seller will compensate the protection buyer in cash for the value of par minus recovery value. An obvious problem with this is that the recovery value must be determined through some market consensus of where the debt of the defaulted entity is trading (dealer poll or, more recently, an auction process, described later).

In a CDS contract settled via physical delivery, since the credit event is not specific to a given security, there is no single bond that needs to be delivered. The protection buyer therefore has some choice over the security that can be delivered and will naturally choose the cheapest available in the market (the "cheapest-to-deliver option"). Obvious choices for cheapest-to-deliver bonds may include those with low coupons (including convertible bonds) and illiquid bonds. Restructuring credit events are particularly significant in this respect, as bonds are more likely to be trading at different levels. The market has evolved to different restructuring options in CDS contracts to try to minimise cheapest-to-deliver risk. The current standards in the U.S. and Europe are modified restructuring (MR) and modified modified restructuring (MMR), respectively. These both include restructuring as a credit event in a CDS contract but limit the securities that can be delivered following such a credit event.

A large proportion of protection buyers do not hold the original risk in the form of bonds. This "naked" CDS position may arise due to pure speculation or may be linked to counterparty risk hedging. There have been efforts and calls to ban naked CDSs and only allow the buying of CDS protection when the buyer holds the underlying debt security (as is the case in insurance contracts where the owner of insurance needs to own the insured risk at the claim time). Aside from the fact that this will make the CDS market inefficient, this can restrict CDS protection being held against credit exposure to hedge counterparty risk. Since future credit exposure is uncertain, it is not clear what an appropriate amount of CDS protection to hold as a hedge would be.[14] An institution may understandably want to buy more CDS protection than their current exposure to mitigate a potential increase in exposure in the future.

Another problem in the CDS market is a *delivery squeeze* that can be created if the amount of notional required to be delivered (total outstanding CDS protection on the reference entity) is large compared with the amount of outstanding debt. In a delivery squeeze, bond prices will increase to reflect a lack of supply and this in turn will suppress the value of the CDS (since the payoff is par less recovery). This is another important consideration in the hedging of counterparty risk since it can create a significant discrepancy between the recovery value of the security itself and the recovery as defined by the CDS contract.

The problems of cheapest-to-deliver options and delivery squeezes have been limited by the adoption of an auction protocol in settling credit events. In 2009, there were a number of changes to CDS documentation and trading practices, aimed at reducing some of the risks described above and improving standardisation. One was the incorporation of auction settlement provisions as the standard settlement method for credit derivatives transactions. The so-called "Big Bang Protocol" allowed this auction to also be included for legacy CDS trades (as long as both counterparties signed up to the Big Bang Protocol). Most major credit events on liquid reference credits should now be settled in this fashion, via a pre-planned auction of defaulted bonds to determine a fair price for cash settlement of all CDSs referencing the credit in question. Whilst this eliminates most basis risks, the problems of settled and final recovery in the hedging of counterparty risk (Figure 14-11) remains.

In Table 14-7 we show recovery values settled following credit events for some CDS auctions in 2008. We see a wide range of recoveries from Fannie Mae and Freddie Mac that were close to 100%, thanks largely to the guarantee from the US government, making this a more technical

[14] For example, even if the exposure to a counterparty was currently zero, an institution may reasonably want to buy CDS protection to hedge a potential increase in the counterparty credit spread.

TABLE 14-7 Recovery Rates for CDS Auctions for Some Credit Events in 2008

The impact of a delivery squeeze can be seen in that Fannie Mae and Freddie Mac subordinated debt traded at higher levels than the senior debt.

Reference Entity	Seniority	Recovery Rate
Fannie Mae	Senior Subordinated	91.5% 99.9%
Freddie Mac	Senior Subordinated	94.0% 98.0%
Washington Mutual		57.0%
Lehman		8.6%
Kaupthing Bank	Senior Subordinated	6.6% 2.4%
Landsbanki	Senior Subordinated	1.3% 0.1%
Glitnir	Senior Subordinated	3.0% 0.1%
Average		38.5%

credit event than Lehman Brothers and Icelandic banks that recovered very little.

The CDS–Bond Basis

It is possible to show theoretically (Duffie, 1999) that, under certain assumptions, a (short) CDS protection position is equivalent to a position in an underlying fixed-rate bond and a payer interest rate swap.[15] This combination of a bond and interest rate swap corresponds to what is known as an asset swap. This implies that spreads, as calculated from the CDS and bond markets, should be similar. However, a variety of technical and fundamental factors means that this relationship will be imperfect. The difference between CDS and bond spreads is known as the CDS–bond basis. A positive (negative) basis is characterised by CDS spreads being higher (lower) than the equivalent bond spreads.[16]

Factors that drive the CDS–bond basis are:

- *Counterparty risk.* CDSs have significant wrong-way counterparty risk (Chapter 16), which tends to make the basis negative.
- *Funding.* The theoretical link between bonds and CDSs supposes that LIBOR funding is possible. Funding at levels in excess of LIBOR will tend to make the basis positive, as CDSs do not require funding. Contributing further to this effect is that shorting cash bonds tends to be difficult, as the bond needs to be sourced in a fairly illiquid and short-dated repo market in which bonds additionally might trade on special, making it expensive to borrow the bond.
- *Credit event definition.* CDS credit events should, in theory, perfectly coincide with the concept of credit-related losses for bondholders. However, credit events are vulnerable to divergence from bond documentation, despite improvements by ISDA in standardising and harmonising CDS legal documentation. Technical credit events may cause CDS protection to pay out on an event that is not considered a default by bondholders. Alternatively, a credit event may not be triggered even though bondholders take credit losses (see comment on Greek debt earlier). The former effect would tend to push the basis into positive territory, whilst the latter would make it negative.
- *Cheapest-to-deliver option.* The delivery option in a CDS contract may have some additional values in certain circumstances, such as restructuring credit events. This would tend to make the basis *positive*.
- *Delivery squeeze.* A delivery squeeze involves a shortage of CDS deliverable debt and would tend to make the basis *negative.*
- *Bonds trading above or below par.* Fixed-rate bonds can trade significantly above or below par because of changes in interest rates. CDS protection is essentially indexed to the par value of a bond and bonds trading above (below) par will tend to make the basis negative (positive). The use of fixed coupon CDS reduces this effect.
- *Accrued interest.* In the event of default, a bond typically does not pay accrued interest for any coupons owed, whereas a CDS does require protection buyers to pay the accrued premium up to the credit event. This will cause the basis to be negative.
- *Other technical factors.* Historically, other technical factors, such as synthetic CDO issuance, have had an impact on the basis.

[15] Specifically, the interest rate swap is not standard, as it must terminate if the underlying credit event in the CDS occurs.

[16] We note that the definition of bond spread is subjective, as it must be defined by some "risk-free" benchmark.

Generally, prior to the global financial crisis, the basis tended to be positive due to effects such as funding and the cheapest-to-deliver option. More recently, the basis has been negative due partially to CDS counterparty risk concerns.

Contingent Credit Default Swaps

In a standard single-name CDS, the protection buyer has protection on a fixed contractual notional amount. Such a contract is reasonably well tailored towards credit exposures arising from instruments such as loans and bonds. For example, \$10m of CDS protection would give protection against holding bonds with par value of \$10m.[17] However, a key aspect of counterparty risk is that the loss as determined by the credit exposure at the credit event time is usually unknown.

A CCDS is an instrument that is the same as a standard single-name[18] CDS but with one key difference, in that the notional amount of protection is referenced to another transaction(s). This underlying transaction can be potentially any product across any asset class. Hence, a CCDS can provide perfect protection against the counterparty risk on a derivative since the protection amount can be linked directly to the exposure of that derivative. Whilst CDSs are generally products which have many applications, CCDSs are products that are tailor-made to hedge counterparty risk. As such, CCDSs potentially allow for the possibility of a complete disentangling of counterparty risk from all other financial risks.

A CCDS represents a contract tailor-made to transfer counterparty risk from one institution to another. However, except in limited cases, CCDSs have not proved particularly popular. Some reasons for this are:

- *Complexity of documentation.* A CCDS must contain a "termsheet within a termsheet" since it must reference the transaction for which the counterparty risk is to be transferred. Confidentiality may also be a problem here since a CCDS counterparty would have information on all trades with the counterparty whose risk is being hedged.
- *No recognition of netting/collateral.* A CCDS typically references a single trade and not a netting set, which would be more relevant. A CCDS referring an entire netting set would be complex and would not cover subsequent trades within that netting set. Additionally, a CCDS does not account for the potential collateralisation of a credit exposure.
- *Double default.* A CCDS is not effective unless the CCDS provider has a very high credit quality and/or is uncorrelated with the original counterparty. These aspects are very hard to achieve, the latter especially so for counterparties of good credit quality.
- *Lack of sellers of protection.* As with the single-name CDS market, there is a lack of sellers of single-name CCDS protection.

There have been attempts to ignite the CCDS market. For example, Novarum group set up a dedicated vehicle to sell fully collateralised CCDS protection in 2009. However, this initiative has not seen great success, probably mainly due to the double default aspect mentioned above. For example, for an OTC derivative dealer to hedge a large component of their CVA with such an entity, they would have to be very certain of this entity's ability to withstand a high default rate environment in order to feel that the hedges were effective. Regulators would need to have the same confidence to allow capital relief and provide a strong credit rating to the protection seller.

CURVE MAPPING

Earlier, we discussed the quantification of risk-neutral default probabilities from the credit spread of a counterparty. Such a credit spread may be derived in a variety of ways from the market prices of bonds, asset swaps and single-name CDSs. However, a key aspect in quantifying CVA is to obtain credit spreads for non-observable names, i.e., those counterparties for which there is no defined credit spread trading in the market.

Whilst using subjective mapping methods to determine a credit spread may seem rather non-scientific, it is generally a necessary process for banks to value illiquid assets, such as bonds and loans, held on their trading books. Furthermore, Basel III capital rules impose similar requirements for capital allocation against CVA, stating (BCBS, 2011): "Whenever such a CDS spread is not available, the bank must use a proxy spread that is appropriate based on the rating, industry and region of the counterparty."

[17] This is only approximately true due to the triggering of a credit event not being aligned with the bond loss as mentioned above, and due to the potentially different recovery values.

[18] We refer here to single-name CCDSs. Index CCDSs will be discussed later.

Banks and authors (e.g., Gregory, 2010) have argued against this requirement on the basis that banks do not attempt to mark-to-market much of their illiquid credit risk (including CVA).

Basics of Mapping

The fundamental aim of credit curve mapping is to use some relevant points to achieve a general curve based on observable market data, as illustrated in Figure 14-13. This illustrates a case where a number of points can be used at various different maturities (as in the case of the secondary bond market). A best fit to these spreads (perhaps with some underlying weighting scheme also used to bias towards the more liquid quotes) gives the entire curve. The classification may be rather broad (e.g., a Single-A curve), in which case there will be a large number of data points to fit but less distinguishing between different counterparties. In contrast, a more granular classification (e.g., rating, sector and geography—for example, a Single-A U.S. utility company) distinguishes better between different counterparties but provides less data for each curve calibration.

We note that this representation is troublesome from a hedging perspective as all points represent hedging instruments. There is also the problem that a recalibration (either periodic or, for example, due to removal of an illiquid data point) will cause a curve shift and a resulting move in CVA with an associated (unhedgeable) PnL impact.

Indices and Classification

Whilst bond spreads provide some mapping information, a key component of a mapping methodology is the link to the hedging of CVA. Credit indices therefore represent a

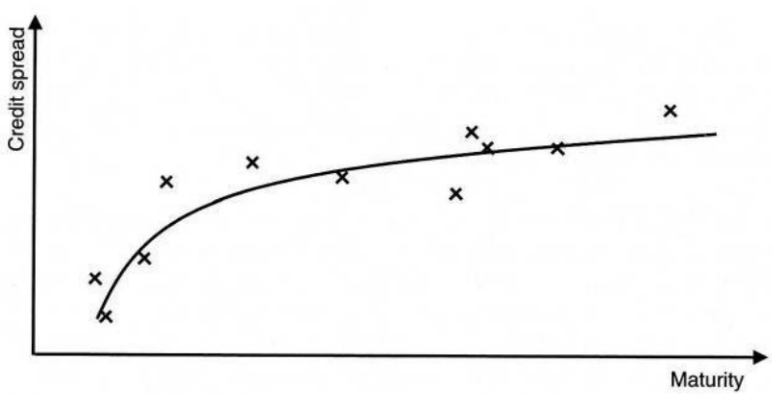


FIGURE 14-13 Illustration of a mapping procedure.

The crosses represent observable spreads as a function of maturity.

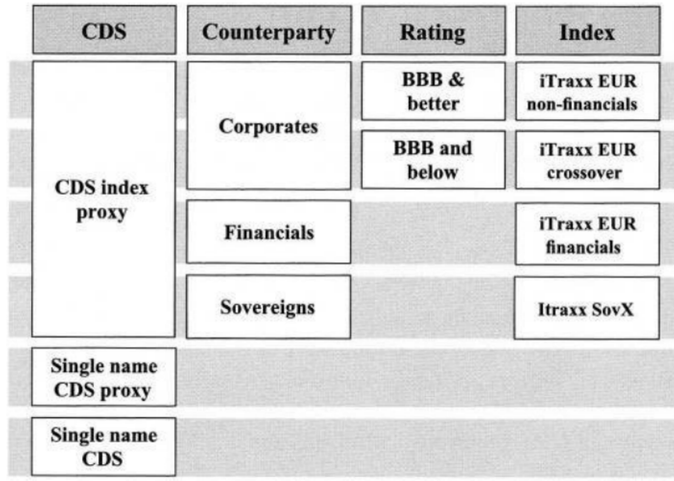


FIGURE 14-14 Illustration of a classification of counterparties according to European credit indices.

better choice for mapping credit curves. An example classification of European counterparties according to credit indices[19] is given in Figure 14-14. Reading from the bottom, the first choice would obviously be to map to a single-name CDS or a relevant proxy such as a parent company. If such information were not available, then the counterparty would be mapped to the relevant index depending on whether it is a corporation, financial, or sovereign entity. Corporations may be further sub-divided according to credit quality.

Note that further, more detailed classifications can be made that are not shown in Figure 14-14. For example, iTraxx SovX is sub-divided into Western Europe (WE) and Central & Eastern European, Middle Eastern and African (CEEMEA). Corporates may also be sub-divided into sectoral indices (in addition to financials and non-financials), such as TMT, industrials, energy, consumers and autos. Whilst these sub-divisions give a more granular representation, they have to be balanced against the available liquidity in the CDS market.

Curve Shape

A final consideration that is relevant is the case where a single-maturity credit spread (typically five years) can be defined (either directly or via

[19] See http://www.markit.com/en/products/data/indices/credit-and-loan-indices/itraxx/itraxx.page

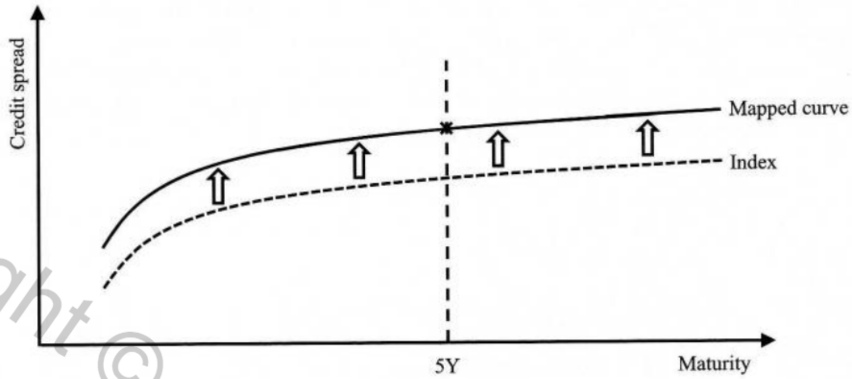


FIGURE 14-15 Illustration of defining a curve shape based on the shape of the relevant index.

The cross shows the 5-year point that is assumed to be known for the curve in question.

some mapping) but the rest of the curve cannot. The obvious solution in such a case is to use the most appropriate index to define the curve shape, as illustrated in Figure 14-15. So, for example, if the 5-year point defined is 130% times the equivalent index maturity, then all points are mapped to 130% of the index curve.

PORTFOLIO CREDIT DERIVATIVES

In this final section, we give a brief overview of portfolio credit derivatives products such as index tranches and collateralised debt obligations (CDOs). A basic understanding of these structures is useful for the discussions on wrong-way counterparty risk in Chapter 16. A more in-depth coverage of portfolio credit derivatives and their uses (and abuses) is given in Tavakoli (2008).

CDS Index Products

Up until 2004, the majority of credit default swaps were written on single names, but thereafter a major impetus to growth and market liquidity of the credit derivative market has been credit default swaps on indices. A credit index can usually be thought of as an equally weighted combination of single-name CDSs and hence the fair premium on the index will be close to the average CDS premium within that index.[20] The two most common credit indices are:

- *DJ iTraxx Europe.* This contains 125 European corporate investment-grade reference entities, which are equally weighted.
- *DJ CDX NA IG.* This contains 125 North American (NA) corporate investment-grade reference entities, which are equally weighted.

Other indices exist for different underlying reference entities and regions but they are less liquid. Indices can be traded in either CDS (unfunded) or CLN[21] (funded) form. Buying CDS protection on $125m of the DJ CDX NA IG index is almost[22] equivalent to buying $1m of CDS protection on each of the underlying reference entities within the index.

An important feature of credit indices is that they "roll" every 6 months. A roll will involve:

- *Adjustment of maturity.* Typical traded maturities are 5, 7 and 10 years. Fixed maturity dates[23] will be used such that the initial maturities are 5.25, 7.25 and 10.25 years. After 6 months, the maturities will have become 4.75, 6.75 and 9.75 and these will be re-set to their original values.
- *Adjustment of portfolio.* Names will be removed from a credit index according to predefined criteria in relation to credit events, ratings downgrades and increases in

[20] This is not quite true for two reasons. First, a theoretical adjustment must be made to the average CDS premium to account for the heterogeneity of the constituents. Second, the index will typically trade at a basis to the average CDS premiums (bid–offer costs will prevent arbitrage of this difference).

[21] Credit-linked note, which is a CDS funded to create a synthetic bond.

[22] Aside from the theoretical adjustment due to a premium mismatch and the fact that the index protection may involve an upfront payment.

[23] International Monetary Market (IMM) dates are used.

individual CDS premiums beyond a certain threshold. The overall aim is to replace defaulted names and maintain a homogenous credit quality. Names removed from the index will be replaced with other names meeting the required criteria.

- *Premium.* In the 6-month period before a roll, the index premium is fixed at a given level of either 100 or 500 bps and trades on the index will involve an upfront payment from one party to the other to compensate for the difference between the fair premium and traded premium. This greatly facilitates unwinding positions and monetising MtM gains (or losses), and is similar to the use of a fixed premium for U.S. CDS contracts discussed earlier. At the roll, the index premium may be reset (to either 100 or 500 bps) depending on its fair theoretical level based on the individual CDS levels at that time.

We note that rolls only influence new trades and not existing ones (which still reference the old index and other terms).

Index Tranches

Following on from the standardisation of credit indices was the development of index tranches. Whilst a credit index references all losses on the underlying names, a tranche will only reference a certain portion of those losses. So, for example, an [X%, Y%] tranche will reference losses between X% and Y% on the underlying index. The "subordination" of the tranche is X% whilst Y% is referred to as the "detachment point." The size of the tranche is $(Y - X)$%. The standard index tranches for the DJ iTraxx Europe and DJ CDX NA indices are illustrated in Figure 14-16. The index tranche that takes the first loss, [0–3%], is referred to as the equity tranche, with the very high-up tranches referred to as senior or super senior and the intermediate tranches referred to as mezzanine.

Irrespective of trading convention, the important aspect of an index tranche is that it covers only a certain range of the losses on the portfolio. Index tranches vary substantially in the risk they constitute: equity tranches carry a large amount of risk and pay attractive returns, whilst tranches that are more senior have far less risk but pay only moderate returns. At the far end, super senior tranches might be considered to have no risk whatsoever (in terms of experiencing losses), but this is a point we analysed in more depth in Chapter 13. Tranching creates a leverage effect since the more junior tranches carry more risk than the index, whilst the most senior tranches[24] have less risk.

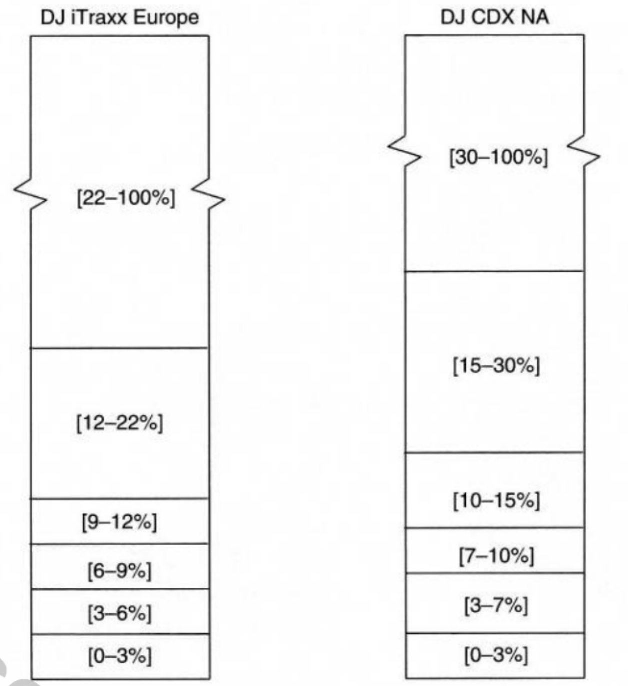


FIGURE 14-16 Illustration of the index tranches corresponding to the DJ iTraxx and DJ CDX North American credit indices.

All tranches are shown to scale except the [22–100%] and [30–100%].

Super Senior Risk

As we shall see in Chapter 16, the more senior a tranche, the more counterparty risk it creates. Not surprisingly then, super senior tranches have created a big headache for the credit market in terms of their counterparty risk. Let us start by asking ourselves how many defaults would cause a loss of either super senior tranche of DJ iTraxx and DJ CDX. We can represent the number of defaults a given tranche can withstand by

$$\text{Number of defaults} = n\frac{X}{(1 - \text{recovery})}, \tag{14.6}$$

where X represents the attachment point of the tranche (%), n is the number of names in the index, and the

[24] Due to its size, usually only the super senior may have a leverage of less than one and all other tranches may be more highly leveraged than the index.

recovery is the (weighted[25]) average recovery rate for the defaults that occur.

Example

How many defaults can the super senior tranches of DJ iTraxx and DJ CDX withstand at assumed average recoveries of 40% and 20%?

From the previous formula, we have for DJ iTraxx:

$125 \times 22\%/(1 - 40\%) = 45.8$ defaults (40% recovery)

$125 \times 22\%/(1 - 20\%) = 34.4$ defaults (20% recovery)

And for DJ CDX:

$125 \times 30\%/(1 - 40\%) = 62.5$ defaults (40% recovery)

$125 \times 30\%/(1 - 20\%) = 46.9$ defaults (20% recovery)

Super senior tranches clearly have very little default risk. Let us consider a super senior tranche of the longest maturity (10 years). From Table 14-2, the Moody's cumulative default probability for the worst investment-grade rating of Triple-B for this period is 6.38%. Then, even assuming the lower 20% recovery, default rates of 4.3 and 5.9 times the historical average would be required to wipe out the subordination on the iTraxx and CDX super senior tranches respectively.[26] This default remoteness has led to terms such as "super Triple-A" or "Quadruple-A" being used to describe the risk on super senior tranches. From the counterparty risk perspective, the important question is: from whom can an institution buy super Triple-A protection?

Collateralised Debt Obligations

There are many different types of collateralised debt obligations. They contain different asset classes and have different structural features. However, the approximate classification of risk defined in the last section (equity, mezzanine, senior) will always follow. For example, any CDO structure will have an associated super senior tranche that will be considered extremely unlikely ever to take credit losses.

CDOs can be broadly divided into two categories:

- *Synthetic CDOs.* Alternatively called collateralised synthetic obligations (CSOs), these are very similar to index tranches except that the underlying portfolio, attachment and detachment points, maturity and other specifics will be bespoke or tailor-made for a given transaction(s). Most commonly, a tranche will be traded in isolation from the rest of the capital structure. Banks have traditionally had large "correlation desks" that trade many different tranches of synthetic CDOs on various different portfolios.
- *Structured finance securities.* This very large class of securitisation structures covers cash CDOs, collateralised loan obligations (CLOs), mortgage-backed securities (MBSs) and CDOs of ABSs. The main difference between these structures and synthetic CDOs is that the structure and tranche losses occur by means of a much more complex mechanism. This means that tranches of these deals cannot be traded in isolation and all tranches must be sold more or less simultaneously[27] as a so-called "full capital structure" transaction.

From the point of view of counterparty risk, the key aspect is that issuers of CDOs need to place (buy protection) on all tranches across the capital structure. In a full capital structure or structured finance-type structure, this is clear from the need to place all of the risk. In a synthetic CDO, it is less obvious but arises because a book cannot be risk-managed effectively unless it has a reasonable balance between equity, mezzanine and senior tranches. Therefore, issuers of CDOs are super senior protection buyers, not necessarily because they think super senior tranches have value but rather because:

- They need to buy protection or place the super senior risk in order to have efficiently distributed the risk. Failure to do this may mean holding onto a very large super senior piece and potentially not being able to recognise P&L on a transaction.

OR

- Buying super senior protection is required as a hedge for other tranche positions. Without going into too much detail, we note that structured product traders may buy a product such as an option or tranche, not because they think it is undervalued, but rather because it allows them to hedge. In options terminology they may pay for the "gamma" (the convexity of the price with respect to market movements). In this

[25] Since the default that actually hits the tranche may have only a fractional impact, as in the previous example.

[26] For example, for iTraxx $34.4/(125 \times 6.38\%) = 4.3$, where the factor of 34.4 is calculated in the above example.

[27] Unless some can be "recycled" and put in the next structure, a practice that has become widely regarded as far from ideal from an investor's perspective.

case, a CDO correlation trader may buy protection on a super senior tranche, not because he thinks it will have a payoff (losses hitting the tranche), but rather because it provides positive gamma.

We will return to these aspects when we show how CDOs fail in Chapter 16.

SUMMARY

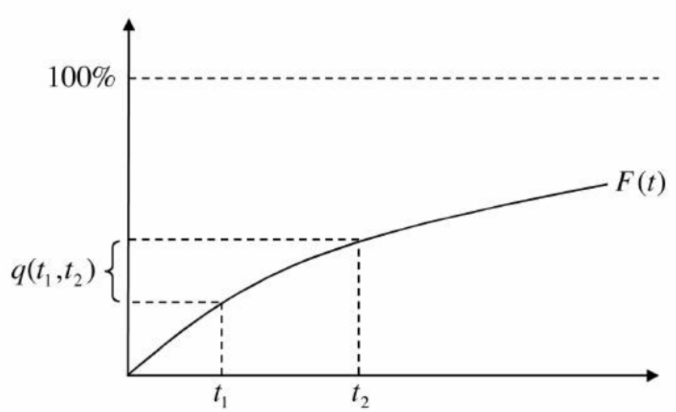
This chapter has been concerned with an overview of default probability, credit spreads and credit derivatives. We have described default probability, estimation methods and the differences between real and risk-neutral default probabilities. The impact of recovery rates has also been discussed. Detail necessary to calculate risk-neutral default probabilities from credit spreads, which will be required in CVA calculations later, has been given. We have described the important credit derivatives instruments that will be essential for discussing wrong-way risk (Chapter 16) and hedging. Finally, we have discussed curve-mapping procedures that are an important component of CVA quantification.

APPENDIX A

Definition of Cumulative Default Probability Function

In defining default probabilities, we define a cumulative default probability function, $F(u)$ which (assuming the entity is not currently in default) gives the probability of default at any point prior to time u. The marginal default probability, which is the probability of a default between two specified future dates, is given by:

$$q(t_1, t_2) = F(t_2) - F(t_1) \qquad (t_1 \leq t_2)$$



The instantaneous default probability is given by the derivative of $F(u)$.

We also use the definition of survival probability as $S(u) = 1 - F(u)$.

APPENDIX B

Mathematics Behind the Default Process and Calculation of Market-Implied Default Probabilities

If default is assumed to be a Poisson process driven by a constant intensity of default, then the cumulative default probability is:

$$F(u) = 1 - \exp[-hu],$$

where h is the intensity of default, often described as the hazard rate. The instantaneous default probability is:

$$\frac{dF(u)}{du} = h\exp[-hu].$$

Since $\exp[-hu]$ gives the probability of **no** default before date u, we can interpret h as a forward instantaneous default probability; the probability of default in an infinitely small period dt conditional on no prior default is hdt.

i) Link from hazard rate to credit spread

We will make the assumption that all cashflows are paid continuously which will simplify the exposition. In practice, calculations must account for the precise timing of cashflows (as is done in Spreadsheet 14-2, for example), although the approximations below are reasonably accurate.[28]

The risky value of receiving a continuous stream of cashflows can be written as:

$$\int_0^T B(u)S(u)du,$$

where $B(u)$ is the risk-free discount factor and $S(u) = 1 - F(u)$ is the survival (no default) probability. The above quantity is often called the risky annuity (or risky duration).

[28] For example, CDS premiums are typically paid quarterly in arrears but an accrued premium is paid in the event of default to compensate the protection seller for the period for which a premium has been paid. Hence, the continuous premium assumption is a good approximation.

The value of receiving protection from a credit default swap (CDS) can be represented as:

$$(1-R)\int_0^T B(u)dF(u) = (1-R)h\int_0^T B(u)S(u)du.$$

The fair CDS spread will be the ratio of the value of default protection divided by the risky annuity (the unit cost of paying for the protection) and we can therefore see that

$$Spread = (1-R)h \text{ or } h = \frac{Spread}{1-R}$$

ii) Simple formulas

Suppose we define the risk-free discount factors via a constant continuously compounded interest rate $B(u) = \exp[-ru]$. We then have closed-form expressions for quantities such as the risky annuity:

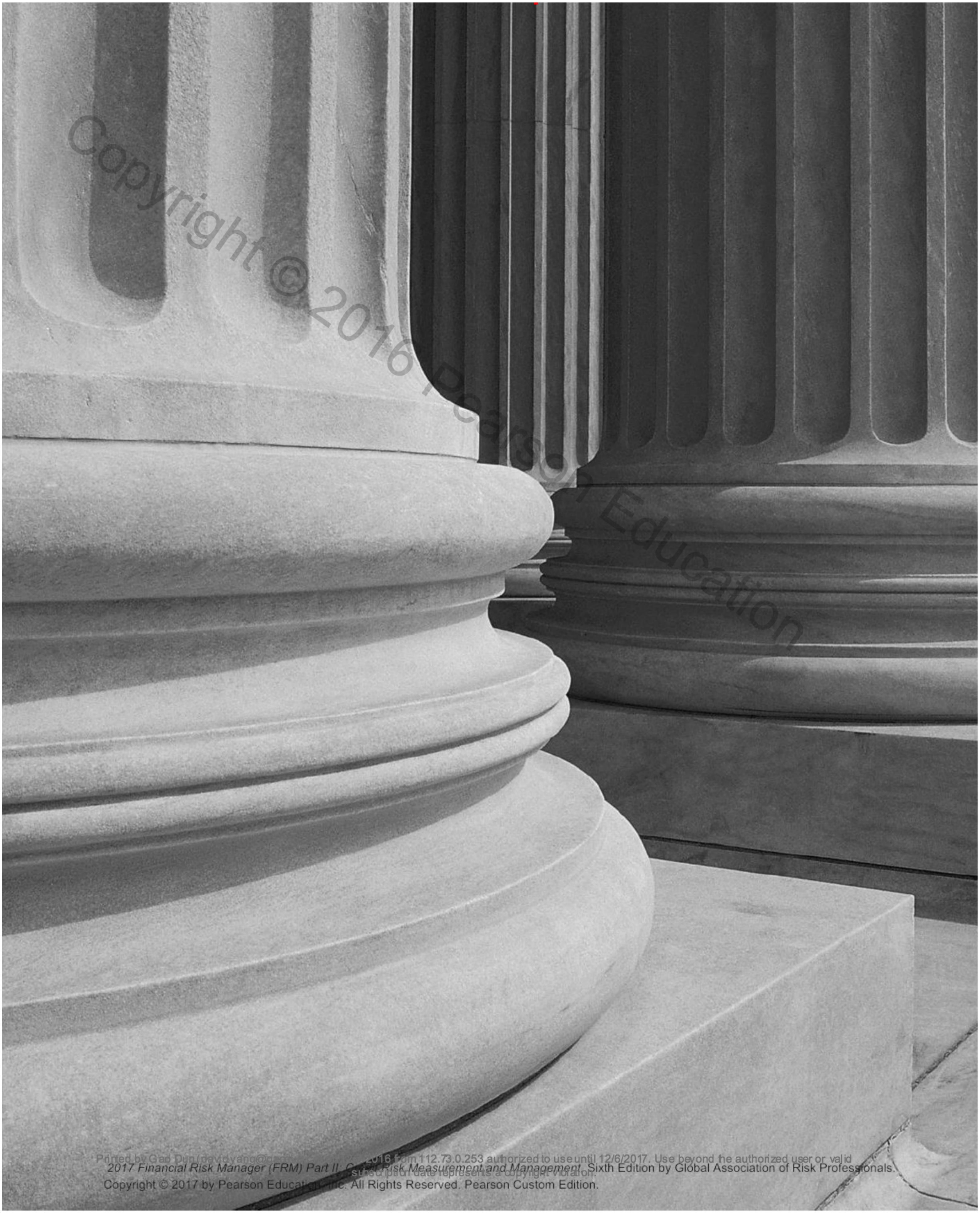
$$\int_t^T \exp\left[-(r+h)u\right]du = \frac{1-\exp\left[-(r+h)u\right]}{r+h}.$$

iii) Incorporating term structure

For a non-constant hazard rate, the survival probability is given by:

$$S(u) = \exp\left[-\int_t^u h(x)dx\right].$$

To allow for a term structure of credit (for example, CDS premiums at different maturities) and indeed a term structure of interest rates, we must choose some functional form for *h*. Such an approach is the credit equivalent of yield curve stripping and was first suggested by Li (1998). The single-name CDS market is mainly based around 5-year instruments and other maturities will be rather illiquid. A standard approach is to choose a piecewise constant representation of the hazard rate to coincide with the maturity dates of the individual CDS quotes. This is illustrated in Spreadsheet 14-2.



Credit Value Adjustment

15

Learning Objectives

After completing this reading you should be able to:

- Explain the motivation for and the challenges of pricing counterparty risk.
- Describe credit value adjustment (CVA).
- Calculate CVA and the CVA spread with no wrong-way risk, netting, or collateralization.
- Evaluate the impact of changes in the credit spread and recovery rate assumptions on CVA.
- Explain how netting can be incorporated into the CVA calculation.
- Define and calculate incremental CVA and marginal CVA, and explain how to convert CVA into a running spread.
- Explain the impact of incorporating collateralization into the CVA calculation.

Excerpt is Chapter 12 of Counterparty Credit Risk and Credit Value Adjustment, *Second Edition, by Jon Gregory.*

> Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.
>
> —Albert Einstein (1879–1955)

The last section focused separately on credit exposure and default probability. Now we proceed to combine these two components in order to address the pricing of counterparty credit risk via CVA.[1] We will see that under certain commonly made assumptions it is relatively simple to combine default probabilities and exposures to arrive at the CVA.

Accurate pricing of counterparty risk involves attaching a value to the risk of all outstanding positions with a given counterparty. This is important in the reporting of accurate earnings information and incentivising trading desks and businesses to trade appropriately. If counterparty risk pricing is combined with a systematic charging of new transactions, then it will also be hedged generated funds that will absorb potential losses in the event that a counterparty defaults. Counterparty risk charges are increasingly commonly associated with hedging costs.

For the purpose of this chapter, we will make three key assumptions that will greatly simplify the initial exposition and calculation of CVA. The key assumptions are:

- *The institution themselves cannot default.* The first assumption corresponds to ignoring the DVA (debt value adjustment) component.
- *Risk-free valuation is straightforward.* We have to assume that the risk-free valuation can be performed. However, this is far from simple due to the lack of a clear discount rate (in the past LIBOR was considered acceptable) and the increased importance of funding.
- *The credit exposure and default probability[2] are independent.* This involves neglecting wrong-way risk, which will be discussed in Chapter 16.

This above separation of concepts should make it easier to explain all the key features around CVA.

Reference papers on the subject of CVA include Sorensen and Bollier (1994), Jarrow and Turnbull (1992, 1995, 1997), Duffie and Huang (1996) and Brigo and Masetti (2005a).

[1] Also sometimes referred to as counterparty value adjustment.

[2] And, also, the recovery value.

DEFINITION OF CVA

Why Pricing CVA Is Not Easy

Pricing the credit risk for an instrument with one-way payments, such as a bond, is relatively straightforward—one simply needs to account for default when discounting the cash flows and add any default payment. However, many derivatives instruments have fixed, floating or contingent cash flows or payments that are made in both directions. This bilateral nature characterises credit exposure and makes the quantification of counterparty risk dramatically more difficult. Whilst this will become clear in the more technical pricing calculations, a simple explanation is provided in Figure 15-1, which compares a bond to a similar swap transaction. In the bond case a given cash flow is fully at risk (its value may be lost entirely) in the event of a default, whereas in the swap case only part of the cash flow will be at risk due to partial cancellation with opposing cash flows. The risk on the swap is clearly smaller due to this effect.[3] However, the fraction of the swap cash flows that are indeed at risk are hard to determine as this

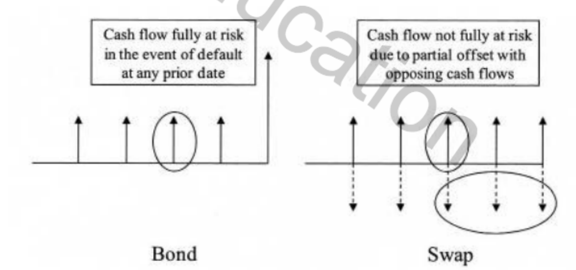


FIGURE 15-1 Illustration of the complexity when pricing the credit (counterparty) risk on a derivative instrument such as a swap, compared with an instrument such as a bond.

In the bond, the cash flow circled is fully at risk (less recovery) in the event of default of the issuer but in the swap the equivalent cash flow is not fully at risk due to the ability to partially offset it with current and future cash flows in the opposite direction (the three dotted cash flows shown circled).

[3] It is also smaller due to the lack of a principal payment, but this is a different point.

depends on many factors such as yield curve shape, forward rates and volatilities.

CVA Formula

We first define the formula for calculating CVA and will discuss after this the motivation and precise use of CVA within an institution. When valuing a financial transaction such as an OTC derivative or repo, counterparty risk must be included. However, it is possible to separate the components according to

$$\text{Risky value} = \text{risk-free value} - \text{CVA}. \qquad \textbf{(15.1)}$$

The above separation is theoretically rigorous, and also extremely useful because the problem of valuing a transaction and computing its counterparty risk can be completely separated. The first implication of this is that it is possible to deal with all CVA components centrally and "transfer price" this away from the originating trader or business. This is critical since it allows separation of responsibilities within a financial institution: one desk is responsible for risk-free valuation and one for the counterparty risk component. Transactions and their associated counterparty risk may then be priced and risk-managed separately. Therefore, for example, a swap trader in a bank need not understand how to price and hedge CVAs[4] as this will be handled by the bank's "CVA desk" who will charge the appropriate CVA for the trade in question.

If this sounds too good to be true, there is a hidden complexity in the seemingly simple Equation (15.1) which is that it is not linear. Due to risk mitigants such as netting and collateral, CVA is not additive with respect to individual transactions. This means that the risky value of a given transaction cannot be calculated individually, as it is defined with respect to other transactions within the *same* netting set. We will therefore have to consider the allocation of CVA just as we considered allocation of exposure in Chapter 13.

[4] Indeed, the trader need know nothing whatsoever about CVA although, since CVA is a charge to their PnL, it is likely they will want at least a basic understanding of what CVA is and how it is calculated.

Nevertheless, under the above assumptions, a standard equation for CVA is

$$\text{CVA} \approx (1 - \text{Rec}) \sum_{i=1}^{m} \text{DF}(t_i)\text{EE}(t_i)\text{PD}(t_{i-1}, t_i). \qquad \textbf{(15.2)}$$

The CVA depends on the following components:

- *Loss given default (1 − Rec).* In the event of counterparty default, some percentage amount of the claim would be recovered; this is the percentage amount of the exposure expected to be lost if the counterparty defaults. Note that LGD = 1 − Rec.
- *Discount factor (DF).* This is the relevant risk-free discount factor. Discounting is relevant since any future losses must be discounted back to the current time.
- *Expected exposure (EE).* The term is the expected exposure (EE) for the relevant dates in the future given by t_i for $i = 0, n$. We will discuss the need to use risk-neutral exposures later.
- *Default probability (PD).* The expression requires the marginal default probability in the interval between date t_{i-1} and t_i. Default probability estimation was covered in Chapter 14.

It should not be a surprise that CVA involves default probability (how likely is the counterparty to default), EE (what is expected to be lost in default) and recovery (what will be recovered). It should also not be a surprise that the formula has a time dimension, since EE and PD can be shown to be rather time inhomogeneous. The formula therefore must integrate over time to take into account the precise distribution of EE and PD (and not just their average values). An illustration of the CVA formula is given in Figure 15-2.

Hence, CVA simply depends on combining components from potentially different sources. For example, an exposure team within a financial institution may compute EE, which is a market risk. The credit department and/or

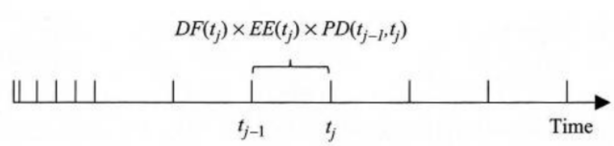


FIGURE 15-2 Illustration of CVA formula.

The component shown is the CVA contribution for a given interval. The formula simply sums up across all intervals and multiplies by the loss given default.

credit derivatives trading desk may provide loss given default and default probability information. Crucially, none of the areas needs to be aware of what the other is doing, as all the components are assumed independent.

A further important advantage of computing CVA via Equation (15.2) is that default enters the expression via default *probability* only. This means that, whilst one may require a simulation framework in order to compute CVA, it is not necessary to simulate default events, only the exposure (EE). This saves significantly on computation time by avoiding the need to simulate relatively rare default events.

SPREADSHEET 15-1

CVA Calculations

To download Spreadsheet 15-1, visit http://www.cvacentral.com/books/credit-value-adjustment/spreadsheets and click Chapter 12 exercises.

We illustrate the above CVA formula with a simple example of a forward contract-type exposure[5] using the simple expression from Chapter 13 and a default probability defined by Equation (14.5). We assume a constant credit spread of 500 bps, a recovery value of 40% and a constant continuously compounded interest rate of 5%.[6] We assume an interval of 0.25 years between the dates in Equation (15.2), which involves evaluation at a total of 20 points. With these assumptions, the expected exposure and marginal default probability are as shown in Figure 15-3. The CVA is calculated to be 0.262%, which is expressed in terms of percentage of notional value (since the EE was expressed in percentage terms).

In terms of the accuracy of the integration, the exact result is 0.254%. One can improve the efficiency by choosing more than 20 points. However, it is also best to approximate the exposure and discount by the average of those at the beginning and end of the period, i.e., $EE(t_j) \rightarrow (EE(t_{j-1}) + EE(t_j))/2$ and $DF(t_j) \rightarrow (DF(t_{j-1}) + DF(t_j))/2$. This gives a more accurate result of 0.253% with the 20 points used above.

We emphasise that, under the assumption of no wrong-way risk, Equation (15.2) provides a very efficient way to compute CVA from components that may already be calculated by a financial institution (exposures, default probabilities, discount factors and loss given default). Historically, for many institutions this route has been a very important way to price counterparty risk in a realistic and practical way.

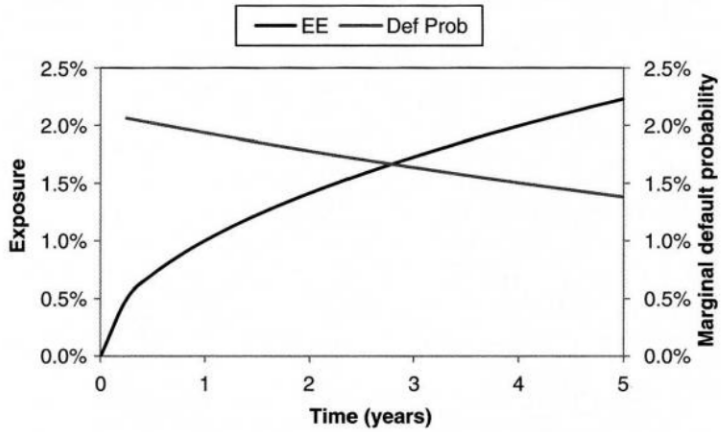


FIGURE 15-3 Illustration of the expected exposure and default probability for the example CVA calculation.

CVA as a Spread

Suppose that instead of computing the CVA as a standalone value, one wanted it to be expressed as a spread (per annum charge). In Appendix 15 we derive an approximate formula for CVA that will be at least of intuitive interest and will also help in expressing CVA as a running spread. The formula assumes that the EE is constant over time and equal to its average value (EPE). This yields the following approximation based on EPE:

$$\text{CVA} = \text{credit spread} \times \text{EPE}, \tag{15.3}$$

where the CVA is expressed in the same units as the credit spread, which should be for the maturity of the instrument in question, and EPE is as defined in Chapter 13.[7] For the example above, the EPE is 1.54% and therefore the CVA approximation is 1.54% × 500 = 7.71 bps.

[5] The expected exposure is given by $EE(t) = \sqrt{t} \times 1\%$ as a percentage of notional.

[6] This means the discount factors are given by $DF(t) = \exp(-5\% \times t)$.

[7] This is the simple average of the EE values in our example, although for non-equal time intervals it would be the weighted average. Discounting is not included in the EPE based on the assumptions used in deriving the approximate formula.

A simple calculation would involve dividing the CVA by the risky annuity[8] value for the maturity in question. For the previous calculation, a risky annuity of 3.65 would be obtained using the simple formula described in Appendix 14B (the accurate result for an interval of 0.25 years is 3.59). From the result above, we would therefore obtain the CVA as a spread, being $0.253\%/3.65 \times 10{,}000 =$ 6.92 bps (per annum).

The approximate calculation works reasonably well in this case. The simple formula is an overestimate because, whilst the EE profile is certainly not constant as assumed, the marginal default probabilities are reasonably constant. This approximate formula tends to be more accurate for swap-like profiles where the symmetry of the profile helps but is less accurate for monotonically increasing profiles such as the one used in the example above.

The approximate formula in Equation (15.2) is often not used for actual calculations but can be useful for intuitive understanding of the drivers of CVA. As counterparty risk became a common component of derivatives transactions from the late 1990s onwards, the above method of representing CVA would be rather common. For example, a bank might tell a corporate client that they would have to pay an extra *X* bps on a swap to cover the "credit charge" or CVA. The simple formula allows the charge to be broken down into the credit component (the credit spread of the counterparty in question) and the market risk component (the exposure, or EPE, in question).

CVA AND EXPOSURE

We have discussed in detail how to quantify exposure, which covers the EE term in Equation (15.2). Institutions may commonly take EE values from a risk management system, even though that system may have been set up for monitoring credit lines and not computing CVA. However, there is one caveat. For quantifying exposure for risk management, one should use the real probability measure whereas for pricing purposes the risk-neutral measure should be used. The use of the risk-neutral versus real probability measure is an important point. We now discuss some aspects of exposure, which relate to the potential need to calculate risk-neutral exposure for CVA purposes.

Exposure and Discounting

In the above, we consider a separate discount factor in order to discount future losses to today, and arrive at a price (the CVA). It is reasonable to do this as long as the exposure is calculated in the correct fashion. A problem could arise, for example, in an interest rate product where, when rates are high a larger discount factor should be used, and vice versa. This convexity effect would mean that we would overestimate the CVA of a payer swap and vice versa for a receiver swap.[9] To solve this problem technically means quantifying the underlying exposure using the "T-forward measure" (Jamshidian, 1997). By doing this, discount factors depend on expected future interest rate values, not on their distribution. Hence, moving the discount factor out of the expectation term (for exposure) is theoretically correct.

Working with separate discount factors may sometimes be convenient. For example, the approximation in Equation (15.2) works only if discounting is done separately.[10] However, often expected exposure for CVA purposes will be discounted during the simulation process.

Risk-Neutral Exposure

For CVA, it may be relevant to calculate a risk-neutral exposure rather than the real-world exposures characterised in Chapter 13. This requires calibration to market, rather than historical, data. For example, interest rate volatilities and mean-reversion parameters would be derived from the prices of interest rate swaptions, caps and floors rather than estimation via historical time series. In addition, the drift of the underlying variables (such as interest rates and FX rates) will need to be calibrated to forward rates, rather than coming from some

[8] The risky annuity represents the value of receiving a unit amount in each period as long as the counterparty does not default.

[9] Since a payer swap has the largest exposure when rates are high and these paths would be discounted according to a lower rate.

[10] In other words, the EPE in Equation (15.2) does not contain any discounting effects.

historical or other real-world analysis.[11] Hence, in terms of risk-neutral exposure, there are two effects to consider which arise from the impact of different volatility and drift assumptions.

We first consider the drift impact on exposure. Taking the base case interest rate swap (Payer IRS GBP 5Y[12]), we compute the expected exposure using the risk-neutral drift (i.e., that implied from the shape of the interest rate curve observed in the market) to compute with the original case, which uses a historical drift. The results are shown in Figure 15-4. Note that, in order to isolate the drift impact, historical volatility is used in both cases.

In this example, the real-world EE is smaller than the NEE due to a negative interest rate "drift" calibrated from historical data. Since the interest rate curve is upwards-sloping (long-term interest rates are higher than short-term rates), the risk-neutral drift is positive, leading to the EE being higher than the NEE. Hence, the difference between using risk-neutral and real-world drift is to "twist" the exposure distribution, so that the risk-neutral EE is greater and the NEE is smaller, compared with the real-world values.

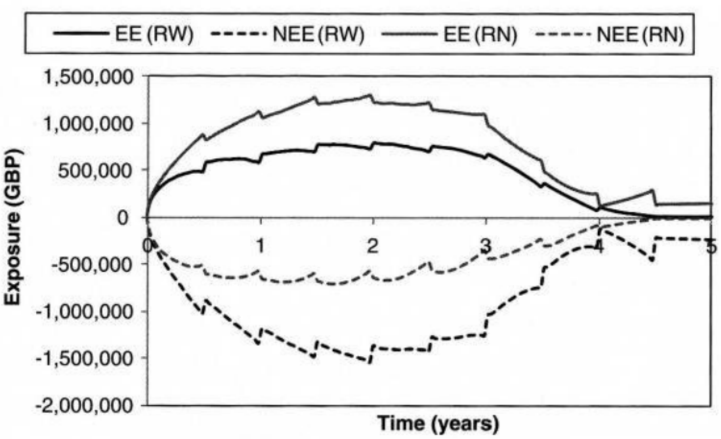


FIGURE 15-4 Illustration of the EE and PFE of a 5-year interest rate swap paying fixed GBP (notional 100m) and receiving floating GBP computed with both real-world (RW) and risk-neutral (RN) simulations.

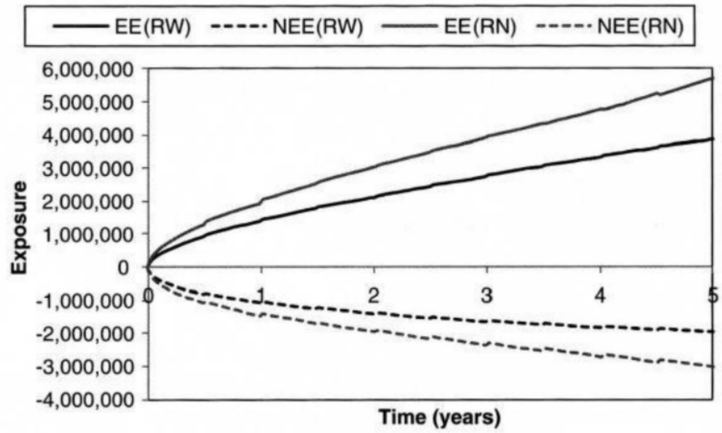


FIGURE 15-5 Illustration of the EE of a 5-year cross-currency swap paying GBP (notional 25m) and receiving USD computed with both real-world (RW) and risk-neutral (RN) simulations.

Now we illustrate the role of volatility. In Figure 15-5, we show the expected exposure of the cross-currency swap described in Chapter 13 under both real-world (historical volatility and drift as discussed in Chapter 13) and risk-neutral (market-implied volatility and drift implied from forward rates) assumptions. Here the main impact is simply that risk-neutral volatilities tend to be higher than real-world ones and hence both the PFE and EE are bigger.

It is important to consider that the higher risk-neutral exposure in this case may be an indication that the market is pricing in a higher level of volatility than is estimated from a real-world (e.g., historical) analysis. In this case, the risk-neutral exposure may be argued to be a superior measure to the real-world one since it represents the future view and not the view of the past. On the other hand, the risk-neutral exposure may simply be systematically higher due to the well-known presence of risk premiums in market parameters.

CVA Semi-Analytical Methods

In the case of some specific product types, it is possible to derive analytical formulas for the CVA. Whilst such formulas are of limited use since they do not account for netting or collateral, they are valuable for quick calculations and an intuitive understanding of CVA.

The first simple example is the CVA of a position that can only have a positive value, such as a long option

[11] Risk-neutral drift may often be used anyway for calculating exposure for risk management purposes.

[12] Payer interest rate swap in GBP with a 5-year maturity and 100m notional.

position with an upfront premium. In this situation, it is possible to show (Appendix 15) that the CVA is simply

$$\text{CVA} \approx \text{LGD} \times F(T) \times V, \quad \textbf{(15.4)}$$

where T is the maturity of the transaction in question and V is its (risk-free) valuation. The term $F(T)$ represents the probability that the counterparty will default during the lifetime of the transaction in question. It is intuitive that one simply multiplies the standard risk-free price by this default probability and corrects for the recovery value.

Moving on to contracts that can have both positive and negative value, the calculation of the CVA of an interest rate swap is considered by Sorensen and Bollier (1994). These authors show that the CVA in this case can be expressed as a function of (reverse) swaptions with different exercise dates. The intuition is that the counterparty might default at any time in the future and, hence, effectively cancel the non-recovered value of the swap, economically equivalent to exercising the reverse swaption.

The swap exposure and swaption analogy is illustrated in Figure 15-6. The expected exposure of the swap will be defined by the interaction between two factors: the swaption payoff and the underlying swap duration (these are the two components in the simple approach given in Equation (13.4)). These quantities respectively increase and decrease monotonically over time. The overall swaption value therefore peaks at an intermediate point.

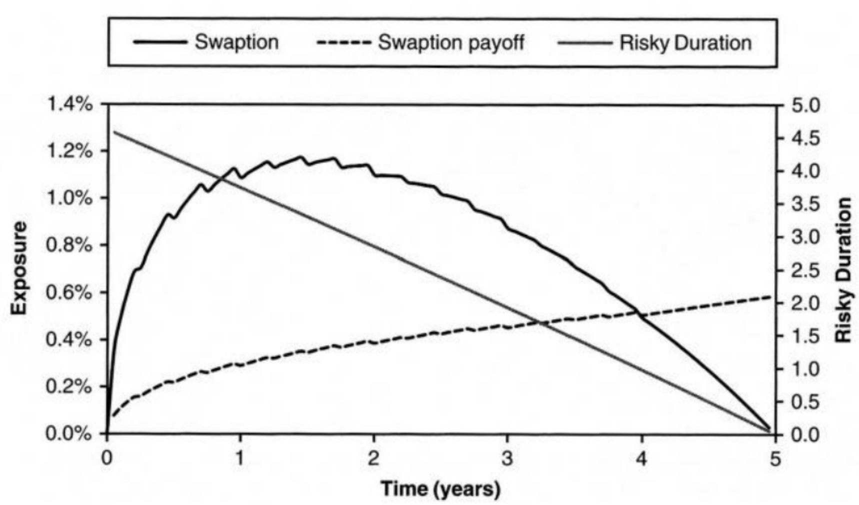


FIGURE 15-6 Illustration of swap EE as defined by swaption values which are given by the product of the swaption payoff and the risky duration value (shown on the secondary y-axis).

SPREADSHEET 15-2

Semi-Analytical Swap CVA

To download Spreadsheet 15-2, visit http://www.cvacentral.com/books/credit-value-adjustment/spreadsheets and click Chapter 12 exercises.

The Sorensen and Bollier formula gives us a very useful insight on CVA calculations, specifically that a CVA calculation will be at least as complex as pricing the underlying product itself. To price the swap CVA, one needs to know about swaption volatility (across time and strike), components far beyond those needed to price the swap itself. The value of the swap does not depend significantly on volatility and yet the CVA for the swap does.

This approach naturally captures effects such as the asymmetry between payer and receiver swap (Figure 15-7) and unequal payment frequencies such as in basis swap (Figure 15-8). In the former case, the receiver (payer) swaptions corresponding to the payer (receiver) swap are in-(out-)of-the-money. In the latter case, the strike of the swaptions moves significantly out-of-the-money when an institution receives a quarterly cash flow whilst not needing (yet) to make a semi-annual one.

The above analogy can be extended to other products where any transaction can be represented as a series of European options. This approach would be the method of choice for evaluating the CVA of a single trade. In some circumstances it can also be extended beyond the single trade level to, for example, a portfolio of single currency swaps as discussed by Brigo and Masetti (2005b). The ability to do this may often be useful, as clients may trade a rather narrow range of underlying products, the exposure of which may be modelled analytically. However, multidimensional netting sets will typically need to be treated in a more generic Monte Carlo-style approach.

IMPACT OF DEFAULT PROBABILITY AND RECOVERY

We now consider the impact of default probability and recovery on CVA. There are several aspects to consider, such as the level

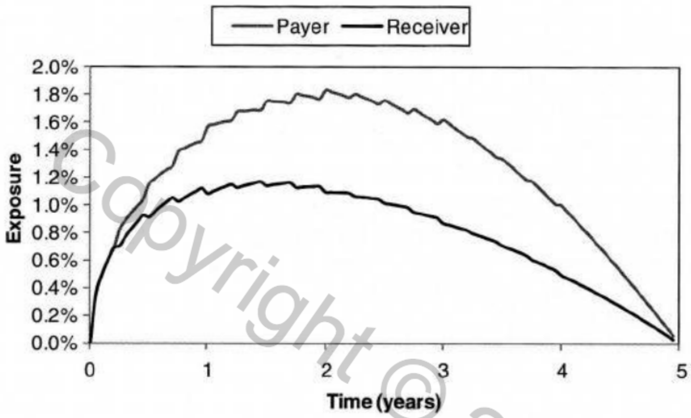


FIGURE 15-7 Illustration of swap EE for payer and receiver swaps as defined by swaption values.

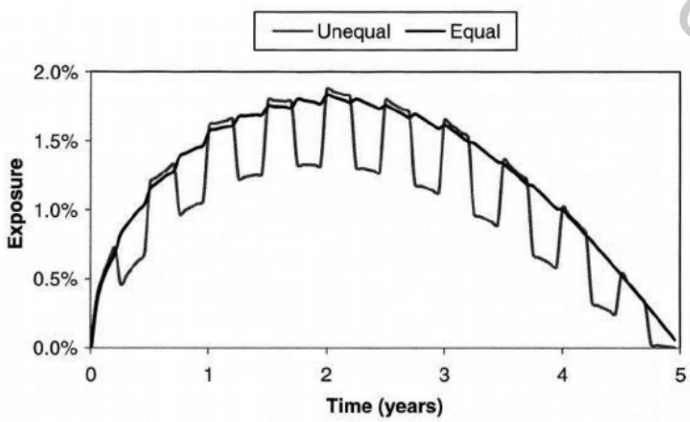


FIGURE 15-8 Illustration of swap EE for unequal (pay semi-annually, receive quarterly) swaps as defined by swaption values.

of credit spreads, the overall shape of the credit curve, the impact of recovery rates and the basis risk arising from recovery rate assumptions. In all the examples below, we will consider the CVA of the same 5-year GBP payer interest rate swap.[13] The base case assumptions will be a flat credit curve of 500 bps and a recovery rate of 40%. The base case CVA is then calculated to be £91,389.

Credit Spread Impact

Let us first review the impact of increasing the credit spread of the counterparty in Table 15-1. The increase in credit spread clearly increases the CVA, but this effect is not linear since default probabilities are bounded by 100%. Another way to understand this is that the "jump to default" risk[14] of this swap is zero, since it has a current value of zero and so an immediate default of the counterparty will not cause any loss. As the credit quality of the counterparty deteriorates, the CVA will obviously increase but at some point, when the counterparty is very close to default, the CVA will decrease again.

Next, we look at the impact of changes in *shape* of the credit curve. In Chapter 14 (e.g., Figure 14-8), we considered upwards-sloping, flat and inverted credit curves all of which assumed a terminal 5-year credit spread of 500 bps. We discussed how, whilst they gave cumulative default probabilities that were approximately the same, the marginal default probabilities differed substantially. For a flat curve, default probability is approximately equally spaced whilst for an upwards (downwards)-sloping curve, defaults are back (front) loaded. We show

TABLE 15-1 CVA of the Base Case IRS as a Function of the Credit Spread of the Counterparty

Spread (bps)	CVA (GBP)
100	20,915
250	49,929
500	92,593
750	129,004
1000	160,033
10,000	289,190
25,000	224,440
50,000	180,455
Default	0

[13] We note that these are not risk-neutral but allow a direct comparison with previous results.

[14] This term is generally used to mean a sudden and immediate default of the counterparty with no other factors changing.

TABLE 15-2 CVA of the Base Case IRS for Different Shapes of Credit Curve. The 5-year credit spread is 500 bps in all cases.

	CVA (GBP)
Upwards-sloping	84,752
Flat	92,593
Downwards-sloping	94,358

TABLE 15-3 CVA of the Base Case IRS for Different Recovery Assumptions. Simultaneous changes in the settled and final recovery ("both") and a 10% settled recovery and 40% final recovery are shown.

Recovery	CVA (GBP)
20% both	96,136
40% both	92,595
60% both	86,003
10%/40%	64,904

the impact of curve shape on the CVA in Table 15-2. Even though the spread at the maturity of the swap (5Y) is the same in all cases, there are quite different results for the different curve shapes. Indeed, going from an upwards- to a downwards-sloping curve increases the CVA by 11%. We note that for EE profiles that are monotonic, such as forward contracts and cross-currency swaps, this impact is typically stronger (for example, for the case represented in Figure 15-3 the corresponding increase is 40%[15]). This illustrates why we emphasised the shape of the credit curve as being an important part of the mapping process.

Recovery Impact

Table 15-3 shows the impact of changing settled and actual recoveries. Recall (Figure 14.11) that the settled recovery is the recovery at the time of default (for example, settled in the CDS auction) whilst the actual recovery is the amount that will actually be received for the claim (i.e., used in Equation (15.2)). Changing both recovery rate assumptions has a reasonably small impact on the CVA since there is a cancellation effect: increasing recovery increases the implied default probability but reduces the resulting loss. Indeed, the simple approximation in Equation (15.3) has no recovery input. The net impact is only a second-order effect, which is negative with increasing recovery, because the implied default probability increase is sub-linear in recovery, but the loss amount is linear. Different assumptions for settled and actual recovery rates will obviously change the CVA more significantly. For example, assuming a 10% recovery for calculating implied default probabilities and a higher 40% actual recovery (similar to Lehman Brother values) gives a much lower CVA.

[15] This is because for such profiles, the maximum exposure occurs at the end of the contract and for a sufficiently upwards-sloping curve, this is also where the maximum default probability occurs. The combination of these two aspects gives a high CVA.

PRICING NEW TRADES USING CVA

Being able to price the stand-alone CVA on a given transaction is useful, but the need to account for risk mitigation such as netting and collateral is critical for any practical use of CVA.

Netting and Incremental CVA

When there is a netting agreement then the impact is likely to reduce the CVA and cannot increase it (this arises from the properties of netting). We therefore know that for a set of netted trades (*NS*):

$$\text{CVA}_{NS} \leq \sum_{i=1}^{n} \text{CVA}_i^{\text{stand-alone}}, \qquad \textbf{(15.5)}$$

where CVA_{NS} is the total CVA of all trades under the netting agreement and $\text{CVA}_i^{\text{stand-alone}}$ is the stand-alone CVA for trade *i*. The above reduction can be substantial and the question then becomes how to allocate the netting benefits to each individual transaction. The most obvious way to do this is to use the concept of *incremental CVA*, analogous to incremental EE. Here the CVA of a transaction *i* is calculated based on the incremental effect this trade has on the netting set:

$$\text{CVA}_i^{\text{incremental}} = \text{CVA}_{NS+i} - \text{CVA}_{NS}. \qquad \textbf{(15.6)}$$

The above formula ensures that the CVA of a given trade is given by its contribution to the overall CVA at the time it is executed. Hence, it makes the most sense when the CVA needs to be charged to individual traders and businesses. The CVA depends on the order in which trades are executed but does not change due to subsequent trades. A CVA desk charging this amount will directly offset the impact on their PnL from the change in CVA from the new trade.

We can derive the following formula for incremental CVA:

$$\mathrm{CVA}_i^{\text{incremental}} = (1 - \mathrm{Rec}) \sum_{j=1}^{m} \mathrm{DF}(t_j)\mathrm{EE}_i^{\text{incremental}}(t_{j-1}, t_j) \quad \mathrm{PD}(t_{j-1}, t_j). \qquad \textbf{(15.7)}$$

This is the same as Equation (15.2), but with the incremental EE replacing the previous stand-alone EE. This should not be surprising since CVA is a linear combination of EE, and netting changes only the exposure and has no impact on recovery values, discount factors or default probabilities.[16] Incremental EE can be negative, due to beneficial netting effects, which will lead to a CVA being negative and, in such a case, it would be possible to transact at a loss due to the overall gain from CVA.

It is worth emphasising, in the relationship defined above, that, due to the properties of EE and netting, the incremental CVA in the presence of netting will never be higher than the stand-alone CVA without netting (except in bilateral CVA cases—see also Duffie and Huang, 1996). The practical result of this is that an institution with existing trades under a netting agreement will be likely to offer conditions that are more favourable to a counterparty with respect to a new trade. Cooper and Mello (1991) first quantified such an impact, showing specifically that a bank that already has a trade with a counterparty can offer a more competitive rate on a forward contract.

The treatment of netting makes the treatment of CVA a complex and often multidimensional problem. Whilst some attempts have been made at handling netting analytically (e.g., Brigo and Masetti, 2005b as noted earlier), CVA calculations incorporating netting typically require a general Monte Carlo simulation for exposure (EE) quantification. However, note that under Equation (15.7), one does not have to simulate default events as mentioned before.

[16] Although we note again that the use of risk-neutral exposure may be considered relevant for CVA purposes.

We will now look at an example of incremental CVA following the previous results for incremental exposure. As before, we consider a 5-year GBP payer interest rate swap (Payer IRS GBP 5Y) and in Table 15-4 consider the CVA under the assumption of four different existing trades with the counterparty.

We can make the following observations:

- The incremental CVA is never higher than the stand-alone CVA (which assumes no netting benefit due to existing trades). This is not surprising since netting could not increase exposure.
- The incremental CVA is only slightly reduced for a very similar existing trade (6-year GBP swap). This follows from the high positive correlation between the two trades.
- The incremental CVA is reduced moderately in the case of a similar swap in a different currency. This is since the trades are still positively correlated.
- The incremental CVA is negative in the last two cases due to the structurally negative correlation. A trader may therefore expect a positive P&L in this situation due to reducing the overall risk to the counterparty in question and may therefore execute a trade with otherwise unfavourable terms.

TABLE 15-4 Incremental CVA Calculations for a 5-Year GBP Swap Paying Fixed (Payer IRS GBP 5Y) with Respect to Four Different Existing Transactions and Compared to the Stand-alone Value. The credit curve is assumed flat at 500 bps with a 40% recovery rate and continuously compounded interest rates of 5% are used.

Existing Trade	Incremental CVA (GBP)
None (stand-alone calculation)	92,593
Payer IRS GBP 6Y	90,076
Payer IRS EUR 5Y	63,832
Receiver IRS EUR 5Y	−42,446
CCS GBPUSD 5Y	−35,801

TABLE 15-5 Illustration of the Breakdown of the CVA of the Interest Rate and Cross-Currency Swap via Incremental (CCS first), Incremental (IRS first) and Marginal. The credit curve is assumed flat or upwards-sloping, recovery rates are 40% and continuously compounded interest rates are 5%.

	Flat Credit Curve			Upwards-Sloping Credit Curve		
	Incremental (IRS first)	**Incremental (CCS first)**	**Marginal**	**Incremental (IRS first)**	**Incremental (CCS first)**	**Marginal**
IRS	92,593	27,133	71,178	84,752	18,995	59,580
CCS	34,098	99,558	55,513	48,902	114,660	74,075
Total	**126,691**	**126,691**	**126,691**	**133,655**	**133,655**	**133,655**

Marginal CVA

We can define *marginal CVA* by simply including the marginal EE in the above formula. Marginal CVA may be useful to break down a CVA for any number of netted trades into trade-level contributions that sum to the total CVA. Whilst it might not be used for pricing new transactions (due to the problem that marginal CVA changes when new trades are executed, implying PnL adjustment to trading books), it may be required for pricing trades transacted at the same time[17] (perhaps due to being part of the same deal) with a given counterparty. Alternatively, marginal CVA is the appropriate way to calculate the trade-level CVA contributions at a given time. This may be useful where a CVA desk is concerned about their exposure to the default of a particular counterparty.

We compute the marginal CVA corresponding to the marginal EE of the interest rate swap (Payer IRS GBP 5Y) and the cross-currency swap (CCS GBPUSD 5Y). We do this for two different credit curves, one flat at 500 bps and one having the form [300 bps, 350 bps, 400 bps, 450 bps, 500 bps] for maturities [1Y, 2Y, 3Y, 4Y, 5Y]. The results are shown in Table 15-5.

We see the effect that the first trade is charged for the majority of the CVA, as seen before, whilst the marginal CVA charges are more balanced. Notice also that, whilst the overall CVA is not changed by much, the breakdown of CVA changes significantly for a differently shaped credit curve. For example, the marginal contribution of the CCS is significantly lower with a flat curve and significantly higher with an upwards-sloping curve. This is because most of the contribution from the CCS to marginal EE comes in the last year of the lifetime, which is where the upwards-sloping curve has the highest default probability.

There are some important practical points to understand when incorporating CVA into trades. We start by looking at various CVA decompositions for the four trades in Table 15-6. It can be seen that incremental CVA depends very much on the ordering of the trades. For example, the incremental CVA of the CCS can be almost 20 times smaller if it is the last and not the first trade to be executed. Clearly, the amount of CVA charged can be very dependent on the timing of the trade. This may be problematic and could possibly lead to "gaming" behaviour by traders. However, whilst the marginal contributions are fair, it is hard to imagine how to get around the problems of charging traders and businesses based on marginal contributions that change as new trades are executed with the counterparty.

CVA as a Spread

Another point to consider when pricing CVA into trades is how to convert an upfront CVA to a running spread CVA. This would facilitate charging a CVA to a client via, for example, adjusting the rate paid on a swap. One simple way to do such a transformation would be to divide the

[17] This could also cover a policy where CVA adjustments are only calculated periodically and several trades have occurred with a given counterparty within that period.

TABLE 15-6 Illustration of the Breakdown of the CVA for Four Trades via Incremental (the ordering of trades given in brackets) and Marginal Contributions. The credit curve is assumed flat at 500 bps, recovery rates are 40% and continuously compounded interest rates are 5%.

	Stand-Alone	Incremental (1-2-3-4)	Incremental (4-1-2-3)	Marginal
Payer IRS GBP 5Y	92,593	92,593	27,133	84,011
Payer IRS GBP 6Y	124,816	122,299	95,520	107,995
Payer IRS EUR 5Y	76,006	37,191	35,694	45,286
CCS GBPUSD 5Y	99,558	5,822	99,558	20,613
Total	**392,973**	**257,905**	**257,905**	**257,905**

CVA by the risky duration for the maturity in question.[18] For example, for the 5Y GBP IRS above (notional 100m), for the stand-alone CVA, we would obtain:

$$\frac{92{,}593}{3.59 \times 100{,}000{,}000} \times 10{,}000 = 2.58 \text{ bps.} \quad \textbf{(15.8)}$$

However, when adding a spread to a contract such as a swap, the problem is non-linear since the spread itself will have an impact on the CVA. The correct value should be calculated recursively (since the spread will be risky also) until the risky MtM of the contract is zero. Hence, we need to solve an equation $V(C^*) = \text{CVA}(C^*)$, where $V(.)$ is the value of the contract for the adjusted rate C^*. This would ensure that the initial value perfectly offsets the CVA and hence C^* is a minimum hurdle for the trade to be profitable. In this case, for the accurate calculation, the relevant spread is 2.34 bps. Obviously, calculating this spread quickly can be an important component. Vrins and Gregory (2011) consider this effect (including the impact of netting and DVA) and show that it is significant in many cases. There are also accurate approximations for computing the correct spread without the need for a recursive solution.[19]

[18] An even simpler way is to use the approximation from Equation (15.3) although, as described above, this can be quite inaccurate in some cases.

[19] Indeed. Vrins and Gregory (2011) show that it is trivial to bound the true spread, which in the example given leads to a range of 2.19–2.67 bps. They also give a reasonably close approximation, which leads to an accurate estimate of 2.36 bps. None of these results requires any additional CVA calculations, as would be required in a full recursive solution.

Another point to emphasise is that the benefit of netting seen in the incremental CVA of a new trade depends also on the relative size of the new transaction. As the transaction size increases, the netting benefit is lost and the CVA will approach the stand-alone value. This is illustrated in Figure 15-9, which shows the incremental CVA of the 5-year IRS EUR payer examined as a function of the relative size of this new transaction. We assume that the existing trades are the other three shown in Table 15-6. The stand-alone and standard incremental CVA values are 76,006 and 35,694,[20] which can be converted approximately into running spreads as in Equation (15.8), giving 1.77 bps and 0.99 bps respectively. For a smaller transaction, the CVA decreases to a lower limit of 0.67 bps whereas for a large transaction size, it approaches the stand-alone value. Clearly, a CVA quote in basis points is only valid for a particular transaction size.

Numerical Issues

Calculating CVA on exotic derivatives can be highly challenging, which is not surprising due to the previous intuition that calculating the CVA on a product is at least as complex (and often more complex) as pricing the product itself. Valuation of exotic products can be rather slow, requiring Monte Carlo or lattice-based modelling. Since each EE value required for calculating CVA requires a rather large number of simulations, this will probably be

[20] This can be seen from the 4-1-2-3 scenario where this trade is considered after the other three.

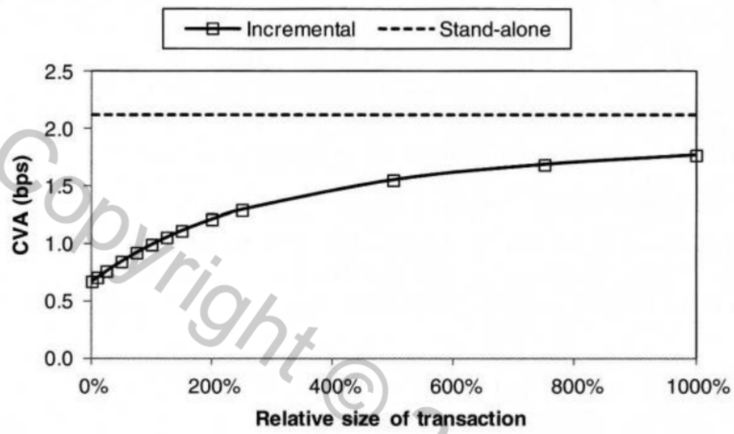


FIGURE 15-9 Incremental CVA (as a spread in basis points per annum) for a 5-year EUR swap paying fixed (Payer IRS EUR 5Y) with respect to the other three trades in Table 15-6.

beyond realistic computational resources. Many pricing functions[21] used by traders may be inadequate to calculate EE.

The CVA calculation as represented by Equation (15.2) is costly due to the large number of calculations of the future value of the underlying trade(s). For example, in the above calculations (as described in Chapter 14), there are 10,000 simulations and 183 time points (representing a point every 10 calendar days for 5 years). This means that all the above CVA estimates are based on 1.83m pricing calls. This is likely to be the bottleneck of the CVA calculation, and the first and most obvious method for improving the efficiency of the CVA calculation will be to speed up the underlying pricing functionality. There are many methods that may achieve this, such as (see also discussion below on exotics):

- Stripping out common functionality (such as cash flow generation and fixings), which does not depend on the underlying market variables at a given point in time.
- Numerical optimisation of pricing functions.
- Use of approximations or grids.
- Parallelisation.

[21] Exotic products in this context could imply any product that does not admit a very simple pricing formula (such as a swap or simple option).

Another aspect to consider when computing CVA is whether to use pathwise or direct simulation. Whilst, for exposure, evaluation of pathwise simulations would seem to be best, it is not clearly the case for CVA. A parallel can be drawn here to pricing synthetic CDOs, which is a similar problem as it involves integration over defaults. Here, practitioners have favoured approaches that simulate defaults directly via the well-known Gaussian copula default time model attributed to Li (2000) rather than, for example, the pathwise default simulation approach of Hull et. al. (2004). In other words, whilst the evaluation of exposure does not favour a direct simulation approach, the evaluation of the default component in CVA *does*.

We consider the above idea by comparing the evaluation of the CVA of the 5Y GBP IRS above with a similar calculation based on a direct simulation approach. In the former case, we have 10,000 paths for the exposure at a total of 183 time steps. In the latter approach, there is no time grid and, instead, default times are drawn randomly in the interval [0, 5Y]. The approach of Li (2000) allows this to be done in a way that is consistent with the underlying cumulative default probability. The exposure is then calculated at each of these points directly. A total of 1.83m default times are generated, so that the number of swap evaluations is the same as in the pathwise case.

The comparison of the CVA estimates is given in Figure 15-10, with error bars representing one standard deviation of uncertainty. We can see that the direct simulation approach is much more accurate for CVA than the pathwise approach for the same number of underlying pricing calls. The reason that the pathwise method is less accurate can be understood as follows. Suppose we generate 10,000 paths that overestimate the interest rate at one year in the future (in other words, due to Monte Carlo noise the average interest rate in the simulation is slightly too high). Then we will tend to overestimate the exposure of the payer swap at this point. However, this is likely to overestimate the exposure at, for example, 18 months, since the interest rate paths six months later are more likely to be positively biased. In the direct simulation approach, this is not a problem since all the default times are drawn independently.

The improvement above is quite dramatic, with the standard deviation 9.7 times smaller in the direct approach. Since Monte Carlo error is approximately proportional to

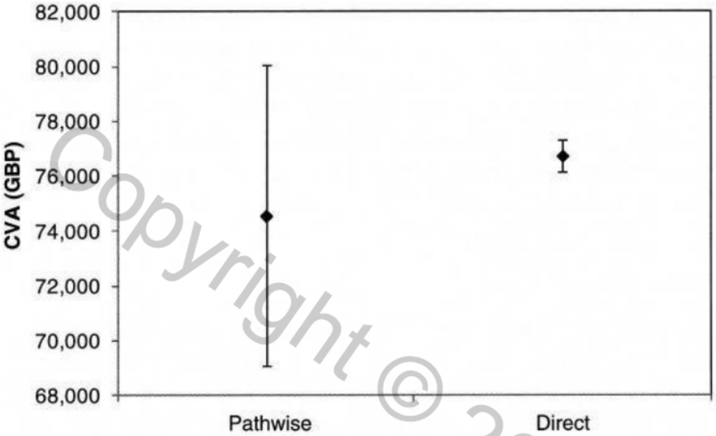


FIGURE 15-10 Estimate of the CVA for the Payer IRS EUR 5Y calculated with pathwise and direct jump to simulation approaches.

In each case, the same numbers of evaluations of the swap are used.

the square root of the number of simulations, this actually represents a speed improvement of 9.7 × 9.7 = 94 times. In other words, we can do 94 times fewer simulations to achieve the same accuracy. Whilst the above may sound appealing, we must consider the overall improvement. Amdahl's law (Amdahl, 1967) gives a simple formula for the overall speedup from improving one component of a calculation. This formula is $((1 - P) + P/S)^{-1}$, where P is the percentage of the calculation that can be improved and S is the relative speed improvement. For example, if 90% (P = 0.9) of the time is spent on pricing function calls and these can be speeded up by 94 times, then the overall improvement is 9.1 times. This is shown in Figure 15-11, illustrating the improvement depending on the proportion of time spent on the valuation. Clearly, P needs to be close to unity for the overall speedup to be good. Furthermore, going from a pathwise to a direct simulation may be more time-consuming. Figure 15-11 also illustrates the impact of the non-valuation stage taking longer, which results in a worse speedup. Overall, we can see that a direct simulation approach

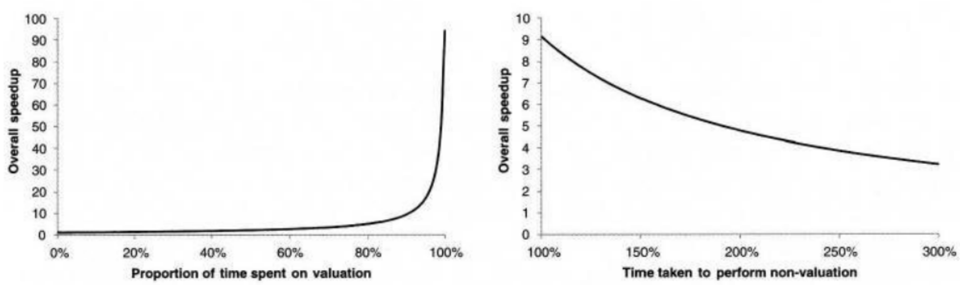


FIGURE 15-11 Illustration of the overall improvement according to the speedup of 94 times in moving from pathwise to direct simulation for CVA computation.

The left-hand graph shows the overall speedup as a function of the proportion of time spent on the valuation stage. The right-hand graph assumes 90% of the time spent on the revaluation and looks at the overall speedup as a function of the increased time to perform the non-valuation components.

for CVA may be faster but this will depend on the precise time spent on different components in the Monte Carlo model.

Path Dependency, Break Clauses and Exotics

Whilst the above idea may allow some speedup in CVA calculations, it will introduce complexities with path-dependent products. Path dependency in CVA calculations presents a problem since, in order to assess a future exposure at a certain date, one must have information about the entire path from now until that date. This aspect was discussed in Chapter 13. Whilst CVA calculations are naturally, and most easily, based on risk-free values, ideally, one should exercise an option based on the risky value (i.e., including CVA). However, this creates a recursive problem where the CVA calculation depends on the exercise decision, which itself depends on the CVA.

Arvantis and Gregory (2001) solve the path-dependent CVA problem for an interest rate swaption with deterministic credit spreads and their results are reported in Table 15-7. We can see that exercising based on the optimal risky value lowers the CVA slightly. This is because it avoids exercising in situations where the risk-free value of the swap is positive but the CVA is greater than this value. We also see the effect is stronger for a larger credit spread.

TABLE 15-7 Illustration of CVA Values for Physically Settled Interest Rate Swaptions Assuming Exercise Based on the Risk-Free and Risky Values. The left-hand column shows the swaption and swap maturity respectively, for example 1Y/5Y indicates a 1-year swaption to exercise into a 5-year swap.

	CDS Curve = 200 bps Flat		CDS Curve = 500 bps Flat	
	Risk-Free Exercise	Risky Exercise	Risk-Free Exercise	Risky Exercise
1Y/5Y	0.117%	0.116%	0.252%	0.245%
2Y/5Y	0.128%	0.127%	0.268%	0.264%
1Y/10Y	0.334%	0.327%	0.690%	0.654%
2Y/10Y	0.355%	0.349%	0.700%	0.679%

Source: Taken from Arvanitis and Gregory (2001)

As shown in Equation (15.2), the calculation of CVA will be approximated with reference to EE calculated at discrete points in time. Whilst this may be acceptable for certain kinds of path dependencies (for example, Bermudan swaptions), exotic derivatives prices are often based on a continuous sampling of quantities (for example, barrier options). Such cases will also require approximations such as those introduced by Lomibao and Zhu (2005), who use a mathematical technique known as a Brownian bridge to calculate probabilities of path-dependent events that are intermediate to actual exposure simulation points.

Regarding exotic products and those with American-style features, there are typically three approaches followed. The first is to use approximations, which may sometimes be upper bounds on the true CVA. Given this, the other uncertainties in quantifying CVA and associated hedging issues, using approximations for exotic products, may not be of great concern. A second, more sophisticated and accurate approach involves using pre-calculated grids to provide the future value of instruments as a function of the underlying variables. This approach works well as long as the dimensionality is not high. Third, American Monte approaches can be used to approximate exposures, handling any exotic feature as well as path dependencies. This is described in detail by Cesari et. al. (2009).

CVA WITH COLLATERAL

Finally, we will consider the impact of collateral on CVA, which follows from the assessment of the impact of collateral. As with netting before, the influence of collateral on the standard CVA formula given in Equation (15.2) is straightforward. Collateral only changes the EE (it does not change the default probability of the counterparty or recovery value) and hence the same formula may be used with the EE based on assumptions of collateralisation. The base case scenario will consider the four trades used previously. This assumes a zero-threshold, two-way CSA with a minimum transfer amount of 100,000 and a rounding of 20,000. For the CVA calculation, a flat credit curve of 500 bps and recovery value of 40% is assumed. The base case CVA without any collateral considered is 257,905 as can be seen, for example, from Table 15-6.

Impact of Margin Period of Risk

We first consider the impact of the margin period of risk on the zero-threshold CVA calculation. The CVA increases, from being very small at a margin period of risk of zero[22] towards the uncollateralised value as shown in Figure 15-12. At a margin period of risk of 30 calendar days, the CVA is almost half the uncollateralised CVA. This is in line with the more conservative assumption of a minimum of 20 business days required in certain circumstances under the Basel III capital rules.

[22] Note that at a margin period of risk of zero, there is still a small CVA due to the minimum transfer amount and rounding.

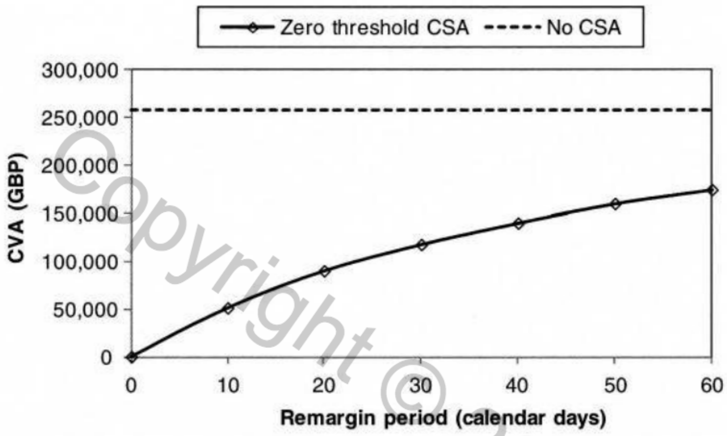


FIGURE 15-12 Impact of the margin period of risk on CVA.

The CVA with no CSA is shown by the dotted line.

Threshold CSAs and Independent Amounts

Figure 15-13 shows the impact of a threshold on various different CSAs. In the case of a one-way CSA, in favour of the counterparty (and therefore against the institution), the overall CVA is increased compared to the uncollateralised CVA (dotted line). A one-way CSA in favour of the institution (for) reduces the CVA significantly. In both one-way CSA cases, the impact of an increasing threshold is to

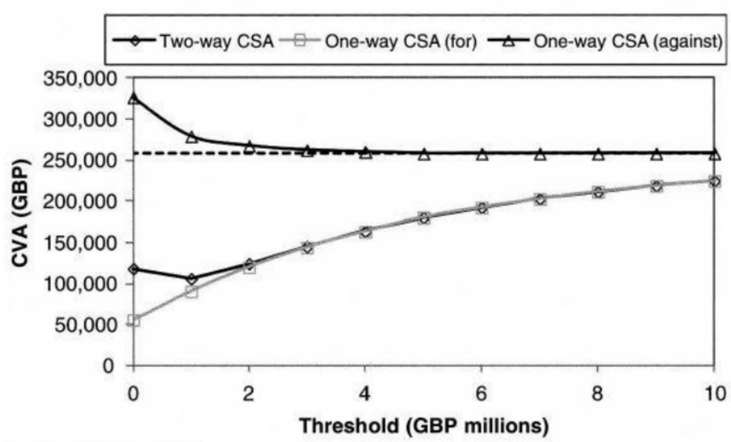


FIGURE 15-13 Impact of the collateral threshold on CVA.

Shown are a two-way CSA, a one-way CSA in the institution's favour (for) and vice versa (against). The dotted line is the uncollateralised CVA.

make the CVA converge to the uncollateralised result. In the case of a two-way CSA the behaviour is not completely monotonic with respect to an increasing threshold such that a (two-way) threshold of $1m appears slightly more beneficial than a zero-threshold CSA. It is interesting to explain this effect in a bit more detail.

This non-monotonic behaviour in the two-way CSA case is related to the discussion on EE being *less* than NEE whilst 95% PFE is *greater* than 5% PFE. Recall that we are dealing with a set of four trades, three of which have a positive sensitivity to overall interest rates. In the zero-threshold case, there are many scenarios where the institution must post a relatively small amount of collateral due to a negative drift (relating to the NEE being greater than the EE). This tends to weaken the benefit of the collateralisation. On the other hand, with a small threshold, many of these scenarios do not result in collateral posting and the ability to mitigate the paths around the 95% PFE, where interest rates are high, outweighs the need to post collateral for the paths around the (smaller) 5% PFE.

Figure 15-14 shows the impact of independent amount and threshold on the CVA. Note that an independent amount can be considered as a negative threshold. We can see an increase from zero, where the independent amount is large, to the uncollateralised CVA (dotted line) where the threshold is large.

In Figure 15-15 we look more carefully at the impact of the independent amount on the CVA. We also show error bars arising from an assumed uncertainty in the margin period of risk of ±10 days (i.e., 20 days or 40 days). Whilst an increase in the independent amount reduces the CVA substantially, the uncertainty over the CVA is relatively greater. With an independent amount, we may believe that the CVA is small but the uncertainty of the estimate is large.

SUMMARY

This chapter has been concerned with the pricing of counterparty risk via CVA. The computation of CVA has been detailed from the commonly made simplification of no wrong-way risk, which assumes that the credit exposure, default of the counterparty and recovery rate

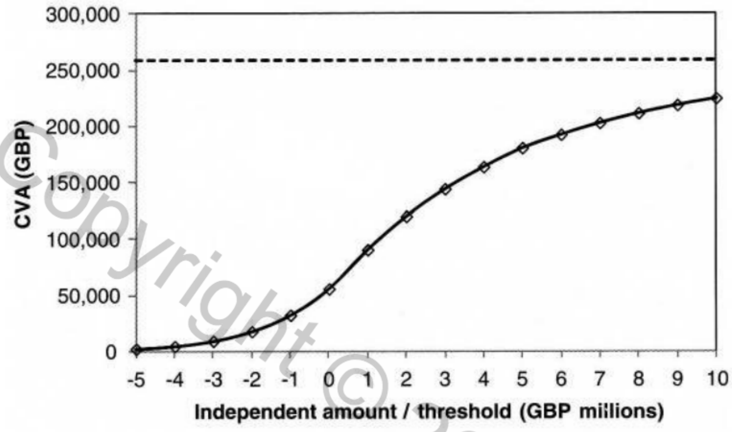


FIGURE 15-14 Impact of the independent amount (negative values) and threshold (positive values) on CVA.

A one-way CSA in the institution's favour is assumed. The dotted line is the uncollateralised CVA.

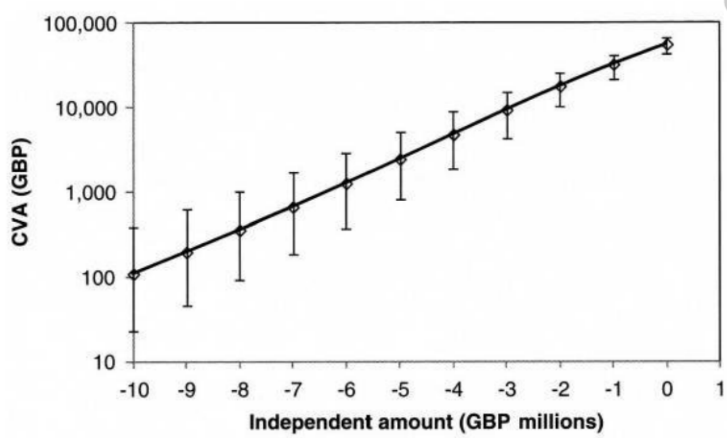


FIGURE 15-15 Impact of the independent amount (represented as a negative value as in the previous figure) on CVA with a logarithmic *y*-axis.

Also shown are error bars corresponding to changing the assumed margin period of risk by ±10 calendar days.

are not related. We have shown the relevant formulas for computing CVA in their simplest possible forms. The concepts of incremental and marginal CVA have been introduced and illustrated in order to provide a means to price new or existing trades. We have discussed the specifics of calculating CVA, including collateral and netting, and covered some more complex aspects such as numerical implementation, exotic products and path dependency.

APPENDIX

CVA Formula for an Option Position

In this case we have a simplification since the exposure of the long option position can never be negative:

$$CVA_{option}(t, T) = (1 - \bar{R})E^Q[I(\tau \leq T)]E^Q[B(t, \tau)V_{option}(\tau, T)],$$
$$= (1 - \bar{R})F(t, T)V_{option}(t, T)$$

where $V_{option}(t,T)$ is the upfront premium for the option. This means that the value of the risky option can be calculated as:

$$V_{option}(t, T) - CVA_{option}(t, T) = V_{option}(t, T) - (1 - \bar{R})F(t, T)$$
$$V_{option}(t, T) = V_{option}(t, T)[1 - F(t, T)] + \bar{R}V_{option}(t, T)F(t, T).$$

With zero recovery we have simply that the risky premium is the risk-free value multiplied by the survival probability over the life of the option.

16 Wrong-Way Risk

Learning Objectives

After completing this reading you should be able to:

- Describe wrong-way risk and contrast it with right-way risk.
- Identify examples of wrong-way risk and examples of right-way risk.

Excerpt is Chapter 15 of Counterparty Credit Risk and Credit Value Adjustment, *Second Edition, by Jon Gregory.*

I never had a slice of bread, particularly large and wide, that did not fall upon the floor, and always on the buttered side.

Newspaper in Norwalk, Ohio, 1841

INTRODUCTION

Previous chapters have been concerned with valuation of counterparty risk and funding via CVA, DVA, and FVA under a key simplifying assumption of no wrong-way risk. Wrong-way risk is the phrase generally used to indicate an unfavourable dependence between exposure and counterparty credit quality—i.e., the exposure is high when the counterparty is more likely to default and vice versa. Whilst it may often be a reasonable assumption to ignore wrong-way risk, its manifestation can be rather subtle and potentially dramatic. In contrast, "right-way" risk can also exist in cases where the dependence between exposure and credit quality is a favourable one. Right-way situations will reduce counterparty risk and CVA.

In this chapter, we will identify causes of wrong-way risk and discuss the associated implications on exposure estimation and quantification of counterparty risk. We will give examples of quantitative approaches used and give specific examples such as forward contracts, options and swaps. We will later discuss wrong-way risk in the credit derivative market and analyse what went so dramatically wrong with CDOs in the global financial crisis. The impact of collateral on wrong-way risk will be analysed and the central clearing implications will be discussed.

OVERVIEW OF WRONG-WAY RISK

Simple Example

Imagine tossing two coins and being asked to assess the probability of getting two heads—that is an easy question to answer.[1] Now suppose that you are told that the coins are linked in some way: the first coin to land can magically have some impact on which way up the other coin lands. Clearly, the question is now much more complex.

In Chapter 15, we saw that CVA could be generally represented as credit spread multiplied by exposure. Indeed, an approximate formula for CVA was simply CVA = credit spread × EPE. However, the multiplication of the default probability (credit spread) and exposure (EPE) terms relies on a key assumption, which is that the different quantities are *independent*. If they are dependent, then the analysis is far more complicated and the relatively simple formulas are no longer appropriate. Essentially, this corresponds to the integration of credit risk (default probability) and market risk (exposure), which is a very complex task. We could have other dependence, such as between loss given default (and equivalently recovery rate) and either exposure or default probability, which will also give rise to other forms of wrong-way risk.

A simple analogy to wrong-way risk is dropping (the default) a piece of buttered bread. Many people believe that in such a case, the bread is most likely to land on the wrong, buttered side (exposure). This is due to "Murphy's Law," which states that "anything that can go wrong, will go wrong." This particular aspect of Murphy's Law has even been empirically tested[2] and, of course, the probability of bread landing butter side down is only 50%.[3] People have a tendency to overweight the times when the bread lands the wrong way against the times they were more fortunate. Since it is in human nature to believe in wrong-way risk, it is rather surprising that it has been significantly underestimated in the derivatives market! The market events of 2007 onwards have illustrated clearly that wrong-way risk can be extremely serious. In financial markets, the bread always falls on the buttered side (or has butter on both sides).

Classic Example and Empirical Evidence

Wrong-way risk is often a natural and unavoidable consequence of financial markets. One of the simplest examples is mortgage providers who, in an economic regression, face both falling property prices and higher default rates by homeowners. In derivatives, examples of trades that obviously contain wrong-way risk across different asset classes, which will be studied in more detail later, are:

- *Put option*. Buying a put option on a stock (or stock index) where the underlying in question has fortunes that are highly correlated to those of the counterparty is an obvious case of wrong-way risk (for example,

[1] It is, of course, 25% from one-half times one-half.

[2] On the English BBC TV science programme Q.E.D. in 1993.

[3] Matthews (1995) has shown that a butter-down landing is indeed more likely, but for reasons of gravitational torque and the height of tables rather than Murphy's Law.

buying a put on one bank's stock from another bank). The put option will only be valuable if the stock goes down, in which case the counter party's credit quality will be likely to be deteriorating. As we shall see later, an out-of-the-money put option will have more wrong-way risk than an in-the-money one. Correspondingly, equity call options should be right-way products.

- *FX forward or cross-currency products*. Any FX contract must be considered in terms of a possible linkage between the relevant FX rate and the default probability of the counterparty. In particular, a potential weakening of the currency received by the counterparty vis-à-vis the paid currency should be a wrong-way risk concern. This would obviously be the case in trading with a sovereign and paying their local currency. Another way to look at a cross-currency swap is that it represents a loan collateralised by the opposite currency in the swap. If this currency weakens dramatically, the value of the collateral is strongly diminished. This linkage could be either way: first, a weakening of the currency could indicate a slow economy and hence a less profitable time for the counterparty. Alternatively, the default of a sovereign or large corporate counterparty may itself precipitate a weakening of its local currency.
- *Interest rate products*. Although this is probably an area with limited wrong-way risk, it is important to consider a relationship between the relevant interest rates and the counterparty default probability. Such a relationship could be considered in either direction: high interest rates may trigger defaults, whereas low interest rates may be indicative of a recession where defaults are more likely.
- *Commodity swaps*. In an oil swap, one party pays cash flows based on a fixed oil price and receives cash flows based on an average spot price of oil over a period. The exposure of payer swap will be high when the price of oil has increased. Suppose the counterparty is an oil company: high oil prices should represent a scenario in which they are performing well. Hence, the contract *should* represent "right-way risk." The right-way risk arises due to hedging (as opposed to speculation). However, it may not always be as clear-cut as this, as we shall see later.
- *Credit default swaps*. When buying protection in a CDS contract, an exposure will be the result of the reference entity's credit spread widening. However, one would prefer that the counterparty's credit spread is not widening also! In the case of a strong relationship between the credit quality of the reference entity and counterparty, clearly there is extreme wrong-way risk. On the other hand, with such a strong relationship, selling CDS protection should be a right-way trade with little or no counterparty risk. In portfolio credit derivatives, this effect becomes more subtle and potentially dramatic and helps to explain the failure of CDOs.

All of the above cases will be considered in more detail later in this chapter.

General empirical evidence supports the presence of wrong-way risk. For example, Duffie (1998) shows a clustering of corporate defaults in the U.S. during periods of falling interest rates. Regarding the FX example, results from Levy and Levin (1999) look at residual currency values upon default of the sovereign and find average values ranging from 17% (Triple-A) to 62% (Triple-C). This implies the amount by which the FX rate involved could jump at the default time of the counterparty.

Losses due to wrong-way risk have also been clearly illustrated. For example, many dealers suffered heavy losses because of wrong-way risk during the Asian crisis of 1997/1998. This was due to a strong link between the default of sovereigns and of corporates and a strong weakening of their local currencies. A decade later, the credit crisis starting in 2007 caused heavy wrong-way risk losses for banks buying insurance from so-called monolines, as discussed later.

Right-Way Risk and Hedging

Right-way risk indicates a beneficial relationship between exposure and default probability that actually *reduces* counterparty risk. Hedges should naturally create right-way risk because the aim of the hedge is to reduce risk, which should in turn mean less uncertainty over counterparty credit quality.

Wrong-way risk *should* be rather rare in an ideal world. Suppose a mining company wishes to hedge (lock in) the price of gold at some date in the future. This can be achieved via a forward contract on gold. When such a contract is in an institution's favour (and against the mining company), the price of gold will be high. Mining companies are not expected to default when gold is expensive. Assuming most counterparties are hedging and not speculating, then they should generate right-way rather than wrong-way risk.

It could be assumed that wrong-way risk will generally be offset by right-way risk. However, we will show later that these assumptions can sometimes be shown to be quite naïve. In the real world, speculation, failed hedges and systemic effects mean that wrong-way risk can occur frequently. Institutions that have exposures to certain market events (such as hedge funds and monolines) will almost surely create wrong-way risk for those trading with them.

Wrong-Way Risk Challenges

Quantifying wrong-way risk will involve somehow modelling the relationship between default probability and exposure. At a high level, there are two potential pitfalls in doing this, which are:

- *Lack (or irrelevance) of historical data*. Unfortunately, wrong-way risk may be subtle and not revealed via any historical time series analysis.
- *Misspecification of relationship*. The way in which the dependency between credit spreads (default probability) and exposure is specified may be inappropriate. For example, rather than being the result of a correlation, it may be the result of a *causality*—a cause-and-effect type relationship between two events.

Suppose an institution makes a statistical study of the correlation between the credit quality of their counterparty and a variable driving the exposure (e.g., an interest rate or FX rate) and finds this correlation is close to zero. There seems to be little evidence of wrong-way risk in this transaction. However, both of the above problems may exist.

Concerning historical data, wrong-way risk by its very nature is extreme and often rather specific. Hence, historical data may not show the relationship. For example, in 2010, the European sovereign crisis began and was accompanied by deterioration in the credit quality of many European sovereigns and a weakening of the euro currency. There is a clear relationship here with sovereign credit spreads widening and their underlying currency weakening. However, historical data did not bear out this relationship, largely since neither the sovereigns concerned nor the currency had ever previously been subject to any adverse credit effects.

Concerning possible misspecification, correlation is only one measure of dependency. It measures only the linear relationship between variables. Suppose one believes that a small move in a market rate will have little or no impact on the credit quality of a counterparty but a much larger move will. This is a second-order relationship that will not be captured by correlation. There may be a causal relationship: for example, the counterparty's credit quality deteriorating significantly moves market variables significantly even though the credit spread of that counterparty previously showed no relationship to the market variable during normal times. It is important to emphasise here, whilst two independent random variables will have zero correlation, the reverse is **not** true. If the correlation between two random variables is measured as zero, then this does not prove that they are independent.[4]

Wrong-Way Risk and CVA

The presence of wrong-way risk will (unsurprisingly) increase CVA. However, the magnitude of this increase will be hard to quantify, as we shall show in some examples. Wrong-way risk also prevents one from using the (relatively) simple formulas used for CVA in Chapter 15. Whilst independence may exist in everyday life, it almost certainly does not in the interconnected and systemic financial markets. All of the aforementioned formulas are therefore wrong.

All is not lost though. We can still use the same CVA expression as long as we calculate the exposure *conditional* upon default of the counterparty. We simply rewrite the expression as

$$\mathrm{CVA} \approx (1-\mathrm{Rec})\sum_{j=1}^{m}\mathrm{DF}(t_j)\mathrm{EE}(t_j \mid t_j=\tau_c)\mathrm{PD}(t_{j-1},t_j) \quad \textbf{(16.1)}$$

where $\mathrm{EE}(t_j|t_j = \tau_C)$ represents the expected exposure at time t_j conditional on this being the counterparty default time (τ_C). This replaces the previous exposure, which was unconditional. As long as we use the conditional exposure[5] in this fashion, everything is correct.

Obviously, calculating the conditional exposure is not at all easy because it depends on the counterparty and future time in question. Two equivalent portfolios of trades with different counterparties will have the same *unconditional* exposure but different *conditional* exposures. Broadly speaking, there are two ways to go about computing conditional exposure:

- Consider the exposure and default of the counterparty together and quantify the economic relationship

[4] A classic example of this is as follows. Suppose a variable X follows a normal distribution. Now choose $Y = X^2$. X and Y have zero correlation but are far from independent.

[5] We note that there are other ways to represent this effect. For example, we could instead look at the conditional default probability, as will be done later.

between them. This method is the "correct" approach but the economic relationship may be extremely hard to define and there may be computation issues in calculating quantities such as CVA in this manner.

- Incorporate wrong-way risk via simple conservative assumptions, "rules of thumb," or simple generic models. This is a much simpler approach that involves minimal effort in the way of systems re-engineering or additional computational requirements.

Simple Example

So exposure should always be computed conditionally on the counterparty default. The correlation is introduced by assuming the exposure follows a normal distribution and that the default time is generated from a normal distribution using the so-called Gaussian copula approach. Under these assumptions, the conditional expected exposure can be calculated directly. This gives the EE at a time *s* under the assumption that the counterparty will have defaulted at time *s*. The relationship between exposure and counterparty default is expressed using a single correlation parameter. This correlation parameter is rather abstract, with no straightforward economic intuition, but it does facilitate a simple way of quantifying and understanding wrong-way risk.

SPREADSHEET 16-1

Simple Wrong-Way Risk Calculation

To download Spreadsheet 16-1, visit http://www.cvacentral.com/books/credit-value-adjustment/spreadsheets and click Chapter 16 exercises.

Let us now consider the impact of wrong-way risk on the example forward contract using the following base case parameters:

$\mu = 0\%$	drift of the value of the forward contract
$\sigma = 10\%$	volatility of the value of the forward contract
$h = 2\%$	hazard rate[6] (default probability) of the counterparty
$\rho = \pm 50\%$	correlation between the value of the forward contract and the default time of the counterparty

[6] See Chapter 14 for the definition of hazard rate.

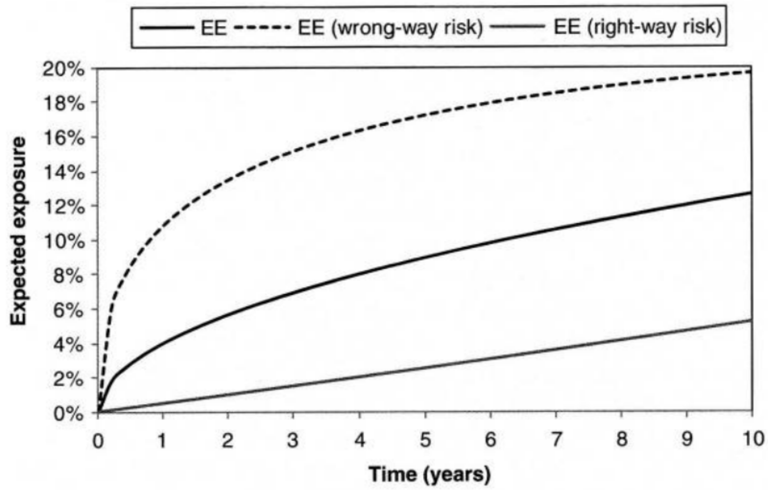


FIGURE 16-1 Illustration of wrong-way and right-way risk EE profiles using the base-case scenario with correlations of 50% and −50%, respectively.

Figure 16-1 shows the impact of wrong-way (and right-way) risk on the EE. We can see that with 50% correlation, wrong-way risk approximately doubles the EE whilst with −50% correlation the impact of right-way risk reduces it by at least half. This is exactly the type of behavior expected: positive correlation between the default probability and exposure increases the conditional expected exposure (default probability is high when exposure is high), which is wrong-way risk. Negative correlation causes right-way risk. Note that since the drift is zero, the negative expected exposure would follow exactly the same trend.

Let us look into this simple model in a bit more detail. Consider now the impact of counterparty default probability on the EE with wrong-way risk. Figure 16-2 shows the EE using three different hazard rates, indicating that the exposure decreases as the credit quality of the counterparty also decreases. This result might seem at first counterintuitive but it makes sense when one considers that for a better credit quality counterparty, default is a less probable event and therefore represents a bigger surprise then it comes. We note an important general conclusion:

Wrong-way risk *increases* as the credit quality of the counterparty *increases*.

Finally, we change the drift of the forward contract to be $\mu = -2\%$ and use a larger hazard rate of $h = 6\%$. The EE profile with and without wrong-way risk is shown in Figure 16-3.

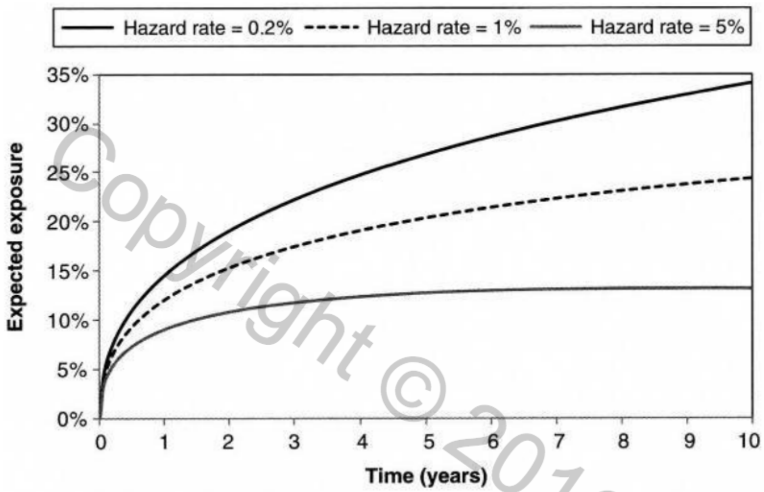


FIGURE 16-2 Illustration of EE under the assumption of wrong-way risk as a function of the hazard rate.

The correlation is assumed to be 50%.

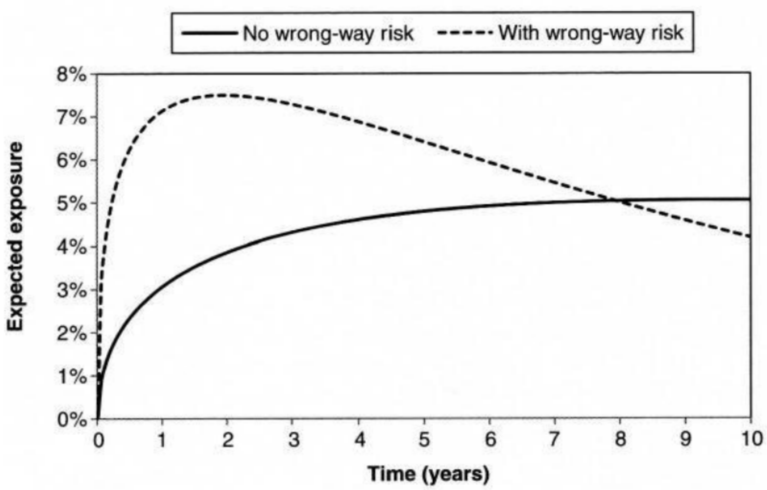


FIGURE 16-3 Illustration of EE with and without the assumption of wrong-way risk for a drift of $\mu = -2\%$ and hazard rate of $h = 6\%$.

Negative drift will reduce the overall exposure, as we can see. However, there is another effect, which is that the wrong-way risk EE is actually smaller than the standard EE after 8 years. This is because counterparty default in later years is not such a surprise as in earlier years (with a hazard rate of 6%, the 8-year default probability is 38%, whilst the 2-year default probability is only 11.3%[7]). Hence, default in early years represents "bad news" whilst in later years default is almost expected! This suggests that wrong-way risk has a term structure effect, with conditional exposure in the shorter term showing a more dramatic effect than in the long term.

PORTFOLIO WRONG-WAY RISK

Broadly speaking, wrong-way risk can be divided into two categories, namely general and specific. This distinction has been made by the Basel committee in regulatory capital rules. General wrong-way risk can be thought of as the general relationship between exposure and default probability due to macroeconomic factors, which is most relevant at the portfolio level. Specific wrong-way risk may be analysed more at the transaction level and often represents more of a structural relationship between the counterparty default probability and the underlying exposure. We will discuss them along similar lines but use the terms portfolio and trade-level wrong-way risk, which do not necessarily coincide with the terms general and specific.

Correlation Approach

The simple approach described previously can readily be extended to the general case. To do this it is necessary to map the exposure distribution at each point in time onto a chosen (e.g., normal) distribution. The most obvious way to do this is to sort the exposures in descending order (although other, more complex approaches can be used, as discussed below) and then map via a quantile mapping procedure. This approach is then the simplest version of the approaches proposed by Garcia-Cespedes et. al. (2010) and Sokol (2010) and is illustrated in Figure 16-4. Due to the mapping of the exposures onto a normal distribution, in the positive correlation case an early default time will lead to a higher exposure, as is the case with wrong-way risk. A negative correlation will have the reverse effect and generate right-way risk. Note that there is no need to recalculate the exposures as the original exposures are used directly. The conditional exposures and corresponding CVA are then calculated easily via Monte Carlo simulation. Other distributional assumptions can also be used.

[7] Recall the simple relationship for the cumulative default probability at time s, being $1 - \exp(-hs)$ where h is the hazard rate.

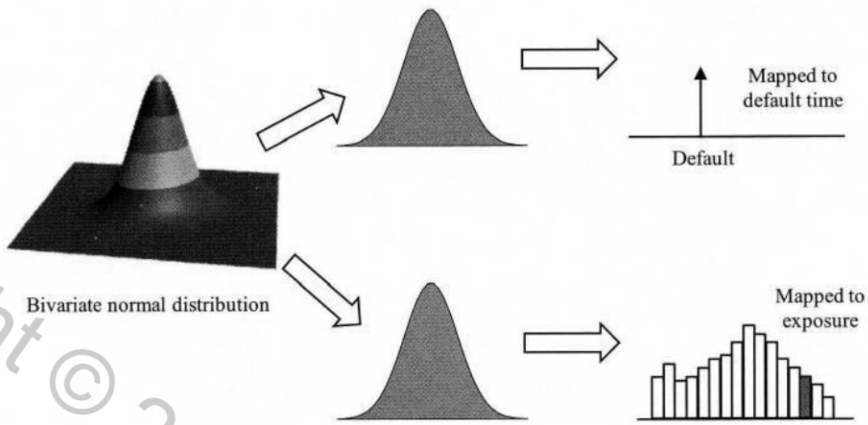


FIGURE 16-4 Illustration of the correlation approach for general wrong-way risk.

A bivariate normal distribution with a certain correlation value drives both default times and exposures. If the correlation is positive, then an early default time will be more likely to lead to a high exposure, as illustrated.

We start with the unilateral case (DVA and wrong-way risk will be discussed later in this section). As before, we assume the counterparty CDS curve is flat at 500 bps and the recovery rate is 40%. The same four-trade portfolio will be used as in previous examples. This corresponds to the unilateral CVA result reported previously in Table 14-1. We will first look at the expected exposure conditional on the counterparty's default, which is shown in Figure 16-5 for both positive and negative correlation values as well as zero correlation. We see the expected impact that positive (negative) correlation leads to a higher (lower) conditional exposure reflecting wrong-way (right-way) risk. As noted before, this effect is stronger for shorter maturities since an early default is more unexpected.

Figure 16-6 shows the (unilateral) CVA as a function of correlation. Negative correlation reduces the CVA due to right-way risk and wrong-way risk, created by positive correlation, increases it. The effect is quite dramatic, with the CVA approximately doubled at 50% correlation.

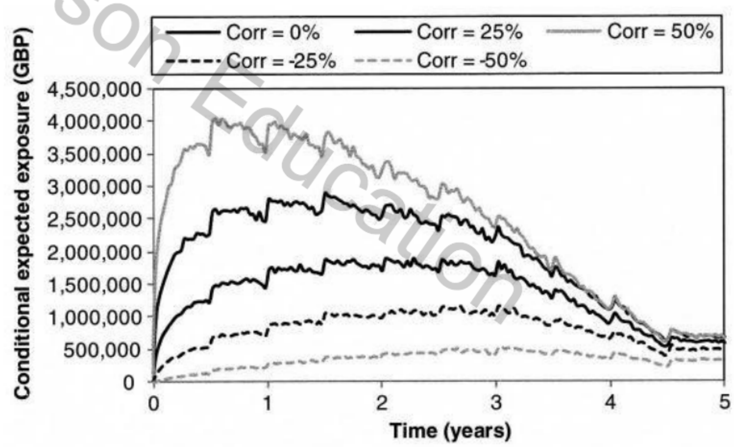


FIGURE 16-5 Conditional expected exposure calculated with various levels of correlation for the four-trade portfolio.

Parametric Approach

Hull and White (2011) have proposed a different approach to the above. Rather than holding the default probability fixed and calculating the conditional expected exposure, they do the reverse (which is equally as valid). The conditional default probability is then defined by linking the hazard rate to the underlying future value of the portfolio. One functional form proposed is $h(t) = \ln[1 + \exp(a(t) + bV(t))]$, where $h(t)$ and $V(t)$ represent the hazard rate and the future value and $a(t)$ and b are parameters.[8] An example of this functional form is given in Figure 16-7, which corresponds approximately to a counterparty CDS

[8] Hull and White also note that the hazard could be related to other variables (such as interest rates). They also propose an additional noise term and a different functional form but note that these aspects do not generally have a significant impact on the results.

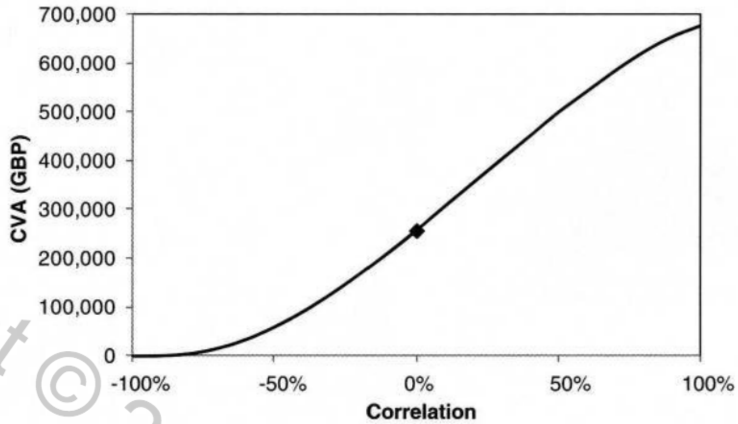


FIGURE 16-6 Unilateral CVA as a function of the correlation between counterparty default time and exposure.

The point marked shows the independence CVA of 257,905.

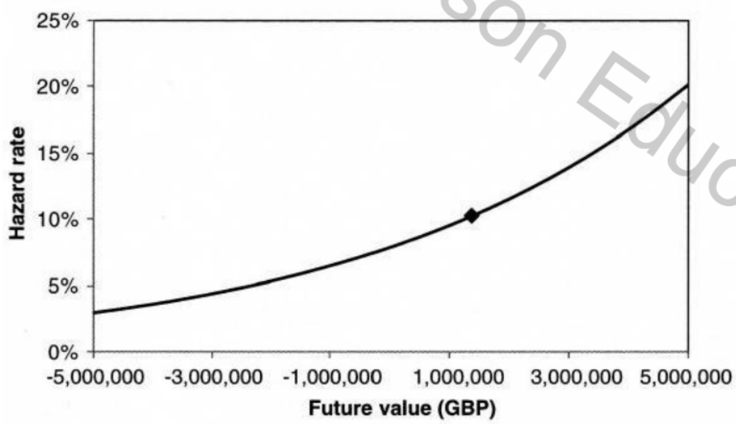


FIGURE 16-7 Illustration of the functional form proposed by Hull and White (2011) in their wrong-way risk approach.

The a(t) function is set constant at -2.5 whilst the b parameter is 2×10^{-7}. The point marked corresponds to the EPE.

spread of 500 bps and recovery rate of 40% (a perfect fit will be used later). To give this some context, the unconditional hazard rate is in this case 8.33%, whilst the value at $V(t) = 0$ is 7.89%. Calculating the hazard rate at the EPE of this portfolio, which corresponds to $V(t) = 1{,}371{,}285$, gives 10.25% (i.e., an increase of around 30%). Note that a negative b value will give the opposite behaviour and create a right-way risk effect.

The function $a(t)$ is most naturally used to fit the term structure of default probability, leaving the single parameter b to define the relationship between exposure and default probability. This can be done numerically as shown by Hull and White (2011) and the calibration required for the four-trade portfolio is shown in Figure 16-8.

Finally, we show the CVA as a function of the b parameter in Figure 16-9. As anticipated, a positive b gives a

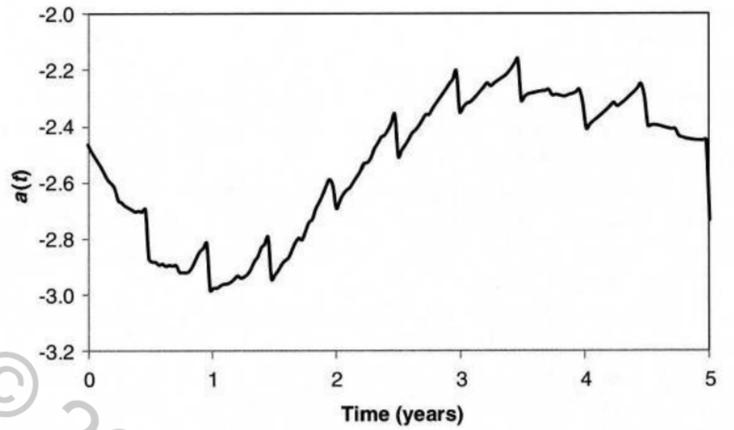


FIGURE 16-8 Illustration of the a(t) function for the four-trade portfolio in the parametric wrong-way risk approach calibrated assuming the b parameter is 2×10^{-7} and the counterparty CDS and recovery rate are 500 bps and 40%, respectively.

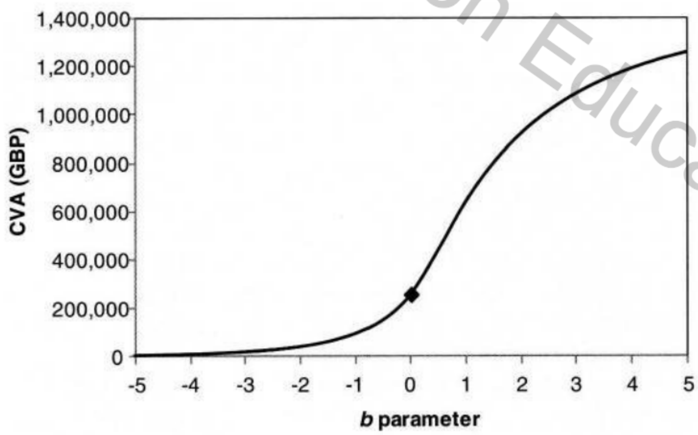


FIGURE 16-9 Unilateral CVA as a function of the b parameter in the Hull and White approach.

The marked point denotes the independence CVA of 257,905.

wrong-way risk effect and a higher CVA whilst a negative value gives the reverse right-way risk effect. The overall profile is similar (although more dramatic) than that given in the correlation model above. In the correlation model, the maximum CVA is 677,261 whilst in Figure 16-9 it can be seen to be significantly exceeding this value. Whether or not this is economically reasonable, it illustrates that 100% correlation should not be taken to imply a limiting case.

Calibration Issues

The correlation and parametric approaches described are relatively simple ways to incorporate general wrong-way risk without a large computational burden and/or having to rerun the underlying exposure simulations. However, the main challenge of such approaches will be calibration of the relevant parameters.

Firstly, regarding the correlation approach, Garcia-Cespedes et. al. (2010) suggest using multifactor models and a principal component approach to calibrate the correlation based on historical data. Discussion and correlation estimates are given by Fleck and Schmidt (2005) and Rosen and Saunders (2010).

For the parametric approach, Hull and White suggest using an intuitive calibration based on a what-if scenario. For example, if the exposure of the portfolio increases to $10 million, what would the spread of the counterparty increase to? Such a question will give a single point that can be used to calibrate the *b* parameter. Alternatively, the parametric relationship can be calibrated directly to historical data. This will involve calculating the portfolio value for dates in the past and looking at the relationship between this and the counterparty's CDS spread (hazard rate). If the portfolio has historically had a high value, which has corresponded to a larger-than-average counterparty CDS spread, then this will indicate some wrong-way risk. This approach obviously requires that the current portfolio of trades with the counterparty is similar in nature to that used in the historical calibration.

It is clear that the calibration of market and credit correlation is a very complex task. There is a significant risk of misspecification, for example, the correlation approach gives a maximum CVA of 677,261 (Figure 16-6) whereas the parametric approach can produce much larger values (Figure 16-9). Furthermore, there is likely to be a substantial error in calibration to historical data. Finally, the historical relationship may be completely meaningless with respect to the future relationship. Indeed, many of the events of the global financial crisis, especially those involving large dependencies, were not in any way borne out in historical data prior to the crisis and/or analyses based only on correlation measures.

DVA and Wrong-Way Risk

We should finally mention the impact of wrong-way risk on bilateral CVA (BCVA). For the purposes of calculating BCVA, the conditional negative expected exposure conditional on the institution's own default is also required. This calculation follows in a similar way to the expected exposure. The symmetry of CVA and DVA implies that if one is affected by wrong-way risk, then the other should

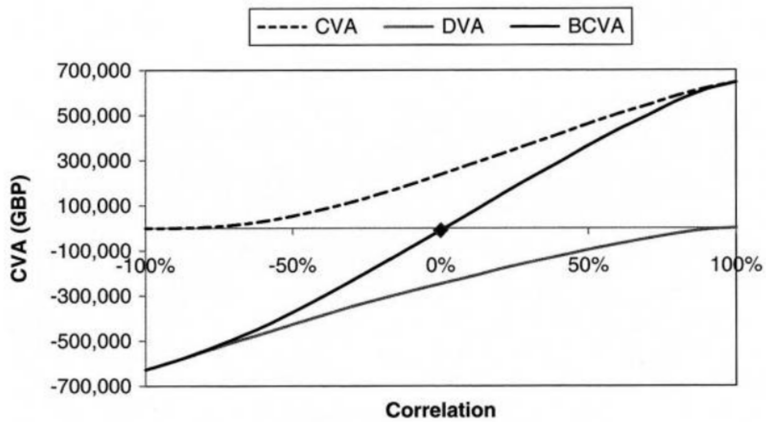


FIGURE 16-10 Bilateral CVA (BCVA) as a function of the correlation between counterparty/institution default time and exposure.

The point marked shows the independence BCVA of −8,791 GBP.

show the effect of right-way risk. There are two obvious cases where the above logic does not work, i.e., one party having wrong-way risk implies that the other party benefits from right-way risk. The first is that the nature of the parties is different, and therefore they are exposed to different risk factors (e.g., a bank and a sovereign). In the interbank market, wrong-way risk and right-way risk are likely to always be side-by-side. However, a bank providing a hedge to an end user may have right-way risk in their trade but the end user will not obviously have wrong-way risk. A second possibility is if the trade payoff is highly asymmetric, so that only one party can have a significant exposure. This is the case in CDS contracts, which are discussed later.

We return to the correlation approach mentioned earlier, and look at the DVA impact. We assume as before that the institution's own CDS spread is 250 bps and their recovery value is 40%. The correlation between the exposure and default times is the same for both the counterparty's and the institution's own default (although these correlations could easily be different, as mentioned earlier). We also assume independence between the default times (again, it would be straightforward to relax this assumption). The results are shown in Figure 16-10. It can be seen that wrong-way risk (positive correlation) has the impact of reducing the DVA whilst right-way risk (negative correlation) reduces the CVA. The overall impact is therefore very strong. For example, at zero correlation (no wrong- or right-way risk), the BCVA is −8,791 GBP, but at just 10% correlation, it has increased to 65,523 GBP, i.e., changed sign and almost an order of magnitude larger!

TRADE-LEVEL WRONG-WAY RISK

We now deal with trade-level wrong-way risk, looking at the different features by asset class. We will illustrate the wide range of wrong-way risk models and the different aspects that are important to consider.

Interest Rates

The relationship between changes in interest rates and default rates have been shown empirically to be generally negative.[9] This means that low interest rates are likely to be accompanied by higher default rates. This is most obviously explained by central bank monetary policy being to keep interest rates low when the economy is in recession and the default rate high. Such an effect clearly leads to wrong- and right-way risk in interest rate products, which we will analyse through an interest rate swap.

An obvious way to proceed in light of the empirical evidence is to correlate interest rates and credit spreads (hazard rates) in the quantification of the CVA on an interest rate product. Such approaches have commonly been used in credit derivative pricing (e.g., see O'Kane, 2008). The previous case corresponds to a negative correlation. We assume a Hull and White (1990) interest rate model[10] with a flat interest rate term structure of 5%. This will give a symmetric exposure profile that will make the wrong- and right-way risk effects easier to identify. We assume a lognormal hazard rate approach so that credit spreads cannot become negative.[11] As before, the counterparty CDS spread and recovery rate are 500 bps and 40%, respectively.

We first show interest rate simulations conditionally on a counterparty default event in Figure 16-11. In the case of zero correlation, these are unaffected by counterparty default and the paths are distributed symmetrically around the starting point of 5%.[12] In the case of negative correlation, the paths are biased downwards towards low interest rates. This happens because low interest rates occur often together with high hazard rates, which leads to a greater chance of default.

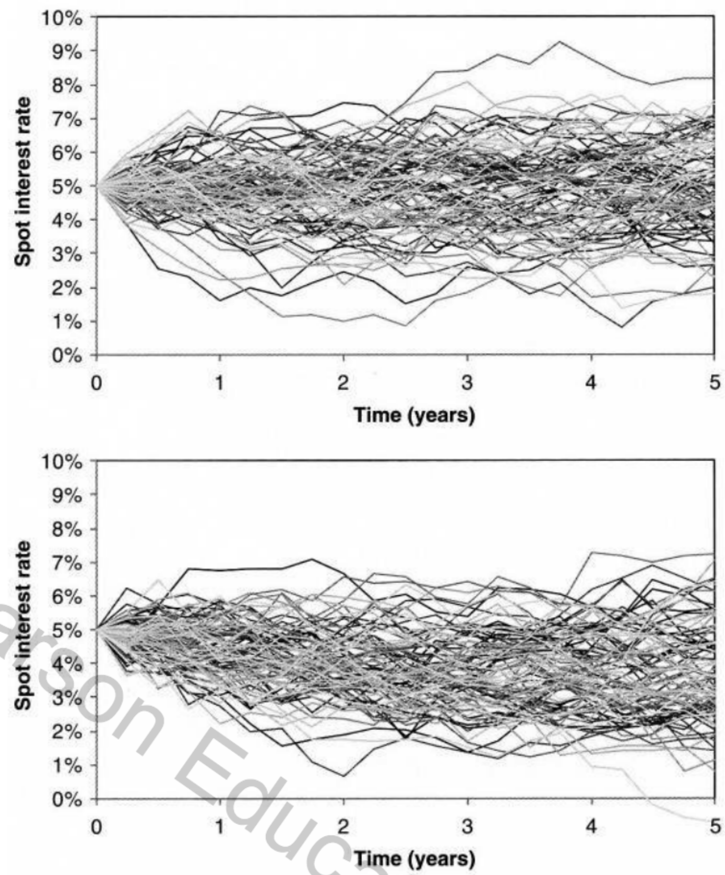


FIGURE 16-11 Interest rate simulations conditional on counterparty default (at some point in the 5-year period) for the correlated interest rate and hazard rate (credit spread) approach.

Cases of zero (top) and −90% (bottom) correlation are used.

In Figure 16-12, we show the future values for a 5-year receiver interest rate swap with negative correlation between interest rates and hazard rates. We see a strong wrong-way risk effect: the swap has an exposure when interest rates are low, which is likely to correspond to a

[9] See, for example, Longstaff and Schwartz (1995). Duffie (1998) and Collin-Dufresne et. al. (2001).

[10] The mean reversion parameter and volatility are set at 0.1 and 1%, respectively.

[11] The volatility used is 50%.

[12] One of the reasons for using a normal interest rate model was to illustrate this.

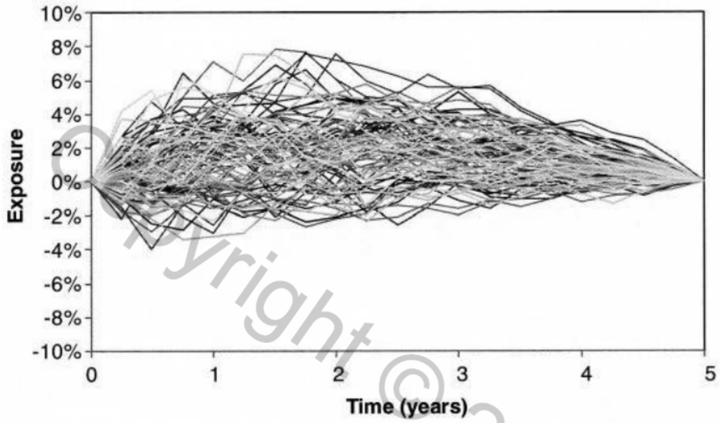


FIGURE 16-12 Future values for a receiver interest rate swap conditional on counterparty default for the correlated interest rate and hazard rate (credit spread) approach.

A correlation of −90% is used. This seemingly large value is chosen to illustrate most clearly the effect.

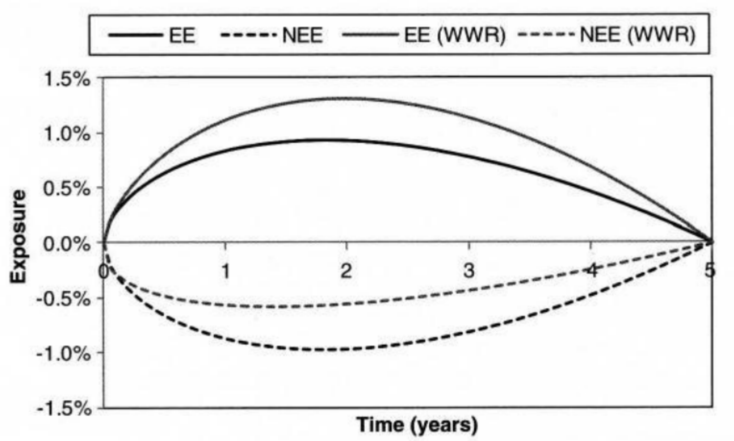


FIGURE 16-13 Expected exposure (EE) and negative expected exposure (NEE) for a receiver interest rate swap conditional on counterparty default for the correlated interest rate and hazard rate (credit spread) approach.

The base case corresponds to an assumed correlation of zero whilst the wrong-way risk (WWR) approach to a correlation of −50%. Note that the EE is computed conditional on the default of the counterparty (500 bps CDS spread assumed) whilst the NEE is conditional on default of the institution (250 bps).

higher hazard rate where counterparty default is more likely. Conditionally on default, its exposure is therefore likely to be positive. The payer swap would, by symmetry, show the reverse behaviour. In a positive correlation environment, the payer swap would be the wrong-way product and the receiver would have right-way risk.

In Figure 16-13, we show the expected exposure (EE) and negative expected exposure (NEE) for the receiver swap in the presence of wrong-way risk. For the purpose of calculating the NEE, the institution's CDS spread is assumed to be 250 bps as previously, with a 40% recovery rate. A correlation of −50% is assumed in both cases (i.e., both the counterparty's and institution's own hazard rates are correlated to interest rates by −50%), although we note that a different choice may be relevant in practice. Wrong-way risk increases the EE and right-way risk reduces the NEE. Note that the right-way risk effect is stronger: this is because default of the institution is less likely and so has a larger impact on the exposure conditional on the default event.

Finally, we show the bilateral CVA (BCVA) contributions as a percentage of the notional value in Figure 16-14. Due to the institution's default probability being approximately half that of the counterparty, the DVA is expected to be approximately half the CVA. However, also important is the fact that the right-way risk is stronger than the wrong-way risk, as discussed earlier. This can be seen from the fact that the maximum DVA is more than half the maximum CVA.[13] We see that the overall BCVA is very sensitive to the correlation, for example being three times bigger at −50% correlation than in the standard case (no wrong- or right-way risk).

The previous example represents the most obvious way to incorporate wrong-way (and right-way) risk. It is computationally more demanding than the standard BCVA approach since defaults must be simulated explicitly via some hazard rate process. However, there are relatively efficient methods for doing this. The correlations required could be calibrated from the market price of interest rate and credit hybrid

[13] The maximum CVA is 0.36% at −100% correlation and the maximum DVA −0.22% at +100% correlation.

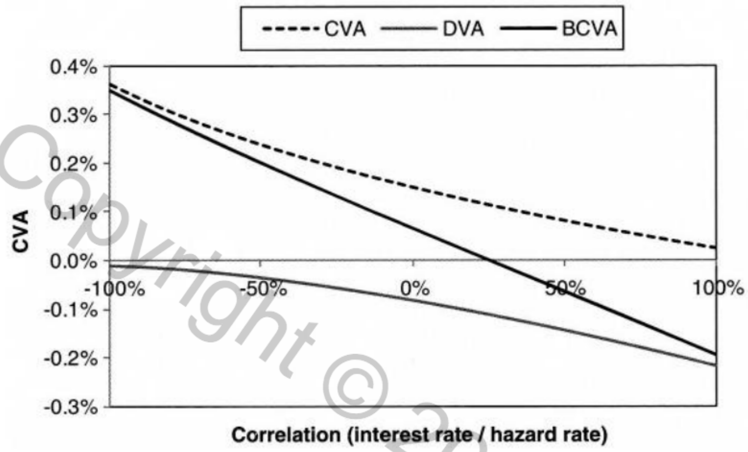


FIGURE 16-14 Bilateral CVA components for a receiver interest rate swap as a function of the correlation between interest rates and hazard rates.

The counterparty and institution CDS spreads are assumed to be 500 and 250 bps, respectively and the recovery rates are 40%.

products or from a historical time series of interest rates and credit spreads.

Nevertheless, there are uncertainties and possible problems with the previous approach. The distributional choices for the interest rate and hazard rates are obviously important. In particular, there is a lack of market information and historical data to calibrate hazard rate processes. Clearly, the estimate of the correlation between interest rates and hazard rates is uncertain. Indeed, since this correlation arises from a recession leading to both higher default rates and causing central banks to lower interest rates, there may also be some inherent time delay. However, the biggest concern should be over the fundamental choice of specifying a dependency between interest rates and hazard rates. In doing so, we assume that a payer swap is a wrong-way (right-way) risk product for negative (positive) correlation whilst a receiver swap will be precisely the reverse (as can be observed from the DVA in Figure 16-6). This is more than a matter of specifying the correct correlation. Empirical evidence is that default rates are high when interest rates are low, as mentioned at the beginning of this section. However, counterparties may also be more likely to default in high-interest rate environments when borrowing costs are high. This could make both payer and receiver swaps wrong-way risk products. A model correlating interest rate *volatility*[14] with default probability would produce this different behaviour.

Foreign Exchange Example

Ehlers and Schönbucher (2006) have considered the impact of a default on FX rates and illustrated cases where a correlation approach (such as the one used in the interest rate case earlier) between the exchange rate and the hazard rate is not able to explain empirical data. The data implies a significant additional jump in the FX rate at default. A simple approach proposed by Levy and Levin (1999) to model FX exposures with wrong-way risk is to assume that the relevant FX rate jumps at the counterparty default time, as illustrated in Figure 16-15. This is a simple approach since the conditional FX rate at default is simply the unconditional value multiplied by some jump factor.[15] The jump factor is often called a residual value (RV) factor of the currency and the assumption is that the currency devalues by an amount (1 − RV) at the counterparty default time and the relevant FX rate jumps accordingly.

The RV approach is most relevant for exposures to sovereigns where their local currency will clearly devalue by a

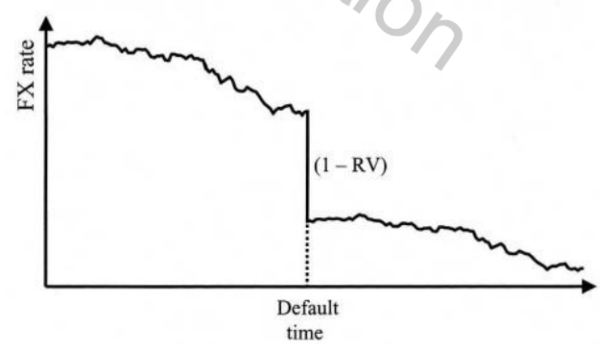


FIGURE 16-15 Illustration of the currency jump approach to wrong-way risk for FX products.

[14] This is analogous to the Merton (1974) idea that shows a relationship between credit spreads and equity *volatility*.

[15] The conditional expected FX rate. $E[FX(s)|s = \tau]$, at the counterparty default time is equal to its unconditional value $E[FX(s)]$ multiplied by a "residual value factor" (RV).

TABLE 16-1 Residual Currency Values (RV) upon Sovereign Default as a Function of the Sovereign Rating Prior to Default

Rating	Residual Value
AAA	17%
AA	17%
A	22%
BBB	27%
BB	41%
B	62%
CCC	62%

Source: From Levy and Levin (1999).

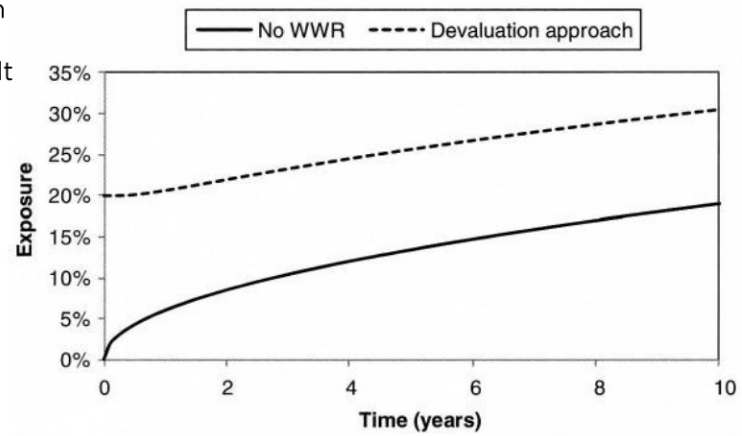


FIGURE 16-16 Illustration of the conditional expected exposure for the devaluation wrong-way risk approach for an FX forward assuming a residual value factor RV = 80%.

The FX volatility is assumed to be 15%.

significant amount in the event they default. Indeed, Levy and Levin (1999) provide estimates of residual currency values by rating upon sovereign default, based on 92 historical default events, which are reproduced in Table 16-1. The RV is larger for better-rated sovereigns, presumably because their default requires a more severe financial shock and the conditional FX rate therefore should move by a greater amount. Such an approach can also be applied to other counterparties, as described by Finger (2000). For example, a default of a large corporate should be expected to have quite a significant impact on their local currency (albeit smaller than that due to sovereign default).

The conditional expected exposure implied by the devaluation approach is shown in Figure 16-16. The impact is fairly time homogeneous, which may be criticised based on the previous observation that wrong-way risk may have a different impact for different future horizons.[16] For example, we may think that an immediate default of a sovereign may produce a large currency jump (small RV in the short term) whereas a later default may be less sudden and therefore lead to a smaller effect (larger RV in the medium to longer term).

The previous approach may seem rather imprecise and ad hoc, and may not be favoured over an approach similar to the correlation one adopted for interest rates described earlier. Whilst the devaluation approach is simple and practical, concern may exist over the inability to characterise the RV factor (and any associated term structure) for a given counterparty. Compared with the ability to estimate FX and credit spread correlation from historical data, this may seem like a bit of a "finger in the air" approach.

In recent years, however, the devaluation approach has been supported by observations in the CDS market. Most CDSs are quoted in U.S. dollars but sometimes simultaneous quotes can be seen in other currencies. For example, Table 16-2 shows the CDS quotes on Italy in both U.S. dollars and euros. These CDS contracts trigger on the same credit event definitions and thus the only difference between them is the currency received on default. There is a large "quanto" effect, with euro-denominated CDS cheaper by around 30% for all maturities. This shows an implied RV of 69% in the event of the default of Italy using 5-year quotes (91/131). This calculation would require adjustment for forward FX and cross-currency basis spreads. Not only is the RV time homogeneous, supporting the earlier approach, but it is also apparent several months before the euro sovereign crisis developed strongly in mid-2011 and Italian credit spreads widened significantly from the levels shown in Table 16-2.

Similar effects during the European sovereign crisis were seen later in 2011. For example, implied RVs of the euro

[16] Although the market data shown later approximately supports this homogeneous assumption.

TABLE 16-2 CDS Quotes (mid-market) on Italy in Both US Dollars and Euros from April 2011

Maturity	USD	EUR
1Y	50	35
2Y	73	57
3Y	96	63
4Y	118	78
5Y	131	91
7Y	137	97
10Y	146	103

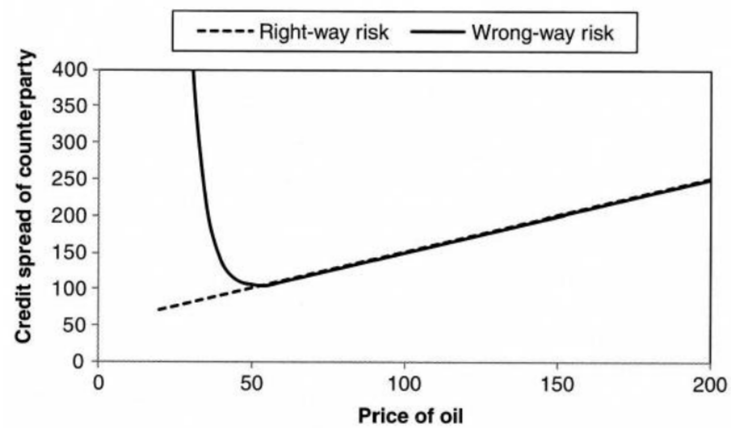


FIGURE 16-17 Schematic illustration of the value of an oil swap versus the credit spread of an airline counterparty.

were 91%, 83%, 80% and 75% for Greece, Italy, Spain and Germany, respectively.[17] This is again consistent with a higher credit quality sovereign creating a stronger impact. The CDS market therefore allows wrong-way risk effect in currencies to be observed and potentially also hedged.

Commodities

Wrong-way risk in commodities can be modelled in a similar way to interest rate products. Brigo et. al. (2008) consider modelling of commodity derivatives CVA in more detail. However, there is another important concept that arises here in certain situations. Consider an institution entering into an oil receiver swap with an airline. Such a contract allows the airline to hedge their exposure to rising oil prices, which is important since aviation fuel represents a very significant cost for the airline industry. From an institution's point of view, such a swap has exposure when the price of oil is low, but at this point, the credit quality of the airline should be sound due to their reduced fuel costs. When the price of oil is high, then the airline may be in a weaker financial situation but this will be the situation of negative exposure. This *should* give rise to right-way risk. However, as illustrated schematically in Figure 16-17, the real situation may be more complex. There is potentially a different linkage here, which is that a low price of oil might mean a severe recession, in which case the airline may have financial troubles. This effect was seen in the global financial crisis. What was originally perceived as right-way risk in the sense of a small fall in the price of oil created wrong-way risk in relation to a more substantial price drop. This is seen on the left-hand side of Figure 16-17.

Note that the above effect should be considered in other asset classes. For example, a slow economy driving low interest rates has the potential to produce a similar effect.

Contingent CDS

One observation from the above approaches to wrong-way risk is that they are generally rather complex modelling problems, with the lack of empirical data and problems with representing dependency creating huge challenges. The only approach that has some clarity is the FX approach mentioned earlier where the simple, economically motivated approach can be calibrated directly to hedging instruments available in the market. This suggests that the main way forward with wrong-way risk is to develop hedging instruments.

In Chapter 14, we described the contingent credit default swap (CCDS), which is a credit derivative instrument particularly designed for the hedging of counterparty risk. Like a credit default swap, a CCDS pays out following a credit event on a reference entity. However, unlike a CDS, which has a fixed notional amount, the CCDS protection buyer has protection indexed to another referenced transaction. Whilst single-name CCDS contracts have existed for a number of years (and the ISDA published standard documentation in 2007), the market has not developed any liquidity due to a shortage of protection sellers.

[17] For example, see "Quanto swaps signal 9 percent Euro drop on Greek default." Bloomberg, June 2010.

More recently, a new kind of CCDS has been developed referencing indices such as CDX, iTraxx and SovX on underlying transactions such as interest rate swaps and cross-currency basis swaps denominated in USD, GBP, EUR and CAD. This may help different banks to hedge differing positions, such as being exposed to high or low interest rates with respect to CVA. However, a key need that has partially driven the emergence of an index-based product is to encourage a wider universe of investors to enter the market, for example, to express a view on the correlation between credit spreads and interest rates. The prices of CCDS products will imply wrong-way risk effects just as the example in Table 16-2, and may be a hedging tool against wrong-way risk problems such as cross-gamma which are otherwise unhedgeable.

Apart from developing liquidity, a key to the success of index CCDS is capital relief under Basel III. Since Basel III considers only credit spread volatility of CVA, hedges linked to market risk components, as CCDSs are, are partly problematic.

Wrong-Way Risk and Collateral

Collateral is typically assessed in terms of its ability to mitigate exposure. Since wrong-way risk essentially causes exposure to increase significantly around the counterparty default time, it could be an important aspect to consider. However, this is very hard to characterise because it is very timing-dependent. If the exposure increases gradually up to the default time then collateral can be received, whereas a jump in exposure deems collateral useless.

To understand the difficulty in characterising the impact of collateral, consider first the approach taken for general wrong-way risk mentioned previously. Recalculating the CVA under the assumptions of a zero-threshold, two-way CSA gives the results shown in Figure 16-18. Interestingly, the collateralised CVA is rather insensitive to wrong-way risk, with the slope of the line being quite shallow. This is because the greater the wrong-way risk, the more collateral that tends to be taken. The relative benefit of collateral is greatest when there is the most wrong-way risk (at +100% correlation) and has a negative impact when there is extreme right-way risk (less than −40% correlation) due to the need to post collateral.

In the previous example, collateral seems to mitigate most of the impact of wrong-way risk as more collateral can be taken in wrong-way risk scenarios. However,

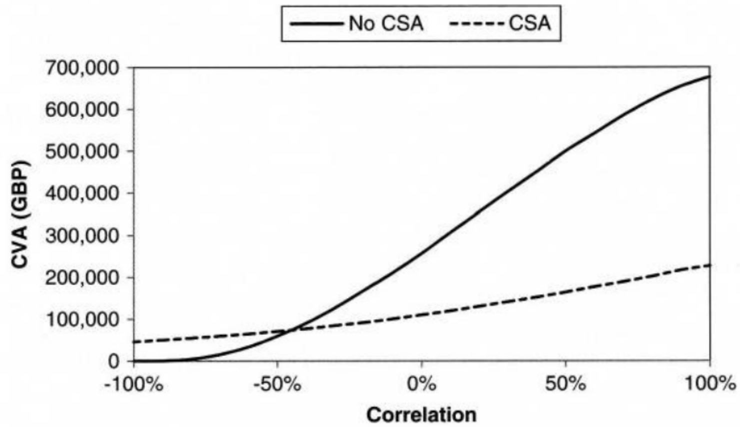


FIGURE 16-18 Combined impact of collateral (via a two-way CSA) and wrong-way risk on the CVA of the portfolio of four swaps considered previously in Figure 16-6.

let us instead consider the impact of collateral in the FX example from earlier. The effect here is obvious, but nevertheless is shown in Figure 16-19. Clearly, the jump effect cannot be collateralised and the exposure cannot be below the assumed devaluation of 20%. In this case, the ability of collateral to reduce wrong-way risk is very limited. If the weakening of currency is gradual, then the exposure can be well collateralised prior to the default. However, if devaluation of a currency is linked very closely to a sovereign default, it may be likely to result in a jump in the FX rate that cannot be collateralised in a timely manner.

Not surprisingly, approaches such as the devaluation approach for FX tend to quantify collateral as being useless whereas more continuous approaches such as the correlation approach described earlier for interest rates (and both approaches described for general wrong-way risk also) suggest that collateral is an effective mitigant against wrong-way risk. The truth is probably somewhere in between, but the quantification is a challenge. A recent paper by Pykhtin and Sokol (2012) considers that the quantification of the benefit of collateral in a wrong-way risk situation must account for jumps and a period of higher volatility during the margin period of risk. They also note that wrong-way risk should be higher for the default of more systemic parties such as banks. Overall, their approach shows that wrong-way risk has a negative impact on the benefit of collateralisation. Interestingly, counterparties that actively use collateral (e.g., banks) tend to be highly systemic and will be subject to these

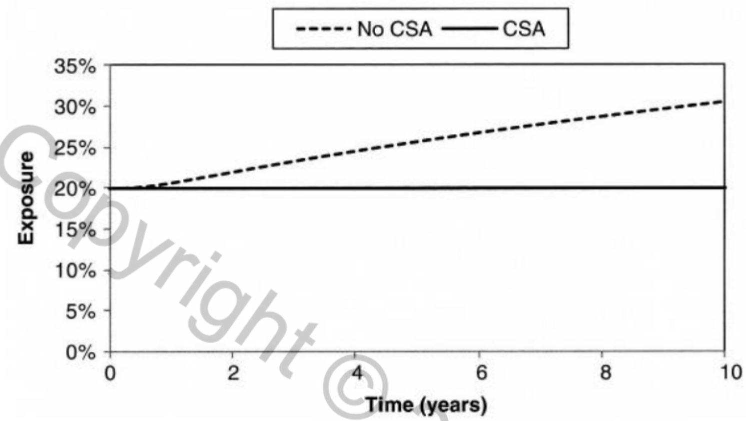


FIGURE 16-19 Impact of collateral (via a two-way CSA) on the conditional expected exposure of the FX forward shown previously in Figure 16-16.

extreme wrong-way risk problems whilst counterparties that are non-systemic (e.g., corporates) often do not post collateral anyway!

We note also that wrong-way risk can be present on collateral itself. This was shown for a fixed-rate bond collateralising a swap. It is also relevant for cash collateral—for example, receiving euro cash from a European sovereign.

WRONG-WAY RISK AND CREDIT DERIVATIVES

Credit derivatives need particular attention as they effectively represent an entire asset class of wrong-way risk. Furthermore, the problems with monoline insurers illustrate the inherent problems with wrong-way risk and credit derivatives. We will analyse the monoline failure in more detail below and explain how wrong-way risk caused such problems. This is not just a historical note: central counterparties intend to clear a significant portion of the credit derivatives market and will therefore have to deal with this wrong-way risk.

Single-Name Credit Derivatives

The wrong-way risk in credit derivatives is a direct consequence of the nature of the products themselves and can lead to serious counterparty risk issues. A protection buyer in a CDS contract has a payoff with respect to a reference entity's default, but is at risk in case the counterparty in the contract suffers a similar fate. The CDS product has a highly asymmetric payoff profile due to being essentially an insurance contract. In addition to this, there is also a correlation effect. Buying CDS protection represents a very definite form of wrong-way risk that is made worse as the correlation between the credit quality of the reference entity and the counterparty increases.

The pricing for a CDS with counterparty risk using a Gaussian copula framework requires valuing the two legs of a CDS contingent on the counterparty surviving (since once the counterparty has defaulted, an institution would neither make premium payments nor receive default payments) and adding the usual term depending on the future value of the CDS contract at the default time. The pricing of CDS counterparty risk is not trivial. However, an elegant solution is provided by Mashal and Naldi (2005), who show that there are upper and lower bounds for the value of protection that can be computed more easily. We will take this approach here and use a simple Monte Carlo simulation to value a CDS with counterparty risk. The upper and lower bounds are generally quite close together and we shall therefore report the average. More details can be found in Gregory (2011).

We will ignore the impact of any collateral in the following analysis. This will be conservative since the use of collateral may be considered to reduce CDS counterparty risk. However, due to the highly contagious and systemic nature of CDS risks, the impact of collateral may be hard to assess and indeed may be quite limited, especially in cases of high correlation. We note also that many protection sellers in the CDS market such as monolines and CDPCs (discussed later) have not traditionally entered into collateral arrangements anyway.

We are interested in the risky value of buying or selling CDS protection as a function of correlation between the reference entity and counterparty (the counterparty is selling protection). We assume that the reference entity CDS spread (without counterparty risk) is 250 bps, whereas the counterparty CDS spread is 500 bps. Both recoveries are assumed to be 40%. We assume that the correlation driving joint defaults can only be positive. It is unlikely that negative correlation would ever be seen except in specific cases (for example, the default of a competitor improves the financial health of a counterparty).

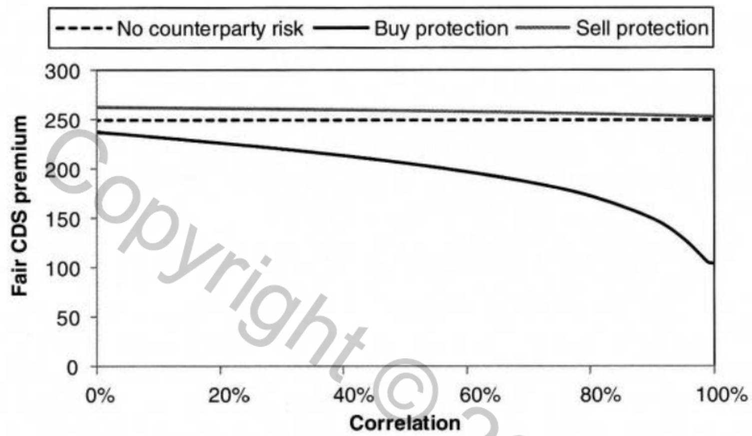


FIGURE 16-20 Fair CDS premium when buying protection to counterparty risk compared with the standard (risk-free) premium.

The counterparty CDS spread is assumed to be 500 bps.

We show the fair premium—i.e., reduced (increased) to account for CVA—that an institution should pay (receive) in order to buy (sell) protection in Figure 16-20. When buying protection we can observe the very strong impact of correlation: one should be willing only to pay around 200 bps at 60% correlation to buy protection compared with paying 250 bps with a "risk-free" counterparty. The CVA in this case is 50 bps (running) or one-fifth of the risk-free CDS premium. At extremely high correlations, the impact is even more severe and the CVA is huge. At a maximum correlation of 100%, the CDS premium is just above 100 bps, which relates entirely to the recovery value.[18] When selling protection, the impact of CVA is much smaller and reduces with increasing correlation due to right-way risk.[19]

Due to the relatively small CVA impact on selling protection, we can see that the bilateral implications of counterparty risk on CDS contracts are relatively small. For these reasons, we will not consider the impact of DVA, although bilateral calculations have been reported by Turnbull (2005).

[18] The premium based only on recovery value (i.e., there is no chance of receiving any default payment) is $250 \times 40\% = 100$ bps.

[19] For zero or low correlation values, the protection seller may possibly suffer losses due to the counterparty defaulting when the CDS has a positive MtM (requiring a somewhat unlikely tightening of the reference entity credit spread). However, for high correlation values, the MtM of the CDS is very likely to be negative at the counterparty default time and (since this amount must still be paid) there is virtually no counterparty risk.

Credit Derivative Indices and Tranches

Structured credit has given rise to even more complex counterparty risk in the form of tranches. There exist many kinds of CDO structure, which are all broadly characterised by their exposure to a certain range of losses on a portfolio. The counterparty risk problem now becomes more complex, since one needs to assess where the counterparty might default compared with all the reference names underlying the portfolio. More details on this can also be found in Turnbull (2005), Pugachevsky (2005) and Gregory (2009b).

We choose tranches according to the standard CDX[20] North American portfolio that are defined by the attachment and detachment points [0%, 3%, 7%, 10%, 15%, 30%, 100%]. Since we are interested only in understanding the qualitative impact of counterparty risk for different tranches, we choose the market standard Gaussian copula model with a fixed correlation parameter of 50%.[21] Due to constraints on the correlation matrix, this means we consider the correlation between the counterparty default and the other names in the portfolio in the range [0, 70%].[22]

We show the impact of counterparty risk across the entire capital structure in Figure 16-21, assuming recovery rates of 10%.[23] In order to compare all tranches on the same scale, we plot the ratio of fair risky premium to risk-free premium: this value will have a maximum at unity and decrease towards the recovery (of the counterparty) as counterparty risk becomes more significant. Indeed, from a counterparty risk perspective, we can view tranching as segregating the counterparty risk: the more senior a tranche, the more risk it contains on a relative basis.

[20] www.markit.com

[21] This does not produce prices close to the market, but the standard approach of "base correlation" used to reproduce market prices does not have an obvious associated way in which to price correctly counterparty risk. We have checked that the qualitative conclusions of these results hold at different correlation levels.

[22] The upper limit for this correlation, due to constraints of positive semi-definitiveness on the correlation matrix, is approximately $\sqrt{50\%} = 70.7\%$.

[23] This is consistent with the low recoveries experienced with some defaulting monoline insurers.

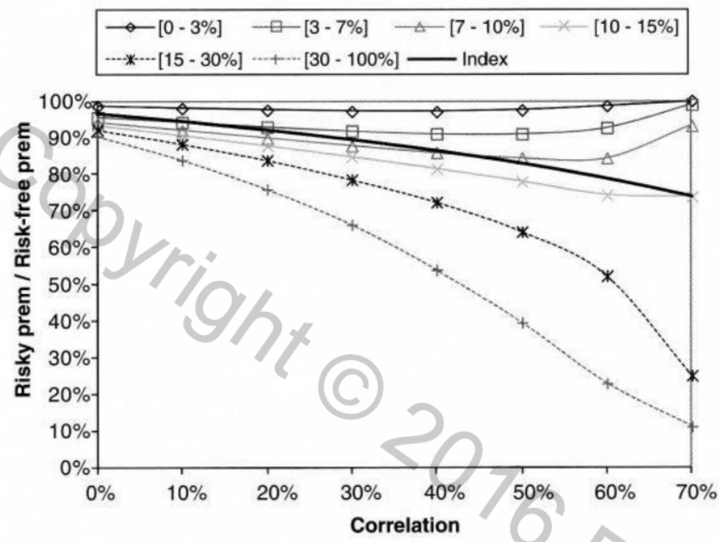


FIGURE 16-21 Impact of counterparty risk across the capital structure for different tranches.

Fair risky tranche premium divided by the risk-free premium for all tranches in the capital structure and compared with the index ([0–100%] tranche). Recovery rates are assumed 10%.

In the analysis of options and wrong-way risk, we concluded that wrong-way risk increases for more out-of-the-money contracts. We now have an analogous conclusion for tranches that wrong-way risk *increases* for tranches that are more senior. The most senior tranche in the capital structure, the super senior [30–100%] represents the most severe problem. Assuming 40% recovery, there needs to be 62.5% (over half the portfolio) defaults[24] before this tranche takes any loss, and so the chance that the counterparty is still around to honour these payments is expected to be much smaller than for other tranches.

Many of the problems in 2008 and 2009 suffered by monolines were caused by high leverage, coupled with the unprecedented increase in value of super senior protection. The credit spreads of monolines widened from 5–10 bps to several hundred basis points. Banks that had bought super senior insurance from monolines had to realise substantial losses due to the increased counterparty risk. Many transactions were unwound, with banks taking substantial losses due effectively to their positive CVA component. In retrospect, it is not surprising that tranches such as the [30–100%] shown previously created severe counterparty risk problems due to their massive wrong-way risk.

The Failure of CDOs

Gregory (2008b) presents a theoretical analysis of the protection purchased by monoline insurers and shows that its value is limited by a number of technical factors. Given the sheer size of these tranches, it is counterparty risk that explains much of the failure of CDOs and synthetic securitisation that led to the global financial crisis. Below, we make a simple presentation on why CDOs can be efficient and create value, but how they are ultimately due to counterparty risk problems.

CDOs come in many forms, such as cash or synthetic, and cover various different assets from corporate to ABS. However, their basic principle is to take the risk on a given credit portfolio and redistribute it via tranches. A typical CDO is represented in Table 16-3. A number of different classes of securities are issued to cover the full portfolio notional. The riskiness of these securities changes from the bottom unrated equity tranche to the top so-called super senior tranche. Although this latter tranche has no rating, it is above the Triple-A rated class A notes and therefore is at least Triple-A or even better (from where the terms super Triple-A and Quadruple-A arose).

We can illustrate the key points with a very simple example of a CDO structure illustrated in Figure 16-22. A portfolio is divided into three tranches—equity, mezzanine and senior—and has a 5-year fixed (bullet) maturity. The underlying credit portfolio is assumed to be 100 bonds with Triple-B ratings. From Table 14-2 we can see that the 5-year BBB default probability is 2.06%. Assuming a loss given default of 60%, this will give an expected loss of 2.06% × 60% = 1.24%. Finally, we know that a BBB portfolio has to compensate for a loss of more than this due to risk and liquidity premiums. The multiplier from the aforementioned study by Hull et. al. (2005) from Table 14-4 is 5.1,[25] which suggests that the overall compensation the investors would receive is in fact 1.24% × 5.1 = 6.29%. We assume the underlying portfolio will provide exactly this amount.[26]

[24] 30% × 125/(1 − 40%). Although it should be noted that a lower recovery rate assumption is probably more relevant in such an extreme situation.

[25] We note that this uses Moody's ratings (Baa) whilst the default probability data in Chapter 14 is from Standard & Poor's. The results are not changed significantly by using Moody's data throughout.

[26] It should be strongly emphasised that all the above numbers are based on empirical analysis over many years of data but the general conclusions are not changed.

TABLE 16-3 Illustration of the Securities Issued from a Typical CDO

Class	Amount	Tranching	Rating	Funding
Super senior	850	[15–100%]	NR	Unfunded
Class A	50	[10–15%]	Aaa/AAA	Funded
Class B	30	[7–10%]	Aa2/AA	Funded
Class C	30	[4–7%]	Baa2/BBB	Funded
Equity	40	[0–4%]	NR	Funded

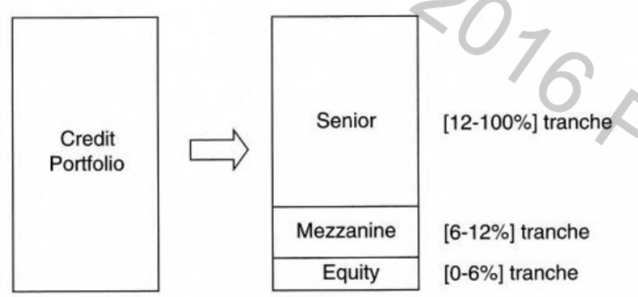


FIGURE 16-22 Simple CDO structure used for the example.

The approximate goal of a CDO is to sell the tranches for less than the return received on the underlying portfolio. In this simple example, this corresponds to paying investors an overall return of less than 6.29% for the equity, mezzanine and senior pieces. In order to sell tranches, they first have to be rated. Assuming an asset correlation of 20%[27] for all names in the portfolio, the tranches would have ratings of CCC, BBB and AAA for the equity, mezzanine and senior respectively. Assuming investors will demand the same return for these investments corresponding to the multiplier in Table 14-4, the economics of the structure are shown in Table 16-4.

The CDO works because most of the risk is sold in the equity tranche, which attracts a relatively low multiplier. It is *relatively* expensive to sell the AAA tranche as the multiplier assumes that for every unit of actual default risk passed on, 16.8 units of return must be paid. However, given the small amount of actual risk that is assessed as being in this tranche, this does not affect the economics of the structure particularly. In Table 16-4, we also show the calculated spreads of the different tranches and the portfolio. Another way to see the value created is via the so-called excess spread, which is the spread paid in versus that paid out. Taking into account the size of each tranche, this is given by $137 - (14 \times 88\%) - (137 \times 6\%) - (1230 \times 6\%) = 43$ bps. This positive excess spread[28] suggests that the overall structure creates a profit. Even if the CDO investors demand a higher relative return for each rating (this was certainly true in the early days of the CDO market), there is enough value in the structure to pay this return.

The above explains how CDOs can work. Their failure could be ascribed to the rating agency models used to rate the tranches as being incorrect. However, there is no evidence from corporate default rates that the correlation assumptions used by rating agencies are too low. Secondly, a higher correlation does not completely ruin the economics of the structure (for example, a 30% correlation in the above example reduces the excess spread from 43 to 27 bps but changing the tranching can substantially improve this[29]).

The true failure of CDOs lies more in counterparty risk. The above does not take into account the counterparty risk in issuing the tranches of the CDO. Whilst the equity and mezzanine tranches can probably be issued on a fully funded basis,[30] the (super) senior tranche will typically be completely unfunded[31] (see Table 16-3). This unfunded tranche then creates the significant counterparty risk that can be seen in Figure 16-21. The relative size of this tranche,[32] the high seniority and the inability of protection sellers (such as monolines to post collateral) makes the risk transfer highly inefficient, as shown by Gregory

[27] This is conservative with respect to the correlations used by rating agencies for corporate names.

[28] The excess spread is not a perfect guide to the profit since it changes over the lifetime of the CDO as defaults occur. However, it is a reasonable guide to the economics of the structure.

[29] The tranches described have not been optimised in any way. For example, by calculating the minimum amount of subordination to achieve a given rating.

[30] The transaction will be a synthetic bond with the investor paying upfront the full notional of the transaction, which is therefore fully collateralised with no counterparty risk.

[31] This will therefore be executed as a credit default swap referencing the underlying tranche. It is therefore subject to counterparty risk.

[32] For example, a typical portfolio size may be around $1 billion, which would make the notional of the senior tranche in this example $880m.

TABLE 16-4 Illustration of the Securities Issued from a Typical CDO

	5-Year Default Probability	Expected Default Loss	Multiplier	Size	Cost	Spread
BBB portfolio	2.06%	1.23%	5.1	100%	**6.29%**	137
AAA tranche	0.07%	0.04%	16.8	88%	0.58%	14
BBB tranche	2.06%	1.23%	5.1	6%	0.38%	137
CCC tranche	56.27%	33.76%	1.3	6%	2.63%	1230
				Total	**3.59%**	

(2008b). The only way to achieve this risk transfer is by counterparties who are not highly leveraged and exposed to senior credit risk (as monolines were). This in turn makes the economics of the transaction less beneficial (since the price paid on the senior tranche will be higher) and severely limits the total amount of such transactions that can be done.

Central Clearing and Wrong-Way Risk

CCPs convert counterparty risk into gap risk and tail risk. A key aim of a CCP is that losses to the default of a clearing member are contained within resources committed by that clearing member (the so-called "defaulter pays" approach). A CCP faces tail and gap risk as illustrated in Figure 16-23 since the initial margin and reserve fund contributions of the defaulting member(s) may be insufficient to cover their losses. This leads to moral hazard (since other CCP members will suffer losses) and potentially even financial insolvency of the CCP.

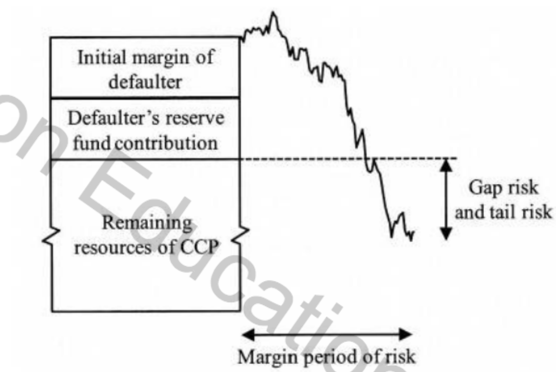


FIGURE 16-23 Illustration of the tail risk faced by a CCP in the event of the default of one or more members.

The effective margin period of risk is as discussed previously and is usually considered by CCPs to be around five business days.

CCPs tend to disassociate credit quality and exposure. Institutions must have a certain *credit* quality to be clearing members but members will then be charged initial margins and reserve fund contributions driven primarily[33] by the *market* risk of their portfolio (that drives the exposure faced by the CCP). In doing this, CCPs are in danger of implicitly ignoring wrong-way risk. The drop-in value represented in Figure 16-23 can be a result of extreme volatility, downward pressure and gap (jump) events. The impact of wrong-way risk is to make all of these aspects more severe when coupled with the default of the counterparty.

For significant wrong-way risk transactions such as CDSs, CCPs have a problem of quantifying the wrong-way risk component in defining initial margins and reserve funds. As with the quantification of wrong-way risk in general, this is far from an easy task. Furthermore, wrong-way risk increases with increasing credit quality, as shown in Figure 16-2 and Table 16-1 (similar arguments are made by Pykhtin and Sokol (2012) in that a large dealer represents more wrong-way risk than a weaker credit quality

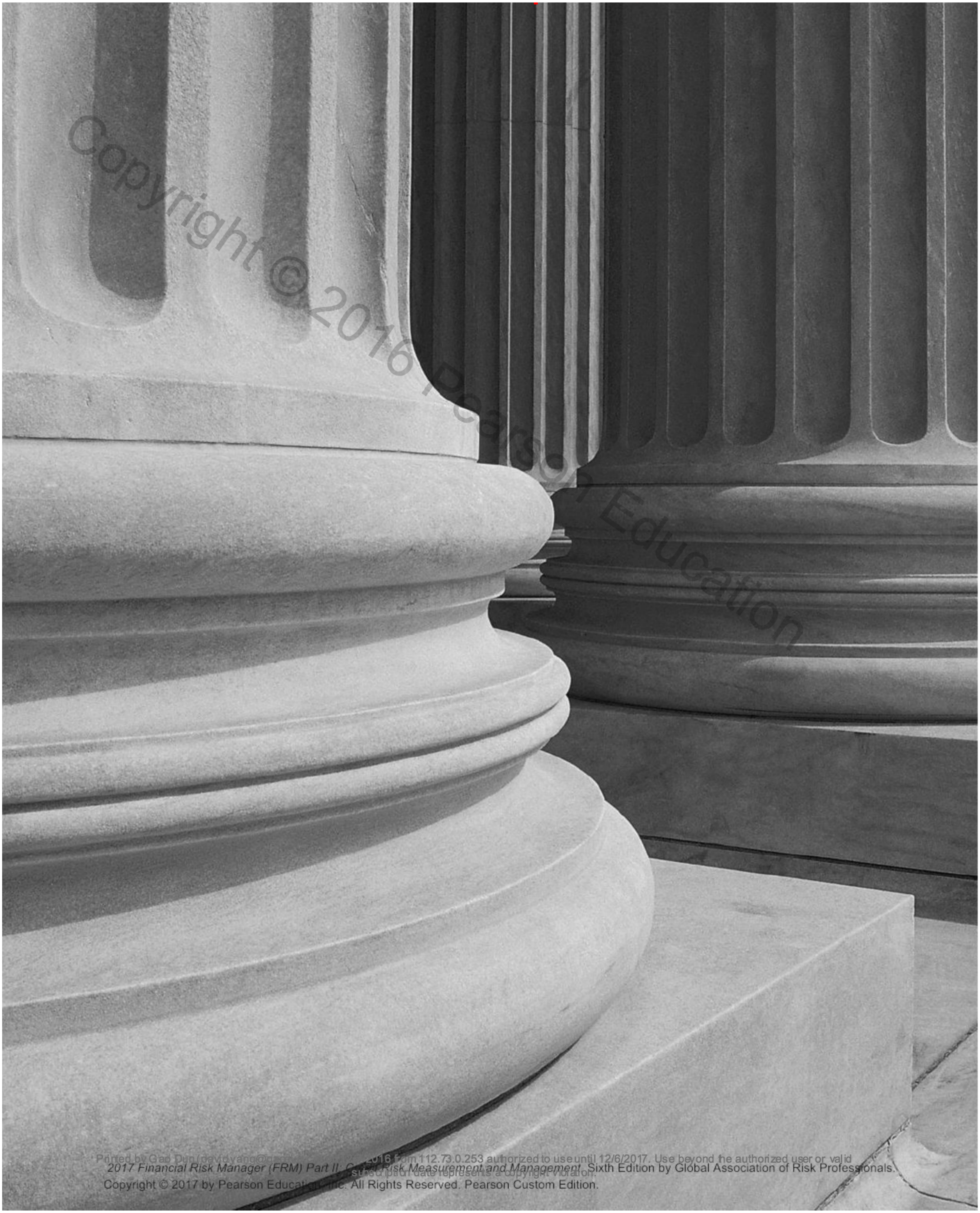
[33] Some CCPs do base margins partially on credit ratings, but this tends to be a secondary impact.

counterparty). These aspects suggest perversely that CCPs should require greater initial margin and reserve fund contributions from better credit quality members.[34]

SUMMARY

In this chapter we have discussed wrong-way counterparty risk, which is a phenomenon caused by the dependence between exposure and default probability. Wrong-way risk is a subtle, but potentially devastating, effect that can increase counterparty risk and CVA substantially. Portfolio and trade-level wrong-way risk have been described. We have examined some classic examples arising in different asset classes (interest rates, FX, equity and commodities) and associated quantitative approaches. Counterparty risk in credit derivatives has been analysed and the failure of CDOs has been linked to this. Finally, we have considered the impact of wrong-way risk on collateral and argued that it represents a very serious concern for central counterparties.

[34] Of course, better credit quality members are less likely to default, but the impact in the event that they do is likely to be more severe.



The Evolution of Stress Testing Counterparty Exposures

17

Learning Objectives

After completing this reading you should be able to:

- Differentiate among current exposure, peak exposure, expected exposure, and expected positive exposure.
- Explain the treatment of counterparty credit risk (CCR) both as a credit risk and as a market risk and describe its implications for trading activities and risk management for a financial institution.
- Describe a stress test that can be performed on a loan portfolio and on a derivative portfolio.
- Calculate the stressed expected loss, the stress loss for the loan portfolio, and the stress loss on a derivative portfolio.
- Describe a stress test that can be performed on CVA.
- Calculate the stressed CVA and the stress loss on CVA.
- Calculate the debt value adjustment (DVA) and explain how stressing DVA enters into aggregating stress tests of CCR.
- Describe the common pitfalls in stress testing CCR.

Excerpt is from "The Evolution of Stress Testing Counterparty Exposures," by David Lynch, reprinted from Stress Testing: Approaches, Methods, and Applications.

2017 Financial Risk Manager (FRM) Part II: Credit Risk Measurement and Management, Sixth Edition by Global Association of Risk Professionals.

The call for better stress testing of counterparty credit risk exposures has been a common occurrence from both regulators and industry in response to financial crises (CRMPG I 1999; CRMPG II 2005; FRB 2011). Despite this call, statistical measures have progressed more rapidly than stress testing. In this chapter we examine how stress testing may be improved by building off the development of the statistical measures. We begin by describing how the measurement of counterparty risk has developed by viewing the risk as a credit risk and as a market risk. The problems this creates for a risk manager who is developing a stress-testing framework for counterparty risk are then identified. Methods to stress-test counterparty risk are described from both a credit risk perspective and from a market risk perspective, starting with the simple case of stressing current exposures to a counterparty. These stress tests are considered from both a portfolio perspective and individual counterparty perspective. Last, some common pitfalls in stress testing counterparty exposures are identified.

THE EVOLUTION OF COUNTERPARTY CREDIT RISK MANAGEMENT

The measurement and management of counterparty credit risk (CCR) has evolved rapidly since the late 1990s. CCR may well be the fastest-changing part of financial risk management over the time period. This is especially true of the statistical measures used in CCR. Despite this quick progress in the evolution of statistical measures of CCR, stress testing of CCR has not evolved nearly as quickly.

In the 1990s a large part of counterparty credit management involved evaluation of the creditworthiness of an institution's derivatives counterparties and tracking the current exposure of the counterparty. In the wake of the Long-Term Capital Management crisis, the Counterparty Risk Management Policy Group cited deficiencies in these areas and also called for use of better measures of CCR. Regulatory capital for CCR consisted of add-ons to current exposure measures (BCBS 1988.) The add-ons were a percentage of the gross notional of derivative transactions with a counterparty. As computer technology has advanced, the ability to model CCR developed quickly and allowed assessments of how the risk would change in the future.

The fast pace of change in CCR modelling can be seen in the progression of statistical measures used to gauge counterparty credit risk. First, potential-exposure models were developed to measure and limit counterparty risk. Second, the potential-exposure models were adapted to expected positive-exposure models that allowed derivatives to be placed in portfolio credit risk models similar to loans (Canabarro, Picoult and Wilde 2003). These two types of models are the hallmark of treating CCR as a credit risk. Pykhtin and Zhu (2007) provide an introduction to these models. The treatment of CCR as credit risk was the predominant framework for measuring and managing CCR from 2000 to 2006 and was established as the basis for regulatory capital as part of Basel II (BCBS 2005). During this time, risk mitigants such as netting agreements and margining were incorporated into the modelling of CCR. The definitions of these exposure measures used in this chapter follow those in BCBS (2005).

- Current exposure is the larger of zero and the market value of a transaction or portfolio of transactions within a netting set, with a counterparty that would be lost upon the default of the counterparty, assuming no recovery on the value of those transactions in bankruptcy. Current exposure is often also called replacement cost.
- Peak exposure is a high-percentile (typically 95% or 99%) of the distribution of exposures at any particular future date before the maturity date of the longest transaction in the netting set. A peak exposure value is typically generated for many future dates up until the longest maturity date of transactions in the netting set.
- Expected exposure is the mean (average) of the distribution of exposures at any particular future date before the longest-maturity transaction in the netting set matures. An expected exposure value is typically generated for many future dates up until the longest maturity date of transactions in the netting set.
- Expected positive exposure (EPE) is the weighted average over time of expected exposures where the weights are the proportion that an individual expected exposure represents of the entire time interval. When calculating the minimum capital requirement, the average is taken over the first year or over the time period of the longest-maturity contract in the netting set.

Furthermore, an unusual problem associated with CCR, that of wrong-way risk, has been identified (Levin and Levy 1999; Finger 2000). Wrong-way risk occurs when the credit quality of the counterparty is correlated with the exposure, so that exposure grows when the counterparty is most likely to default. When exposure is fixed as is the case for a loan, this does not occur, so adaptation of techniques used in other areas of risk management is more difficult.

At the same time, the treatment of CCR as a market risk was developing, but was largely relegated to pricing in a credit valuation adjustment (CVA), prior to the financial crisis of 2007–9. This was first described for Swaps (Sorensen and Bollier 1994; Duffie and Huang 1996) and has since become widespread due to the accounting requirement of FAS 157 (FASB 2006). The complexities of risk-managing this price aspect of a derivatives portfolio did not become apparent until the crisis. Prior to the crisis, credit spreads for financial institutions were relatively stable and the CVA was a small portion of the valuation of banks' derivatives portfolios. During the crisis, both credit spreads and exposure amounts for derivative transactions experienced wide swings, and the combined effect resulted in both large losses and large, unusual gains. Financial institutions are just now beginning to develop their frameworks to risk-manage CVA. The regulatory capital framework has adopted a CVA charge to account for this source of risk (BCBS 2011).

The treatment of CCR as a credit risk or CCR as a market risk has implications for the organisation of a financial institution's trading activities and the risk-management disciplines (Picoult 2005; Canabarro 2009). Both treatments are valid ways to manage the portfolio, but adoption of one view alone leaves a financial institution blind to the risk from the other view. If CCR is treated as a credit risk, a bank can still be exposed to changes in CVA. A financial institution may establish PFE limits and manage its default risk through collateral and netting, but it still must include CVA in the valuation of its derivatives portfolio. Inattention to this could lead to balance-sheet surprises. If CCR is treated as a market risk, dynamically hedging its CVA to limit its market risk losses, it remains exposed to large drops in creditworthiness or the sudden default of one of its counterparties. A derivatives dealer is forced to consider both aspects.

The view of CCR has implications for how the risk is managed as well. The traditional credit risk view is that the credit risk of the counterparty can be managed at inception or through collateral arrangements set up in advance, but there is little that can be done once the trades are in place. At default the financial institution must replace the trades of the defaulting counterparty in the market all at once in order to rebalance its book. A large emphasis is placed on risk mitigants and credit evaluation as a result.

The view of CCR as a market risk allows that its counterparty credit risk can be hedged. Instead of waiting until the counterparty defaults to replace the contracts, the financial institution will replace the trades with a counterparty in the market before it defaults by buying the positions in proportion to the counterparty's probability of default. Thus a counterparty with a low probability of default will have few of its trades replaced in advance by the financial institution, but, as its credit quality deteriorates, a larger proportion of those trades will be replaced by moving them to other counterparties. At default, the financial institution will have already replaced the trades and the default itself would be a non-event.

IMPLICATIONS FOR STRESS TESTING

The dual nature of CCR leads to many measures that capture some important aspects of CCR. On the credit risk side, there are the important measures of exposure: current exposure, peak exposure and expected exposure. On the market risk side there is the valuation aspect coming from CVA, and there is the risk generated by changes in the CVA, as measured by VaR of CVA, for example. This creates a dazzling array of information that can be difficult to interpret and understand at both portfolio and counterparty levels. The search for a concise answer to the question "What is my counterparty credit risk?" is difficult enough, but an equally difficult question is "What CCR measures should I stress?"

When confronted with the question of stress testing for CCR, the multiplicity of risk measures means that stress testing is a complicated endeavour. To illustrate this complexity we can compare the number of stresses that a bank may run on its market risk portfolio with the number of similar stresses a bank would run on its counterparty credit risk portfolio. In market risk, running an equity crash stress test may result in one or two stress numbers: an instantaneous loss on the current portfolio and potentially a stress VaR loss. A risk manager can easily consider the implications of this stress.

In contrast, the CCR manager would have to run this stress at the portfolio level and at the counterparty level, and would have to consider CCR as both a credit risk and a market risk. The number of stress-test results would be at least twice the number of counterparties plus one.[1] The number of stress-test results would at least double again if the risk manager stressed risk measures in addition to considering instantaneous shocks.[2] The number of stress values that can be produced can bewilder even the most diligent risk manager, and overwhelm IT resources.

Despite this array of potential stress results, a risk manager must stress-test counterparty exposures to arrive at a comprehensive view of the risk of the financial institution's portfolio.[3] This chapter provides a description of the types of stress tests that can be run to get a picture of the CCR in a financial institution's derivative portfolio.

STRESS TESTING CURRENT EXPOSURE

The most common stress tests used in counterparty credit are stresses of current exposure. To create a stressed current value, the bank assumes a scenario of underlying risk-factor changes and reprices the portfolio under that scenario. Generally speaking, a financial institution applies these stresses to each counterparty. It is common practice for banks to report their top counterparties with the largest current exposure to senior management in one table, and then follow that table with their top counterparties, with the largest stressed current exposure placed under each scenario in separate tables.

For example, Table 17-1 shows an example of what a financial institution's report on its equity crash stress test for current exposure might look like. The table lists the top 10 counterparties by their exposure to an equity market crash of 25%. It shows the following categories: the counterparty rating, market value of the trades with the counterparty, collateral, current exposure, and stressed current exposure after the stress is applied but before any collateral is collected. This provides a snapshot of which counterparties a CCR manager should be concerned about in the event of a large drop in equity markets. A financial institution would construct similar tables for other stresses representing credit events or interest-rate shocks. These tables would likely list different counterparties as being exposed to the stress scenario, since it is unlikely that the counterparty with the most exposure to an equity crash is the same as the counterparty with the most exposure to a shock in interest rates.

[1] The stresses are run for each counterparty and at the aggregate portfolio level. The stress may also be run for various subportfolios, divided by region or industry, for example. These would have to be run in both a credit and market risk context.

[2] It might increase even more since there are multiple risk measures of importance in CCR.

[3] This is included in regulatory guidance on stress testing for counterparty credit risk, for example in SR 11-10 (Federal Reserve Board 2011).

TABLE 17-1 Current Exposure Stress Test: Equity Crash

	Scenario: Equity Market Down 25%				
($MM)	**Rating**	**MtM**	**Collateral**	**Current Exposure**	**Stressed Current Exposure**
Counterparty A	A	0.5	0	0.5	303
Counterparty B	AA	100	0	100	220
Counterparty C	AA	35	0	35	119
Counterparty D	BBB	20	20	0	76
Counterparty E	BBB	600	600	0	75
Counterparty F	A	−5	0	0	68
Counterparty G	A	−10	0	0	50
Counterparty H	BB	−50	0	0	24
Counterparty I	A	35	20	15	17
Counterparty J	BB	24	24	0	11

This type of stress testing is quite useful, and financial institutions have been conducting it for some time. It allows the bank to identify which counterparties would be of concern in such a stress event, and also how much the counterparty would owe the financial institution under the scenario. However, stress tests of current exposure have a few problems. First, aggregation of the results is problematic, and, second, it does not account for the credit quality of the counterparties. Also, it provides no information on wrong-way risk.

While the individual counterparty results are meaningful, there is no meaningful way to aggregate these stress exposures without incorporating further information. If we were to sum the exposures to arrive at an aggregate stress exposure, this would represent the loss that would occur if every counterparty defaulted in the stress scenario. Unless the scenario were the Apocalypse, this would clearly be an exaggeration of the losses. Other attempts to aggregate these results are also flawed. For example, running the stressed current exposure through a portfolio credit risk model would also be incorrect, since expected exposures, not current exposures, should go through a portfolio credit risk model (Canabarro, Picoult, Wilde 2003). Table 17-1 does not provide an aggregate stressed amount as a result.

The stressed current exposures also do not take into account the credit quality of the counterparty. This should be clear from the outset, since it accounts only for the value of the trades with the counterparty and not the counterparty's willingness or ability to pay. This is an important deficiency since a US$200 million exposure to a start-up hedge fund is very different from a US$200 million exposure to an AAA corporate. While we could imagine a limit structure for stressed current exposure that takes into account the credit quality of the counterparty, most financial institutions have not gone down this path for stressed current exposure. The degree of difficulty involved in doing this for each scenario and each rating category is daunting, mostly because the statistical measures such as peak exposure provide a more consistent way to limit exposure by counterparties who may be exposed to different scenarios. From Table 17-1, it is unclear whether the CCR manager should be more concerned about Counterparty C or Counterparty D in the stress event. While Counterparty C has a larger stressed current exposure than Counterparty D, Counterparty C has a better credit quality.

Last, stress tests of current exposure provide little insight into wrong-way risk. As a measure of exposure that omits the credit quality of the counterparty, these stress tests without additional information cannot provide any insight into the correlation of exposure with credit quality. Stresses of current exposure are useful for monitoring exposures to individual counterparties, but do not provide either a portfolio outlook or incorporate a credit quality.

STRESS TESTING THE LOAN EQUIVALENT

To stress-test in the credit framework for CCR, we first have to describe a typical stress test that would be performed on a loan portfolio. The typical framework for loans is to analyse how expected losses would change under a stress.

For credit provisioning, we might look at an unconditional expected loss across a pool of loan counterparties. Expected loss for any one counterparty is the product of the probability of default, p_i, where this may depend on other variables, exposure at default, ead_i, and loss-given default, lgd_i. The expected loss for the pool of loan counterparties is:

$$EL = \sum_{i=1}^{N} p_i \cdot ead_i \cdot lgd_i$$

A stress test could take exposure at default and loss-given default as deterministic and focus on stresses where the probability of default is subject to a stress. In this case, the probability of default is taken to be a function of other variables; these variables may represent an important exchange rate or an unemployment rate, for example. In this case, the stressed expected loss is calculated conditional on some of the variables affecting the probability of default being set to their stressed values; the stressed probability of default is denoted p^s_i; and the stressed expected loss is:

$$EL_s = \sum_{i=1}^{N} p_i^s \cdot ead_i \cdot lgd_i$$

The stress loss for the loan portfolio is $EL_s - EL$. A financial institution can generate stress tests in this framework rather easily. It can simply increase the probability of defaults, or it can stress the variables that these probabilities of defaults depend on. These variables are typically macroeconomic variables or balance-sheet items for the counterparty. The stress losses can be generated for individual loan counterparties as well as at an aggregate level.

This framework can be adapted for CCR treated as a credit risk. In this case the probability of default and loss-given default of the counterparty are treated the same, but now exposure at default is stochastic and depends on the levels of market variables. EPE multiplied by an alpha factor (Picoult 2005; Wilde 2005) is the value that allows CCR exposures to be placed in a portfolio credit model along with loans and arrive at a high-percentile loss for the portfolio of exposures (both loan and derivatives).[4] The same procedure is applied here and EPE is used in an expected-loss model. In this case expected loss and expected loss conditional on a stress for derivatives counterparties are:

$$EL = \sum_{i=1}^{N} p_i \cdot \alpha \cdot epe_i \cdot lgd_i$$
$$EL_s = \sum_{i=1}^{N} p_i^s \cdot \alpha \cdot epe_i^s \cdot lgd_i$$

Stress losses on the derivatives portfolio can be calculated similarly to the loan portfolio case. A financial institution can stress the probability of default similarly to the loan case by stressing probability of default or the variables that affect probability of default, including company balance-sheet values, macroeconomic indicators and values of financial instruments. It can also combine the stress losses on the loan portfolio and the stress losses on its derivatives portfolio by adding these stress losses together.

Table 17-2 shows the results of a typical stress test that could be run that would shock the probability of default

[4] Alpha typically depends on the quantile at which we measure economic capital. In this case it would be the alpha calculated at the expected loss. For this reason it may differ from the alpha used for economic or regulatory capital calculations.

TABLE 17-2 PD Stress: Dotcom Crash

	PD (%)	EPE (US$m)	LGD (%)	EL (US$m)	Stressed PD (%)	Stressed EL (US$m)	Stress loss (US$m)
Counterparty AA	0.05	213.00	0.70	0.08	0.50	0.77	0.69
Counterparty BB	0.03	202.50	0.60	0.04	0.30	0.38	0.34
Counterparty CC	0.45	75.00	0.70	0.24	0.62	0.34	0.09
Counterparty DD	0.90	30.00	0.65	0.18	1.20	0.24	0.06
Counterparty EE	1.05	10.00	0.75	0.08	1.40	0.11	0.03
Counterparty FF	0.09	157.00	0.50	0.07	0.12	0.10	0.02
Counterparty GG	0.98	68.00	0.70	0.48	1.02	0.50	0.02
Counterparty HH	2.17	3.00	0.34	0.02	3.00	0.03	0.01
Counterparty II	0.03	150.00	0.20	0.01	0.05	0.02	0.01
Counterparty JJ	0.50	50.00	0.60	0.15	0.50	0.15	0.00
Aggregate				**1.36**		**2.63**	**1.27**